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Review

Advancements in Physics-Informed Neural Networks for Laminated Composites: A Comprehensive Review

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Abstract: Physics-Informed Neural Networks (PINNs) integrate physics principles with machine learning, offering innovative solutions for complex modeling challenges. Laminated composites, characterized by their anisotropic behavior, multi-layered structures, and intricate interlayer interactions, pose significant challenges for traditional computational methods. PINNs address these issues by embedding governing physical laws directly into neural network architectures, enabling efficient and accurate modeling. This review provides a comprehensive overview of PINNs applied to laminated composites, highlighting advanced methodologies such as hybrid PINNs, k-space PINNs, Theory-Constrained PINNs, optimal PINNs, and disjointed PINNs. Key applications, including structural health monitoring (SHM), structural analysis, stress-strain and failure analysis, and multi-scale modeling, are explored to illustrate how PINNs optimize material configurations and enhance structural reliability. Additionally, this review examines the challenges associated with deploying PINNs and identifies future directions to further advance their capabilities. By bridging the gap between classical physics-based models and data-driven techniques, this review advances the understanding of PINN methodologies for laminated composites and underscores their transformative role in addressing modeling complexities and solving real-world problems.

Keywords: physics-informed neural networks; laminated composites; structural health monitoring; multi-scale modeling; structural analysis; composite material optimization

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1. Introduction

PINNs represent a transformative approach to computational modeling by embedding governing physical principles into the neural network training process [1–3]. Unlike traditional data-driven models, PINNs integrate domain-specific knowledge through equations such as partial differential equations (PDEs) and ordinary differential equations (ODEs), hence establishing the integration of physics-based and data-driven approaches [4,5]. This synergy has made PINNs a powerful tool for addressing complex problems in fields such as fluid dynamics, material behavior prediction, and inverse problem-solving [6,7]. Traditional numerical methods, including finite element methods (FEM) and finite difference methods (FDM), have long been the standard for analyzing composite materials [8,9].

While these methods are reliable, they often face limitations in computational efficiency and adaptability, particularly in scenarios involving incomplete or sparse data. However, these approaches often face limitations in computational efficiency and adaptability, particularly in cases involving incomplete data. PINNs offer a compelling alternative by combining the rigor of classical numerical techniques with the adaptability of machine learning. This fusion not only improves computational efficiency but also broadens the applicability of computational models to previously intractable problems [6,10,11].

In the domain of laminated composite materials, PINNs have shown exceptional promise. Laminated composites, composed of multiple layers of materials with complementary properties, are integral to advanced engineering applications [12]. Their superior strength-to-weight ratio, durability, and flexibility have driven their adoption in industries such as aerospace [13], automotive [14], renewable energy [15], and healthcare [16]. However, the inherent complexities of composite materials, including anisotropy, heterogeneity, and multilayered structures, pose significant challenges for traditional computational methods, which often rely on extensive experimental data and entail high computational costs [17,18]. PINNs address these challenges by embedding the governing physical laws of composite mechanics, such as stress equilibrium, energy conservation, and material compatibility, directly into their neural network architecture. For example, in laminated composites, PINNs can efficiently model vibration modes [19], predict structural responses under varying loads [20], and simulate failure mechanisms such as delamination or cracking [21]. This makes PINNs particularly well-suited for multi-scale modeling, where capturing interactions between microstructural and macroscopic behaviors is critical [22]. The increasing research interest in PINNs and their application to laminated composite materials is evident from the publication trend shown in Figure 1. This trend is based on a keyword search combining relevant terms related to laminated composites, PINNs, and application-specific methodologies.

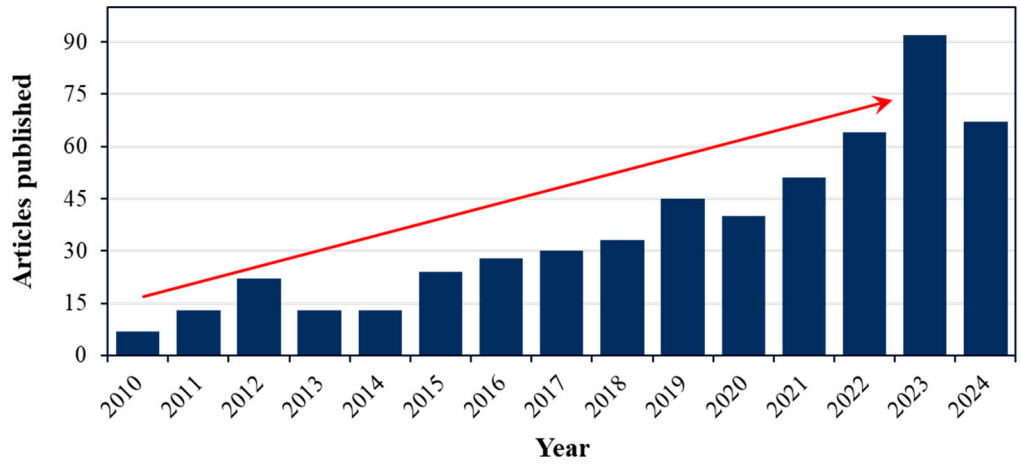


Figure 1. The publication trend based on a keyword search of "laminated composites OR composite laminates OR composite structures" combined with "Physics-Informed Neural Networks OR Physics Informed Neural Networks OR Physics-based" and application-specific terms like "structural health monitoring, multi-scale modeling, structural analysis, or stress-strain prediction" highlights the growing research interest in these topics. (Source: Web of Science. Last updated: 2 December 2024).

Beyond traditional modeling, PINNs enable real-time analysis and adaptability, making them ideal for SHM, predictive maintenance, and adaptive design systems [23]. Their capacity for continuous evaluation of structural integrity allows early damage detection, failure prediction, and maintenance optimization, enhancing safety and cost efficiency [24]. For composite materials, this involves integrating boundary conditions, anisotropic proper-

ties, and stress-strain relationships into the optimization process. Recent advancements in PINN methodologies have further expanded their efficiency and applicability. Spectral methods like k-space PINNs are used to analyze vibration phenomena with high precision, leveraging wavenumber-domain techniques to reconstruct vibrational modes [19]. Hybrid PINNs combine the computational efficiency of traditional PINNs with additional data-driven approaches, reducing training time and enhancing scalability [20]. Theory-Constrained PINNs embed domain-specific theoretical principles, ensuring compliance with the mechanics of laminated composites and improving prediction accuracy [25]. Disjointed PINNs utilize domain-decomposed architectures to localize analysis while maintaining global consistency, making them particularly effective for large-scale or heterogeneous systems [26]. These advancements in PINN architectures not only improve computational performance but also contribute to the design of sustainable and efficient composites [27], reducing material waste [28] and energy consumption [29].

Despite these advancements, challenges remain. Data scarcity, a common issue in modeling composite materials, persists even with the physics-driven nature of PINNs. Validation against sparse or noisy experimental data can complicate the assessment of model accuracy [30]. Computational efficiency is another concern, particularly for large-scale or multi-scale problems. Training PINNs requires optimizing complex loss functions, which can be computationally expensive and sensitive to hyperparameter tuning. Additionally, incorporating intricate boundary conditions and domain constraints, such as interfacial properties and layer-wise continuity, demands innovative approaches to maintain physical realism [31]. Figure 2 illustrates how challenges such as data scarcity, computational demands, multi-physics coupling, nonlinear behaviors, and multi-scale modeling complexity are addressed by various PINN methodologies. By integrating governing equations and deep neural networks, methodologies like traditional PINNs, Theory-Constrained PINNs, k-space PINNs, disjointed PINNs, and hybrid PINNs provide effective approaches to tackle these challenges.

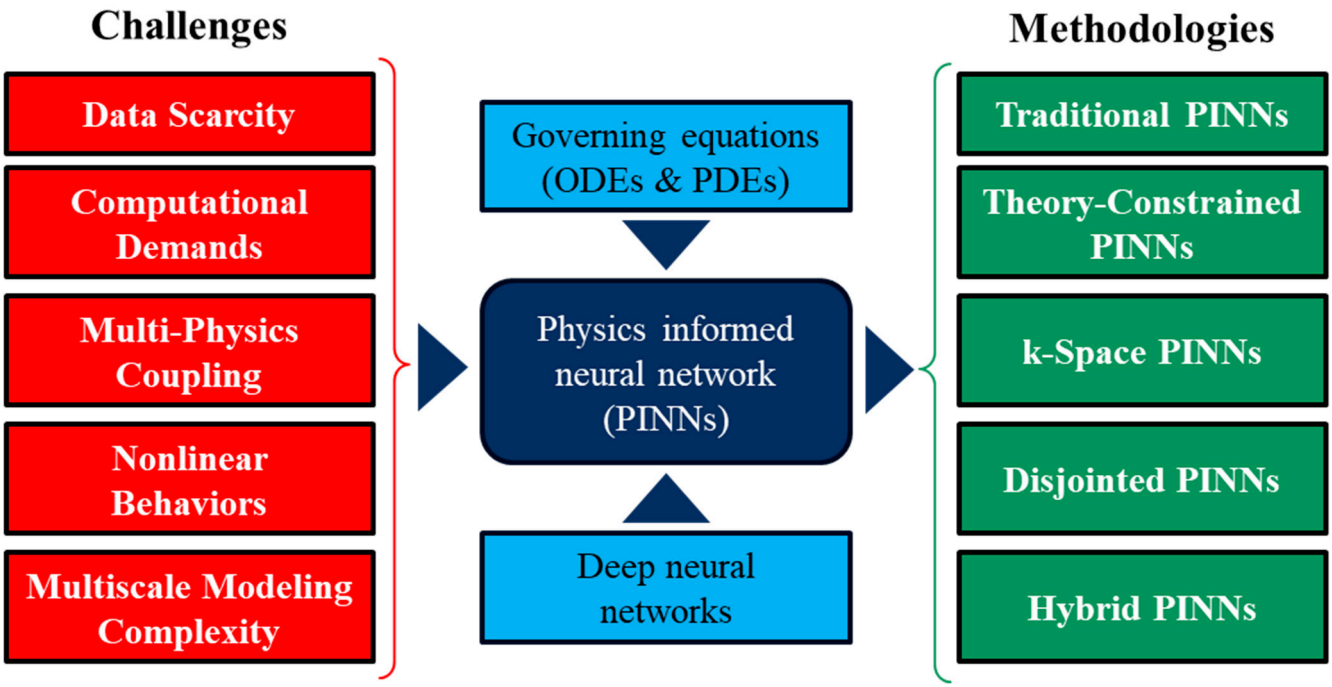


Figure 2. Overview of challenges in modeling composite materials and the corresponding Physics-Informed Neural Network (PINN) methodologies employed to address them.

