

Article

Optimizing Mine Ventilation Systems: An Advanced Mixed-Integer Linear Programming Model

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Abstract

In the underground mine ventilation area, the absence of robust solutions for nonlinear programming models has impeded progress for decades. To overcome the enduring difficulty of solving nonlinear optimization models for mine ventilation optimization, a major technical bottleneck, we first develop an advanced linear optimization technique. This method transforms the nonlinear ventilation optimization and regulation model into a linear control model, avoiding the limitation of difficulty in solving the nonlinear mathematical model. The linear strategy opens up a new solution idea for the nonlinear calculation of the mine ventilation optimization and regulation. Furthermore, this study introduces evaluation metrics for ventilation scheme quality, including minimal energy consumption, fewest adjustment points, and optimal placement of these points, enhancing flexibility in ventilation network optimization. By analyzing the ventilation model control objectives and constraints, we formulated a linear optimization model and developed a multi-objective mixed-integer programming model for ventilation network optimization. This paper constructs and verifies a calculation example model for mine ventilation optimization, assessing its reliability based on airflow distribution calculations.

Keywords: mine ventilation; mathematical optimization; mixed integer programming; ventilation optimization; airflow regulation

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1. Introduction

The problem of ventilation optimization and regulation is one of the core technical difficulties of mine intelligent ventilation systems [1–3]. Ventilation optimization and regulation require meeting the dynamic and demand-driven airflow distribution at various times underground. It involves utilizing the fluid network-based ventilation optimization theory to develop a ventilation optimization scheme that satisfies the actual safety and production requirements of mines and is technically reasonable, reliable, and economically optimized. This approach aims to adjust the air quantity and pressure distribution within the ventilation network, ensuring the mine's ventilation system operates safely, reliably, stably, and economically. The modeling of ventilation systems, particularly in underground mines and large spaces, has evolved significantly to address critical challenges

like energy consumption, health and safety risks, and environmental control, employing methods ranging from numerical simulation software [4] to sophisticated machine learning algorithms [5,6].

The mathematical model for ventilation optimization and regulation with unknown airflow characteristics is inherently nonlinear. Due to the significant challenges in solving nonlinear problems, the problem of integrated ventilation optimization and regulation has remained predominantly in the theoretical research phase over recent decades, with leading mine ventilation software (iVent V1.1.0), both domestically and internationally, yet unable to offer effective solutions. In particular, for complex mine ventilation systems, the difficulty of solving large-scale nonlinear mathematical models increases exponentially, making it challenging to obtain reliable solution results using existing optimization strategies [7].

When solving nonlinear ventilation optimization models, such systems typically contain quadratic terms in both objectives and constraints, representing nonlinear nonconvex optimization problems. Due to the inherent complexity of solving nonlinear problems and the uncertainties in both regulation positions and ways, there is currently no reliable solution method available. A large number of scholars have dealt with the computational performance problem of nonlinear optimization by introducing genetic algorithm [7–10] (GA), ant colony algorithm [11–14] (ACO), simulated annealing algorithm [15–17] (SA), and taboo search algorithm [18–20] (Tabu Search, TS), as well as other computationally intelligent, bionic computing algorithms to deal with the computational performance problems of nonlinear optimization. Sui et al. [21] applied the Harmonic Search Optimization Algorithm, which is inspired by the principles of simulated musical performance, to investigate optimization solutions for mine ventilation networks. Conversely, Ge Hengqing [22] from the China University of Mining and Technology employed the Particle Swarm Optimization (PSO) algorithm to address the optimization challenges of complex coal mine ventilation systems, aiming to solve the multi-objective nonlinear constrained optimization model of large-scale complex ventilation networks, which is challenging to handle with precise solution methods. Furthermore, additional researchers [23–27] have explored various approaches to nonlinear programming, proposing methods such as altering the constraint model, reducing model complexity, and utilizing alternative intelligent optimization algorithms to address the optimal regulation of ventilation networks. Yuanping Huang from China University of Mining and Technology [28] examined the feasibility of employing the constrained variable scale method to tackle the corresponding mathematical model, considering the limitations inherent in existing solution methods for the optimal regulation of mine ventilation networks. In summary, despite the numerous studies on solution methods for nonlinear planning models in ventilation network optimization, challenges such as complex matrix operations, reliance on specific assumptions, sensitivity to initial values, and algorithmic inefficiencies persist. Consequently, there remains a lack of efficient and reliable solution methods that can effectively address the complexity inherent in mine ventilation network models.

While the mathematical model for ventilation optimization and regulation with known airflow is linear, the optimization scheme derived from this two-step regulation strategy [29,30] relies solely on the optimization results of the current airflow distribution (rather than an optimal regulation solution), resulting in significant limitations in the obtained optimization scheme. To address the limitations of the two-step optimization scheme, which must adhere to the existing airflow distribution results and the challenges in solving the nonlinear model of ventilation optimization and regulation with unknown air quantity, this paper introduces a novel approach: the linearization of the ventilation optimization and regulation model through a discretization strategy of the variables. Furthermore, an integrated regulation and optimization method for the ventilation network

based on mixed-integer programming is proposed, offering a new perspective for the optimal regulation of the ventilation network. The solution results obtained from the optimized model can be used to guide the control of ventilation equipment [31].

2. Multi-Objective Nonlinear Optimization Model

2.1. Basic Mathematical Framework

The objective function and hard constraints in traditional linear or nonlinear programming problems for ventilation networks can be converted into objective constraints by incorporating target values and deviation variables. Nevertheless, for practical ventilation network optimization issues, decision-makers must establish the specific form of the objective function based on the problem type and real-world requirements.

A mathematical model of objective programming can be designed as follows. The general form of the objective programming method is expressed as follows,

$$\begin{aligned} \min Z &= \sum_{l=1}^L P_l \sum_{k=1}^K (\omega_{lk}^+ d_k^+ + \omega_{lk}^- d_k^-) \\ \text{s.t.} \quad &\begin{cases} \sum_{j=1}^n c_{kj} x_j - d_k^+ + d_k^- = g_k, \quad k = 1, 2, \dots, K \\ \sum_{j=1}^n a_{ij} x_j \leq (=, \geq) b_i, \quad i = 1, 2, \dots, m \\ x_j \geq 0, \quad j = 1, 2, \dots, n \\ d_k^+ \geq 0, \quad k = 1, 2, \dots, K \\ d_k^- \geq 0, \quad k = 1, 2, \dots, K \end{cases} \end{aligned} \quad (1)$$

where L is the number of layers of objectives with different priorities in the mathematical model of objective programming. P_l is the prioritization factor of the objectives in layer l , which preferentially guarantees objectives with higher priority, satisfying $P_l \gg P_{l+1}$. K is the number of objectives in the mathematical model of objective programming. d_k^+ and d_k^- are the positive and negative bias variables for the k_{th} objective. ω_{lk}^+ and ω_{lk}^- are the weight coefficients of the k_{th} objective with the l_{th} level of priority. x_j is the j_{th} decision variable. The rest of the parameters are those related to the decision variables. a_{ij} , b_i , c_{kj} , and g_k are constant coefficients, determined by the specific application background.

