

Article

UAV Formation for Cargo Transport by PID Control with Neural Compensation

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Abstract

Unmanned Aerial Vehicles (UAVs) are known to have limited payloads, which challenges their widespread use in transporting heavy goods. Meanwhile, collaboration between multiple UAVs in performing such a task may be a promising solution. To address the issues associated with the simultaneous use of UAVs, this paper presents a formation control system for transporting a payload suspended via a cable using two UAVs. The control structure is based on a layered scheme that combines a null-space-based kinematic controller with a PID controller associated with each UAV (quadcopters) with a neural correction system. The null-space supervisor controller is designed to generate the desired velocity for the UAV system to maintain formation. This proposal aims to avoid obstacles, balance the weight distribution across each vehicle, and also reduce the payload trajectory tracking error. The PID controller associated with the neural correction system receives these desired speeds and performs dynamic compensation, taking into account parametric uncertainties and dynamic disturbances caused by the movement of the payload coupled to the UAV systems. The stability analysis of the entire control system is performed using Lyapunov theory. Detailed dynamic models of each UAV in the system, the flexible cables, and the payload are presented in a realistic scenario. Finally, numerical simulations demonstrate the good performance of the UAV system control in formation.

Keywords: hierarchical null-space structure; collision avoidance; trajectory tracking; PID control; neural control

MSC: 37M05



Academic Editor: Shiqiang Liu

Received: 11 July 2025

Revised: 6 August 2025

Accepted: 12 August 2025

Published: 18 August 2025

Citation: Boubaker, S.; Vacca, C.; Rosales, C.; Kamel, S.; Alsubaei, F.S.; Rossomando, F. UAV Formation for Cargo Transport by PID Control with Neural Compensation. *Mathematics* **2025**, *13*, 2650. <https://doi.org/10.3390/math13162650>

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1. Introduction

1.1. Motivation

Payload transportation with UAV systems (alone or in formation) is revolutionizing logistics and global trade by offering an efficient, fast, and sustainable solution for the delivery of goods. This technology overcomes geographical limitations, such as remote or difficult-to-access areas, and significantly reduces delivery times compared to traditional methods. In addition, UAV systems minimize the carbon footprint by operating on electric power and optimize operating costs by eliminating the need for complex infrastructure.

The versatility of these systems makes them ideal for sectors such as healthcare, where they deliver critical medicines and supplies in real time, or e-commerce, where delivery is streamlined and service improved. In a world that demands immediacy and sustainability, cargo transportation with UAV systems is not only an innovation but a strategic necessity that will be in greater demand in the future. Methods of cargo transportation with drones can be classified according to the type of drone, the delivery mechanism, and the purpose of transportation. Below, we mention the main existing methods based on the extensive information available. These include fixed-wing drones, which use wings to generate lift, similar to airplanes. They are ideal for long distances and heavier payloads due to their energy efficiency [1]. Multi-rotor drones (quadcopters, hexacopters, etc.) are the most common for short-distance deliveries. They use multiple propellers for precision maneuvering [2]. Hybrid drones (VTOL—Vertical Take-Off and Landing) combine fixed-wing and multirotor characteristics, allowing vertical take-offs and long-distance flights [3], ideal for cargo transport in mixed terrain (urban–rural area). Within the above classification we can also mention heavy-lift drones [4], specifically designed to transport larger loads (up to hundreds of kilograms). They are usually used in industrial or military logistics. And finally we can mention the transport of cargo with several drones in a collaborative way [5]. In this case, groups of drones work in coordination to transport larger loads or multiple packages simultaneously. The present research work proposes the use of drones in a coordinated manner to transport cargo while following pre-established trajectories, avoiding obstacles and maintaining formation in complex environments, among which we can mention [6–8]. As an application in drone swarming, we can cite the work of Garcia-Aunon [9], which, although the authors focus on UAV systems for search and rescue, explores the potential for cargo transport with task coordination in complex environments. This work applies distributed decision making to ensure training stability. In contrast, in [10], the author presents the formation control of UAV systems, including its application to cargo systems. The work demonstrates through simulations that the proposed control can be used for cargo transportation and obstacle avoidance, which includes adaptability in dynamic environments. This work addresses the transport of a payload suspended by cables, which, when suspended by cables from two UAVs (quadrotors), can be considered an underactuated formation, because the cable-suspended payload increases the number of degrees of freedom (d.o.f.) and the number of control signals is constant. This situation is due to the fact that the payload increases the number of degrees of freedom of the formation, which generates an increase in the dynamics of the entire system (center of gravity position, sway, cable flexibility, etc.). To model and control the complete system formed by the UAV system and the payload, the following approaches are taken into account: The first one is based on modeling using the Newton–Euler [11] or Euler–Lagrange [12] equations. Instead the second [13] uses the Euler–Lagrange formalism, where a single UAV–utility payload system suspended from a cable is obtained. The proposed control technique uses two different control laws: the first one dampens the oscillation angle of the load and the second one does not apply to the UAV. A decrease in the oscillation of the payload suspended by the cable was demonstrated. In this work, the problem lies in having an accurate model of the vehicle (in this case a quadcopter) and the details of the cargo being transported, which is not always possible. In the scientific literature, control techniques based on neural networks [14] for partially unknown dynamics have been presented, as well as the works [15,16], which use fuzzy control systems and adaptive control with satisfactory results. Following this line of work, Ref. [8] is based on formation control using passivity techniques for multiple UAVs with a payload suspended by cables. This work does not consider uncertainties in the parameters, which can generate significant errors in real environments. In the case of [17,18], they work on the same problem, but in this

case they take into account parametric uncertainties and include climatic factors as external disturbances. Here, the proposals are robust controllers designed using Lyapunov and adaptive fuzzy systems, respectively. Likewise, Ref. [19] also considers suspended payload transport that compensates for the uncertainty of the payload mass while positioning the load at a reference position, considering the vertically aligned supporting cable, using an adaptive backward-cost controller. In contrast, in Ref. [20] geometric control for the transport of a rigid body suspended by a cable is presented, where the unified payload dynamics and UAVs are part of the control technique. As mentioned above, some papers in the literature use compensation schemes for dynamics not considered in the kinematic control scheme. They aim for reference speed tracking where the vehicle dynamics are not completely known. In [21] a cascade controller is designed, where a kinematic controller calculates the speed references needed to achieve the control objectives without considering the vehicle dynamics, and at the same time an additional dynamic controller receives the references from the kinematic one to achieve perfect speed tracking, obtaining the desired speeds by the considered vehicle. In addition, a similar scheme for non-holonomic robots is presented in [22] that includes an adaptive neural technique with sliding mode control to compensate for the unmodeled dynamics of the robot. This work takes into account the contributions of previous work by [7]. It presents formation control (UAV + payload) for cargo transport that includes an obstacle avoidance system based on null-space control, where each UAV has dynamic control to minimize the dynamic variations in the load. In contrast, this new proposal shows an adaptive correction to the PID controllers of each UAV, where these are cascaded with the kinematic controller of the load transport formation to ensure perfect velocity tracking. The neural correction is developed on an artificial neural network of the fully tuned radial basis function (RBF-FT) type. And it is not necessary to know the dynamic model of the system formed by the UAV quadrotors and the payload; in addition it imposes a correction on the control action at the output of the PIDs, providing an adaptive capability to a system that has a static controller. Another great advantage of this proposal is that it can be extended to formations with more than two vehicles in a simple way through an approach similar to the one explained in [6].

1.2. Contributions

The main contributions of this research work can be summarized in the following items:

- (i) The feasibility of a cascade control system based on null-space kinematic control together with internal dynamic PID control with neural compensation.
- (ii) The PID internal control proposal with its adaptable neural system that allows the dynamics to be adjusted to any type of payload.
- (iii) Simplicity of implementation, using each vehicle's internal PID.
- (iv) A freight transport system in formation that can be expanded to a larger number of vehicles.

1.3. Organization

To present this proposal, the paper is organized into the following sections, starting with Section 2, which shows how the system is constituted with the null-space kinematics, and Section 4, which shows the representation of the complete UAV model. Next, Section 5 discusses the proposed dynamic control system (static PID and adaptive neural network). And Section 6 includes the corresponding stability analysis. Then, in Section 7 some simulation results are presented, which allow one to observe the behavior of the proposed static–adaptive system. Finally, the conclusions are presented in Section 8.

