mathematics

MDPI

Article

Fuzzy TOPSIS Reinvented: Retaining Linguistic Information Through Interval-Valued Analysis

Abdolhanan Aminoroaya [1], Abdollah Hadi-Vencheh [1,*], Ali Jamshidi [1] and Amir Karbassi Yazdi [2]

[1] Department of Mathematics, Isf. C., Islamic Azad University, Isfahan 39998-8155, Iran; aminoroaya@iau.ac.ir (A.A.); a.jamshidi@iau.ac.ir (A.J.)
[2] Departamento de Ingenieria, Industrial y de Sistemas, Facultad de Ingenieria, Universidad de Tarapaca, Arica 1010069, Chile; akarbassiy@academicos.uta.cl
* Correspondence: ahvencheh@iau.ac.ir

Abstract

In real-world decision-making situations, experts often rely on subjective and imprecise judgments, frequently expressed using linguistic terms. While fuzzy logic offers a valuable tool to capture and process such uncertainty, traditional methods often convert fuzzy inputs into crisp values too early in the process. This premature defuzzification can result in significant loss of information and reduced interpretability. To address this issue, the present study introduces an enhanced fuzzy TOPSIS model that utilizes expected interval representations instead of early crisp transformation. This approach allows the original fuzzy data to be preserved throughout the analysis, leading to more transparent, realistic, and informative decision outcomes. The practical application of the proposed method is demonstrated through a supplier selection case study, which illustrates the model's capability to handle real-world, complex, and qualitative decision environments. By explicitly linking the method to this domain, the study provides a concrete anchor for practitioners and decision-makers seeking transparent and robust evaluation tools.

Keywords: decision processes; multiple criteria decision-making (MCDM); TOPSIS; expected interval; linguistic variable; supplier selection

MSC: 90B50

check for updates

Academic Editors: Pavel Anselmo Álvarez and Ernesto León-Castro

Received: 26 July 2025
Revised: 22 August 2025
Accepted: 25 August 2025
Published: 2 September 2025

Citation: Aminoroaya, A.; Hadi-Vencheh, A.; Jamshidi, A.; Karbassi Yazdi, A. Fuzzy TOPSIS Reinvented: Retaining Linguistic Information Through Interval-Valued Analysis. *Mathematics* **2025**, *13*, 2819. https://doi.org/10.3390/math13172819

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

In today's increasingly complex decision-making environments, individuals and organizations often need to assess multiple alternatives under varying degrees of uncertainty. Such uncertainty frequently stems from the imprecise, vague, or subjective nature of human judgments, which are commonly expressed through linguistic terms rather than exact numerical values. Traditional mathematical models struggle to accommodate this ambiguity, prompting the integration of fuzzy set theory, first introduced by Zadeh [1], as a powerful tool for handling uncertainty in decision-making contexts. Among the various multi-criteria decision-making (MCDM) methods, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has gained popularity due to its straightforward logic, computational efficiency, and ease of implementation [2].

In this study, TOPSIS was specifically chosen over other MCDM methods such as VIKOR, MARCOS, ARAS, and EDAS because of its simplicity, ability to efficiently handle interval-valued data, intuitive ranking process, and compatibility with preserving linguistic information in fuzzy environments. Unlike earlier works on fuzzy TOPSIS and its

variants (e.g., interval-valued fuzzy TOPSIS, D-TOPSIS, Z-TOPSIS), our approach avoids early defuzzification, retains the full semantic richness of expert judgments, and remains computationally simple for large-scale decision problems. The motivation behind this study lies in bridging that gap. Our aim is to design a fuzzy TOPSIS model that not only avoids early defuzzification, but also remains computationally simple and applicable to group decision-making problems. The proposed model introduces the concept of expected interval-based evaluation, allowing all fuzzy assessments to retain their semantic depth until the final ranking stage. This approach improves the transparency of the process and ensures greater alignment with human reasoning patterns. The novelty of this paper lies in its ability to balance two critical needs: maintaining fuzzy data integrity and preserving analytical clarity. Through a structured decision-making model and a real-world example, we demonstrate that our method enhances both interpretability and practical usability, particularly in settings involving multiple experts and qualitative judgments. This makes the approach suitable for a broad range of applications in engineering, management, and public policy.

The remainder of this paper is organized as follows. Section 2 presents the theoretical background and a brief review of related literature in fuzzy MCDM and TOPSIS methods. Section 3 introduces the proposed interval-based fuzzy TOPSIS approach, detailing its mathematical formulation and procedural steps. Section 4 provides a numerical case study to demonstrate the model's applicability and performance. Section 5 compares the results with existing methods to highlight the advantages of the proposed approach. Finally, Section 6 concludes the paper with key findings and suggestions for future research.

2. Literature Review

2.1. Classical TOPSIS Approaches

Multi-criteria decision-making (MCDM) refers to a set of techniques used for evaluating and ranking multiple alternatives based on a set of criteria. These techniques have gained significant attention in fields such as engineering, management, supply chain, and public administration, where decisions must often be made under conflicting priorities [3]. Among the classical methods, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) has been widely applied due to its conceptual simplicity and ability to reflect decision-maker preferences. The original TOPSIS approach assumes that all criteria evaluations are available in crisp numerical form. However, in many real-world problems, the input data are often uncertain, vague, or qualitative in nature. To address this limitation, fuzzy set theory has been incorporated into MCDM methods, resulting in the development of fuzzy TOPSIS [4,5]. This integration allows decision-makers to use linguistic variables (e.g., "very good," "medium," "poor") to express their preferences, which are then translated into fuzzy numbers for processing. Over the years, several extensions to fuzzy TOPSIS have been proposed to improve its accuracy and adaptability to complex decision environments. Some notable approaches have focused on enhancing the ranking process or on modeling group decision-making scenarios. For instance, Abootalebi et al. [6] introduced a modified ranking mechanism that improves the separation of alternatives in fuzzy environments, while Sadabadi et al. [7] proposed a fuzzy TOPSIS index that better accounts for the distribution of evaluations and avoids misleading prioritizations in group settings. Despite these efforts, a common issue remains: most fuzzy TOPSIS variants require early defuzzification, which reduces the richness and interpretability of fuzzy input [8]. This can weaken the model's ability to reflect true preferences, especially in complex or expert-driven contexts. To overcome this, some researchers have advocated for the use of interval-valued fuzzy sets or hesitant fuzzy data, allowing more information to be retained throughout the evaluation process [9]. The motivation behind the current

study lies in bridging the gap between simplicity and information preservation. While many advanced models provide higher precision, they often do so at the cost of increased computational burden or limited interpretability. The approach proposed in this paper seeks to maintain the integrity of fuzzy data without sacrificing the usability of the model.

