

Article

Optimal Inventory and Pricing Strategies for Integrated Supply Chains of Growing Items Under Carbon Emission Policies

Mehak Sharma ¹, Mandeep Mittal ² , Divya Agarwal ¹ , Anil Dhanda ¹, Rekha Guchhait ^{3,4} 
and Mitali Sarkar ^{5,*} 

¹ Amity Institute of Applied Sciences, Amity University, Noida 201313, Uttar Pradesh, India; mehak.sharma1@s.amity.edu (M.S.); dagarwal1@amity.edu (D.A.); anil.dhanda1@s.amity.edu (A.D.)

² School of Computer Science Engineering and Technology, Bennett University, Noida 201310, Uttar Pradesh, India; mandeep.mittal@bennett.edu.in

³ Department of Global Business & Trade, Joongbu University, Goyang 10279, Republic of Korea; rekha@jmail.ac.kr or rg.rekhaguchhait@gmail.com

⁴ Center for Global Health Research, Saveetha Medical College, Saveetha Institute of Medical and Technical Sciences, Saveetha University, Chennai 600077, Tamil Nadu, India

⁵ Department of Hospitality and Tourism Management, Sejong University, 209 Neungdong-ro (Gunja-dong), Gwangjin-gu, Seoul 05006, Republic of Korea

* Correspondence: mitali@sejong.ac.kr or mitalisarkar.ms@gmail.com

Abstract: This study investigates inventory management and pricing techniques in a two-tier supply chain where newborn items are grown, slaughtered, and transported to retailers for consumer sale. This study assesses how certain carbon regulations can enhance or hinder profitability for suppliers and retailers, demonstrating the interdependence of their financial performance in connection to environmental regulations. A mathematical model considers demand as impacted by unit weight, selling price, and storage duration, with consumption patterns as a power function of these variables. This paper examines demand dynamics and proposes a solution for optimizing crucial factors such as the number of newborn items, the retailer's selling price, and operating cycle time to increase profitability while maintaining excellent customer service.

Keywords: growing items inventory; integrated supply chain system; two-level permissible delay in payments (trade-credit); and carbon emission regulations

MSC: 90B05



Academic Editor: Mauro Passacantando

Received: 12 March 2025

Revised: 11 April 2025

Accepted: 23 April 2025

Published: 9 May 2025

Citation: Sharma, M.; Mittal, M.; Agarwal, D.; Dhanda, A.; Guchhait, R.; Sarkar, M. Optimal Inventory and Pricing Strategies for Integrated Supply Chains of Growing Items Under Carbon Emission Policies. *Mathematics* **2025**, *13*, 1567. <https://doi.org/10.3390/math13101567>

Copyright: © 2025 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Collaborations between businesses and their partners are critical in today's economy. Businesses comprehend that the performance of their partners heavily impacts the quality of their products and services. This interconnected network of partners is recognized as a supply chain (SC) in which numerous firms collaborate to provide goods or services to customers [1]. Effectively controlling the activity of these SC participants is critical for satisfying client expectations.

Supply chain management (SCM) refers to various operations involving various stakeholders, including raw material suppliers and end customers. Customer demand drives all SC activities. Thus, effective inventory management is critical for satisfying that demand and guaranteeing customer satisfaction. Proper inventory management can reduce costs throughout the system, leading to increased profitability [2]. Furthermore, a significant relationship exists between pricing and demand, i.e., lower prices usually result

in increased demand. That is, improving pricing tactics can significantly boost profitability. Effective pricing, a crucial revenue management component, relies on efficient inventory management procedures. Consequently, combining inventory and revenue management methods is critical for improving SC performance.

In modern business transactions, a credit term is provided by suppliers, which aids inventory management by allowing retailers to settle outstanding accounts. Suppliers frequently provide a grace period during which retailers can postpone payment without incurring interest costs. This arrangement allows retailers to generate revenue from sales while possibly earning interest by investing those funds. However, suppliers might incur interest charges if payments are delayed beyond this grace period. Delayed payments can lower inventory holding costs. Trade-credit from suppliers motivates retailers to purchase more and acts as a marketing tool, luring new consumers who see it as an alternative to bulk discounts. As a result, suppliers and retailers rely largely on trade-credit.

The obligations of SCM must shift from traditional methods to incorporate novel concepts [3]. Inventory management often assumes that items remain unmodified throughout storage. However, this is not always the case. For example, the quality of inventory might decline or increase with time. While this idea is well explored in inventory management, the issue of growth—defined as increases in inventory levels owing to weight gain—receives far less scholarly attention. Farming, fishery, poultry, and livestock industries all experience growth as inventory goes through breeding and consumption. During breeding, inventory grows until slaughter, which is consumed to meet the demand. Managing the inventory of growing things is vital since they are susceptible to illnesses and quality loss. Poor assessments can lead to financial losses along with threats to consumer health. After slaughter, these goods become perishable. Optimizing inventory and pricing for such commodities can minimize waste and unnecessary expenditure.

The retailer's activities produce large carbon emissions, necessitating taxes based on certain carbon restrictions by the authorities. This study answers numerous essential questions as follows: (i) How is the per unit weight price for a retailer affected by the carbon laws? (ii) How can the optimal permissible time be determined under carbon regulations? (iii) How is the integrated profitability affected by carbon regulations? The presented model primarily contributes to the research with integrated optimal pricing, permissible payment delay, and ordering policies for newborn growing items, all designed to maximize the overall profit while complying with carbon limits.

Section 2 provides a brief overview of the current literature in the remaining paper, while Section 3 outlines the notations and assumptions underpinning the model. The utilized mathematical framework is described in Section 4. The methodology for obtaining solutions is explained in Section 5. Section 6 examines the numerical results, while Section 7 delves into sensitivity analysis and its implications for management. Lastly, the work is concluded in Section 8, and several avenues are suggested for additional research.

2. Contextual Analysis of the Literature

The existing literature has been reviewed considering four aspects: (1) growing items inventory, (2) integrated SC system, (3) two-level permissible delay in payments (trade-credit), and (4) carbon emissions regulations.

Khalilpourazari and Pasandideh [4] developed a mathematical model to solve operational restrictions for businesses that manage various types of growing goods, utilizing hybrid meta-heuristic algorithms to identify the ideal number of newborn items to acquire. Nobil et al. [5] incorporated shortages into their economic order quantity (EOQ) model for poultry producers. Sarkar et al. [6] built an SCM model to reduce emissions from the system as well as emissions. Pourmohammad-Zia and Karimi [7] included degradation

in their analysis, outlining the best feeding and consuming periods for slaughtered items. Alfares and Afzal [8] investigated growing products with defective quality and full back-order. Furthermore, Mittal and Sharma's [9] study in 2021 advanced the knowledge of how financial regulations may strategically influence the purchase behavior in SCs with growing products, while Sharma and Mittal [10] examined its influence on selecting items of different quality.

Yu et al. [11] utilized profit-sharing contracts to improve coordination within an SC, especially when the supplier faces considerable upfront expenses. Moreover, revenue-sharing contracts and collaborative investment in inventory technology were implemented by Zhang et al. [12] to reduce degradation. These studies found that coordinating techniques boost system profitability. However, three-echelon supply networks were under-researched due to their complexity, as emphasized by Cai et al. [13] in their study, encompassing a supplier, distributor, and retailer. A three-tier SC was examined by Sebatjane and Adetunji [14] for growing commodities, taking the consumption rate to be both price- and freshness-sensitive, implementing an exponential function for price effect and a linear function for freshness over storage time. Sebatjane and Adetunji [15] then investigated a model emphasizing maximum lifespan-sensitive degradation rates, ignoring expiry date influences.

Researchers have extensively investigated the best ordering procedures for businesses operating on fixed credit durations across various scenarios. Huang et al. [16] conducted research on collaborative inventory management between suppliers and customers to optimize order quantities and scheduling to improve SC efficiency. Mahata et al. [17] studied inventory models created particularly for items with a short shelf life, highlighting the importance of timely replenishment to avoid spoiling losses. Mahata [18] examined the issues that retailers confront when physical or operational constraints limit their capacity to hold or handle goods, requiring careful order quantity planning. Vandana and Sharma [19], for example, expanded the credit term to incorporate payment delays based on purchase quantity. In certain circumstances, if the retailer's purchase fulfills a minimum amount, the supplier will provide a defined credit period; otherwise, immediate payment is needed. Furthermore, some works suggest partial payment delays to stimulate larger purchases, in which the retailer pays a certain percentage beforehand and the balance at the end of the credit term [18]. Seifert and Seifert [20] provided extensive assessments of trade-credit policies.

In the late 1960s, authorities began implementing pollution control rules to minimize human-caused greenhouse gas emissions. Product handling operations in logistics facilities account for about 13% of total emissions, as shown by the World Economic Forum in 2009. The product storage operations in UK warehouses produce around 10.2 million tons of CO₂ [21]. An effective strategy was employed by Hua et al. [22] to investigate how trading mechanisms affect firm expenses when certain carbon caps are imposed. Chen et al. [23] investigated how a carbon cap policy affects the optimum ordering policy, considering purchased items as an emissions source. Toptal et al. [24] examined purchase and investment strategies for lowering carbon emissions under three tax schemes. Hovelaque and Bironneau [25] investigated how carbon pricing influences consumer behavior in the context of carbon price legislation. Lin and Sarker [26] incorporated certain rules regarding carbon taxes to determine optimal ordering strategies for low-quality goods. Li and Hai [27] developed effective reorder regulations considering various retailers utilizing a single warehouse, factoring carbon tax into the overall costs of buying and maintaining inventory. Dhanda et al. [28] evaluated the implications of carbon emission regulations on fresh food SCs, emphasizing degrading products of varying quality.

Alamri [29], Chang and Tseng [30], and De-la-Cruz-Márquez et al. [31] analyzed sustainable SC models for growing and perishable commodities, considering trade-credit, price, carbon emissions, imperfect quality, and replenishment techniques to optimize inventory and financial decisions.

Research Gap

This research draws on these models, particularly the work by Nobil et al. [5] as a basis for incorporating growth dynamics and quality into inventory management in SCs with growing products. Building upon these insights, it addresses the complexities of a three-echelon SCM, particularly in managing growing products and integrating inventory, trade-credit, and carbon emissions considerations. This research further extends the previous studies by considering the dual challenge of carbon emissions and product degradation, particularly in growing SCs. The work by Hua et al. [22] and Dhanda et al. [28] is a foundation for understanding how carbon pricing can impact SC decisions, which is further integrated with trade-credit and growth dynamics.

Building on the previous major contributions, this study intends to support an integrated business-focused model by providing optimal integrated price, payment delay, and ordering policies for newborn growing goods. These strategies aim to optimize the integrated profit while considering power demand patterns and different carbon emission regulations. The gap in the emerging literature is addressed in the presented research by comparing its findings to previous research and offering contributions in Table 1.

Table 1. Author contributions in the existing literature.