The objective programming method aims to minimize the sum of the products of weights and deviations for each sub-objective. When the weight of a sub-objective is significant, its deviation should be minimized to ensure that higher-priority objectives are prioritized.

The fundamental steps involved in constructing a mathematical model for optimizing a ventilation system using multi-objective planning are as follows.

- (1) Identify each objective and constraint of the optimization model based on the specific requirements of the ventilation system optimization problem.
- (2) Convert some hard constraints into objective constraints based on the actual requirements of the optimization problem.
- (3) Establish priority factors and weighting coefficients for each objective according to their relative importance in the optimization problem.
- (4) Define the final objective function, constraints, and range of decision variables for the optimization model.
- (5) Solve the optimization model using appropriate methods to develop an optimal scheme for the ventilation system.

2.2. Nonlinear Optimization Model

- (1) Theoretical Assumptions

To enable practical mathematical modeling and solutions, several key theoretical assumptions are made. Airflow is treated as incompressible, assuming constant air density throughout the network, which simplifies the mass and energy equations. The analysis assumes steady-state conditions, meaning airflow rates and pressures do not change with time. The constant air density assumption is applied, ignoring density variations along airways. Finally, airway resistance (r) is considered constant for a given airway segment, dependent solely on its physical characteristics (length, cross-section, perimeter, roughness) and independent of the airflow quantity passing through it. These assumptions allow the complex physical network to be represented as a solvable system of non-linear equations based on the fundamental laws.

(2) Fundamental Laws of Airflow

The solution of mine ventilation networks relies on three fundamental airflow laws governing steady-state flow. The Law of Mass Conservation states that at any node in the network, the sum of airflow rates entering the node equals the sum leaving it. Kirchhoff's Second Law dictates that around any closed loop (circuit) in the network, the algebraic sum of the pressure losses equals the algebraic sum of the applied ventilation pressures (from fans or natural ventilation). The Ventilation Resistance Law defines the relationship between airflow quantity (Q) and pressure loss (h) for a specific airway (edge), typically expressed as $h = rQ^2$ (where r is the airway resistance), describing the energy dissipation due to friction and shock losses.

(3) Nonlinear mathematical model

The basic mathematical model for optimizing airflow quantity distribution in a ventilation network is as follows

$$\begin{aligned} \min Z &= \sum_{j=1}^N q_j (r_j q_j^2 + \Delta h_j - h_{N,j}) \\ \text{s.t.} &\begin{cases} \sum_{j=1}^N a_{ij} q_j = 0, & i = 1, 2, \dots, J-1 \\ \sum_{j=1}^N b_{ij} h_j = 0, & i = 1, 2, \dots, M \\ q_j \geq 0, & j = 1, 2, \dots, N \end{cases} \end{aligned} \quad (2)$$

where z denotes the minimum ventilation energy consumption target. N is the number of airways of the ventilation network. M is the number of independent circuits (loops) of the ventilation network. r_j is the air resistance of the j_{th} airway, $N.s^2/m^8$. q_j is the airflow of the j_{th} airway, m^3/s . Δh_j is the air pressure regulation value of the j_{th} airway, Pa. $h_{N,j}$ is the natural air pressure of the j_{th} airway, Pa. a_{ij} denotes the relationship between nodes and airways. b_{ij} denotes the relationship between the airway and the loop.

The decision variables of the mathematical model are the airflow quantity q_j of all unknown airflow airways and the regulated air pressure values Δh_j of all airways. Therefore, the above basic mathematical model for mine ventilation optimization is a non-convex, nonlinear programming model.

3. Mixed-Integer Linear Programming Model

3.1. Linearization Strategy of Nonlinear Model

To circumvent the presence of nonlinear decision variables (unknown airflow) in the optimization mathematical model, this paper eliminates the nonlinear terms by introducing binary integer variables (0–1 variables). It is assumed that the airway airflow is constrained by both the airflow regulation accuracy and the range constraints for the j_{th} airway, allowing

it to take values from the set $\{q_{j,1}, q_{j,2}, \dots, q_{j,k}, \dots, q_{j,K_j}\}$, where K_j denotes the number of permissible values for the j_{th} airway airflow.

Define the binary integer variable $n_{j,k}$ to indicate whether the j_{th} airway airflow assumes the value of $q_{j,k}$ or not, i.e.,

$$n_{j,k} = \begin{cases} 1, & q_j = q_{j,k} \\ 0, & q_j \neq q_{j,k} \end{cases}, 1 \leq k \leq K_j \quad (3)$$

where q_j is the airflow of the j_{th} airway, m^3/s . $q_{j,k}$ represents the k_{th} possible air quantity value for the j_{th} airway, which is a constant.

To limit the value of $n_{j,k}$, $n_{j,k}$ should satisfy

$$\begin{cases} q_j = \sum_{k=1}^{K_j} n_{j,k} q_{j,k} \\ \sum_{k=1}^{K_j} n_{j,k} = 1 \\ n_{j,k} \geq 0, 1 \leq k \leq K_j \end{cases}, 1 \leq j \leq N \quad (4)$$

where N is the number of airways of the ventilation network.

There exists an implicit condition in the above equation that one and only one of the K_j $n_{j,k}$ variables has a value of 1, i.e., the value of the j_{th} airway air quantity is necessarily one of $\{q_{j,1}, q_{j,2}, \dots, q_{j,k}, \dots, q_{j,K_j}\}$.

Specifically, the K_j 0–1 integer variables $n_{j,k}$ of the j_{th} airway satisfy the following characterization

$$\begin{cases} n_{j,k} = n_{j,k}^2 \\ n_{j,k} \times n_{j,k'} = 0, k \neq k' \\ n_{j,k} \times n_{j,k'} = n_{j,k}, k = k' \end{cases} \quad (5)$$

To eliminate the nonlinear variable q_j from the mathematical model, it is essential to explore linear expressions for q_j , q_j^2 , and q_j^3 . Through derivation, it has been determined that the following equation can be used to replace the nonlinear variable q_j

$$\begin{cases} q_j = \sum_{k=1}^{K_j} n_{j,k} q_{j,k} \\ q_j^2 = \sum_{k=1}^{K_j} n_{j,k} q_{j,k}^2 \\ q_j^3 = \sum_{k=1}^{K_j} n_{j,k} q_{j,k}^3 \end{cases}, 1 \leq j \leq N \quad (6)$$

where

$$\begin{aligned} q_j^2 &= \left[\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right]^2 = \left[\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right] \left[\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right] = \sum_{k=1}^{K_j} n_{j,k} q_{j,k}^2 \\ q_j^3 &= \left[\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right]^3 = \left[\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right] \left[\sum_{k=1}^{K_j} n_{j,k} q_{j,k}^2 \right] = \sum_{k=1}^{K_j} n_{j,k} q_{j,k}^3 \end{aligned} \quad (7)$$

3.2. Mathematical Modeling

The mathematical model of mixed-integer programming for mine ventilation network optimization is designed as follows.

