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Article

Quantum Computing for Intelligent Transportation Systems: VQE-Based Traffic Routing and EV Charging Scheduling

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Abstract

Complex optimization problems, such as traffic routing and electric vehicle (EV) charging scheduling, are becoming increasingly challenging for intelligent transportation systems (ITSs), in particular as computational resources are limited and network conditions evolve frequently. This paper explores a quantum computing approach to address these issues by proposing a hybrid quantum-classical (HQC) workflow that leverages the variational quantum eigensolver (VQE), an algorithm particularly well suited for execution on noisy intermediate-scale quantum (NISQ) hardware. To this end, the EV charging scheduling and traffic routing problems are both reformulated as binary optimization problems and then encoded into Ising Hamiltonians. Within each VQE iteration, a parametrized quantum circuit (PQC) is prepared and measured on the quantum processor to evaluate the Hamiltonian's expectation value, while a classical optimizer—such as COBYLA, SPSA, Adam, or RMSProp—updates the circuit parameters until convergence. In order to find optimal or nearly optimal solutions, VQE uses PQCs in combination with classical optimization algorithms to iteratively minimize the problem Hamiltonian. Simulation results exhibit that the VQE-based method increases the efficiency of EV charging coordination and improves route selection performance. These results demonstrate how quantum computing will potentially advance optimization algorithms for next-generation ITSs, representing a practical step toward quantum-assisted mobility solutions.

Keywords: intelligent transportation systems; quantum computing; variational quantum optimization algorithms

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1. Introduction

Intelligent transportation systems (ITSs) constitute a crucial element of modern smart city infrastructure, assisting in optimizing traffic management, minimizing environmental impact, and facilitating in identifying scalable solutions concerning urban mobility [1]. However, rapid urbanization has rendered that transportation networks operate intensely, resulting in increased traffic congestion, higher emissions, and enhanced complexity associated with integrating electric vehicle (EV) infrastructures [2]. In particular, ITSs are confronted with numerous multifaceted challenges spanning technical, resource-related, managerial, interoperability, economic, and personal domains [3]. At the technical level, robust, secure, and efficient infrastructures are essential for handling the immense datasets generated by urban mobility networks. From the management and resource perspective, it remains challenging to maintain and coordinate large connected vehicle fleets across diverse geographic areas while safeguarding against cybersecurity threats. Furthermore, interoperability issues and ongoing standardization efforts aimed at seamless integration with existing transportation systems continue to pose significant hurdles. Additionally, personal concerns regarding privacy and data protection, along with broader socioeconomic implications such as potential disruptions to employment within the transportation sector, further complicate the ITS adoption. Within these resource-constrained settings, traffic routing optimization and EV charging scheduling mark two prominent computationally intensive problems [4,5]. Although various classical optimization techniques have been extensively employed to address these problems, their effectiveness is fundamentally limited by the exponential scaling inherent in combinatorial optimization problems that characterize real-world transportation systems [6]. The computational complexity of these problems, which frequently exhibit NP-hard characteristics, necessitates the investigation of alternative computational methodologies [7]. Due to this computational constraint, research into advanced computational paradigms has been spurred, with quantum computing emerging as a promising approach for addressing the inherent complexity of large-scale transportation optimization problems, which has potentially superior efficiency compared with classical methods [8].

Quantum computing has emerged as a promising computational paradigm for solving complex optimization problems within ITSs, particularly those related to vehicle routing and traffic management [9]. Prior research has extensively applied quantum annealing to the capacitated vehicle routing problem and its extensions—such as the dynamic multi-depot capacitated vehicle routing problem—to model these challenges as quadratic unconstrained binary optimization problems, showing notable improvements in solution efficiency when executed on D-Wave hardware [10]. Similarly, the quantum approximate optimization algorithm has been utilized to solve the heterogeneous vehicle routing problem, where simulation studies indicate that the required number of qubits grows quadratically with the number of customers, underscoring significant scalability limitations in real-world applications [11]. Moreover, quantum-inspired hybrid algorithms combining classical heuristics with quantum annealing have achieved practical gains in dynamic routing and signal-timing optimization, outperforming several traditional methods on benchmark instances [12]. Together, these prior studies illustrate the transformative potential of quantum methods in ITS contexts while underscoring urgent challenges surrounding algorithm scalability, hardware fragility, and error resilience. However, existing implementations have predominantly relied on simulations via platforms like *PennyLane*, leaving practical hardware-level performance largely unexplored. Several works have demonstrated the application of quantum annealing to real-world traffic optimization using D-Wave hardware, including formulations of quadratic unconstrained binary optimization models for traffic signal control and real-time routing, showing promising benefits in congestion reduction and solution quality compared with fixed-cycle methods [13]. Analogously, studies deploying a quantum approximate optimization algorithm on noisy intermediate-scale quantum (NISQ) devices have revealed that circuit depth, noise, and qubit counts critically influence performance, with hardware implementations often exhibiting substantial degradation compared with ideal simulations [14]. A recent noise-aware distributed quantum approximate optimization algorithm framework further highlights how decomposing large problems and incorporating error mitigation strategies can improve scalability and accuracy on current hardware [15]. These findings highlight the need for either a preliminary hardware demonstration or a detailed analysis of noise effects, error mitigation, and circuit design to enhance the practical relevance and technical robustness.