Authors	Imperfect Quality Items	Growing Items	Trade-Credit Policy	Demand	Carbon Emissions
Khalilpourazari et al. [4]		✓		Constant	
Sebatjane and Adetunji [32]	✓	✓		Constant	
Pourmohammad et al. [7]		✓		Constant	
Mittal and Sharma [9]		✓	✓	Constant	
Sarkar and Bhuniya [33]	✓			Variable	
Sharma and Mittal [10]	✓	✓	✓	Constant	
Alfares and Afzal [8]	✓	✓		Constant	
Sebatjane and Adetunji [14]		✓		Price and freshness	
Mahata et al. [18]			✓	Constant	
Garai and Sarkar [34]				Fuzzy	
Moon et al. [35]	✓			Variable	✓
Mahata et al. [17]			✓	Price sensitive	
Mahapatra et al. [36]	✓			Selling and advertisement	✓
Hovelaque and Bironneau [25]				Carbon emissions	✓
Lin et al. [26]	✓			Constant	✓
Sarkar et al. [37]				Price	✓
Present study	✓	✓	✓	Price and time-dependent	✓

3. Nomenclature and Assumptions

This section provides notation used in the paper and associative assumptions to build the model.

3.1. Nomenclature

The following nomenclature is adopted in the presented model as given in Table 2.

Table 2. Nomenclature.

Symbol	Description
Parameters	
P_s	Per unit supplier's selling cost (\$/kg)
O_r	Retailer's fixed ordering cost (\$)
F_r	Retailer's fixed transportation cost (\$)
I_r	Per unit retailer's inspection cost (\$)
H_r	Per unit retailer's holding cost (\$/kg/year)
V	Per unit selling price of defective items (\$)
γ	Fraction of defective items
α	Scale parameter of the time-sensitive demand where $\alpha \in (-\infty, \infty)$ (kg/day)
β	Index of power demand pattern in customers' demand where $\beta > 0$
u	Highest potential customers' demand rate without impacts of price and time (kg/day)
v	Price-sensitive parameter in customer's demand ($v > 0$)
w	Price index based on customer's demand ($w \geq 1$)
$s1$	Highest possible selling price of each growing item (\$/kg)
φ_1	Supplier's inventory cycle length (years)
φ_2	Screening period (years)
U_i	Weight of each newborn growing item initially (kg)
U_t	Weight of each newborn growing item at any time t (kg)
O_s	Supplier's fixed ordering cost (\$)
P_c	Per unit supplier's purchasing cost (\$/kg)
I_s	Per unit supplier's opportunity cost (\$/kg)
F_s	Per unit feeding cost of newborn items (\$/kg)
M	Retailer's trade-credit period offered by the supplier (years)
n	Customer's trade-credit period offered by the retailer (years)
I_E	Retailer's earned interest (%/year)
I_P	Retailer's paid interest (%/year)
b	Constant of integration (>0)
η	Asymptotic weight of each growing item when the feeding period tends to infinity (>0) (kg)
θ	Growth rate (>0)
$\hat{P}$	Carbon emissions from purchasing growing items (ton/kg/year)
$\hat{O}$	Carbon emissions from ordering growing items (ton/kg/year)
$\hat{H}$	Carbon emissions from holding growing items (ton/kg/year)
$\hat{T}$	Carbon emissions from transporting growing items (ton/kg/year)
$\hat{c}$	Cap for carbon emissions (ton/kg/year)
$\hat{z}_1$	Tax on carbon emissions (\$/ton/kg)
$\hat{z}_2$	Market price of carbon emissions (\$/ton/kg)
Decision Variables	
X	Number of newborn items purchased by the supplier (integer)
s	Selling price of good quality items (\$/kg)
φ	Retailer's inventory cycle length (years)

3.2. Assumptions

- The supplier's inventory techniques do not involve stock-outs.
- Carbon emissions are accounted for in all inventory activities, resulting in higher expenses due to various carbon regulations enforced by authorities.
- The customer demand structure, $D(t, s)$, is a nonlinear function of both selling price per weight unit and storage time, stated in a separable and additive manner. The demand function is represented as $D(t, s) = u - vs^w + \alpha\beta\left(\frac{t}{\varphi}\right)^{\beta-1}$, where $u > 0$, $v > 0$, $w \geq 1$, $-\infty < \alpha < \infty$, $\beta > 0$, and $s \leq s1$. The highest selling price possible for each weight unit is taken to be $s1 = \left(\frac{u}{v}\right)^{\frac{1}{w}}$.

- iv. The credit period enables retailers to collect revenue from sales while earning interest on their capital without paying any interest expenses. This structure gives them more financial freedom and improves their overall profitability.

4. Mathematical Model

Consider that the supplier orders X newborn growing items of a specific type from an outside supplier, each having an initial weight of U_i . The total initial weight of these items upon delivery is XU_i . These items are then fed and nurtured throughout the growth time φ_1 , using a three-parameter logistic growth function

$$U(t) = \frac{\eta}{1 + be^{-\theta t}} \quad (1)$$

where $U(t)$ represents each item's weight at time t , $\eta > 0$ is the asymptotic weight, $b > 0$ is the integration constant, and $\theta > 0$ is the growth rate.

When each item achieves the required weight U_t after feeding and growing, it is slaughtered, signaling the end of the growing phase φ_1 , after which it is sold to the retailer for further inspection, as shown in Figure 1.

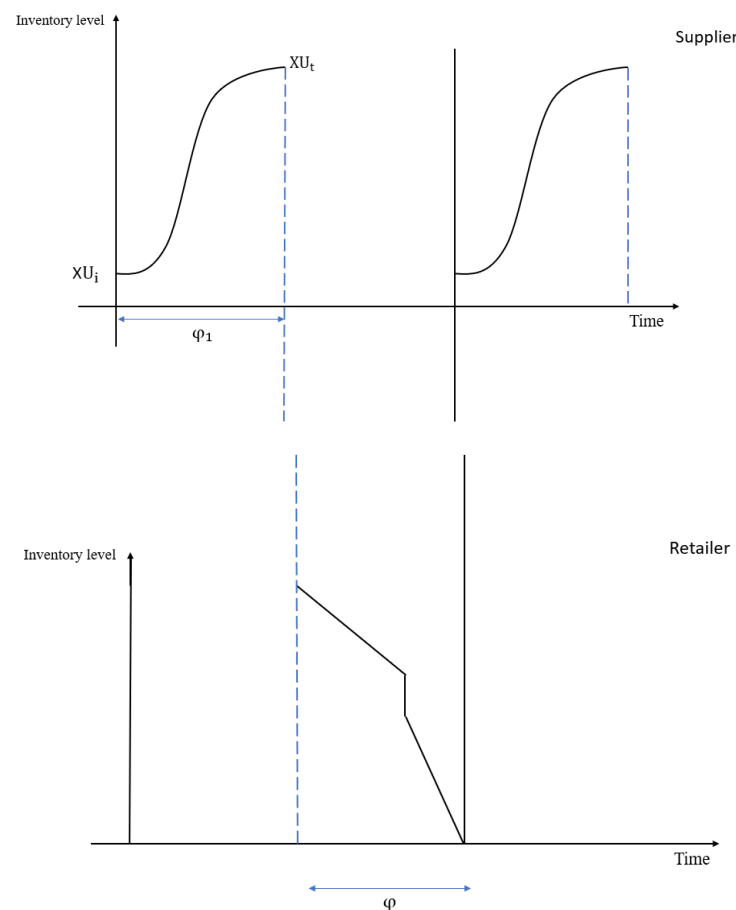


Figure 1. Integrated inventory system.

Hence, Equation (1) yields

$$U_t = U(\varphi_1) = \frac{\eta}{1 + be^{-\theta \varphi_1}} \quad (2)$$

which gives the length of the growing period as

$$\varphi_1 = -\frac{1}{\theta} \text{Log} \left(\frac{\eta - U_t}{bU_t} \right). \quad (3)$$

Following slaughter, the retailer commences the inspection with the initial inventory XU_t units of weight after which the items are sold to the consumers. The differential equation below describes the on-hand inventory weight, $I(t)$, throughout the time $t \in [0, \varphi]$.

$$I'(t) = -D(t, s) = - \left\{ u - vs^w + \alpha \beta \left(\frac{t}{\varphi} \right)^{\beta-1} \right\}, \quad 0 < t < \varphi$$

where the auxiliary conditions are $I(0) = XU_t$ and $I(\varphi) = 0$.

On solving the above equation, the on-hand inventory weight for time $t \in [0, \varphi]$ is given by

$$I(t) = (u - vs^w)(\varphi - t) + \alpha \varphi \left\{ 1 - \left(\frac{t}{\varphi} \right)^{\beta} \right\}, \quad 0 \leq t \leq \varphi. \quad (4)$$

Hence, the initial inventory weight for consumption is given by

$$XU_t = I(0) = \int_0^{\varphi} D(t, s) dt = (\alpha + u - vs^w) \varphi \quad (5)$$

which gives the retailer's entire cycle length as

$$\varphi = \frac{\eta X}{(1 + be^{-\theta \varphi_1})(\alpha + u - vs^w)} \quad (6)$$

4.1. Supplier's Profit Components

The supplier's profit function components are calculated to understand how credit financing influences their overall profitability:

- i. Purchasing Cost: Since the supplier's purchase cost is P_c per unit weight of a newborn item, the total purchase cost is given by

$$PC_s = P_c XU_i.$$

- ii. Ordering Cost: The supplier's fixed ordering cost is given by

$$OC_s = O_s.$$

- iii. Feeding cost: Since the supplier feeds and nurtures the newborn items for the time period 0 to φ_1 , the total feeding cost is

$$FC_s = F_s X \int_0^{\varphi_1} U(t) dt = F_s X \left\{ \eta \varphi_1 + \frac{\eta}{\theta} \left(\text{Log} [1 + be^{-\theta \varphi_1}] - \text{Log} [1 + b] \right) \right\}.$$

- iv. Opportunity Cost: When a supplier offers a retailer a credit period, they face an opportunity cost in the form of lost interest income, that is, the supplier misses out on the opportunity to invest that money elsewhere, which is given by

$$OPC_s = I_s P_c \varphi M (1 - \gamma) (\alpha + u - vs^w).$$

- v. Sales Revenue: Since the supplier earns P_s per unit weight of a grown item, the total sales revenue is

$$SR_s = P_s (\alpha + u - vs^w) \varphi.$$

Hence, the overall profit of the supplier is given by

$$PR_s = SR_s - (PC_s + OC_s + FC_s + OPC_s).$$

$$PR_s(X, s, \varphi) = P_s(\alpha + u - vs^w)\varphi - P_c XU_i - O_s - I_s P_c \varphi M(1 - \gamma)(\alpha + u - vs^w) - F_s X \left\{ \eta \varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \right\} \quad (7)$$

4.2. Retailer's Profit Components

The retailer's profit function components include revenue from good quality and imperfect items and the total costs associated with acquiring them.

