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# Quantifying Uncertainty of Insurance Claims Based on Expert Judgments

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**Abstract:** In Bayesian statistics, prior specification has an important role in determining the quality of posterior estimates. We use expert judgments to quantify uncertain quantities and produce appropriate prior distribution. The aim of this study was to quantify the uncertainty of life insurance claims, especially on the policy owner's age, as it is the main factor determining the insurance premium. A one-day workshop was conducted to elicit expert judgments from those who have experience in accepting claims. Four experts from different insurance companies were involved in the workshop. The elicitation protocol used in this study was The Sheffield Elicitation Framework (SHELF), which produces four different statistical distributions for each expert. A linear pooling method was used to aggregate the distributions to obtain the consensus distribution among experts. The consensus distribution suggested that the majority of policy owners will make a claim at the age of 54 years old.

**Keywords:** expert judgment; elicitation; statistical distribution; prior distribution; life insurance

**MSC:** 62P05

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## 1. Introduction

The mortality rate is a critical determinant of life expectancy, functioning as a key indicator of population health and societal well-being. A lower mortality rate contributes to a higher proportion of individuals in the working-age population, thereby enhancing per capita income, improve life quality and enhance health [1]. Mortality is also one of the main issues and trends in global health. Maternal and child mortality are in the attention of World Health Organization (WHO). According to WHO, the maternal mortality rate in the world was extremely high, with 287,000 women losing their lives during and after pregnancy in 2020. The maternal mortality rate was an estimated 223 deaths per 100,000 live births, where almost 95% of all maternal mortality occurred in low- and lower middle-income countries. Meanwhile, the under-five-years-old mortality rate was 37 deaths per 1000 live births in 2022. The global adult mortality rate was 191 for men and 121 for women per 1000 individuals in 2021. Furthermore, noncommunicable disease (NCD) is the leading cause of death worldwide, accounting for 41 million deaths annually, or 74% of all deaths worldwide (WHO, 2024). NCDs primarily consist of cardiovascular illnesses (e.g., heart attacks and stroke), cancers, chronic respiratory illnesses (e.g., asthma and chronic obstructive lung disease) and diabetes.

Given the fact of the global mortality rate and risk, individuals should prioritize maintaining their health to support longevity. Given the inherent uncertainties of life,

including the potential for illnesses, accidents, and other unforeseen causes of death, there is a growing awareness of the need for adequate protection and preparedness. Regarding lifetime uncertainty, a model of uncertain lifetime called the consumer's net asset was pioneered by [2] and reviewed by [3]. This model employs utility function and can be expanded for human capital as well [4]. The relationship between the annuity program and mortality risk is explored by [5]. The utility function and annuity are also used in the theory of life insurance. The main goal of owning a life insurance policy is to guard against the financial difficulties that could arise from the policy owner's untimely passing [6]. Among some factors that influence life insurance premium, age is one of the most determining elements [7,8]. Age is a crucial factor in studying mortality and premiums in life insurance as mortality rates significantly vary across different age groups [9]. Life insurance companies will look at the age of the prospective insurance participants to decide the insurance rate. Nevertheless, due to the uncertainty of lifetime, lifespan and other mortality risk, it is not guaranteed that younger insurance participants will submit claims within a long period of time, while older participants will submit claims more quickly. Therefore, an insurance participant's age in life when making a claim is uncertain.

This study aims to investigate the uncertainty of age when claiming life insurance by employing expert judgment. Expert judgment is used in this study as exact information about the claim age is limited and highly depends on the history of claim, but the future claim in unknown.

The involvement of experts in quantifying uncertainty was proposed by [10] and is crucial in decision making when data are limited or unavailable. To obtain information, opinion, or judgment from experts about uncertain quantities can be named as elicitation. This type of collecting information has also been studied in the field of psychology. A practical guide on elicitation was primarily provided by [11]. The method of elicitation such as general elicitation and application-specific elicitation was studied [12,13]. Furthermore, the elicited information of expert is expressed in the form of statistical distributions [14]. The general protocol of elicitation and some types of elicitation are discussed in [15,16]. In its development, there are some protocols that can be used in elicitation process. As mentioned in [17], there are at least three different protocols: classical model [10,18], SHELF protocol [19,20], and IDEA protocol [21]. A review of these methods can be found in [22] and the strategy of applying the protocol is presented in [23].

The process of eliciting expert judgments on claim age are adapted with the mortality phenomena in Indonesia. The mortality rate is used as a basis of constructing questions in the elicitation workshop. According to the Long Form Population Census 2020 (Long Form SP2020) conducted by Statistics Indonesia (BPS) in the period 2010 to 2020, the mortality rate in Indonesia is 6.4 indicating there would be 6–7 deaths among 1000 individuals. This rate is moderate compared to other developing countries. When the mortality rate is classified by age group—children, adults, and the elderly—the distribution is as shown in Figure 1.

