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Data-Driven Spatial Zoning and Differential Pricing for Large Commercial Complex Parking

Yuwei Yang [1,2,*], Honggang Zhang [3], Jun Chen [4] and Jiao Ye [5]

[1] College of General Aviation and Flight, Nanjing University of Aeronautics and Astronautics, Jiangning District, Nanjing 211189, China
[2] Hangzhou International Urbanology Research Center and Center for Zhejiang Urban Governance Studies, Hangzhou 311121, China
[3] Department of Logistics and Maritime Studies, Faculty of Business, The Hong Kong Polytechnic University, Kowloon, Hong Kong, China; honggang.zhang@polyu.edu.hk
[4] School of Mechano-Electronic Engineering, Suzhou Polytechnic University, Suzhou 215104, China; 92418@jssvc.edu.cn
[5] Institute of Future Networks, Southern University of Science and Technology, Shenzhen 518000, China; yejiao23@163.com
* Correspondence: yuweiy@nuaa.edu.cn

Abstract

This study presents a data-driven framework for optimizing parking space allocation and pricing in large commercial complexes, addressing persistent spatial imbalances in occupancy between high- and low-demand zones. A mixed Logit (ML) model with interaction terms is estimated from stated preference survey data to capture heterogeneous user preferences across trip purposes. A dual clustering algorithm is then applied to generate spatially coherent pricing zones, integrating geometric, functional, and occupancy-based attributes. Two differential pricing strategies are formulated: an administered model with regulatory price bounds and a market-based model without such constraints. Both pricing models are solved using an improved multi-objective Particle Swarm Optimization–Grey Wolf Optimizer (PSO–GWO) algorithm that jointly optimizes spatial zoning and zone–time pricing schedules. Using data from the Kingmo Complex in Nanjing, China, the results show that both strategies significantly reduce spatio-temporal occupancy variance and improve utilization balance. The administered strategy reduces variance by up to 67% on weekdays, with only a 1% increase in revenue, making it suitable for contexts prioritizing regulatory compliance and price stability. In contrast, the market-based strategy reduces variance by over 40% while generating substantially higher revenue, particularly during periods of high and uneven demand. The proposed framework demonstrates the potential of integrating behavioral modeling, spatial clustering, and multi-objective optimization to improve parking efficiency. The findings provide practical guidance for operators and policymakers seeking to implement adaptive pricing strategies in large-scale parking facilities.

Keywords: parking pricing; spatial zoning; mixed logit model; administered differential pricing; market-based differential pricing

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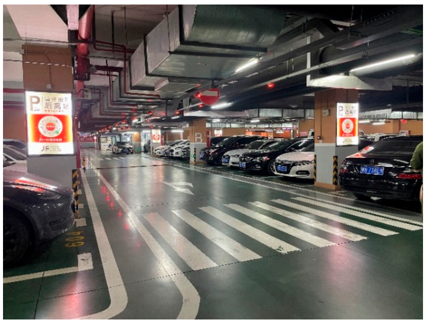
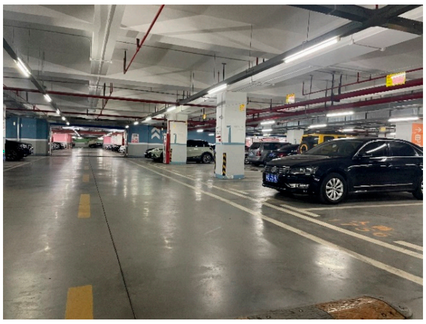
1. Introduction

The rapid expansion of urban commercial complexes has significantly intensified parking demand in densely built city centers. These complexes typically integrate diverse land uses, such as retail, office, dining, and leisure. They attract high volumes of vehicular traffic and generate concentrated, time-sensitive parking needs. Field investigations indicate that, despite the provision of high-capacity parking facilities, the internal utilization of spaces is often spatially imbalanced. Spaces near primary entrances and anchor stores tend to remain continuously occupied, whereas less accessible zones are persistently underutilized. This spatial heterogeneity in occupancy directly affects parking efficiency and user experience, highlighting the need for more refined allocation and pricing strategies.

Traditional pricing strategies in most facilities follow a uniform, one-size-fits-all approach, which fails to respond to the spatio-temporal variation in parking demand. This has been empirically demonstrated in recent urban studies showing that differentiated pricing can more effectively reduce congestion and rebalance parking occupancy compared to uniform pricing [1]. This mismatch often causes congestion in high-demand zones and idle capacity in low-demand areas, thereby reducing overall efficiency [2]. Therefore, a systematic approach to spatial partitioning and differential pricing is essential for rebalancing demand and improving utilization. As shown in Figure 1a,b, empirical observations in a large-scale complex reveal clear variation inside the facility: high-saturation areas near core entrances remain congested throughout peak periods, whereas lower-saturation areas exhibit prolonged underuse. These conditions are not effectively addressed through the uniform pricing strategies currently employed in most facilities. Simulation-based analyses have shown that incorporating spatial location differences into pricing design can significantly improve both parking utilization and user satisfaction [3].



(**a**) high-saturation areas



(**b**) lower-saturation areas

Figure 1. Current issues of allocated parking lots for large complexes.

Conventional approaches to resolving such issues have largely relied on supply-side measures, such as expanding physical capacity or upgrading parking infrastructure. These methods are increasingly constrained by space limitations and economic feasibility and do not address the core mismatch between user preferences and spatial resource distribution. As an alternative, demand-side pricing strategies—particularly differentiated pricing—have gained recognition for their ability to influence user behavior and redistribute parking demand.

Although differentiated pricing has received growing attention in both academic research and urban parking practice, its implementations remain limited by several critical challenges. First, most current strategies operate at a coarse spatial granularity, typically applying different rates by urban subdistrict, which overlooks the finer variation in parking space inside the facility value caused by differences in proximity, visibility, or accessibility [4]. Second, while user behavior has been extensively studied in terms of parking lot selection, few models account for behavioral heterogeneity in parking space choice inside facilities, particularly with respect to spatial and pricing attributes [2,5–8]. Third, spatial zoning methods used to support differentiated pricing are often guided by managerial intuition, lacking algorithmic rigor and limiting transparency and scalability [9,10]. These limitations collectively underscore the need for a more structured, data-driven, and behaviorally informed approach to pricing design in large-scale parking facilities.

To address these challenges, this study focuses on internal resource allocation and pricing differentiation within a single large-scale complex. By emphasizing micro-level demand imbalances, spatial variation in parking space attributes, and user behavioral heterogeneity, the research aims to formulate a pricing strategy that not only enhances utilization efficiency within the facility but also better accommodates diverse user preferences.

This paper proposes an integrated framework for the design and optimization of differentiated pricing strategies tailored to complex parking environments. The framework is structured into three major stages. First, a mixed Logit model is constructed to capture user choice behavior in response to parking space attributes. Second, we employ a dual clustering algorithm rather than a conventional single clustering method. While single clustering may generate fragmented or spatially discontinuous groups, the dual clustering framework jointly considers attribute homogeneity and spatial contiguity, producing zones that are both interpretable and operable [11]. Building on this, the dual clustering algorithm is applied to partition the facility into spatially homogeneous pricing zones, using data on spatial layout, functional connectivity, and occupancy patterns. Third, a bilevel optimization model is formulated to jointly consider societal objectives—such as occupancy balancing—and operator goals like revenue maximization [12]. The upper-level model proposes zone-level pricing guidance, while the lower-level model simulates user responses and resulting space allocation [13].

The contributions of this study are both theoretical and practical, and can be summarized as follows:

Behavioral modeling: A mixed Logit model with interaction terms is developed to capture intra-facility heterogeneity in parking space choice. The specification reflects variations in sensitivity to attributes such as price and search time, as well as the combined effects of multiple attributes.

Spatial zoning: An improved dual clustering algorithm is introduced, which simultaneously ensures attribute homogeneity and spatial contiguity. This method produces zones that are analytically robust and practically operable for differentiated pricing.

Pricing modeling: Two bilevel optimization models are formulated to design differentiated pricing strategies for large-scale commercial complex parking facilities. The zonal differentiated guidance pricing model minimizes the deviation from existing prices while simultaneously improving spatio-temporal occupancy balance, ensuring compatibility with current pricing practices. In contrast, the zonal differentiated autonomous pricing model jointly optimizes spatio-temporal occupancy balance and revenue maximization, providing a more flexible but operator-oriented strategy. Together, these models establish a rigorous foundation for evaluating trade-offs between policy feasibility and operational efficiency.

Empirical validation: The proposed methodology is validated through a case study based on real-world data from a large commercial complex parking facility. The results demonstrate its feasibility and applicability.

The remainder of this paper is organized as follows. Section 2 reviews relevant literature on parking behavior modeling, spatial partitioning, and differentiated pricing. Section 3 presents the methodological framework, including user behavior modeling, clustering algorithms, and the optimization structure. Section 4 reports the case study and analyzes the outcomes. Section 5 concludes with key findings and future research directions.

2. Literature Review

This section reviews three research streams related to parking pricing strategies, user choice behavior, and spatial zoning methods.

2.1. Pricing Strategies and Optimization Models

Parking pricing serves as a critical mechanism for regulating demand and improving facility-level efficiency. Traditional flat-rate pricing fails to capture temporal and spatial variations, often leading to overcrowding in popular zones and the underutilization of peripheral parking spaces [14]. Recent studies advocate dynamic or differentiated pricing strategies to address such issues more effectively [1].

Pricing strategies can be broadly classified into flat-rate and differential pricing approaches. Flat-rate strategies determine rates based on average occupancy levels or managerial judgment, whereas differential strategies adjust prices in real time according to current occupancy or demand intensity [2]. While dynamic pricing improves adaptability, it also demands extensive sensing infrastructure and real-time control systems, which limit implementation. Hybrid approaches offer a balanced solution by combining static pricing with periodic updates [13].

Optimization modeling plays a central role in the design of pricing strategies. Linear and mixed-integer programming have been widely used to maximize revenue, optimize utilization, or balance fairness [15]. Among them, bilevel optimization has become a popular framework, modeling operators' pricing decisions at the upper level and user responses at the lower level [16]. Notably, Yang et al. [11] integrated pricing design with facility layout and user behavior modeling, demonstrating improved performance compared with uniform pricing schemes.

However, several limitations persist. First, most pricing studies rely on zonal-level strategies, lacking granularity at the individual parking space level. Second, while user response is modeled, sensitivity to spatial attributes or behavioral variability is often simplified. Third, zoning for pricing is typically predefined rather than optimized, hindering adaptability.

In summary, existing work does not fully integrate pricing decisions with user heterogeneity and spatial microstructure, limiting the efficacy of differentiated strategies.

2.2. Parking Space Choice Behavior Modeling

Understanding how users choose parking spaces is essential for demand-responsive management. Logit-based discrete choice models dominate this field, including multinomial Logit (MNL), nested Logit (NL), mixed Logit (ML), and more recently, cross-nested Logit (CNL) and hybrid choice models [17–20].

Hunt and Teply [17] applied a nested Logit model to explain commuter parking space choices, showing how grouping similar parking alternatives can improve predictive validity over standard MNL. Borgers and Timmermans [18] modeled parking behavior within a facility using a two-level nested Logit model, in which travelers first select a parking area and then choose a specific parking space within it. The study found that walking distance to ticket machines and exits, as well as whether the space is a corner space, significantly influences parking choices.

User heterogeneity is also a critical consideration in modeling. Middelkoop [19] applied a suite of discrete choice models, including mixed-multinomial Logit and its extensions to account for both observed and unobserved preference variation. Meng et al. [21] specifically studied university parking behavior, showing that students and staff respond differently to price and walking distance. Similarly, Khaliq and Janssens [22] incorporated micro-scale attributes like perceived safety, congestion, and space dimensions into their MNL model for agent-based simulations.

Recent efforts emphasize the need to go beyond observed heterogeneity. Li et al. [23] integrated latent variables like risk perception into parking models via PCA-Mixed Logit, while Li et al. [24] compared Logit, regret minimization, and prospect theory frameworks to capture underlying cognitive diversity. Meanwhile, Le et al. [20] applied the cross-nested Logit model to resolve overlap across spatial alternatives—highlighting its flexibility for intra-facility modeling. Qin et al. [25] also studied parking at large airports using nested Logit structures to capture joint location–mode decisions, reaffirming the need to jointly consider spatial and functional layout.

Despite growing efforts in discrete choice modeling, most studies still emphasize parking facility selection rather than fine-grained parking space choice. Intra-facility choice behaviors, which are shaped by subtle spatial cues and personal trade-offs, remain underexplored. Furthermore, limited research integrates behavioral heterogeneity with spatial pricing design, leaving a gap in linking user decision-making to real-time space allocation and economic incentives. These issues call for integrative approaches that jointly model user preferences and space-level management strategies.

2.3. Spatial Zoning Methods

Spatial zoning provides the structural foundation for differentiated pricing and demand management. In practice, many parking facilities divide space heuristically by floor, entrance, or aisle geometry. Such methods, though simple, lack empirical grounding and often fail to reflect the actual spatial variation in value, walking distance, or user demand [2].

To address this, recent studies have applied data-driven zoning techniques. Traditional K-means and hierarchical clustering are widely used due to their simplicity [26], but their ability to preserve spatial contiguity and behavioral relevance is limited. More advanced approaches, such as dual clustering [27] and constrained clustering [28], account for spatial adjacency, usage similarity, and logical constraints. These methods offer improved interpretability and alignment with operational needs.

In the scenario of parking, Yang et al. [11] developed a bilevel optimization model integrating clustering-based zoning with pricing design. Their results show that coupling spatial zoning with user response models enhances both revenue and occupancy balance. Moreover, Duong et al. [28] embedded logical constraints into spatial clustering, showing promise for structured environments such as parking facilities. Liu and Chen [27] and Ke et al. [29] proposed dual clustering strategies that maintain both geometric and functional cohesion. These methods are better suited for high-dimensional spatial behavior integration.

Despite methodological advancements, existing zoning strategies still face several critical limitations. First, many zoning schemes are based on fixed geometric layouts or management heuristics, which fail to reflect continuous spatial gradients in parking space value or occupancy patterns. Second, most clustering models do not incorporate user choice behavior, resulting in weak coupling between spatial partitioning and actual demand. Third, the methodological systems remain fragmented—many approaches optimize spatial, functional, or behavioral aspects separately, lacking an integrated framework. To bridge these gaps, this study proposes a dual-clustering framework that jointly considers spatial configuration, occupancy distribution, and user preference heterogeneity, providing a structured foundation for refined zone-level pricing optimization.

3. Mathematical Formulations and Methods

This study proposes a three-stage methodological framework to design differentiated pricing strategies within large-scale parking facilities. First, a mixed Logit model is developed to capture user preferences and behavioral heterogeneity with respect to parking space attributes. Second, a dual clustering algorithm is introduced to partition the facility into spatially coherent pricing zones by integrating geometric, functional, and occupancy-based factors. Third, a bilevel optimization model is formulated to determine optimal zone-level prices, balancing operator objectives and user responses. These modules are sequentially interconnected: user behavior modeling informs both clustering and pricing design, while spatial zoning provides structural input for the pricing model.