2. Problem Description

In cargo transport, two UAV vehicles will be used in formation (UAV¹–payload–UAV²). The payload will be considered as a point mass attached to both vehicles by means of supporting cables m (see Figure 1). We define $\mathbf{x}_i^w = [x_i^w, y_i^w, z_i^w]^T$ as the position of the center of gravity (c.g.) of the i -th UAV in the global reference frame. Each quadrotor (i -th UAV) has a yaw angle indicated by ψ_i^w , also referred to the global reference frame. Based on these details, we can indicate the complete configuration of the system by $\mathbf{q} = [\mathbf{x}_1^{wT}, \psi_1^w, \mathbf{x}_2^{wT}, \psi_2^w]^T$. In this framework, we must also indicate the supporting cables, which have a length ℓ and hold the load from the two quadrotors.

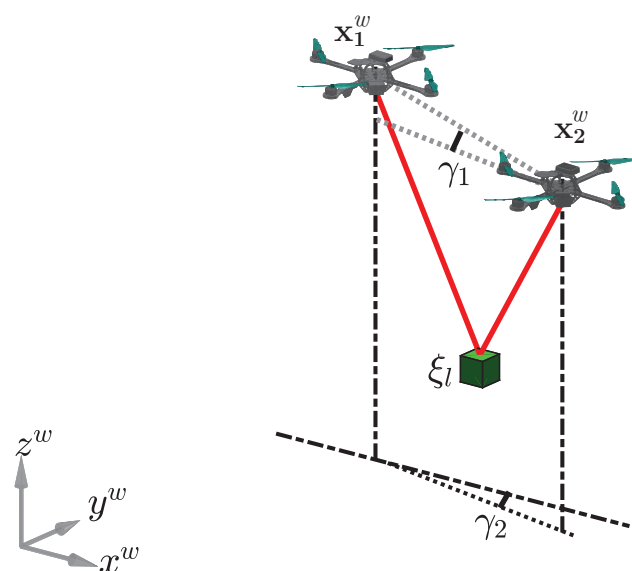


Figure 1. Figure of payload transport scheme using two AscTec Hummingbird quadrotors, where both vehicles are connected to the payload by flexible cables.

To meet the payload transport objective, the three-element formation (UAV¹–payload–UAV²) must guarantee the following objectives (see [23]):

- (i) Avoid collisions (obstacle avoidance).
- (ii) Payload positioning $\xi_\ell = [x_\ell^w, y_\ell^w, z_\ell^w]^T$ must follow a reference trajectory as closely as possible, indicated by $\xi_\ell^* = \xi_\ell^{w*}(t) = [x_\ell^{w*}, y_\ell^{w*}, z_\ell^{w*}]^T$.
- (iii) The distance between two transport quadrotors is indicated by $d = \|\mathbf{x}_1^w - \mathbf{x}_2^w\|$; this distance must be maintained within safe ranges. $d_m \leq d \leq d_M$.
- (iv) For balanced distribution of the payload weight, the angle of elevation $\gamma_1 = \arcsin((z_2^w - z_1^w)/d)$ must ensure that $\check{f}_1/\rho = \check{f}_2/(1 - \rho)$, where $\check{f}_i$ is the tension force of the cables on each quadrotor, which was filtered out. And $0 < \rho < 1$ balances the weight ratio of the payload suspended by each quadrotor.
- (v) For tangential cargo transport, the yaw orientation of the formation line must be $\gamma_2 = \text{atan2}(y_2^w - y_1^w, x_2^w - x_1^w)$ and the transport line of the formation must be tangential to the trajectory.
- (vi) The yaw orientation of each quadrotor must allow for forward movement.

As can be seen from the above list, obstacle avoidance is the item with the highest priority, and it is activated as soon as the positive artificial potential function V exceeds a critical threshold. The potential function must be $V = 0$ when there are no obstacles in the vicinity of the formation line and redefines obstacle avoidance trajectories with its contour lines ($\dot{V} = 0$). Therefore, the main objective of null-space technique control is to navigate in the region with $V = 0$ (see [23] for more details). It can be shown that

$$\dot{V} = \mathbf{J}_1 \dot{\mathbf{q}} + a_1,$$

where $\mathbf{J}_1 = \mathbf{J}_1(\mathbf{q})$ is the Jacobian function of the task and a_1 is a residual value. Next, obstacles are avoided by taking into account the null-space controller priorities.

$$\mathbf{v}_c^{(1)} = \mathbf{J}_1^\dagger (-k_V V - a_1),$$

where $\mathbf{J}_1^\dagger$ denotes the pseudoinverse matrix of $\mathbf{J}_1$.

The other priority tasks can be checked simultaneously if there are no obstacles in the vicinity of the path to be followed. Therefore, there are eight independent task variables.

$\xi = [x_\ell^w, y_\ell^w, z_\ell^w, d, \gamma_1, \gamma_2, \psi_1^w, \psi_2^w]^T$, as in the vector $\mathbf{q}$ of the system configuration. The velocities of these variables are linearly related to $\dot{\mathbf{q}}$ as follows (see [23] for more details):

$$\dot{\xi} = \mathbf{J}_2 \dot{\mathbf{q}} + \mathbf{a}_2, \quad (1)$$

where $\mathbf{J}_2 = \mathbf{J}_2(\mathbf{q})$ is the Jacobian of the predefined task and $\mathbf{a}_2$ is a residual value vector. Therefore, these predefined tasks in order of priority can be implemented using the null-space-based kinematic controller defined by

$$\mathbf{v}_c^{(2)} = \mathbf{J}_2^\dagger (\nu_x - \mathbf{a}_2),$$

where ν_x is given by (see [23])

$$\nu_x = \begin{pmatrix} \xi_\ell^* + \mathbf{K}_{\ell,1} \tan^{-1}(\mathbf{K}_{\ell,2}(\xi_\ell^* - \xi_\ell)) \\ \tilde{d} \\ \dot{\gamma}_1^* + k_{\gamma_1,1} \tan^{-1}(k_{\gamma_1,2}(\gamma_1^* - \gamma_1)) \\ \dot{\gamma}_2^* + k_{\gamma_2,1} \tan^{-1}(k_{\gamma_2,2}(\gamma_2^* - \gamma_2)) \\ \dot{\psi}_1^{w*} + k_{\psi_1,1} \tan^{-1}(k_{\psi_1,2}(\psi_1^{w*} - \psi_1)) \\ \dot{\psi}_2^{w*} + k_{\psi_2,1} \tan^{-1}(k_{\psi_2,2}(\psi_2^{w*} - \psi_2)) \end{pmatrix}$$

with

$$\tilde{d} = \begin{cases} -k_{d,1} \tan^{-1}(k_{d,2}(d - d_m)) & \text{if } d < d_m, \\ 0 & \text{if } d_m \leq d \leq d_M, \\ -k_{d,1} \tan^{-1}(k_{d,2}(d - d_M)) & \text{if } d > d_M, \end{cases}$$

$$\gamma_{1,d} = \gamma_1 + k_{\gamma_1,1} \tan^{-1} \left(k_{\gamma_1,2} \left(\frac{\check{f}_1}{\rho} - \frac{\check{f}_2}{1-\rho} \right) \right).$$

Finally, the kinematic controller (based on the null-space technique) must fulfill the greatest number of tasks to complete the transport objective, which is indicated by

$$\mathbf{v}_c = \mathbf{v}_c^{(1)} + (\mathbf{I}_8 - \mathbf{J}_1^\dagger \mathbf{J}_1) \mathbf{v}_c^{(2)}. \quad (2)$$

The reference speeds generated by the kinematics (2) are the speed references for the dynamics of each quadrotor. These reference velocities $\mathbf{v}_c$ are not reached instantaneously by each quadrotor due to intervening dynamics (UAVs and payload system). Therefore, it is necessary to use a cascade dynamics control system that acts in conjunction with the formation control (Figure 2) so that the velocity tracking error of each element of the formation decreases as time tends to infinity.

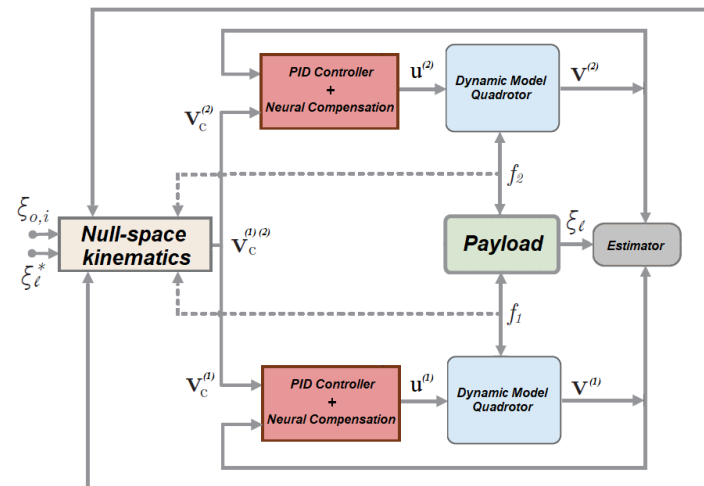


Figure 2. Kinematic formation controller framework.