As shown in the figure, the death rate for the 0–14 years age group is 1.71 meaning that there would be 1–2 deaths among 1000 individuals under 15 years old. The death rate for the second group (15–59) is 2.64, which indicates 2–3 would die in the range of productive ages. Lastly, the death rate for people aged 60 or more rises around 10 times to 26.1 compared to the prior age groups, which shows that the mortality of elderly people is 26–27 among 1000 individuals.

In addition to the age-specific death rate, it is interesting to investigate the cause of the death obtained from a similar census. According to Figure 2, non-infectious disease is the major cause of death in Indonesia, which is 87.15%. The second cause is accident, which is

a quite small percentage (2.82%). Infectious disease is the rarest cause of mortality with a percentage of only 0.89%.

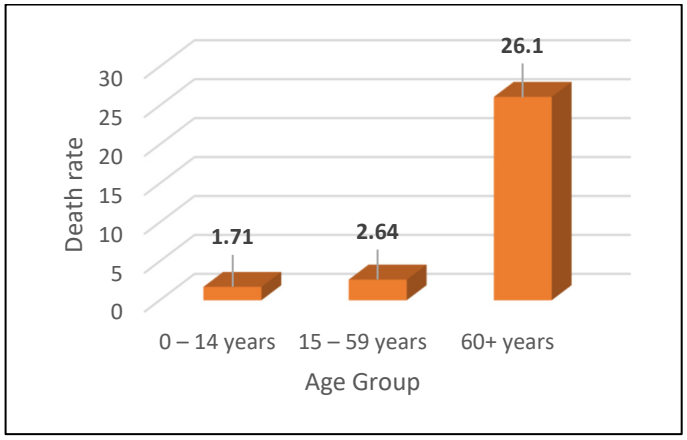



**Figure 1.** Age-specific death rate in Indonesia.

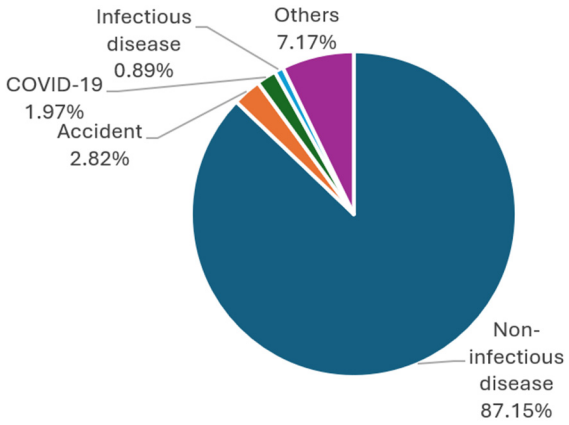



**Figure 2.** The causes of death for Indonesian people from 2010 to 2020.

Furthermore, we are interested in the probability of death in the interval of ages which can be used to quantify age when an policy owner makes a claim. Based on the Indonesian mortality table, the probability of death can be seen in Figure 3.

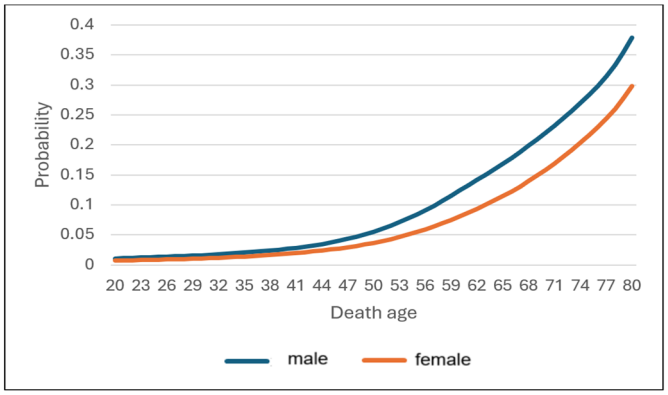



**Figure 3.** The probability of death for male and female based on age in the interval between 20 and 80 years.

Figure 3 illustrates the probability of death across the ages 20 to 80, with separate lines representing the mortality risks for males (blue line) and females (orange line). The data reveal several key patterns that reflect the differences in life expectancy and mortality rates between genders over this age range. Both the male and female probability-of-death lines

exhibit a gradual increase with age, indicating that the risk of death rises as individuals get older. However, the overall probability of death for males is consistently higher than that for females across all age intervals. This aligns with general population trends where men, on average, have shorter life expectancies than women.

In the younger age ranges, the probability of death for both males and females remains relatively low and increases slowly. For males, the probability reaches 0.05 (5%) around the age of 48, while for females, it takes a slightly longer time, reaching the same probability at age 52. This suggests that, in early adulthood, the mortality risk is low for both genders, but men face a slightly higher risk than women during these years. After this point, the probability of death for both genders begin to rise more rapidly, reflecting the increasing mortality risk as people age. For men, the probability jumps to 0.25 (25%) by the age of 73, while for women, it reaches the same level at age 77. This exponential rise in death probability is typical of aging populations, where the risk of death accelerates significantly as individuals approach advanced age. The gender gap remains consistent, with men facing a higher mortality risk at every age point.

