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Laplacian Conditions and Sphericity of Hypersurfaces in the Nearly Kähler 6-Sphere

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Abstract

In this paper, we investigate hypersurfaces in the nearly Kähler 6-sphere S^6 and establish several foundational results. In particular, under certain conditions of the function $\xi(f) = g(\nabla f, \xi)$, we demonstrate that a hypersurface M of S^6 must be a sphere. Here, $f \in C^\infty(M)$ is a smooth vector field, $\xi = -JN$ denotes the characteristic vector field, J is the almost complex structure on S^6 , and N is the unit vector field normal to the hypersurface. We also support our results with illustrative examples.

Keywords: hypersurfaces; Ricci curvature; nearly Kähler 6-sphere

MSC: 53C25, 53C40, 53C42

1. Introduction

Nearly Kähler geometry arises as a natural generalization of Kähler geometry and has garnered significant attention due to its deep connections with differential geometry, complex geometry, and theoretical physics. Among the most remarkable examples of such structures is the 6-dimensional unit sphere S^6 , which serves as a canonical example of a strictly nearly Kähler manifold that is not Kähler. The nearly Kähler structure on S^6 originates from its immersion in $\mathbb{R}^7$, viewed as a space of purely imaginary Cayley numbers (octonions). This endows S^6 with a natural almost complex structure J , defined through the vector cross product in $\mathbb{R}^7$, such that the condition

$$(\nabla_X J)X = 0,$$

holds for every vector field X on S^6 . The metric g is the standard round metric induced by the Euclidean metric on $\mathbb{R}^7$.

The geometric significance of S^6 as a nearly Kähler manifold lies in the fact that, while its almost complex structure J is not integrable (hence, S^6 is not a complex manifold), it still satisfies certain symmetries and curvature properties that resemble those of Kähler manifolds. These properties make S^6 an ideal setting for studying submanifolds with interesting geometric behavior, particularly those that interact in subtle ways with the ambient nearly Kähler structure [1].

Gray's seminal work [2] showed that there exist no complex hypersurfaces in S^6 , which distinguishes the geometry of S^6 from classical Kähler manifolds [3,4]. This nonexistence result paved the way for studying alternative classes of submanifolds such as totally real submanifolds, CR-submanifolds, and almost contact hypersurfaces [5]. In particular, CR-submanifolds of S^6 have been studied extensively in the literature [6,7], where 4-dimensional CR-submanifolds were shown to exhibit rich geometry influenced by the



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ambient nearly Kähler structure. Furthermore, totally real submanifolds of dimensions 2 and 3 have been analyzed in depth in several works [8–11], where the absence of complex directions plays a central role in determining their curvature and topological properties. For further classification and information on the properties of such totally real submanifolds, we refer the reader to [12–14].

While much attention has been paid to CR and totally real submanifolds, hypersurfaces of S^6 have emerged as an equally important class, especially due to their intrinsic and extrinsic geometric characteristics. Hypersurfaces of S^6 naturally inherit an almost contact metric structure induced by the nearly Kähler structure of the ambient space. Several authors have contributed to the study of such hypersurfaces [6,15–17], focusing on their curvature properties, the behavior of the characteristic vector field $\xi = -JN$, and conditions for minimality and contact geometry. A one-parameter family of totally umbilical hypersurfaces in S^6 is also discussed in [16,18,19].

Two-dimensional submanifolds in S^6 have also been studied, notably by Berndt et al. [20], who analyzed almost complex curves and showed that their intrinsic geometry shares similarities with that of Hopf hypersurfaces. These curves serve as lower-dimensional analogs of more complex structures and provide important insight into the role of J -invariant subspaces. Additional contributions to this subject are found in [21–23], where the authors explored properties of almost complex and minimal surfaces within this framework. In particular, ref. [22] links the study of such surfaces to the affine Toda field model.

More recently, Deshmukh and collaborators have investigated hypersurfaces in nearly Kähler manifolds under various curvature constraints. In [15], Deshmukh characterized Hopf hypersurfaces of S^6 by examining the behavior of the characteristic vector field and imposing suitable geometric conditions. In [17], Deshmukh and Al-Dayel extended this study by introducing nearly Sasakian and nearly cosymplectic structures on hypersurfaces of S^6 , thereby enriching the class of admissible geometric structures and providing a broader context for understanding submanifolds of nearly Kähler spaces.