Recent advances in quantum computing have led to the development of robust quantum computational frameworks capable of addressing complex optimization problems while exceeding the capabilities of classical algorithms, especially through hybrid quantum-classical (HQC) methodologies [16]. In particular, a noteworthy advancement within this paradigm is the variational quantum eigensolver (VQE), which exploits fundamental quantum mechanical phenomena including superposition and entanglement in conjunction with classical optimization algorithms to approximate the ground-state energies of complex Hamiltonians [17]. The effectiveness of VQE has been demonstrated across multiple application domains, including quantum chemistry and combinatorial optimization problems related to transportation networks. Ref. [18] investigated VQE's application to the vehicle routing problem in transportation logistics, demonstrating its potential for logistics optimization within current NISQ hardware constraints. Concurrently, quadratic unconstrained binary optimization formulations mapped to Ising Hamiltonians have been used to address unmanned aerial vehicle collision avoidance wherein a conditional value at risk-enhanced VQE has shown robust performance for real-time operational scenarios [19]. The reliability of VQE has been further validated through quantum chemistry applications, where optimized ansatz designs and noise mitigation strategies have enabled accurate molecular ground-state energy estimations [20].

The development of specialized VQE variants for constrained and large-scale optimization challenges have been prioritized in emerging research directions. A major methodological advancement is the VQE with a constraint framework, which integrates Lagrangian duality with perturbed primal-dual optimization techniques to enable effective resolution of complex constrained problems, such as quadratic unconstrained binary optimization, MaxCut, stochastic and deterministic quadratic constrained binary optimization, and linear programming formulations on NISQ devices [21]. Furthermore, the modified deep VQE framework has improved local basis constructions and introduced penalty-based methods for computing low-energy excited states in quantum chemistry, primarily focusing periodic material systems [22]. In generalized assignment problems in vehicular network contexts, conditional values at risk-based VQE implementations have shown substantial performance improvements over classical computational approaches [23]. In addition, the integration of quantum support vector machines with VQE has demonstrated improved computational efficiency in vehicle routing applications, highlighting the critical role of encoding methodologies in maintaining a balance between parametrized quantum circuit (PQC) complexity and solution precision [24]. Despite these notable developments, there is still a significant opportunity for impactful research contributions because the application of VQE to traffic routing and EV charging scheduling problems within ITS remains substantially unexplored.

This paper addresses the computationally intensive problems of traffic routing and EV charging scheduling while explicitly exploring the VQE-based optimization framework for ITS applications. The methodology reformulates both the aforementioned optimization problems as binary polynomial formulations and systematically encodes them into Ising Hamiltonians, thereby enabling operational quantum computational processing. The framework uses PQCs (i.e., ansatzes) as variational trial states, which are iteratively refined using an HQC optimization algorithm, which is designed to minimize the system Hamiltonian's expectation value. The optimization objectives include minimizing traffic congestion and effectively scheduling EVs charging demands across the transportation network. Notwithstanding NISQ limitations, the VQE-based framework demonstrates considerable performance advantages, achieving improvements in route optimization efficiency and EV charging coordination capabilities. These empirical findings conclusively establish the technical viability and practical applicability of integrating HQC computational methodologies within existing ITS architectures, thereby constituting a substantive advancement toward quantum-enhanced algorithmic optimization for next-generation intelligent urban transportation ecosystems. The primary contributions are summarized as follows.