A life insurance claim will be made when a policy owner passes away. Due to the uncertainty regarding a policy owner's death, the insurance company must be prepared whenever a policy owner makes a claim. This is because the uncertainty surrounding a policy owner's death leads to the uncertainty of the time of claim, which relates to the policy's monetary value based on time. As a result, insurance companies need to calculate the probability of a policy owner's mortality.

There are some different ways of determining the probability of death: using the mortality table, which is the tabulation of age and a basic function involving the number of surviving individuals, the number of individuals who die in the specific age interval, and the probability of death; alternatively, the determination of the probability of death can also use a distributional approach. In this case, survival function based on mortality laws can be employed. Some notable mortality laws were introduced by De-Moivre in 1729 (1-parameter formula), Gompertz in 1825 and Weibull in 1939 (two-parameter formula), and Makeham in 1860 (three-parameter and four-parameter formulas). Recent studies on the use of mortality distribution for life insurance based on mortality laws were conducted by [24,25].

Both approaches, mortality tables and mortality distribution, rely on historical data. Mortality tables utilize past data tabulated to represent the desired population mortality state, whereas mortality laws require aligned historical population data to estimate distribution parameters. In this study, the quantification of claim age uncertainty is not based on historical data as it may not be appropriate to depict the overall claim age distribution in the future. Instead, we employ expert opinions based on their experience and insight into current distribution trends which can also be used for prior distribution in the future.

The main contribution of this paper lies in its innovative approach to quantifying uncertainty where information is limited or unknown. By addressing this challenge, the study provides a practical framework for managing uncertainty, offering valuable insights for applications in some fields. Specifically, it equips life insurance companies with crucial information about the expected age mean of claims. This enables life insurance companies to better anticipate and prepare for any claim proposals, enhancing their ability to allocate resources effectively and manage risks proactively. Ultimately, the findings support improved decision making and operational readiness in the face of uncertainty. In addition, the claim distribution obtained from expert judgments will be utilized to evaluate mortality figures in Indonesia.

## 2. Materials and Methods

Data were obtained through an Elicitation workshop which was held in July 2024. The workshop was attended by four experts from four different insurance companies in Indonesia. The experts involved in the workshop are the people who are in charge of receiving insurance claims for each company and have sufficient experience working in the company. The representatives of four experts are expected to represent insurance companies in Indonesia. Unfortunately, the names of the companies are kept anonymous on request from the experts. Therefore, in the rest of the paper, we will call the four experts Expert A, B, C, and D. The workshop aimed at quantifying the age mean of policy owners when they make claims. In addition, the percentage of claim numbers based on policy owners' age was also collected.

In this study, we use the elicitation method to handle subjective expert opinions. Elicitation is defined as the process of expressing expert knowledge for an uncertain quantity in the form of a probability distribution [26]. Elicitation can also be interpreted as an opportunity to express subjective opinion on a true but uncertain quantity through a probability distribution. Accordingly, the objective of elicitation is to create probability distributions that appropriately represent expert knowledge [15]. The distribution obtained from elicitation is typically subjective based on expert belief, and hence, different experts might provide different probability distributions based on their own judgment and experience. Therefore, an elicitation protocol is required to guarantee that distributions produced are scientifically valid. We utilize Sheffield Elicitation Framework (SHELF) protocol to elicit expert judgments on uncertainty in insurance claim ages. SHELF is used in this study because we aim to have a consensus distribution of the different types of distributions experts may produce. We chose not to incorporate an iterative refinement process, as seen in the IDEA protocol, because this study allows experts the freedom to provide judgments solely based on their own experience. Furthermore, since each expert's judgment is considered to contribute equally to the consensus distribution, we assign equal weight to all experts. Therefore, we do not adopt the classical model, which assigns different weights to experts based on their performance.

Let us suppose that there is a true but uncertain quantity $X$ (claim age). Experts will be asked for their judgment on some values to the best of their understanding and experience about $X$. Firstly, experts need to determine the range of the uncertainty of $X$ by judging the minimum $(a)$ and the maximum value $(b)$. It is expected that $X$ is likely to happen inside the range of $[a, b]$ and unlikely to happen outside this range. Secondly, experts are requested to determine other values, namely, the lower value ($L$), middle value ($M$), and upper value ($U$), which are corresponding to the cumulative probabilities $Q_1$, $Q_2$, and $Q_3$ [17]. Values $L$, $M$, and $U$ need to fulfill the following probabilities:

$$P(X \leq L) = Q_1;\ P(X \leq M) = Q_2;\ P(X \leq U) = Q_3$$

In this case, $Q_1$, $Q_2$, and $Q_3$ are 25%, 50%, and 75%, respectively. Next, the cumulative distribution function of each quantile will form a full CDF of $X$. We can also derive the probability distribution function (pdf) for $X$ and estimate the parameter distribution. Let us suppose that the expert distribution has a parameter $\boldsymbol{\theta} = (\theta_1, \theta_2)$, then the CDF for the first value, second value, and third value provided by the expert is $F(L; \boldsymbol{\theta})$, $F(M; \boldsymbol{\theta})$, and $F(U; \boldsymbol{\theta})$, respectively. The function of $\boldsymbol{\theta}$ combining the three CDFs is $H(\boldsymbol{\theta})$ as follows:

$$H(\boldsymbol{\theta}) = (F(L; \boldsymbol{\theta}) - Q_1)^2 + (F(M; \boldsymbol{\theta}) - Q_2)^2 + (F(U; \boldsymbol{\theta}) - Q_3)^2$$