3. Numerical Simulation

UAV Payload Considerations

In order to perform the validation of the proposed control technique, it is first necessary to know the parameters of the UAV (quadrotor). This vehicle has six degrees of freedom (d.o.f.); it is also necessary to know the mathematical model used to perform the simulation of the payload-carrying cables. This would give realism to the complete model. For this purpose, the Kelvin–Voigt cable model [24] of point masses joined by flexible joints and damping was used. With this model, realistic behavior of the payload is obtained with waving and rolling motions when suspended from the UAVs.

The models were implemented in C++ language and were connected to the Matlab R2021b environment using shared memory resources. Both null-space control and PID control with neural tuning were programmed using Matlab instructions. The following table presents the parameters used in the simulation (Table 1). The number of point masses used to study the dynamic behavior of each load-bearing cable is a parameter set by the designer. The higher the number of point masses, the more realistic the level of detail of the simulation. For the load-carrying model, 40 point masses per cable were used, the ends of which were connected to the payload. With this number of point masses, a resolution of 10 cm was obtained.

Table 1. Cable parameters and payload.

Parameters	Valor
Number of supporting cables	2
Number of mass particles per s. cable	40
Mass of support cable	0.001 [kg]
Cable diameter	0.0005 [m]
Cable drag coefficient	1.0
Mass of payload	0.4 [kg]
Payload edge length	0.1 [m]
Payload drag coefficient	1.05
Spring length	0.08 [m]
Elasticity index	5000 [N/m]
Spring friction	0.6 [N·s/m]
Gravity acceleration	9.7917 [m/s ²]
Air density	1.151 [kg/m ³]
Air friction	0.02 [N·s/m]
Soil repulsion	100 [N/m]
Soil friction	0.2 [N·s/m]
Soil absorption	0.05 [N·s/m]

4. UAV Mathematical Model

The controller for each vehicle is based on a mathematical model of a multirotor aerial vehicle, of which there is abundant information in scientific works. Therefore, two mathematical models are presented below to represent the dynamic behavior of the aircraft. The first is a simplified model valid for non-aggressive maneuvers that was published in [25] and is the one used for the controller design. Thus, the author considers that a commercial drone always has a controller on board to stabilize it, and the pitch (ϕ) and roll (θ) angles are small enough in non-aggressive maneuvers to be considered zero, and only the yaw (ψ) angle is taken into account.

$$\dot{\mathbf{v}} = \mathbf{K}_u \mathbf{u} - \mathbf{K}_v \mathbf{v} \quad (3)$$

The above Equation (3) is the representation of the UAV model with four degrees of freedom considered in relation to an inertial frame. In addition, $\mathbf{u} = [u_x, u_y, u_z, u_\psi]$ are normalized control signals $\mathbf{u} \in [0, 1]$. The matrix $\mathbf{R}$ is the rotation matrix. The dynamic parameters of the model are considered grouped in positive diagonal matrices $\mathbf{K}_u = \text{diag}[k_{u1}, k_{u2}, k_{u3}, k_{u4}]$ and $\mathbf{K}_v = \text{diag}[k_{v1}, k_{v2}, k_{v3}, k_{v4}]$. These parameters (k_i being $i : 1 \dots 8$) contain information about the dynamic model of the drone.

To validate the proposal, a more complex dynamic model, as shown in [26], is considered for the Hummingbird quadrotor from Ascending Technologies (Figure 3). This quadrotor has a plus configuration; i.e., the x^b axis has the same direction as one arm of the quadrotor body. The model for the quadrotor was obtained using the Newton–Euler equations

$$\begin{aligned} \dot{u} &= -g \sin(\theta) + rv - qw \\ \dot{v} &= g \sin(\phi) \cos(\theta) - ru + pw \\ \dot{w} &= \frac{1}{m} F_z + F_\ell + g \cos(\phi) \cos(\theta) + qu - pv \\ \dot{p} &= \frac{1}{I_{xx}} (L + L_\ell + (I_{yy} - I_{zz})qr) \\ \dot{q} &= \frac{1}{I_{yy}} (M + M_\ell + (I_{zz} - I_{xx})pr) \\ \dot{r} &= \frac{1}{I_{zz}} (N + N_\ell + (I_{xx} - I_{yy})pq) \end{aligned} \quad (4)$$

where the variables u , v , and w and p , q , and r are the linear and angular velocities of the frame with respect to the moving frame, g is the acceleration of gravity, and L , M , and N are the moments caused by the four propellers about the axes x^b , y^b , and z^b , respectively. The four propellers produce a vertical force $F_i = k_F \omega_i^2$. In addition, they create a moment $M_i = k_M \omega_i^2$ around the center of gravity of the quadrotor.

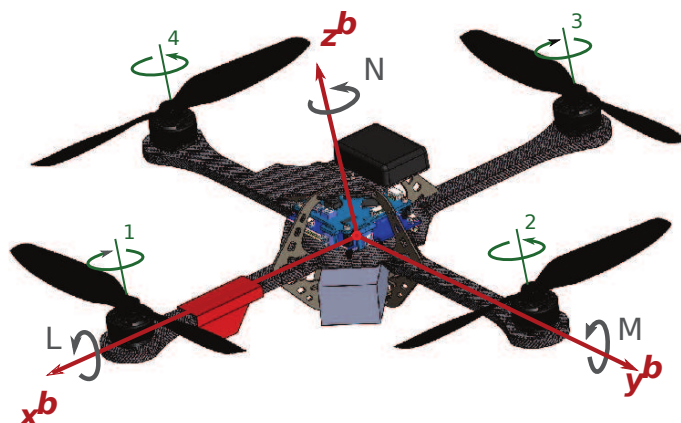


Figure 3. The Hummingbird quadrotor from AscTec company.

5. Dynamic Controller Design

The main idea of this work is to provide the internal controller of the UAV with adaptive capability, so that the internal controllers of the vehicles have the ability to self-adjust and maintain the desired trajectory independently of the dynamic variations produced by the load. An adaptive neural correction based on Lyapunov theory will be defined in the PID control of the UAV, whose main function is to improve the performance of the system in a closed loop. And finally, as defined in the previous section, the null-space control coordinates the tasks between the vehicle and the payload. The mathematical model (3) presents a simplified representation for the quadrotor dynamics. Disturbances caused by unmodeled dynamics and external disturbances can be accounted for in the dynamic model with an extra term δ .

$$\dot{\mathbf{v}} = \mathbf{K}_u \mathbf{u} - \mathbf{K}_v \mathbf{v} + \delta \quad (5)$$

It is assumed that δ and its derivative are functions bounded by a positive real value δ_0 .

$$\|\dot{\delta}(t)\| \leq \delta_0 \quad (6)$$

Based on the above, an adaptive controller is proposed. The control law consists of a static PID controller and an adaptive neural correction system based on Lyapunov theory. To continue with the analysis, the outputs of the kinematic controller $\mathbf{v}_c^i$ correspond to the velocity references of each vehicle; therefore $\mathbf{v}_c^i = \mathbf{v}_{ref}$, recalling that the supra-index i is the indicative of UAV 1 or 2.

$$\mathbf{e} = \mathbf{v}_{ref} - \mathbf{v} \quad (7)$$

Deriving the equation of error (7), we obtain

$$\dot{\mathbf{e}} = \dot{\mathbf{v}}_{ref} - \dot{\mathbf{v}} \quad (8)$$

Now, an auxiliary variable χ , which will be used to adjust the control action, is defined as follows:

$$\chi = \dot{\mathbf{v}}_{ref} - (\mathbf{K}_u \mathbf{u} - \mathbf{K}_v \mathbf{v}) \quad (9)$$

From the above Equation (9), we obtain the control action $\mathbf{u}$, which will depend on the model parameters. Next, rewriting (9) leads to

$$\mathbf{u} = \mathbf{K}_u^{-1} [(\dot{\mathbf{v}}_{ref} - \chi) + \mathbf{K}_v \mathbf{v}] \quad (10)$$

If we consider χ as a PID controller, the variable χ is expressed as follows:

$$\begin{cases} \chi = -\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (11)$$

with $\boldsymbol{\mu}$ being an auxiliary variable for the calculation of the integral action. Now substituting Equation (11) into (10) yields the following equation:

$$\begin{cases} \mathbf{u} = \mathbf{K}_u^{-1} [(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref}) + \mathbf{K}_v \mathbf{v}] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (12)$$