3.2.1. Optimization Objectives

(1) Minimum ventilation energy consumption objective

The minimum ventilation energy consumption objective can be expressed as

$$\min z_1 = \omega_1 \sum_{f \in F} q_f h_f = \sum_{j=1}^N q_j h'_{r,j} = \omega_1 \sum_{f \in F} \left(\sum_{k=1}^{K_f} n_{f,k} q_{f,k} \right) h_f \quad (8)$$

where z_1 denotes the minimum ventilation energy consumption. ω_1 denotes the weight coefficient of the minimum ventilation energy consumption objective. F is the set of all fan airways f , including main and auxiliary fans. q_f is the fan airflow of the fan airway f , m^3/s . h_f is the fan air pressure of the fan airway f , Pa. N is the number of airways of the ventilation network. $h'_{r,j}$ is the algebraic sum of the ventilation resistance of the j_{th} airway, Pa. $h'_{r,j}$ can be determined by $h'_{r,j} = h_{r,j} + \Delta h_j - h_{N,j}$. $h_{r,j}$ is the air pressure of the j_{th} airway, Pa. $h_{r,j} = r_j q_j^2$. r_j is the air resistance of the j_{th} airway, $\text{N}\cdot\text{s}^2/\text{m}^8$. q_j is the airflow of the j_{th} airway, m^3/s . Δh_j is the air pressure regulation value of the j_{th} airway, Pa. $h_{N,j}$ is the natural air pressure of the j_{th} airway, Pa. K_f represents the number of air quantity values for the fan airway f . $q_{f,k}$ is to denote the k_{th} possible air quantity that takes the value of fan airway f , which is a constant. The 0–1 integer variable $n_{f,k}$ indicates whether or not the fan airway f air quantity takes the value $q_{f,k}$.

(2) Optimal regulation position objective

The optimal regulation position objective can be expressed as

$$\min z_2 = \omega_2 \sum_{j=1}^N n_{j,a} |s_j| \quad (9)$$

where z_2 denotes the optimal position for mine ventilation regulation. ω_2 denotes the weight coefficient of the best regulation position target. s_j denotes the number of regulation levels of the j_{th} airway, which is a constant value set by the user. $n_{j,a}$ denotes whether the j_{th} airway needs to be regulated or not. $n_{j,a}$ satisfies $n_{j,a} = \begin{cases} 0, & \text{if } |\Delta h_j| \leq \rho_j \\ 1, & \text{if } |\Delta h_j| > \rho_j \end{cases}$. Δh_j is the air pressure regulation value of the j_{th} airway, Pa. ρ_j denotes the amount of regulation (regulation factor) that the j_{th} airway is allowed to ignore, satisfying $\rho_j > 0$.

The airway regulation level constructed in this article satisfies the following characteristics. The default value of the airway regulation level is zero, indicating that the airway is an adjustable airway that allows any type of regulation way. When the sign of the airway regulation level is positive and the value is larger, it indicates that the airway is less adjustable in terms of increasing resistance. When the sign of the airway regulation level is negative and the value is smaller, it indicates that the airway is less adjustable in terms of increasing energy or decreasing resistance.

To facilitate the solution of the mathematical model, a mixed-integer programming approach is introduced to define $n_{j,a}$ as a 0–1 integer variable indicating whether the j_{th} airway needs to be regulated or not. To denote $|\Delta h_j|$ in $n_{j,a}$, $\Delta h'_j$ is introduced to denote the absolute value of Δh_j . $\Delta h'_j$ satisfies the following conditions.

$$\begin{cases} \Delta h'_j \geq \Delta h_j \\ \Delta h'_j \geq -\Delta h_j \end{cases} \quad (10)$$

Subject to the above conditions, there exists an implicit constraint that $\Delta h'_j \geq 0$. To limit the size of $\Delta h'_j$, the priority hierarchy method is used to introduce an objective constraint with the highest priority to ensure that $\Delta h'_j = |\Delta h_j|$.

$$\min z_0 = \omega_0 \Delta h'_j, j = 1, 2, \dots, N \quad (11)$$

where z_0 denotes the additional target that limits the air pressure regulation value variable. ω_0 denotes the weight coefficient (taking a larger value) of the variable limiting the air pressure regulation value.

The above additional objectives must be satisfied preferentially; otherwise, the reliability of the values taken by the 0–1 integer variable $n_{j,a}$ will be compromised. The 0–1 integer variable $n_{j,a}$ needs to satisfy the following conditions:

$$\begin{cases} |\Delta h_j| - \rho_j \leq n_{\max,a} n_{j,a} \\ \rho_j n_{j,a} \leq |\Delta h_j| \end{cases} \quad (12)$$

$$\begin{cases} \Delta h'_j - \rho_j \leq n_{\max,a} n_{j,a} \\ \rho_j n_{j,a} \leq \Delta h'_j \end{cases}$$

In the equation, $n_{\max,a}$ can be set to a sufficiently large normal value to ensure that $\Delta h'_j - \rho_j \leq n_{\max,a}$. Under the constraints of the above conditions, the binary integer variable $n_{(j,a)}$ satisfies: $n_{j,a} = \begin{cases} 0, & \text{if } |\Delta h_j| \leq \rho_j \\ 1, & \text{if } |\Delta h_j| > \rho_j \end{cases}$, where $\rho_j > 0$.

(3) Optimal regulation way objective

The optimal regulation way objective can be expressed as

$$\min z_3 = \omega_3 \left(\sum_{j=1, s_j > 0}^N n_{j,b} s_j - \sum_{j=1, s_j < 0}^N n_{j,c} s_j \right) \quad (13)$$

where z_3 denotes the target of the best regulation mode. ω_3 denotes the weight coefficient of the best regulation mode objective. s_j denotes the number of regulation levels of the j_{th} airway, which is a constant set by the user. $n_{j,b}$ denotes whether the j_{th} airway needs to be regulated with incremental resistance. $n_{j,b}$ is satisfied by $n_{j,b} = \begin{cases} 0, & \text{if } \Delta h_j \leq \rho_j \\ 1, & \text{if } \Delta h_j > \rho_j \end{cases}$.

$n_{j,c}$ indicates whether the j_{th} airway needs to be regulated for energy enhancement or resistance reduction.

$n_{j,c}$ satisfies $n_{j,c} = \begin{cases} 0, & \text{if } \Delta h_j \geq -\rho_j \\ 1, & \text{if } \Delta h_j < -\rho_j \end{cases}$; Δh_j is the air pressure regulation value of the j_{th} airway. ρ_j denotes the amount of regulation (regulation factor) that the j_{th} airway is allowed to ignore, satisfying $\rho_j > 0$.

To facilitate the solution of the mathematical model, the mixed integer programming method is introduced to define $n_{j,b}$ as a 0–1 integer variable indicating whether the j_{th} airway needs to be regulated for increasing resistance. $n_{j,c}$ is defined as a 0–1 integer variable indicating whether the j_{th} airway needs to be regulated for increasing energy or decreasing resistance.