- i. Purchasing Cost: Since the retailer's purchase cost is P_s per unit weight of a grown item, the total purchase cost is given by

$$PC_R = P_s(\alpha + u - vs^w)\varphi.$$

- ii. Ordering Cost: The retailer's fixed ordering cost is given by

$$OC_R = O_r.$$

- iii. Transportation Cost: Since the retailer has to transport the items from the supplier at the cost of F_r per unit weight, the total transportation cost is given by

$$TC_R = F_r(\alpha + u - vs^w)\varphi.$$

- iv. Inspection Cost: The retailer starts inspecting the items as the order is received from the supplier at the rate of I_r per unit weight; hence, the total inspection cost is

$$IC_R = I_r(\alpha + u - vs^w)\varphi.$$

- v. Holding Cost: The retailer holds the inventory at the rate of H_r per unit weight, until inspection is completed, and items are sold to the customers; hence, the holding cost is

$$HC_R = H_r \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - \left(\frac{t}{\varphi}\right)^\beta)) dt.$$

- vi. Carbon Emissions Cost: The retailer bears the cost of producing emissions through various activities involving purchasing ($\hat{P}$), ordering ($\hat{O}$), holding ($\hat{H}$), and transporting ($\hat{T}$) the items from the supplier. Hence, the total carbon emissions cost will be

$$CE = \hat{P}(\alpha + u - vs^w)\varphi + \hat{O} + \hat{H} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - \left(\frac{t}{\varphi}\right)^\beta)) dt + \hat{T}(\alpha + u - vs^w)\varphi.$$

- vii. Sales Revenue: The retailer makes revenue by selling good quality items at s per unit item and imperfect quality items at a discounted price of V per unit item. Hence, the total sales revenue is

$$SR_R = (s(1 - \gamma) + V\gamma) \int_0^\varphi \left((u - vs^w) + \alpha \beta \left(\frac{t}{\varphi}\right)^{\beta-1} \right) dt.$$

Hence, the overall retailer's profit is

$$PR_R = SR_R - (PC_R + OC_R + TC_R + IC_R + HC_R + CE).$$

$$\begin{aligned}
 PR_R(s, \varphi) = & (s(1 - \gamma) + V\gamma) \int_0^\varphi \left((u - vs^w) + \alpha\beta \left(\frac{t}{\varphi} \right)^{\beta-1} \right) dt - P_s(\alpha + u - vs^w)\varphi - O_r - \\
 & F_r(\alpha + u - vs^w)\varphi - I_r(\alpha + u - vs^w)\varphi - H_r \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt - \\
 & (\hat{P}(\alpha + u - vs^w)\varphi + \hat{O} + \hat{H} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt + \hat{T}(\alpha + u - vs^w)\varphi)
 \end{aligned} \quad (8)$$

4.3. Trade-Credit Policy

Considering that the supplier provides the retailer a credit period of M years to settle payments, while the retailer further offers n years of credit period to the customer, the following cases of trade-credit are taken as

- i. $0 \leq N \leq \varphi_2 \leq \varphi \leq M$
- ii. $0 \leq \varphi_2 \leq N \leq \varphi \leq M$
- iii. $0 \leq \varphi_2 \leq \varphi \leq N \leq M$
- iv. $0 \leq N \leq \varphi_2 \leq M \leq \varphi$
- v. $0 \leq \varphi_2 \leq N \leq M \leq \varphi$
- vi. $0 \leq N \leq M \leq \varphi_2 \leq \varphi$.

Case 1: $0 \leq N \leq \varphi_2 \leq \varphi \leq M$

Figure 2 depicts how the buyer receives interest from 0 to M based on the sales revenue earned. However, in this case, the buyer does not pay any interest to the seller for unsold items since the cycle length is shorter than the credit period, and the account is settled by time M .

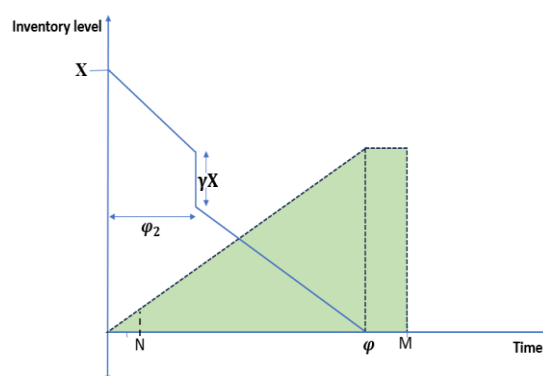


Figure 2. Time-weighted inventory for Case 1.

Interest Earned: $(s(1 - \gamma) + V\gamma)I_E \int_n^\varphi \int_0^t ((u - vs^w) + \alpha\beta \left(\frac{x}{\varphi} \right)^{\beta-1}) dx dt + sI_E(M - \varphi) \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt.$

Interest Paid: 0.

The combined overall profit per unit time is calculated as

$$TPU_1(X, s, \varphi) = \frac{PR_S(X, s, \varphi) + PR_R(s, \varphi)}{\varphi}.$$

$$\begin{aligned}
TPU_1(X, s, \varphi) = & P_s(\alpha + u - vs^w) - \frac{P_c XU_i}{\varphi} - \frac{O_s}{\varphi} - I_s P_c M(1 - \gamma)(\alpha + u - vs^w) - \frac{F_s X}{\varphi} \{ \eta \varphi_1 + \\
& \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \} + \frac{(s(1-\gamma) + V\gamma)}{\varphi} \int_0^\varphi \left((u - vs^w) + \alpha \beta \left(\frac{t}{\varphi} \right)^{\beta-1} \right) dt - P_s(\alpha + u - vs^w) - \frac{O_r}{\varphi} - \\
& F_r(\alpha + u - vs^w) - I_r(\alpha + u - vs^w) - \frac{H_r}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt - (\hat{P}(\alpha + u - vs^w) + \\
& \frac{\hat{Q}}{\varphi} + \frac{\hat{H}}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt + \hat{T}(\alpha + u - vs^w)) + \frac{(s(1-\gamma) + V\gamma)}{\varphi} I_E \int_n^\varphi \int_0^t ((u - vs^w) + \\
& \alpha \beta \left(\frac{x}{\varphi} \right)^{\beta-1}) dx dt + \frac{s I_E (M - \varphi)}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt
\end{aligned} \quad (9)$$

Case 2: $0 \leq \varphi_2 \leq N \leq \varphi \leq M$

Figure 3 depicts how the buyer receives interest from 0 to M based on the sales revenue earned. However, in this case, the buyer does not pay any interest to the seller for unsold items since the cycle length is shorter than the credit period, and the account is settled by time M .

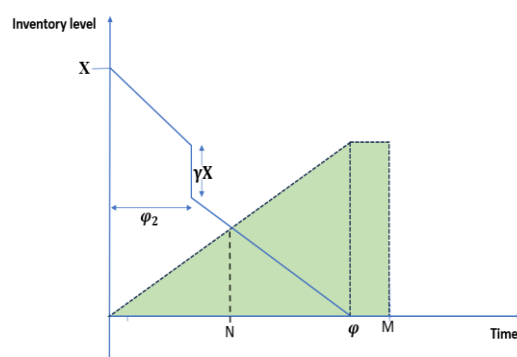


Figure 3. Time-weighted inventory for Case 2.

$$\begin{aligned}
\text{Interest Earned: } & (s(1 - \gamma) + V\gamma) I_E \int_n^\varphi \int_0^t \left((u - vs^w) + \alpha \beta \left(\frac{x}{\varphi} \right)^{\beta-1} \right) dx dt + s I_E (M - \\
& \varphi) \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt.
\end{aligned}$$

Interest Paid: 0.

The combined overall profit per unit time is calculated as

$$\begin{aligned}
TPU_2(X, s, \varphi) = & P_s(\alpha + u - vs^w) - \frac{P_c XU_i}{\varphi} - \frac{O_s}{\varphi} - I_s P_c M(1 - \gamma)(\alpha + u - vs^w) - \frac{F_s X}{\varphi} \{ \eta \varphi_1 + \\
& \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \} + \frac{(s(1-\gamma) + V\gamma)}{\varphi} \int_0^\varphi ((u - vs^w) + \alpha \beta \left(\frac{t}{\varphi} \right)^{\beta-1}) dt - P_s(\alpha + u - vs^w) - \frac{O_r}{\varphi} - \\
& F_r(\alpha + u - vs^w) - I_r(\alpha + u - vs^w) - \frac{H_r}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt - (\hat{P}(\alpha + u - vs^w) + \\
& \frac{\hat{Q}}{\varphi} + \frac{\hat{H}}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt + \hat{T}(\alpha + u - vs^w)) + \frac{(s(1-\gamma) + V\gamma)}{\varphi} I_E \int_n^\varphi \int_0^t ((u - vs^w) + \\
& \alpha \beta \left(\frac{x}{\varphi} \right)^{\beta-1}) dx dt + \frac{s I_E (M - \varphi)}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt.
\end{aligned} \quad (10)$$

Case 3: $0 \leq \varphi_2 \leq \varphi \leq N \leq M$

Figure 4 depicts how the buyer receives interest from 0 to M based on the sales revenue earned. However, in this case, the buyer does not pay any interest to the seller for unsold items since the cycle length is shorter than the credit period, and the account is settled by time M .

$$\text{Interest Earned: } s I_E (M - \varphi) \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt.$$

Interest Paid: 0.

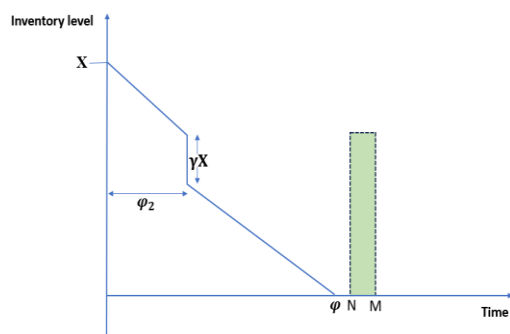


Figure 4. Time-weighted inventory for Case 3.

The combined overall profit per unit of time is calculated as

$$\begin{aligned}
 TPU_3(X, s, \varphi) = & P_s(\alpha + u - vs^w) - \frac{P_c X U_i}{\varphi} - \frac{O_s}{\varphi} - I_s P_c M(1 - \gamma)(\alpha + u - vs^w) - \frac{F_s X}{\varphi} \{ \eta \varphi_1 + \\
 & \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \} + \frac{(s(1-\gamma) + V\gamma)}{\varphi} \int_0^\varphi ((u - vs^w) + \alpha \beta (\frac{t}{\varphi})^{\beta-1}) dt - P_s(\alpha + u - vs^w) - \frac{O_r}{\varphi} - \\
 & F_r(\alpha + u - vs^w) - I_r(\alpha + u - vs^w) - \frac{H_r}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt - (\hat{P}(\alpha + u - vs^w) + \\
 & \frac{\hat{Q}}{\varphi} + \frac{\hat{H}}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt + \hat{T}(\alpha + u - vs^w)) + \frac{s I_E (M - \varphi)}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \\
 & \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt.
 \end{aligned} \quad (11)$$

Case 4: $0 \leq N \leq \varphi_2 \leq M \leq \varphi$

Figure 5 shows that the buyer receives interest on the money received from selling the items between time 0 and M . However, the buyer continues to possess some items after M and must pay interest to the vendor on those items till φ . Hence, the buyer must manage the interest earned simultaneously with the interest paid to achieve a balanced financial outcome.