The estimate of parameter $\boldsymbol{\theta}$ is obtained by minimizing $H(\boldsymbol{\theta})$ as follows:

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{arg\ min} H(\boldsymbol{\theta})$$

The minimization process of the function $H(\boldsymbol{\theta})$ is carried out numerically using the Nelder–Mead optimization method to obtain the best estimate [17]. The steps for Nelder–Mead optimization are as follows:

1. Determine the objective function $H(\boldsymbol{\theta})$, where $\boldsymbol{\theta} \in \mathbb{R}^n$, and $n$ is the number of parameters.
2. Evaluate the vertex $H_i = H(\boldsymbol{\theta_i})$, for $i = 1, 2, \ldots, n+1$. The collection of $\boldsymbol{\theta_i}$ is called simplex ($S$).
3. Sort the vertices obtained from Step 2 in ascending order, $H_1 \leq H_2 \leq \ldots \leq H_n \leq H_{n+1}$.
4. Compute the centroid of the simplex, $\overline{x}(j)$, by excluding vertex $j$ using formula $\overline{\mathbf{x}}(j) = \frac{1}{n}\sum_{i=1;i\neq j}^{n+1} \boldsymbol{\theta_i}$
5. Compute the new vertex, $\mathbf{x}(\rho, j)$, by considering the centroid and a reflection factor $\rho$ using formula $\mathbf{x}(\rho, j) = (1+\rho)\,\overline{\mathbf{x}}_j - \rho\boldsymbol{\theta}_j$.
6. Replace the worst vertex $\boldsymbol{\theta}_{n+1}$ with the new vertex $\mathbf{x}(\rho, j)$.
7. Repeat the process until the standard deviation of $S <$ tolerance value.

### *2.1. Statistical Distribution for Quantifying Expert Judgment*

Quantifying expert judgments in elicitation often involves fitting statistical distributions that capture the uncertainty and subjective opinions expressed by experts. Some of the statistical distributions that can be used to quantify expert opinions are Beta distribution, Gamma distribution, and normal distribution.

1. Beta distribution

Beta distribution is ideal for modeling probabilities or proportions, as it is defined on the interval [0, 1]. It can take various shapes (e.g., uniform, skewed, U-shaped), making it versatile for representing expert beliefs about probabilities. The distribution has two parameters, $\alpha$ dan $\beta$, which are scale and shape parameters, respectively. The probability density function of the Beta distribution is

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}\ ; 0 \leq x \leq 1$$

and zero elsewhere. $\Gamma$ is the gamma function, which can written as

$$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$$

or in the factorial form $\Gamma(n) = (n-1)!$. The use of Beta distribution in eliciting expert opinions had been assessed by [27], and how to assess with this distribution is discussed in [28].

2. Gamma distribution

The Gamma distribution is particularly well-suited for modeling insurance claims due to its flexibility in representing positively skewed data. This characteristic is essential in the insurance field, where claim amounts often display significant variability and skewness. The pdf of the Gamma distribution is expressed as

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1}e^{-x/\beta};\ 0 < x < \infty,$$

and zero for elsewhere, where $\alpha$ is the shape parameter, and $\beta$ is the rate parameter.

The ability of the Gamma distribution to capture a wide range of shapes—from exponential-like for $\alpha = 1$, to more bell-shaped as $\alpha$ increases—allows it to align with expert opinions that express various degrees of uncertainty about the potential claim age.

Sometimes, it is possible that the expert distribution is the mirror of the Gamma distribution, meaning that the distributions have opposite shapes. Technically, if $X \sim$ Gamma$(\alpha, \beta)$, then $Y = -X$ has a mirror Gamma distribution $(\alpha, \beta)$ with pdf

$$f(y) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}}(-y)^{\alpha-1}e^{y\beta}; -\infty < y < 0$$

3. Normal distribution

In certain situations, where the claim judgment is reasonably symmetrical and devoid of notable skewness or heavy tails, the normal distribution may be appropriate for fitting insurance claims.

If $X$ is the age where an policy owner made a claim with mean $\mu$ and varians $\sigma^2$, the pdf of the normal distribution can be expressed as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; -\infty \leq x \leq \infty$$

and zero elsewhere.