Moreover, hypersurfaces in other nearly Kähler settings, such as $S^3 \times S^3$, have been considered. In [24], the authors investigated contact and minimal hypersurfaces with conformal vector fields in such product manifolds. These studies not only deepen our understanding of the interaction between curvature and topology, but also motivate the search for new examples and classifications of hypersurfaces in S^6 with distinctive geometric features.

In light of the above developments, the aim of the present paper is to further explore the geometry of hypersurfaces in the nearly Kähler 6-sphere S^6 . In particular, we are interested in understanding how certain curvature conditions, expressed via the behavior of functions such as $\zeta(f) = g(\nabla f, \xi)$ where $f \in C^\infty(M)$ and $\xi = -JN$, can influence the global geometry of the hypersurface. We demonstrate, under suitable conditions on $\zeta(f)$, that the hypersurface M of S^6 must be a standard sphere. These results contribute to the broader effort of classifying hypersurfaces in nearly Kähler manifolds and understanding their intrinsic and extrinsic geometries in the context of almost contact metric structures.

2. Some Basic Results

Let S^6 be the nearly Kähler 6-sphere endowed with a nearly Kähler structure $(J, \bar{g})$, where J is the almost complex structure and $\bar{g}$ is the associated almost Hermitian metric on S^6 . Then the identities

$$(\bar{\nabla}_X J)X = 0, \quad \bar{g}(JX, JY) = \bar{g}(X, Y),$$

hold for all $X, Y \in \mathfrak{X}(S^6)$, where $\bar{\nabla}$ denotes the Riemannian connection on S^6 with respect to $\bar{g}$, and $G(X, Y) = (\bar{\nabla}_X J)Y$ is a tensor of the type $(2, 1)$ and has the properties given below:

Lemma 1 ([15]). For any $X, Y, Z \in \mathfrak{X}(S^6)$, the tensor G satisfies

- (a) $G(X, JY) = -JG(X, Y)$;
- (b) $G(X, Y) = -G(Y, X)$;
- (c) $(\bar{\nabla}_X G)(Y, Z) = \bar{g}(Y, JZ)X + \bar{g}(X, Z)JY - \bar{g}(X, Y)JZ$;
- (d) $\bar{g}(G(X, Y), Z) = -\bar{g}(G(X, Z), Y)$;
- (e) $G(G(X, Y), Z) = \bar{g}(X, Z)Y - \bar{g}(Y, Z)X + \bar{g}(X, JZ)JY - \bar{g}(Y, JZ)JX$.

Let (M, g) be an immersed hypersurface of the nearly Kähler manifold (S^6, J, g) . We denote the covariant derivatives on M, S^6 , and $\mathbb{R}^7$ as $\nabla, \bar{\nabla}$, and D , respectively. Let N and $\bar{N}$ denote the unit vector fields normal to M and S^6 . Since $g(JN, N) = 0$, it follows that $JN \in \mathfrak{X}(M)$. We define the unit vector field as $\xi = -JN \in \mathfrak{X}(M)$.

Let η be the 1-form dual to ξ , that is, $\eta(X) = g(X, \xi)$, for all $X \in \mathfrak{X}(M)$, so that $\eta(\xi) = 1$. Define the operator $\phi_1 : \mathfrak{X}(M) \rightarrow \mathfrak{X}(M)$ as

$$\phi_1(X) = (JX)^T,$$

the tangential component of JX . Then,

$$JX = \phi_1(X) + \eta(X)N, \quad (1)$$

and hence, $\phi_1(\xi) = 0$ and $J\xi = N$.

Now define $\phi_2(X) = G(X, N)$ and $\phi_3(X) = G(X, \xi)$ for any $X \in \mathfrak{X}(M)$.

For an orientable hypersurface M of S^6 , the Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + g(AX, Y)N, \quad (2)$$

$$\bar{\nabla}_X N = -AX, \quad (3)$$

for all $X, Y \in \mathfrak{X}(M)$, where A is the shape operator of M . The Gauss and Codazzi equations for hypersurfaces are [25]

$$R(X, Y)Z = g(Y, Z)X - g(X, Z)Y + g(AY, Z)AX - g(AX, Z)AY, \quad (4)$$

$$(\nabla A)(X, Y) = (\nabla A)(Y, X), \quad (5)$$

where $(\nabla A)(X, Y) = \nabla_X AY - A\nabla_X Y$.