If we consider $\hat{\mathbf{K}}_u$ and $\hat{\mathbf{K}}_v$ as the known parameters of the vehicle, the new control law is expressed as follows:

$$\begin{cases} \mathbf{u} = \hat{\mathbf{K}}_u^{-1} [(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref}) + \hat{\mathbf{K}}_v \mathbf{v}] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (13)$$

To obtain the differential equation of the control errors, the model Equation (5) must be considered, to which we add and subtract the term $\hat{\mathbf{K}}_u \mathbf{u}$. It is then expressed as follows:

$$\dot{\mathbf{v}} = \mathbf{K}_u \mathbf{u} - \mathbf{K}_v \mathbf{v} + \delta + \hat{\mathbf{K}}_u \mathbf{u} - \hat{\mathbf{K}}_u \mathbf{u} \quad (14)$$

Substituting Equation (13) into Equation (14), we obtain the error dynamics of the form

$$\begin{cases} \dot{\mathbf{v}} = \mathbf{K}_u \mathbf{u} - \mathbf{K}_v \mathbf{v} + \delta + [(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref}) + \hat{\mathbf{K}}_v \mathbf{v}] - \hat{\mathbf{K}}_u \mathbf{u} \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (15)$$

Rearranging the above equation yields

$$\begin{cases} \dot{\mathbf{v}} = (\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref}) - \tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (16)$$

The matrices $\tilde{\mathbf{K}}_v = \mathbf{K}_v - \hat{\mathbf{K}}_v$ and $\tilde{\mathbf{K}}_u = \mathbf{K}_u - \hat{\mathbf{K}}_u$ are defined. Subsequently, the variable $\dot{\mathbf{v}}_{ref}$ is cleared and a rearrangement of (16) is obtained:

$$\begin{cases} \dot{\mathbf{v}} - \dot{\mathbf{v}}_{ref} = \mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} - \tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (17)$$

Considering Equation (8) leads to

$$\begin{cases} \dot{\mathbf{e}} = \mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} - \tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (18)$$

By subtracting $\dot{\mathbf{e}}$ from both members of the equation, we obtain the following:

$$\begin{cases} \dot{\mathbf{e}} = (\mathbf{K}_d + \mathbf{I})^{-1} \left(-\mathbf{K}_p \mathbf{e} - \boldsymbol{\mu} - (-\tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta) \right) \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (19)$$

From the above Equation (19), it is necessary to obtain a method to correct the control error due to external disturbances δ and variations in the vehicle parameters $\tilde{\mathbf{K}}_v$ and $\tilde{\mathbf{K}}_u$. To solve this problem, it is proposed to add an adaptive term to the static PID control action (19), which aims to reduce the effect of dynamic differences and disturbances due to string tension. Figure 4 shows the proposed control strategy applied to the dynamics of each vehicle.

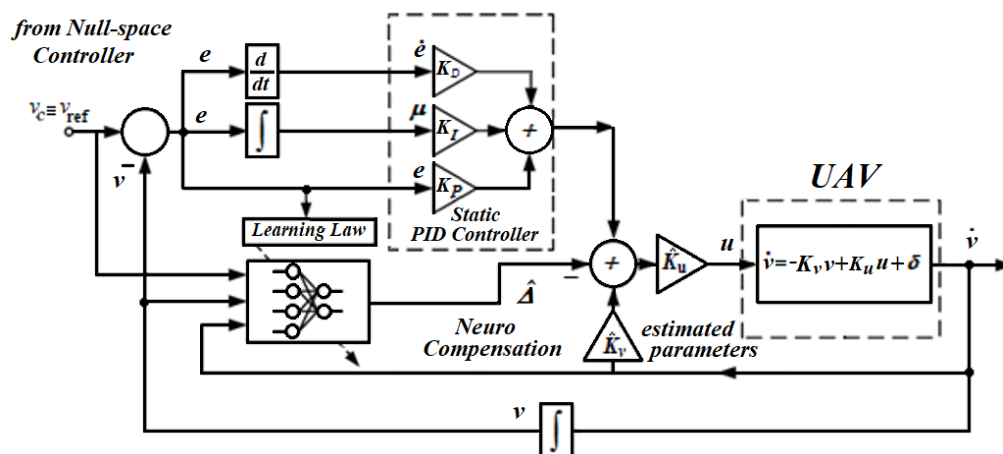


Figure 4. Dynamic control structure.

Neural Correction System Design

In this section we implement a tuning technique based on adaptive neural network functions (ANN); the structure of the network is of the RBF type (radial basis functions), which are mainly used to map input vector functions to real numbers [27]. In this work, the ANN is in charge of improving the control action of the main PID ($\mathbf{u}$) to compensate for dynamic variations with respect to the known dynamics ($\tilde{\mathbf{K}}_v$ and $\tilde{\mathbf{K}}_u$); for this action the PID gains are kept constant. To implement this control strategy, a regressor vector, fed with system signals, must be defined that tends to improve the response of the proposed strategy; the regressor vector is defined as $\boldsymbol{\zeta} = [\mathbf{v}, \mathbf{v}_{ref}, \dot{\mathbf{v}}]$. In this case, the dynamic variation $\Delta = -\tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta$ will be approximated by an ANN [27], with on-line parameterization. Here, the Δ approximation is expressed as follows:

$$\Delta = -\tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta = \sum_{k=1}^m w_k^{*T} \exp\left(-\frac{1}{\sigma_k^{*2}} \|\boldsymbol{\zeta} - \mathbf{c}_k^*\|\right) + \varepsilon(\mathbf{v}) = \dots = \mathbf{w}^{*T} \boldsymbol{\varphi}(\boldsymbol{\zeta}, \sigma^*, \mathbf{c}^*) + \varepsilon(\mathbf{v}) \quad (20)$$

where $\mathbf{w}^*$ is a vector of optimal weights $m \times n$ (m denotes the number of hidden neurons) and $\boldsymbol{\varphi}$ is $m \times 1$. Each Gaussian function is determined by the vector of centers $\mathbf{c}^*$ and the widths of the bells σ^* , and finally ε is the smallest value of the neural network approximation error. From now on φ^* is written instead of $\boldsymbol{\varphi}(\boldsymbol{\zeta}, \sigma^*, \mathbf{c}^*)$ to improve readability.

$$\Delta = \mathbf{w}^{*T} \boldsymbol{\varphi}^* + \varepsilon(\mathbf{v}) \quad (21)$$

At this point it is necessary to define a compensation term that is added to the control law to reduce the control error. This compensation can be stated as follows:

$$\hat{\Delta} = \hat{\mathbf{w}}^T \hat{\boldsymbol{\varphi}} \quad (22)$$

Adding the neural correction to the control action (13) leads to

$$\begin{cases} \mathbf{u} = \hat{\mathbf{K}}_u^{-1} \left[\left(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref} \right) + \hat{\mathbf{K}}_v \mathbf{v} - \hat{\Delta} \right] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (23)$$

Substituting (22), (23) can be written as follows:

$$\begin{cases} \mathbf{u} = \hat{\mathbf{K}}_u^{-1} \left[\left(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref} \right) + \hat{\mathbf{K}}_v \mathbf{v} - (\hat{\mathbf{w}}^T \hat{\boldsymbol{\varphi}}) \right] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (24)$$

Now, defining the weight vector error $\tilde{\mathbf{w}}$ as the difference between $\mathbf{w}^*$ (optimal) and $\mathbf{w}$ (instantaneous), and in the same way for $\tilde{\boldsymbol{\varphi}} = \boldsymbol{\varphi}^* - \boldsymbol{\varphi}$, (23) can be written as follows:

$$\begin{cases} \mathbf{u} = \hat{\mathbf{K}}_u^{-1} \left[\left(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref} \right) + \hat{\mathbf{K}}_v \mathbf{v} - \dots \right. \\ \left. - (\mathbf{w}^{*T} \boldsymbol{\varphi}^* + \tilde{\mathbf{w}}^T \boldsymbol{\varphi}^* + \mathbf{w}^{*T} \tilde{\boldsymbol{\varphi}} + \tilde{\mathbf{w}}^T \tilde{\boldsymbol{\varphi}}) \right] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (25)$$

Substituting into (14) and reordering yield

$$\begin{cases} \dot{\mathbf{v}} = -\tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta + \left(\mathbf{K}_p \mathbf{e} + \mathbf{K}_d \dot{\mathbf{e}} + \boldsymbol{\mu} + \dot{\mathbf{v}}_{ref} \right) - \dots \\ - (\mathbf{w}^{*T} \boldsymbol{\varphi}^* + \tilde{\mathbf{w}}^T \boldsymbol{\varphi}^* + \mathbf{w}^{*T} \tilde{\boldsymbol{\varphi}} + \tilde{\mathbf{w}}^T \tilde{\boldsymbol{\varphi}}); \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (26)$$