The original TOPSIS method, introduced by Hwang and Yoon [2], remains a widely endorsed MCDM approach due to its clarity and low computational cost. However, researchers have noted drawbacks such as sensitivity to normalization techniques and the possibility of rank reversal when alternatives are modified. For example, the decision-making framework which combines color spectrum visualization with D-TOPSIS in rough environments demonstrates enhanced interpretability and stability, but requires careful parameter tuning [2].

2.2. Fuzzy and Uncertainty-Based Extensions

Fuzzy extensions of TOPSIS improve its ability to handle linguistic or vague inputs. A notable example is the use of interval-valued Fermatean fuzzy TOPSIS for COVID-19 vaccine evaluation, which maintains uncertainty throughout the analysis but increases model complexity [10]. Another study employing Z-numbers introduces reliability-based group consensus, a compelling method for decision environments where evaluator confidence matters, though it may suffer from computational effort [11]. A critical methodological review highlights that Euclidean distance may misrepresent fuzzy differences, advising more suitable distance measures in fuzzy TOPSIS applications [12].

2.3. Hybrid MODELS

Hybrid approaches attempt to harness the strengths of multiple frameworks. A hybrid DEA-TOPSIS model (TEA-IS) for assessing healthcare system performance in China combines efficiency analysis with ranking clarity, improving benchmarking over time, but also increasing implementation complexity [13]. Separately, a hybrid inventory classification model using alternative factor extraction improves resilience against multicollinearity, yet relies heavily on domain-specific parameter calibration [14].

2.4. Domain-Specific Applications

TOPSIS has been adapted successfully across sectors. In supply chain decision-making, a modified TOPSIS algorithm was designed to alleviate ranking instability, yielding more consistent results in logistics and transportation contexts. Meanwhile, interval-valued Fermatean fuzzy TOPSIS was utilized to benchmark COVID-19 vaccines, demonstrating practical value in health policy despite its higher abstraction level [10].

With the rise of fuzzy extensions of TOPSIS, many researchers have attempted to improve its applicability to real-world decision problems involving linguistic and subjective inputs [15]. Despite these advancements, a recurring limitation persists in traditional fuzzy TOPSIS models: the early defuzzification of fuzzy inputs [16]. This transformation, often introduced to simplify computations, compromises the richness of original data and reduces the model's ability to capture nuanced expert opinions. Moreover, it undermines the very advantage that fuzzy logic offers, representing uncertainty without prematurely forcing precise values [17,18]. Recent studies have begun to address this by preserving fuzziness through interval-based or hesitant fuzzy representations [19,20], yet the computational frameworks of many such models remain either overly complex or insufficiently validated through real applications [21].

3. Theoretical Background

This section presents the theoretical foundations and essential prerequisites necessary for understanding the proposed method. It begins by introducing the fundamental concepts,

key definitions, and relevant terminologies related to the problem under study. Next, it outlines the theoretical frameworks that form the basis of the research. The aim is to equip the reader with the conceptual and analytical tools needed to follow the methodology and to highlight the logical link between established theory and the innovations introduced in this work.

3.1. Interval Arithmetic

Interval arithmetic provides a flexible framework for representing and manipulating uncertain data without the need for early defuzzification. An interval number is typically defined as $[a] = [a_1, a_2]$, where a_1 and a_2 represent the lower and upper bounds of the interval, capturing the range of possible values that an imprecise quantity might assume. This representation is particularly suitable for modeling expert opinions and linguistic terms in multi-criteria decision-making (MCDM) contexts. Basic arithmetic operations between two interval numbers $[a] = [a_1, a_2]$ and [b] = $[b_1, b_2]$ are carried out using the following rules [22]:

(1) [a] + [b] = $[a_1 + b_1, a_2 + b_2]$
(2) [a] − [b] = $[a_1 - b_2, a_2 - b_1]$
(3) [a]. [b] = [min $\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$, $max\{a_1b_1, a_1b_2, a_2b_1, a_2b_2\}$]
(4) [a]/[b] = $[a_1, a_2]$. $[1/b_2, 1/b_1]$ $(0 \notin [b])$

The use of interval arithmetic in fuzzy decision-making allows uncertain evaluations to be processed without collapsing them into crisp values too early. This helps preserve the full semantic range of expert assessments, thereby producing more reliable and interpretable results. Sevastjanov and Dymova [22] demonstrated that interval-based modeling provides greater robustness in group decision-making, while Yang et al. [19] recently emphasized its advantages in handling inconsistency and preference variability in large-scale fuzzy MCDM problems.

3.2. Transformation of Fuzzy Numbers to Expected Intervals

In fuzzy decision-making models, trapezoidal fuzzy numbers (TFNs) are frequently used to represent linguistic terms. These fuzzy numbers, defined by four parameters $\widetilde{a} = (a_1, a_2, a_3, a_4)$, offer a convenient way to express uncertainty through their continuous membership functions. While defuzzification is often used to simplify these fuzzy numbers into crisp values, doing so at early stages can lead to significant information loss. To avoid this, the expected interval representation has been proposed as an alternative that retains the core uncertainty of the fuzzy numbers while enabling numerical computation. Based on the method introduced by Heilpern [23], the expected interval of a trapezoidal fuzzy number $\widetilde{a}$ can be expressed as

$$EI\ (\widetilde{a}) = [\frac{1}{2}(a_1 + a_2),\ \ \frac{1}{2}(a_3 + a_4)] \tag{1}$$

This transformation effectively captures the central spread of the fuzzy number without collapsing it into a single value. It has been shown to preserve more semantic information compared to early defuzzification methods, especially when evaluating expert-based linguistic inputs [24]. This interval representation forms the basis for the computations in our proposed method, allowing fuzzy data to remain interpretable and structurally intact throughout the decision-making process.