This review delves into the application of PINNs in composite material analysis, with a specific emphasis on laminated composites. It examines fundamental theories, advanced methodologies tailored for composite modeling, and learning strategies designed to enhance computational efficiency and predictive accuracy. Key applications discussed include the structural analysis of plates and shells, stress-strain behavior and failure prediction, multi-scale modeling, and SHM of composites. The review also addresses persistent challenges in deploying PINNs for laminated composites, including data scarcity, computational demands, and boundary condition complexities. Future directions are outlined to advance the integration of PINNs into composite material research, highlighting innovative methodologies and potential breakthroughs. By providing a comprehensive overview, this review highlights the transformative potential of PINNs in revolutionizing composite analysis and optimizing material design across scales.

2. Fundamental of PINNs

PINNs provide a powerful framework for solving PDEs and ODEs, which are fundamental to modeling physical phenomena across scientific and engineering domains [11,32]. The primary objective of PINNs is to approximate the solution of a system of one or more differential equations, potentially nonlinear, using a mathematical formulation of these equations directly into the neural network architecture. A generic nonlinear differential equation solvable by PINNs can be represented as [33]:

$$v(y)_\tau = \mathcal{L}v(y) = 0,\ y \in \Lambda, \tau \in [0, \Theta] \tag{1}$$

where v represents the solution of the system, v_τ is its derivative with respect to the time variable τ within the interval $[0, \Theta]$, $\mathcal{L}$ is a non-linear differential operator, and y is an independent variable, possibly multi-dimensional, defined over the domain Λ. A common reference problem for PINNs, as explored in [34], is the Poisson equation:

$$\nabla^2 v(a, b) = g(a, b), (a, b) \in [0, 1] \times [0, 1] \tag{2}$$

This equation, while linear in nature, is widely used in scientific applications and serves as an illustrative example for testing PINN capabilities. In this case, PINNs encode Dirichlet boundary conditions on the unit square domain to approximate the solution $v(a, b)$. Despite its focus on a linear problem, PINNs are capable of handling more complex, nonlinear systems. At its core, a PINN combines two neural network components: a surrogate network (or approximator) and a residual network (Figure 3). The surrogate network approximates the solution at collocation points (a, b) within the simulation domain. These collocation points represent spatial inputs where the surrogate predicts dependent variables, such as velocity components (u, v) and pressure (p). The residual network encodes the governing physical equations and computes residual errors, which are minimized during training to ensure that the approximations satisfy the underlying physical laws. For instance, in the context of fluid dynamics, the Navier–Stokes equations are used to model incompressible flows, with the residuals defined as the following equations [35]:

$$\begin{aligned} \epsilon_1 &= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{1}{\rho}\frac{\partial \rho}{\partial x} - \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \epsilon_2 &= u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{1}{\rho}\frac{\partial \rho}{\partial y} - \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \\ \epsilon_3 &= \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \end{aligned} \tag{3}$$

Here, ϵ_3 enforces mass conservation (continuity), while ϵ_1 and ϵ_2 correspond to the momentum conservation equations in the x and y directions, respectively.

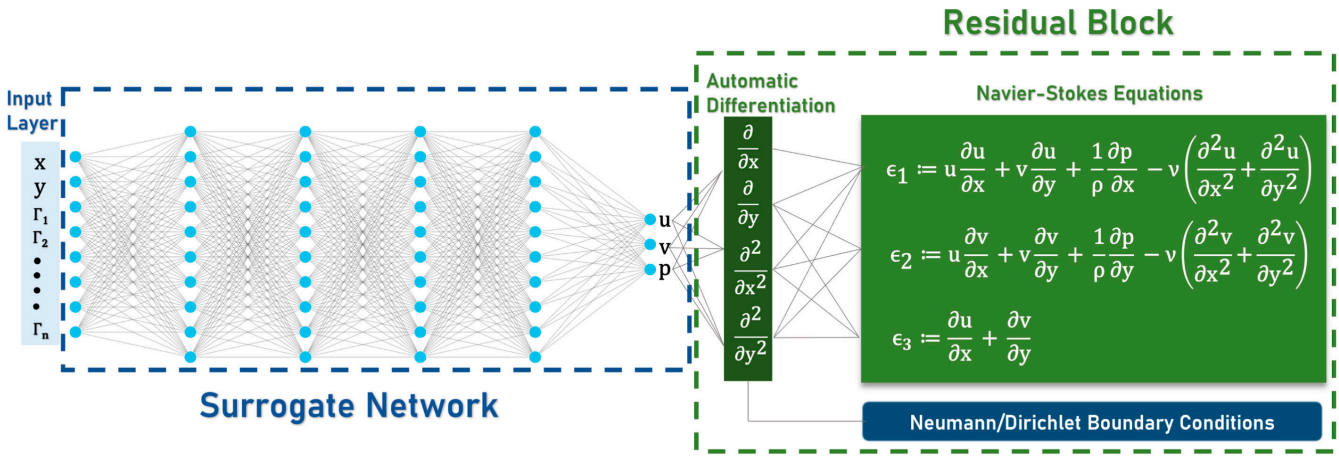


Figure 3. Architecture of a PINN, consisting of a surrogate network and a residual block. The surrogate network approximates the solution by predicting dependent variables (e.g., u, v, p) at collocation points, while the residual block enforces governing equations, such as the Navier–Stokes equations, using automatic differentiation. Boundary conditions (e.g., Neumann/Dirichlet) are integrated to ensure physically consistent solutions [35].

The surrogate network processes inputs (x, y) through multiple layers of neurons [36]. It maps inputs using affine-linear transformations (W) followed by scalar non-linear activation functions (b):

$$\widetilde{v}(y) = W_m \circ b \circ W_{m-1} \circ b \circ \ldots \circ b \circ W_2 \circ b \circ W_1(y) \tag{4}$$

Common activation functions in PINNs include Rectified Linear Unit (ReLU), tanh, swish, sine, and sigmoid. Each affine transformation is characterized by a weight matrix M_k and a bias vector c_k:

$$V_k y_k = M_k y_k + c_k \tag{5}$$

In PINNs, weights are typically initialized using the Xavier (or Glorot) initialization method [37], ensuring stable training dynamics. The residual network evaluates the governing equations to calculate the loss function, which guides the surrogate network's optimization. The optimization process minimizes the Mean Squared Error (MSE) of the residual:

$$MSE_s = \frac{1}{N_{a_i,b_i}} \sum_{i=1}^{N_{(a_i,\ b_i)}} |s(a_i, b_i)|^2 \tag{6}$$

where N_{a_i,b_i} is the number of collocation points. In PINNs, the collocation points constitute the training dataset. These collocation points form the training dataset, and the accuracy of the solution depends on their distribution and density. Through the collaboration of the surrogate and residual networks, PINNs effectively approximate solutions to PDEs while ensuring they adhere to governing physical principles. This combination of data-driven learning and physics-informed constraints makes PINNs a versatile and robust tool for solving a wide range of scientific and engineering problems.

3. PINN Methodologies for Laminated Composites

Over time, several PINNs methodologies have been developed to enhance the efficiency and applicability in composite material research. These methodologies range from traditional frameworks to hybrid approaches that incorporate additional optimization techniques, such as domain decomposition and advanced learning strategies. Emerging methods like Theory-Constrained PINNs, optimal PINNs, and k-space PINNs further

expand the scope of applications by improving accuracy and computational performance. This section explores the advancements in PINN methodologies for laminated composites.

3.1. Traditional and Hybrid Approaches in PINNs for Laminated Composites

In the modeling of laminated composites, traditional PINNs embed governing physical laws directly into the neural network's loss function [38]. This approach enables accurate predictions of critical properties such as stress, strain, deformation, and failure mechanisms while ensuring compliance with governing equations like elasticity and wave propagation. Laminated composites, characterized by their complex, layered structures and anisotropic properties, benefit from PINNs' ability to capture intricate behaviors such as stress distribution, deflection under load, and failure initiation [39,40]. However, traditional PINNs often face challenges in fully capturing the multifaceted nature of these materials, particularly when relying on a single framework that combines data-driven and physics-based constraints.

Wang et al. [41] developed a PINN framework to predict the bending behavior of clamped laminated composite plates by embedding governing physical laws into the model's loss function. The framework utilizes Classical Laminated Plate Theory (CLPT) and the energy method to construct the loss function, aligning predictions with well-established mechanics principles. This approach minimizes the need for extensive datasets, addressing the limitations of purely data-driven methods. The model's predictions were validated against analytical solutions from CLPT and finite element (FE) results, demonstrating its capability to deliver accurate and computationally efficient outcomes. However, the reliance on simplified CLPT assumptions, which neglect shear deformation effects, restricts its applicability to thick laminates or nonlinear material behaviors. Additionally, the study focuses exclusively on clamped rectangular plates, limiting its generalizability to other geometries and boundary conditions. Furthermore, while the method reduces data requirements, training PINNs for large-scale or highly detailed composite models remains computationally intensive.

To address the limitations of traditional PINNs, Barcelona Moreno et al. [42] proposed a hybrid PINN methodology that integrates the computational efficiency of the Extreme Learning Machine (ELM) algorithm. This advanced framework enhances the modeling of complex, multilayered composite structures by embedding the Sublaminate Generalized Unified Formulation (S-GUF) into the neural network. Unlike traditional PINNs, which rely on gradient-based optimization, the hybrid methodology leverages ELM to solve minimum-norm least squares problems, significantly reducing training time. The framework supports both strong-form formulations, enforcing equilibrium and compatibility conditions at collocation points, and weak-form formulations, minimizing total potential energy to improve numerical stability. A major contribution of this study is its extension to arbitrary geometries, overcoming the geometric constraints of earlier methods. Results demonstrate the hybrid PINN–ELM framework's ability to maintain high accuracy and efficiency, offering a robust and scalable solution for laminated composite analysis. Despite its successes, this approach introduces new challenges, such as sensitivity to parameter selection and potential numerical stability issues under complex loading conditions, warranting further investigation.

Expanding on the advancements in hybrid methodologies, Yan et al. [20] introduced a novel PINN framework integrated with ELM for solving direct and inverse problems in linear elasticity for composite thin-walled structures. This framework addresses the computational inefficiencies of traditional PINNs by leveraging ELM's single-step optimization, eliminating the need for iterative parameter updates in gradient-based learning. Training times were significantly reduced with the hybrid PINN–ELM, achieving approximately

0.9 s per iteration. In contrast, traditional PINNs required over 40,000 GBL iterations, resulting in a computational time reduction of several orders of magnitude. It incorporates a domain decomposition strategy that divides the computational domain into subregions, improving both accuracy and scalability for complex structural assemblies, such as plates and shells. Key features of this framework include its applicability to static and dynamic problems, such as free vibration and buckling analysis, as well as its utility in solving inverse problems like identifying stacking sequences in variable-stiffness composite plates. The results, validated against analytical and FE solutions, demonstrate the framework's effectiveness in predicting displacement fields, stress distributions, and eigenmodes while significantly reducing computational costs. However, limitations remain, including restrictions to shallow network architectures imposed by ELM and numerical sensitivity issues, such as poor conditioning in eigenvalue problems with large neuron counts or specific random initializations. Despite these challenges, the hybrid PINN–ELM framework represents a significant advancement in addressing the scalability and efficiency challenges of traditional PINNs, offering a versatile and robust tool for laminated composite analysis.