In the case, where the ideal symmetry of X does not hold, the skew normal distribution can be used as an alternative. The skew normal distribution serves as an effective model to capture the asymmetry typically observed in claim amounts, where a higher frequency of smaller claims is accompanied by a gradual tail of larger, less common claims. The pdf for a skew normal distribution is given by

$$f(x) = (x; \xi, \omega, \alpha) = \frac{2}{\omega}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\alpha\frac{x-\xi}{\omega}\right)$$

where $\xi$ is the location parameter, $\omega$ is the scale parameter, and $\alpha$ is the shape parameter. Here, $\phi(z) = \frac{1}{\sqrt{2\pi}}e^{-\frac{z^2}{2}}$ represents the probability density function of the standard normal distribution, while $\Phi(z)$ denotes its cumulative distribution function.

### *2.2. Linear Pooling: A Synthesis of Expert Judgments*

Linear pooling is a method used to aggregate expert judgment in the form of statistical probability distributions onto a consensus probability distribution. A foundational critique and overview of various methods for combining probability distributions, including linear pooling can be found in [29]. Linear pooling involves the weighted average of the probability density functions (PDFs) constructed from expert opinions. In addition, linear pooling is still superior in comparison with other methods in terms of its practical use [30]. Weighted pooling, on the other hand, can work better, but an extra step in expert testing is required. The idea of pooling is to combine these individual distributions that reflect the collective judgment. The method assumes that each expert's judgment represents a valid piece of information.

In addition, linear pooling provides a more statistical perspective, particularly in the context of elicitation [14]. Linear pooling is not just an averaging technique but also a way to propagate uncertainty through the aggregation process. By combining expert opinions via linear pooling, the resulting distribution serves as a coherent summary of the experts' beliefs about a particular uncertain quantity, integrating both the central tendencies and uncertainties reflected in the individual distributions.

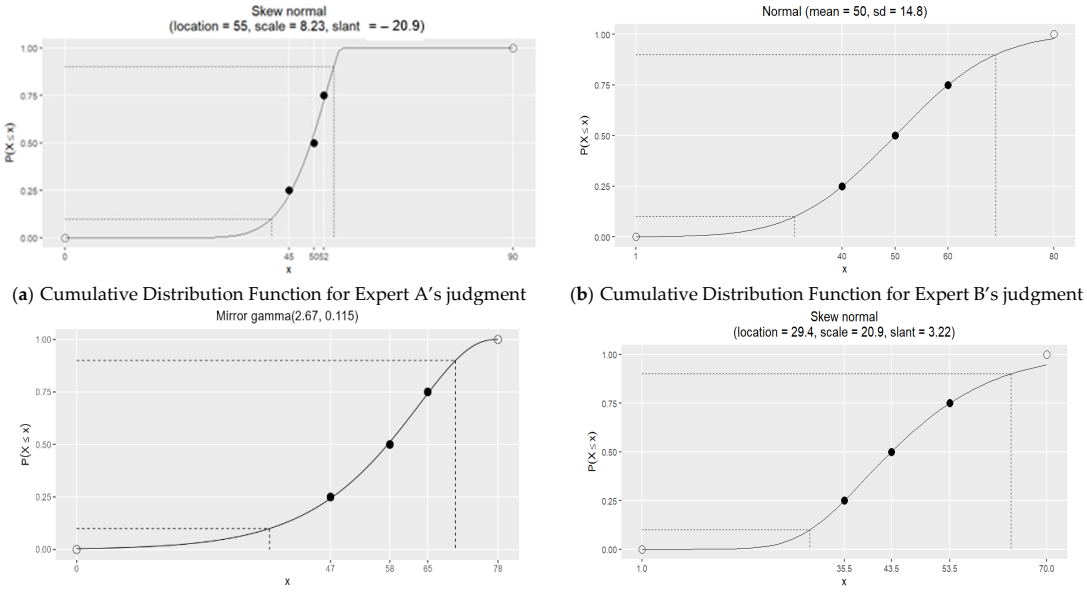
Assume we have $n$ experts providing probability distributions $P_1(x)$, $P_2(x)$, ..., $P_n(x)$ over the same domain $x$. The combined distribution $P_c(x)$ using linear pooling is defined as

$$P_c(x) = \sum_{i=1}^{n} w_i P_i(x)$$

where $P_i(x)$ is the probability distribution function given by the $i$th expert, and $w_i$ is the weight allocated to the $i$th expert.

## 3. Results

Four experts, A, B, C, and D, are required to provide judgments based on their experience in the form of cumulative percentages. The judgment is about the age mean of policy owner when making a life insurance claim. Three quantiles, $Q_1$, $Q_2$, and $Q_3$, are recorded from the judgment along with the minimum and the maximum ages of insured person in their companies. The minimum and the maximum ages serve as the lower and upper bounds that define the range of uncertainty quantification. The elicited values of the claimant's age in the form of quantiles are then fitted to a statistical distribution. Each expert will have their own statistical distribution that is best fitted to their judgment. The cumulative distribution function and the probability density function of each expert are presented in Figures 4 and 5, respectively.