3.3. Modeling Linguistic Variables as Fuzzy Numbers

In this study, both the importance weights of the criteria and the ratings assigned to qualitative attributes are expressed using linguistic variables. This approach is particu-

larly beneficial in decision environments where precise numerical data are unavailable or where expert judgments are inherently vague. Linguistic terms provide a structured yet flexible way to represent subjective opinions using natural language. These linguistic expressions are systematically converted into trapezoidal fuzzy numbers, enabling their integration into the fuzzy MCDM framework. The corresponding fuzzy scales are illustrated in Tables 1 and 2, based on the commonly adopted mappings [25]. This transformation ensures that qualitative assessments are processed consistently and remain mathematically tractable within the decision model.

Table 1. Linguistic variables for rates.

Very Poor (VP)	(0,1,1,2)
Poor (P)	(1,2,2,3)
Medium Poor (MP)	(2,3,4,5)
Fair (F)	(4,5,5,6)
Medium Good (MG)	(5,6,7,8)
Good (G)	(7,8,8,9)
Very Good (VG)	(8,9,9,10)

Table 2. Linguistic variables for the importance of each criterion.

Very Low (VL)	(0,0.1,0.1,0.2)
Low (L)	(0.1,0.2,0.2,0.3)
Medium Low (ML)	(0.2,0.3,0.4,0.5)
Medium (M)	(0.4,0.5,0.5,0.6)
Medium High (MH)	(0.5,0.6,0.7,0.8)
High (H)	(0.7,0.8,0.8,0.9)
Very High (VH)	(0.8,0.9,0.9,1)

It is supposed through this paper that the DMs use the linguistic variables shown in Tables 1 and 2 to evaluate the importance of the criteria and the ratings of alternatives with respect to criteria.

4. Proposed Method

To address the limitations of early defuzzification in conventional fuzzy TOPSIS models, this study proposes an enhanced interval-based extension of the FTOPSIS method. In our approach, fuzzy ratings and weights expressed as linguistic variables are first translated into trapezoidal fuzzy numbers and then transformed into expected intervals. This preserves the fuzzy nature of the data and allows for more comprehensive comparison among alternatives.

4.1. Linguistic Evaluation and Aggregation

Linguistic variables are particularly useful when decision-makers (DMs) provide subjective assessments under ambiguity. Tables of linguistic scales are used to convert terms such as Good, Medium, or Very Poor into trapezoidal fuzzy numbers. For group decision-making, individual fuzzy evaluations from multiple DMs are aggregated to a collective fuzzy rating. The aggregation process ensures that the collective fuzzy range encompasses all individual opinions, enhancing inclusivity [26]. Let the aggregated fuzzy ratings and weights be represented as R_{ij} and w_j. These trapezoidal fuzzy numbers are then converted to expected intervals using the formulation by [23]. The resulting interval decision matrix D and weight vector W reflect the expected value range of each fuzzy input.

4.2. Interval-Based Fuzzy TOPSIS Algorithm

Let $x_{ij} = [x_{ij}^l, x_{ij}^u]$ be the $(i,\ j)$-th component of the decision matrix and $w_j = [w_j^l, w_j^u]$. The proposed method extends the classical TOPSIS approach to handle interval-valued fuzzy data, ensuring that uncertainty is preserved throughout the ranking process. Let the normalized interval decision matrix be denoted as $N = [n_{ij}]_{m\times n}$, where

$$n_{ij} = [n_{ij}^l, n_{ij}^u]; i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{2}$$

Step 1. For each criterion j, the normalization of the interval value is defined as

$$[n_{ij}^l, n_{ij}^u] = [x_{ij}^l / c_j^+, x_{ij}^u / c_j^+]; i = 1, 2, \ldots, m,\ j \in B, \text{ and } c_j^+ = \max_i x_{ij}^u \tag{3}$$

$$[n_{ij}^l, n_{ij}^u] = [a_j^- / x_{ij}^u, a_j^- / x_{ij}^l]; i = 1, 2, \ldots, m,\ j \in C, \text{ and } a_j^- = \min_i x_{ij}^l. \tag{4}$$

Step 2. The weighted normalized interval matrix V is computed as

$$V = [v_{ij}]_{m\times n}, \tag{5}$$

$$v_{ij} = [v_{ij}^l, v_{ij}^u];\ i = 1, 2, \ldots,\ m, j = 1, 2, \ldots,\ n. \tag{6}$$

In which

$$[v_{ij}^l, v_{ij}^u] = [n_{ij}^l, n_{ij}^u].[w_j^l, w_j^u];\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{7}$$

Step 3. The positive ideal solution and negative ideal solution are defined as

$$A^+ = \{v_1^+, v_2^+, \ldots,\ v_n^+\} \quad , \quad A^- = \{v_1^-, v_2^-, \ldots,\ v_n^-\}$$

where

$$v_j^+ = \max_i\{v_{ij}^u\},\ \ v_j^- = \min_i\{v_{ij}^l\};\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{8}$$

Step 4. The distance of each alternative from the ideal solution $\left[D_{i2}^+, D_{i1}^+\right]$ is defined by

$$D_{i1}^+ = \sqrt{\sum_{j=1}^n (v_{ij}^l - v_j^+)^2} \text{ and } D_{i2}^+ = \sqrt{\sum_{j=1}^n (v_{ij}^u - v_j^+)^2},\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{9}$$

Similarly, the separation from the negative ideal solution is given by $\left[D_{i1}^-, D_{i2}^-\right]$, which is computed by

$$D_{i1}^- = \sqrt{\sum_{j=1}^n (v_{ij}^l - v_j^-)^2} \text{ and } D_{i2}^- = \sqrt{\sum_{j=1}^n (v_{ij}^u - v_j^-)^2},\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n. \tag{10}$$

It is worth indicating that in this way we lose less information (data values) than when just converting immediately to crisp values.