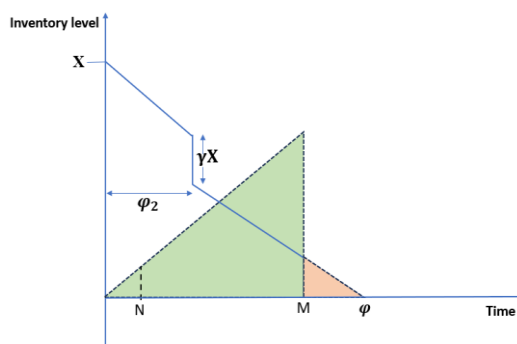


Figure 5. Time-weighted inventory for Case 4.

Interest Earned: $(s(1 - \gamma) + V\gamma) I_E \int_N^M \int_0^t ((u - vs^w) + \alpha \beta (\frac{x}{\varphi})^{\beta-1}) dx dt.$

Interest Paid: $P_s I_P \int_M^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt.$

The combined overall profit per unit of time is calculated as

$$\begin{aligned}
 TPU_4(X, s, \varphi) = & P_s(\alpha + u - vs^w) - \frac{P_c X U_i}{\varphi} - \frac{O_s}{\varphi} - I_s P_c M(1 - \gamma)(\alpha + u - vs^w) - \frac{F_s X}{\varphi} \{ \eta \varphi_1 + \\
 & \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \} + \frac{(s(1-\gamma) + V\gamma)}{\varphi} \int_0^\varphi ((u - vs^w) + \alpha \beta (\frac{t}{\varphi})^{\beta-1}) dt - P_s(\alpha + u - vs^w) - \frac{O_r}{\varphi} - \\
 & F_r(\alpha + u - vs^w) - I_r(\alpha + u - vs^w) - \frac{H_r}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt - (\hat{P}(\alpha + u - vs^w) + \\
 & \frac{\hat{Q}}{\varphi} + \frac{\hat{H}}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt + (\alpha + u - vs^w)) + \frac{(s(1-\gamma) + V\gamma)}{\varphi} I_E \int_N^M \int_0^t ((u - vs^w) + \\
 & \alpha \beta (\frac{x}{\varphi})^{\beta-1}) dx dt - \frac{P_s I_P}{\varphi} \int_M^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt.
 \end{aligned} \quad (12)$$

Case 5: $0 \leq \varphi_2 \leq N \leq M \leq \varphi$

Figure 6 depicts how the buyer earns interest on the money received from selling items between time 0 and M . However, the buyer continues to retain certain products after M and must pay interest to the vendor until φ .

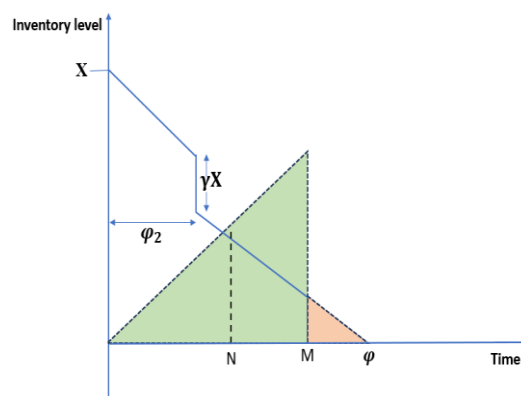


Figure 6. Time-weighted inventory for Case 5.

$$\text{Interest Earned: } (s(1 - \gamma) + V\gamma)I_E \int_n^M \int_0^t \left((u - vs^w) + \alpha\beta \left(\frac{x}{\varphi} \right)^{\beta-1} \right) dx dt.$$

$$\text{Interest Paid: } P_s I_P \int_M^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt.$$

The combined overall profit per unit of time is calculated as

$$\begin{aligned} TPU_5(X, s, \varphi) = & P_s(\alpha + u - vs^w) - \frac{P_c X U_i}{\varphi} - \frac{O_s}{\varphi} - I_s P_c M(1 - \gamma)(\alpha + u - vs^w) - \frac{F_s X}{\varphi} \{ \eta \varphi_1 + \\ & \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \} + \frac{(s(1-\gamma) + V\gamma)}{\varphi} \int_0^\varphi ((u - vs^w) + \alpha\beta \left(\frac{t}{\varphi} \right)^{\beta-1}) dt - P_s(\alpha + u - vs^w) - \frac{O_r}{\varphi} - \\ & E_r(\alpha + u - vs^w) - I_r(\alpha + u - vs^w) - \frac{H_r}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt - (\hat{P}(\alpha + u - vs^w) + \\ & \frac{\hat{O}}{\varphi} + \frac{\hat{H}}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt + \hat{T}(\alpha + u - vs^w)) + \frac{(s(1-\gamma) + V\gamma)}{\varphi} I_E \int_n^M \int_0^t ((u - vs^w) + \\ & \alpha\beta \left(\frac{x}{\varphi} \right)^{\beta-1}) dx dt - \frac{P_s I_P}{\varphi} \int_M^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt. \end{aligned} \quad (13)$$

Case 6: $0 \leq N \leq M \leq \varphi_2 \leq \varphi$

Figure 7 depicts how the buyer earns interest on the money received from selling items between time 0 and M . However, the buyer continues to retain certain products after M and must pay interest to the vendor until φ .

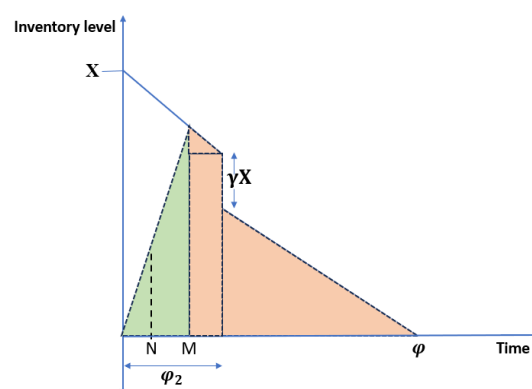


Figure 7. Time-weighted inventory for Case 6.

$$\text{Interest Earned: } (s(1 - \gamma) + V\gamma)I_E \int_n^M \int_0^t \left((u - vs^w) + \alpha\beta \left(\frac{x}{\varphi} \right)^{\beta-1} \right) dx dt.$$

$$\text{Interest Paid: } P_s I_P \int_M^\varphi ((u - vs^w)(\varphi - t) + \alpha\varphi \left(1 - \left(\frac{t}{\varphi} \right)^\beta \right)) dt.$$

The combined overall profit per unit time is calculated as

$$\begin{aligned}
TPU_6(X, s, \varphi) = & P_s(\alpha + u - vs^w) - \frac{P_c XU_i}{\varphi} - \frac{O_s}{\varphi} - I_s P_c M(1 - \gamma)(\alpha + u - vs^w) - \frac{F_s X}{\varphi} \{ \eta \varphi_1 + \\
& \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta \varphi_1}] - \text{Log}[1 + b]) \} + \frac{(s(1-\gamma)+V\gamma)}{\varphi} \int_0^\varphi ((u - vs^w) + \alpha \beta (\frac{t}{\varphi})^{\beta-1}) dt - P_s(\alpha + u - vs^w) - \frac{O_r}{\varphi} - \\
& F_r(\alpha + u - vs^w) - I_r(\alpha + u - vs^w) - \frac{H_r}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt - (\hat{P}(\alpha + u - vs^w) + \\
& \frac{\hat{O}}{\varphi} + \frac{\hat{H}}{\varphi} \int_0^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt + \hat{T}(\alpha + u - vs^w)) + \frac{(s(1-\gamma)+V\gamma)}{\varphi} I_E \int_n^M \int_0^t ((u - vs^w) + \\
& \alpha \beta (\frac{x}{\varphi})^{\beta-1}) dx dt - \frac{P_s I_P}{\varphi} \int_M^\varphi ((u - vs^w)(\varphi - t) + \alpha \varphi (1 - (\frac{t}{\varphi})^\beta)) dt.
\end{aligned} \quad (14)$$

4.4. Implementing Carbon Emissions Policies

- i. A carbon tax policy imposes additional expenditures on the SC by imposing taxes based on the carbon emissions produced. By representing the problem numerically, where $\hat{z}_1$ represents the tax amount levied for each unit of carbon released per unit of time, companies can better understand the financial implications and develop strategies to mitigate these costs while promoting sustainable practices within their operations. Hence,

$$TP_i(X, s, \varphi) = \frac{PR_S(X, s, \varphi) + PR_R(s, \varphi) + IE_i - IP_i - \hat{z}_1 CE}{\varphi} \text{ where } 1 \leq i \leq 6. \quad (15)$$

- ii. A carbon cap policy imposes critical restrictions on SCs by limiting total carbon emissions. The produced emissions throughout the SC must stay below this limit. Using $\hat{c}$ as the maximum permissible carbon emissions per unit of time, $TP_i(X, s, \varphi)$ may be represented analytically as

$$TP_i(X, s, \varphi) = \frac{PR_S(X, s, \varphi) + PR_R(s, \varphi) + IE_i - IP_i}{\varphi} \text{ where } 1 \leq i \leq 6. \quad (16)$$

- iii. A carbon cap-and-trade policy effectively allows companies to trade carbon credits, encouraging businesses to innovate and find ways to lower their emissions. Companies that reduce their emissions below the cap can sell their extra credits to other businesses failing to meet their targets. This allows additional money because these credits may be sold at current market pricing. If a company exceeds its emissions limit, it must purchase more credits from the market to offset the excess emissions. By defining $\hat{c}$ as the emissions cap per unit time and $\hat{z}_2$ as the market price for selling carbon emissions, $TP(X, s, \varphi)$ may be mathematically expressed as

$$TP_i(X, s, \varphi) = \frac{PR_S(X, s, \varphi) + PR_R(s, \varphi) + IE_i - IP_i - \hat{z}_2 CE + \hat{z}_2 \hat{c}}{\varphi} \text{ where } 1 \leq i \leq 6. \quad (17)$$

5. Solution Procedure

The necessary conditions required for the maximization of $TPU_i(X, s, \varphi)$ in each case is defined by $\frac{\partial [TPU_i(X, s, \varphi)]}{\partial X} = 0$, $\frac{\partial [TPU_i(X, s, \varphi)]}{\partial s} = 0$, and $\frac{\partial [TPU_i(X, s, \varphi)]}{\partial \varphi} = 0$, which provide the optimal values for X , s , and φ . This objective is achieved by following a specific methodology from Mahato et al. [38].