Rearranging Equation (26) again and recalling that $\mathbf{K}_S = (\mathbf{I} + \mathbf{K}_d)^{-1}$,

$$\begin{cases} \dot{\mathbf{e}} = \mathbf{K}_S \left[-\mathbf{K}_p \mathbf{e} - \boldsymbol{\mu} - \left(-\tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta \right) + \dots \right. \\ \quad \left. + (\mathbf{w}^{*T} \boldsymbol{\varphi}^* + \tilde{\mathbf{w}}^T \boldsymbol{\varphi}^* + \mathbf{w}^{*T} \tilde{\boldsymbol{\varphi}} + \tilde{\mathbf{w}}^T \tilde{\boldsymbol{\varphi}}) \right] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (27)$$

The neural network tends to minimize the error ([27]); therefore,

$$\left(-\tilde{\mathbf{K}}_v \mathbf{v} + \tilde{\mathbf{K}}_u \mathbf{u} + \delta \right) = \mathbf{w}^{*T} \boldsymbol{\varphi}^* + \varepsilon(v) \quad (28)$$

And further considering that $\tilde{\mathbf{w}}^T \tilde{\boldsymbol{\varphi}} + \varepsilon(v) = \varepsilon'(v)$, since the product $\tilde{\mathbf{w}}^T \tilde{\boldsymbol{\varphi}}$ equals the product of two errors, it can be considered as part of the minimum error of the neural network. Therefore, Equation (27) is expressed as follows:

$$\begin{cases} \dot{\mathbf{e}} = \mathbf{K}_S \left[-\mathbf{K}_p \mathbf{e} - \boldsymbol{\mu} + \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \tilde{\boldsymbol{\varphi}} + \varepsilon'(v) \right) \right] \\ \dot{\boldsymbol{\mu}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (29)$$

The equilibrium point of the above equation is $\begin{pmatrix} \mathbf{e} & \boldsymbol{\mu} \end{pmatrix}^T = \begin{pmatrix} 0 & \boldsymbol{\mu}^* \end{pmatrix}^T$, and then the equilibrium point $\mathbf{v} = \mathbf{v}_{ref}$; therefore, the balance is $\begin{pmatrix} 0 & \varepsilon'(\mathbf{v}_{ref}) \end{pmatrix}^T$. Then moving the equilibrium point at the origin yields

$$\tilde{\boldsymbol{\mu}} = \boldsymbol{\mu} - \varepsilon'(\mathbf{v}_{ref}) \quad (30)$$

Substituting (30) into (29) leads to

$$\begin{cases} \dot{\mathbf{e}} = \mathbf{K}_S \left[-\mathbf{K}_p \mathbf{e} - \tilde{\boldsymbol{\mu}} - \varepsilon'(\mathbf{v}_{ref}) + \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \tilde{\boldsymbol{\varphi}} + \varepsilon'(\mathbf{v}) \right) \right] \\ \dot{\tilde{\boldsymbol{\mu}}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (31)$$

or

$$\begin{cases} \dot{\mathbf{e}} = \mathbf{K}_S \left[-\mathbf{K}_p \mathbf{e} - \tilde{\boldsymbol{\mu}} + \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \tilde{\boldsymbol{\varphi}} + \tilde{\varepsilon}' \right) \right] \\ \dot{\tilde{\boldsymbol{\mu}}} = \mathbf{K}_i \mathbf{e} \end{cases} \quad (32)$$

where $\tilde{\varepsilon}' = \varepsilon'(v) - \varepsilon'(\mathbf{v}_{ref})$; this error equation will be used to determine the stability of the system.

6. Stability Analysis and Update Rules

We propose a candidate Lyapunov function of the form

$$\mathcal{L} = \frac{1}{2} \left[\mathbf{e}^T \mathbf{K}_i \mathbf{K}_p \mathbf{e} + (\tilde{\boldsymbol{\mu}})^T \mathbf{K}_p \mathbf{K}_S \tilde{\boldsymbol{\mu}} + tr \left(\frac{\tilde{\mathbf{w}}^T \tilde{\mathbf{w}}}{\alpha} \right) + \frac{\tilde{\mathbf{c}}^T \tilde{\mathbf{c}}}{\beta} + \frac{\tilde{\boldsymbol{\sigma}}^T \tilde{\boldsymbol{\sigma}}}{\gamma} \right] \quad (33)$$

Deriving the above Equation (33) yields

$$\dot{\mathcal{L}} = \left[\mathbf{e}^T \mathbf{K}_i \mathbf{K}_p \dot{\mathbf{e}} + \tilde{\boldsymbol{\mu}}^T \mathbf{K}_p \mathbf{K}_S \dot{\tilde{\boldsymbol{\mu}}} + tr \left(\frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha} \right) + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\gamma} \right] \quad (34)$$

Substituting Equation (32) into (34) and taking into account that $\mathbf{K}_p$ is a diagonal matrix ($\mathbf{K}_p^T \mathbf{K}_p = \mathbf{K}_p^2$) leads to

$$\begin{aligned}\dot{\mathbf{L}} = & \left(-\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} - \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \tilde{\boldsymbol{\mu}} + \dots \right. \\ & + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \tilde{\boldsymbol{\varphi}} + \tilde{\boldsymbol{\varepsilon}}' \right) + \dots \\ & \left. + \tilde{\boldsymbol{\mu}}^T \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} + \text{tr} \left(\frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha} \right) + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\gamma} \right)\end{aligned}\quad (35)$$

Reordering yields

$$\begin{aligned}\dot{\mathbf{L}} = & \left[-\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \tilde{\boldsymbol{\varphi}} + \tilde{\boldsymbol{\varepsilon}}' \right) + \dots \right. \\ & \left. + \text{tr} \left(\frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha} \right) + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\gamma} \right]\end{aligned}\quad (36)$$

Now using an approximation for the function $(\tilde{\varphi})$ based on the Taylor series, it can be expressed as the Taylor expansion of (φ^*) , which applies at the points $(\mathbf{c}^*) = \mathbf{c}$ and $(\sigma^*) = (\sigma)$,

$$\varphi^*(\zeta, \sigma^*, \mathbf{c}^*) = \varphi(\zeta, \sigma, \mathbf{c}) + \mathbf{Z}^T \tilde{\mathbf{c}} + \boldsymbol{\Gamma}^T \tilde{\sigma} + \mathbf{O}(\zeta, \tilde{\sigma}, \tilde{\mathbf{c}}) \quad (37)$$

where $\mathbf{O}(\cdot)$ denotes the higher-order terms in a Taylor series expansion and $\mathbf{Z}$ and $\boldsymbol{\Gamma}$ are derivatives of $\varphi^*(\zeta, \sigma^*, \mathbf{c}^*)$ with respect to σ , which can be expressed as $\mathbf{Z}^T = \left[\frac{\partial \varphi^*(\zeta, \sigma^*, \mathbf{c}^*)}{\partial \mathbf{c}^*} \right]$ and $\boldsymbol{\Gamma}^T = \left[\frac{\partial \varphi^*(\zeta, \sigma^*, \mathbf{c}^*)}{\partial \sigma^*} \right]$ evaluated at the points $(\mathbf{c}^*) = \mathbf{c}$ and $(\sigma^*) = (\sigma)$. Now from (37) we obtain

$$\tilde{\boldsymbol{\varphi}}(\zeta, \tilde{\sigma}, \tilde{\mathbf{c}}) = \mathbf{Z}^T \tilde{\mathbf{c}} + \boldsymbol{\Gamma}^T \tilde{\sigma} + \mathbf{O}(\zeta, \tilde{\sigma}, \tilde{\mathbf{c}}) \quad (38)$$