Step 5. The relative closeness of the $i-$th alternative to the ideal solution is shown as $R_i^* = [Rc_{li},\ Rc_{ui}]$, where

$$Rc_{li} = \frac{D_{i1}^-}{D_{i1}^- + D_{i1}^+},\ \ Rc_{ui} = \frac{D_{i2}^-}{D_{i2}^- + D_{i2}^+} \tag{11}$$

Step 6. Now we judge on alternatives based on R_i^* interval numbers, and corresponding to each alternative we set a linguistic variable. For this purpose, first note that R_i^* is a subset of [0, 1]; a simple way to judge is that we portion, for instance, the [0, 1] into five

sub-intervals as follows. This approach enhances interpretability and supports decision transparency [27]. We then classify the alternatives in five classes, as shown in Table 3.

Table 3. Approval status.

Relative Closeness (Rc_i^*)	**Linguistic Variable**
$Rc_i^* \in [0, 0.2)$	Do not recommend
$Rc_i^* \in [0.2, 0.4)$	Recommend with high risk
$Rc_i^* \in [0.4, 0.6)$	Recommend with low risk
$Rc_i^* \in [0.6, 0.8)$	Approved
$Rc_i^* \in [0.8, 1.0)$	Approved and preferred

The Figure 1 provides a visual summary of the main stages, from collecting linguistic evaluations to ranking alternatives using the proposed approach.

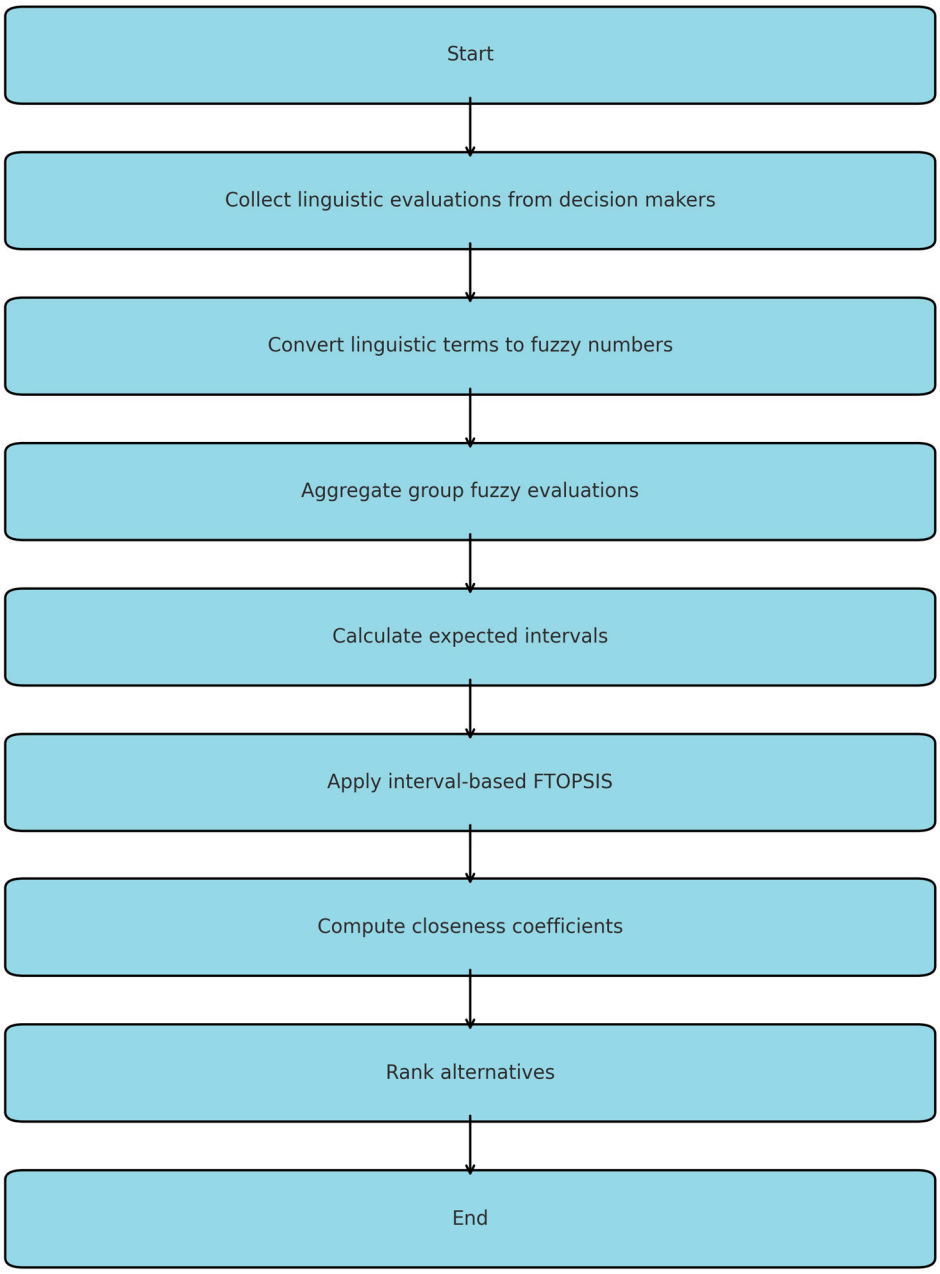

Figure 1. The flowchart of the proposed approach.

4.3. Comparative Advantage

The proposed interval-based fuzzy TOPSIS method offers several advantages over existing models:

No Early Defuzzification: Avoids premature loss of information, unlike earlier models such as those by Wang and Elhag [28].

Linguistic Output Classification: Enhances transparency by translating scores into meaningful qualitative judgments.

Improved Computational Efficiency: Eliminates the need for solving non-linear programs, making it scalable for large decision matrices.

Interval Sensitivity: Maintains uncertainty throughout, aligning with recent research directions in fuzzy decision systems [29]

Compared to the method of Yue [30], which uses a possibility-based ranking and complementary matrices, our approach is both simpler to implement and more intuitive for group decision-making under uncertainty.

