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Editorial

The Need for Expanding the Formal Modeling Toolkit of the Social Sciences

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The effectiveness of mathematical and computational modeling (i.e., formal modeling) in any science revolves around finding qualitative regularities and patterns in nature whose logical structure may be correctly captured by some mathematical structure. The idea is similar to that encountered in the axiomatic theory of measurement [1] where scales are defined in terms of homomorphisms from qualitative structures consisting of world objects (e.g., rods) and relevant qualitative relations/operations (e.g., concatenation) to real number structures with corresponding relations/operations. However, in the case of formal models, the structures are not merely real number structures but are generalized to those consisting of a set of suitable mathematical objects and a set of their corresponding relations. This may seem simple at first sight, but the task is particularly challenging for the social sciences (cognitive psychology included) given the scarcity of known robust deterministic regularities with respect to human behavior. Notwithstanding, in some areas of cognitive research (e.g., perception, categorization, and memory research) such regularities can occasionally be found even though they may fall short of the high lawfulness standards associated with the physical sciences [2].

Unfortunately, the effectiveness of mathematical modeling does not depend solely on the strength of the qualitative patterns underlying the phenomenon under study. Other factors should be addressed to advance formal modeling in both the social sciences and the physical sciences. Central to these factors is what I shall label the *metatheory problem*: namely, that no one has yet developed a rigorous science of modeling, or perhaps better stated, a metatheory of modeling that formalizes the nature and process of formal modeling as a function of the type of qualitative structure underlying the phenomenon being modeled. Such a theory should, in the spirit of maximizing rigor, first take the form of a set of axioms or first principles on modeling, including rigorous definitions for the different types of models and their components. Although this is a difficult challenge, it is a crucial step toward resolving key issues in model selection.

One of these key issues involves the appropriateness of a model. Often, the toolkit of a modeler does not contain the formal structure necessary to best describe the qualitative structure in question. It has been a misfortune for the social and behavioral sciences that formal modeling has predominantly involved the use of classical probability theory and statistics with a great deal of emphasis on Bayesian statistics when modeling the human mind. This trend seems natural given the mentioned lack of deterministic regularities associated with the phenomena studied and given that the quantitative roots of scientific psychology are strongly grounded on data analysis. But it also has diverted from requiring basic training for quantitative psychologists in the four "big" branches of mathematics:

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analysis, topology, algebra, and metamathematics. In this context, I am, of course, referring to the same basic training expected from chemists and physicists and sometimes from biologists.

Moreover, this one-dimensional quantitative training in probability and statistics has often driven those under its influence to attempt to "fit the square peg in the round hole" when modeling psychological phenomena. This is unfortunate because a limited mathematical toolkit reduces the likelihood of developing valid models. Furthermore, as suggested, such a toolkit is available from the plethora of languages, theories, structures, and objects that make up modern mathematics. In addition, those who prefer probabilities as outputs for their models may easily transform results from the chosen deterministic mathematical structure into such probabilistic terms. Thus, the goal should be to find the most appropriate mathematical structure and then impose probabilities on it rather than to start from probabilistic assumptions that are not appropriate to make about the modeled phenomenon in the first place. Please understand, I do not mean to suggest that effective non-statistical and non-probabilistic models of cognitive phenomena do not exist. However, the number of such models is a small fraction of the number of formal models in the extant literature, partly because of an unfamiliarity by formal modelers in the social sciences with theories and objects from the four big branches of mathematics.

The final impediment that I wish to discuss has plagued all formal models in the social sciences in the most severe way. I refer to it as the *context problem*. Since the beginning of scientific psychology in the late 19th century, it has become apparent that humans have an inherent predisposition for finding relationships between stimuli. This is not surprising since it is through such discovery that information becomes manageable for human survival. The downside to this process is that, in detecting relationships between objects and events in the world, humans are also influenced by the contextual effects that are borne from such relations. Simply put, humans are sensitive to the influences exerted by the surroundings of the stimulus of interest. This means that the phenomenon in question may not be effectively predicted without first factoring these effects into our models. Thus, formal models should be equipped with the necessary components to account for these effects.

This finally brings me to the purpose of volume 1 of the Special Issue on mathematical and computational models of cognition. First, the volume is meant to introduce mathematicians from all fields of mathematics to the types of psychological phenomena we study as cognitive scientists and to some of the mathematical structures used thus far to model such phenomena. From this standpoint, the volume is an open invitation for mathematicians of all types to identify mathematical objects, relations, operations, and operators from their domain of expertise that may be useful in alternatively representing and/or describing the phenomena studied in the articles.

Second, and more importantly, the volume focuses on the context problem (as described earlier) from five different perspectives. Specifically, in the article by Vigo et al. (Contribution 1), the authors explore a candidate law of reaction times that accounts for context effects by identifying the key structural relationships between alternatives in choice sets as defined in Generalized Invariance Structure Theory [3]. On the other hand, Townsend & Liu (Contribution 2) further develop the meta-theory known as Systems Factorial Technology, which examines selective influence in psychological research as a means for better understanding model selection and context effects. In contrast, Schweickert & Zheng (Contribution 3) investigate selective influence using Multinomial Processing Trees. This approach facilitates the determination of context effects. On the other hand, Waddup et al. (Contribution 4) take a more direct and metatheoretical approach to the problem of context by examining the sensitivity to context in human interactions using a prisoner's dilemma task without Nash's equilibrium. Finally, Bamber (Contribution 5) ex-

plores properties of rationality when the Dutch Book argument is transported to a different environmental context.

These five papers illustrate that only by enriching the mathematical toolkit of social scientists in terms of alternative mathematical approaches, theories, structures, and objects beyond those found in ordinary statistics and probability theory can we hope to find the most appropriate formal representations and descriptions of cognitive phenomena.

Conflicts of Interest: The author declares no conflict of interest.

List of Contributions

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