- i. Differentiate $TPU_i(X, s, \varphi)$ ($1 \leq i \leq 6$) partially with respect to s and φ for any fixed X , and then put it equal to 0 to optimize the variables to maximize TPU_i .
- ii. However, it is impossible to analytically find the values of s and φ . Therefore, the values are calculated in the following section through a numerical example.
- iii. The second optimality condition to demonstrate the concavity of $TPU_i(X, s, \varphi)$ in each case is given by $\frac{\partial^2 [TPU_i(X, s, \varphi)]}{\partial X^2} \leq 0$, $\frac{\partial^2 [TPU_i(X, s, \varphi)]}{\partial s^2} \leq 0$, and $\frac{\partial^2 [TPU_i(X, s, \varphi)]}{\partial \varphi^2} \leq 0$.

(Refer to Appendix A.)

6. Numerical Example

This example applies the parametric values from Mahato et al. [38] and Sebatjane [39] and the extra needed data in order to comply with the given model: $F_r = \$1$, $V = 40$, $\eta = 6.87$ kg, $b = 120$, $U_i = 0.064$ kg, $U_t = 1.5$ kg, $X = 148$, $\theta = 0.12$, $I_s = \$0.25$ kg, $u = 243$ kg/day, $v = 1$, $w = 1.25$, $\alpha = 10$ kg/day, $\beta = 2$, $\gamma = 0.02$, $P_s = \$24$ /kg, $H_r = \$2.5$ /kg/year, $O_r = \$300$, $\hat{P} = 0.5$ ton/kg/year, $\hat{O} = 0.5$ ton/kg/year, $\hat{H} = 0.4$ ton/kg/year, $\hat{T} = 0.04$ ton/kg/year, $I_E = 0.05$, $I_P = 0.08$, $P_c = \$12$ /kg, $O_s = \$1000$, $F_s = \$0.5$, $\hat{z}_1 = \$1.25$ ton/kg, $\hat{c} = 2000$ ton/kg/year, $\hat{z}_2 = \$2.5$ ton/kg.

Table 3 presents an assessment of all six cases, highlighting how s^* and φ^* affect TPU . By understanding the correlation between these variables and integrated profit, SC managers can make educated decisions that improve their efficiency and profitability. It can be seen from Figures 8 and 9 that Case 2 is the most profitable, where TPU reaches its maximum value (\$4770.06/year) when the optimal values of $s^* = \$47$ /kg and $\varphi^* = 3.03$ years.

Table 3. Comparative analysis of decision variables and TPU .

Cases	s^* (\$/kg)	φ^* (years)	TP_R (\$/cycle)	TP_S (\$/cycle)	TPU (\$/cycle)
1	66	3.6	9475.36	2865.67	3428.06
2	47	3.03	8168.02	6285.25	4770.06
3	46.5	2.21	8352.2	7885.03	4689.39
4	46	2.8	6226.67	6280.85	4466.97
5	47	2.9	6329.27	6367.68	4378.26
6	46	2.85	5715.67	6587.15	4316.76

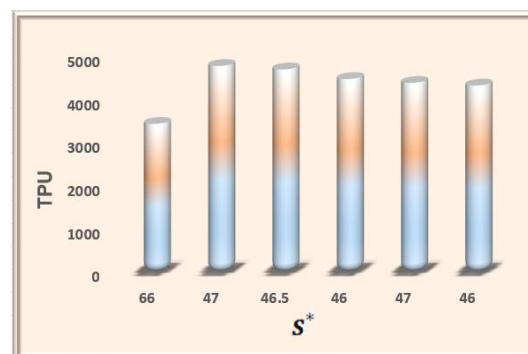


Figure 8. Values of s^* and TPU in all cases.

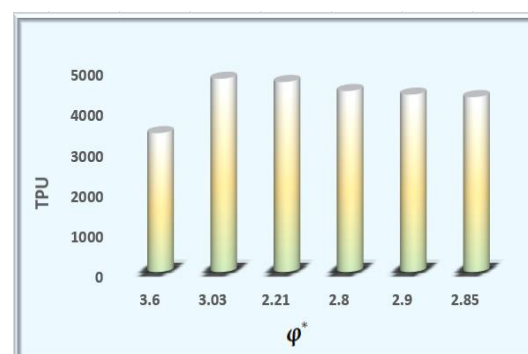


Figure 9. Values of φ^* and TPU in all cases.

Table 4 indicates that combining a cap with the flexibility of trading licenses yields the most profitable or efficient results. Imposing a strict cap on emissions, even without

trading, is more successful than directly taxing emissions. It can be said that charging emissions without imposing restrictions or allowing for market-based flexibility is less effective at maximizing profit.

Table 4. Integrated profit under different carbon emission policies.

Policy	$\hat{z}_1$ (\$/ton/kg)	$\hat{c}$ (ton/kg/year)	$\hat{z}_2$ (\$/ton/kg)	TPU (\$/cycle)
Carbon tax	1.25	0	0	4732.28
Carbon cap	0	2000	0	4921.15
Carbon cap and trade	0	2000	2.5	6193.58

Table 5 represents different cases under which various restrictions on carbon emissions are evaluated. The carbon cap-and-trade policy provides the highest values across all scenarios, with a particularly noticeable rise in Case 2 (\$6193.58/cycle). Even the smallest value for this policy (\$5846.27/cycle in Case 6) exceeds the greatest values of the other two policies, implying that it is the most resilient and robust policy option among those analyzed in this study. For all three policies, Case 2 appears to provide the best results, which might imply that the conditions in Case 2 are particularly favorable for these policies.

Table 5. Comparison of integrated profit (\$/cycle) for all cases with carbon emissions policies.

Policy	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
Carbon tax	3406.99	4732.28	4656.68	4429.83	4341.35	4279.3
Carbon cap	3512.35	4921.15	4820.29	4615.54	4525.9	4466.71
Carbon cap and trade	4690.52	6193.58	5492.71	6029.83	5880.94	5846.27

7. Sensitivity Investigation

A sensitive investigation is undertaken based on the above numerical example to study the impact of certain parameters (γ , α , β , $\hat{H}$, $\hat{O}$, $\hat{P}$, $\hat{T}$, I_e) on the selling price per unit weight (s^*), cycle length (φ^*), and TPU. Table 6 offers an insightful overview of the analysis results.

Table 6. Impact of changes in key parameters on TPU.

Parameters	% of Change	s^*	φ^*	TPU
γ	−40	47.2366	3.04211	6034.30
	−20	47.1804	3.04149	6032.24
	+20	47.059	3.03654	6030.03
	+40	46.9939	3.03472	6029.01
α	−40	46.4436	3.07327	6009.49
	−20	46.7709	3.04818	6103.38
	+20	47.3542	3.01862	6281.29
	+40	47.6131	2.98854	6380.35
β	−40	47.1967	3.06737	6187.41
	−20	47.1371	3.05138	6188.05
	+20	47.1018	3.02744	6190.86
	+40	47.0878	3.0188	6192.02

Table 6. Cont.

Parameters	% of Change	s^*	φ^*	TPU
$\hat{H}$	−40	47.0408	3.08359	6244.86
	−20	47.0788	3.06071	6216.89
	+20	47.1553	3.05991	6136.41
	+40	47.1938	2.9952	6135.44
$\hat{O}$	−40	47.1181	3.03826	6189.61
	−20	47.1181	3.0383	6189.47
	20	47.1182	3.0384	6189.25
	40	47.1182	3.0385	6189.11
$\hat{P}$	−40	47.0338	3.03662	6254.83
	−20	47.0760	3.03749	6222.06
	+20	47.1604	3.03923	6156.70
	+40	47.2025	3.0401	6124.10
$\hat{T}$	−40	47.1114	3.03822	6194.58
	−20	47.1148	3.03829	6191.97
	+20	47.1215	3.03843	6186.74
	+40	47.1249	3.0385	6184.12
I_e	−40	47.3184	3.18697	5853.21
	−20	47.2115	3.10584	6023.54
	+20	47.0356	2.98132	6351.47
	+40	46.9618	2.93247	6510.53

Managerial Implications

- Figure 10 shows that, as γ changes, the relationship between selling price and cycle length with TPU becomes nonlinear. Increasing the selling price usually raises TPU , but the impact reduces as prices rise, demonstrating an optimal pricing range. Similarly, while a longer cycle length initially increases TPU , it eventually decreases TPU , implying that there is an optimal shorter cycle length for maximizing profits.
- Figure 11 shows that as α increases from −40% to 40%, increasing the selling price usually raises TPU , but the impact reduces as prices rise. Similarly, while a longer cycle length initially increases TPU , it eventually decreases TPU , implying that there is an optimal shorter cycle length for maximizing profits.
- Figure 12 shows that as β increases from −40% to 40%, increasing the selling price decreases TPU , but the impact reduces as prices reduce. Similarly, while a longer cycle length initially decreases TPU , it eventually increases TPU , implying that there is an optimal shorter cycle length for maximizing profits.
- Figure 13 shows that as $\hat{H}$ increases from −40% to 40%, increasing the selling price decreases TPU , while decreasing the cycle length reduces TPU .
- Figures 14–16 show that as there is a change in $\hat{O}$, $\hat{P}$, and $\hat{T}$ from −40% to 40%, both the selling price and cycle length have a negative impact on TPU . However, as the selling price and cycle length increase, TPU eventually decreases.
- Figure 17 shows that both the selling price and cycle length have a negative impact on TPU as I_e increases, which implies that a decrease in the selling price and cycle length leads to an increase in TPU .

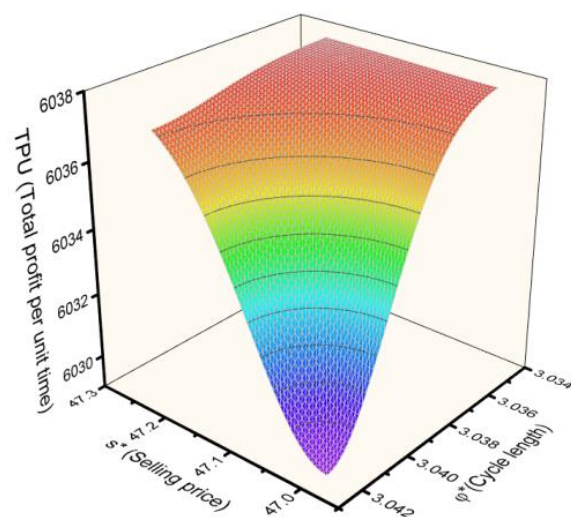


Figure 10. Effect of change in γ on TPU .