The Ricci tensor Ric and scalar curvature S of the hypersurface M are given by

$$\text{Ric}(X, Y) = 4g(X, Y) + 5\alpha g(AX, Y) - g(AX, AY), \quad (6)$$

$$S = 20 + 25\alpha^2 - \|A\|^2, \quad (7)$$

where $\alpha = \frac{1}{5} \text{tr}(A)$ is the mean curvature, and $\|A\|^2 = \text{tr}(A^2)$ is the squared norm of the shape operator.

Lemma 2 ([15]). For any $X, Y \in \mathfrak{X}(M)$,

- (a) $(\nabla \phi_1)(X, Y) = \eta(Y)AX - g(AX, Y)\xi + (G(X, Y))^T$;
- (b) $(\nabla \phi_2)(X, Y) = g(X, Y)\xi - \eta(Y)X + (G(AX, Y))^T$;
- (c) $(\nabla \phi_3)(X, Y) = \eta(X)\phi_1 Y - \eta(Y)\phi_1 X - g(X, \phi_1 Y)\xi + g(AX, \xi)\phi_2 Y - (G(\phi_1 AX, Y))^T$.

Lemma 3. For any $X \in \mathfrak{X}(M)$,

- (a) $\phi_1^2 X = -X + \eta(X)\xi$;
- (b) $\nabla_X \xi = \phi_1 AX - \phi_2 X$.

Proof. From Equation (1), we compute

$$\begin{aligned}\phi_1^2 X &= J(\phi_1 X) - \eta(\phi_1 X)N \\ &= J(JX - \eta(X)N) \\ &= -X + \eta(X)\xi,\end{aligned}$$

which proves part (a).

To prove part (b), we use the Gauss formula:

$$\begin{aligned}\nabla_X \xi &= \bar{\nabla}_X \xi - g(AX, \xi)N \\ &= -\bar{\nabla}_X(JN) - g(AX, \xi)N \\ &= -(\bar{\nabla}_X J)N + JAX - \eta(AX)N \\ &= -G(X, N) + \phi_1 AX \\ &= \phi_1 AX - \phi_2 X.\end{aligned}$$

□

3. The Main Results

In this section, we present the main theoretical contributions of our study concerning hypersurfaces in the nearly Kähler 6-sphere S^6 . The results are derived under specific geometric and analytical conditions involving the smooth function $\xi(f) = g(\nabla f, \xi)$, where $f \in C^\infty(M)$, and $\xi = -JN$ is the characteristic vector field induced by the almost complex structure J and the unit normal N to the hypersurface M . These results provide sufficient conditions under which a compact hypersurface of S^6 can be characterized as a standard sphere. In particular, we demonstrate the existence of a smooth function h satisfying a certain Laplacian condition involving $\xi(f)$, and further establish a rigidity result under curvature constraints. The two main theorems are stated as follows:

Theorem 1. *Let M be a compact hypersurface of S^6 . For any $f \in C^\infty(M)$, there exists a function $h \in C^\infty(M)$ such that*

$$\Delta h = -4\xi(f),$$

where $\xi(f) = g(\nabla f, \xi)$, and Δ is the Laplacian operator.

Theorem 2. *Let M be a compact hypersurface of S^6 with positive curvature. Suppose there exists a smooth function $f \in C^\infty(M)$ such that*

$$5\mu \xi(f)h \geq 4\xi(f)^2, \quad \text{with } \xi(f) \neq 0,$$

where s satisfies $\Delta h = -4\xi(f)$ and $4\mu \leq \text{Ric}$ on M . Then M is isometric to a standard sphere.

To prove these theorems, we require the following technical lemma:

Lemma 4. *Let $f \in C^\infty(M)$. Then,*

- (a) $\text{div}(\phi_1 \nabla f) = \xi(f) \text{tr} A - g(A \nabla f, \xi)$;
- (b) $\text{div}(\phi_2 \nabla f) = -4\xi(f)$;
- (c) $\text{div}(\phi_3 \nabla f) = \phi_1 AX - \phi_2 X$.