One notable advancement is the Physics-Informed Neural Operator (PINO) framework, introduced by Meng et al. [43]. This approach represents a significant departure from traditional PINNs by leveraging Fourier Neural Operators (FNOs) to establish function-to-function mappings, as illustrated in Figure 4. Unlike conventional PINNs, which rely on point-to-point mappings of spatiotemporal coordinates $(t_i,\ x_j)$ to outputs $(\mathrm{T}(t_i,\ x_j))$, PINO directly maps entire cure cycles $T_a(t)$ to temperature fields $\mathrm{T}(t, x)$. This enables efficient handling of parametric coupled PDEs, achieving unparalleled computational efficiency. The computational time comparison between FC–PINN and PINO was conducted for a thermochemical curing analysis of laminated composites, a common aerospace scenario. Figure 4a illustrates the governing physics, where the temperature field $T(x, t)$ is described by the following heat transfer equations:

$$h(T_a - T_{bc1}) = k_c \frac{\partial T}{\partial x} \tag{7}$$

$$\rho_c C_c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_c \frac{\partial}{\partial x}\right) + \dot{Q} \tag{8}$$

$$\rho_t C_t \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k_t \frac{\partial T}{\partial x}\right) \tag{9}$$

In these equations, h represents the heat transfer coefficient, T_a is the autoclave temperature, and T_{bc1} is the boundary temperature at the composite interface. The parameters ρ_c, C_c, and k_c refer to the density, specific heat, and thermal conductivity of the composite, while ρ_t, C_t, and k_t represent the corresponding values for the tool material. The term $\dot{Q}$ accounts for the internal heat generation due to exothermic resin curing reactions.

The system considered is a 1D laminated composite-to-tool configuration with a 30 mm CFRP composite bonded to a 20 mm Invar tool. Boundary conditions simulated autoclave curing, with a Dirichlet boundary condition applied at the composite-tool interface and a Robin boundary condition imposed at the tool surface. While Figure 4b shows that FC–PINN relied on dense collocation points (100 spatial points $\times$ 500 time steps), resulting in a training time of 2370 s, Figure 4c highlights how PINO's function-to-function mapping reduced computational time to just 84 s while maintaining accuracy. These conditions reflect real-world curing processes in thick CFRP laminates, where accurate and efficient temperature predictions are critical to preventing thermal gradients, under-curing, and residual stresses. PINO's superior performance makes it a scalable and robust solution for thermochemical curing optimization, achieving two orders of magnitude faster training compared to FC–PINNs.

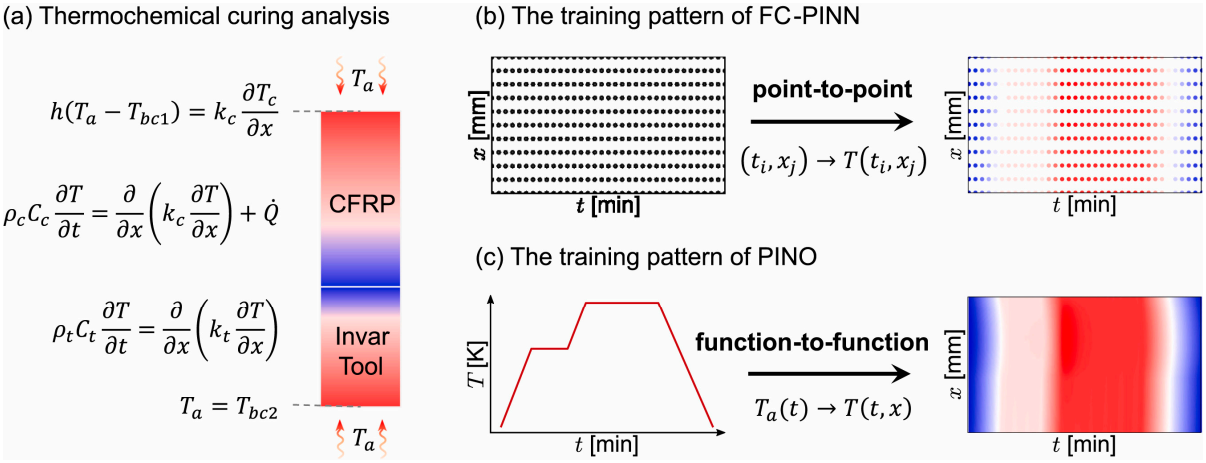


Figure 4. Comparison of training patterns in traditional fully connected PINNs (FC–PINNs) and the Physics-Informed Neural Operator (PINO) framework. (**a**) Thermochemical curing analysis setup with boundary conditions. (**b**) FC–PINNs rely on point-to-point mappings of spatiotemporal coordinates to outputs. (**c**) PINO utilizes function-to-function mappings, directly linking cure cycles to temperature fields, enabling more efficient and scalable training [43].

3.2. Theory-Constrained PINNs

Theory-Constrained PINNs are advanced models that embed composite-specific theories directly into the neural network's learning process [44,45]. Unlike standard PINNs, which incorporate general physical laws like elasticity or continuity, Theory-Constrained PINNs enforce detailed theoretical principles unique to the behavior of laminated composites [46]. This approach ensures that the network's predictions accurately reflect the mechanical responses of composites under diverse conditions while maintaining alignment with established composite mechanics.

Li et al. [25] developed a Theory-Constrained PINN framework tailored for inverse material parameter identification in laminated composites. The framework embeds First-Order Shear Deformation Theory (FSDT) directly into the loss function, ensuring predictions adhere to the governing mechanics of laminated composites. This integration of theoretical principles with data-driven modeling exemplifies the core concept of Theory-Constrained PINNs, where domain-specific knowledge is used to guide the learning process. As shown in Figure 5, Inputs, the framework maps displacement inputs (U_1, U_2, U_3) to material parameters, including elastic moduli (E1, E2), Poisson's ratio, and shear modulus. These parameters are used to compute stiffness coefficients (A_{11}, A_{12}, A_{22}, A_{16}, A_{26}, A_{66}) while combining data and physics-based losses (ζ_{Data} and $\zeta_{Physics}$) for accurate predictions. To enhance efficiency, the study incorporates transfer learning, leveraging pre-trained models to accelerate training on datasets with varying loading conditions. This significantly reduces computational costs while maintaining high accuracy. Validation on carbon and glass fiber-reinforced composites subjected to thermal-force coupling and uniform pressure demonstrated prediction errors within 0.1%, highlighting the precision and scalability of the framework. However, challenges remain in applying this approach to highly nonlinear problems or heterogeneous composites, where the simplicity of FSDT may limit its accuracy.

Yao et al. [21] developed a Theory-Constrained PINN framework for predicting fatigue delamination growth (FDG) in composite laminates, explicitly incorporating fiber-bridging effects. The framework embeds a physics-based empirical model into the neural network's loss function to ensure compliance with established mechanics principles and enhance the interpretability of the machine learning process. The embedded empirical model is represented as:

$$\frac{db}{dM} = K(b - b_0)\left(\Delta\sqrt{H}\right)^p \tag{10}$$

where K and p are Paris curve-fitting parameters calibrated using experimental fatigue data. $\Delta\sqrt{H}$ is Strain Energy Release Rate (SERR) of a fatigue cycle, which is determined using the Modified Compliance Calibration (MCC) method outlined in ASTM D5528 [47]. The fatigue crack growth rate db/dM is calculated using the ASTM E647 [48] standard's seven-point incremental polynomial method. By embedding this semi-empirical fatigue model, the PINN framework integrates physical constraints, balancing them with data-driven regression to capture the loading-history dependence of FDG, including fiber bridging effects. Experimental validation demonstrated the framework's superior prediction accuracy compared to traditional artificial neural networks (ANNs) and semi-empirical models, showcasing the capability of Theory-Constrained PINNs to merge domain-specific physics with machine learning for robust and interpretable fatigue modeling in composite laminates. Despite these strengths, the model's reliance on predefined Paris parameters limits its generalizability to other composite systems without recalibration, and its computational cost remains high for large-scale simulations.

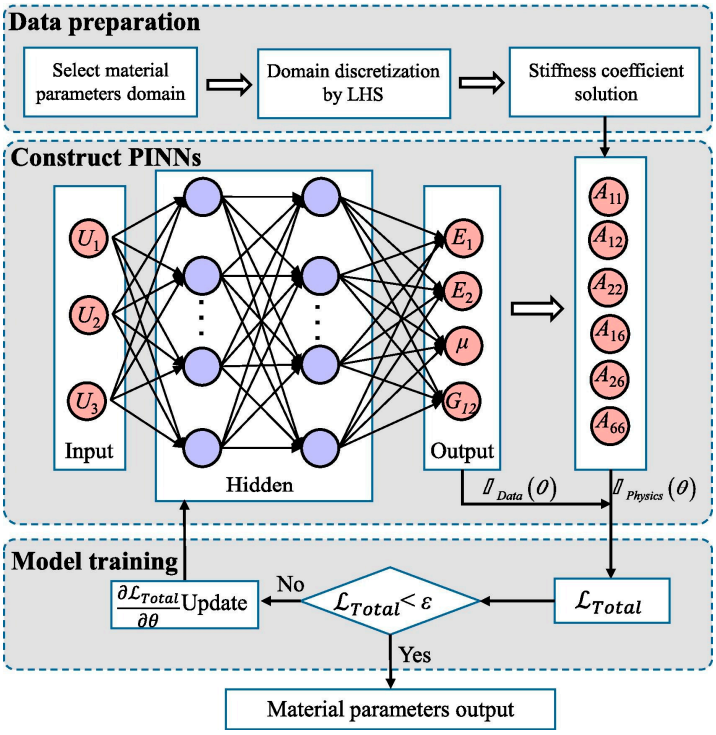


Figure 5. Schematic representation of the PINN framework for inverse material parameter identification in laminated composites [25].