From (38) the higher-order term $\mathbf{O}(\cdot)$ is bounded by

$$\begin{aligned}\|\mathbf{O}(\zeta, \tilde{\sigma}, \tilde{\mathbf{c}})\| &= \left\| \tilde{\boldsymbol{\varphi}}(\zeta, \tilde{\sigma}, \tilde{\mathbf{c}}) - \mathbf{Z}^T \tilde{\mathbf{c}} - \boldsymbol{\Gamma}^T \tilde{\sigma} \right\| \leq \dots \\ &\leq \|\tilde{\boldsymbol{\varphi}}(\zeta, \tilde{\sigma}, \tilde{\mathbf{c}})\| + \|\mathbf{Z}^T \tilde{\mathbf{c}}\| + \|\boldsymbol{\Gamma}^T \tilde{\sigma}\| \leq k_1 + k_2 \|\tilde{\mathbf{c}}\| + k_3 \|\tilde{\sigma}\| \leq \mathbf{O}_{\max}\end{aligned}\quad (39)$$

where k_1, k_2 and k_3 are constants due to the fact that the Gaussian functions (RBF) and their derivatives are bounded by constants (k_1, k_2 and k_3). Substituting (38) into (36), we obtain

$$\begin{aligned}\dot{\mathbf{L}} = & -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \dots \\ & + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T (\mathbf{Z}^T \tilde{\mathbf{c}} + \boldsymbol{\Gamma}^T \tilde{\sigma} + \mathbf{O}(\cdot)) + \tilde{\boldsymbol{\varepsilon}}' \right) + \dots \\ & + \text{tr} \left(\frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha} \right) + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\gamma} = \dots \\ & = -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \dots \\ & + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \mathbf{Z}^T \tilde{\mathbf{c}} + \mathbf{w}^T \boldsymbol{\Gamma}^T \tilde{\sigma} + \mathbf{w}^T \mathbf{O}(\cdot) + \tilde{\boldsymbol{\varepsilon}}' \right) + \dots \\ & + \text{tr} \left(\frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha} \right) + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta} + \frac{\tilde{\boldsymbol{\sigma}}^T \dot{\tilde{\boldsymbol{\sigma}}}}{\gamma}\end{aligned}\quad (40)$$

Define the sum $\mathbf{w}^T \mathbf{O}(\cdot) + \tilde{\boldsymbol{\varepsilon}}' = \bar{\boldsymbol{\varepsilon}}$, where this sum is supposed to be bounded by

$$\|\bar{\boldsymbol{\varepsilon}}\| \leq k = cte \quad (41)$$

where $\bar{\boldsymbol{\varepsilon}}$ represents the approximation error of the neural compensation. Using neural network-based approximation theory, the inherent approximation error $\bar{\boldsymbol{\varepsilon}}$ can be arbitrarily reduced by increasing the number of m neurons, and then it is reasonable to assume that $\bar{\boldsymbol{\varepsilon}}$ is bounded by a constant k ; thus (40) can be rewritten as follows:

$$\begin{aligned}\dot{\mathbf{z}} = & -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \dots \\ & + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p (\tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \mathbf{w}^T \mathbf{Z}^T \tilde{\mathbf{c}} + \mathbf{w}^T \Gamma^T \tilde{\sigma} + k) + \dots \\ & + tr\left(\frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha}\right) + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta} + \frac{\tilde{\sigma}^T \dot{\tilde{\sigma}}}{\gamma}\end{aligned}\quad (42)$$

Group and rearrange (42) such that

$$\begin{aligned}\dot{\mathbf{z}} = & -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \left[tr\left(\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \tilde{\mathbf{w}}^T \boldsymbol{\varphi} + \frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha}\right) + \dots \right. \\ & + \left(\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \mathbf{w}^T \mathbf{Z}^T \tilde{\mathbf{c}} + \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta}\right) + \dots \\ & \left. + \left(\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \mathbf{w}^T \Gamma^T \tilde{\sigma} + \frac{\tilde{\sigma}^T \dot{\tilde{\sigma}}}{\gamma}\right) \right] + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \bar{\mathbf{e}}\end{aligned}\quad (43)$$

From the above equation we know that

$$\begin{aligned}\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \mathbf{w}^T \mathbf{Z}^T \tilde{\mathbf{c}} &= \left(\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \mathbf{w}^T \mathbf{Z}^T \tilde{\mathbf{c}}\right)^T = \tilde{\mathbf{c}}^T \mathbf{Z} \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} \\ \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \mathbf{w}^T \Gamma^T \tilde{\sigma} &= \left(\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \mathbf{w}^T \Gamma^T \tilde{\sigma}\right)^T = \tilde{\sigma}^T \Gamma \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e}\end{aligned}\quad (44)$$

Applying the cyclic property of the trace, we know that

$$tr\left(\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \tilde{\mathbf{w}}^T \boldsymbol{\varphi}\right) = tr\left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p\right)\quad (45)$$

Substituting (44) and (45) into (43) yields

$$\begin{aligned}\dot{\mathbf{z}} = & -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \left[tr\left(\tilde{\mathbf{w}}^T \boldsymbol{\varphi} \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p + \frac{\tilde{\mathbf{w}}^T \dot{\tilde{\mathbf{w}}}}{\alpha}\right) + \dots \right. \\ & + \left(\tilde{\mathbf{c}}^T \mathbf{Z} \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} - \frac{\tilde{\mathbf{c}}^T \dot{\tilde{\mathbf{c}}}}{\beta}\right) + \left(\tilde{\sigma}^T \Gamma \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} + \frac{\tilde{\sigma}^T \dot{\tilde{\sigma}}}{\gamma}\right) \left. \right] + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \bar{\mathbf{e}}\end{aligned}\quad (46)$$

Clearing $\tilde{\mathbf{w}}^T$, $\tilde{\mathbf{c}}^T$, and $\tilde{\sigma}^T$, Equation (46) can be expressed as follows:

$$\begin{aligned}\dot{\mathbf{z}} = & -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \left[tr\left(\tilde{\mathbf{w}}^T \left(\boldsymbol{\varphi} \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p + \frac{\dot{\tilde{\mathbf{w}}}}{\alpha}\right)\right) + \dots \right. \\ & + \tilde{\mathbf{c}}^T \left(\mathbf{Z} \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} - \frac{\dot{\tilde{\mathbf{c}}}}{\beta}\right) + \tilde{\sigma}^T \left(\Gamma \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} + \frac{\dot{\tilde{\sigma}}}{\gamma}\right) \left. \right] + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \bar{\mathbf{e}}\end{aligned}\quad (47)$$

Equalizing the terms in parentheses to zero and clearing the derivatives, they are expressed as follows:

$$\dot{\tilde{\mathbf{w}}} = -\alpha \boldsymbol{\varphi} \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p\quad (48)$$

$$\dot{\tilde{\mathbf{c}}} = -\beta \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e}\quad (49)$$

$$\dot{\tilde{\sigma}} = -\gamma \Gamma \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e}\quad (50)$$

Substituting each of the laws of adjustment (48)–(50) in (47) gives

$$\dot{\mathbf{z}} = -\mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p^2 \mathbf{e} + \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \bar{\mathbf{e}}\quad (51)$$

To prove the convergence of the control proposal, we must show that (51) is negative; therefore, we must find the conditions for this equation to be negative, considering

$$\begin{aligned}\dot{\mathbf{z}} \leq & -\lambda_{\min}(\mathbf{K}_i) \lambda_{\min}(\mathbf{K}_S) \left[(\lambda_{\min}(\mathbf{K}_p))^2 \|\mathbf{e}\|^2 + \|\mathbf{e}\|k \right] = \dots \\ = & -\lambda_{\min}(\mathbf{K}_i) \lambda_{\min}(\mathbf{K}_S) \lambda_{\min}(\mathbf{K}_p) \|\mathbf{e}\| \left(\|\mathbf{e}\| - \frac{k}{\lambda_{\min}(\mathbf{K}_p)} \right) \leq 0\end{aligned}\quad (52)$$

Therefore, in order for $\dot{\mathcal{L}} \leq 0$,

$$\|\mathbf{e}\| > \frac{k}{\lambda_{\min}(\mathbf{K}_p)} = \psi \quad (53)$$

Now considering $\tilde{\mathbf{w}} = \mathbf{w}^* - \mathbf{w}$, $\tilde{\mathbf{c}} = \mathbf{c}^* - \mathbf{c}$, and $\tilde{\sigma} = \sigma^* - \sigma$ and considering $\dot{\mathbf{w}}^* = 0$, $\dot{\mathbf{c}}^* = 0$, and $\dot{\sigma}^* = 0$, the adjustment rules are expressed as follows:

$$\dot{\mathbf{w}} = \alpha \varphi \mathbf{e}^T \mathbf{K}_i \mathbf{K}_S \mathbf{K}_p \quad (54)$$

$$\dot{\mathbf{c}} = \beta \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} \quad (55)$$

$$\dot{\sigma} = \gamma \Gamma \mathbf{w} \mathbf{K}_p \mathbf{K}_S \mathbf{K}_i \mathbf{e} \quad (56)$$

Equation (53) implies that the control error rules are ultimately value-limited:

$$\psi = \frac{k}{\lambda_{\min}(\mathbf{K}_p)} \quad (57)$$

It can be affirmed that the proposed control technique presents a uniformly bounded stability, whose dimension is defined by Equation (57).

7. Simulation Results

To evaluate the performance of the proposed dynamic compensation strategy for multi-UAV cooperative payload transport, a set of numerical simulations was carried out in MATLAB. The aerial system consists of two quadrotors collaboratively transporting a suspended load via rigid cables. The control architecture is divided into two layers: a kinematic formation controller that provides reference velocity commands and a dynamic controller responsible for generating the actual control inputs to each UAV, taking into account their dynamics and external perturbations.