3.1. Modeling Method for User Parking Selection Behavior

This study applies discrete choice modeling to quantify individual parking space selection within large-scale facilities. The modeling process comprises three key components: (1) specification of explanatory variables, (2) unlabeled experimental design, and (3) mixed Logit estimation that incorporates random coefficients and interaction terms. The overall objective is to capture individual-level preference heterogeneity while avoiding label-related biases inherent in traditional stated preference (SP) experiments, thereby enhancing both the behavioral interpretability and the operational applicability of the model for zoning and pricing strategies.

3.1.1. Specification of Explanatory Variables

The selection of explanatory variables was informed by an extensive literature review and refined through a structured questionnaire design. While various spatial, economic, and contextual factors can influence parking space choices (e.g., floor level or wall adjacency), it is neither practical nor necessary to include all possible variables. This study focuses on five core explanatory variables that are both significant in influencing behavior and practical to quantify in the experimental design:

Parking fee: the total expected charge for occupying the space, measured in RMB per hour [11].

Type of parking space: whether the space is mechanical or standard, with mechanical spaces requiring additional time.

Search time: the estimated time spent navigating within the facility to locate the space, including delays caused by other vehicles or inter-floor transitions [30].

Walking time: the time needed to walk from the parked vehicle to the internal destination (e.g., elevator or escalator) [31].

Trip purpose: a categorical variable representing either leisure or commuting, to capture variation in urgency and behavioral flexibility [32].

The attributes and their levels considered in this study are summarized in Table 1.

Table 1. Attribute variable level table.

Attribute Variables		**Attribute Level**
Personal socioeconomic attributes entry 2	Gender	Male: 1; Female: 2
	Age	18–25: 1; 26–35: 2; 36–49: 3; 50 and over: 4
	Education level	High school and below: 1; Associate degree: 2; Undergraduate: 3; Graduate and above: 4
	Driving experience	Less than 1 year: 1; 1–3 years: 2; 3–5 years: 3; 5–10 years: 4; More than 10 years: 5
	Monthly income	Less than 3000 yuan: 1; 3000–5000 yuan: 2; 5000–10,000 yuan: 3; More than 10,000 yuan: 4

Table 1. *Cont.*

Attribute Variables		Attribute Level
Parking-related attributes	Parking duration	Less than 15 min: 1; 15 min to 1 h: 2; 1–2 h: 3; 2–4 h: 4; More than 4 h: 5
Parking space-related attributes	Search time	1 min; 8 min; 15 min
	Walking time	1 min; 8 min; 15 min
	Whether it is a mechanical parking space	Yes: 1; No: 0
	Parking fee	5 yuan/h; 8 yuan/h; 11 yuan/h

3.1.2. Unlabeled Experiment Design

To elicit parking space choice behavior within a systematically designed choice scenario, this study adopts an unlabeled stated preference (SP) experiment. Unlike labeled designs that assign fixed identifiers to each alternative, the unlabeled format mitigates label-induced biases by directing respondents' attention exclusively to attribute levels [33]. This is particularly valuable in the scenario of large-scale parking facilities, such as commercial complexes with more than 300 or even 500 spaces, where labeling individual alternatives would dramatically increase the number of choice options. Such expansion not only imposes an excessive cognitive burden on respondents but also reduces attribute variation across tasks, thereby weakening the model's capacity to recover reliable preference structures. In contrast, the unlabeled format enables all alternatives to share a common utility function, improving both model parsimony and estimation efficiency.

To ensure design quality, this study applies a D-efficient experimental design method using an iterative Excel-based tool developed by Prof. John Rose [34]. The tool minimizes the determinant of the asymptotic variance–covariance (AVC) matrix, thereby enhancing the precision of parameter estimates for a fixed sample size [35]. The final design includes six choice scenarios for each of the two trip purposes, with each scenario presenting three unlabeled alternatives defined solely by their attribute levels (e.g., walking time, search time, parking fee, and parking space type).

This structure reduces respondent fatigue and improves data reliability, while also enabling the estimated utility parameters to generalize across different facility types and spatial layouts without relying on scenario-specific labels or identifiers.

3.1.3. Mixed Logit Model Estimation

Parking space choices are influenced by multiple behavioral factors, including sensitivity to cost, walking time, and space type. These preferences often vary significantly across individuals and trip purposes. Moreover, each respondent completes multiple-choice tasks, resulting in repeated observations that violate the independence assumptions of traditional Logit models. In particular, the multinomial Logit (MNL) model assumes the Independence of Irrelevant Alternatives (IIA), implying that the relative odds of choosing between any two alternatives are unaffected by the presence or attributes of other options. However, in parking space scenarios, this assumption is often unrealistic, as alternatives are not perceived as equally substitutable.

Parking space choices are influenced by a variety of behavioral factors, including sensitivity to cost, walking distance, and space type. These preferences exhibit significant heterogeneity across individuals and trip purposes. In addition, each respondent completes multiple-choice tasks, resulting in repeated observations that violate the independence assumptions of traditional Logit models.

To address these limitations, this study estimates a Mixed Logit model, which relaxes the IIA assumption and allows for random taste heterogeneity by specifying certain coefficients as random variables. Each coefficient is drawn from a statistical distribution, typically normal, with the mean representing the average strength of user preference and the standard deviation capturing the degree of heterogeneity across individuals. The choice probability of an individual i selecting an alternative j is modeled as the weighted average of Logit probabilities under varying coefficients:

$$P_{ij} = \int L_{ij}(\beta_j) f(\beta_j|\theta) d\beta_j, \tag{1}$$

where $f(\beta_j|\theta)$ is the probability density function of coefficients β, parameterized by distributional parameters θ, and $L_{ij}(\beta_j)$ is the standard Logit probability under the coefficient vector β.

$$L_{ij} = \frac{e^{U_{ij}(\beta_j, x_i)}}{\sum_{j=1}^{C} e^{U_{ij}(\beta_j, x_{ij})}}, \tag{2}$$

where x_{ij} is the vector of observed attributes associated with alternative j for individual i, and x_i is the vector of individual-specific characteristics.

The utility function U_{ij} for each alternative includes observed variables and random coefficients:

$$U_{ij} = \beta_1 \times Fee_j + \beta_2 \times Mec_j + \beta_3 \times Cruising_j + \beta_4 \times Walking_j + \varepsilon_{ij}, \tag{3}$$

where Fee_j is the unit parking fee (RMB/h), $Fee_j = \sum_i Price_{ij} \times Time_i$, Mec_j is a dummy variable for mechanical space, $Cruising_j$ is estimated search time, $Walking_j$ is walking time to destination $\beta_k \sim N\left(\overline{\beta_k}, \sigma^2_{\beta_k}\right)$, and $k = 1, 2, 3, 4$ are normally distributed random coefficients.

While the standard mixed Logit specification captures heterogeneity across basic parking attributes, it assumes independence between explanatory variables. In reality, sensitivities to price, search time, and walking distance often vary systematically across demographic groups. For instance, price sensitivity may depend on income, and tolerance for walking distance may differ by age or gender. To account for such interactions, the specification is extended to incorporate interaction terms between user socio-demographics and parking attributes. This allows the model to better capture preference heterogeneity and improve prediction accuracy.

The extended utility functions are estimated separately for two trip purposes:

For commuting:

$$U_{ij}^{(c)} = \sum_{k \in x_c} \beta_k^{(c)} X_{ijk} + \sum_{m \in I_c} \gamma_m^{(c)} (Z_{im} X_{ijm}) + \varepsilon_{ij}^{(c)}, \tag{4}$$

where $x_c = \{Fee, Mec, SeaTime, WalkTime\}$ denotes hourly parking fee (CNY/h), mechanical space dummy, estimated search time (minutes), and walking time to the destination (minutes). The interaction set is $I_c = \{Age2 \times SeaTime, Gender \times Fee, Age2 \times Fee, Gender \times WalkTime\}$. Z_{im} represents the socio-demographic characteristics of individual i, including age group, gender, and income, which interact with parking attributes.

For leisure:

$$U_{ij}^{(l)} = \sum_{k \in x_l} \beta_k^{(l)} X_{ijk} + \sum_{m \in I_l} \gamma_m^{(l)} (Z_{im} X_{ijm}) + \varepsilon_{ij}^{(l)} \tag{5}$$

where $x_l = \{Fee, Mec, SeaTime, WalkTime\}$, defined as above. The interaction set is $I_l = \{Age1 \times Mec, Income1 \times Fee, Age2 \times Fee, Gender \times WalkTime, Gender \times Mec\}$. Z_{im} represents the socio-demographic characteristics of individual i, including age group, gender, and income, which interact with parking attributes.

All coefficients β and γ are treated as random parameters following normal distributions; their means represent average marginal utilities, and their standard deviations capture taste heterogeneity. The error terms ε_{ij} are assumed to be independently and identically distributed, following the standard Logit specification.

This model reflects how socio-demographic factors modify sensitivities to parking attributes, thereby enhancing behavioral realism in the parking space allocation model used in subsequent optimization. This study adopts the maximum simulated likelihood method (MSL), implemented via the Python 3.7 Biogeme v3.2 [36].

3.2. Spatial Zoning Algorithm

This study proposes an improved dual clustering algorithm to divide parking spaces into pricing zones. The resulting zoning design supports the development of pricing strategies while ensuring spatial continuity, attribute consistency, and balanced zone sizes.

3.2.1. Design Principles

The spatial zoning of parking facilities is guided by three core principles to ensure practical feasibility and behavioral relevance in pricing strategy implementation:

Spatial contiguity: Parking spaces within the same zone must be spatially connected without being divided by spaces from other zones. This ensures that each pricing zone forms a cohesive spatial unit, thereby facilitating user recognition and administrative management.

Attribute consistency: Within each zone, non-spatial attributes (e.g., space type, walking time) should remain as consistent as possible, while inter-zone differences should be maximized. This promotes internally consistent pricing rules and avoids user confusion caused by heterogeneous conditions within the same pricing zone.

Balanced zone size: Each pricing zone should include a moderate and manageable number of parking spaces. Overly large or small zones may lead to difficulties in policy implementation and reduce pricing effectiveness. The aim is to align zone sizes with facility layout and operational constraints.

These principles provide the foundation for improving the dual clustering algorithm, making it suitable for complex parking facilities such as those attached to large commercial complexes.

3.2.2. Algorithmic Modifications

Although the original dual clustering algorithm effectively integrates optimization and constraint domains [27], it has two main limitations when applied to parking space zoning in large-scale facilities. First, the use of complete-linkage hierarchical clustering in the optimization domain suffers from irreversible cluster formation and poor scalability. Second, the algorithm lacks an effective mechanism to control zone sizes, which contradicts the zoning principle of moderate and balanced region scales outlined.

To address these issues, two major modifications are introduced:

Replacing hierarchical clustering with K-medoids: The optimization domain now applies the K-medoids algorithm instead of complete-linkage hierarchical clustering. K-medoids reduces the impact of outliers and offers greater computational efficiency. Moreover, unlike hierarchical methods, it allows for more flexible and iterative updates, improving the quality and reversibility of the clustering results [37].

Adding a recursive size adjustment procedure: A post-processing step is appended to the algorithm to adjust the number of spaces in each zone. Parking zones are first evaluated based on their deviation from the target zone size (as defined by managerial preferences). Zones exceeding or falling short of this threshold are identified as donors or recipients. Then, spatially adjacent spaces are iteratively reallocated from oversized to undersized zones, subject to a distance threshold based on facility corridor widths. This ensures that boundaries remain aligned with the physical layout while achieving size balance.

A detailed algorithmic workflow is illustrated in Figure 2 and summarized in the following steps:

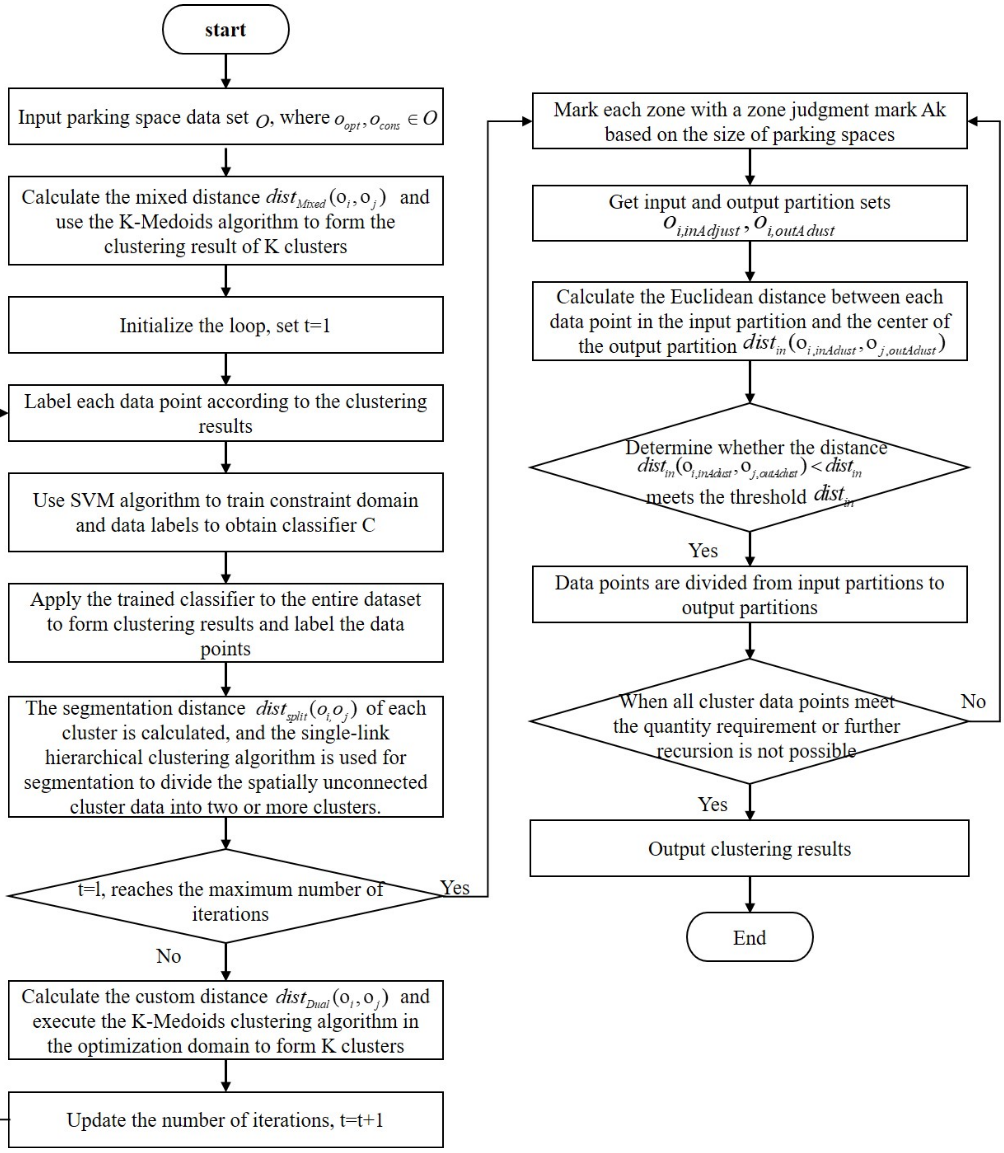


Figure 2. Workflow of the improved dual clustering algorithm.