(**a**) Cumulative Distribution Function for Expert A's judgment

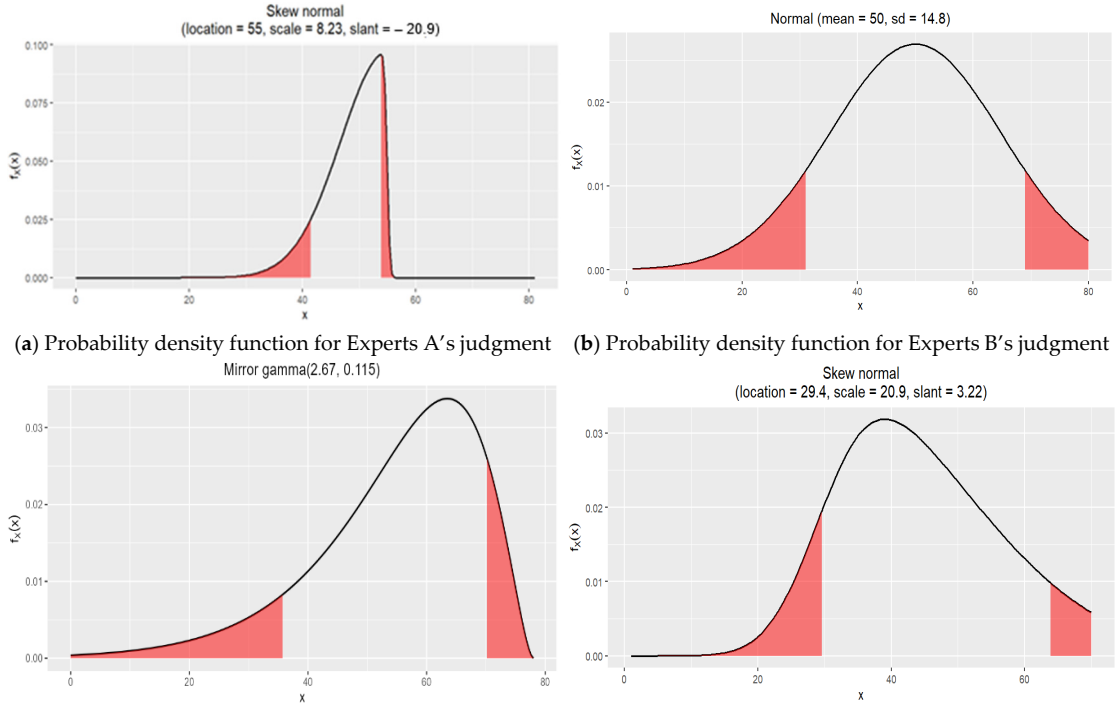
(**b**) Cumulative Distribution Function for Expert B's judgment

(**c**) Cumulative Distribution Function for Expert C's judgment

(**d**) Cumulative Distribution Function for Expert D's judgment

**Figure 4.** Fitted cumulative distribution function of policy owner average ages when making claim based on different expert judgments. (**a**–**d**) represent CDFs for experts A, B, C, and D, respectively.

Figure 4 shows the cumulative distribution function (CDF) of the expert judgment on the average age of the policy owner when making an insurance claim. The CDF produced from Expert A's judgment is quite steep. The CDF indicates that 25% of claims are expected to occur by the age of 45, 50% of claims will be made by policy owners aged 50, and 75% of claims will be made by an insured person by the age of 52. In addition, the range of policy owner ages in company A is between 0 and 90 years old, which is quite wide.




(**a**) Probability density function for Experts A's judgment (**b**) Probability density function for Experts B's judgment

(**c**) Probability density function for Experts C's judgment (**d**) Probability density function for Experts D's judgment

**Figure 5.** Fitted probability density function of policy owner average ages when making a claim, based on different expert judgments. (**a**–**d**) represent pdfs for expert A, B, C, and D, respectively.

Meanwhile, the CDF of Expert B is more sloping than that of Expert A. Expert B states that 25% of claims are made by a policy owner aged 40, 50% of claims are made by an insured person aged 50, and 75% of claims will be made by people aged 60. Additionally, the range of policy owner ages in company B is between 1 and 80 years. Based on this CDF, we infer that the average age of a claim begins when someone reaches the age of 40 and will increase in number when reaching 50 years old. Many policy owners make a claim when aged 60, which is relevant to the mortality rate in Indonesia.

On the other hand, Expert C provides judgment represented by a CDF in Figure 4c. The CDF shows that 25% of claims are made when policy owner's age is 47, 50% of claims will be made when the policy owner's age is 58, and 75% of claims will be made by the age of 65. In addition, the CDF shows that the youngest policy owners are newborn babies, and the oldest policy owner is 78 years old. These age claim trends are somewhat longer compared to company B, as shown by the quantile ages being around 5 to 8 years older.

Finally, the CDF for expert D illustrates that 25% of claims are expected to occur when a policy owner in company D reach 35.5 years old. A total of 50% of claims will be made by the age of 43.5 years old, and 75% of claims will be made by 53.5 years old. The ages of a policy owner in company D are between 1 and 70 years old. The features of this claim are relatively similar to those of company B, even though the quantile age is 3.5 years older.