The 0–1 integer variable $n_{j,b}$ needs to satisfy the following conditions,

$$\begin{cases} \Delta h_j - \rho_j \leq n_{\max,b} n_{j,b} \\ n_{j,b} + n_{j,c} = n_{j,a} \end{cases} \quad (14)$$

In the equation, $n_{max,b}$ can be set to a large positive constant to ensure that $\Delta h_j - \rho_j \leq n_{max,b}$.

The 0–1 integer variable $n_{j,c}$ needs to satisfy the following conditions

$$\begin{cases} -(\Delta h_j + \rho_j) \leq n_{max,c} n_{j,c} \\ n_{j,b} + n_{j,c} = n_{j,a} \end{cases} \quad (15)$$

In the equation, $n_{max,c}$ can be set to a large positive constant to ensure $-(\Delta h_j + \rho_j) \leq n_{max,c}$.

(4) Minimum number of regulations objective

The objectives of the minimum number of regulations can be expressed as

$$\min z_4 = \omega_4 \sum_{j=1}^N n_{j,a} \quad (16)$$

where z_4 denotes the minimum number of regulation objectives. ω_4 denotes the weight coefficient of the minimum number of regulation objectives. $n_{j,a}$ denotes whether the j_{th} airway needs to be regulated. $n_{j,a}$ satisfies $n_{j,a} = \begin{cases} 0, & \text{if } |\Delta h_j| \leq \rho_j \\ 1, & \text{if } |\Delta h_j| > \rho_j \end{cases}$. Δh_j is the regulated value of air pressure in the j_{th} airway. ρ_j denotes the amount of regulation (regulation factor) that the j_{th} airway is allowed to ignore, satisfying $\rho_j > 0$.

3.2.2. Constraints

(1) Air quantity balance constraints

The ventilation network airflow regulation optimization scheme must satisfy the node airflow balance condition, i.e., the algebraic sum of the airway airflows into and out of any node in the ventilation network is zero.

$$\sum_{j=1}^N a_{ij} \left(\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right) = 0, i = 1, 2, \dots, J-1 \quad (17)$$

where N is the number of airways of the ventilation network. J is the number of nodes of the ventilation network. a_{ij} denotes the relationship between nodes and airways. a_{ij} satisfies

$$a_{ij} = \begin{cases} 0, & \text{if node } i \text{ isn't adjacent to airway } j \\ -1, & \text{if node } i \text{ adjacent to airway } j, \\ & \text{and } i \text{ is the end point of airway } j \\ 1, & \text{if node } i \text{ is adjacent to airway } j, \\ & \text{and } i \text{ is the origin of the airway} \end{cases} \quad (18)$$

(2) Air pressure balance constraints

The ventilation network airflow regulation optimization scheme must satisfy the loop air pressure balance condition, i.e., the algebraic sum of the airway air pressures in any loop in the ventilation network is zero.

$$\sum_{j=1}^N b_{ij} h_j = 0, i = 1, 2, \dots, M \quad (19)$$

where M is the number of independent loops in the ventilation network, $M = N - J + 1$. h_j is the algebraic sum of the air pressure of the j_{th} airway, $h_j = r_j q_j^2 + \Delta h_j - h_{f,j} - h_{N,j}$.

q_j is the airflow of the j_{th} airway. r_j is the air resistance of the j_{th} airway. Δh_j is the air pressure regulation value of the j_{th} airway. $h_{f,j}$ is the fan air pressure of the j_{th} airway. $h_{N,j}$ is the natural air pressure of the j_{th} airway. b_{ij} denotes the relationship between the airway and the loop. b_{ij} satisfies

$$b_{ij} = \begin{cases} 0, & \text{if airway } j \text{ isn't in independent loop } i \\ -1, & \text{if airway } j \text{ is within an independent loop } i \text{ and is reversed from the loop} \\ 1, & \text{if airway } j \text{ is within an independent loop } i \text{ and in the same direction as the loop} \end{cases} \quad (20)$$

(3) Regulation position constraints

When the j_{th} airway does not allow the installation of regulating facilities (non-adjustable airway), then

$$\Delta h_j = 0 \quad (21)$$

It is worth noting that a negligible range of regulation tolerance can be set for non-adjustable airways without affecting the effectiveness of regulation, so that the j_{th} airway is not allowed to install a regulation facility constraint can be expressed as

$$-\rho'_j \leq \Delta h_j \leq \rho''_j \quad (22)$$

where ρ'_j denotes the amount of energy-enhancing or resistance-reducing regulation (regulation factor) that the j_{th} airway is allowed to ignore, satisfying $\rho'_j > 0$. ρ''_j denotes the amount of energy gain regulation (regulation factor) that the j_{th} airway is allowed to ignore, satisfying $\rho''_j > 0$.

(4) Regulation way constraints

The j_{th} airway regulation constraint

$$\Delta h_{j,min} \leq \Delta h_j \leq \Delta h_{j,max} \quad (23)$$

where $\Delta h_{j,min}$ is the lower limit of the adjustable air pressure of the j_{th} airway; $\Delta h_{j,max}$ is the upper limit of the adjustable air pressure of the j_{th} airway.

When the j_{th} airway only allows resistance to increase regulation constraints, then

$$\Delta h_j \geq 0 \quad (24)$$

When the j_{th} airway only allows energy-enhancing regulation (or drag-reducing regulation) constraints,

$$\Delta h_j \leq 0 \quad (25)$$

When constructing the actual regulation way constraints, it is not advisable to set constraints that restrict specific regulation ways for a large number of airways; otherwise, the optimization model may be unsolvable.

To set the regulation way constraint conditions, the attributes of the allowed airway regulation way, the upper limit of the airway adjustable air pressure, and the lower limit of the airway adjustable air pressure can be added to the airways of the ventilation network.

(5) Regulation number constraints

To reduce the cost of ventilation system control and simplify the management process of ventilation control facilities, the control scheme of the optimization model should minimize the number of regulations.

$$\sum_{j=1}^N n_{j,a} \leq N_a \quad (26)$$

In the formula, N_a is the number of airways allowed to be adjusted in the ventilation network. $n_{j,a}$ indicates whether the j_{th} airway needs to be adjusted. $n_{j,a}$ satisfies $n_{j,a} = \begin{cases} 0, & \text{if } |\Delta h_j| \leq \rho_j \\ 1, & \text{if } |\Delta h_j| > \rho_j \end{cases}$. Δh_j is the air pressure adjustment value of the j_{th} airway. ρ_j indicates the adjustment amount (adjustment factor) allowed to be ignored for the j_{th} airway, satisfying $\rho_j > 0$. In real-world mine ventilation design, N_a is typically limited to 3–10 critical airways to balance control effectiveness with operational stability, as excessive regulators introduce complexity and maintenance burdens.

(6) Fan operating condition constraints

When the fan selection has been determined and the fan cannot be changed, the simulation of the fan operating conditions can be determined by the fan characteristic curve, and the fan characteristic curve is integrated into the mathematical model constraint conditions.