As shown in Figure 2, the improved dual clustering procedure follows an iterative cycle alternating between the attribute domain and the spatial domain. In each iteration, K-medoids is first employed to update robust cluster medoids in the attribute space, thereby reducing sensitivity to outliers. A Support Vector Machine (SVM) classifier is then used to refine the separating boundaries between high- and low-demand zones based on occupancy and accessibility features, ensuring behavioral consistency. In the spatial domain, a recursive size adjustment procedure is applied to balance zone sizes and enforce contiguity. This cycle repeats until the zone configuration converges, yielding clusters that are both homogeneous in attributes and coherent in space, thus enhancing robustness, behavioral realism, and practical applicability in large-scale commercial parking facilities.

3.2.3. Evaluation Indicators

Numerous clustering evaluation methods have been developed to assess clustering performance for datasets of varying scales and structures. However, in the original dual clustering method, only partial parameter settings were specified, such as assigning equal weights of 0.5 to both the optimization and constraint domains to avoid bias. Given that the dataset used in the study was structurally homogeneous and its application scenario differed significantly from that of this study, such fixed settings may not be directly applicable to large commercial complex parking facilities.

Therefore, this study adopts a grid search approach to systematically combine five key parameters ($dist_{in}$, K, w, α and $ratio$). In line with the spatial zoning principles and to address the operational characteristics of large-scale commercial parking facilities, two indicators are developed: one measures spatial coherence and the other assesses zone size balance. These indicators enable the identification of optimal parameter combinations for different facility types, thereby narrowing the search space in practical deployments and enhancing the operational applicability and scalability of the proposed method.

Ratio of External and Internal Distances ($REID$)

This indicator draws on the concept of the Dunn Validity Index (DVI), which evaluates clustering validity by taking the ratio of the minimum inter-cluster distance to the maximum intra-cluster distance. A higher DVI indicates greater separation between clusters and tighter cohesion within clusters. However, increasing the number of clusters can reduce inter-cluster distances and increase intra-cluster distances, making DVI unsuitable for evaluating parameter combinations in this study.

To address this, the $REID$ is proposed as an adaptation of the DVI, calculated as follows:

$$REID = -\frac{external\_dist}{internal\_dist}, \tag{6}$$

$$external\_singledist = \sum_{i,j=1}^{K} \sqrt{\left(Mean(x_i) - Mean\left(x_j\right)\right)^2 + \left(Mean(y_i) - Mean\left(y_j\right)\right)^2}, \tag{7}$$

$$external\_dist = \frac{\sum_{i=1}^{K} external\_singledist}{K^2 - K}, \tag{8}$$

$$internal\_singledist = Mean\left(\sqrt{(x_i - \overline{x})^2 + (y_i - \overline{y})^2}\right), \tag{9}$$

$$internal\_dist = Mean\left(\sum_{i=1}^{K} internal\_singledist\right), \tag{10}$$

where $external\_dist$ is the average inter-cluster distance, calculated as the mean of all $external\_singledist$, $internal\_dist$ is the mean intra-cluster distance across all clusters, $internal\_singledist$ is the average distance between each parking space within cluster i and the centroid of that cluster, $external\_singledist$ is the Euclidean distance between the centroids of clusters i and j.

A smaller $REID$ value indicates larger inter-zone distances and smaller intra-zone distances, reflecting better clustering quality; conversely, a larger $REID$ suggests poorer performance.

Parking Distribution Entropy (PDE)

Inspired by Shannon's information entropy, which measures system uncertainty, this indicator evaluates the balance of parking space quantities among clusters [38]. The entropy is calculated as follows:

$$H = -\sum_{i=1}^{K} p_{ij} log_2 p_{ij}, \tag{11}$$

where p_{ij} is the probability that a data point in cluster i belongs to cluster j; K is the number of clusters.

In this study, the ratio of the number of parking spaces in each zone to the total number of parking spaces is used as the probability in information entropy. Drawing on the concept of "information entropy," this study proposes the parking distribution entropy (PDE) to assess the balance of parking spaces across zones. A higher entropy indicates greater overall dislocation and a more balanced distribution of parking spaces across zones. Because this variable is correlated with the number of zones, it is necessary to eliminate the influence of the number of zones when using this metric to evaluate the balance of clustering results for different parameter combinations.

When the number of data points within each cluster is equal, the entropy is minimized ($p_{ij} = \frac{1}{K}$). The parking distribution entropy (PDE) can be calculated as follows:

$$PDE = -\sum_{i=1}^{K} \frac{1}{K} log_2 \frac{1}{K} = log_2 K, \tag{12}$$

When the number of data points in each cluster is different, the value of PDE will be smaller than the derivation of (12), so $PDE \in [0, log_2 K]$. In order to use PDE to compare clustering results with different numbers of clusters, it is necessary to constrain the range of PDE so that $PDE'(x) \in [0, 1]$, then:

$$PDE' = \frac{2^{PDE}}{K}, \tag{13}$$

$$PDE' = \frac{2^{log_2 K}}{K} = \frac{K}{K} = 1, \tag{14}$$

Under the assumptions, $PDE = 1$, meaning that a larger value indicates a more balanced number of data points within each cluster after clustering. Therefore, this chapter refines entropy to the parking distribution entropy (PDE) to measure the balance of the number of parking spaces in each region after clustering. The practical significance of this approach is that when the parking distribution entropy is close to 1, the distribution of parking in each region is more balanced, leading to better clustering results.

3.2.4. Parameter Selection

To determine the optimal spatial zoning configuration, five key parameters are considered: the distance threshold from output spaces to input zones $dist_{in}$, the number of clusters K, the initial weight of the optimization domain in the mixed distance calculation w, the incremental optimization-domain weight α, and the allowable fluctuation ratio for zone sizes $ratio$. Their candidate values are listed in Table 2.

Table 2. Parameter variable value range.

Parameter	Symbol	Candidate Values
Distance threshold from output spaces to input zones	$dist_{in}$	{1, 2, 3}
Number of clusters	K	{3, 4, . . ., 10}
Initial optimization-domain weight	w	{0.3, 0.4, 0.5}
Incremental optimization-domain weight	α	{0.3, 0.4, 0.5}
Zone size fluctuation ratio	$ratio$	{0.1, 0.2}

A grid search procedure is employed to exhaustively combine these parameters and evaluate each configuration using the two performance indicators defined in Section 3.2.3: the Parking Distribution Entropy (PDE) and the Ratio of External and Internal Distances (REID). The dual-clustering algorithm is executed for each parameter combination, with a maximum recursion depth of 1000 and 2000 iterations. Combinations that fail to converge are recorded and excluded from further analysis.

For each feasible parameter set, the spatial zoning result is generated, and its PDE and REID values are computed. The grid search produces a large number of feasible configurations, which are jointly evaluated using the concept of the Pareto frontier [39]. A solution is considered Pareto-optimal if no other configuration can improve one indicator without deteriorating the other. When two parameter sets produce identical REID and PDE values, their spatial zoning outcomes are identical, and any duplicate solutions are removed.

The resulting unique set of Pareto-optimal parameters constitutes the candidate set for defining pricing units in the differentiated pricing strategy and serves as the spatial zoning basis for the optimal strategy.

3.3. Model Development

3.3.1. Model Assumptions

To develop a differential pricing strategy that balances resource utilization and operational objectives, this study proposes a bi-level programming model, with the upper level for price optimization and the lower level for berth allocation based on utility maximization. The following assumptions are adopted in this study:

1. The probability of parking space choice follows a Logit distribution;
2. Illegal parking during peak hours is excluded, assuming total vehicles do not exceed the planned capacity;
3. Parking revenue is solely derived from parking fees;
4. Parking demand follows a Poisson distribution;
5. Changes in trip mode or destination due to price changes are not considered;
6. Information on prices, berth status, and spatial layout is fully transparent to users;
7. Users can estimate parking duration and cost, and select the shortest route when driving/walking.

3.3.2. Symbol and Variables

The main symbols used in this study are summarized in Table 3:

Table 3. Symbol explanation.

Symbol	Meaning
Z	Total number of zones
T	$T = \frac{24h}{\lvert period \rvert}$ Total number of time periods
z	$z \in \{1, 2, \ldots\ldots, Z\}$
t	$t \in \{1, 2, \ldots\ldots, T\}$
$period$	Set of time period (hour)
n_z	$\sum_{z=1}^{Z} n_z = N$, The total number of parking spaces in the zone z
o_z^t	The spatio-temporal occupancy of a zone z in a time period t
N	Total number of parking spaces
C	Total daily parking demand
P_{ij}	The probability that parker i chooses a parking space j
P_i	Total fee per parking event for the parker i
Pt_i	Parking duration per parking event for the parker i
T_{limit}	Maximum chargeable duration for a single parking event
p_z^t	Parking fee in the zone z at the time t
$p_{zuplimit}^t$	Upper bound of the unit parking fee in the zone z at the time t
$p_{zfloorlimit}^t$	Lower bound of the unit parking fee in the zone z at the time t

Table 3. *Cont.*

Symbol	Meaning
$\left\lvert \overrightarrow{Time} \right\rvert$	The time period experienced by a single parking event
R	Set of spatial zoning results $R = \{r_1, r_2, \ldots\ldots, r_{list}\}$
Sec_t	Size in the time period t
bp	Current parking unit fee for parking lots
P_{limit}	Maximum total price per parking event
$p^k_{WDuplimit} p^k_{WEuplimit}$	weekday and weekend price upper bounds for parking spaces k
$SeaTime_k$	Search time for a parking space k
$WalkTime_k$	Walking time for the parking space k
Mec_k	Type of parking space k

3.3.3. Definition of Spatial–Temporal Occupancy Rate

To simultaneously measure the balanced utilization of parking resources in both temporal and spatial dimensions, this study introduces the spatial–temporal occupancy rate (STOR). Unlike conventional occupancy rates that focus on a single time period or zone, the spatial–temporal occupancy rate captures the variation in utilization across zones and time periods.

Assume that the total number of time periods for the differential pricing strategy is T, and the total number of spatial zones is Z, as illustrated in Figure 3a.

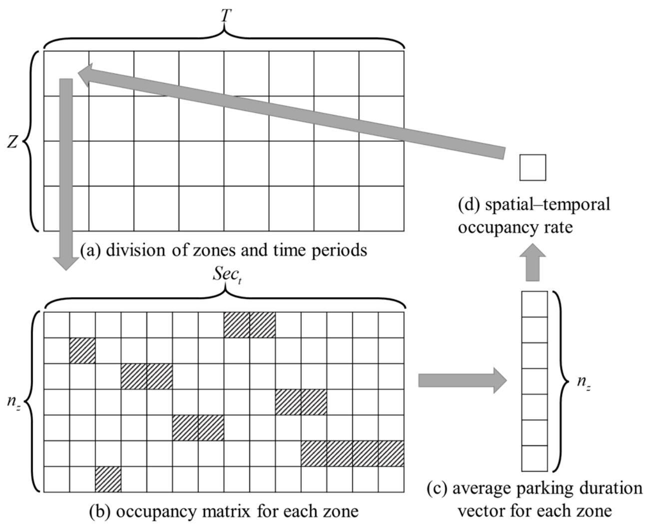


Figure 3. Calculation ideas for spatial–temporal occupancy rate.

For each zone in each time period, its occupancy status can be represented by an Occupancy Matrix, with rows corresponding to the number of berths in the zone and columns corresponding to the total seconds in the time period, as shown in Figure 3b.

Based on this, the STOR calculation proceeds as follows:

1. Construct parking spaces status matrix: For each time period, a status matrix of size "number of parking spaces in the zone z × total seconds Sec_t in the time period t" is created, where gray cells indicate an occupied parking space at that second, and white cells indicate a vacant parking space, as shown in Figure 3b.
2. Calculate average parking duration: For each zone, the actual parking durations of all occupied parking spaces are averaged to obtain a mean parking time vector, as shown in Figure 3c.

3. Calculate zone-level spatial–temporal occupancy rate: Divide the average parking duration by the number of parking spaces n_z in the zone to obtain the occupancy rate for that zone in the given time period, as shown in Figure 3d.
4. Calculate the variance of occupancy rates: For each time period, compute the variance of occupancy rates across all zones. A smaller variance indicates a more balanced distribution of parking resources.
5. Calculate total spatial–temporal occupancy rate: Sum the variances across all time periods to obtain the total STOR for the analysis period.

3.3.4. Administered a Differential Pricing Model

In practice, differential pricing for parking can be implemented under two distinct regulatory scenarios:

Administered pricing strategy applies when the pricing authority sets or approves parking rates primarily to guide user behavior, such as balancing demand across zones and time periods, while keeping prices within regulated bounds and close to the current standard. Revenue maximization is not the primary goal, and the operator's flexibility is limited.

Market-based pricing strategy applies when the operator has autonomy to adjust prices in response to market conditions, aiming to improve both resource utilization and revenue.

Based on the administered pricing strategy, the differential pricing model is developed from the perspective of the government, which emphasizes the role of parking facilities as part of urban transport infrastructure more than the operator does. The upper-level objectives are to

(i) Minimize the variance of spatial–temporal occupancy rates across zones within the parking facility;
(ii) Minimize the deviation of parking fees from the current charging standard.

The decision variables in the integrated bi-level optimization framework consist of the spatial zoning configuration R and the unit parking price p_z^t for each zone z and time period t. The variable R represents the spatial zoning of the parking facility and is selected from the unique parameter solutions, which serve as the candidate set in the iterative optimization process to determine the optimal configuration. The variable p_z^t represents the current parking unit fee for parking lots; when the continuous parking duration exceeds the maximum chargeable time limit, the total fee is capped at the amount corresponding to this limit. This requirement is enforced through the single-parking-event cost constraint in Equation (20). The spatial–temporal occupancy rates o_z^t are jointly determined by the parking space choice model and the spatial zoning configuration: the parking space choice model allocates demand to individual spaces, while the zoning configuration determines the zone to which each berth belongs.

The optimization framework follows an iterative bi-level process, where the upper-level model evaluates the objectives based on o_z^t computed by the lower-level model, and updates p_z^t and R until convergence. The process can be summarized by the following pseudocode:

In this bi-level structure, the upper level determines the unit prices p_z^t for each zone z and time period t together with the spatial zoning configuration R. These decisions are passed to the lower-level allocation model, which computes the mixed Logit choice probabilities P_{ij} and allocates parking demand to spaces. The resulting zone–time occupancies o_z^t are then aggregated and fed back to the upper level. Based on these occupancy outcomes, the upper level evaluates its objectives—minimizing the spatio-temporal variance of occupancy and controlling the deviation of parking fees from the baseline—and updates p_z^t and R. This process is repeated until convergence, as summarized in Algorithm 1.

Algorithm 1. Decision Variable Calculation Method

Input: Spatial partitioning result $R = \{r_1, r_2, \ldots, r_{list}\}$; Total number of clusters in a set of partitioning results Z; Probability P_{ij} of a parker i selecting parking space j; Parking demand $C = \{C_1, C_2, \ldots, C_T\}$; Time period set $T = \{t_1, t_2, \ldots, T\}$.

Process:

```
for r ← each R do
o_z^t = 0;
  for t ← each T do
    for z ← each Z do
      for i ← each C_T do
        Choice_i = argmax{P_ij}
          Determine the zone z to which Choice_i belongs;
      end for
          update o_z^t, Choice_i = z
          computer the average parking duration within zone z during period t
          divide by the number of spaces n_z in zone z
          yields o_z^t for zone z and period t
    end for
  end for
end for
```

Output: o_z^t: The spatial–temporal occupancy of zone z in time period t.