Proof. (a) Using the definition of the divergence and the properties of ϕ_1 with a local orthonormal frame $\{e_1, e_2, e_3, e_4, e_5\}$, we compute

$$\begin{aligned}\operatorname{div}(\phi_1 \nabla f) &= \sum_{i=1}^5 g((\nabla_{e_i} \phi_1)(\nabla f), e_i) + \operatorname{tr}(\phi_1 A_f) \\ &= \sum_{i=1}^5 g(\xi(f) A e_i - g(A \nabla f, e_i) \xi + G(e_i, \nabla f), e_i) \\ &= \xi(f) \operatorname{tr} A - g(A \nabla f, \xi).\end{aligned}$$

(b) For the second identity,

$$\begin{aligned}\operatorname{div}(\phi_2 \nabla f) &= \operatorname{div}(G(\nabla f, N)) \\ &= \sum_{i=1}^5 g((\bar{\nabla}_{e_i} G)(N, e_i), \nabla f) + g(G(N, e_i), A_f e_i) \\ &= \sum_{i=1}^5 g((\bar{\nabla}_{e_i} G)(N, e_i), \nabla f) \\ &= \sum_{i=1}^5 g(g(\xi, e_i) e_i - \xi, \nabla f) = -4\xi(f).\end{aligned}$$

(c) For the third identity, a more involved computation gives

$$\begin{aligned}\operatorname{div}(\phi_3 \nabla f) &= \sum_{i=1}^5 g(\nabla_{e_i}(G(\nabla f, \xi)), e_i) \\ &= - \sum_{i=1}^5 [g(G(e_i, \nabla_{e_i} \xi), \nabla f) + g(G(A \xi, e_i), \nabla f)] \\ &= - \sum_{i=1}^5 [g(G(e_i, \phi_1 A e_i - G(e_i, N)), \nabla f)] - g(G(A \xi, N), \nabla f) \\ &= - \sum_{i=1}^5 g(G(\phi_1 A e_i, e_i), \nabla f) - g(\phi_2 A \xi, \nabla f).\end{aligned}$$

□

It is also well known that for any vector field $X \in \mathfrak{X}(M)$, there exists a decomposition

$$X = u + \nabla h,$$

where $u \in \mathfrak{X}(M)$ satisfies $\operatorname{div}(u) = 0$ and $h \in C^\infty(M)$.

Proof of Theorem 1. Let $f \in C^\infty(M)$ be any smooth function. Then $\phi_2 \nabla f \in \mathfrak{X}(M)$, and from the previous lemma, we have

$$\operatorname{div}(\phi_2 \nabla f) = -4\xi(f),$$

where $\xi(f) = g(\nabla f, \xi)$.

Since the divergence of a vector field can be represented as the Laplacian of a potential function plus a divergence-free component, there exist a vector field $u \in \mathfrak{X}(M)$ and a smooth function $h \in C^\infty(M)$ such that

$$\phi_2 \nabla f = u + \nabla h \quad \text{with} \quad \operatorname{div}(u) = 0.$$

Taking the divergence of both sides gives

$$\operatorname{div}(\phi_2 \nabla f) = \operatorname{div}(u) + \Delta h = \Delta h.$$

Thus, we obtain

$$\Delta h = -4\zeta(f),$$

which completes the proof. $\square$

Proof of Theorem 2. Let $f, h \in C^\infty(M)$ such that $\Delta h = -4\zeta(f)$. Then, according to Bochner's formula, we have

$$\int_M \left\{ \operatorname{Ric}(\nabla h, \nabla h) + \|A_h\|^2 - 16\zeta(f)^2 \right\} = 0,$$

where $A_h(X) = A(\nabla h, X)$ for any $X \in \Gamma(TM)$.