The fan operating condition constraint can be expressed as

$$h_f = a_0 + a_1 q_f + a_2 q_f^2 = a_0 + a_1 \sum_{k=1}^{K_f} n_{f,k} q_{f,k} + a_2 \sum_{k=1}^{K_f} n_{f,k} q_{f,k}^2 \quad (27)$$

where a_0 , a_1 , a_2 are the fan characteristic curve fitting coefficients. q_f is the fan air quantity of the fan airway f . h_f is the fan air pressure of the fan airway f . K_f , $n_{f,k}$, and $q_{f,k}$ are constant values as described in the section of linearization strategy.

3.3. Mathematical Model Solution

The integrated control optimization model of the ventilation network based on mixed-integer programming can be expressed as

$$\begin{aligned} \min Z = & \omega_0 \sum_{j=1}^N \Delta h'_j + \omega_1 \sum_{f \in F} \left(\sum_{k=1}^{K_f} n_{f,k} q_{f,k} \right) h_f + \omega_2 \sum_{j=1}^N n_{j,a} |s_j| \\ & + \omega_3 \left(\sum_{j=1, s_j > 0}^N n_{j,b} s_j - \sum_{j=1, s_j < 0}^N n_{j,c} s_j \right) + \omega_4 \sum_{j=1}^N n_{j,a} \end{aligned} \quad (28)$$

The decision variables in the mathematical model are the regulated air pressure values Δh_j and air quantity values $n_{j,k}$ of all airways, as well as the auxiliary decision variables $\Delta h'_j$, $n_{j,a}$, $n_{j,b}$, and $n_{j,c}$. Since the objective function and constraints are both linear functions, the corresponding mathematical model is a linear mixed integer programming model.

The air quantity range of the j_{th} airway is determined according to the on-demand airflow distribution constraint, air speed range constraint, air flow direction constraint, and effective air quantity constraint. When a smaller air quantity interval is used, the number of decision variables of the hybrid optimization model will increase significantly, which has a greater impact on the solver's solution performance.

$$\begin{aligned}
 & \left\{ \begin{aligned}
 & \sum_{j=1}^N a_{ij} \left(\sum_{k=1}^{K_j} n_{j,k} q_{j,k} \right) = 0, \quad i = 1, 2, \dots, J-1 \\
 & \sum_{j=1}^N b_{ij} h_j = 0, \quad i = 1, 2, \dots, M \\
 & -\rho'_j \leq \Delta h_j \leq \rho''_j \\
 & \Delta h_{j,\min} \leq \Delta h_j \leq \Delta h_{j,\max} \\
 & \sum_{j=1}^N n_{j,a} \leq N_a \\
 & \Delta h'_j \geq \Delta h_j \\
 & \Delta h'_j \geq -\Delta h_j \\
 & \Delta h'_j - \rho_j \leq n_{\max,a} n_{j,a} \\
 & \rho_j n_{j,a} \leq \Delta h'_j \\
 & \Delta h_j - \rho_j \leq n_{\max,b} n_{j,b} \\
 & -(\Delta h_j + \rho_j) \leq n_{\max,c} n_{j,c} \\
 & n_{j,b} + n_{j,c} = n_{j,a} \\
 & h_f = a_0 + a_1 q_f + a_2 q_f^2 \\
 & q_j = \sum_{k=1}^{K_j} n_{j,k} q_{j,k} \\
 & \sum_{k=1}^{K_j} n_{j,k} = 1
 \end{aligned} \right. \quad (29)
 \end{aligned}$$

4. Case Studies

4.1. A Simple Case

4.1.1. Ventilation Network

To verify the effectiveness of the integrated regulation and optimization method of the ventilation network based on mixed-integer programming, regulation, and optimization simulation experiments are carried out on the ventilation network shown in Figure 1. The ventilation network contains 21 airways and 14 nodes, of which the 12th and 15th airways are the air demand airways, and the air demand is $30 \text{ m}^3/\text{s}$. The 21st airway is the fan airway, the fan model is K40-8-No25, the blade installation angle is 32° , and its fan characteristic curve is $H_f = -0.1014Q_f^2 + 13.90Q_f + 853.95$. The basic parameters of the ventilation network are shown in Table 1. From Table 1, it can be seen that No. 12 and No. 15 air-demanding airways can no longer meet the requirements of the air-demanding quantity needed for production operations, and it is necessary to optimize and regulate the ventilation network.

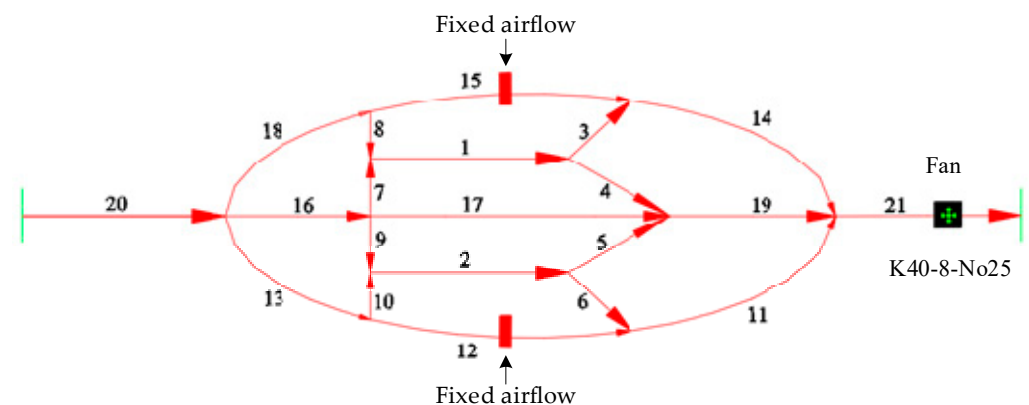


Figure 1. Example of optimizing ventilation network air quantity control.

Table 1. Basic parameters of the ventilation network.

Number	Type	$r \text{ N.s}^2/\text{m}^8$	$q \text{ m}^3/\text{s}$	$h \text{ Pa}$	s_j
1	General	0.03	27.416	22.631	−2
2	General	0.036	25.199	22.594	−3
3	General	0.012	18.643	4.084	0
4	General	0.018	8.774	1.356	−1
5	General	0.016	9.491	1.442	−1
6	General	0.013	15.708	3.189	−2
7	General	0.008	12.774	1.302	−5
8	General	0.007	14.642	1.396	−2
9	General	0.008	12.532	1.253	−5
10	General	0.007	12.666	1.045	−2
11	General	0.027	41.681	47.726	−2
12	Fixed airflow	0.04	25.973	26.827	0
13	General	0.034	38.64	50.799	−1
14	General	0.024	44.098	46.744	0
15	Fixed airflow	0.043	25.455	28.111	0
16	General	0.022	47.909	50.591	100
17	General	0.05	22.602	25.288	−2
18	General	0.031	40.097	50.496	−1
19	General	0.03	40.867	49.472	−1
20	General	0.028	126.646	450.053	100
21	Fan	0.026	126.646	−575.404	100

Note: The number of adjustment levels refers to how many airway regulation levels the user sets based on the airway's adjustability. A higher absolute value means the airway is less adjustable. Specifically, a larger positive value indicates the airway is less adjustable for increasing resistance regulation, while a larger negative value means it is less adjustable for increasing energy regulation or decreasing resistance regulation.