Mea et al. [49] present a Physics-Informed Deep Learning (PIDL) framework that extends the principles of Theory-Constrained PINNs by embedding governing physical laws into a self-supervised segmentation process for composite materials. Unlike traditional PINNs, which rely on predefined boundary conditions, this methodology replaces explicit boundary definitions with supervised learning from matrix mechanical response data, ensuring adherence to physical principles. The framework leverages convolutional neural network architectures, such as U-Net, to segment composite material phases while simultaneously predicting stress fields based on multi-objective loss functions. These loss functions integrate physics-informed constraints with supervised matrix response data, ensuring consistency with stress-strain relationships in composite mechanics. By generalizing across

different loading scenarios and mechanical tests, the model offers scalability and adaptability for a variety of composite material analyses. The inclusion of transfer learning enhances the framework's efficiency, enabling stress prediction and segmentation for previously unseen data with minimal additional training. However, the reliance on matrix-specific response data limits its applicability to composite systems with significant heterogeneity, and the computational cost of U-Net architectures for large-scale data remains a challenge.

The exploration of Theory-Constrained PINNs demonstrates their transformative potential in addressing complex challenges in laminated composite modeling. By embedding composite-specific theoretical principles, such as FSDT and fatigue mechanics, these frameworks bridge the gap between domain-specific knowledge and data-driven learning. Studies have shown that Theory-Constrained PINNs excel in providing accurate, physically consistent predictions for material parameter identification, damage progression, and stress-strain relationships in composites. The incorporation of advanced techniques, such as transfer learning and multi-objective loss functions, further enhances their scalability and computational efficiency. However, challenges persist, including the need to address highly nonlinear behaviors, heterogeneous material properties, and the computational demand of large-scale implementations. Despite these limitations, Theory-Constrained PINNs offer a robust and scalable solution for integrating physics-based constraints with machine learning, paving the way for advancements in composite analysis and engineering applications.

3.3. Optimal PINN

Optimal PINN is a type of framework that addresses the different areas of research in model architecture design, training duration, computational efficiency, time for prediction, and accuracy of artificial neural networks and results for models with high accuracy and least computational resources [4,50]. Shen et al. [51] demonstrated the effectiveness of an optimal PINN framework for solving thermoelasticity problems in laminated composite plates. As illustrated in Figure 6, the framework accurately predicts stress distributions across the composite layers, with results comparable to those obtained using the State Space Finite Element Method (B-SSFE) and traditional FEM. The stress contour plots validate the framework's ability to deliver precise predictions while significantly reducing computational complexity. The optimal PINN leverages its meshless architecture to streamline the analysis process, eliminating the need for traditional discretization and reducing computational costs without compromising accuracy. By embedding governing equations directly into the neural network's loss function, the framework ensures physical consistency in predictions. This efficiency aligns with the principles of optimal PINNs, which aim to achieve high performance with reduced computational resources. The study reinforces the suitability of optimal PINNs for dynamic and thermoelastic analysis, providing a scalable and robust alternative to conventional numerical methods for laminated composites. However, the study faces challenges in handling complex boundary conditions, sensitivity to hyperparameters, and scalability to large-scale or highly nonlinear composite systems.

Tan et al. [52] present an optimal PINN method for analyzing the dynamic characteristics of functionally graded graphene nanoplatelet-reinforced composite (FG-GNPRC) One-Variable Edge Plates (OVEP). By utilizing classical mechanical theory, the framework transforms complex 3D vibration problems into simplified 1D representations, significantly reducing input data, model complexity, and computational costs while maintaining high predictive accuracy. The optimal PINN is designed to predict time-dependent vibration amplitudes under various dynamic scenarios, demonstrating high precision with R^2 scores exceeding 0.9999 and computational speeds up to 12 times faster than traditional numerical

methods. The PINN employs a neural network with L layers, where the transformation within each layer is defined by:

$$H^k = Q^k B^{k-1} + c^k,\ B^k = \sigma\left(H^k\right) \tag{11}$$

where H^k represents the pre-activation value at layer k, computed as a linear combination of the previous layer's outputs (B^{k-1}) weighted by Q^k and adjusted by biases c^k, and B^k is the output (activation) of layer k, obtained by applying the activation function σ to H^k. To ensure compliance with governing equations, the PINN framework incorporates a composite loss function:

$$\mathcal{L}_{total} = \lambda_1 \mathcal{L}_{\mathrm{bc}} + \lambda_2 \mathcal{L}_{\mathrm{ode}} + \lambda_3 \mathcal{L}_{\mathrm{dt}} \tag{12}$$

where $\mathcal{L}_{\mathrm{bc}}$, $\mathcal{L}_{\mathrm{ode}}$ and $\mathcal{L}_{\mathrm{dt}}$ enforce boundary conditions, governing equations, and observational data consistency, respectively. This structure enables the PINN to effectively operate in environments with incomplete or noisy data. Although originally designed for FG-GNPRC plates, the framework can be tailored to laminated composites by adapting the governing equations and constraints to match the specific mechanical behaviors and dynamics of these materials.

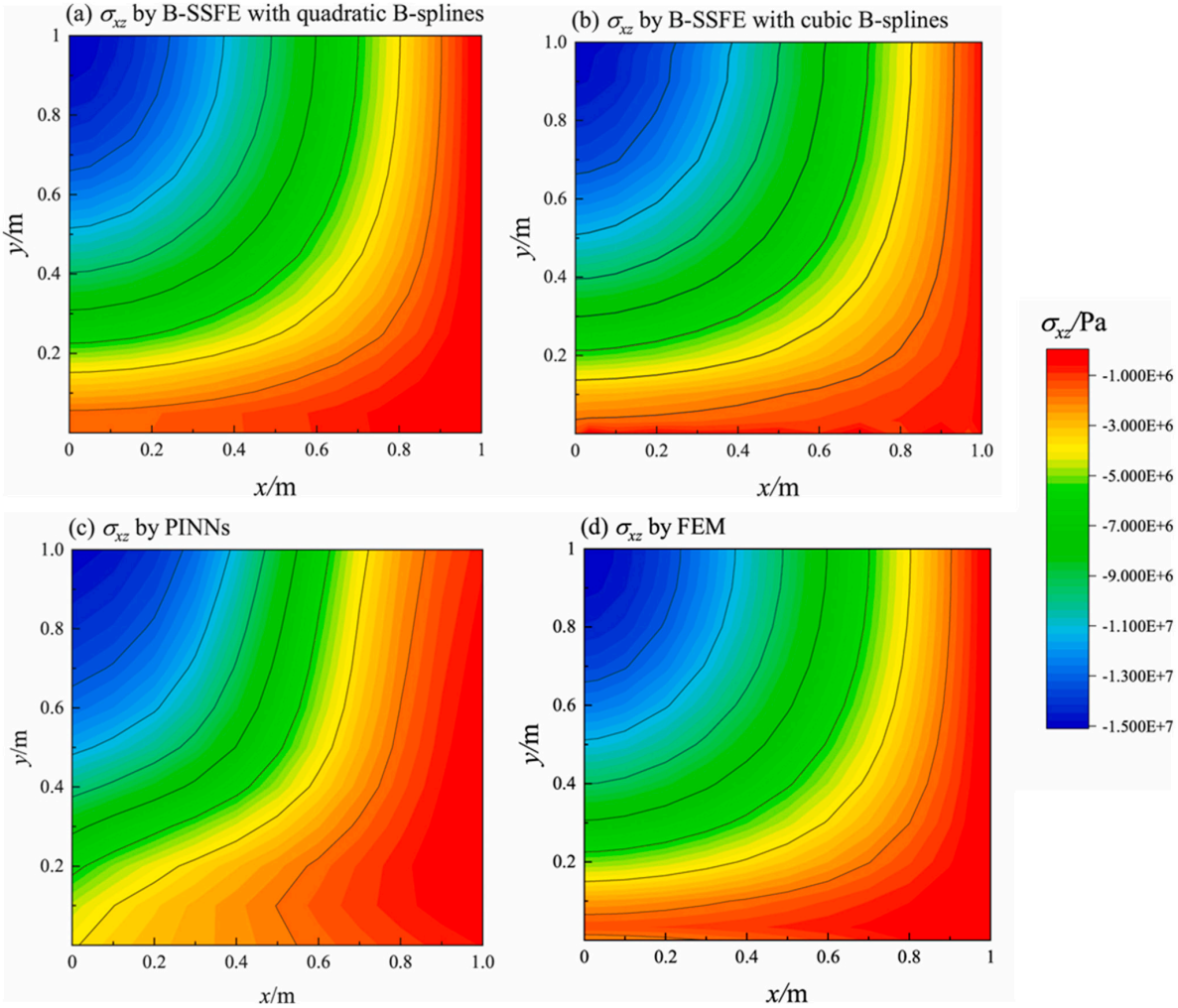


Figure 6. Comparison of stress contour plots for laminated composite plates in thermoelastic analysis predicted by different methods: (**a**) B-SSFE with quadratic B-splines, (**b**) B-SSFE with cubic B-splines, (**c**) Optimal PINN framework, and (**d**) traditional FEM [51].

Optimal PINNs demonstrate their transformative potential in addressing computational challenges in dynamic analyses of composite materials. By leveraging dimensionality reduction techniques, such as transforming 3D vibration problems into 1D representations, these frameworks significantly enhance efficiency while maintaining high predictive accuracy. The integration of physics-based constraints through composite loss functions ensures compliance with governing equations, even in data-scarce or noisy environments. Furthermore, their adaptability to various material systems, including nanocomposites and laminated composites, underscores their versatility across engineering domains. However, challenges such as reliance on precise governing equations, sensitivity to hyperparameters, and scalability to highly nonlinear and multi-physics problems remain areas for further exploration. Optimal PINNs pave the way for efficient, accurate, and scalable solutions in the design and analysis of advanced composite materials.

3.4. k-Space PINN

The k-space PINN framework is an advanced PINN tailored for compressed spectral mapping of vibrations, providing interpretable insights into the wavenumber composition and symmetry of the response function [53–55]. Saeid et al. developed an innovative k-space Physics-Informed Neural Network (k-PINN) framework tailored for the vibrational analysis of laminated composites. This framework employs Fourier basis functions to construct the response function, expressed as [19]:

$$\hat{V}(a,b,g) = \sum_{p_a=-M_g\times B_a}^{M_g\times B_a} \sum_{p_b=-M_g\times B_b}^{M_g\times B_b} \widetilde{V}(q_a(p_a), q_b(p_b), g) exp(2\pi i(q_a(p_a)a + q_b(p_b)b)) \quad (13)$$

where $\left(\widetilde{V}(q_a(p_a), q_b(p_b), g)\right)$ represents the Fourier coefficients in k-space, and $q_a(p_a)$ and $q_b(p_b)$ are wavenumbers. This spectral representation allows the k-PINN framework to decompose the solution into real and imaginary Fourier components, thereby effectively learning and reconstructing vibrational responses in the frequency domain. By adjusting the complexity of the response through its spectral components, k-PINN overcomes the spectral bias typically encountered in conventional PINNs, which struggle to capture high-frequency oscillations in solutions.