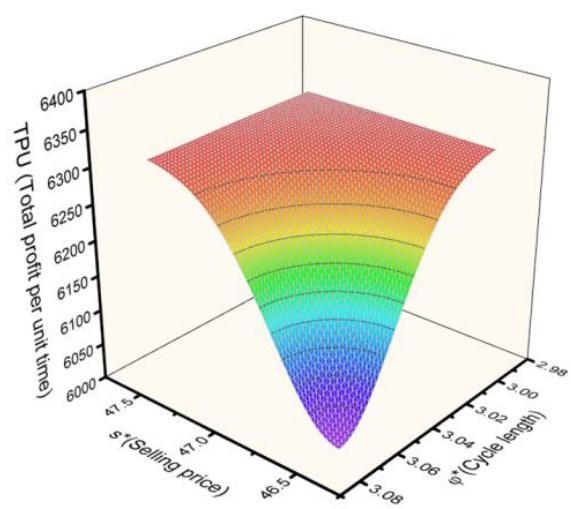


Figure 11. Effect of change in α on TPU .

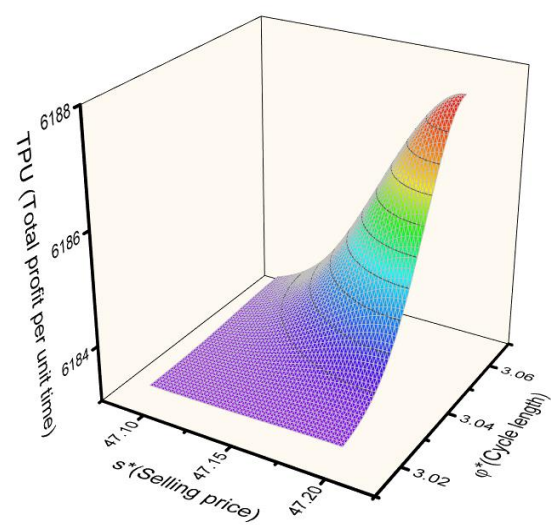


Figure 12. Effect of change in β on TPU .

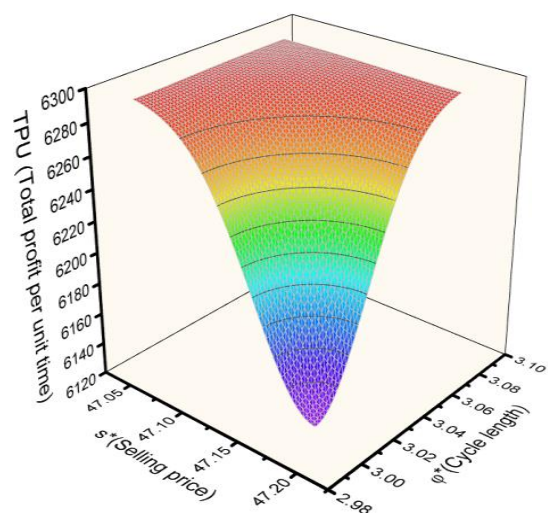


Figure 13. Effect of change in $\hat{H}$ on TPU .

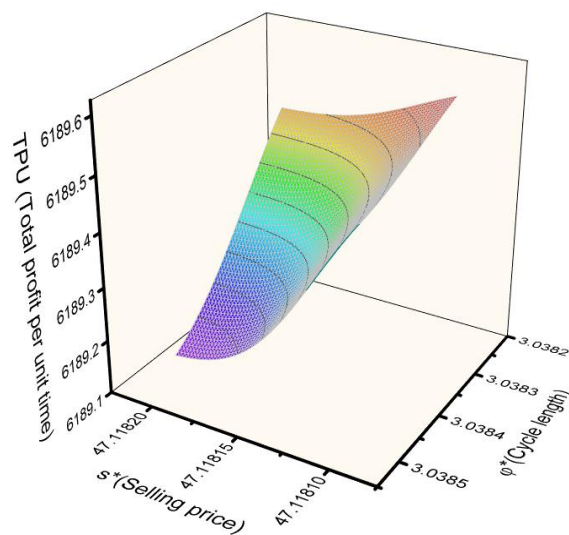


Figure 14. Effect of change in $\hat{O}$ on TPU .

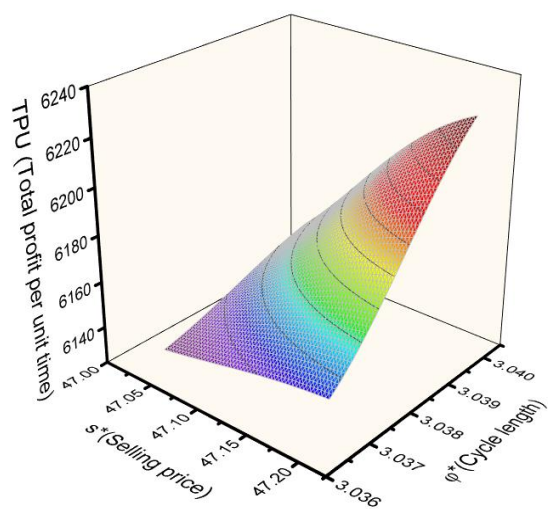


Figure 15. Effect of change in $\hat{P}$ on TPU .

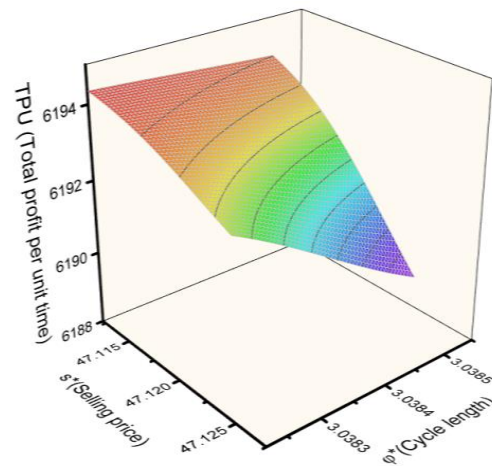


Figure 16. Effect of change in $\hat{T}$ on TPU.

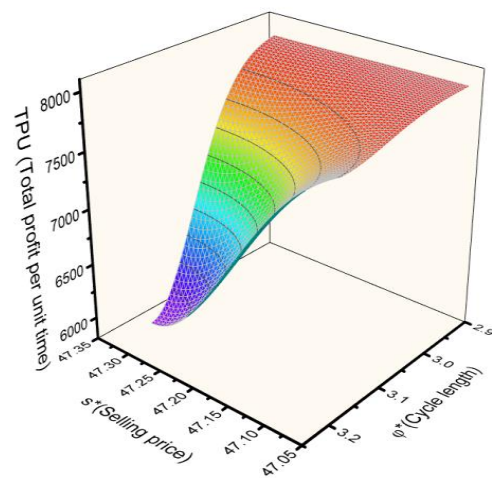


Figure 17. Effect of change in I_c on TPU.

8. Conclusions

This study examined the interactions between suppliers and retailers, specifically how their relationships impact the management of stocks along with pricing decisions. This study intended to provide insights into how varying levels of trade-credit might influence a company's financial dynamics and operational strategies. The emphasis on items that grow and are then processed highlights the complexity of managing perishable or time-sensitive inventories, which necessitates careful planning and forecasting. This paper looked at how retailers manage their stock levels while balancing customer demand and supplier restrictions, especially in a volatile market environment, and the influence of various carbon restrictions on their integrated profit. The mathematical model highlighted how two primary factors, the per unit weight selling price and the duration for which items are stored, impact product demand. This approach recognized that pricing strategies and storage times substantially impact customer behavior and purchase decisions. The model employed a power function to describe how demand fluctuates in response to the selling price and storage duration. This implied that minor changes in either aspect might result in varied amounts of demand, providing flexibility in predicting and planning. The model might simulate various customer responses by incorporating different scenarios within the demand function. This agility was critical for organizations operating in volatile market situations. Based on the analytical results, an algorithm was presented to balance the optimal values of the number of newborn items that are brought to the market based on the expected demand, the best price for each unit weight of the items, and the time between

product releases or restocking periods to optimize sales while ensuring that clients have timely access to the items.

The algorithm's effectiveness was shown using a numerical example, and various insights are presented into how optimal techniques change with variations in system parameters. As γ increased from -40 to 40% , it led to a slight decrease in TPU , s^* , and φ^* . However, an increase in α led to a considerable increase in TPU from $\$6009.49/\text{cycle}$ to $\$6380.35/\text{cycle}$, while φ^* decreased and s^* increased. Changes in β had a negligible effect on TPU , remaining relatively stable around $\$6190/\text{cycle}$, whereas s^* and φ^* exhibited a modest decrease. TPU dropped from $\$6244.86/\text{cycle}$ to $\$6135.44/\text{cycle}$ as $\hat{H}$ increases, while it remained stable across different levels of $\hat{O}$. TPU declined from $\$6254.83/\text{cycle}$ to $\$6124.10/\text{cycle}$ as $\hat{P}$ increased, while s^* increased slightly. s^* and φ^* varied minimally as $\hat{T}$ changed, while leading to a decrease in TPU . As I_e increased by 40% , s^* and φ^* decreased while TPU rose significantly from $\$5853.21/\text{cycle}$ to $\$6510.53/\text{cycle}$. Overall, parameters α and I_e had the biggest impact on total profit per unit time (TPU), with larger values resulting in significant increases in profitability.

Future Implication

While this research focuses on a three-echelon SCM, future studies can extend this model to multi-echelon systems, considering additional layers such as wholesalers or transportation providers. Future research can incorporate dynamic and uncertain demand models, especially considering the volatility in the pricing and demand for growing products. Future research may look into distinct growth curves, feeding functions, deterioration patterns, and cash flow [40]. Furthermore, adding backordering and unpredictable demand into the assumptions may be beneficial. Advertisement policies [41] for deteriorating nature-type products [42] can be more effective for further studies, along with managing any defective products within the system using additional investment [6]. Another potential area of research is the study of growth within a three-level SCM [43].