Simulations consider various realistic conditions, including aerodynamic drag, external wind disturbances, and payload-induced oscillations. The navigation strategy is developed with the cargo designated as the point of interest, as the primary objective is cargo transportation, requiring the cargo to track the reference trajectory. The simulations employ a 0.4 kg payload, which exceeds the individual carrying capacity of each drone (0.2 kg maximum for the Hummingbird quadrotor), thus necessitating cooperative transport to accomplish the lifting task. An inclined lemniscate trajectory is employed as the reference path due to its inherent complexity, which provides a comprehensive benchmark for evaluating system performance. Figure 5 shows the trajectory of the payload and both UAVs in 3D space, illustrating that the proposed control law effectively maintains formation and ensures smooth transport of the load along the desired path. In addition, two obstacles are introduced along the intended path of the formation: one static and one dynamic. The static obstacle is positioned directly in the trajectory, forcing a deviation that must be executed while maintaining the payload's stability. The dynamic obstacle follows a circular trajectory that intersects the formation's reference, introducing time-varying constraints that require agile and anticipative adjustments. The control errors are presented in Figure 6. Task 1 corresponds to obstacle avoidance, with the potential field values demonstrating the effectiveness of the avoidance algorithm through consistently small magnitudes. Despite the presence of dynamic obstacles, the proposed control strategy maintains tracking performance by integrating obstacle motion prediction based on velocity estimation into the control framework.

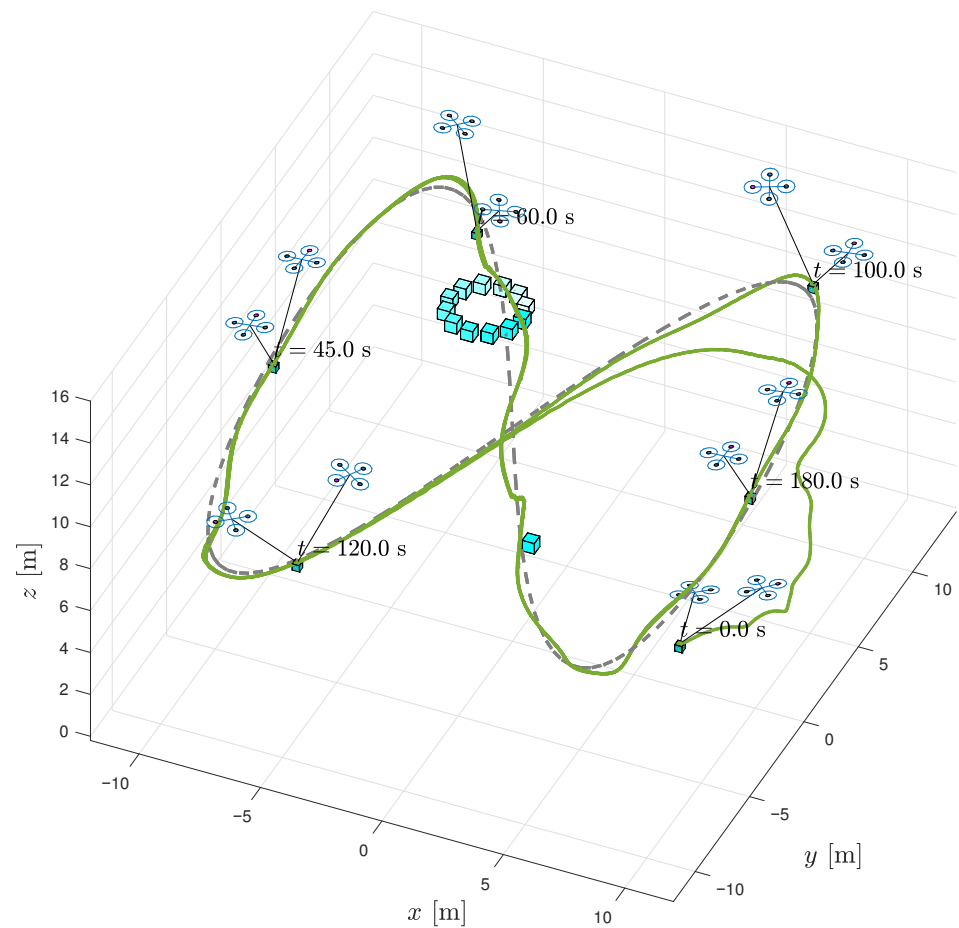


Figure 5. Three-dimensional trajectories of the two UAVs and the payload during the cooperative transport task. The desired trajectory is shown in dashed green. The proposed controller ensures coordinated motion while maintaining formation and minimizing payload swing.

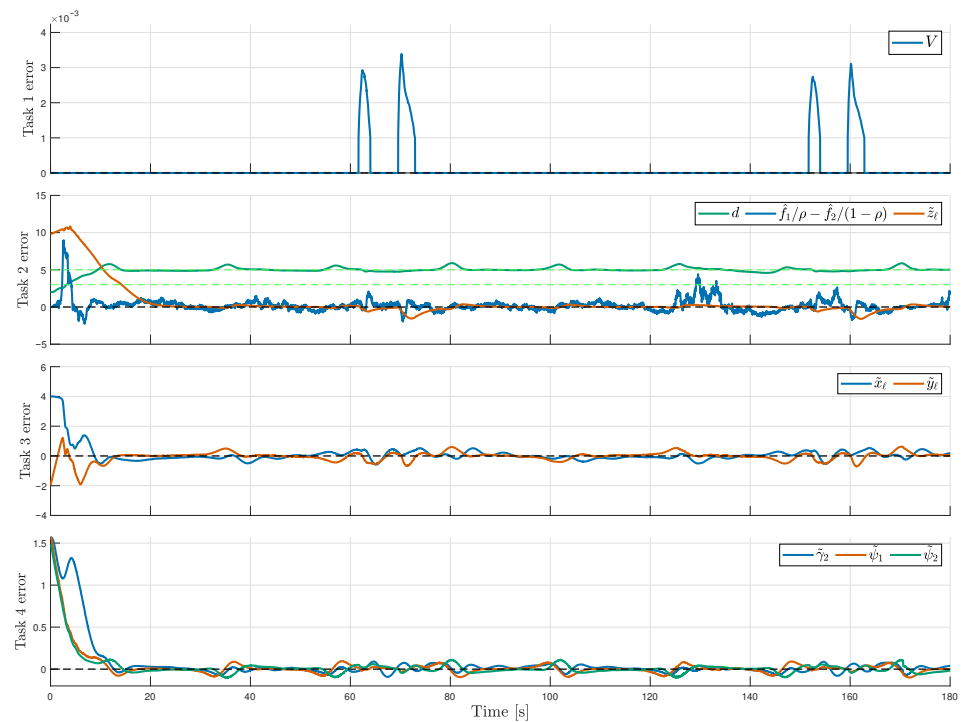


Figure 6. Evolution of the control errors for the four tasks defined in the kinematic training controller. It is observed that each of them remains at values close to zero.

Task 2 encompasses the control errors of three formation variables: the relative distance between UAVs, the altitude tracking error for the reference trajectory, and the load distribution balance between the two vehicles. The results show that the relative distance between vehicles converges to and remains within the specified reference bounds. The altitude tracking error approaches zero, indicating precise vertical positioning control. The load distribution analysis reveals that both vehicles effectively share equal proportions of the payload weight, confirming balanced cooperative lifting. Figure 7 presents a comparison between the reference velocity commands computed by the kinematic controller and the measured vehicle velocities. The close agreement between reference and actual velocities demonstrates the superior tracking performance achieved by the proposed dynamic control framework, with tracking errors maintained at negligible levels.