5. Illustrative Example

To demonstrate the practicality and efficiency of the proposed interval-based fuzzy TOPSIS method, a real-world case study adapted from Bin Azim et al. [31] is presented. The scenario involves selecting the most appropriate supplier for a high-tech manufacturing firm that depends on reliable component sourcing.

5.1. Problem Description

A company aims to select the most suitable supplier for critical product components. After initial screening, five supplier candidates remain. A decision-making committee composed of three experts evaluates each supplier based on five benefit criteria:

1. Supplier's profitability (C1);
2. Relationship closeness (C2);
3. Technological capability (C3);
4. Conformance quality (C4);
5. Conflict resolution ability (C5).

Experts assess both the importance of each criterion and the performance of each supplier using predefined linguistic terms. These qualitative judgments are converted into trapezoidal fuzzy numbers based on the fuzzy linguistic scales shown in Tables 4 and 5. The individual assessments are then aggregated using Equations (4) and (5), and the final combined judgments are reported in Table 6.

Table 4. Importance weight of criteria based on three DMs.

Criteria	**DMs**		
	D_1	D_2	D_3
C_1	H	H	H
C_2	VH	VH	VH
C_3	VH	VH	H
C_4	H	H	H
C_5	H	H	H

Table 5. DMs' assessments based on each criterion.

Criteria	Suppliers	DMs		
		D_1	D_2	D_3
C_1	A_1	MG	MG	MG
	A_2	G	G	G
	A_3	VG	VG	G
	A_4	G	G	G
	A_5	MG	MG	MG
C_2	A_1	MG	MG	VG
	A_2	VG	VG	VG
	A_3	VG	G	G
	A_4	G	G	MG
	A_5	MG	G	G
C_3	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	VG	VG	G
	A_4	MG	MG	G
	A_5	MG	MG	MG
C_4	A_1	G	G	G
	A_2	G	VG	VG
	A_3	VG	VG	VG
	A_4	G	G	G
	A_5	MG	MG	G
C_5	A_1	G	G	G
	A_2	VG	VG	VG
	A_3	G	VG	G
	A_4	G	G	VG
	A_5	MG	MG	MG

Table 6. The fuzzy decision matrix and fuzzy weights.

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	(5,6,7,8)	(5,7,8,10)	(7,8,8,9)	(7,8,8,9)	(7,8,8,9)
A_2	(7,8,8,9)	(8,9,10,10)	(8,9,10,10)	(7,8.7,9.3,10)	(8,9,10,10)
A_3	(7,8.7,9.3,10)	(7,8.3,8.7,10)	(7,8.7,9.3,10)	(8,9,10,10)	(7,8.3,8.7,10)
A_4	(7,8,8,9)	(5,7.3,7.7,9)	(5,6.7,7.3,9)	(7,8,8,9)	(7,8.3,8.7,10)
A_5	(5,6,7,8)	(5,7.3,7.7,9)	(5,6,7,8)	(5,6.7,7.3,9)	(5,6,7,8)
Weighted	(0.7,0.8,0.8,0.9)	(0.8,0.9,1,1)	(0.7,0.87,0.93,1)	(0.7,0.8,0.8,0.9)	(0.7,0.8,0.8,0.9)

5.2. Implementation of the Proposed Method

The proposed interval-based fuzzy TOPSIS approach is applied to the problem as follows.

Step 1: Using Equation (2), the expected interval decision matrix is generated, as presented in Table 7.

Table 7. The expected interval matrix and weights.

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	[5.5,7.5]	[6,9]	[7.5,8.5]	[7.5,8.5]	[7.5,8.5]
A_2	[7.5,8.5]	[8.5,10]	[8.5,10]	[7.85,9.65]	[8.5,10]
A_3	[7.85,9.65]	[7.65,9.35]	[7.85,9.65]	[8.5,10]	[7.65,9.35]
A_4	[7.5,8.5]	[6.15,8.35]	[5.85,8.15]	[7.5,8.5]	[7.65,9.35]
A_5	[5.5,7.5]	[6.15,8.35]	[5.5,7.5]	[5.85,8.15]	[5.5,7.5]
Weighted	[0.75,0.85]	[0.85,1]	[0.785,0.965]	[0.75,0.85]	[0.75,0.85]

Step 2: The normalized version of this matrix is calculated (Table 8).

Table 8. The normalized expected interval matrix.

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	[0.5699,0.7772]	[0.6,0.9]	[0.75,0.85]	[0.75,0.85]	[0.75,0.85]
A_2	[0.7772,0.8808]	[0.85,1]	[0.85,1]	[0.785,0.965]	[0.85,1]
A_3	[0.8135,1]	[0.765,0.935]	[0.785,0.965]	[0.85,1]	[0.765,0.935]
A_4	[0.7772,0.8808]	[0.615.0.835]	[0.585,0.815]	[0.75,0.85]	[0.765,0.935]
A_5	[0.5699,0.7772]	[0.615,0.835]	[0.55,0.75]	[0.585,0.815]	[0.55,0.75]

Step 3: By incorporating the aggregated weights, the weighted normalized expected interval matrix is formed (Table 9).

Table 9. The weighted normalized expected interval matrix.

Alternative	C_1	C_2	C_3	C_4	C_5
A_1	[0.4275,0.6606]	[0.51,0.9]	[0.5888,0.8202]	[0.5625,0.7225]	[0.5625,0.7225]
A_2	[0.5829,0.7487]	[0.7225,1]	[0.6673,0.965]	[0.5887,0.8203]	[0.6375,0.85]
A_3	[0.6101,0.85]	[0.6502,0.935]	[0.6162,0.9312]	[0.6375,0.85]	[0.5737,0.7947]
A_4	[0.5829,0.7487]	[0.5227,0.835]	[0.4592,0.7865]	[0.5625,0.7225]	[0.5737,0.7947]
A_5	[0.4275,0.6606]	[0.5227,0.835]	[0.4318,0.7238]	[0.4387,0.6927]	[0.4125,0.6375]

Step 4: The positive and negative ideal solutions are determined using Equation (8) as follows:

A^+ = {0.85, 1, 0.965, 0.85, 0.85},

A^- = {0.4275, 0.51, 0.4318, 0.4387, 0.4125}.