Author Contributions: M.S. (Mehak Sharma): conceptualization, visualization, data curation, review, methodology, writing—original draft, software; M.M.: conceptualization, visualization, investigation, supervision, writing—review and editing, software D.A.: methodology, writing—review and editing, investigation, software; A.D.: supervision, writing—review and editing; R.G.: visualization, writing—review and editing; M.S. (Mitali Sarkar): resources, formal analysis. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Data is provided within the manuscript.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Case 1: $0 \leq N \leq \varphi_2 \leq \varphi \leq M$

$$\begin{aligned} \frac{\partial[TPU_i(X,s,\varphi)]}{\partial\varphi} &= \frac{1}{\varphi^2} (O_r + O_s + P_c X U_i + F_S X (\eta \varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + b e^{-\theta \varphi_1}] - \text{Log}[1 + b]))) - H_r \left(\frac{u - v s^w}{2} + \frac{\alpha \beta}{\beta + 1} \right) + \\ &\quad \frac{\hat{O}}{\varphi^2} - \hat{H} \left(\frac{u - v s^w}{2} + \frac{\alpha \beta}{\beta + 1} \right) + (s(1 - \gamma) + V\gamma) I_E \left[\frac{u - v s^w}{2} + \frac{\alpha}{\beta + 1} + \frac{n^2(u - v s^w)}{2\varphi^2} + \frac{\alpha \beta n^{\beta+1}}{\varphi^{\beta+1}(\beta+1)} \right] - s I_E (\alpha + u - v s^w) \\ \frac{\partial[TPU_i(X,s,\varphi)]}{\partial s} &= -v w s^{w-1} (s(1 - \gamma) + V\gamma + P_s) + (1 - \gamma)(\alpha + u - v s^w) + (P_s + F_r + I_r + I_s P_c M(1 - \\ &\quad \gamma)) v w s^{w-1} + v w s^{w-1} \left(\frac{H_r \varphi}{2} \right) + v w s^{w-1} \left(\hat{P} + \hat{T} + \frac{\hat{H} \varphi}{2} \right) + I_E (M - \varphi) (-s v w s^{w-1} + (\alpha + u - v s^w)) + I_E \left\{ \left(\frac{\varphi}{2} - \right. \right. \\ &\quad \left. \left. \frac{n^2}{2\varphi} \right) [- (s(1 - \gamma) + V\gamma) v w s^{w-1} + (1 - \gamma)(u - v s^w)] + (1 - \gamma) \left[\frac{\alpha \varphi}{\beta + 1} - \frac{\alpha n^{\beta+1}}{\varphi^{\beta}(\beta+1)} \right] \right\} \end{aligned}$$

Case 2: $0 \leq \varphi_2 \leq N \leq \varphi \leq M$

$$\begin{aligned}\frac{\partial[TPU_i(X,s,\varphi)]}{\partial\varphi} &= \frac{1}{\varphi^2} (O_r + O_s + P_c XU_i + F_s X(\eta\varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta\varphi_1}] - \text{Log}[1 + b]))) - H_r \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + \\ &\quad \frac{\dot{O}}{\varphi^2} - \hat{H} \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + (s(1 - \gamma) + V\gamma) I_E \left[\frac{u - vs^w}{2} + \frac{\alpha}{\beta+1} + \frac{n^2(u - vs^w)}{2\varphi^2} + \frac{\alpha\beta n^{\beta+1}}{\varphi^{\beta+1}(\beta+1)} \right] - s I_E(\alpha + u - vs^w) \\ \frac{\partial[TPU_i(X,s,\varphi)]}{\partial s} &= -vws^{w-1}(s(1 - \gamma) + V\gamma + P_s) + (1 - \gamma)(\alpha + u - vs^w) + (P_s + F_r + I_r + I_s P_c M(1 - \\ &\quad \gamma))vws^{w-1} + vws^{w-1}(\frac{H_r\varphi}{2}) + vws^{w-1}(\hat{P} + \hat{T} + \frac{\hat{H}\varphi}{2}) + I_E(M - \varphi)(-s vws^{w-1} + (\alpha + u - vs^w)) + I_E\{(\frac{\varphi}{2} - \\ &\quad \frac{n^2}{2\varphi})[-(s(1 - \gamma) + V\gamma)vws^{w-1} + (1 - \gamma)(u - vs^w)] + (1 - \gamma)[\frac{\alpha\varphi}{\beta+1} - \frac{\alpha n^{\beta+1}}{\varphi^{\beta}(\beta+1)}]\}\end{aligned}$$

Case 3: $0 \leq \varphi_2 \leq \varphi \leq N \leq M$

$$\begin{aligned}\frac{\partial[TPU_i(X,s,\varphi)]}{\partial\varphi} &= \frac{1}{\varphi^2} (O_r + O_s + P_c XU_i + F_s X(\eta\varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta\varphi_1}] - \text{Log}[1 + b]))) - H_r \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + \\ &\quad \frac{\dot{O}}{\varphi^2} - \hat{H} \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) - s I_E(\alpha + u - vs^w) \\ \frac{\partial[TPU_i(X,s,\varphi)]}{\partial s} &= -vws^{w-1}(s(1 - \gamma) + V\gamma + P_s) + (1 - \gamma)(\alpha + u - vs^w) + (P_s + F_r + I_r + I_s P_c M(1 - \\ &\quad \gamma))vws^{w-1} + vws^{w-1}(\frac{H_r\varphi}{2}) + vws^{w-1}(\hat{P} + \hat{T} + \frac{\hat{H}\varphi}{2}) + I_E(M - \varphi)(-s vws^{w-1} + (\alpha + u - vs^w))\end{aligned}$$

Case 4: $0 \leq N \leq \varphi_2 \leq M \leq \varphi$

$$\begin{aligned}\frac{\partial[TPU_i(X,s,\varphi)]}{\partial\varphi} &= \frac{1}{\varphi^2} (O_r + O_s + P_c XU_i + F_s X(\eta\varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta\varphi_1}] - \text{Log}[1 + b]))) - H_r \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + \\ &\quad \frac{\dot{O}}{\varphi^2} - \hat{H} \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + I_E(s(1 - \gamma) + V\gamma) \left[\frac{(u - vs^w)(M^2 - n^2)}{2\varphi^2} + \frac{\alpha\beta(M^{\beta+1} - n^{\beta+1})}{(\beta+1)\varphi^{\beta+1}} \right] - P_s I_P \left[\frac{u - vs^w}{2} \left(1 - \frac{M^2}{\varphi^2} \right) + \right. \\ &\quad \left. \frac{\alpha\beta}{\beta+1} \left(1 - \frac{M^{\beta+1}}{\varphi^{\beta+1}} \right) \right] \\ \frac{\partial[TPU_i(X,s,\varphi)]}{\partial s} &= -vws^{w-1}(s(1 - \gamma) + V\gamma + P_s) + (1 - \gamma)(\alpha + u - vs^w) + (P_s + F_r + I_r + I_s P_c M(1 - \\ &\quad \gamma))vws^{w-1} + vws^{w-1}(\frac{H_r\varphi}{2}) + vws^{w-1}(\hat{P} + \hat{T} + \frac{\hat{H}\varphi}{2}) + \frac{I_E}{2\varphi} (M^2 - n^2) [(1 - \gamma)(u - vs^w) - vws^{w-1}(s(1 - \gamma) + \\ &\quad V\gamma)] + \frac{I_E \alpha (1 - \gamma)}{(\beta+1)\varphi^{\beta}} (M^{\beta+1} - n^{\beta+1}) + P_s I_P [vws^{w-1}(\frac{\varphi}{2} - M + \frac{M^2}{2\varphi})]\end{aligned}$$

Case 5: $0 \leq \varphi_2 \leq N \leq M \leq \varphi$

$$\begin{aligned}\frac{\partial[TPU_i(X,s,\varphi)]}{\partial\varphi} &= \frac{1}{\varphi^2} (O_r + O_s + P_c XU_i + F_s X(\eta\varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta\varphi_1}] - \text{Log}[1 + b]))) - H_r \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + \\ &\quad \frac{\dot{O}}{\varphi^2} - \hat{H} \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + I_E(s(1 - \gamma) + V\gamma) \left[\frac{(u - vs^w)(M^2 - n^2)}{2\varphi^2} + \frac{\alpha\beta(M^{\beta+1} - n^{\beta+1})}{(\beta+1)\varphi^{\beta+1}} \right] - P_s I_P \left[\frac{u - vs^w}{2} \left(1 - \frac{M^2}{\varphi^2} \right) + \right. \\ &\quad \left. \frac{\alpha\beta}{\beta+1} \left(1 - \frac{M^{\beta+1}}{\varphi^{\beta+1}} \right) \right] \\ \frac{\partial[TPU_i(X,s,\varphi)]}{\partial s} &= -vws^{w-1}(s(1 - \gamma) + V\gamma + P_s) + (1 - \gamma)(\alpha + u - vs^w) + (P_s + F_r + I_r + I_s P_c M(1 - \\ &\quad \gamma))vws^{w-1} + vws^{w-1}(\frac{H_r\varphi}{2}) + vws^{w-1}(\hat{P} + \hat{T} + \frac{\hat{H}\varphi}{2}) + \frac{I_E}{2\varphi} (M^2 - n^2) [(1 - \gamma)(u - vs^w) - vws^{w-1}(s(1 - \gamma) + \\ &\quad V\gamma)] + \frac{I_E \alpha (1 - \gamma)}{(\beta+1)\varphi^{\beta}} (M^{\beta+1} - n^{\beta+1}) + P_s I_P [vws^{w-1}(\frac{\varphi}{2} - M + \frac{M^2}{2\varphi})]\end{aligned}$$

Case 6: $0 \leq N \leq M \leq \varphi_2 \leq \varphi$

$$\begin{aligned}\frac{\partial[TPU_i(X,s,\varphi)]}{\partial\varphi} &= \frac{1}{\varphi^2} (O_r + O_s + P_c XU_i + F_s X(\eta\varphi_1 + \frac{\eta}{\theta} (\text{Log}[1 + be^{-\theta\varphi_1}] - \text{Log}[1 + b]))) - H_r \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + \\ &\quad \frac{\dot{O}}{\varphi^2} - \hat{H} \left(\frac{u - vs^w}{2} + \frac{\alpha\beta}{\beta+1} \right) + I_E(s(1 - \gamma) + V\gamma) \left[\frac{(u - vs^w)(M^2 - n^2)}{2\varphi^2} + \frac{\alpha\beta(M^{\beta+1} - n^{\beta+1})}{(\beta+1)\varphi^{\beta+1}} \right] - P_s I_P \left[\frac{u - vs^w}{2} \left(1 - \frac{M^2}{\varphi^2} \right) + \right. \\ &\quad \left. \frac{\alpha\beta}{\beta+1} \left(1 - \frac{M^{\beta+1}}{\varphi^{\beta+1}} \right) \right] \\ \frac{\partial[TPU_i(X,s,\varphi)]}{\partial s} &= -vws^{w-1}(s(1 - \gamma) + V\gamma + P_s) + (1 - \gamma)(\alpha + u - vs^w) + (P_s + F_r + I_r + I_s P_c M(1 - \\ &\quad \gamma))vws^{w-1} + vws^{w-1}(\frac{H_r\varphi}{2}) + vws^{w-1}(\hat{P} + \hat{T} + \frac{\hat{H}\varphi}{2}) + \frac{I_E}{2\varphi} (M^2 - n^2) [(1 - \gamma)(u - vs^w) - vws^{w-1}(s(1 - \gamma) + \\ &\quad V\gamma)] + \frac{I_E \alpha (1 - \gamma)}{(\beta+1)\varphi^{\beta}} (M^{\beta+1} - n^{\beta+1}) + P_s I_P [vws^{w-1}(\frac{\varphi}{2} - M + \frac{M^2}{2\varphi})]\end{aligned}$$