- We describe the VQE computational workflow and implementation details, including the Hamiltonian formulation, ansatz design, HQC optimization, and measurement.
- We apply the VQE framework to two critical optimization case studies within ITSs: optimal traffic routing and EV charging scheduling. The simulation results and comparative analyses demonstrate performance improvements over conventional methods, highlighting the potential of quantum-assisted optimization strategies for transportation networks.

This article is organized as follows: Section 2 describes the VQE algorithm. Section 3 details the VQE's application to optimal traffic routing and EV charging scheduling problems. Section 4 provides performance analysis of the experimental results. Finally, Section 5 concludes with future insights.

2. Methods

Specifically tailored for NISQ computers, the VQE algorithm is an HQC algorithm that solves challenging optimization problems. The VQE methodology effectively combines the computational reliability and convergence guarantees of classical optimization techniques with the inherent ability of quantum computing in exploring exponentially large solution spaces. This hybrid approach demonstrates exceptional effectiveness for complex optimization applications, such as resource allocation, traffic routing optimization, and EV charging scheduling (see Figure 1). In practice, the VQE algorithm works by following these main steps as detailed below.

- **Problem Formulation as Quantum Optimization:**
  The target optimization problem, which may include task assignment, vehicle routing, or operational scheduling, is transformed into a Hamiltonian ($\boldsymbol{H}$). The optimization problem's objective function and constraints are encoded by this Hamiltonian, while quantifying the energy or cost associated with each potential solution configuration. The optimization problem is usually formulated as a binary polynomial and subsequently mapped to an Ising Hamiltonian, a canonical form particularly well suited for quantum computing.
- **Quantum Trial State Construction via Ansatz Design:**
  A variational quantum state is constructed through a PQC known as an ansatz. This ansatz comprises quantum gates, including single-qubit rotation operations and two-qubit entangling gates, enabling the coherent exploration of the solution space through quantum superposition. The circuit contains adjustable parameters $\boldsymbol{\theta}$, which determine the quantum state configuration. These variational parameters are initialized through random assignment or heuristic-based methods to create the initial trial state.
- **HQC Optimization:**
  In order to prepare the trial state $|\psi(\boldsymbol{\theta})\rangle$, the quantum computing platform implements the PQC. Then, it measures the expectation value of the problem Hamiltonian, which represents the evaluation of the energy or cost function for the current parameter configuration. The expectation value is computed as

  $$E(\boldsymbol{\theta}) = \langle\psi(\boldsymbol{\theta})|\,\boldsymbol{H}\,|\psi(\boldsymbol{\theta})\rangle\,. \tag{1}$$

  The optimization objective is to minimize this energy functional. A classical optimization algorithm (such as COBYLA or SPSA) systematically adjusts the variational parameters $\boldsymbol{\theta}$ to reduce the expectation value through iterative refinement. Through the adoption of this HQC feedback mechanism, configurations that yield minimal energy values—that is, optimal or nearly optimal solutions—can be systematically explored in the parameter space.

- **Solution Extraction and Validation:**
  The optimized parameters ($\boldsymbol{\theta}_{\text{opt}}$) are decoded to produce a binary solution vector that represents the ideal configuration for the initial optimization problem upon algorithmic convergence, which is determined by the stabilization of energy reduction. In relation to the initial problem specification, a thorough validation process confirms constraint satisfaction and solution viability. After this, the quantum-derived solution is analyzed and processed into practical recommendations for real-world application.