This zoning-based administered differential pricing model is formulated as a bi-level programming problem: the upper level minimizes total spatial–temporal occupancy variance and price deviation from the baseline, while the lower level is a dynamic parking space allocation model based on the utility maximization theory, with parameters estimated from the mixed Logit model.

The upper-level bi-objective functions are as follows:

$$min\ STOR = \sum_{t=1}^{T} var\left(\left\{o_1^t, \ldots, o_z^t\right\}\right), \tag{15}$$

$$min \sum_{t=1}^{T} \sum_{z=1}^{Z} p_z^t - bp, \tag{16}$$

Subject to:

$$o_z^t \leq 1, \tag{17}$$

$$p_{zuplimit}^t \geq p_z^t \geq p_{zfloorlimit}^t, \tag{18}$$

$$p_{zuplimit}^t = \begin{cases} \mathrm{Max}\left(p_{WDuplimit}^k\right), weekday \\ \mathrm{Max}\left(p_{WEuplimit}^k\right), weekend \end{cases} \tag{19}$$

$$\begin{cases} \overrightarrow{Time} \times \left[p_z^{t1}, p_z^{t2}, \ldots\ldots, p_z^{tq}\right]^T, & \left|\overrightarrow{Time}\right| \leq T_{limit} \\ \left[t_1, t_2, \ldots\ldots, T_{limit}\right] \times \left[p_z^{t1}, p_z^{t2}, \ldots\ldots, p_z^{T_{limit}}\right]^T, & \left|\vec{t}\right| > T_{limit} \end{cases} \tag{20}$$

Equations (15) and (16) correspond to the two upper-level objectives described above, while Equation (17) constrains the spatio-temporal occupancy rates of all zones in each time period. Equations (18) and (19) specify the pricing bounds: (18) imposes upper and lower limits on unit parking fees, and (19) provides distinct upper bounds for weekdays and weekends. Finally, Equation (20) limits the total charge for a single parking event, ensuring that when the parking duration exceeds the maximum chargeable time, only the fee for that limit is applied.

The lower-level problem is a dynamic parking space allocation model, in which parking demand is assigned to individual parking spaces based on the utility maximization principle. The lower-level model relies on the mixed Logit choice probabilities defined in Section 3.2. For clarity, the relevant equations are restated here. The mixed Logit probability that user i selects space j is:

$$P_{ij} = \int L_{ij}(\beta_j) f(\beta_j|\theta) d\beta_j, \tag{21}$$

$$L_{ij} = \frac{e^{U_{ij}(\beta_j, x_i)}}{\sum_{j=1}^{C} e^{U_{ij}(\beta_j, x_{ij})}}, \tag{22}$$

$$max\ U_{ij} = \alpha Fee_j + \beta Mec_j + \delta SeaTime_j + \gamma WalkTime_j + \varepsilon_{ij} \tag{23}$$

Here, Fee_j represents the total parking cost incurred by user i for selecting space j. It is calculated consistently with the total cost constraint defined in (20), i.e., as the unit price p_z^t determined by the upper level multiplied by the effective parking duration, capped by the maximum chargeable time limit.

All coefficients in the lower-level model are derived from the estimation results of the mixed Logit model. These probabilities P_{ij} are aggregated across users to derive the expected zone–time occupancy o_z^t, which feeds back into the upper-level objectives.

3.3.5. Market-Based Differential Pricing Model

The market-based differential pricing strategy applies when the parking operator has the autonomy to adjust prices dynamically within an approved range, responding to market conditions such as demand patterns and occupancy rates. Unlike the administered pricing strategy, the operator's objectives include not only improving the balance of resource utilization but also maximizing economic returns. In this market-based model, no minimum price floor is imposed, thereby allowing prices to be set as low as zero when necessary. This flexibility aims to enhance occupancy balance by stimulating demand in underutilized zones, even if it means temporarily offering free parking during certain periods.

In this scenario, the upper-level objectives are

(i) Minimize the variance of spatial–temporal occupancy rates across zones within the parking facility.
(ii) Maximize the total parking revenue.

This zoning-based market differential pricing model is also formulated as a bi-level programming problem: the upper level simultaneously minimizes total spatial–temporal occupancy variance and maximizes total revenue; the lower level is a dynamic parking space allocation model based on the utility maximization theory, with parameters estimated from a mixed Logit model including interaction terms.

$$min \sum_{t=1}^{T} var(\{o_1^t, \ldots, o_z^t\}), \tag{24}$$

$$max \sum_{z=1}^{Z} \sum_{t=1}^{T} o_z^t p_z^t, \tag{25}$$

Subject to:

$$p_{zuplimit}^t \geq p_z^t \geq 0, \tag{26}$$

Subject to: the same constraints as in the administered differential pricing model, i.e., Equations (17)–(20).

The lower-level dynamic berth allocation model is identical to that described in Section 3.3.4, with the allocation results determining the spatial–temporal occupancy rates o_z^t for the upper-level objectives.

3.3.6. Solution Method

The bilevel differential pricing models formulated in this study are Non-deterministic Polynomial-time hard (NP-hard) problems that cannot be efficiently solved by exact approaches such as vertex enumeration, KKT reformulations, penalty functions, or branch-and-bound [40]. To address this challenge, an improved hybrid algorithm integrating Particle Swarm Optimization (PSO) and Grey Wolf Optimizer (GWO) is employed. This hybrid PSO–GWO combines the strong global exploration ability of GWO with the effective local exploitation capability of PSO, thereby accelerating convergence and enhancing solution stability compared with either algorithm alone [41,42].

Recent research has also demonstrated the continued advancement of PSO–GWO and related hybrid swarm intelligence methods in solving complex multi-objective optimization problems across transportation and infrastructure management. For example, PSO–GWO hybrids have been employed in smart city emergency service systems to optimize multi-objective vehicle routing under dynamic traffic conditions, showing superior convergence and balance among objectives compared with single algorithms [43]. Similar approaches have been adopted for multi-modal facility layout optimization and urban logistics distribution networks considering congestion, time windows, and environmental impacts, further confirming the robustness of swarm-based hybrid methods in large-scale transportation scheduling [44,45]. In addition, PSO–GWO frameworks have been applied to energy–transport integration problems such as smart grid reconfiguration with electric vehicles and distributed generation planning, where they effectively balance system losses, stability, and cost efficiency [46,47]. Together, these studies highlight the versatility and effectiveness of PSO–GWO in addressing multi-objective challenges with structural and operational constraints, supporting its application to the bilevel differential pricing models in large-scale parking facilities considered in this study.

For multi-objective optimization, conflicts among objectives make it difficult to evaluate the superiority of candidate solutions. To overcome this issue, the concept of Pareto optimality is introduced [48]. Using non-dominated sorting, the population is divided into multiple hierarchical fronts, where solutions within the same front are mutually non-dominated. In addition, the crowding-distance measure is applied to preserve diversity within the Pareto front and to ensure well-distributed solutions across the objective space. Explorers are further introduced to expand the search space, avoiding premature convergence and covering different regions of the objective space.

The overall procedure of the improved PSO–GWO algorithm is summarized in Figure 4, which illustrates the initialization, leader updating, velocity–position updating, fitness evaluation, and termination stages.

After initialization, the top three leaders guide the global search following the GWO mechanism, while PSO operators refine local search through velocity–position updates. Fitness is evaluated based on the bilevel objectives, which integrate spatio-temporal occupancy balance and revenue maximization. Non-dominated sorting and crowding-distance mechanisms are then applied to maintain a well-distributed Pareto front. This iterative process continues until the termination criterion is satisfied, defined either by the maximum number of iterations or by the stabilization of the solution set.

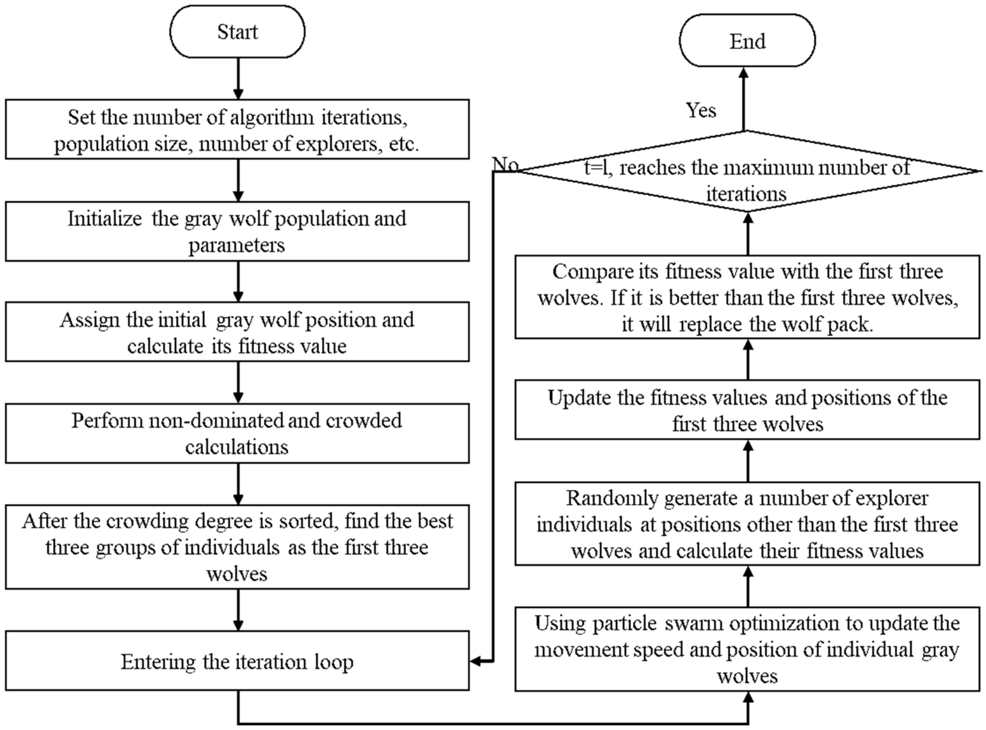


Figure 4. Flowchart of the PSO–GWO algorithm.

4. Results

4.1. Case Description and Data Preparation

To evaluate the performance of the proposed differential pricing models, a case study was conducted using the parking facility of the Kingmo Complex, located in the Baijia Lake commercial district of Nanjing, China. The facility serves as a representative example of large-scale parking infrastructure in a mixed-use urban environment, accommodating a combination of retail, dining, entertainment, and office functions that generate diverse temporal parking demand patterns.

The Kingmo Complex comprises five floors of commercial space, including one underground floor and four above-ground floors. Its parking facility is located on the second to fourth basement levels (B2–B4). The dataset was provided with the consent of the facility management authority, anonymized prior to analysis, with license plate numbers anonymized and all personally identifiable information removed, and used solely for academic research purposes. The B2 level contains 562 parking spaces, and the B3 level contains 590 parking spaces. Both B2 and B3 are equipped with self-service payment machines featuring reverse cruising functions, allowing for occupancy monitoring that meets the requirements of this study. In contrast, the B4 level is generally not open to short-term parking and does not support occupancy monitoring. Consequently, B4-level spaces are excluded from the analysis. For ease of reference in the subsequent zoning analysis and illustrations, the B2 and B3 levels are labeled as B1 and B2, respectively. This notation does not alter the actual physical floor designation but is used solely to simplify presentation and mapping in the analytical process.

Parking data were collected from 00:00:00 on 16 November 2021 to 23:59:59 on 30 November 2021 through the facility's monitoring system, supplemented by on-site patrol surveys. Three types of data were obtained: (1) static facility data, including the layout of internal facilities, entrances and exits, parking space arrangements, and internal road network diagrams; (2) vehicle entry–exit records at the facility level, including license plate numbers, entry and exit times, and entry and exit locations; and (3) space-level entry–exit records, including license plate numbers, space identifiers, and timestamps for entering and leaving individual spaces. The internal structure of the parking facility is shown in Figure 5.

Figure 5. Parking Lot Internal Structure.

Before applying the model, the raw datasets were processed to ensure consistency and reliability. First, incomplete or erroneous records, such as missing license plates, undefined space identifiers, or invalid timestamps, were removed. Second, abnormal timestamps, such as negative or excessively long durations, were corrected or discarded based on logical constraints. Third, license plate recognition errors were corrected by cross-referencing with the on-site patrol survey records. Fourth, inconsistencies in space identifiers were resolved, and duplicate records were eliminated. Finally, datasets (2) and (3) were matched to reconstruct complete parking chains for each vehicle.

To establish the temporal segmentation used as an input to the differential pricing models, vehicle inflow, vehicle outflow, and saturation were analyzed for both weekdays and weekends, as shown in Figure 6. All three indicators exhibited distinct daily variation with multiple peaks, but the timing and intensity of these peaks differed between weekdays and weekends.

For vehicle inflow as Figure 6, weekday demand showed three peaks at approximately 09:00, 12:00, and 18:30, corresponding to commuting and meal periods. Weekend peaks occurred later, at around 12:00, 17:00–18:00, and 19:00, reflecting leisure trips and later opening hours of commercial facilities. The highest weekend inflow reached about 250 vehicles per 15 min, roughly twice the weekday peak.

For vehicle outflow as Figure 6, weekends displayed a two-peak pattern with the main departure peak between 20:30 and 22:00, occurring later than the main inflow peak and indicating prolonged parking durations. Weekday outflows followed a single pronounced peak ending earlier in the evening, consistent with commuting patterns.

Saturation levels confirmed these differences in Figure 6. Weekend peaks exceeded a saturation ratio of 1.1, indicating illegal parking and reduced internal circulation efficiency, while weekday peaks remained around 0.6 and lasted for shorter periods.

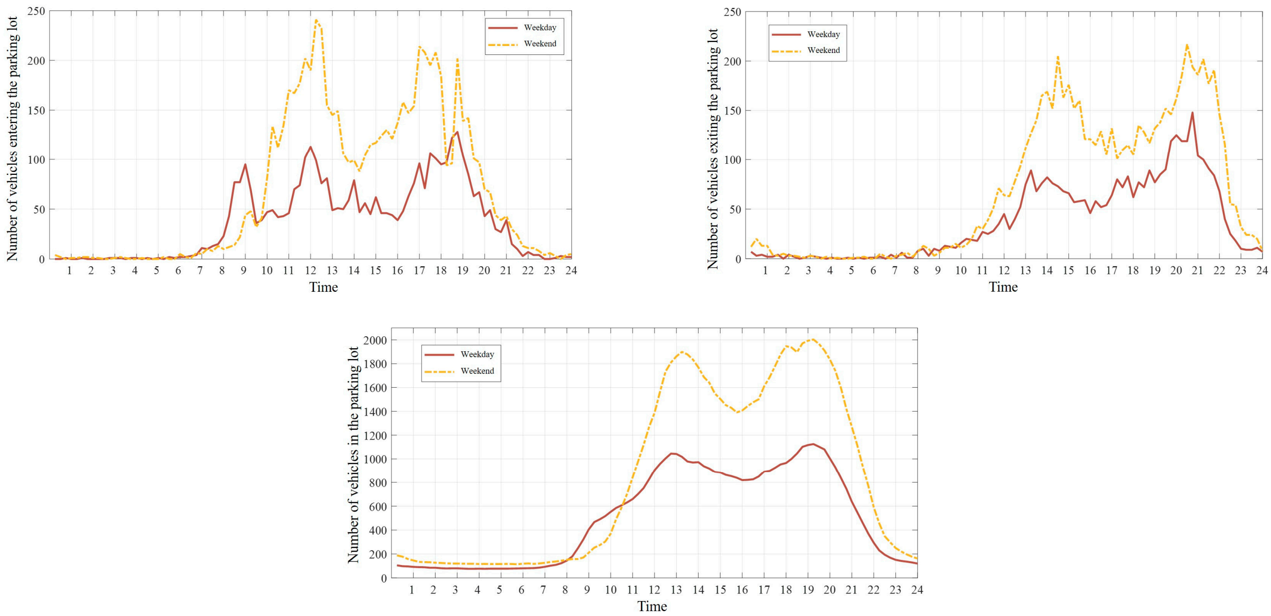


Figure 6. Statistics on vehicle parking.