For laminated composites, the k-PINN framework demonstrates exceptional computational efficiency and scalability, as illustrated in Figure 7. In Figure 7a, k-PINN achieves significantly lower computational times per epoch, with a time of approximately 50 ms for a neural network (NN) width of 128, compared to over 300 ms for traditional PINNs. Even as the NN width increases to 1024, k-PINN maintains a time of around 100 ms, whereas traditional PINNs require nearly 700 ms. In terms of memory efficiency, Figure 7b shows that k-PINNs efficiently manage large-scale problems with up to 16x more collocation points (21,600 points) while maintaining a computational time of approximately 240 ms. In contrast, traditional PINNs fail due to memory limitations at this scale. The scalability of k-PINNs is demonstrated in Figure 7c, where the computational time increases linearly with the number of k-space components. Even with 4 components (5124 total), the time remains under 160 ms per epoch, highlighting the method's ability to handle detailed spectral representations efficiently. Finally, Figure 7d underscores the minimal computational overhead of k-PINNs, with times of 96 ms for a single neural network and 110 ms when managing two networks simultaneously, showcasing their superior efficiency for solving vibrational problems in laminated composites. The k-PINN framework's ability to operate in the frequency domain not only enhances accuracy for problems involving high-frequency oscillations but also ensures computational robustness and scalability. These advantages position k-PINN as a superior alternative to conventional PINNs for laminated composite vibrational analysis.

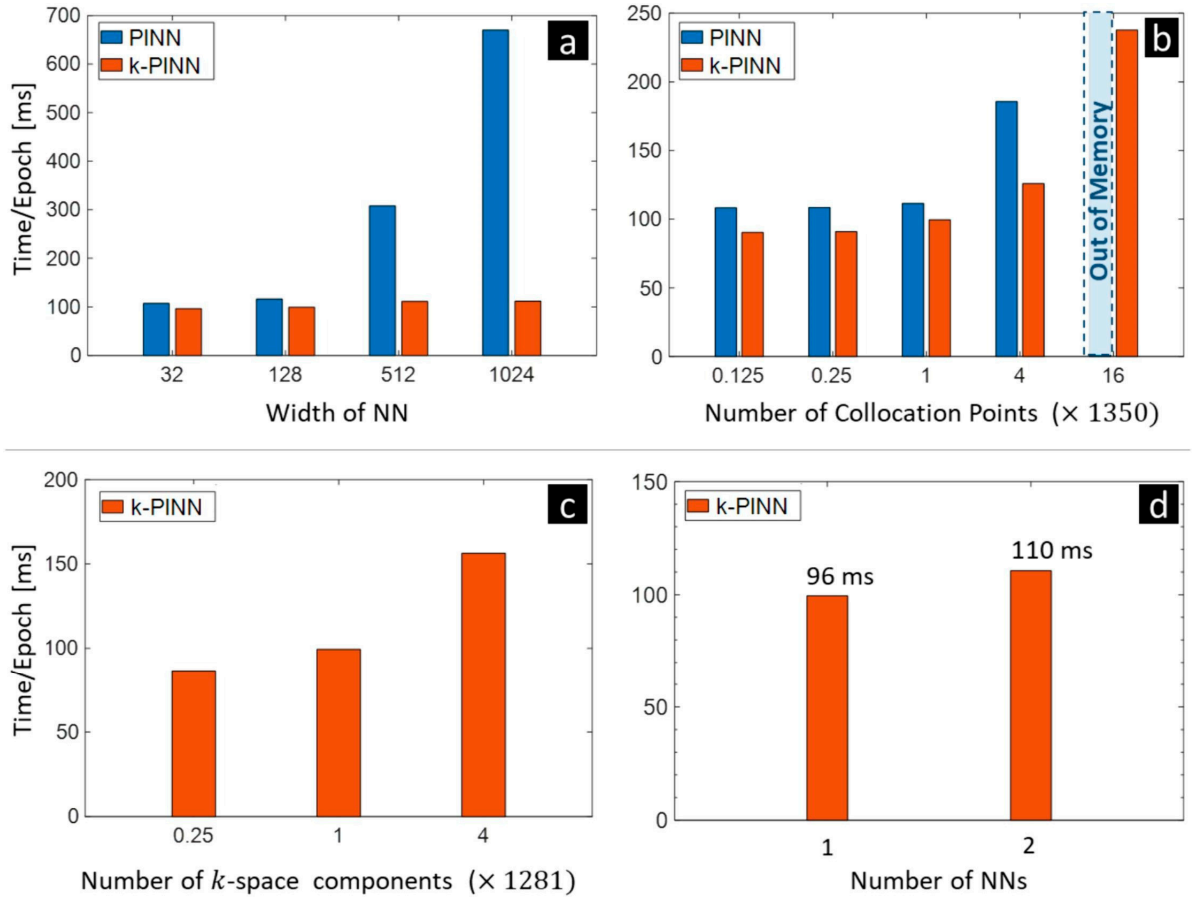


Figure 7. Performance comparison between traditional PINNs and k-PINNs: (**a**) Computational time per epoch as a function of neural network width, demonstrating k-PINN's efficiency at larger network widths; (**b**) Scalability with collocation points, where traditional PINNs face memory limitations, but k-PINNs maintain performance; (**c**) Computational cost of k-PINNs for varying numbers of k-space components, showing favorable scaling; and (**d**) Minimal computational overhead of k-PINNs with one or two neural networks [19].

3.5. Disjointed PINN

Disjointed PINNs represent an advanced methodology where the computational domain is divided into separate subdomains, each governed by localized PINN architectures, enabling efficient handling of complex, multi-scale, or heterogeneous systems while ensuring continuity and consistency at the interfaces [56–58]. Disjointed PINNs are specifically designed to address challenges associated with modeling multi-physics processes in laminated composites. Traditional PINNs struggle with coupled phenomena that exhibit distinct behaviors, such as the interaction between heat conduction and chemical reactions during composite curing. Disjointed PINNs overcome this limitation by decoupling the solution into separate subnetworks, each optimized for a specific process. This modular approach allows each subnetwork to focus on a narrower domain, improving convergence and stability during training. Additionally, sequential training strategies and specialized loss functions ensure that disjointed PINNs effectively capture complex physical interactions, such as temperature gradients and exothermic curing, with high accuracy and computational efficiency. This makes disjointed PINNs an ideal solution for analyzing thermochemical processes in composite manufacturing.

Recently, Sina et al. [26] introduce a disjointed PINN framework tailored for modeling the thermochemical evolution of composite-tool systems during autoclave curing. The system involves two coupled processes: (1) heat conduction from the autoclave environ-

ment and the exothermic reaction of the composite, and (2) temperature-dependent curing kinetics of the composite material. Traditional PINNs face challenges in capturing both phenomena within a single framework due to sharp discontinuities at material interfaces and differences in temporal and spatial scales.

To address these challenges, the study employs disjointed subnetworks for temperature (T) and degree of cure (α). Each subnetwork independently approximates its respective variable, as illustrated in Figure 8. The input layer of the network processes spatial (x) and temporal (t) coordinates, which are propagated through fully connected hidden layers to output the temperature (T) and degree of cure (α). The total loss function is composed of several terms:

$$\mathcal{L}_{\mathcal{N}} = \mathcal{L}_{\alpha} + \mathcal{L}_{\mathcal{T}} + \mathcal{L}_{\alpha_0} + \mathcal{L}_{\mathcal{T}_0} + \mathcal{L}_{\mathcal{T}_{\mathrm{bc}_1}} + \mathcal{L}_{\mathcal{T}_{\mathrm{bc}_2}} \tag{14}$$

where the terms correspond to losses for PDE constraints ($\mathcal{L}_{\alpha}$, $\mathcal{L}_{\mathcal{T}}$), initial conditions ($\mathcal{L}_{\alpha_0}$, $\mathcal{L}_{\mathcal{T}_0}$), and boundary conditions ($\mathcal{L}_{\mathcal{T}_{\mathrm{bc}_1}}$, $\mathcal{L}_{\mathcal{T}_{\mathrm{bc}_2}}$). Each loss term ensures that the model adheres to physical laws, initial states, and boundary conditions.

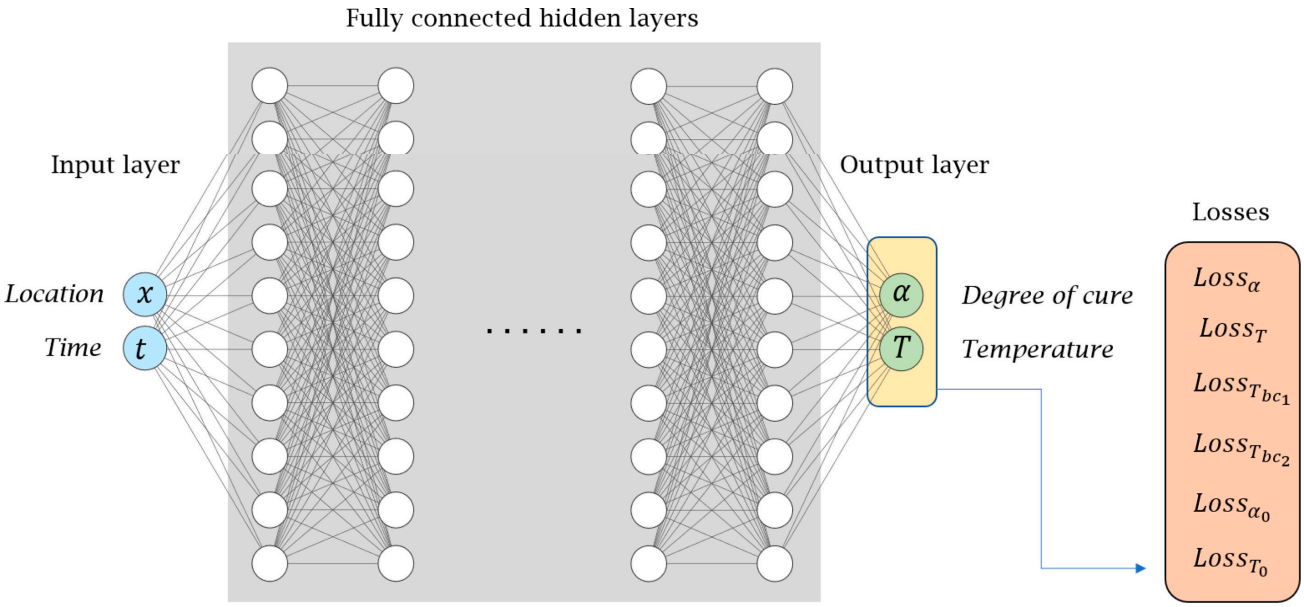


Figure 8. Architecture of the disjointed PINN framework for modeling the thermochemical evolution of composite-tool systems during autoclave curing [26].