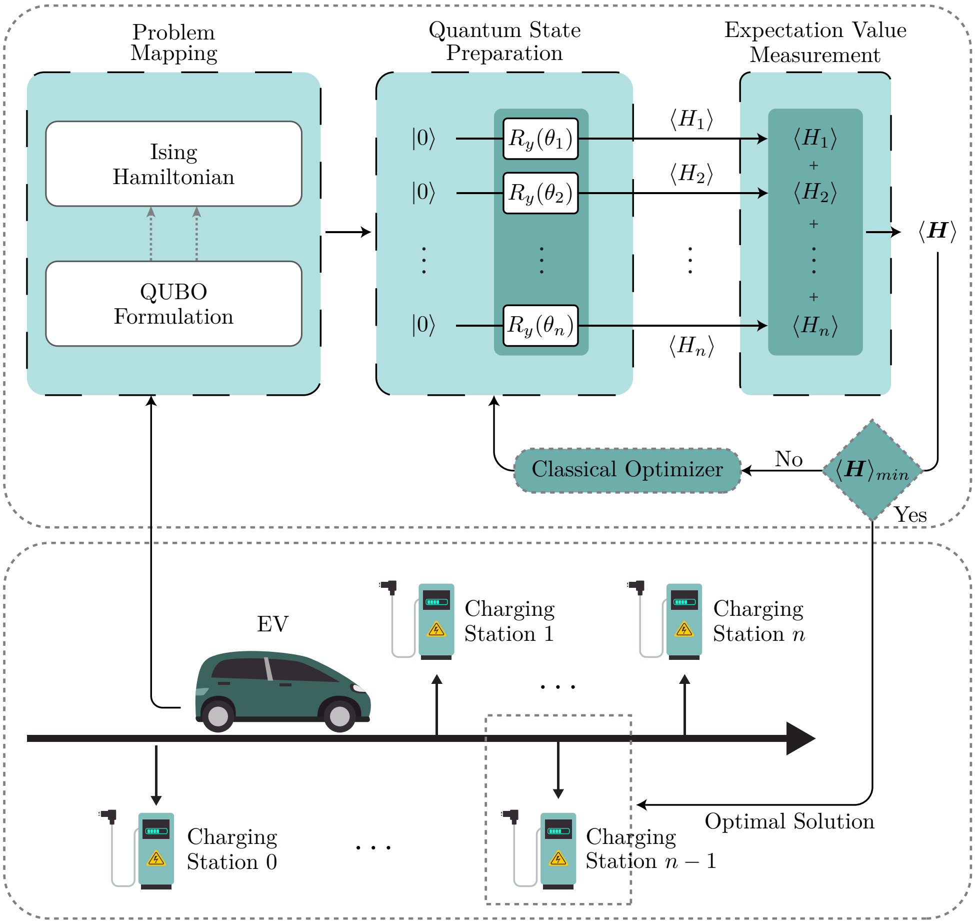


Figure 1. Quantum computing for ITS: VQE methodology applied to EV charging scheduling.

3. Results

In the following, we demonstrate the application of VQE to the optimization problems of optimal traffic routing and EV charging scheduling.

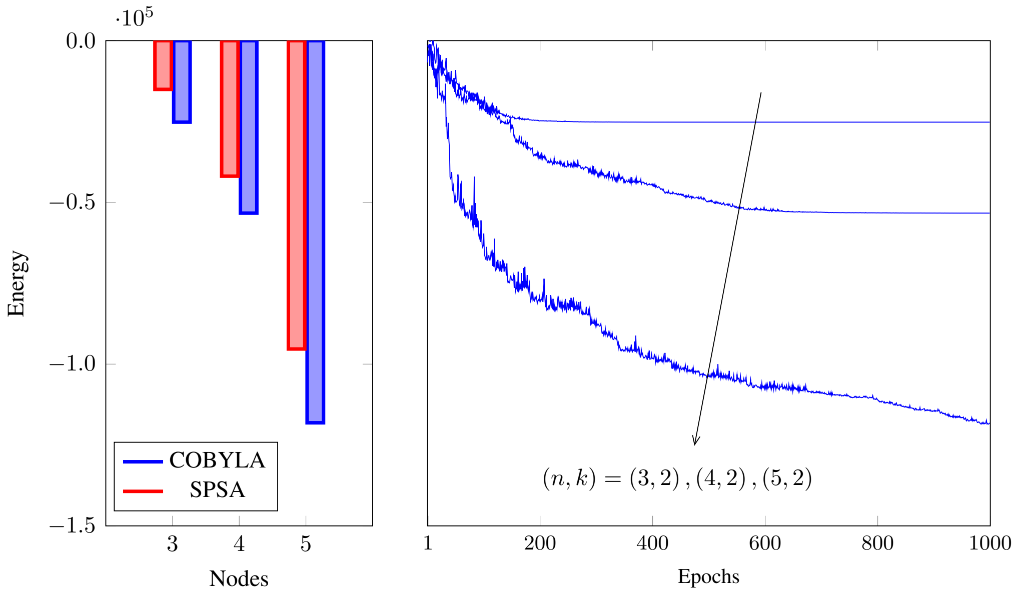
3.1. VQE for Optimal Traffic Routing

In this case study, the Miller–Tucker–Zemlin formulation is utilized to explore how VQE can be used to solve a constrained version of the vehicle routing problem. In the optimization scenario, two vehicles in a fleet are assigned to visit a series of predetermined locations exactly once, making sure that each vehicle returns to a designated depot at the end of its route. Such formulations hold significant practical implications for real-life logistics and mobility management, as they represent fundamental challenges in ITSs.