Based on the processed dataset, three key indicators were analyzed to determine time segmentation: vehicle inflow, vehicle outflow, and saturation. According these indicators, the day was divided into eight time periods to capture distinct demand regimes: (1) 07:00–09:00, morning commuting peak; (2) 09:00–11:00, post-commuting shoulder period; (3) 11:00–13:00, midday peak; (4) 13:00–16:00, afternoon shoulder period; (5) 16:00–17:00, pre-evening peak; (6) 17:00–19:00, evening dining peak; (7) 19:00–21:00, post-dining leisure peak; and (8) 21:00–24:00, late-night off-peak. This segmentation ensures that subsequent optimization reflects the temporal heterogeneity of parking demand.

For this case study, the following parameter and constraint settings apply uniformly to all optimization models, as shown in Table 4:

Table 4. Parameter and Constraint Settings.

Variable	Define	Value	Explain
T	Total number of time periods	$T = 8$	see above
$period$	Set of time period (hour)	$period = \{0–9,\ 9–11,\ 11–13,\ 13–16,\ 16–20,\ 20–21,\ 21–22,\ 22–24\}$	
N	Total number of parking spaces	1152	Total number of parking spaces in the Kingmo Complex
C	Total daily parking demand	35,705	Survey demand for the day
T_{limit}	Maximum chargeable duration for a single parking event	6	In accordance with the parking fee regulations of Nanjing for public commercial parking facilities
$p^t_{zuplimit}$	Upper bound of the unit parking fee in the zone z at the time t	20	based on Nanjing's maximum temporary on-street parking rate and consistent with parking fees in several other major Chinese cities, such as Shanghai, Beijing, where rates have reached or exceeded this level
$p^t_{z\,floorlimit}$	Lower bound of the unit parking fee in the zone z at the time t	3	corresponding to the current parking unit fee for parking lots

Table 4. *Cont.*

Variable	Define	Value	Explain
bp	Current parking unit fee for parking lots	3	corresponding to the current parking unit fee for parking lots
P_{limit}	Maximum total price per parking event	$6 \times 20 = 120$	$T_{limit} \cdot p^{t}_{zuplimit}$

4.2. Mixed Logit Model Estimation and Analysis

4.2.1. Data Collection and Sample Description

To estimate the mixed logit model for parking space choice, a stated preference (SP) survey was conducted. The survey design and implementation followed established practices in discrete choice modeling, ensuring that the collected data could capture user preferences with respect to parking space attributes under different hypothetical scenarios.

The SP questionnaire consisted of three sections. The first section gathered respondents' socio-demographic information, including gender, age, occupation, household income, and driving experience. The second section presented a series of hypothetical choice scenarios that were designed separately for two trip purposes, namely shopping and leisure, and commuting, in order to capture differences in urgency and behavioral flexibility. In each scenario, respondents were asked to choose between parking spaces with varying attributes, including parking fee, type of parking space, search time, walking time, and the likely parking duration expected by the user under the given trip purpose.

The survey covered both weekdays and weekends, ensuring that temporal variations in parking demand were reflected in the dataset. A total of 410 questionnaires were distributed, and 384 valid responses were retained after data cleaning, yielding a valid response rate of 93.7%. Responses were deemed invalid if key information was missing, if the choice tasks were left incomplete, or if inconsistent answers were provided across scenarios. Among the valid responses, 53.1% were collected on weekdays and 46.9% on weekends, providing a balanced representation of different temporal demand conditions.

In terms of respondent characteristics, 53.96% of participants were male and 46.04% were female; the majority were aged between 25 and 49 years; and the predominant trip purposes were shopping, dining, and leisure. Parking duration patterns differed markedly between trip purposes. For shopping and leisure trips, 82.66% of respondents parked for between 1 and 4 h, consistent with the typical time requirements of such activities. For commuting trips, 72.81% of respondents parked for more than 4 h, reflecting the longer durations associated with work-related stays. These descriptive statistics are summarized in Table 5, which also presents the distribution of other socio-demographic variables and parking behaviors.

Table 5. Summary of Questionnaire Results.

Survey Project	Options	Proportion
gender	Male	53.96%
	Female	46.04%
Age	18–25 years old	16.27%
	25–35 years old	28.48%
	36–49 years old	28.48%
	50 years old and above	26.77%
Monthly Income	Under 3000 yuan	17.34%
	3000–5000 yuan	19.27%
	5000–10,000 yuan	31.69%
	Over 10,000 yuan	31.39%

Table 5. *Cont.*

Survey Project	Options	Proportion
Driving Experience	Under 1 year	16.06%
	1–3 years	11.35%
	3–5 years	11.13%
	5–10 years	18.20%
	Over 10 years	43.25%
Parking duration when the purpose of the trip is leisure	15 min or less	5.35%
	15 min to 1 h	8.99%
	1–2 h	42.83%
	2–4 h	39.83%
	Over 4 h	3.00%
Parking duration when the purpose of the trip is commuting	15 min or less	5.14%
	15 min to 1 h	6.42%
	1–2 h	9.21%
	2–4 h	6.42%
	Over 4 h	72.81%

The final sample thus provided a robust and representative basis for the estimation of the mixed logit model, enabling the analysis of how parking space attributes influence user choice behavior under different pricing and facility configurations.

4.2.2. Model Estimation Results

The mixed logit model was estimated separately for the two trip purposes, leisure and commuting, in order to capture heterogeneity in parking space choice behavior. Table 6 presents the estimation results, including the mean coefficient, standard deviation of the random parameter distribution, and significance level for each explanatory variable. The random parameters were assumed to follow a normal distribution, allowing for individual-specific taste variations.

Table 6. Parking space selection utility estimation results of the cross-term ML model.

Value	Commuting		Leisure	
	Coefficient	*p*-Value	Coefficient	*p*-Value
B_Fee	−0.158	0	−0.348	0
B_Fee std.dev.	0.116	0	0.374	0
B_Mec	−0.68	0	−0.858	0
B_ Mec std.dev.	1.41	0	1.42	0
B_SeaTime	−0.104	0	−0.082	0.001
B_SeaTime std.dev.	0.125	0.002	0.141	0.004
B_WalkTime	−0.181	0	−0.27	0
B_WalkTime std.dev.	-	-	0.266	0.018
B_Age1_Mec	-	-	−0.591	0.002
B_Age2_Fee	0.031	0.008	0.085	0.018
B_Age1_Mec	-	-	−0.591	0.002
B_Age2_Fee	0.031	0.008	0.085	0.018
B_Age2_SeaTime	0.036	0.01	-	-
B_Income1_Fee	-	-	−0.071	0.038
B_Gender_Fee	0.034	0.004	-	-
B_Gender_Mec	-	-	0.479	0.021
B_Gender_WalkTime	0.048	0.043	0.116	0.004
Total observations	2802(467 × 6)			
Parameters	11		13	

Table 6. *Cont.*

Value	Commuting		Leisure	
	Coefficient	*p*-Value	Coefficient	*p*-Value
Null model likelihood estimate	−3176.751		−3078.312	
Model likelihood estimate	−2573.509		−2576.351	
Model fit	0.190		0.163	

The mixed logit model with interaction terms was estimated separately for commuting and leisure to capture preference heterogeneity across trip purposes. Respondents were grouped by age into eighteen to thirty-five and above thirty-five, and by income into below five thousand CNY per month and above five thousand CNY per month. Table 6 reports the estimated coefficients, standard errors, and significance levels for all main effects and interaction terms. The results show that the two trip purposes yield different sets of significant interactions, which indicates distinct preference structures. For attributes that enter the utility with interactions, the effective coefficient follows the corresponding calculation rule. For example, the coefficient on the parking fee differs by age group. Drivers above thirty-five years old face a different fee coefficient from drivers aged eighteen to thirty-five, as implied by the age-fee interaction.

For the commuting scenario, heterogeneity appears only along gender and age. Both B_Age2_Fee and B_Age2_SeaTime are positive, which indicates that drivers above thirty-five years old are more willing to accept higher parking fees and longer search time. This aligns with the interpretation that older commuters usually have more structured time management and greater wealth accumulation, and thus are less pressed to minimize in-facility search. Gender also affects the sensitivity to the fee and walking time. Male drivers accept higher fees and longer walking times compared with female drivers. In contrast, in the shopping and leisure scenario, the fee parameter does not vary by gender, which implies similar price sensitivity between male and female users in that scenario.

For the leisure scenario, the set of significant parameters is richer than in commuting, which suggests stronger heterogeneity in preferences across user groups. The coefficient B_Age1_Mec equals −0.591, which means that drivers under thirty-five are less willing to choose mechanical parking spaces than older users, likely due to lower experience with mechanical equipment and higher perceived inconvenience. The coefficient B_Age2_Fee is positive, which indicates that drivers above thirty-five are more tolerant of higher fees when parking for leisure activities. Income also matters in this scenario. The coefficient B_Income1_Fee equals −0.071, which shows that lower-income users are more price sensitive and are less likely to choose higher fee spaces than higher-income users. The coefficient B_Gender_Mec equals 0.479, which means that male drivers are more accepting of mechanical spaces than female drivers. Male users also show greater tolerance for longer walking times, and this pattern holds for both trip purposes.

In terms of model fit, the mixed logit with interaction terms achieves better goodness of fit than the multinomial logit model and the standard mixed logit without interactions. The interaction specification captures user heterogeneity more accurately, thereby supporting more precise parking space allocation in the lower-level model. The estimated parameters are applied in the lower-level parking space allocation, which in turn determines the spatio-temporal occupancy rates used by the upper-level optimization in the administered and market-based pricing models.

4.3. Administered Differential Pricing Model Results

4.3.1. Unique Solution Set of Zoning Parameters

Based on the parking space attribute dataset of the Kingmo Complex parking facility, the spatial clustering method developed in Section 3.2 was applied to generate a unique solution set of key zoning parameters. Each solution in this set corresponds to a distinct combination of zoning parameters that satisfies the clustering constraints and achieves an optimal trade-off between two zoning performance indicators: the Ratio of External and Internal Distances (REID) and the Parking Distribution Entropy (PDE). The complete unique solution set is presented in Table 7.

Table 7. Unique solution set of key zoning parameters for the Kingmo Complex.

No.	$dist_{in}$	K	w	α	*ratio*	PDE-REID	REID	PDE
1	1	6	0.3	0.3	0.1	0.1442	0.7618	0.6176
2	1	6	0.4	0.4	0.1	0.0616	0.9440	0.8824
3	2	6	0.5	0.5	0.1	0.0015	0.9757	0.9742
4	2	9	0.5	0.4	0.1	0.0529	0.4932	0.4403
5	3	6	0.4	0.4	0.1	0.0512	0.9558	0.9046
6	3	6	0.4	0.6	0.1	0.0000	1.0000	1.0000
7	3	6	0.5	0.4	0.1	0.1460	0.8943	0.7482
8	3	6	0.5	0.5	0.1	0.0197	0.9956	0.9760
9	3	8	0.3	0.4	0.1	0.1224	0.2240	0.1016
10	3	8	0.5	0.3	0.1	0.1014	0.6319	0.5305
11	3	8	0.5	0.5	0.1	−0.0051	0.3939	0.3990
12	3	9	0.5	0.4	0.1	0.0549	0.5191	0.4642
13	3	9	0.5	0.4	0.2	0.0552	0.5138	0.4586
14	3	10	0.5	0.7	0.1	0.0000	0.0000	0.0000

By varying the relative weights assigned to REID and PDE in the optimization process, a series of non-dominated solutions was obtained. These solutions form the Pareto front shown in Figure 7, where different colored circles represent non-dominated solutions obtained during optimization. The figure illustrates the inherent trade-off between the two indicators: improving one generally leads to a deterioration in the other.

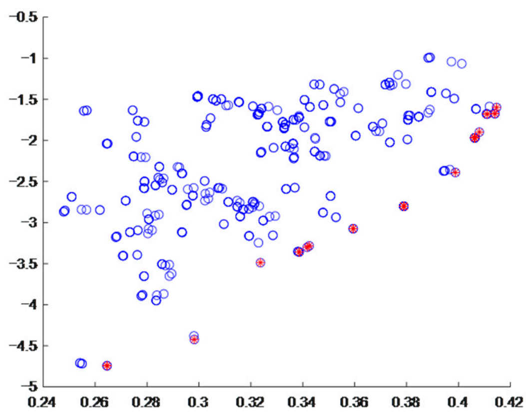


Figure 7. Pareto Front for the Kingmo Complex Parking Lot.

This unique solution set will serve as one of the inputs to the differential pricing model settings to determine the optimal spatial zoning configuration and the corresponding time-dependent zone-level parking fees.

4.3.2. Administered Differential Pricing Results

The administered differential pricing model described in Section 3.3.4 was solved by treating both the zoning parameters and the parking fees as joint decision variables. The model took as inputs the unique solution set of key zoning parameters from Section 4.3.1, the dataset of the Kingmo Complex parking facility, the constraint and parameter settings specified in Section 4.1, and the mixed logit estimation results from Section 4.2. The optimization aimed to identify the combination of zoning configuration and time-dependent pricing that achieves the best balance between the two upper-level objectives.

The optimization was performed for 500 iterations using the improved multi-objective PSO-GWO algorithm. The convergence process for the administered differential pricing model is shown in Figure 8, illustrating stable convergence under both weekdays and weekend demand conditions.

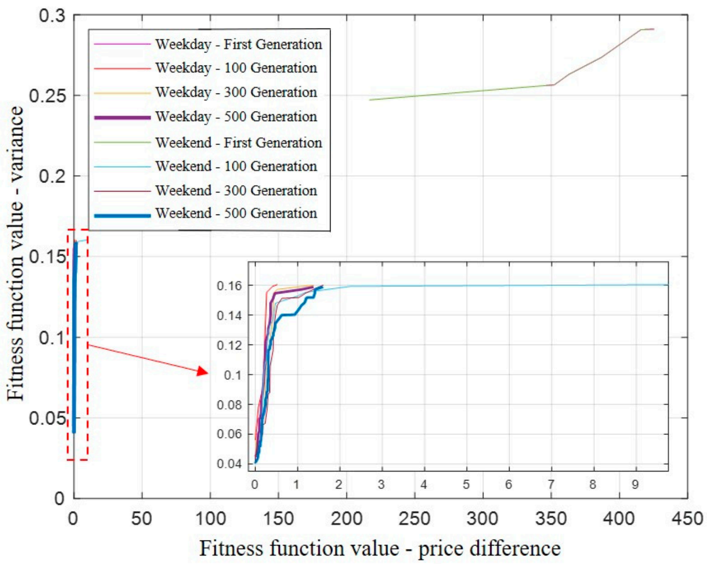


Figure 8. Convergence Process of Administered Differential Pricing Model.