The architecture clearly shows how the framework incorporates these losses to optimize the training process. Sequential training alternates between optimizing the temperature and degree of cure networks, ensuring stable convergence. This approach successfully reduces all loss terms to below 10^{-5} within a limited number of iterations. Validation on case studies involving varying material thicknesses and thermal boundary conditions demonstrates the framework's accuracy in predicting temperature and curing behavior. The disjointed architecture explicitly captures discontinuities at composite-tool interfaces, accurately modeling temperature gradients and heat flux. Compared to finite element methods, the disjointed PINN approach achieves comparable accuracy while significantly reducing computational costs. This modular framework not only addresses the challenges of multi-physics modeling in laminated composites but also highlights the broader potential of disjointed PINNs in applications where coupling distinct physical phenomena is crucial.

3.6. Comparison of PINN Methodologies for Laminated Composites

PINNs have revolutionized the modeling of laminated composites by integrating physics-based principles into data-driven machine-learning frameworks. These methodolo-

gies have evolved to tackle challenges inherent in analyzing complex composite materials, such as nonlinear behavior, multi-scale interactions, and intricate boundary conditions. Traditional PINNs embed governing equations like elasticity and wave propagation directly into loss functions, ensuring physical consistency in predictions. However, as the complexity of composite systems increases, these conventional approaches often struggle with computational inefficiency and scalability issues, necessitating advancements in PINN methodologies.

To address these limitations, hybrid and domain-specific approaches have emerged, enhancing both computational efficiency and predictive accuracy. Hybrid PINNs integrate optimization techniques and advanced numerical formulations to reduce training times and improve scalability for large-scale composite models. Domain-specific approaches, such as Theory-Constrained PINNs, incorporate established theories unique to laminated composites into the learning process, ensuring compliance with mechanics principles and enabling accurate predictions for critical parameters like stress, strain, and damage evolution. These advancements have expanded the applicability of PINNs to a wider range of composite structures and loading conditions, though challenges remain in handling highly nonlinear phenomena and heterogeneous material properties.

Emerging methodologies such as optimal PINNs, k-space PINNs, and disjointed PINNs represent the next frontier in the application of PINNs to laminated composites. These frameworks focus on reducing computational costs while maintaining high accuracy, leveraging techniques like dimensionality reduction, spectral mapping, and modular network architectures. Optimal PINNs streamline data representation to achieve faster convergence, while k-space PINNs utilize Fourier-based spectral representations to capture complex vibrational behaviors. Disjointed PINNs decouple multi-physics problems into modular subnetworks, enabling efficient modeling of coupled processes such as thermochemical evolution during composite manufacturing. Despite their promise, these advanced methodologies face challenges such as scalability to highly nonlinear systems, parameter optimization, and integration across multiple physics domains. Continued innovation in PINN frameworks will be crucial for advancing the analysis, design, and optimization of laminated composite materials. Table 1 provides a comprehensive overview of various PINN methodologies for laminated composites, summarizing their key contributions, accuracy, computational efficiency, and limitations.

Table 1. Comparative summary of PINN methodologies for laminated composites.

Methodology	Contribution	Accuracy	Computational Cost	Limitations	Ref.
Traditional PINNs	Predict bending behavior of laminated composites using CLPT, validated against finite element results.	Max displacement error (RMSE) range: 1.35–2.36%	Training stabilized at 5000–10,000	Limited to simple geometries; neglects shear deformation effects.	[41]
Hybrid PINNs	Combines PINNs with ELM for faster convergence and complex geometries.	Displacement Error: 7.2×10^{-3}; Frequency error: 0.0063%	Training time: 0.9 s/iteration (~40× faster)	Sensitive to parameter tuning; instability under complex loading.	[20]
Theory-Constrained PINNs	Embeds domain-specific theories (e.g., FSDT) into loss functions for accuracy.	Error < 0.1%	Convergence time reduced upto 5 times as compared to conventional models	High computational cost for large-scale systems.	[25]
Optimal PINNs	Reduces dimensionality (3D to 1D) for faster convergence and efficiency.	Stress error: <0.1%. $R^2 > 0.9999$	Speed-up: 12× faster; Training ~180 s	Sensitive to hyperparameters; challenges with multi-physics problems.	[51]
k-Space PINNs	Fourier-based spectral methods for efficient vibrational mode analysis.	Mode shape error: 1.44×10^{-3} to 2.22×10^{-3}	Training time: 85 s (10 modes)	Requires accurate spectral formulations; optimization challenges.	[19]
Disjointed PINNs	Decouples coupled thermochemical processes into separate subnetworks.	Relative error: <1% for temperature predictions	Training time: 2370 s; Transfer learning: 45 s	Computational overhead for sequential training; sharp interface issues.	[26]

4. Learning and Optimizing Strategies in PINN

This section explores the various learning and optimization strategies employed in PINNs to enhance their training efficiency, robustness, and accuracy, particularly for complex applications like laminated composites. These strategies encompass adaptive optimization techniques, multi-objective optimization frameworks, and hyperparameter tuning tailored to physics-guided neural networks. By addressing the challenges of convergence, computational cost, and balancing data-driven learning with physics-based constraints, these methodologies aim to advance the capabilities of PINNs in modeling intricate behaviors of composite materials. The subsequent subsections delve into specific techniques, providing insights into their implementation and impact on performance.

4.1. Adaptive Optimization Techniques

Adaptive optimization techniques are renowned for achieving superior convergence compared to vanilla gradient methods [59,60]. Unlike traditional gradient schemes [61], adaptive algorithms mimic the behavior of second-order methods by adapting to the global geometry of the loss function, thereby enhancing performance. These methods, such as Adam optimization, are well-suited for problems involving large datasets or parameters, non-stationary objectives, and noisy or sparse gradients [62]. Adam optimization, in particular, employs adaptive estimates of lower-order moments, making it computationally efficient, memory-efficient, and robust across a variety of optimization challenges. Its intuitive hyperparameters typically require minimal tuning, allowing for straightforward implementation and favorable comparisons to other stochastic optimization methods [63].

The Adaptive Moment Estimation (Adam) algorithm was implemented in this study as a training rule for PINNs in the context of a laminated composite analysis [20]. The iterative process involves evaluating the loss function $\mathcal{L}$ and its gradient $\nabla\mathcal{L}$, followed by recursive updates to the internal parameters. The parameters are iteratively adjusted as follows:

$$\begin{gathered} p_{(t)} = \gamma_1 p_{(t-1)} + (1-\gamma_1)\nabla\mathcal{J}(\Phi_{(t)}) \\ q_{(t)} = \gamma_2 q_{(t-1)} + (1-\gamma_2)\nabla^2\mathcal{J}(\Phi_{(t)}) \\ \hat{p}_{(t)} = \frac{p_{(t)}}{1-\gamma_1^t},\ \hat{q}_{(t)} = \frac{q_{(t)}}{1-\gamma_2^t}, \quad \Phi_{(t+1)} = \Phi_{(t)} - \frac{\mu}{\sqrt{\hat{q}_{(t)}}+\zeta}\hat{p}_{(t)} \end{gathered} \tag{15}$$

where $\Phi_{(t)}$ represents the parameters at iteration t, γ_1 and γ_2 are decay rates and μ is the learning rate.

In another study, Adam optimization was implemented in the training of PINNs for laminated composites to enhance convergence and predictive accuracy across multi-scale datasets [64]. As shown in Figure 9, the training loss for the low-fidelity PINN model rapidly decreases over 50,000 epochs, demonstrating the optimizer's capability to efficiently handle approximate representations of the complex physics in laminated composites. For the high-fidelity PINN model, transfer learning is applied, fine-tuning the last network layers and achieving convergence within just 10,000 epochs. The inset in Figure 7 highlights the optimizer's efficiency, showing smooth and consistent loss reduction for high-fidelity data. This integration of Adam optimization enables accurate predictions of critical laminated composite behaviors, such as stress, deformation, and failure mechanisms, while significantly reducing computational costs. By leveraging adaptive moment estimation, Adam optimization ensures stable parameter updates even in noisy or incomplete datasets, underscoring its suitability for modeling laminated composites in diverse applications.

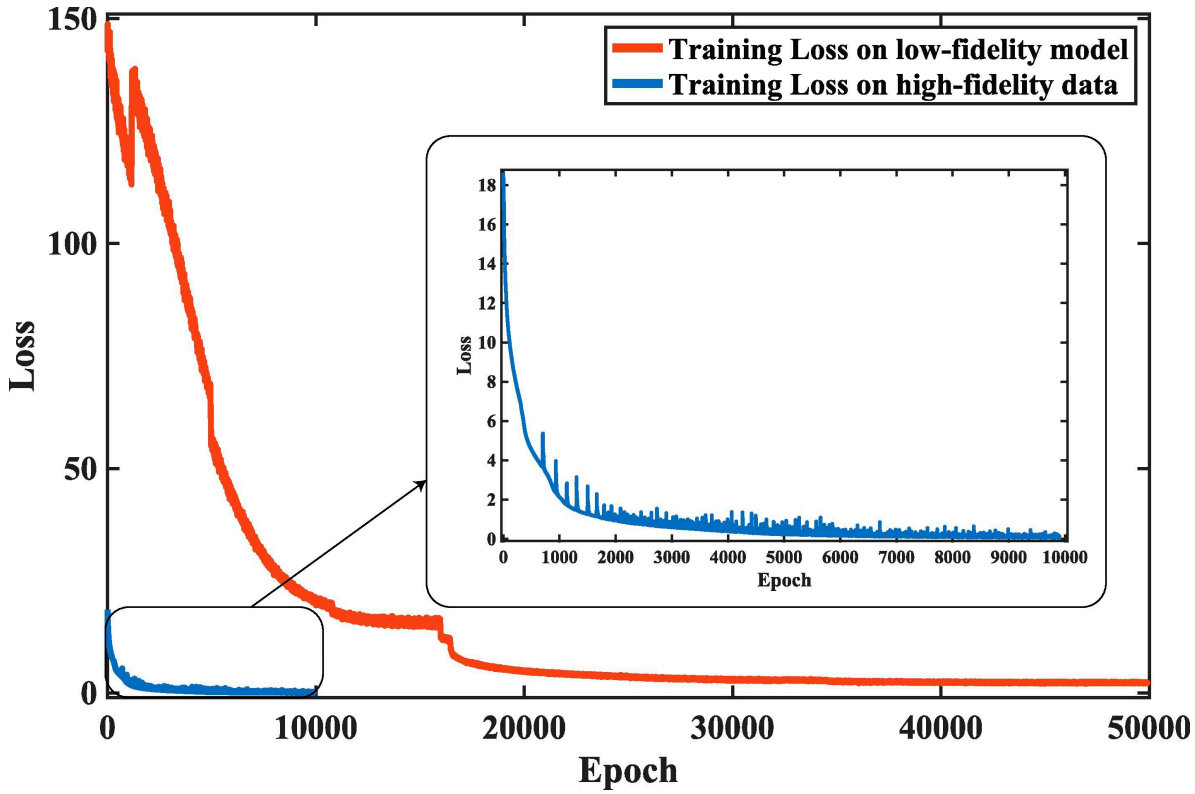


Figure 9. Performance summary plot of the optimization algorithm during low- and high-fidelity training [64].