4.1.2. Optimization Results

The above ventilation network model is optimized using the model proposed in this paper, and the program is prepared using Python software (3.12.11) called Gurobi for solving the problem. The program was run on a computer with 12th Gen Intel (R) Core (TM) i7-12700 2.10 GHz CPU and 32G RAM. The optimized condition of the ventilator is (145.4 m³/s, 731.296 Pa) with a power of 106.33 kW, and the adjusting dampers are installed on airways 5, 6, 7, 12, 17, and 18 for drag increase adjustment, respectively. The optimized ventilation network parameters are shown in Table 2.

Table 2. Basic parameters of the optimized ventilation network.

Number	Before Optimization $q \text{ (m}^3/\text{s)}$	Before Optimization $h \text{ (Pa)}$	After Optimization $q \text{ (m}^3/\text{s)}$	After Optimization $h \text{ (Pa)}$	$\Delta h \text{ (Pa)}$
1	27.416	22.631	16.800	8.467	0
2	25.199	22.594	58.400	122.780	0
3	18.643	4.084	15.300	2.809	0
4	8.774	1.356	1.500	0.041	0
5	9.491	1.442	28.400	593.038	580.133
6	15.708	3.189	30.000	544.079	532.379
7	12.774	1.302	12.600	718.481	717.211
8	14.642	1.396	4.200	0.123	0
9	12.532	1.253	37.200	11.071	0
10	12.666	1.045	21.200	3.146	0

Table 2. Cont.

Number	Before Optimization q (m ³ /s)	Before Optimization h (Pa)	After Optimization q (m ³ /s)	After Optimization h (Pa)	Δh (Pa)
11	41.681	47.726	60.000	97.200	0
12	25.973	26.827	30.000	668.905	665.305
13	38.64	50.799	51.200	89.129	0
14	44.098	46.744	45.300	49.250	0
15	25.455	28.111	30.000	3.600	0
16	47.909	50.591	60.000	79.200	0
17	22.602	25.288	10.200	723.058	717.856
18	40.097	50.496	34.200	798.562	762.303
19	40.867	49.472	40.100	48.240	0
20	126.646	450.053	145.400	591.952	0
21	126.646	−575.404	145.400	−181.626	0

Before optimization, the power of the main fan in the ventilation network was 125.11 kW. After applying the optimization and control method based on mixed-integer programming, the power has been reduced by 15.01%. This effectively decreases the energy consumption of the ventilation network. Additionally, the optimization process requires the installation of only six regulating dampers to adjust the ventilation network, ensuring that the airflow in airways No. 12 and No. 15 meets the production operation requirements.

4.2. A Complex Case

4.2.1. Ventilation Network

In a metal mine ventilation network shown in Figure 2, there are 186 airways and 129 nodes, of which airways 21, 73, 111, 154, and 156 are air demand airways, with air demand of 10 m³/s, 24 m³/s, 20 m³/s, 10 m³/s, and 12 m³/s, respectively, and some of the air demand airways do not meet the requirements of the air demand. 1, 2, 3, 4, 5, 6, 7, and 16 are fan airways, and the fan models are shown in Table 3. Airways Nos. 1, 2, 3, 4, 5, 6, 7, and 16 are the fan airways, and the fan models are shown in Table 3. Some of the basic roadway parameter information is shown in Table 4.

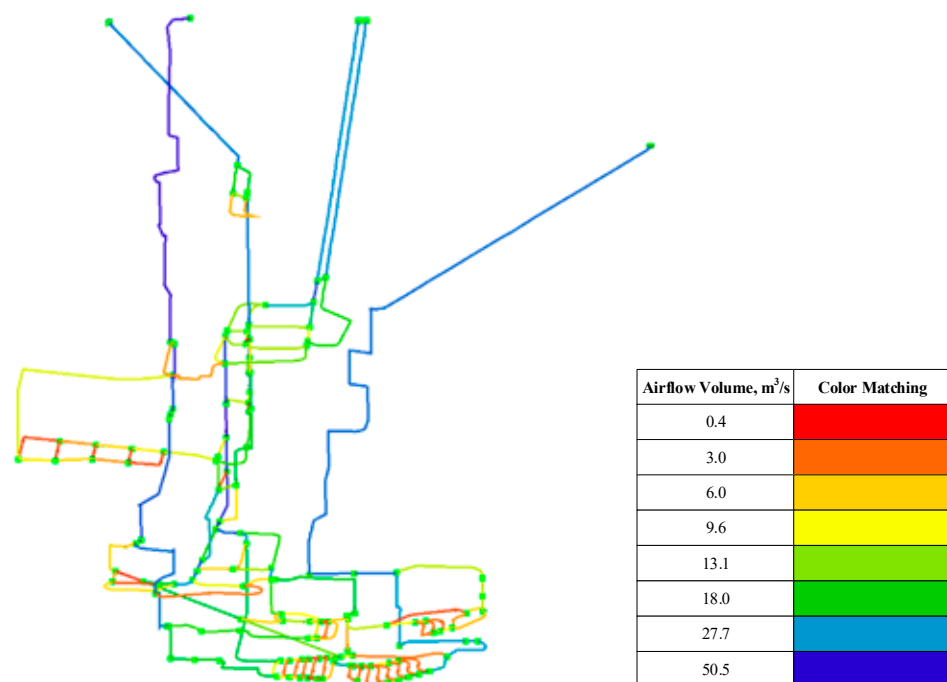


Figure 2. A metal mine ventilation network graph.

Table 3. Fan parameters.

Fan Roadway	Fan Models	Fan Characteristic Curve			Before Optimization		After Optimization	
		a_0	a_1	a_2	Q (m ³ /s)	H (Pa)	Q (m ³ /s)	H (Pa)
1	K40-4-No15	1038.6	50.055	−0.9013	956.3	57.1	59	854.4
2	K40-8-No18	408.78	18.13	−0.5344	196.5	43.1	43	200.3
3	K40-8-No18	408.78	18.13	−0.5344	196.5	43.1	43	200.3
4	K40-6-No12	398.88	21.483	−2.6334	131.4	15.0	15	128.6
5	K40-6-No13	433.66	24.514	−2.039	153.7	19.2	20	108.3
6	K40-6-No15	421.54	39.077	−0.8905	825.3	27.2	43	455.3
7	K40-4-No15	1038.6	50.055	−0.9013	1634.8	38.2	40	1598.7
16	K40-4-No10	617.87	33.808	−5.3003	348.368	11	13	161.6

Table 4. Part of the roadway foundation parameters.