Step 5: The distances of each supplier from the ideal and anti-ideal solutions are computed using Equations (9) and (10), and the results are summarized in Table 10. These distances are represented as intervals to maintain the uncertainty of original assessments.

Table 10. The distance from the ideal solution and negative ideal solution.

Alternative	$[D^+_{i2}, D^+_{i1}]$	$[D^-_{i1}, D^-_{i2}]$
A_1	[0.3152,0.8518]	[0.2499,0.7307]
A_2	[0.1056,0.592]	[0.4448,0.9821]
A_3	[0.0918,0.6504]	[0.3906,0.9612]
A_4	[0.2978,0.8449]	[0.2577,0.7492]
A_5	[0.4372,1.0253]	[0.0128,0.6003]

Step 6: Interval-valued closeness coefficients are computed and shown in Tables 11 and 12.

Table 11. The interval of relative closeness.

Alternative	$[Rc_l, Rc_u]$
A_1	[0.2269,0.6987]
A_2	[0.4291,0.9029]
A_3	[0.3752,0.9129]
A_4	[0.2337,0.7156]
A_5	[0.0123,0.5786]

Table 12. Approval status rate.

Relative Closeness (Rc_i^*)	**Assessment Status**
$R_i^* \in [0, 0.2)$	Do not recommend
$R_i^* \in [0.2, 0.4)$	Recommend with high risk
$R_i^* \in [0.4, 0.6)$	Recommend with low risk
$R_i^* \in [0.6, 0.8)$	Approved
$R_i^* \in [0.8, 1.0)$	Approved and preferred

5.3. Linguistic Interpretation

Instead of relying solely on numeric closeness scores, the interval [0, 1] is divided into five equal sub-intervals, each associated with a linguistic category (i.e., Rejected, Low-Risk Recommended, Approved, etc.). Table 12 summarizes the linguistic decision rules used for classification. Based on these classifications, the suppliers are assigned to appropriate categories. If two suppliers fall into the same class, their ranking is refined by directly comparing their relative closeness values, as described in Equation (11). In this case, Suppliers A and B are categorized under Class IV (Approved). Suppliers C and D fall into Class III (Recommend with low risk). Supplier E is assigned to Class II (Recommend with caution).

To better illustrate the results of the proposed method, two visual aids are provided in this section. These figures aim to present the relative performance of each supplier and their respective rankings based on the calculated closeness coefficients. The inclusion of these graphics facilitates a clearer understanding of the differences among alternatives and supports more informed decision-making.

Figure 2 shows a bar chart comparing the closeness coefficients of all suppliers, making it easy to observe which alternatives are closer to the positive ideal solution. Figure 3 depicts the ranking of suppliers, arranged from the highest to lowest closeness coefficient, providing a direct view of the preferred order for selection. Together, these visuals complement the tabular data and enhance the interpretability of the findings.

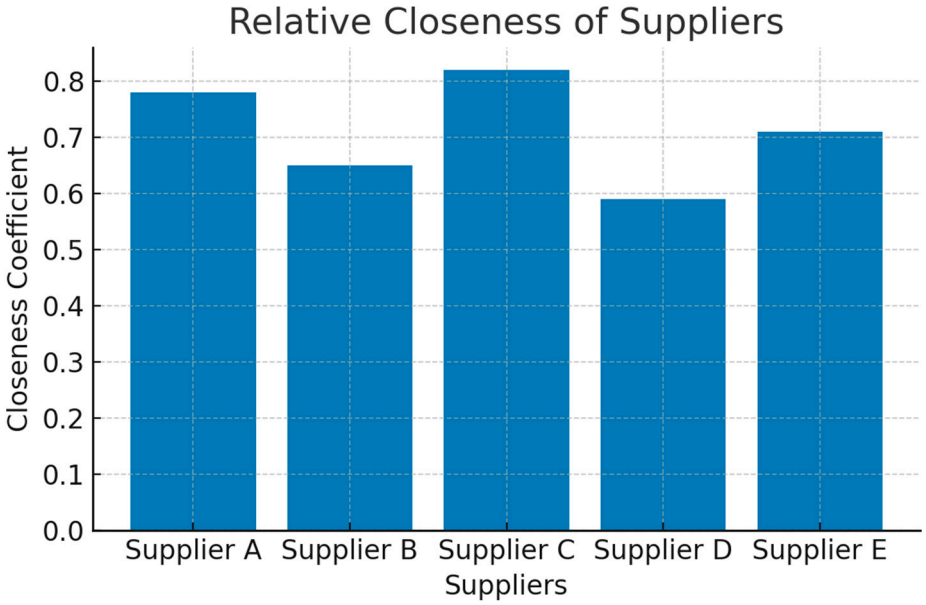

Figure 2. Relative closeness of suppliers.

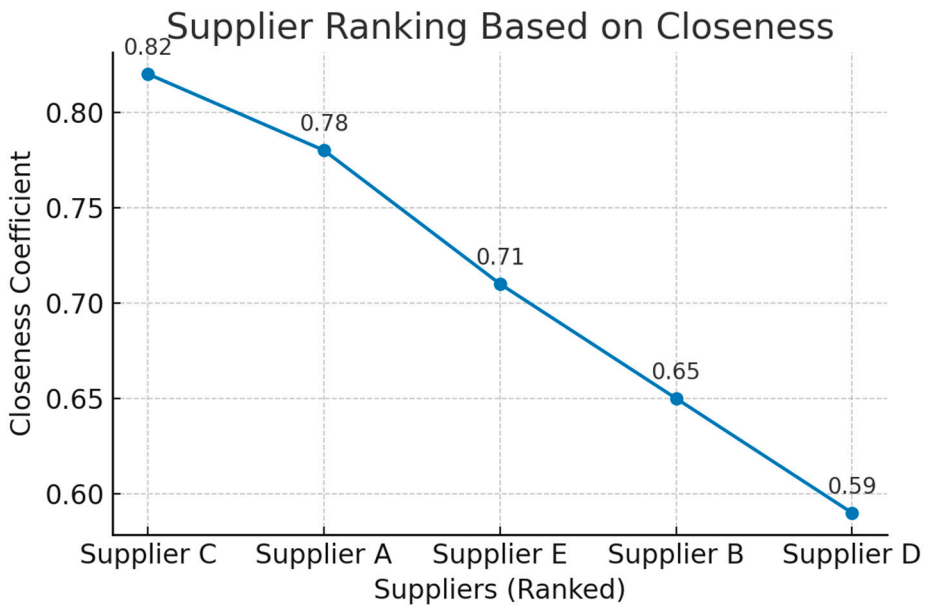

Figure 3. Supplier ranking based on closeness coefficient.