All optimization results from the iterative process were aggregated to construct the Pareto front with respect to the two objectives, total parking revenue and the sum of spatial–temporal occupancy variances. Both objectives were normalized to remove scale effects. For the default equal-weight case, the optimal solution was selected by computing the difference between the normalized revenue and the normalized variance sum. The solution with the maximum difference was identified as the most balanced trade-off point on the Pareto front.

For the equal-weight case, the optimal solution corresponds to the zoning parameter combination $dist_{in} = 3,\ K = 6,\ w = 0.5,\ \alpha = 0.4, ratio = 0.1$. The resulting spatial zoning configuration is illustrated in Figure 9. The optimized hourly parking fees for each zone across the eight time periods are reported in Table 8.

Distinct spatial and temporal differences can be observed from the pricing results, as shown in Table 8. Zone 5 shows the highest prices during peak hours, reflecting persistent excess demand near the core entrances. In contrast, the prices in other zones remain relatively stable, indicating that large-scale adjustments are not required. These findings indicate that the optimization framework effectively achieves the upper-level objective of balancing spatio-temporal occupancy primarily through targeted adjustments in high-demand areas, while maintaining overall price stability across the facility.

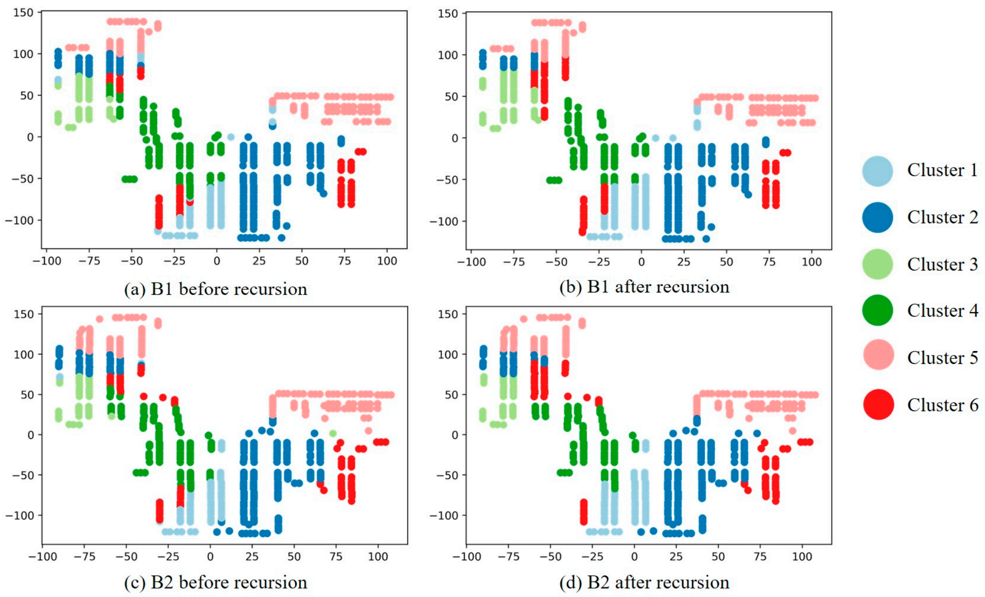


Figure 9. Optimal Spatial Zoning Configuration for the Kingmo Complex.

Table 8. Administered Differential Pricing Strategy for the Kingmo Complex (CNY/hour).

Weekdays							
Time Period	**Time**	**Zone 1**	**Zone 2**	**Zone 3**	**Zone 4**	**Zone 5**	**Zone 6**
1	00:00–09:00	3.00	3.00	3.00	3.00	3.00	3.00
2	09:00–11:00	3.00	3.00	3.00	3.00	3.00	3.00
3	11:00–13:00	3.00	3.00	3.00	3.00	3.46	3.00
4	13:00–16:00	3.00	3.00	3.00	3.00	3.00	3.00
5	16:00–20:00	3.00	3.00	3.00	3.00	3.00	3.00
6	20:00–21:00	3.00	3.00	3.00	3.00	3.00	3.00
7	21:00–22:00	3.00	3.00	3.00	3.00	3.00	3.00
8	22:00–24:00	3.00	3.00	3.00	3.00	3.00	3.00
Weekend							
Time Period	**Time**	**Zone 1**	**Zone 2**	**Zone 3**	**Zone 4**	**Zone 5**	**Zone 6**
1	00:00–09:00	3.00	3.00	3.00	3.00	3.00	3.00
2	09:00–11:00	3.00	3.00	3.00	3.00	3.00	3.00
3	11:00–13:00	3.00	3.00	3.00	3.00	3.00	3.00
4	13:00–16:00	3.00	3.00	3.00	3.00	3.00	3.00
5	16:00–20:00	3.00	3.00	3.00	3.00	3.00	3.00
6	20:00–21:00	3.00	3.00	3.00	3.00	3.43	3.00
7	21:00–22:00	3.00	3.00	3.00	3.00	3.00	3.00
8	22:00–24:00	3.00	3.00	3.00	3.00	3.00	3.00

The optimized fee schedule reflects the administered pricing principle: modest increases in high-demand periods and more accessible zones, coupled with lower fees in low-demand periods or peripheral zones to redistribute demand, while maintaining compliance with regulatory price bounds and the maximum chargeable duration.

In addition to pricing results, the spatial characteristics of each zone in the optimal configuration are summarized in Table 9.

Table 9. Attributes of Parking Spaces in Each Zone.

Cluster	Number	Convenience	Accessibility	Floor	Number of Mechanical Spaces
1	156	36.80	200.27	1.51	0
2	326	40.14	206.78	1.52	0
3	95	34.69	176.98	1.40	0
4	192	31.89	156.97	1.50	0
5	213	77.05	223.88	1.54	0
6	170	27.07	206.58	1.48	0

The clusters satisfy connectivity constraints and exhibit relatively balanced sizes. Cluster 1 and Cluster 4 share the same elevator lobby and are located on the same side of the entrances; however, Cluster 4 has the highest accessibility score due to its more compact footprint. Cluster 2 is the largest and benefits from adjacency to both an elevator and an entrance, although its average accessibility and convenience scores are moderate. Cluster 3 contains the fewest parking spaces, evenly distributed across two levels around the B1 and B2 corridors. Cluster 5 has the lowest accessibility and convenience scores, being located in a relatively isolated section without direct elevator access. Cluster 6 ranks highest in convenience, owing to its smaller size and proximity to multiple elevator lobbies. Overall, the clustering results in Table 9 reflect meaningful spatial heterogeneity: zones near entrances and elevator lobbies consistently show higher accessibility and convenience, while peripheral areas exhibit lower scores. Such differentiation provides a rational basis for implementing zonal pricing, as it directly links prices to observable spatial attributes. This differentiated pricing supports a more balanced utilization of parking resources across both space and time, consistent with the upper-level objectives of the administered pricing model.

This differentiated pricing supports a more balanced utilization of parking resources across both space and time, consistent with the upper-level objectives of the administered pricing model.

4.3.3. Result Analysis

Figure 10 compares the pre- and post-optimization performance in terms of the sum of spatial–temporal occupancy variances and total parking revenue.

On weekdays, the total spatial–temporal occupancy variance over the day decreases by 67.17%, while the average hourly price increases by only 0.46 CNY. Daily revenue rises from 36,142 CNY to 36,483 CNY, a gain of 0.94%. On weekends, the variance decreases by 69.21%, with an average hourly price increase of only 0.43 CNY, and daily revenue rises from 42,999 CNY to 43,434 CNY, a gain of 1.01%. The weekend scenario achieves slightly greater variance reduction with similar or smaller price adjustments, although a few time periods exhibit marginally higher variance after optimization due to the model's focus on minimizing the total daily variance.

An analysis of the optimization results by time period, as informed by the spatial–temporal occupancy rates in Table 10, reveals that:

Time period 1 (00:00–09:00): Demand is low across all zones, and occupancy rates are uniformly low. Any single vehicle's choice can noticeably affect the variance, but the model's daily variance minimization goal means that in this period, variance is adjusted primarily through the lower-level dynamic allocation model without changing prices.

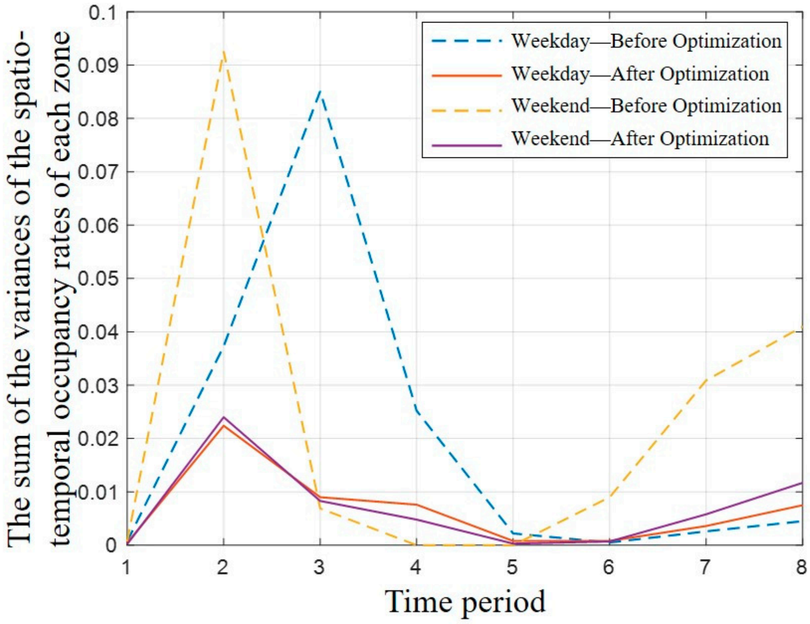


Figure 10. Comparison of Administered Differential Pricing Model of Kingmo Complex Parking Lot before and After Optimization.

Time period 2 (09:00–11:00): Variance decreases sharply in both scenarios, by 40.06% on weekdays and 74.12% on weekends. Price changes are minimal, with demand redistribution achieved mainly by reallocating users from the most convenient and accessible zones (3, 4, and 6) to zones 1, 2, and 5.

Time periods 3–5 (11:00–20:00): These intervals include the daily demand peaks. On weekdays, variance reduction is substantial, but on weekends, the effect is smaller due to uniformly high occupancy rates across all zones. In weekday period 3, zone 5′s high demand leads to the highest observed price, with effects persisting into later periods. High saturation limits the scope for further variance reduction.

Time periods 6–8 (20:00–24:00): Demand declines from the peak. On weekdays, variance reduction is modest, whereas on weekends the effect is stronger due to the higher volume of evening leisure trips. In the weekend scenario, both dynamic allocation and small price changes help redistribute demand, improving occupancy balance.

Overall, the Kingmo Complex case demonstrates that a well-designed zoning configuration, combined with small and strategically targeted price adjustments, can markedly reduce spatial–temporal occupancy variance while keeping price increases minimal. The weekend scenario shows greater relative improvement in variance reduction, partly due to the more flexible trip purposes and later peaking demand patterns.

Table 10. Spatial–Temporal Occupancy Rate Table of Each Time Period of Kingmo Complex Parking Lot.

Time Period	Time	Weekdays-Zone 1		Weekdays-Zone 2		Weekdays-Zone 3		Weekdays-Zone 4		Weekdays-Zone 5		Weekdays-Zone 6		Spatial–Temporal Occupancy Rate Comparison	
		Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization
1	00:00–09:00	0.0070	0.0168	0.0107	0.0182	0.0125	0.0000	0.0722	0.0179	0.0016	0.0558	0.0265	0.0089	0.0007	0.0004
2	09:00–11:00	0.3016	0.4025	0.3359	0.4920	0.5397	0.1258	0.6514	0.3933	0.1104	0.4019	0.4796	0.1581	0.0373	0.0224
3	11:00–13:00	0.8387	0.8239	0.7581	0.8112	0.9768	0.7633	0.9479	0.7108	0.1939	0.7023	0.8896	0.5647	0.0851	0.0090
4	13:00–16:00	0.9922	0.9358	0.9833	0.9152	0.9987	0.9667	0.9962	0.8649	0.6038	0.9976	0.9920	0.7538	0.0252	0.0076
5	16:00–20:00	0.9976	0.9766	0.9912	0.9716	0.9989	0.9801	0.9971	0.9549	0.8827	0.9992	0.9967	0.9179	0.0022	0.0008
6	20:00–21:00	0.9494	0.9643	0.9546	0.9519	0.9832	0.9372	0.9850	0.9275	0.9263	0.9920	0.9734	0.9155	0.0005	0.0008
7	21:00–22:00	0.8034	0.8543	0.8156	0.8247	0.8793	0.7821	0.9168	0.8339	0.7809	0.9480	0.8259	0.7908	0.0026	0.0036
8	22:00–24:00	0.5886	0.6154	0.5784	0.6034	0.5918	0.5144	0.7106	0.6012	0.5025	0.7664	0.6115	0.5499	0.0045	0.0075
Time Period	Time	Weekend-Zone 1		Weekend-Zone 2		Weekend-Zone 3		Weekend-Zone 4		Weekend-Zone 5		Weekend-Zone6		Patio-Temporal Occupancy Rate Comparison	
1	00:00–09:00	0.0000	0.0164	0.0000	0.0178	0.0000	0.0000	0.0000	0.0176	0.0657	0.0460	0.0000	0.0088	0.0007	0.0002
2	09:00–11:00	0.1912	0.4186	0.2125	0.5048	0.0728	0.1351	0.1447	0.4026	0.8661	0.4461	0.0591	0.1682	0.0926	0.0240
3	11:00–13:00	0.9184	0.8341	0.9106	0.8261	0.8795	0.7903	0.8097	0.7400	0.9981	0.7588	0.7660	0.5859	0.0069	0.0083
4	13:00–16:00	0.9970	0.9397	0.9951	0.9449	0.9955	0.9672	0.9942	0.9287	0.9991	0.9979	0.9905	0.7970	0.0000	0.0048
5	16:00–20:00	0.9935	0.9850	0.9910	0.9823	0.9878	0.9927	0.9845	0.9821	0.9993	0.9992	0.9809	0.9463	0.0000	0.0003
6	20:00–21:00	0.8390	0.9457	0.8516	0.9701	0.7437	0.9263	0.7716	0.9424	0.9917	0.9979	0.7405	0.9378	0.0090	0.0007
7	21:00–22:00	0.6224	0.8083	0.6230	0.8891	0.4662	0.7680	0.5056	0.7992	0.9477	0.9722	0.5161	0.8029	0.0309	0.0058
8	22:00–24:00	0.3262	0.5562	0.3112	0.6356	0.2178	0.5565	0.2531	0.5441	0.7625	0.8106	0.2719	0.5234	0.0410	0.0117

4.4. Market-Based Differential Pricing Model Results

4.4.1. Market-Based Differential Pricing Results

The market-based differential pricing model presented in Section 3.3.5 was applied to the Kingmo Complex parking facility, with both the spatial zoning configuration and the parking fees optimized simultaneously. The optimization utilized the unique zoning parameter solutions from Section 4.3.1, combined with the facility's attribute dataset, the parameter and constraint settings outlined in Section 4.1, and the mixed logit estimates from Section 4.2.

The problem was solved using the improved multi-objective PSO-GWO algorithm, set to run for 500 iterations. The convergence curves for both weekdays and weekend scenarios are shown in Figure 11, indicating stable performance of the algorithm under the market-based objective structure.