References

- Datta, A.; Dey, B.K.; Bhuniya, S.; Sangal, I.; Mandal, B.; Sarkar, M.; Guchhait, R.; Sarkar, B.; Ganguly, B. Adaptation of e-commerce retailing to enhance customer satisfaction within a dynamical system under transfer of risk. *J. Retail. Consum. Serv.* **2025**, *84*, 104129. [\[CrossRef\]](#)
- Dey, B.K.; Bhuniya, S.; Sarkar, B. Involvement of controllable lead time and variable demand for a smart manufacturing system under a supply chain management. *Expert Syst. Appl.* **2021**, *184*, 115464. [\[CrossRef\]](#)
- Guchhait, R.; Sarkar, B. Economic evaluation of an outsourced fourth-party logistics (4PL) under a flexible production system. *Int. J. Prod. Econ.* **2025**, *279*, 109440. [\[CrossRef\]](#)
- Khalilpourazari, S.; Pasandideh, S.H.R. Modeling and optimization of multi-item multi-constrained EOQ model for growing items. *Knowl. Based Syst.* **2019**, *164*, 150–162. [\[CrossRef\]](#)
- Nobil, A.H.; Sedigh, A.H.A.; Cárdenas-Barrón, L.E. A generalized economic order quantity inventory model with shortage: Case study of a poultry farmer. *Arab. J. Sci. Eng.* **2019**, *44*, 2653–2663. [\[CrossRef\]](#)
- Sarkar, B.; Sao, S.; Ghosh, S.K. Smart production and photocatalytic ultraviolet (PUV) wastewater treatment effect on a textile supply chain management. *Int. J. Prod. Econ.* **2025**, *283*, 109557. [\[CrossRef\]](#)
- Pourmohammad-Zia, N.; Karimi, B. Optimal replenishment and breeding policies for growing items. *Arab. J. Sci. Eng.* **2020**, *45*, 7005–7015. [\[CrossRef\]](#)
- Alfares, H.K.; Afzal, A.R. An economic order quantity model for growing items with imperfect quality and shortages. *Arab. J. Sci. Eng.* **2021**, *46*, 1863–1875. [\[CrossRef\]](#)
- Mittal, M.; Sharma, M. Economic ordering policies for growing items (poultry) with trade-credit financing. *Int. J. Appl. Comput. Math.* **2021**, *7*, 39. [\[CrossRef\]](#)
- Sharma, M.; Mittal, M. Effect of credit financing on the supply chain for imperfect growing items. *RAIRO-Oper. Res.* **2022**, *56*, 2903–2917. [\[CrossRef\]](#)
- Yu, J.C.; Lin, Y.S.; Wang, K.J. Coordination-based inventory management for deteriorating items in a two-echelon supply chain with profit sharing. *Int. J. Syst. Sci.* **2013**, *44*, 1587–1601. [\[CrossRef\]](#)
- Zhang, J.; Liu, G.; Zhang, Q.; Bai, Z. Coordinating a supply chain for deteriorating items with a revenue sharing and cooperative investment contract. *Omega* **2015**, *56*, 37–49. [\[CrossRef\]](#)
- Cai, X.; Chen, J.; Xiao, Y.; Xu, X.; Yu, G. Fresh-product supply chain management with logistics outsourcing. *Omega* **2013**, *41*, 752–765. [\[CrossRef\]](#)
- Sebatjane, M.; Adetunji, O. A three-echelon supply chain for economic growing quantity model with price-and freshness-dependent demand: Pricing, ordering and shipment decisions. *Oper. Res. Perspect.* **2020**, *7*, 100153. [\[CrossRef\]](#)
- Sebatjane, M.; Adetunji, O. Optimal inventory replenishment and shipment policies in a three-echelon supply chain for growing items with expiration dates. *Opsearch* **2022**, *59*, 809–838. [\[CrossRef\]](#)
- Huang, C.K.; Tsai, D.M.; Wu, J.C.; Chung, K.J. An integrated vendor-buyer inventory model with order-processing cost reduction and permissible delay in payments. *Eur. J. Oper. Res.* **2010**, *202*, 473–478. [\[CrossRef\]](#)
- Mahata, P.; Mahata, G.C.; De, S.K. An economic order quantity model under two-level partial trade credit for time varying deteriorating items. *J. Oper. Res.* **2018**, *7*, 1–17. [\[CrossRef\]](#)
- Mahata, G.C. Partial trade credit policy of retailer in economic order quantity models for deteriorating items with expiration dates and price-sensitive demand. *J. Math. Model. Algorithms Oper. Res.* **2015**, *14*, 363–392. [\[CrossRef\]](#)
- Vandana; Sharma, B.K. An EOQ model for retailer's partial permissible delay in payment linked to order quantity with shortages. *Math. Comput. Simul.* **2016**, *125*, 99–112. [\[CrossRef\]](#)
- Seifert, D.; Seifert, R.W.; Protopappa-Sieke, M. A review of trade credit literature: Opportunities for research in operations. *Eur. J. Oper. Res.* **2013**, *231*, 245–256. [\[CrossRef\]](#)
- Ries, J.M.; Grosse, E.H.; Fichtinger, J. Environmental impact of warehousing: A scenario analysis for the United States. *Int. J. Prod. Res.* **2017**, *55*, 6485–6499. [\[CrossRef\]](#)
- Hua, G.; Cheng, T.C.E.; Wang, S. Managing carbon footprints in inventory management. *Int. J. Prod. Econ.* **2011**, *132*, 178–185. [\[CrossRef\]](#)
- Chen, X.; Benjaafar, S.; Elomri, A. The carbon-constrained EOQ. *Oper. Res. Lett.* **2013**, *41*, 172–179. [\[CrossRef\]](#)
- Toptal, A.; Özlü, H.; Konur, D. Joint decisions on inventory replenishment and emission reduction investment under different emission regulations. *Int. J. Prod. Res.* **2014**, *52*, 243–269. [\[CrossRef\]](#)
- Hovelaque, V.; Bironneau, L. The carbon-constrained EOQ model with carbon emission dependent demand. *Int. J. Prod. Econ.* **2015**, *164*, 285–291. [\[CrossRef\]](#)
- Lin, T.Y.; Sarker, B.R. A pull system inventory model with carbon tax policies and imperfect quality items. *Appl. Math. Model.* **2017**, *50*, 450–462. [\[CrossRef\]](#)
- Li, Z.; Hai, J. Inventory management for one warehouse multi-retailer systems with carbon emission costs. *Comput. Ind. Eng.* **2019**, *130*, 565–574. [\[CrossRef\]](#)

28. Dhanda, A.; Mittal, M.; Chawla, S.; Prasad, J. Impact of Carbon Emission Policy on Fresh Food Supply Chain Model for Deteriorating Imperfect Quality Items. *Int. J. Math. Eng. Manag. Sci.* **2024**, *9*, 516–536. [\[CrossRef\]](#)
29. Alamri, O.A. Sustainable Supply Chain Model for Defective Growing Items (Fishery) with Trade Credit Policy and Fuzzy Learning Effect. *Axioms* **2023**, *12*, 436. [\[CrossRef\]](#)
30. Chang, C.-T.; Tseng, Y.-T. The Impacts of Payment Schemes and Carbon Emission Policies on Replenishment and Pricing Decisions for Perishable Products in a Supply Chain. *Mathematics* **2024**, *12*, 1033. [\[CrossRef\]](#)
31. De-la-Cruz-Márquez, C.G.; Cárdenas-Barrón, L.E.; Porter, J.D.; Loera-Hernández, I.J.; Smith, N.R.; Céspedes-Mota, A.; Treviño-Garza, G.; Bourguet-Díaz, R.E. An Inventory Model for Growing Items When the Demand Is Price Sensitive with Imperfect Quality, Inspection Errors, Carbon Emissions, and Planned Backorders. *Mathematics* **2023**, *11*, 4421. [\[CrossRef\]](#)
32. Sebatjane, M.; Adetunji, O. Economic order quantity model for growing items with imperfect quality. *Oper. Res. Perspect.* **2019**, *6*, 100088. [\[CrossRef\]](#)
33. Sarkar, B.; Bhuniya, S. A sustainable flexible manufacturing–remanufacturing model with improved service and green investment under variable demand. *Expert Syst. Appl.* **2022**, *202*, 117154. [\[CrossRef\]](#)
34. Garai, A.; Sarkar, B. Economically independent reverse logistics of customer-centric closed-loop supply chain for herbal medicines and biofuel. *J. Clean. Prod.* **2022**, *334*, 129977. [\[CrossRef\]](#)
35. Moon, I.; Yun, W.Y.; Sarkar, B. Effects of variable setup cost, reliability, and production costs under controlled carbon emissions in a reliable production system. *Eur. J. Ind. Eng.* **2022**, *16*, 371–397. [\[CrossRef\]](#)
36. Mahapatra, A.S.; Sengupta, S.; Dasgupta, A.; Sarkar, B.; Goswami, R.T. What is the impact of demand patterns on integrated online-offline and buy-online-pickup in-store (BOPS) retail in a smart supply chain management? *J. Retail. Consum. Serv.* **2025**, *82*, 104093. [\[CrossRef\]](#)
37. Sarkar, B.; Tayyab, M.; Kim, N.; Habib, M.S. Optimal production delivery policies for supplier and manufacturer in a constrained closed-loop supply chain for returnable transport packaging through metaheuristic approach. *Comput. Ind. Eng.* **2019**, *135*, 987–1003. [\[CrossRef\]](#)
38. Mahato, C.; De, S.K.; Mahata, G.C. Joint pricing and inventory management for growing items in a supply chain under trade credit. *Soft Comput.* **2021**, *25*, 7271–7295. [\[CrossRef\]](#)
39. Sebatjane, M. Three-echelon circular economic production–inventory model for deteriorating items with imperfect quality and carbon emissions considerations under various emissions policies. *Expert Syst. Appl.* **2024**, *252*, 124162. [\[CrossRef\]](#)
40. Kumar, S.; Sigroha, M.; Kumar, N.; Kumari, M.; Sarkar, B. How does the retail price maintain trade-credit management with continuous investment to support the cash flow? *J. Retail. Consum. Serv.* **2025**, *83*, 104116. [\[CrossRef\]](#)
41. Sarkar, B.; Omair, M.; Kim, N. A cooperative advertising collaboration policy in supply chain management under uncertain conditions. *Appl. Soft Comput.* **2020**, *88*, 105948. [\[CrossRef\]](#)
42. Sarkar, B.; Debnath, A.; Chiu, A.S.; Ahmed, W. Circular economy-driven two-stage supply chain management for nullifying waste. *J. Clean. Prod.* **2022**, *339*, 130513. [\[CrossRef\]](#)
43. Habib, M.S.; Asghar, O.; Hussain, A.; Imran, M.; Mughal, M.P.; Sarkar, B. A robust possibilistic programming approach toward animal fat-based biodiesel supply chain network design under uncertain environment. *J. Clean. Prod.* **2021**, *278*, 122403. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.