5.4. Comparison with Other Methods

To evaluate the performance and added value of the proposed interval-based fuzzy TOPSIS method, it is compared against the conventional fuzzy TOPSIS (FTOPSIS) approach under identical conditions. Both models are applied to the same supplier selection problem using identical input data and evaluation criteria. Although the final ranking of the alternatives remains consistent across both methods, the proposed approach demonstrates distinct advantages in interpretability and information retention. Specifically, the interval-based formulation allows decision-makers to capture and reflect the underlying uncertainty present in expert judgments, rather than collapsing fuzzy data into crisp scores too early in the process. This leads to more transparent and meaningful decision outcomes, particularly in cases where ambiguity or subjectivity is prominent. Moreover, the linguistic classification of closeness coefficients in the proposed method enhances the expressiveness of results, offering a clear justification for recommendations based on both qualitative and quantitative reasoning. These enhancements make the proposed approach a more suitable tool for real-world applications involving imprecise or subjective information. Recent studies have emphasized the importance of such qualitative interpretability in group decision-making [32,33], highlighting the benefits of our approach in practical scenarios.

6. Conclusions

This paper proposed an enhanced fuzzy TOPSIS framework grounded in the transformation of fuzzy values into expected intervals, aimed at addressing the limitations of early defuzzification in conventional methods. By retaining the fuzzy nature of linguistic evaluations throughout the analysis, the proposed model offers a more accurate and nuanced reflection of subjective judgments. Unlike classical approaches that prematurely reduce fuzziness, the interval-based formulation captures the inherent uncertainty in expert opinions, preserving both the lower and upper bounds of evaluations. This results in a decision-making process that is not only more robust but also more interpretable, especially when supported by a linguistic classification scheme for ranking alternatives. The case study highlighted that, while the proposed and classical models may yield the same final rankings, the interpretive depth, transparency, and resilience to uncertainty provided by the new method are considerably superior. Furthermore, the use of interval arithmetic simplifies computational effort compared to non-linear programming solutions, enhancing

scalability for large-scale MCDM problems. The effectiveness of the proposed method aligns with recent trends in decision science, emphasizing the need for techniques that balance theoretical rigor with practical applicability and explainability [32,34]. The proposed method demonstrated improved interpretability and robustness in a supplier selection case study, outperforming conventional fuzzy TOPSIS in preserving linguistic information and offering clearer decision support. Despite its strengths, the method has certain limitations, including dependence on well-defined linguistic scales and sensitivity to the boundaries set for closeness interval classification. These factors may affect results in contexts where expert agreement on scales is difficult to achieve. Future research may extend the model to incorporate more sophisticated fuzzy structures, such as intuitionistic fuzzy sets, type-2 fuzzy logic, and hesitant fuzzy environments. Applications in domains like environmental impact assessment, healthcare prioritization, and policy analysis can further validate and enhance the versatility of the approach.

Author Contributions: Resources, visualization, methodology, writing—original draft preparation: A.A.; supervision, project administration, writing—original draft preparation: A.H.-V.; data curation, validation, formal, writing—review and editing: A.J.; conceptualization, software, investigation: A.K.Y. All authors have read and agreed to the published version of the manuscript.

Funding: Amir Karbassi Yazdi thanks the financial support from Fortalecimiento Grupos de Investigación UTA N° 8764-25.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Acknowledgments: The authors would like to thank two anonymous referees for their helpful comments and suggestions which improved the first draft of this study.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Hwang, C.-L.; Yoon, K. Methods for multiple attribute decision making. In *Multiple Attribute Decision Making*; Springer: Berlin/Heidelberg, Germany, 1981; pp. 58–191.
3. Han, F.; Alkhawaji, R.N.; Shafieezadeh, M.M. Evaluating sustainable water management strategies using TOPSIS and fuzzy TOPSIS methods. *Appl. Water Sci.* **2025**, *15*, 4. [CrossRef]
4. Bottani, E.; Rizzi, A. A fuzzy TOPSIS methodology to support outsourcing of logistics services. *Supply Chain. Manag. Int. J.* **2006**, *11*, 294–308. [CrossRef]
5. Junior, F.R.L.; Osiro, L.; Carpinetti, L.C.R. A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection. *Appl. Soft Comput.* **2014**, *21*, 194–209. [CrossRef]
6. Mokhtarian, M.N.; Hadi-Vencheh, A. A new fuzzy TOPSIS method based on left and right scores: An application for determining an industrial zone for dairy products factory. *Appl. Soft Comput.* **2012**, *12*, 2496–2505. [CrossRef]
7. Sadabadi, S.A.; Hadi-Vencheh, A.; Jamshidi, A.; Jalali, M. An Improved Fuzzy TOPSIS Method with a New Ranking Index. *Int. J. Inf. Technol. Decis. Mak.* **2022**, *21*, 615–641. [CrossRef]
8. Keikha, A. Generalized hesitant fuzzy numbers and their application in solving MADM problems based on TOPSIS method. *Soft Comput.* **2022**, *26*, 4673–4683. [CrossRef]
9. Agrawal, S.; Singh, R.K.; Murtaza, Q. Prioritizing critical success factors for reverse logistics implementation using fuzzy-TOPSIS methodology. *J. Ind. Eng. Int.* **2016**, *12*, 15–27. [CrossRef]
10. Ilieva, G.; Yankova, T. Extension of interval-valued Fermatean fuzzy TOPSIS for evaluating and benchmarking COVID-19 vaccines. *Mathematics* **2022**, *10*, 3514. [CrossRef]
11. Yaakob, A.; Gegov, A.E. Interactive TOPSIS based group decision making methodology using Z-Numbers. *Int. J. Comput. Intell. Syst.* **2016**, *9*, 311–324. [CrossRef]
12. Kirişci, M. New cosine similarity and distance measures for Fermatean fuzzy sets and TOPSIS approach. *Knowl. Inf. Syst.* **2023**, *65*, 855–868. [CrossRef] [PubMed]