In the case of non-local PINN for laminated composites [65], the optimization process effectively incorporates the Adam optimizer combination with a full-batch training scheme. Training begins with an initial learning rate of 1×10^{-4}, which is progressively reduced through an exponential decay schedule to enhance convergence. Additionally, material properties, such as stiffness and stress parameters, are initialized with random values to ensure robust learning across diverse conditions. The Adam optimizer's adaptive moment estimation facilitates efficient updates of network parameters, enabling the non-local PINN framework to capture complex spatial dependencies and ensure accurate predictions of mechanical behaviors in laminated composites. This approach proves particularly advantageous in dealing with the anisotropic and heterogeneous nature of these materials.

4.2. Multi-Objective Optimization

Multi-objective optimization is a mathematical and computational approach aimed at simultaneously optimizing two or more conflicting objectives, balancing trade-offs to identify a set of optimal solutions, commonly known as the Pareto front, that offer the best possible compromises for decision-making in complex systems [66–68]. Multi-objective optimization strategies have emerged as pivotal tools in PINNs for laminated composites, addressing the need to balance multiple objectives such as boundary conditions, governing equations, and observational data [69]. A widely utilized approach is the weighted-sum scheme, which consolidates these objectives into a unified optimization framework by assigning dynamic weights to individual loss terms. This enables the optimization process to accommodate varying priorities or constraints, allowing for the exploration of diverse solution spaces or the selection of a single optimized outcome.

For laminated composites, Multi-objective optimization is particularly significant due to the complex physical phenomena that demand accurate representation, including stress-strain relationships, deformation dynamics, and failure mechanisms [70]. The total

loss function in PINNs is commonly defined by key components such as $\mathcal{L}_{\mathcal{N}_\alpha}$ and $\mathcal{L}_{\mathcal{N}_\mathcal{T}}$, which enforce adherence to critical physical principles, initial conditions, and boundary constraints. These loss terms are expressed as:

$$\begin{aligned} \mathcal{L}_{\mathcal{N}_\alpha} &= \mathcal{L}_\alpha + \omega_{\alpha_0}\mathcal{L}_{\alpha_0} \\ \mathcal{L}_{\mathcal{N}_\mathcal{T}} &= \mathcal{L}_\mathcal{T} + \omega_{T_0}\mathcal{L}_{\mathcal{T}_0} + \omega_{T_{\mathrm{bc}_1}}\mathcal{L}_{\mathcal{T}_{\mathrm{bc}_2}} + \omega_{T_{\mathrm{bc}_2}}\mathcal{L}_{\mathcal{T}_{\mathrm{bc}_2}} \end{aligned} \tag{16}$$

where ω represents the weight associated with each term. These weights play a crucial role in balancing the contributions of individual loss components, avoiding overemphasis on any single objective.

To address challenges such as unbalanced gradients that hinder optimization convergence, adaptive weight tuning is employed [26]. This technique dynamically adjusts the weights during training, ensuring robust gradient flow dynamics and stable optimization. The updated scaling weight $\omega^{\mathrm{e}+1}$ for each loss term is calculated at epoch e as:

$$\begin{aligned} \omega^{e+1} &= \beta\omega^e + (1-\beta)\hat{\omega}^{e+1} \\ \hat{\omega}_{\alpha_0}^{e+1} &= \frac{1}{\omega_{\alpha_0}^e}\frac{max(|\nabla_\theta\mathcal{L}_\alpha(\theta^e)|)}{\mathrm{mean}\left(|\nabla_\theta\mathcal{L}_{\alpha_0}(\theta^e)|\right)} \text{ for } \mathcal{N}_\alpha \\ \hat{\omega}_i^{e+1} &= \frac{1}{\omega_i^e}\frac{max(|\nabla_\theta\mathcal{L}_\mathcal{T}(\theta^e)|)}{\mathrm{mean}(|\nabla_\theta\mathcal{L}_i(\theta^e)|)} \text{ for } \mathcal{N}_\mathcal{T}, \text{where } i \in \{T_0, T_{\mathrm{bc}_1}, T_{\mathrm{bc}_2}\} \end{aligned} \tag{17}$$

where β is a smoothing factor, and $\hat{\omega}_{\mathrm{i}}^{\mathrm{e}+1}$ represents the normalized gradient ratio for each loss term.

This adaptive approach significantly enhances the robustness and accuracy of PINNs for laminated composites by mitigating issues like gradient stiffness and imbalance. Multi-objective optimization strategies ensure reliable and efficient optimization, enabling PINNs to effectively model intricate, coupled physical phenomena. These advancements are crucial for the precise analysis of mechanical and thermal behaviors in laminated composites, facilitating applications across aerospace, marine, and structural engineering domains.

4.3. Hyperparameter Tuning for Physics-Informed Networks

Hyperparameter tuning is the process of systematically optimizing the parameters that control the training of a machine learning model, aiming to enhance its performance and generalization by finding the best combination of values for parameters such as learning rate, batch size, and network architecture [71–73]. Hyperparameter tuning is a critical process in enhancing the efficiency and accuracy of PINNs when applied to laminated composites [20]. Laminated composites present unique challenges due to their anisotropic and layered structure, which demands precise modeling of stress-strain relationships, deformation dynamics, and interlaminar interactions. To address these challenges, hyperparameter tuning ensures optimal performance of the PINN by adjusting parameters that govern the training process, such as learning rates, loss weights, and activation functions.

A recent study highlights the role of hyperparameter tuning in refining PINN performance for composite curing processes, focusing on spatio-temporal modeling of critical parameters like degree of cure and temperature [74]. In this context, loss weights assigned to physical laws, boundary conditions, and observational data were carefully balanced to capture complex interactions without overemphasizing individual phenomena. For instance, specific tuning ensured proper stress transfer and deformation compatibility at layer interfaces, addressing interlaminar complexities inherent in laminated composites. Additionally, dynamic learning rate adjustments using advanced optimization algorithms, such as the Adam optimizer, mitigated issues of slow convergence or overshooting solutions. Activation functions, another critical hyperparameter, were tuned to improve the network's ability to model nonlinear behaviors like damage progression and large

deformations. Functions like hyperbolic tangent and ReLU were chosen for their capacity to approximate complex mappings between inputs and outputs effectively. This careful selection enhanced the network's flexibility in capturing the intricate physical relationships in laminated composites. This process is exemplified in Figure 10, which demonstrates the PiNDiff model's predictions for temperature contours in laminated composites. The figure highlights the importance of hyperparameter tuning in achieving high fidelity to experimental ground truth, showcasing the ability of PINNs to effectively model the thermal behavior of composites.

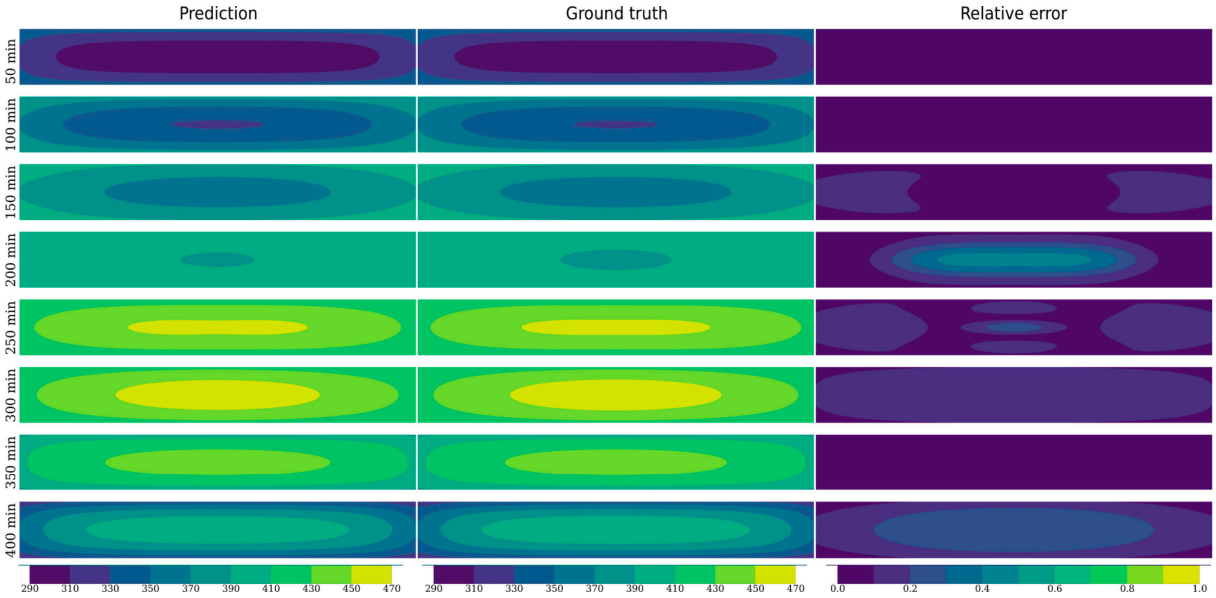


Figure 10. Temperature contour predictions in laminated composites using the PiNDiff model, demonstrating the impact of hyperparameter tuning on achieving high fidelity to experimental ground truth [74].

4.4. Comparison of Optimization Strategies

Learning and optimizing strategies in PINNs have proven essential for effectively modeling the intricate behaviors of laminated composites. These strategies include adaptive optimization techniques, multi-objective optimization frameworks, and hyperparameter tuning, each addressing specific challenges posed by the anisotropic, layered structures of composites. Adaptive optimization methods, such as the Adam optimizer, have been pivotal in ensuring efficient convergence during the training of PINNs. By dynamically adjusting parameter updates, these techniques manage noisy and incomplete datasets, enabling accurate predictions of stress-strain relationships, deformation dynamics, and interlaminar stress transfer. However, challenges such as selecting appropriate hyperparameters and avoiding convergence issues in high-dimensional spaces remain.

Multi-objective optimization further enhances PINNs' capabilities by balancing multiple loss terms that represent physical laws, boundary conditions, and observational data. Adaptive weight tuning ensures the robust handling of gradient flow dynamics, mitigating imbalances that hinder convergence. This approach is particularly beneficial for capturing the coupled physical phenomena in laminated composites, such as thermal and mechanical interactions. Despite its strengths, Multi-objective optimization strategies require careful weight initialization and adaptive adjustment to optimize performance effectively. Hyperparameter tuning represents another critical strategy in improving the accuracy and efficiency of PINNs for laminated composites. Parameters like learning rates, loss weights, and activation functions are optimized to enhance the network's ability to model nonlin-

ear behaviors, such as damage progression and large deformations. Dynamic learning rate schedules and tailored activation functions, such as ReLU and hyperbolic tangent, have been employed to capture the complex physical relationships in laminated composites. While these methods significantly enhance performance, they demand extensive computational resources and meticulous trial-and-error procedures.