This can be rearranged as

$$\int_M \left\{ \left(\|A_h\|^2 - \frac{16}{5}\zeta(f)^2 \right) + \operatorname{Ric}(\nabla h, \nabla h) - \frac{64}{5}\zeta(f)^2 \right\} = 0.$$

Now, suppose the Ricci curvature satisfies the bounds $4\mu \leq \operatorname{Ric} \leq 4\beta$ for constants μ, β . Then,

$$\int_M \left\{ \left(\|A_h\|^2 - \frac{16}{5}\zeta(f)^2 \right) + 16\mu\zeta(f)h - \frac{64}{5}\zeta(f)^2 \right\} \leq 0,$$

which implies that

$$\int_M \left\{ \left(\|A_h\|^2 - \frac{16}{5}\zeta(f)^2 \right) + \frac{16}{5} \left(5\mu\zeta(f)h - 4\zeta(f)^2 \right) \right\} \leq 0.$$

Assume that the condition $5\mu\zeta(f)h \geq 4\zeta(f)^2$ holds on M . Then the integrand is non-negative, and so the entire integral must be zero. Therefore,

$$\|A_h\|^2 = \frac{16}{5}\zeta(f)^2 \quad \text{and} \quad 5\mu\zeta(f)h = 4\zeta(f)^2.$$

From the second equality, we obtain $\zeta(f) = \frac{5}{4}\mu h$. Substituting this into the first, we get

$$A_h = -\frac{4}{5}\zeta(f) \cdot I = -\mu h \cdot I,$$

i.e., the shape operator is proportional to the identity.

Since M has positive curvature, this implies that the hypersurface is totally umbilical and satisfies $A_h = -c^2 h \cdot I$ for some constant $c^2 = \mu > 0$. Thus, M is isometric to the standard sphere $S^5(c)$ for some constant $c > 0$. $\square$

Example 1 (Geodesic Hypersphere as a Hypersurface in the Nearly Kähler S^6).

Let $S^6 \subset \mathbb{R}^7$ denote the standard 6-dimensional unit sphere endowed with the nearly Kähler structure (J, g) , where J is the canonical almost complex structure on S^6 . We construct a family of real hypersurfaces in S^6 by intersecting it with the hyperplane $x_7 = r$, where $r \in (-1, 1)$ is a fixed real number.

Define

$$M_r = \left\{ x = (x_1, \dots, x_7) \in S^6 \subset \mathbb{R}^7 \mid x_7 = r \right\}.$$

Then M_r is a 5-dimensional submanifold of S^6 , since

$$\sum_{i=1}^6 x_i^2 = 1 - r^2,$$

and thus,

$$M_r \cong S^5(\sqrt{1-r^2}).$$

Each M_r is a compact, connected, real hypersurface in S^6 , often referred to as a geodesic hypersphere centered at the pole in the x_7 -direction.

Further, we notice that M_r is a totally umbilical hypersurface in S^6 , and its shape operator A satisfies

$$A = -\frac{r}{\sqrt{1-r^2}} I,$$

where I is the identity transformation in the tangent space.

The mean curvature of M_r is given by

$$H = -5\frac{r}{\sqrt{1-r^2}}.$$

In particular, M_r is minimal if and only if $r = 0$, corresponding to the equatorial hypersphere.

The unit normal vector field ν of M_r satisfies $J\nu = \xi \in TM_r$. Since $A\xi = \lambda\xi$, where λ is a constant, the structure vector field ξ is a principal direction. Therefore, M_r is a Hopf hypersurface in the nearly Kähler S^6 [25].

Example 2 (Explicit Example for Theorem 1).

Consider the nearly Kähler 6-sphere $S^6 \subset \mathbb{R}^7$, equipped with the standard metric g and the nearly Kähler structure J induced from the octonionic cross product. Define the compact hypersurface

$$M := S^5 = S^6 \cap \{x_7 = 0\},$$

i.e., the equatorial 5-sphere. M is totally geodesic in S^6 and inherits a Riemannian metric from S^6 .

Let us define a smooth function $f \in C^\infty(M)$ as

$$f(x) = x_1, \quad x = (x_1, x_2, \dots, x_6, 0) \in M.$$

We now construct a function $h \in C^\infty(M)$ such that $\Delta h = -4\xi(f)$.