13. Antunes, J.J.M.; Hadi-Vencheh, A.; Jamshidi, A.; Tan, Y.; Wanke, P. TEA-IS: A hybrid DEA-TOPSIS approach for assessing performance and synergy in Chinese health care. *Decis. Support. Syst.* **2023**, *171*, 113916. [CrossRef]
14. Gay, B.A.; Pastick, N.J.; Züfle, A.E.; Armstrong, A.H.; Miner, K.R.; Qu, J.J. Investigating permafrost carbon dynamics in Alaska with artificial intelligence. *Environ. Res. Lett.* **2023**, *18*, 125001. [CrossRef]
15. Sarwar, M. Decision-making approaches based on color spectrum and D-TOPSIS method under rough environment. *Comput. Appl. Math.* **2020**, *39*, 291. [CrossRef]
16. Zeydan, M.; Çolpan, C. A new decision support system for performance measurement using combined fuzzy TOPSIS/DEA approach. *Int. J. Prod. Res.* **2009**, *47*, 4327–4349. [CrossRef]
17. Haddad, A.N.; da Costa, B.B.F.; de Andrade, L.S.; Hammad, A.; Soares, C.A.P. Application of Fuzzy-TOPSIS Method in Supporting Supplier Selection with Focus on HSE Criteria: A Case Study in the Oil and Gas Industry. *Infrastructures* **2021**, *6*, 105. [CrossRef]
18. Jain, V.; Sangaiah, A.K.; Sakhuja, S.; Thoduka, N.; Aggarwal, R. Supplier selection using fuzzy AHP and TOPSIS: A case study in the Indian automotive industry. *Neural Comput. Appl.* **2018**, *29*, 555–564. [CrossRef]
19. Yang, Y.; Zhang, C.; Zhao, Q.; Zhang, Y. A sustainability evaluation framework for the urban energy Internet using the Fermatean fuzzy Aczel-Alsina hybrid MCDM method. *Expert. Syst. Appl.* **2024**, *238*, 122115. [CrossRef]
20. Roy, P.K.; Shaw, K.; Ishizaka, A. Developing an integrated fuzzy credit rating system for SMEs using fuzzy-BWM and fuzzy-TOPSIS-Sort-C. *Ann. Oper. Res.* **2023**, *325*, 1197–1229. [CrossRef]
21. Yazdi, A.K.; Haddadi, M. Prioritising critical successful factors of knowledge management in insurance companies. *Int. J. Oper. Res.* **2018**, *31*, 281. [CrossRef]
22. Sevastjanov, P.; Dymova, L. A new method for solving interval and fuzzy equations: Linear case. *Inf. Sci.* **2009**, *179*, 925–937. [CrossRef]
23. Heilpern, S. The expected value of a fuzzy number. *Fuzzy Sets Syst.* **1992**, *47*, 81–86. [CrossRef]
24. Jiménez, M.; Arenas, M.; Bilbao, A.; Rodrı'guez, M.V. Linear programming with fuzzy parameters: An interactive method resolution. *Eur. J. Oper. Res.* **2007**, *177*, 1599–1609. [CrossRef]
25. Kumar, S.; Kumar, S.; Barman, A.G. Supplier selection using fuzzy TOPSIS multi criteria model for a small scale steel manufacturing unit. *Procedia Comput. Sci.* **2018**, *133*, 905–912. [CrossRef]
26. Vaníček, J.; Vrana, I.; Aly, S. Fuzzy aggregation and averaging for group decision making: A generalization and survey. *Knowl. Based Syst.* **2009**, *22*, 79–84. [CrossRef]
27. Sudjono, S.S.; Hakam, D.F.; Wasesa, M. Advancing towards Indonesia's net zero emission goals: An in-depth multi-criteria decision making (MCDM) analysis of ship-loader operations in maritime transportation using interval type-2 fuzzy AHP and TOPSIS methods. *Energy Rep.* **2025**, *14*, 552–565. [CrossRef]
28. Wang, Y.-M.; Elhag, T.M.S. Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. *Expert. Syst. Appl.* **2006**, *31*, 309–319. [CrossRef]
29. Perçin, S. Circular supplier selection using interval-valued intuitionistic fuzzy sets. *Environ. Dev. Sustain.* **2022**, *24*, 5551–5581. [CrossRef]
30. Yue, Z. An extended TOPSIS for determining weights of decision makers with interval numbers. *Knowl. Based Syst.* **2011**, *24*, 146–153. [CrossRef]
31. Azim, A.B.; Ali, A.; Khan, A.S.; Awwad, F.A.; Ismail, E.A.A.; Ali, S. Assessing indoor positioning system: A q-spherical fuzzy rough TOPSIS analysis. *Heliyon* **2024**, *10*, e31018. [CrossRef]
32. Liu, H.-C.; Wang, L.-E.; Li, Z.; Hu, Y.-P. Improving Risk Evaluation in FMEA With Cloud Model and Hierarchical TOPSIS Method. *IEEE Trans. Fuzzy Syst.* **2019**, *27*, 84–95. [CrossRef]
33. Wanke, P.; Azad, M.A.K.; Yazdi, A.K.; Birau, F.R.; Spulbar, C.M. Revisiting camels rating system and the performance of Asean banks: A comprehensive mcdm/z-numbers approach. *IEEE Access* **2022**, *10*, 54098–54109. [CrossRef]
34. Ezeji, I.N.; Adigun, M.O.; Oki, O. Strategic Selection of Application Area for Optimizing Computational Complexity in Explainable Decision Support System Using Multi-Criteria Decision Analysis (MCDA). *J. Syst. Cybern. Inf.* **2025**, *23*, 36–47. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.