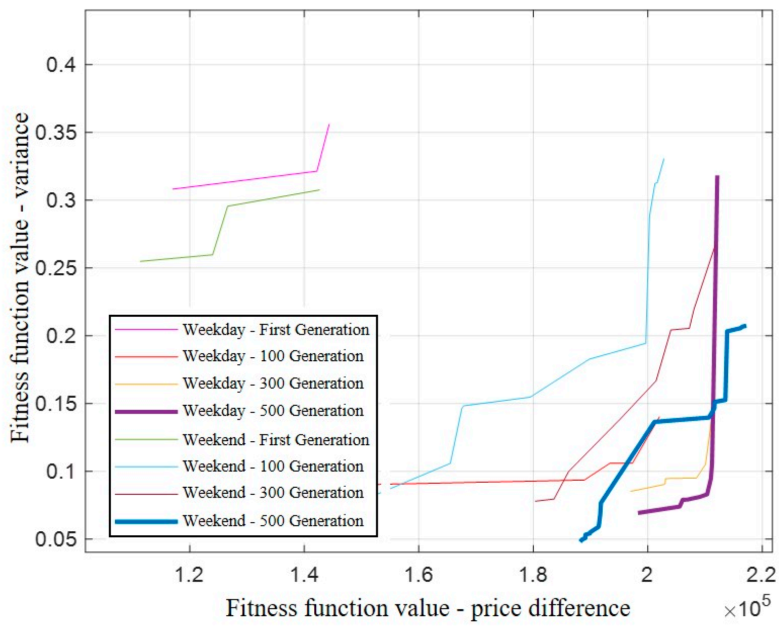


Figure 11. Convergence Process of Market-based Differential Pricing Model.

From the resulting solution set, the Pareto-optimal configurations were identified with respect to total parking revenue and the sum of spatial–temporal occupancy variances. For the equal-weight case (0.5 for each objective), the selected optimal configuration matched the spatial zoning obtained in Section 4.3.2 (Figure 9), ensuring consistency across pricing strategies.

The optimized market-based fee schedules for weekdays and weekends are summarized in Table 11. The weekday schedule features more frequent and larger price adjustments than the weekend schedule, reflecting the predominance of commuting demand during weekdays and its relatively low price elasticity. In contrast, weekend adjustments are more limited in the early time period, consistent with leisure-oriented trip patterns.

As reported in Table 11, Zone 5 exhibits the highest prices across both weekdays and weekends, reaching 20 CNY/h during peak periods, consistent with its persistent excess demand near the core entrances. In contrast, Zone 2 shows relatively stable prices across periods, reflecting its moderate accessibility and balanced occupancy levels. Temporal variation is also evident: during weekday peaks (09:00–11:00 and 16:00–20:00), multiple zones experience price increases, whereas weekend adjustments are concentrated in the midday period.

Table 11. Market-based Differential Pricing Strategy for the Kingmo Complex (CNY/h).

Weekdays							
Time Period	**Time**	**Zone 1**	**Zone 2**	**Zone 3**	**Zone 4**	**Zone 5**	**Zone 6**
1	00:00–09:00	20.00	0.00	20.00	20.00	20.00	20.00
2	09:00–11:00	0.00	20.00	20.00	0.00	20.00	20.00
3	11:00–13:00	20.00	20.00	20.00	17.22	20.00	0.00
4	13:00–16:00	20.00	20.00	20.00	19.85	0.00	20.00
5	16:00–20:00	20.00	0.00	20.00	20.00	20.00	20.00
6	20:00–21:00	0.00	20.00	0.00	20.00	20.00	20.00
7	21:00–22:00	20.00	20.00	20.00	20.00	0.00	19.49
8	22:00–24:00	20.00	20.00	20.00	20.00	20.00	20.00
Weekend							
Time Period	**Time**	**Zone 1**	**Zone 2**	**Zone 3**	**Zone 4**	**Zone 5**	**Zone 6**
1	00:00–09:00	0.00	0.00	20.00	19.85	20.00	20.00
2	09:00–11:00	20.00	0.00	0.00	0.00	20.00	20.00
3	11:00–13:00	20.00	20.00	20.00	0.00	0.00	0.00
4	13:00–16:00	20.00	20.00	19.93	0.00	0.00	20.00
5	16:00–20:00	0.00	0.00	20.00	20.00	20.00	0.00
6	20:00–21:00	0.00	20.00	20.00	20.00	0.00	20.00
7	21:00–22:00	20.00	20.00	20.00	0.00	0.00	0.00
8	22:00–24:00	20.00	20.00	20.00	20.00	0.00	20.00

Note: The current price for each zone and time period in the parking lot is 3 yuan/h.

Compared with the administered pricing strategy, the market-based strategy provides greater flexibility in price setting, resulting in more pronounced adjustments. Figures 12 and 13 further illustrate these results by comparing the optimized prices with the baseline uniform rate. The figures comparison highlights which zones and time periods deviate most strongly from the baseline, making the spatial and temporal differentiation more evident. These deviations are consistent with underlying demand structures: weekday peaks align with commuting flows characterized by lower price elasticity, while weekend adjustments correspond to leisure and entertainment activities with more flexible timing.

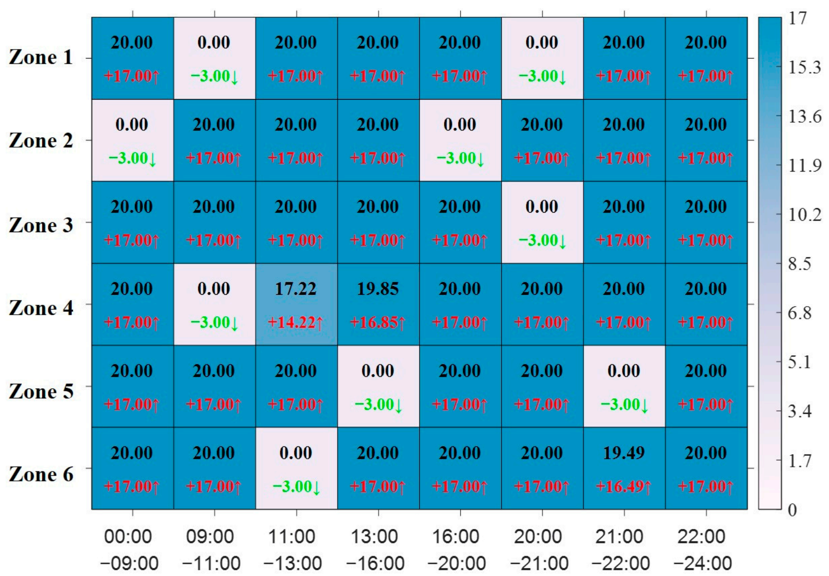


Figure 12. Changes in Weekday Market-based Differentiated Pricing Strategy.

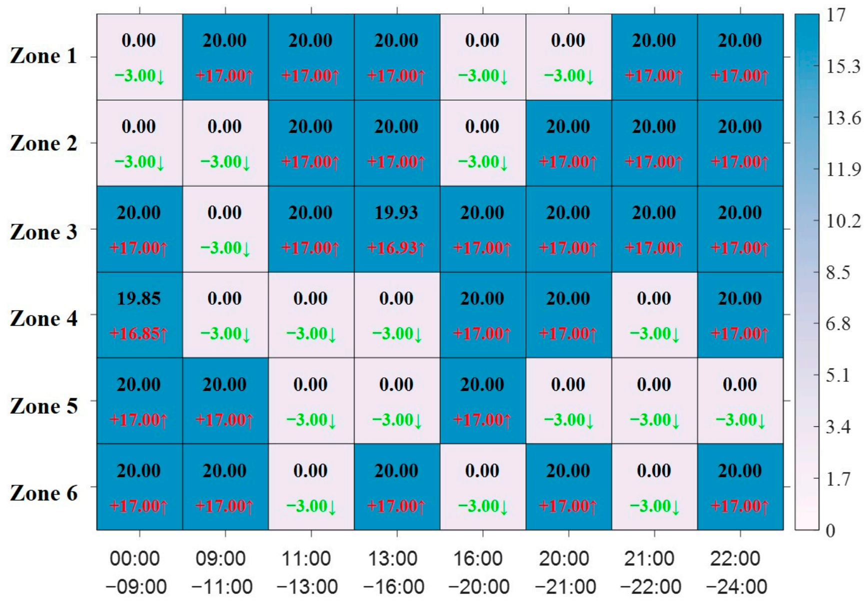


Figure 13. Changes in Weekend Market-based Differentiated Pricing Strategy.

Under weekday conditions, price increases are observed in a larger number of zones and time periods, particularly during the morning and evening peaks, reflecting the higher proportion of commuting demand. Under weekend conditions, fewer adjustments are made, and these are concentrated in the midday and evening periods, consistent with leisure and entertainment activity patterns. The contrast highlights how the market-based strategy adapts pricing to distinct temporal demand structures while maintaining spatial differentiation.

4.4.2. Result Analysis

The performance of the market-based differential pricing model under weekday and weekend conditions is summarized in Figure 14. The results show that, compared with the baseline uniform-rate policy, the optimized market-based strategy significantly improves the spatial balance of occupancy while achieving substantial revenue growth.

For weekdays, the total spatial–temporal occupancy variance is reduced by 43.15%, and the daily parking revenue increases from 36,273 CNY to 210,358 CNY, with 81.71% attributable to the price increase effect. Under weekend conditions, the variance is reduced by 70.23%, and revenue rises from 43,680 CNY to 189,087 CNY, with 76.90% of the increase coming from higher prices. These results confirm the model's capacity to meet both upper-level objectives in the market-based scenario, although with a stronger revenue impact than in the administered pricing strategy.

An analysis of the optimization results by time period, based on the spatial–temporal occupancy rates in Table 12, reveals that the following:

Time period 1 (00:00–09:00): Occupancy variance is reduced in both weekday and weekend conditions. The greater pricing flexibility of the market-based model allows larger fee changes than in the administered strategy, although the immediate impact within this low-demand period remains limited. Most of the benefit comes from influencing user choices in subsequent periods.

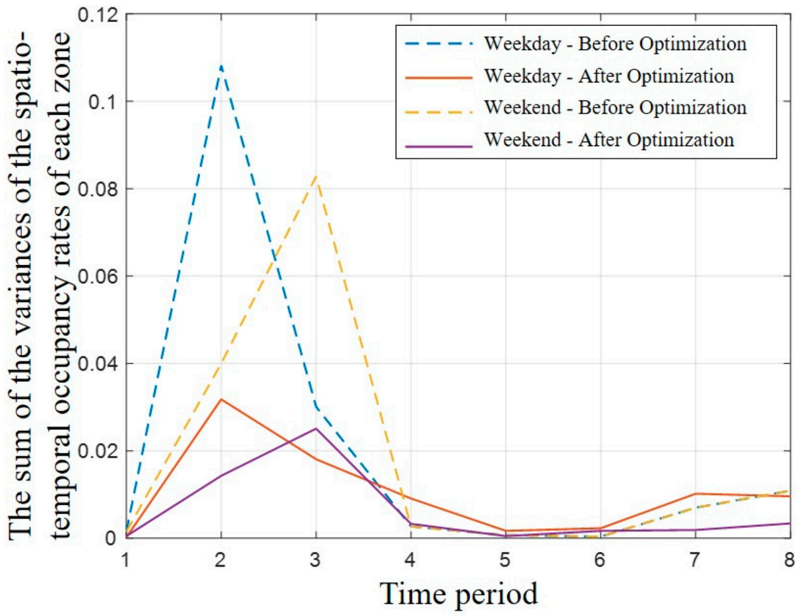


Figure 14. Comparison of Market-based Differential Pricing Model of Kingmo Complex Parking Lot before and After Optimization.

Time period 2 (09:00–11:00): Variance reduction is substantial, with weekend improvements exceeding weekday levels. Significant fee differences redirect demand from high-accessibility zones to peripheral zones, leveraging the absence of constraints on price changes.

Time periods 3–5 (11:00–20:00): These periods correspond to the main demand peaks. In the weekday scenario, strong differentiation between zones delivers notable revenue gains while still reducing variance. In the weekend scenario, high saturation limits further variance improvement, but upper-bound pricing in popular zones sustains high revenue.

Time periods 6–7 (20:00–22:00): The weekend scenario achieves more pronounced variance reduction because evening leisure demand remains strong, allowing redistribution between zones. In the weekday scenario, demand falls earlier, reducing the scope for redistribution.

Time period 8 (22:00–24:00): Price adjustments are mainly observed in the weekend scenario, targeting late-night demand to maximize revenue while maintaining occupancy balance. On weekdays, demand during this period is too low to warrant significant changes.

Overall, the market-based differential pricing strategy demonstrates greater capacity than the administered approach to adjust fees in response to demand variations, achieving substantial revenue gains while improving or maintaining occupancy balance, particularly during periods of high and uneven demand.

Table 12. Spatial–Temporal Occupancy Rate Table of Each Time Period of Kingmo Complex Parking Lot.

Time Period	Time	Weekdays-Zone1		Weekdays-Zone 2		Weekdays-Zone 3		Weekdays-Zone 4		Weekdays-Zone 5		Weekdays-Zone 6		Spatial–Temporal Occupancy Rate Comparison	
		Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization	Before Optimization	After Optimization
1	00:00–09:00	0.0056	0.0122	0.0027	0.0163	0.0000	0.0000	0.0042	0.0151	0.0984	0.0286	0.0010	0.0426	0.0015	0.0002
2	09:00–11:00	0.4066	0.4819	0.6535	0.5678	0.3070	0.1863	0.0958	0.4258	0.1816	0.1231	0.4505	0.2470	0.0400	0.0318
3	11:00–13:00	0.7954	0.8220	0.7959	0.7924	0.7133	0.7148	0.5944	0.6620	0.9966	0.8327	0.1520	0.4770	0.0828	0.0181
4	13:00–16:00	0.9620	0.9236	0.9403	0.8820	0.9699	0.9690	0.9041	0.8355	0.9993	0.9972	0.8541	0.7362	0.0027	0.0091
5	16:00–20:00	0.9824	0.9856	0.9740	0.9554	0.9839	0.9970	0.9468	0.9768	0.9989	0.9975	0.9296	0.8890	0.0007	0.0017
6	20:00–21:00	0.9590	0.9828	0.9693	0.9012	0.9531	0.9825	0.9553	0.9774	0.9970	0.9685	0.9429	0.8742	0.0004	0.0023
7	21:00–22:00	0.8120	0.9258	0.8207	0.7226	0.7449	0.8890	0.7607	0.9197	0.9764	0.8441	0.7844	0.6931	0.0070	0.0102
8	22:00–24:00	0.5777	0.6128	0.5802	0.4885	0.5366	0.5871	0.5006	0.6549	0.7868	0.7469	0.5214	0.4974	0.0109	0.0096
Time Period	**Time**	**Weekend-Zone 1**		**Weekend-Zone 2**		**Weekend-Zone 3**		**Weekend-Zone 4**		**Weekend-Zone 5**		**Weekend-Zone6**		**Patio-Temporal Occupancy Rate Comparison**	
1	00:00–09:00	0.0050	0.0120	0.0025	0.0103	0.0000	0.0124	0.0037	0.0639	0.0975	0.0000	0.0008	0.0244	0.0015	0.0005
2	09:00–11:00	0.2854	0.4976	0.3722	0.2891	0.1083	0.4466	0.2258	0.6101	0.9886	0.3107	0.0942	0.4194	0.1103	0.0143
3	11:00–13:00	0.7777	0.8516	0.7782	0.5283	0.6833	0.8417	0.5680	0.8320	0.9961	0.9974	0.4673	0.7129	0.0342	0.0251
4	13:00–16:00	0.9393	0.9951	0.9141	0.8916	0.9648	0.9990	0.8116	0.9872	0.9971	0.8693	0.7464	0.9786	0.0093	0.0033
5	16:00–20:00	0.9859	0.9941	0.9798	0.9854	0.9920	0.9994	0.9591	0.9994	0.9991	0.9400	0.9390	0.9982	0.0005	0.0005
6	20:00–21:00	0.9479	0.8984	0.9595	0.9030	0.9383	0.9885	0.9364	0.9968	0.9966	0.9298	0.9589	0.9469	0.0005	0.0017
7	21:00–22:00	0.8184	0.8653	0.8596	0.8129	0.7222	0.8687	0.7932	0.8841	0.9845	0.7892	0.7719	0.7866	0.0082	0.0019
8	22:00–24:00	0.5814	0.6513	0.6207	0.6270	0.4897	0.5586	0.5563	0.5869	0.8334	0.5559	0.5186	0.4877	0.0152	0.0034

5. Conclusions

This research proposes and validates an integrated framework for spatial zoning and differential pricing in large commercial parking facilities. By combining mixed Logit modeling of user choice behavior with dual clustering–based zoning and bilevel pricing optimization, the approach effectively addresses spatial occupancy imbalances. Application to the Kingmo Complex demonstrates three key findings:

(i) incorporating user heterogeneity improves the accuracy of demand redistribution in both administered and market-based pricing scenarios;
(ii) the administered strategy achieves balanced utilization with minimal revenue fluctuation, reducing spatio-temporal occupancy variance by about 67% on weekdays and increasing revenue by only 1%, making it suitable where regulatory compliance and price stability are priorities;
(iii) the market-based strategy offers greater flexibility in fee adjustment, reducing variance by over 40% and maintaining or improving occupancy balance, while enabling significant revenue increases, especially during peak and uneven demand periods.