The formulation introduces continuous auxiliary variables at each node to impose constraints that guarantee that each vehicle follows a complete, acyclic route, preventing subtour formation—a prevalent issue in vehicle routing. Incorporating these constraints transforms the original routing problem into a constrained binary polynomial optimization model. This reformulation makes the problem suitable for HQC computational approaches, such as VQE, enabling efficient problem solving under current NISQ hardware limitations.

The VQE implemented here comprises controlled-phase rotation gates and single-qubit rotation gates around the Pauli-Y axis, parameterized by a vector of adjustable variational parameters $\boldsymbol{\theta}$. Energy evaluation is carried out through measurements in the computational basis, with the expectation value of each Hamiltonian term combined as a weighted sum to obtain the total system energy. A classical optimization algorithm then iteratively updates the parameters $\boldsymbol{\theta}$ to minimize this energy, thereby approximating the optimal (ground-state) configuration.

Routing solutions are determined by analyzing the final sampling distribution from the optimized PQC upon convergence. In order to validate the methodology, examples of two vehicles ($k = 2$) operating across routing networks with varying degrees of complexity—that is, networks with $n = 3, 4, 5$ nodes—are systematically evaluated, as shown in Figure 2.



Nodes	Qubits	Parameters	Depth
3	6	24	13
4	12	48	19
5	20	80	27

Figure 2. Traffic optimal routing: Comparison of COBYLA and SPSA optimizers for the VQE performance upon application to the vehicle routing problem under $n = 3, 4, 5$ nodes and $k = 2$ vehicles showing energy convergence over 1000 epochs (top left). Ground-state energy reduction after each iteration for the COBYLA optimizer for the aforementioned node and vehicle configurations (n, k). The table describes the variational quantum circuit configurations, displaying the number of qubits, number of parameters, and circuit depth.

3.2. *VQE for EV Charging Scheduling*

In this case study, we address the EV charging scheduling problem using quadratic unconstrained binary optimization. In this scenario, we consider multiple EV charging stations that serve multiple vehicles, each requiring charging within specific time windows, with constraints on available charging slots and station capacity.

Specifically, we assume that there are five charging stations, each represented by an edge. Each station has two outlets, but due to constraints, only one outlet per station can be active at a time, shown in Figure 3a. Vehicles awaiting charging are represented as nodes, where each node is also associated with a power capacity. Additionally, when a vehicle is charging at a node, it cannot distribute power to its forward adjacent nodes, which is crucial for effective load management and maintaining network stability. Therefore, optimal scheduling is required to ensure that all parked vehicles are charged without disrupting the power distribution. If the node has a vehicle being charged, then $x_i = 0$; else, $x_i = 1$. The problem is formulated as an optimization challenge, where the goal is to maximize the objective function's value, such that more cars can be charged at once. Each edge (station) gives rise to a constraint as

$$1 + x_i + x_j - 2x_i x_j, \tag{2}$$

where the term is zero when exactly one of x_i or x_j is zero; i.e., only one vehicle is charging, which is desirable. In our case, the charging conflict pairs are nodes (cars) as $(1,4),(2,3),(4,5),(3,4),(3,6)$. We can write the function that acts as our quadratic unconstrained binary optimization formulation of the given EV charging scheduling problem as follows:

$$\begin{aligned} f(x_1,\ldots,x_6) = {} & (1 + x_1 + x_4 - 2x_1x_4) + (1 + x_2 + x_3 - 2x_2x_3) \\ & + (1 + x_4 + x_5 - 2x_4x_5) + (1 + x_3 + x_4 - 2x_3x_4) \\ & + (1 + x_3 + x_6 - 2x_3x_6). \end{aligned} \tag{3}$$