On the round sphere $S^5 \subset \mathbb{R}^6$, the Laplacian of a coordinate function is well known:

$$\Delta x_i = -5x_i \quad \text{for } i = 1, \dots, 6.$$

We aim to solve

$$\Delta h = -4g(\nabla f, \xi) = -4g(e_1, \xi).$$

Assume, for simplicity, that at each point $x \in M$, the structure vector field $\xi(x)$ is given by a fixed linear combination of the frame vectors:

$$\xi(x) = a_1(x)e_1 + a_2(x)e_2 + \dots + a_6(x)e_6,$$

where $a_i(x) \in C^\infty(M)$ and $\sum a_i(x)^2 = 1$ (since ξ is a unit vector). Then

$$g(e_1, \xi(x)) = a_1(x),$$

and we set

$$h(x) = \frac{4}{5}a_1(x).$$

Then

$$\Delta h = \frac{4}{5}\Delta a_1(x) \approx -4a_1(x) = -4g(e_1, \tilde{\zeta}(x)) = -4\tilde{\zeta}(f),$$

provided $a_1(x)$ satisfies $\Delta a_1 = -5a_1$. One such choice is $a_1(x) = x_1$, since x_1 is an eigenfunction of the Laplacian on S^5 with eigenvalue -5 . So we set the following:

$$h(x) = \frac{4}{5}x_1.$$

Then

$$\Delta h = \frac{4}{5}\Delta x_1 = \frac{4}{5}(-5x_1) = -4x_1 = -4\tilde{\zeta}(f),$$

since $\nabla f = e_1$, $\tilde{\zeta} = x_1e_1 + \dots$, and $g(\nabla f, \tilde{\zeta}) = x_1$.

Hence, the constructed function

$$h(x) = \frac{4}{5}x_1$$

is a smooth function on $M = S^5$ that satisfies the required condition

$$\Delta h = -4\tilde{\zeta}(f),$$

verifying Theorem 1 concretely.

4. Conclusions and Future Work

In this paper, we have explored the geometry of hypersurfaces in the nearly Kähler 6-sphere S^6 , focusing particularly on the behavior of the function $\tilde{\zeta}(f) = g(\nabla f, \tilde{\zeta})$, where $\tilde{\zeta} = -JN$ is the characteristic vector field. By analyzing conditions under which the Laplacian of a related scalar function leads to geometric rigidity, we have shown that certain hypersurfaces must necessarily be standard spheres. These results contribute to the broader understanding of submanifold theory in nearly Kähler geometry and provide new insights into the interplay between curvature, complex structures, and scalar function behavior.

As directions for future research, one could consider extending these results to other classes of almost Hermitian manifolds, particularly those with torsion or weaker integrability conditions. Another promising direction is to study the influence of additional geometric structures, such as conformal or CR-structures, on the rigidity of hypersurfaces. Moreover, examining the behavior of $\tilde{\zeta}(f)$ and related Laplace-type equations under various curve [26,27] and curvature constraints may yield further classification results or pave the way to applications in geometric analysis and mathematical physics.

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References

1. Banaru, M.B. On the six-dimensional sphere with a nearly Kählerian structure. *J. Math. Sci.* **2020**, *245*, 553–567. [[CrossRef](#)]
2. Gray, A. Almost complex submanifolds of the six sphere. *Proc. Am. Math. Soc.* **1969**, *20*, 277–279. [[CrossRef](#)]
3. Kirichenko, V.F.; Stepanova, L.V. The geometry of hypersurfaces of quasi-Kählerian manifolds. *Uspekhi Mat. Nauk.* **1995**, *50*, 213–214.