From the CDFs in Figure 4, we can obtain the probability density function for each expert to know what type of statistical distribution fitted to each expert. Although there are some possible distributions that can be fitted, we present the best-fit model for each expert.

Figure 5 describes the probability density function of the average age of the policy owner when making an insurance claim for four different experts. According to Figure 5, the distribution fitted for the judgment given by Expert A is a skew normal distribution with a mode of 55 years old. This means that the majority of policy owners in company A will make claims at the age of 55 years old. The 95% confidence interval of the distribution is [42, 55], which implies that the policy owner will make a claim in the interval between 42

and 55 years old. In addition, it is unlikely that the policy owner will make a claim before 25 years old or after 60 years old.

The distribution for Expert B is normal with a mean of 50 and standard deviation of 14.8, indicating the average age of a policy owner when making a claim in company B is 50 years old. Expert B beliefs with 95% degree of confidence that the policy owner will make a claim between the ages of 32 and 68 years old. On the other hand, the distribution elicited for Expert C is a mirror of the Gamma distribution (26.7, 0.115) with a mode of 64 years old, indicating that the majority of claims will be made at that age. Moreover, Expert C has a 95% confidence that claims will be proposed by policy owners between the ages of 36 and 70 years old. Lastly, Expert D has a similar distribution to Expert A, but with different parameters and opposite skewness. The distribution for Expert D is right skew, while Expert A is left skew, meaning that Expert D believes that there is a higher probability of a claim be proposed by a younger policy owner. As we can see in Figure 2, it is believed that most claims are made by people aged 38 years old. Furthermore, with 95% confidence, Expert D believes that a claim will be proposed by the policy owner between the ages of 30 and 64 years old.

To verify whether the distribution aligns with experts' beliefs and is sensible, feedback is required. Feedback quantiles are used to evaluate the age mean in resulting distributions, which is computed from each expert's distribution, as shown in Table 1. Quantiles 1%, 10%, 90%, and 99% are used to justify the age mean [17]. The 1% quantile for Expert A is 33.6, meaning that he or she believes that it is very unlikely that claims will be made by a policy owner whose age is 34 years old. This belief is quite different from other experts, where Expert B believes that it is unlikely that a claim will be made by a teenage policy owner. Experts C and D have a similar belief that it is unlikely that a claim will be made by children aged under 10. Additionally, based on expert distributions, Expert A and Expert D have a similar belief that almost all policy owners will make a claim when aged in their 50s, as seen from the 0.9 quantiles. On the other hand, Expert B and Expert C believe that the majority of claims will be made by a policy owner aged in their 70s. These two groups of belief represent quite different judgments of a high probability of claiming insurance in their distributions. In relation to the mortality rates in Figure 1, this fact partially supports the figure, and from the point of view of insurance, the mortality rate will also be high at the age of 50.

**Table 1.** Feedback quantiles for expert distributions.

| Quantiles | Age Means | | | |
|---|---|---|---|---|
| | **Expert A** | **Expert B** | **Expert C** | **Expert D** |
| 0.01 | 33.6 | 15.5 | 9.77 | 9.12 |
| 0.1 | 41.4 | 31 | 35.7 | 26 |
| 0.9 | 53.9 | 69 | 70.1 | 58.6 |
| 0.99 | 55.1 | 84.5 | 75.2 | 69 |

The differences in distributions between the experts are normal since their experiences in handling life insurance claims differ between insurance companies, being really dependent on the behavior of policy owners when making claims. To address the differences in expert distributions, we use linear pooling to achieve consensus. The consensus distribution obtained by linear pooling is presented in Figure 6.

Figure 6 shows the consensus distribution of aggregating four distinct expert distributions using linear pooling. The peak of the distribution is on average the age of 54 years old with the density value 0.0438. This means that by consensus of the four experts, life insurance claims in Indonesia will be submitted mostly by people aged 54 years old. The consensus distribution is likely to be much influenced by the distribution of Expert A as it

tends to have left skew and a sharp top shape. The density of 0.025 is reached when the policy owner is aged 42 and 55 years old, indicating these ages have high density.