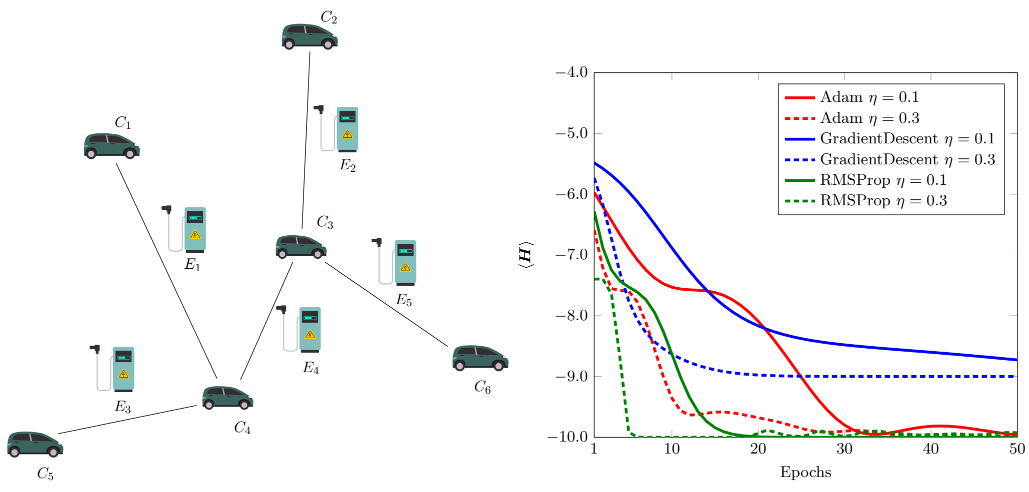


Figure 3. EV charging scheduling: (**a**) case scenario of our problem and (**b**) convergence of expectation value of the $\boldsymbol{H}$ plotted as a function of epochs for different classical optimizers with learning rate $\eta = 0.1, 0.3$.

This can be further simplified as

$$\begin{aligned} f(x_1,\ldots,x_6) = {} & 5 + x_1 + x_2 + 3x_3 + 3x_4 + x_5 + x_6 - 2x_1x_4 \\ & - 2x_2x_3 - 2x_4x_5 - 2x_3x_4 - 2x_3x_6. \end{aligned} \tag{4}$$

In order to implement this optimization problem on a quantum computer, it is necessary to convert the binary variables $x_i \in \{0,1\}$ into spin variables $z_i \in \{-1,+1\}$, which are compatible with the Ising model commonly used in quantum annealers and variational quantum algorithms. This transformation is achieved using the change in variables $x_i = (1 - z_i)/2$. Applying this substitution to the quadratic unconstrained binary optimization's objective function results in a new expression entirely in terms of the spin variables z_i, which can then be used as the cost Hamiltonian in VQE. Hence, after change in variable, the equation now becomes

$$f(z_1, \ldots, z_6) = 0.5(15 - z_1 z_4 - z_2 z_3 - z_3 z_4 - z_3 z_6 - z_4 z_5). \tag{5}$$

The provided function can be transformed into a Hamiltonian. This can be performed by substituting the z_i terms with the Pauli operator $\boldsymbol{Z}$, the identity matrix $\boldsymbol{I}$ for any scalar, and the dot product by a tensor product in the expression. This transformation can be expressed as follows:

$$\boldsymbol{H} = 0.5(15\,\boldsymbol{I} - [\mathbf{Z}_1 \otimes \mathbf{Z}_4] - [\mathbf{Z}_2 \otimes \mathbf{Z}_3] - [\mathbf{Z}_3 \otimes \mathbf{Z}_4] - [\mathbf{Z}_3 \otimes \mathbf{Z}_6] - [\mathbf{Z}_4 \otimes \mathbf{Z}_5]). \tag{6}$$

It is often more convenient to formulate optimization tasks as minimization problems. Accordingly, the objective can be reformulated by reversing the sign of each term in (6), thereby casting the problem into a minimization problem.