4. Stepanova, L.V.; Banarau, M.B.; Banaru, G.A. On geometry of QS-hypersurfaces of Kählerian manifolds. *Sib. Elektron. Mat. Izv.* **2018**, *15*, 815–822.
5. Banaru, M.B. Almost contact metric hypersurfaces with type number 0 or 1 in nearly-Kählerian manifolds. *Mosc. Univ. Math. Bull.* **2014**, *69*, 132–134. [[CrossRef](#)]
6. Hashimoto, H.; Mashimo, K. On some 3-dimensional CR submanifolds in S^6 . *Nagoya Math. J.* **1999**, *156*, 171–185. [[CrossRef](#)]
7. Sekigawa, K. Almost complex submanifolds of a 6-dimensional sphere. *Kodai Math. J.* **1983**, *6*, 174–185. [[CrossRef](#)]
8. Deshmukh, S. Totally real submanifolds in a 6-sphere. *Mich. Math. J.* **1991**, *38*, 349–353. [[CrossRef](#)]
9. Deshmukh, S. Totally real surfaces in S^6 . *Tamkang J. Math.* **1992**, *23*, 11–14. [[CrossRef](#)]
10. Dillen, F.; Vrancken, L. Totally real submanifolds in $S^6(1)$ satisfying Chens equality. *Trans. Amer. Math. Soc.* **1996**, *348*, 1633–1646. [[CrossRef](#)]
11. Ejiri, N. Totally real submanifolds in a 6-sphere. *Proc. Amer. Math. Soc.* **1981**, *83*, 759–763. [[CrossRef](#)]
12. Deshmukh, S. Totally real submanifolds of S^6 . *Nihonkai Math. J.* **1990**, *1*, 193–201.
13. Deshmukh, S. Characterizing spheres by conformal vector fields. *Ann. Univ. Ferrara Sez. VII Sci. Mat.* **2010**, *56*, 231–236. [[CrossRef](#)]
14. Deshmukh, S. Minimal hypersurfaces in a nearly Kähler 6-sphere. *J. Geom. Phys.* **2010**, *60*, 623–625. [[CrossRef](#)]
15. Deshmukh, S. Real Hypersurfaces in Nearly Kähler 6-sphere. *Mediterr. J. Math.* **2014**, *11*, 789–797. [[CrossRef](#)]
16. Bae, J.; Park, J.H.; Sekigawa, K. A one parameter family of totally umbilical hypersurfaces in nearly Kähler 6-sphere. *arXiv* **2017**, arXiv:1709.00177.
17. Deshmukh, S. Ibrahim Al-Dayel, A note on nearly Sasakian and nearly Cosymplectic structures of 5-dimensional spheres. *Int. Electron. J. Geom.* **2018**, *11*, 90–95. [[CrossRef](#)]
18. Abu-Saleem, A.; Banaru, M.B.; Banaru, G.A. A note on 2-hypersurfaces of the nearly Kaehlerian six-sphere. *Bul. Acad. Ştiinţe Repub. Mold. Mat.* **2017**, *3*, 107–114.
19. Banaru, M.B. On the type number of nearly-cosymplectic hypersurfaces in nearly-Kaehlerian manifolds. *Fundam. Prikl. Mat.* **2002**, *8*, 357–364.
20. Berndt, J.; Bolton, J.; Woodward, L.M. Almost complex curves and Hopf hypersurfaces in the nearly Kahler 6-sphere. *Geom. Dedicata* **1995**, *56*, 237–247. [[CrossRef](#)]
21. Bolton, J.; Vrancken, L.; Woodward, L.M. On almost complex curves in the nearly Kahler 6-sphere. *Quart. J. Math. Oxford Ser.* **1994**, *45*, 407–427. [[CrossRef](#)]
22. Bolton, J.; Pedit, F.; Woodward, L. Minimal surfaces and the ane Toda field model. *J. Reine Angew. Math.* **1995**, *459*, 119–150.
23. Bolton, J.; Vrancken, L.; Woodward, L.M. Totally real minimal surfaces with non-circular ellipse of curvature in the nearly Kahler S^6 . *J. London Math. Soc.* **1997**, *56*, 625–644. [[CrossRef](#)]
24. Yang, B.; Zhu, Q. Hypersurfaces in nearly Kähler manifold $S^3 \times S^3$. *Proc. Math. Sci.* **2017**, *127*, 707–717. [[CrossRef](#)]
25. Deshmukh, S.; Al-Solamy, F.R. Hopf hypersurfaces in nearly Kähler 6-sphere. *Balkan J. Geom. Appl.* **2008**, *13*, 38–46.
26. Zhu, Y.; Li, Y.; Eren, K.; Ersoy, S. Sweeping Surfaces of Polynomial Curves in Euclidean 3-space. *Analele Stiintifice Univ. Ovidius Constanta Ser. Math.* **2025**, *33*, 293–311.
27. Çalışkan, A.; Eren, K.; Ersoy, S. Dual Magnetic Curves and Flux ruled surfaces. *Int. J. Geom. Methods Mod. Phys.* **2025**, *22*, 2450273. [[CrossRef](#)]

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