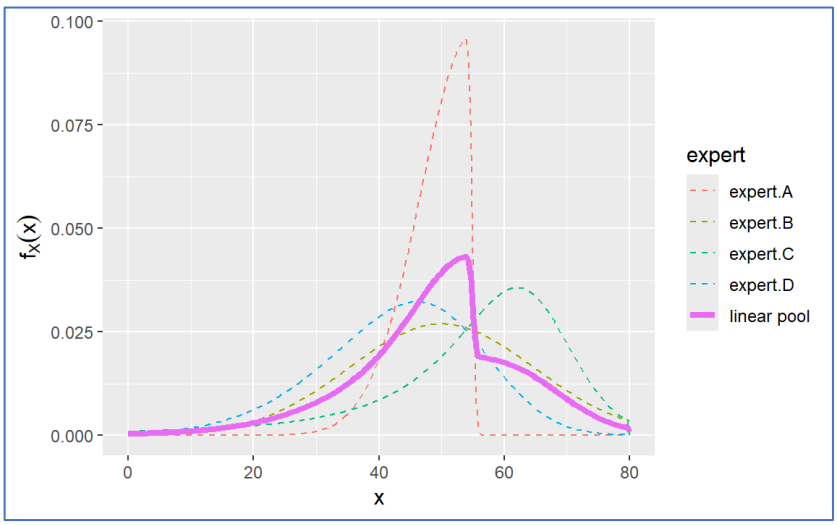



**Figure 6.** Consensus distribution on policy owner's age mean via linear pooling.

The expert judgment distribution is subsequently compared with the currently available life insurance claim data to validate against the actual conditions. An analysis based on the claim data reveals that the majority of claims are made when the policy owner's age ranges from 52 to 57 years, as shown in Figure 7. The actual data support the expert judgment distribution where most policy owners are believed to make claims at the age of 54 years old.

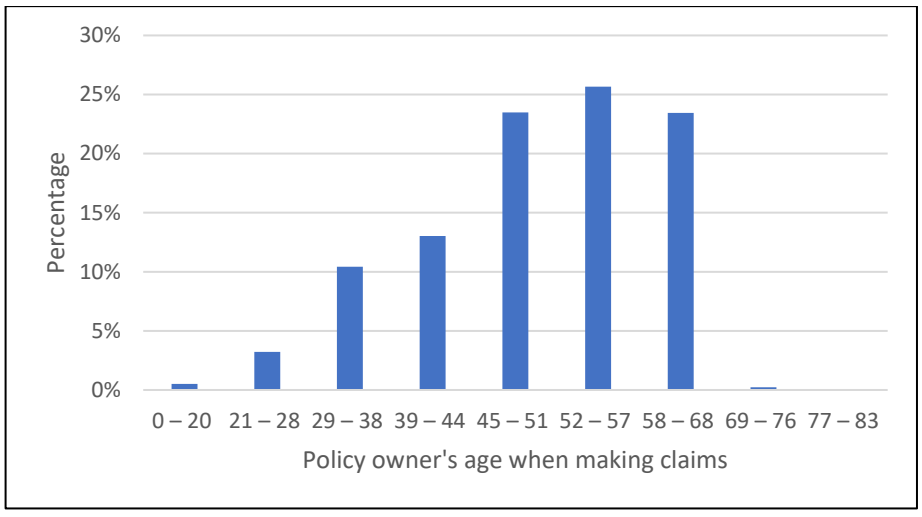



**Figure 7.** Distribution of life insurance claims in Indonesia based on available data.

The analysis reveals an intriguing relationship between mortality rates and insurance claim distributions in Indonesia. The mortality data indicate that fatalities predominantly occur among individuals aged 60 and above, reflecting the higher vulnerability of older populations. In contrast, expert judgments on insurance claim distributions suggest that most claims are expected around the age of 54. This discrepancy highlights the role of risk perception, as experts may anchor their assessments on the perceived likelihood of claims near retirement age, a critical life stage for financial planning. Interestingly, actual claim data show a concentration of claims among individuals aged 52–57, bridging the gap between expert expectations and mortality trends. This alignment suggests that insurance claims are primarily driven by mortality risk, as claims are typically made when policy

owners pass away. Overall, the interplay between mortality rates, expert perceptions, and actual claim data underscores the importance of integrating demographic insights with behavioral factors to refine life insurance risk models. This result does not consider the cause of mortality of the policyowner and purely considers the point of view of life insurance claims. Further research is require to explore the underlying causes of mortality and involve health experts to provide deeper insights into mortality in Indonesia.

## 4. Conclusions

This study investigates uncertainty regarding the average age of people claiming on their insurance using an expert judgment approach. The result of elicitation from four different experts indicates that they have different judgments. The consensus distribution indicates that policy owners will make claims mostly at the age of 54 years old. This finding is relevant to the probability of Indonesian people dying aged 54 being around 0.05. Even though this probability is small compared with those aged 60 or more, the consensus distribution suggests that people aged 54 have a higher risk of mortality due to their activities, which might leave them vulnerable to suffer from disease or accident. People aged 60 or more may make a claim on their insurance, but the probability of this is less compared to those aged 54. Feedback quantiles indicate that there is certainly a lower probability of younger individuals claiming on their insurance up to the age of 26. The available insurance claim data support our findings in that most of the claims (a quarter of claims) will be made by insured persons aged 52–57 years old. These results are particularly valuable for insurance companies in Indonesia, helping them develop policies and strategies for life insurance based on the average age of policy owners when making a claim.

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**Data Availability Statement:** The data supporting the findings of this study are available from the corresponding author upon reasonable request. The data are stored in the repository https://github.com/budhihandoko-unpad/expert_elicitation.git (accessed on 15 December 2024).

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