4. Discussion

For the first case study, the comparative performance of two classical optimization algorithms—SPSA and COBYLA—within the VQE framework applied to traffic routing optimization is presented by the experimental results shown in Figure 2. In all network configurations ($n = 3, 4, 5$ nodes with $k = 2$ vehicles), the convergence analysis shows that COBYLA has better optimization characteristics than SPSA, achieving lower energy values with higher stability levels. This confirms the results of several studies showing that COBYLA is more effective than SPSA in terms of energy minimization and convergence speed in HQC optimization scenarios, particularly for constrained problems [25]. However, owing to its highly efficient gradient estimation capabilities under uncertainty, SPSA exhibits superior robustness for high-dimensional parameter spaces and noisy quantum hardware implementations, rendering it practical for NISQ applications [26]. As indicated in the table, a fundamental challenge that is consistent with the existing literature on NISQ algorithms is the progressive increase in computational resource requirements as problem complexity scales, including qubit count, variational parameters, and PQC depth [27]. Such characteristics of resource scaling highlight how important it is to develop effective ansatz architectures and traditional optimization techniques for practical quantum advantage. Finally, the observed convergence behavior, where even larger problem instances demonstrate effective energy minimization despite higher initial values, validates the VQE framework's potential scalability within current NISQ hardware constraints [28].

For the second case study of the EV charging scheduling problem, we ran the experiment on a quantum simulator provided by *PennyLane*. In our experimental setup, a parameterized quantum circuit consisting of 6 qubits (same as the number of nodes) was employed. Each qubit underwent a $\boldsymbol{R_y}$ gate rotation, with the rotation angle θ being iteratively optimized for 50 epochs. The circuit configuration is depicted in Figure 1, where initial parameter values were selected randomly. We utilized three different types of classical optimizers for the same problem with a variable learning rate η to learn about their performance. Specifically, we employed the classical optimizers, i.e., Adam, Gradient Descent, and RMSProp. Their respective performances are depicted in Figure 3b. The Adam optimizer demonstrates robust performance with both learning rates, converging to the minimum value of the cost function around epoch 30, which is around -10. RMSProp exhibits the fastest convergence, especially at $\eta = 0.3$, achieving the minimum value in fewer than 10 epochs. In contrast, Gradient Descent shows the slowest convergence.

With $\eta = 0.1$, it fails to reach the optimal value within the evaluated epochs, while a higher learning rate $\eta = 0.3$ improves its performance, which is a trend seen in all optimizers. Overall, adaptive optimizers (Adam and RMSProp) with a larger learning rate η outperform standard Gradient Descent with a smaller η in terms of both convergence speed and final performance. In our problem, the ground state of the Hamiltonian corresponds to the optimal solution of the quadratic unconstrained binary optimization problem, which in our case is approximately -10, as shown in Figure 3b). To retrieve this solution, we sampled the optimized configuration of the quantum circuit. In our circuit, qubits 1 to 6 represent nodes in the order $[1, 4, 2, 3, 6, 5]$, since *PennyLane* internally registers the wires based on their order of first occurrence in the Hamiltonian definition. Upon sampling the final quantum state corresponding to the minimum energy configuration, we obtained the binary solution $[0\ 1\ 1\ 0\ 1\ 0]$, indicating that nodes (or car) 1, 3, and 5 are scheduled to charge during the first time slot, followed by nodes 2, 4, and 6 in the subsequent slot, which is an optimal solution to our scheduling problem. Furthermore, this problem only utilizes a small circuit depth, which is well suited for current NISQ computers. Therefore, the aforementioned results can be effectively demonstrated on quantum hardware, especially for small to medium-scale quadratic unconstrained binary optimization problems.