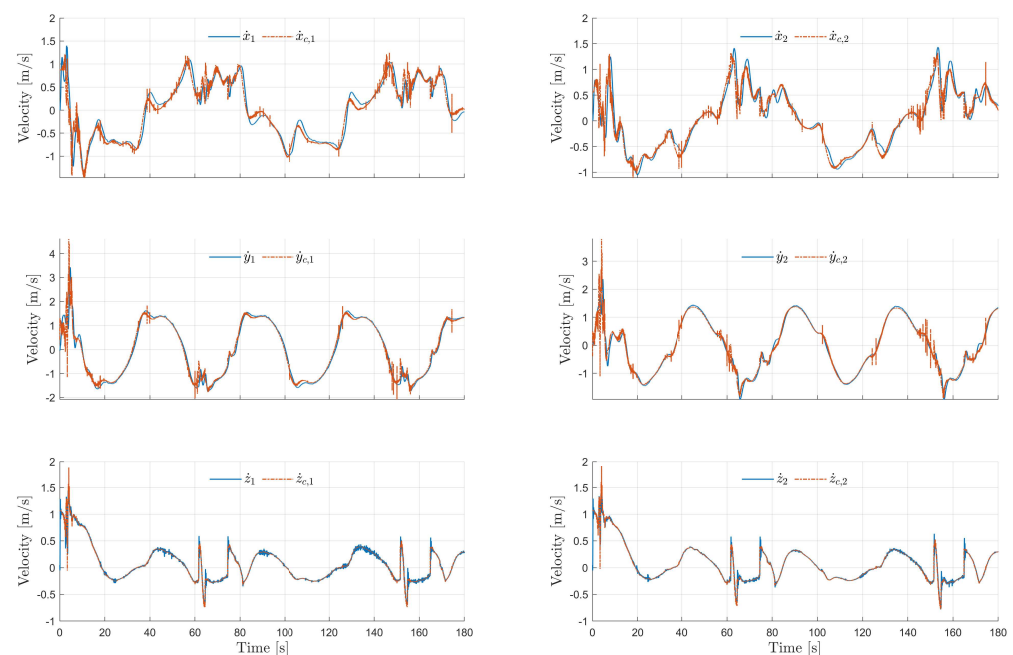


Figure 7. Velocity references calculated by the formation kinematic controller and the real velocities of each UAV.

The neural compensation mechanism proposed in this work is designed to operate entirely online, without any pre-training phase. At the beginning of each simulation, the RBF network parameters—including centers, synaptic weights, and variances—are initialized with random values. During operation, the network adapts in real time to minimize the velocity tracking errors between the references generated by the kinematic controller and the actual UAV velocities. This online adaptation enables the controller to effectively compensate for unmodeled dynamics, parameter variations, and external disturbances, ensuring smooth and coordinated motion of the aerial formation. Figure 8 illustrates the evolution of the neural compensation terms for each UAV across all degrees of freedom, highlighting the adaptive behavior of the RBF network as it learns to complement the baseline PID controller during the transport task.

A subsequent simulation was carried out to evaluate the performance of the proposed control strategy under the influence of external disturbances. In this scenario, a constant wind field with a velocity of approximately 20 km/h was applied along the positive X-axis direction. This perturbation impacted all elements of the cooperative transport system, including the UAVs, the suspended payload, and the connecting cables, introducing additional aerodynamic forces and coupling effects that challenge the system's stability and tracking accuracy.

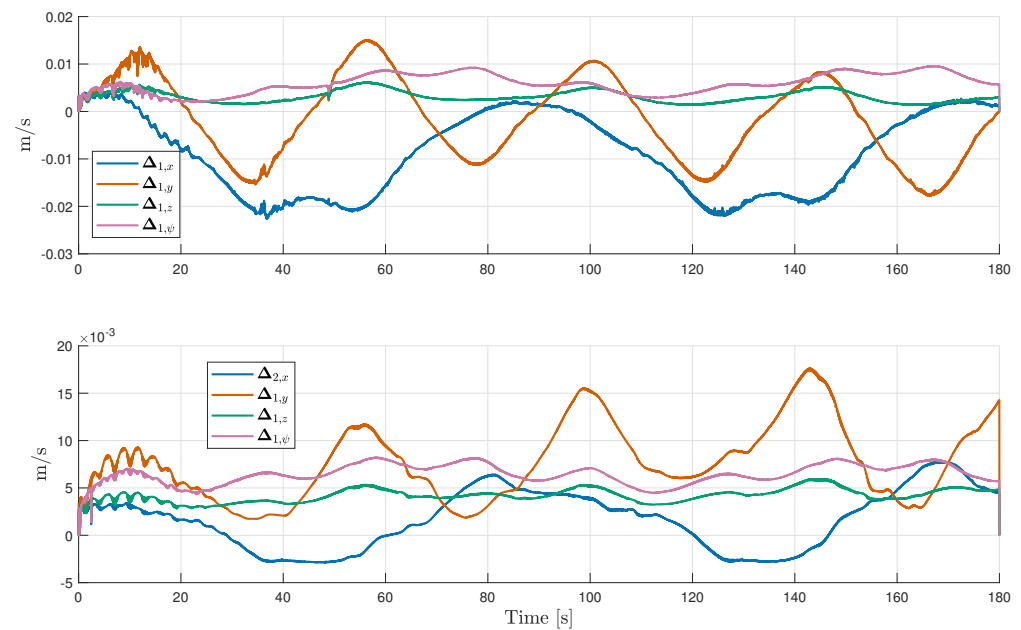


Figure 8. Time evolution of the neural compensation terms generated by the RBF network for each UAV during the cooperative transport task. The top subplot corresponds to quadrotor 1 and the bottom subplot to quadrotor 2.

The results highlight the significant influence of wind on the formation, particularly on the payload's oscillatory behavior and the tension dynamics in the suspension cables. Despite these challenges, the control architecture maintained coordinated transport with bounded errors and demonstrated robust compensation capabilities. Figure 9 illustrates the velocity tracking errors of the UAVs in all degrees of freedom, showing that the combined PID and neural compensation effectively mitigates the wind-induced deviations and ensures smooth cooperative motion.



Figure 9. Formation control errors occur when the system is disturbed by a wind of approximately 20 km/h in the positive x -axis direction.

It is easy to see that the proposed control technique offers advantages for cargo transport in formation. This same technique can be extended to more than three UAVs for cargo transport in formation. Looking at the figures, it can be seen that dynamic control (PID + RBF-FT) can compensate for external disturbances such as wind, as well as dynamic variations in the center of mass caused by cargo oscillation. This compensation also includes unmodeled dynamics; remember that the control was designed based on a reduced model of the UAV's dynamics (3). These experiments demonstrate the viability of the proposed control technique and that the null-space-based kinematics technique complements PID control with neural compensation perfectly in a cascade control configuration. In addition to the simplicity of design, it includes an adaptable complement for control.

8. Conclusions

In this work, a novel control framework for cooperative transport of suspended payloads using two quadrotor UAVs has been developed and analyzed. The proposed approach is scalable and can be extended to formations involving a larger number of aerial vehicles. The control architecture is implemented as a cascaded system, with a high-level kinematic controller based on null-space projections generating velocity references for the dynamic controllers of each UAV. The kinematic controller addresses multiple objectives simultaneously, including trajectory tracking, obstacle avoidance, inter-UAV distance regulation, balanced load distribution, formation orientation, and individual yaw alignment of the quadrotors. These objectives are organized hierarchically as prioritized tasks within the null-space framework, ensuring coordinated and adaptive formation control.

As a key contribution, this work introduces an adaptive neural compensation mechanism that enhances the baseline PID dynamic controllers of the UAVs. The neural module, implemented as an RBF-FT (Radial Basis Function–Fuzzy Tuning) network, operates online to minimize velocity tracking errors between the references generated by the kinematic controller and the actual UAV velocities. By adapting its parameters in real time, the neural compensator effectively mitigates the effects of unmodeled dynamics, parameter variations, and external disturbances, thereby improving the robustness and precision of the overall system.

The proposed method has been validated through high-fidelity numerical simulations under realistic dynamic conditions and complex navigation environments. The results demonstrate the ability of the control strategy to maintain formation integrity, ensure stable payload transport, and achieve robust performance in the presence of disturbances such as wind and obstacle interactions. These findings highlight the potential of the approach for advancing cooperative aerial manipulation and transport tasks in real-world applications.

Author Contributions: Conceptualization, F.R., S.B., F.S.A., S.K., and C.V.; data curation, S.K. and C.V.; formal analysis, S.B. and F.R.; funding acquisition, S.B.; investigation, S.B., F.S.A., and C.R.; methodology, F.R.; project administration, S.B.; resources, S.B. and S.K.; software, C.V.; supervision, S.B. and F.R.; validation, S.B., C.R., and F.S.A.; writing—original draft, F.R. and C.R. writing—review and editing, F.S.A. All authors have read and agreed to the published version of the manuscript.

Funding: Deputyship for Research and Innovation, Ministry of Education in Saudi Arabia, project number MoE-IF-UJ-R2-22-20772-2.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding author.

Acknowledgments: The authors extend their appreciation to the Deputyship for Research and Innovation, Ministry of Education in Saudi Arabia for funding this research work through the project number MoE-IF-UJ-R2-22-20772-2.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

VTOL	Vertical Take-Off and Landing
UAV	Unmanned Aerial Vehicle
PID	Proportional–Integral–Derivative (controller)
RBF-FT	Radial Basis Function—Fully Tuned
ANN	Adaptive Neural Network

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