Number	Type	r N.s ² /m ⁸	q m ³ /s	Regulator Levels	Airway Number	Airway Category	r kg/m ³	q m ³ /s	Regulator Levels
1	Fan	0.0038	57.1	100	57	General	0.0626	18.8	2
2	Fan	0.0113	43.1	100	66	Intake airflow	0.2209	34.6	1
3	Fan	0.0055	43.1	100	73	Fixed airflow	0.0095	24.0	−1
4	Fan	0.0391	15.0	100	81	General	0.0314	3.0	−2
5	Fan	0.0249	19.2	100	83	General	0.0240	3.0	0
6	Fan	0.0789	27.2	100	99	General	0.0040	23.4	−3
7	Fan	0.0409	38.2	100	111	Fixed airflow	0.1130	19.8	−4
16	Fan	0.0470	11.0	100	119	General	0.0438	12.6	−1
17	Return airflow	0.1370	57.1	2	128	Intake airflow	0.2955	30.7	−4
21	Fixed airflow	0.0277	10.0	1	129	Intake airflow	0.3109	30.1	−3
26	General	0.0095	9.7	−2	130	General	0.0276	11.5	−3
27	General	0.0031	4.7	2	131	General	0.0840	4.0	−2
37	General	0.0634	24.2	0	134	General	0.1039	15.7	−2
42	General	0.0185	5.9	−4	154	Fixed airflow	0.0252	9.0	5
44	General	0.0245	9.6	−4	156	Fixed airflow	0.0122	11.5	1
54	General	0.0124	4.5	2	168	General	0.2250	6.6	−2
55	General	0.0093	1.5	2	178	Return airflow	1.2300	38.2	−5

4.2.2. Optimization Results

We implemented the regulation optimization program and tested it on a Windows 64-bit computer equipped with a 3.20 GHz Intel (R) Core (TM) i9-14900K processor, 64.0 GB RAM, and an NVIDIA GeForce RTX 4080 GPU. The program is developed by the Python language and uses the GUROBI solver (12.0.3) to solve the mixed-integer programming model.

As a comparison, the genetic algorithm (GA) and the differential evolution (DE) algorithm are implemented to solve the nonlinear optimization model. The program is developed by PlatEMO (4.13) [32] (A MATLAB (R2024a) Platform for Evolutionary Multi-Objective Optimization). Unfortunately, due to the complexity of the model, the iterative process of evolutionary algorithms tends to diverge, resulting in no feasible solution. Figure 3 demonstrates the solution process in this case of ventilation network.

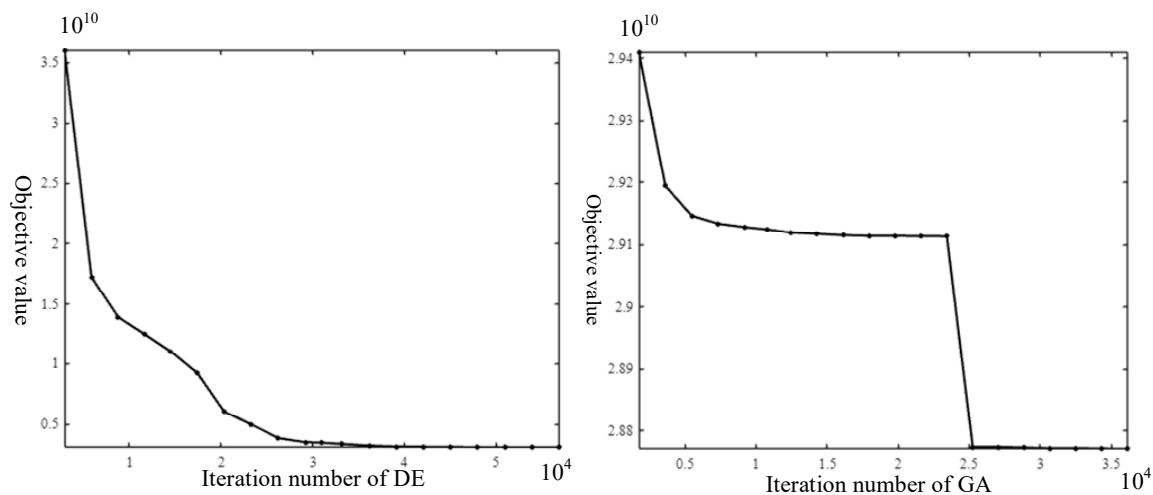


Figure 3. Solution process of nonlinear optimization model.

Different regulation optimization schemes can be obtained by adjusting the objective weights and re-solving the mathematical model. Adjusting the weight coefficients of objectives can distinguish the importance differences among various objectives. The specific values of these weight coefficients are generally determined by the users based on specific regulatory requirements and actual circumstances. During practical calculations, it has been found that the regulation position objectives and regulation number objectives are crucial factors in determining the feasibility of ventilation optimization schemes. Therefore, higher weights should be assigned to these corresponding objectives.

Besides the objective weights, the airflow regulation accuracy (discrete interval value of linearization strategy) is the key parameter that affects optimization results. Smaller airflow regulation accuracy intervals will significantly increase the scale of integer variables in the mathematical model, while larger intervals may compromise the reliability of the solution results. It is recommended to set the airflow regulation accuracy within the interval of (0.1, 2.0). Figure 4 demonstrates the relationship between the airflow regulation accuracy and the number of integer variables in this case of ventilation network.

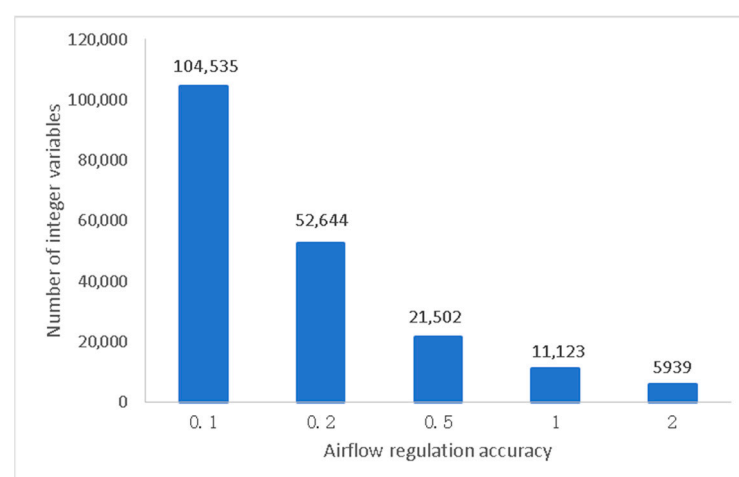


Figure 4. Airflow regulation accuracy versus number of integer variables.

Based on the mathematical model of mixed integer programming proposed in this paper, the ventilation network of this mine is optimized with the airflow regulation accuracy set to $1.0 \text{ m}^3/\text{s}$. The optimizer utilizes the airway-and-board method to calculate the regulation result. When the number of integer variables exceeds tens of thousands,

the solution process of the mathematical model typically requires tens of hours. As a compromise strategy, suboptimal solutions can be obtained by setting a predefined gap value between feasible solutions and the relaxed optimal solution. Figure 5 illustrates the convergence process of the solver under different airflow regulation accuracy settings.