While the aforementioned case studies currently focus on relatively small problem sizes (up to 5 nodes for routing and 6 qubits for EV scheduling), scaling to realistic ITS networks remains a significant challenge. In these instances, the HQC optimization reached near-optimal solutions, e.g., finding the ground-state minimum corresponding to the optimal EV charging schedule and optimal route. The circuits required 6–20 qubits (for 3–5 nodes) with shallow depths (13–27 layers of gates), and convergence took on the order of 50–1000 iterative epochs of the classical optimizer. Nevertheless, classical optimization techniques currently outperform these quantum methods by a substantial margin. For instance, modern mixed-integer linear programming solvers and branch-and-cut algorithms comfortably handle vehicle routing problems with more than 100 customers, often finding exact solutions within a few hours [29]. Additionally, advanced metaheuristics such as adaptive large neighborhood search or variable neighborhood search regularly deliver near-optimal solutions for routing problems of several hundred nodes within minutes, even when faced with complex routing constraints such as time windows or pickup-and-delivery tasks [30]. Similarly, classical mixed-integer linear programming and heuristic-based methods quickly optimize EV charging schedules for dozens of vehicles and chargers [31]. While the VQE approach provides a promising proof of concept for quantum optimization in ITS, it neither outperforms classical mixed-integer linear programming or metaheuristics in solution quality nor approaches their scalability. In practical NISQ terms, classical solvers and heuristics remain the primary choice for traffic routing and EV charging scheduling, with quantum computing algorithms limited to small-scale demonstrations until hardware advances—such as higher qubit counts and lower noise—enable it to tackle ITS's realistic problem sizes. In this regard, a recent HQC algorithmic research demonstrated a 13-node vehicle routing problem requiring 156 qubits, but with circuit-level optimization and problem decomposition to achieve major reductions in circuit depth and gate count, showing potential feasibility on near-term quantum hardware [32]. Additionally, VQE and related algorithms reveal that increasing qubit counts negatively impact circuit depth, introduce greater noise, and increase control overhead, thus requiring advanced techniques like ansatz optimization and error mitigation to remain practical [28]. While generic ansatz designs using rotations and controlled-phase gates are common, recent HQC approaches have introduced specialized ansatzes such as the quantum alternating operator ansatz, explicitly tailored for structural constraints in vehicle routing and EV charging scheduling [33]. These tailored ansatz designs, incorporating domain-specific constraints like one-hot encoding and vehicle capacity limitations, significantly reduce circuit depth and improve algorithm convergence, illustrating the importance of embedding ITS-specific problem structures directly into PQC architectures. Therefore, addressing larger ITS problems will necessitate thoughtful PQC design consideration while considering both hardware constraints and the potential benefits of tailored HQC architectures.

5. Conclusions and Future Outlook

In this work, we explored the VQE algorithm in addressing critical optimization tasks concerning ITSs, with particular emphasis on traffic routing and EV charging scheduling. By formulating these combinatorial problems into Ising Hamiltonians, the HQC computational framework was effectively leveraged to identify optimal or near-optimal solutions. When compared with conventional classical optimization techniques, the simulation-based evaluations showed improved performance in traffic route optimization and EV charging coordination. These results endorse the practicality and potential of quantum computing approaches for easing the computationally demanding problems that are associated with managing transportation infrastructure. Consequently, this study lays essential groundwork for integrating quantum optimization frameworks into next-generation ITSs, particularly those requiring responsive real-time decision-making capabilities.

In order to decrease PQC depth and improve the quality of obtained solutions, future research should focus on developing specialized variational ansatzes tailored to the inherent structural characteristics of ITS networks. To bridge the gap between idealized simulation environments and real-world quantum hardware implementations, comprehensive error mitigation strategies specific for NISQ devices need comprehensive investigations. Achieving near-term practical deployments will require advancing HQC algorithms, which systematically distribute computational tasks between quantum processors and classical computational resources. Furthermore, to precisely define the operational bounds for quantum advantage in ITS optimization scenarios, comprehensive scalability studies across a range of network sizes and complexity levels are necessary. Experimental validation on current NISQ hardware platforms will finally offer crucial insights regarding realistic performance capabilities and practical constraints of quantum-enhanced optimization methodologies.

As ITSs increasingly adopt digital twin technologies and interconnected infrastructures, they face growing cybersecurity challenges, including data breaches, denial-of-service attacks, and the exploitation of digital models [34]. Quantum computing presents a dual-edged scenario for these systems—while it threatens to undermine traditional encryption methods, it also opens doors to enhanced security through post-quantum cryptography and sophisticated anomaly detection capabilities. Therefore, future ITSs developments must carefully assess the security risks that quantum computing introduces against the protective benefits it can provide. Based on recent advances in scalable IoT authentication frameworks, upcoming research should explore how permissioned blockchain technology can strengthen authentication processes and safeguard privacy in IoT environments [35]. Additionally, combining blockchain approaches with quantum-enhanced features offers a promising strategy to safeguard ITS applications from quantum-based threats while ensuring strong data integrity and maintaining system efficiency.

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Data Availability Statement: The original contributions presented in the study are included in the article; further inquiries can be directed to the corresponding authors.

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Abbreviations

The following abbreviations are used in this manuscript:

EV	electric Vehicle
HQC	hybrid quantum-classical
ITS	intelligent transportation system
NISQ	noisy intermediate-scale quantum
PQC	parametrized quantum circuit
VQE	variational quantum eigensolver

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