The framework's adaptability allows its application to other large-scale parking facilities with diverse demand patterns, including those with multiple floors and more complex spatial structures. Future research could extend validation across multiple complexes in different cities and policy contexts to further demonstrate generality, and refine behavioral models to incorporate additional factors such as perceived safety, site familiarity, and search frustration. In addition, the framework could be extended upstream by integrating demand forecasting into the pricing model, thereby providing facility managers with end-to-end decision support and enhancing its practical applicability.

From a practical perspective, implementation requires a sufficiently precise parking management system capable of parking space-level data acquisition and monitoring. Once zoning results are generated, facility managers should provide clear physical signage or guidance so that drivers can readily distinguish between pricing zones. The management system must also support parking space-level pricing assignment and updates, and integrate with sensing and payment devices to ensure billing accuracy. Finally, local user acceptance should be considered in policy design and evaluation.

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References

1. Zhou, X.; Lv, M.; Ji, Y.; Zhang, S.; Liu, Y. Pricing Curb Parking: Differentiated Parking Fees or Cash Rewards? *Transp. Policy* **2023**, *142*, 46–58. [CrossRef]
2. Mingardo, G.; Vermeulen, S.; Bornioli, A. Parking Pricing Strategies and Behaviour: Evidence from the Netherlands. *Transp. Res. Part A Policy Pract.* **2022**, *157*, 185–197. [CrossRef]
3. Mei, Z.; Feng, C.; Kong, L.; Zhang, L.; Chen, J. Assessment of Different Parking Pricing Strategies: A Simulation-Based Analysis. *Sustainability* **2020**, *12*, 2056. [CrossRef]

4. Wu, Y.; He, Q.-C.; Wang, X. Competitive Spatial Pricing for Urban Parking Systems: Network Structures and Asymmetric Information. *IISE Trans.* **2021**, *54*, 186–197. [CrossRef]
5. Zhang, Z.; Zhang, F.; Liu, W.; Yang, H. On the Service Differentiation for Parking Sharing. *Transp. Res. Part C Emerg. Technol.* **2025**, *170*, 104915. [CrossRef]
6. Geva, S.; Fulman, N.; Ben-Elia, E. Getting the Prices Right: Drivers' Cruising Choices in a Serious Parking Game. *Transp. Res. Part A Policy Pract.* **2022**, *165*, 54–75. [CrossRef]
7. Gomari, S.; Knoth, C.; Antoniou, C. Cluster Analysis of Parking Behaviour: A Case Study in Munich. *Transp. Res. Procedia* **2021**, *52*, 485–492. [CrossRef]
8. Tu, L.; Ma, Z.; Huang, B. Analysis and Prediction of Differential Parking Behaviors. In Proceedings of the 2019 IEEE Intl Conf on Dependable, Autonomic and Secure Computing, Intl Conf on Pervasive Intelligence and Computing, Intl Conf on Cloud and Big Data Computing, Intl Conf on Cyber Science and Technology Congress (DASC/PiCom/CBDCom/CyberSciTech), Fukuoka, Japan, 5–8 August 2019; IEEE: Piscataway, NJ, USA, 2019; pp. 44–49.
9. Jioudi, B.; Sabir, E.; Moutaouakkil, F.; Medromi, H. Congestion Awareness Meets Zone-Based Pricing Policies for Efficient Urban Parking. *IEEE Access* **2019**, *7*, 161510–161523. [CrossRef]
10. Fulman, N.; Benenson, I. Spatially-Explicit Toolset for Establishing and Assessing Heterogeneous Parking Prices in the Smart City. *ISPRS Ann. Photogramm. Remote Sens. Spat. Inf. Sci.* **2020**, *6*, 63–70. [CrossRef]
11. Yang, Y.; Chen, J.; Ye, J.; Chen, J.; Luo, Y. Joint Optimization of Facility Layout and Spatially Differential Parking Pricing for Parking Lots. *Transp. Res. Rec. J. Transp. Res. Board* **2023**, *2677*, 241–257. [CrossRef]
12. Tian, Z.; Yu, B.; Shi, B.; Zhang, M.; Yao, B. Parking Lot Pricing Optimization Strategy Considering Autonomous Vehicle User Choice Behavior. *J. Transp. Eng. Part A Syst.* **2024**, *150*, 04023140. [CrossRef]
13. Ye, Q.; Cai, Z.; Hu, S. Optimizing Urban Parking Pricing with a Dual Dynamic Evolution Model for Multimodal Networks. In Proceedings of the 2024 IEEE 27th International Conference on Intelligent Transportation Systems (ITSC), Edmonton, AB, Canada, 24 September 2024; IEEE: Edmonton, AB, Canada, 2024; pp. 2812–2817.
14. Shoup, D.C. Cruising for Parking. *Transp. Policy* **2006**, *13*, 479–486. [CrossRef]
15. Soppert, M.; Steinhardt, C.; Müller, C.; Gönsch, J. Static Pricing Optimization in Shared Mobility Systems Under the Consideration of Network Effects. *Inf. Syst. Econ. eJ.* **2020**, *28*, 1. [CrossRef]
16. Bayih, S.H.; Tilahun, S.L. Dynamic Vehicle Parking Pricing: A Bilevel Optimization Approach. *Oper. Res. Int. J.* **2025**, *25*, 21. [CrossRef]
17. Hunt, J.D.; Teply, S. A Nested Logit Model of Parking Location Choice. *Transp. Res. Part B Methodol.* **1993**, *27*, 253–265. [CrossRef]
18. Waerden, P.J.H.J.; Borgers, A.; Timmermans, H.J.P. Travelers Micro-Behavior at Parking Lots-A Model of Parking Choice Behvior. In Proceedings of the 82nd Annual Meeting of the Transportation Research Board, Washington, DC, USA, 12–16 January 2003.
19. van Middelkoop, H.J. Modeling Parking Choices Considering User Heterogeneity. Bachelor's Thesis, Erasmus University, Rotterdam, The Netherlands, 2018.
20. Le, B.L.; Mai, T.; Ta, T.A.; Ha, M.H.; Vu, D.M. Competitive Facility Location under Cross-Nested Logit Customer Choice Model: Hardness and Exact Approaches. *arXiv* **2024**, arXiv:2408.02925. [CrossRef]
21. Meng, F.; Du, Y.; Chong Li, Y.; Wong, S.C. Modeling Heterogeneous Parking Choice Behavior on University Campuses. *Transp. Plan. Technol.* **2018**, *41*, 154–169. [CrossRef]
22. Khaliq, A.; Van Der Waerden, P.; Janssens, D. A Discrete Choice Approach to Define Individual Parking Choice Behaviour for the Parkagent Model. In Proceedings of the 23rd International Conference on Urban Transport and the Environment, Rome, Italy, 5–7 September 2016; WIT Press: Southampton, UK, 2018; Volume 176, pp. 493–502.
23. Li, X.; Xie, B.; Wang, X.; Li, G.; Yao, Z. Parking Choice Behavior of Urban Village Residents Considering Parking Risk: An Integrated Modeling Approach. *Case Stud. Transp. Policy* **2024**, *15*, 101145. [CrossRef]
24. A Study on the Decision-Making Heterogeneity of Parking Mode Choice. In *Smart Innovation, Systems and Technologies*; Springer: Singapore, 2019; pp. 74–83. ISBN 978-981-13-7541-5.
25. Qin, H.; Gao, J.; Chen, Y.; Wu, S.; Zhang, K. Analysis on the Parking Behavior of a Large Airport Based on the Nested Logit Model. In Proceedings of the CICTP 2016, Shanghai, China, 6–9 July 2016; American Society of Civil Engineers: Shanghai, China, 2016; pp. 2103–2115.
26. Soppert, M.; Steinhardt, C.; Müller, C.; Gönsch, J. Differentiated Pricing of Shared Mobility Systems Considering Network Effects. *Transp. Sci.* **2022**, *56*, 1279–1303. [CrossRef]
27. Lin, C.-R.; Liu, K.-H.; Chen, M.-S. Dual Clustering: Integrating Data Clustering over Optimization and Constraint Domains. *IEEE Trans. Knowl. Data Eng.* **2005**, *17*, 628–637. [CrossRef]
28. Duong, K.-C.; Vrain, C. Constrained Clustering by Constraint Programming. *Artif. Intell.* **2017**, *244*, 70–94. [CrossRef]
29. Ke, X.; Deng, X.; Chen, Y. A Partitioned GeoCA Based on Dual-Constraint Spatial Cluster and Its Effect on the Accuracy of Simulating Result. *Yaogan Xuebao J. Remote Sens.* **2011**, *15*, 512–517.

30. Van Ommeren, J.N.; Wentink, D.; Rietveld, P. Empirical Evidence on Cruising for Parking. *Transp. Res. Part A Policy Pract.* **2012**, *46*, 123–130. [CrossRef]
31. Ruisong, Y.; Meiping, Y.; Xiaoguang, Y. Study on Driver's Parking Location Choice Behavior Considering Drivers' Information Acquisition. In Proceedings of the 2009 Second International Conference on Intelligent Computation Technology and Automation, Washington, DC, USA, 10–11 October 2009; IEEE: Piscataway, NJ, USA, 2009; Volume 3, pp. 764–770.
32. Becker, N.; Carmi, N. Changing Trip Behavior in a Higher Education Institution: The Role of Parking Fees. *Int. J. Sustain. Transp.* **2019**, *13*, 268–277. [CrossRef]
33. Jin, W.; Jiang, H.; Liu, Y.; Klampfl, E. Do Labeled versus Unlabeled Treatments of Alternatives' Names Influence Stated Choice Outputs? Results from a Mode Choice Study. *PLoS ONE* **2017**, *12*, e0178826. [CrossRef]
34. Rose, J.M.; Scarpa, R.; Bliemer, M.C. *Incorporating Model Uncertainty into the Generation of Efficient Stated Choice Experiments: A Model Averaging Approach*; The University of Sydney: Sydney, NSW, Australia, 2009.
35. Puckett, S.M.; Rose, J.M. Observed Efficiency of a D-Optimal Design in an Interactive Agency Choice Experiment. In Proceedings of the Choice Modelling: The State-of-the-Art and The State-of-Practice: Proceedings from the Inaugural International Choice Modelling Conference, Harrogate, UK, 30 March–1 April 2009; Emerald Group Publishing Limited: Leeds, UK, 2010; pp. 163–193.
36. Bierlaire, M. *A Short Introduction to PandasBiogeme*; Technical Report TRANSP-OR 200605; Transport and Mobility Laboratory, ENAC, EPFL: Écublens, Switzerland, 2020.
37. Tiwari, M.; Singh, R. Comparative Investigation of K-Means and k-Medoid Algorithm on Iris Data. *Int. J. Eng. Res. Dev.* **2012**, *4*, 69–72.
38. Shannon, C.E. A Mathematical Theory of Communication. *Bell Syst. Tech. J.* **1948**, *27*, 379–423. [CrossRef]
39. Pareto, V. The New Theories of Economics. *J. Political Econ.* **1897**, *5*, 485–502. [CrossRef]
40. Jeroslow, R.G. The Polynomial Hierarchy and a Simple Model for Competitive Analysis. *Math. Program.* **1985**, *32*, 146–164. [CrossRef]
41. Şenel, F.A.; Gökçe, F.; Yüksel, A.S.; Yiğit, T. A Novel Hybrid PSO–GWO Algorithm for Optimization Problems. *Eng. Comput.* **2019**, *35*, 1359–1373. [CrossRef]
42. Singh, N.; Singh, S.B. Hybrid Algorithm of Particle Swarm Optimization and Grey Wolf Optimizer for Improving Convergence Performance. *J. Appl. Math.* **2017**, *2017*, 1–15. [CrossRef]
43. Sareddy, M.R. Enhancing Smart City Healthcare with Hybrid Swarm Optimization: A Comparison of MFO-PSO and ACO Approaches. *IRO J. Sustain. Wirel. Syst.* **2025**, *7*, 1–18. [CrossRef]
44. Ma, J.; Han, Z.; Deng, Q.; Huang, Y.; Feng, J. New Hybrid Algorithm Combining Multiple Transportation Modes for an Environmental Protection Workshop Layout. *J. Ambient. Intell. Hum. Comput.* **2023**, *14*, 14189–14208. [CrossRef]
45. Li, K.; Li, D.; Ma, H.Q. An Improved Discrete Particle Swarm Optimization Approach for a Multi-Objective Optimization Model of an Urban Logistics Distribution Network Considering Traffic Congestion. *Adv. Prod. Eng. Manag.* **2023**, *18*, 211–224. [CrossRef]
46. Nguyen, T.L.; Nguyen, Q.A. A Multi-Objective PSO-GWO Approach for Smart Grid Reconfiguration with Renewable Energy and Electric Vehicles. *Energies* **2025**, *18*, 2020. [CrossRef]
47. Alyu, A.B.; Salau, A.O.; Khan, B.; Eneh, J.N. Hybrid GWO-PSO Based Optimal Placement and Sizing of Multiple PV-DG Units for Power Loss Reduction and Voltage Profile Improvement. *Sci. Rep.* **2023**, *13*, 6903. [CrossRef] [PubMed]
48. Deb, K.; Pratap, A.; Agarwal, S.; Meyarivan, T. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. *IEEE Trans. Evol. Comput.* **2002**, *6*, 182–197. [CrossRef]