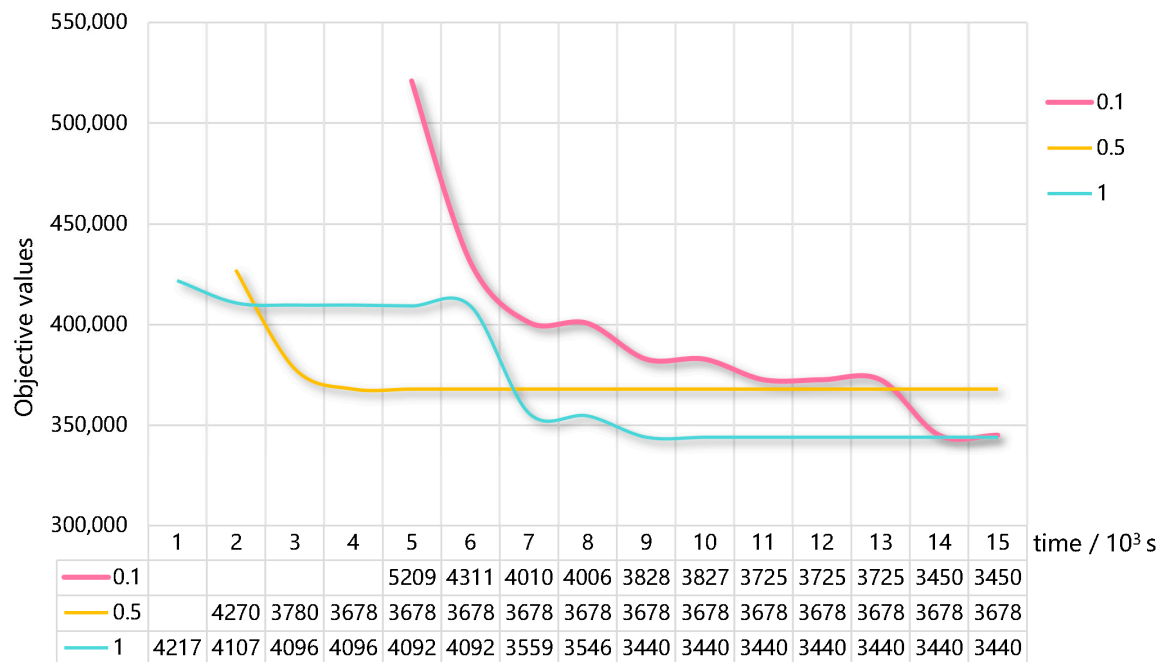


Figure 5. Convergence analysis of the solution process using different airflow regulation optimization accuracy.

The comparison of the main ventilation fan parameters before and after the optimization is shown in Table 3, and the optimal ventilation network control scheme is shown in Table 5. As shown in Table 3, the total power of the fan before optimization is 165.31 kW, and the total power of the fan after optimization is 157.36 kW, and the energy consumption of the ventilation network is reduced by 4.81%. As shown in Table 5, the air demand of the demand airways of the optimized ventilation network can be satisfied, and the optimization of the whole ventilation network can be completed only by adjusting the resistance of the corresponding airways. The optimized ventilation network also reduces the ventilation energy consumption while meeting the requirements of air demand, which is of great significance to the safe production and energy saving of the mine.

Table 5. Ventilation network regulation program.

Number	Type	r kg/m ³	Before Optimization Q (m ³ /s)	After Optimization Q (m ³ /s)	Δh (Pa)	Regulator Levels
26	General	0.0095	9.7	17	−61.4	−2
27	General	0.0031	4.7	33	−119.6	2
37	General	0.0634	24.2	39	−62.3	0
42	General	0.0185	5.9	1	−44.4	−4
44	General	0.0245	9.6	45	−144.0	−4
54	General	0.0124	4.5	3	98.7	2
55	General	0.0093	1.5	3	97.3	2
57	General	0.0626	18.8	12	78.7	2
66	Intake airflow	0.2209	34.6	57	−664.2	1
81	General	0.0314	3.0	1	1.1	−2

Table 5. Cont.

Number	Type	r kg/m ³	Before Optimization Q (m ³ /s)	After Optimization Q (m ³ /s)	Δh (Pa)	Regulator Levels
83	General	0.0240	3.0	1	1.5	0
99	General	0.0040	23.4	18	−47.4	−3
119	General	0.0438	12.6	1	20.3	−1
129	Intake airflow	0.3109	30.1	21	24.5	−3
130	General	0.0276	11.5	36	−55.2	−3
131	General	0.0840	4.0	9	155.8	−2
134	General	0.1039	15.7	1	43.4	−2
168	General	0.2250	6.6	20	−91.2	−2
178	Return airflow	1.2300	38.2	41	−748.5	−5

5. Conclusions and Discussion

Due to the limitations of the solution methods of nonlinear multi-objective optimization mathematical models and the specific constraints of the actual mine ventilation network optimization problem, there is currently no universally applicable mine ventilation optimization control model. This paper first transformed the nonlinear ventilation optimization control model into a linear control model through the variable discretization strategy, avoiding the limitation of the difficulty in solving nonlinear mathematical models and opening up a new solution for the mine ventilation network optimization control calculation. Based on the analysis of the ventilation model control objectives and control constraints, the mathematical formula of the control optimization model was derived, and the reliability of the ventilation network model control was verified through ventilation control examples.

The ventilation system optimization model only optimizes the adjustment method with the minimum energy consumption as the objective, which is usually not the best adjustment method and is generally difficult to directly apply to the actual ventilation adjustment process underground. In the actual mine ventilation optimization and regulation problem, there is usually more than one indicator to measure the quality of the adjustment plan. In addition to the minimum ventilation energy consumption, it is also necessary to consider the cost of the adjustment facilities and the feasibility of the adjustment plan. Constraints such as the adjustment position, adjustment method, and number of adjustments are also considered. The ventilation network air quantity regulation optimization mathematical model proposed in this paper constructs target constraints such as minimum ventilation energy consumption, the minimum number of adjustment points, and the optimal adjustment point position, which improves the flexibility of the ventilation network optimization adjustment plan.

The variable discretization process introduces a large number of 0–1 integer variables. Although it reduces the complexity of solving nonlinear models and improves the model convergence efficiency, it increases the solution space of linear models. The efficient solution of large-scale linear control models will become an important research direction for the optimization and regulation of mine ventilation. To improve the solution efficiency, one feasible idea is to reduce the number of 0–1 integer variables by limiting the range of ventilation prior constraints, such as the air quantity value. Another feasible strategy is to use a heuristic search-based method to optimize the airway and bound calculation process of the mixed integer programming mathematical model. This includes designing smarter node selection strategies (such as those based on estimated lower bounds or problem-specific heuristic information), variable branching strategies (prioritizing the branching of variables with a significant impact on the objective), and developing efficient cutting plane generation methods or pre-solving techniques. We have conducted a lot of research on the optimization of large-scale mixed-integer programming mathematical

models. In the future, we will further optimize the efficiency of solving complex ventilation network air volume regulation models based on the above strategies.

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