Collectively, these strategies enable PINNs to address the multifaceted challenges of laminated composite analysis, delivering robust, accurate, and computationally efficient solutions. By leveraging advancements in learning and optimization, PINNs are poised to revolutionize applications in aerospace, marine, and structural engineering, where precise modeling of composite behaviors is critical. Table 2 presents a summary of various optimization strategies applied to PINNs for laminated composites, highlighting their corresponding models, impacts on predictive performance, and associated challenges. These strategies address key challenges in training efficiency, convergence stability, and accurate representation of complex physical behaviors in laminated composite materials.

Table 2. Summary of optimization strategies for PINNs in laminated composites.

Optimization Strategy	Impact on PINN	Accuracy	Computational Efficiency	Challenges	References
Adam Optimization	Improved convergence efficiency and accuracy for complex physical problems.	Localization error reduced from 10.32 cm to 0.5075 cm	Training time reduced by ~80% compared to low-fidelity PINNs.	Due to its adaptive nature, Adam can potentially be overfit on small or noisy datasets. Adam's implementation of regularization may not behave the same way as standard weight decay, leading to inconsistent results in some applications.	[64]
Multi-objective Optimization	Balanced loss functions to enforce physical constraints and optimize coupled physics.	Relative error reduced from 17.9% to 1.27%	50% fewer iterations required for convergence.	Difficulty in initializing and dynamically adjusting weights; risk of gradient imbalance impacting convergence stability.	[70]
Hyperparameters Tuning	Enhanced predictive accuracy and computational efficiency through optimized settings.	Relative error improved to 0.01–0.05	Training time reduced by ~40% through hyperparameter optimization.	Demanding computational resources for extensive trial-and-error tuning; sensitivity to improper loss weight balancing.	[74]

5. Applications of PINNs in Composite Analysis

This section explores the key applications of PINNs in composite analysis, emphasizing their role in structural analysis of plates and shells, SHM (including damage prediction), stress-strain evaluation, and multi-scale modeling.

5.1. Structural Analysis of Plates and Shells

PINNs are revolutionizing the structural analysis of composite plates and shells by offering a robust, meshless alternative to traditional computational methods like the FEM [75] or the B-Spline Finite Element Method [76]. By embedding physical laws and governing equations directly into their architecture, PINNs provide an efficient and accurate framework for tackling the complexities of composite structures. Their ability to handle diverse geometries and boundary conditions with reduced preprocessing and computational overhead makes them highly versatile for a range of applications in composite analysis.

Yan et al. [20] introduced a hybrid framework combining PINNs and Extreme Learning Machines (ELMs) to analyze the structural behavior of composite plates and shells. This framework employs domain decomposition and substructuring techniques, enabling efficient analysis of assemblies comprising multiple plates and shell elements with complex geometries. A critical component of this methodology is its ability to incorporate the Donnell shell theory, which governs the static and dynamic equilibrium of shell structures.

Figure 11 provides a geometrical representation of a cylindrical shell analyzed using this framework. The axes and vectors illustrate the orthogonal curvilinear coordinate system aligned with the shell's mid-surface. This configuration enables accurate modeling of stresses, displacements, and deformation patterns under various loading scenarios. By integrating governing equations and boundary conditions directly into the PINN framework, the approach minimizes computational overhead and improves predictive accuracy. The hybrid PINN–ELM method demonstrates significant potential in aerospace and marine applications, particularly for optimizing lightweight composite designs while ensuring structural reliability.

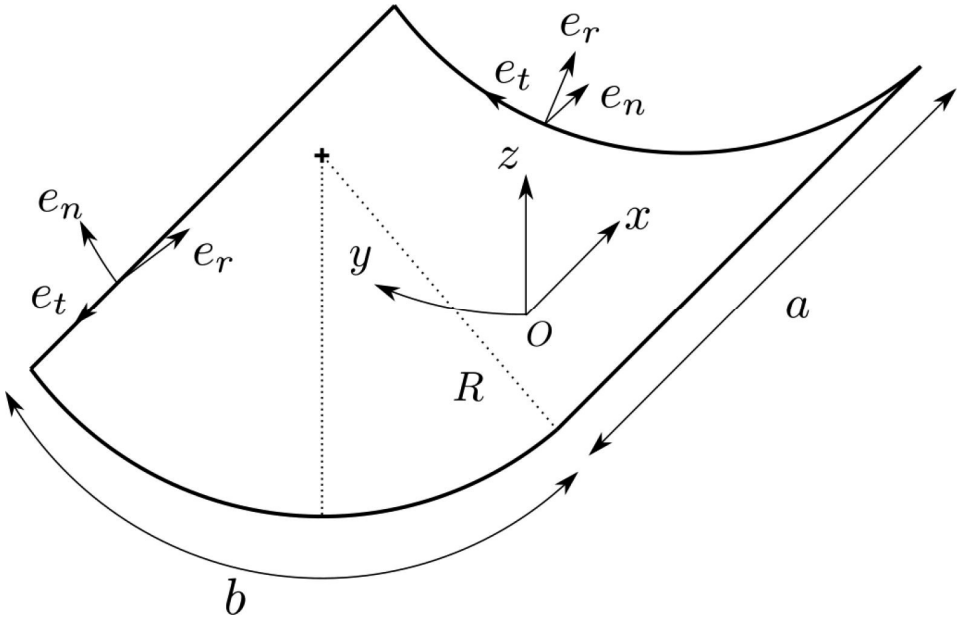


Figure 11. Geometrical representation of a cylindrical shell, facilitating accurate modeling of stresses, displacements, and deformation in structural analysis using the PINN framework [20].

Jan et al. [10] demonstrated the power of PINNs for structural analysis in modeling the small-strain deformation of thin shells using Naghdi shell theory. The study showcased PINNs' unique ability to address geometrically non-Euclidean domains, such as curved shells, without requiring traditional mesh setups. By directly integrating physical laws, the approach overcame common challenges in structural analysis, such as locking phenomena, which often hinder FEM solutions for thin shells. Validation against several benchmark problems highlighted the computational efficiency and accuracy of PINNs. For the hyperbolic paraboloid with partial clamping, the PINN framework accurately predicted displacement fields, achieving convergence with FEM solutions under complex boundary conditions. In the Scordelis-Lo roof benchmark, the PINN effectively resolved displacement fields, achieving a displacement value of -0.297, closely matching the reference solution of -0.3024 and avoiding locking issues common in FEM. Additionally, the framework modeled a fully clamped hemisphere under a concentrated load, replicating FEM predictions across varying geometries, showcasing its robustness in handling stringent boundary conditions. These case studies emphasize the ability of PINNs to deliver accurate and computationally efficient solutions for structural analysis in scenarios where traditional methods face limitations due to meshing complexities, high computational costs, or challenging boundary conditions.

Shen et al. [51] extended the application of PINNs to laminated composite plates under thermal loads. By leveraging the meshless capabilities of PINNs, the study achieved significant reductions in preprocessing and demonstrated flexibility in managing complex

boundary conditions. The framework was validated against the State Space Finite Element Method (SSFEM), delivering comparable accuracy in predicting temperature distributions and thermoelastic responses. Notably, PINNs excelled in computational efficiency for problems involving intricate stress and deformation patterns across composite layers, highlighting their potential for thermoelastic modeling while minimizing reliance on computationally intensive mesh-based techniques.

Challenges in applying PINNs to the structural analysis of composite plates and shells persist despite their significant advantages. One primary limitation lies in the integration of highly complex governing equations, such as Donnell and Naghdi shell theories, directly into the loss functions of the neural network. While PINNs offer a meshless and efficient alternative, ensuring convergence and accuracy for intricate geometries, multi-domain assemblies, and coupled phenomena remains computationally intensive. The effectiveness of PINNs often depends on the availability of high-quality training data or well-posed boundary conditions, particularly for problems involving highly localized stress fields, nonlinear behaviors, or defects. Although PINNs reduce reliance on large datasets by embedding physical laws into their formulation, accurate and high-resolution data can significantly improve model performance in real-world applications. This is particularly relevant for cases involving anisotropic materials or multi-physics interactions, such as thermal stresses in laminated composites, where sparse or noisy data can hinder predictive accuracy. Another challenge is the accurate modeling of nonlinear material behaviors and multi-physics interactions, such as thermal stresses in laminated composites, which require advanced formulations and tuning of hyperparameters. Moreover, while PINNs reduce the reliance on meshing, they sometimes struggle to capture fine-scale details in highly anisotropic or layered composites, especially under dynamic or multi-scale loading conditions. Addressing these limitations will require advancements in hybrid methodologies, improved optimization algorithms, and adaptive frameworks that balance computational efficiency with predictive robustness, paving the way for more reliable applications in aerospace, marine, and structural engineering.

5.2. SHM of Laminated Composites

SHM is a critical aspect of ensuring the safety, durability, and reliability of laminated composite structures, particularly in aerospace, automotive, and marine applications [77,78]. Traditional SHM methods often rely on extensive sensor networks and computationally intensive simulations, which can be time-consuming and limited in scalability. PINNs offer a novel approach by integrating data-driven learning with governing physical laws, providing an efficient, accurate, and interpretable framework for monitoring damage in laminated composites.

Xu et al. [24] developed a hybrid framework for SHM of laminated composites that combines PINNs with guided Lamb wave analysis, as illustrated in Figure 12. The framework begins with a Physical Module, where piezoelectric sensors mounted on CFRP laminates capture Lamb wave signals. These guided waves, sensitive to defects such as delamination and matrix cracking, propagate through the composite structure and serve as diagnostic inputs. Stiffness degradation models and damage indices are incorporated to quantify the structural integrity, allowing for the characterization of damage mechanisms. In the Feature Extraction Module, advanced time–frequency analysis and automatic feature extraction techniques are applied to the Lamb wave signals. This step identifies critical patterns and distortions in the waveforms caused by structural defects. These features are then processed through the PINN framework, which incorporates physics-based constraints, such as wave propagation equations, directly into its loss function. This ensures that the model adheres to fundamental physical principles, enabling accurate predictions even with

limited datasets. The Diagnosis Module integrates the extracted features with a hybrid physics-data-driven approach. This approach leverages physics-based loss functions to combine observed features and physical constraints, enabling the PINN to diagnose damage effectively. This approach enables the PINN to diagnose the type, location, and severity of defects with high accuracy. Notably, the framework achieved an accuracy improvement from less than 80% to over 90% in detecting damage conditions across different CFRP layups. The physics-guided approach ensures better generalization to unseen structural configurations, enhancing real-time damage detection and localization capabilities.

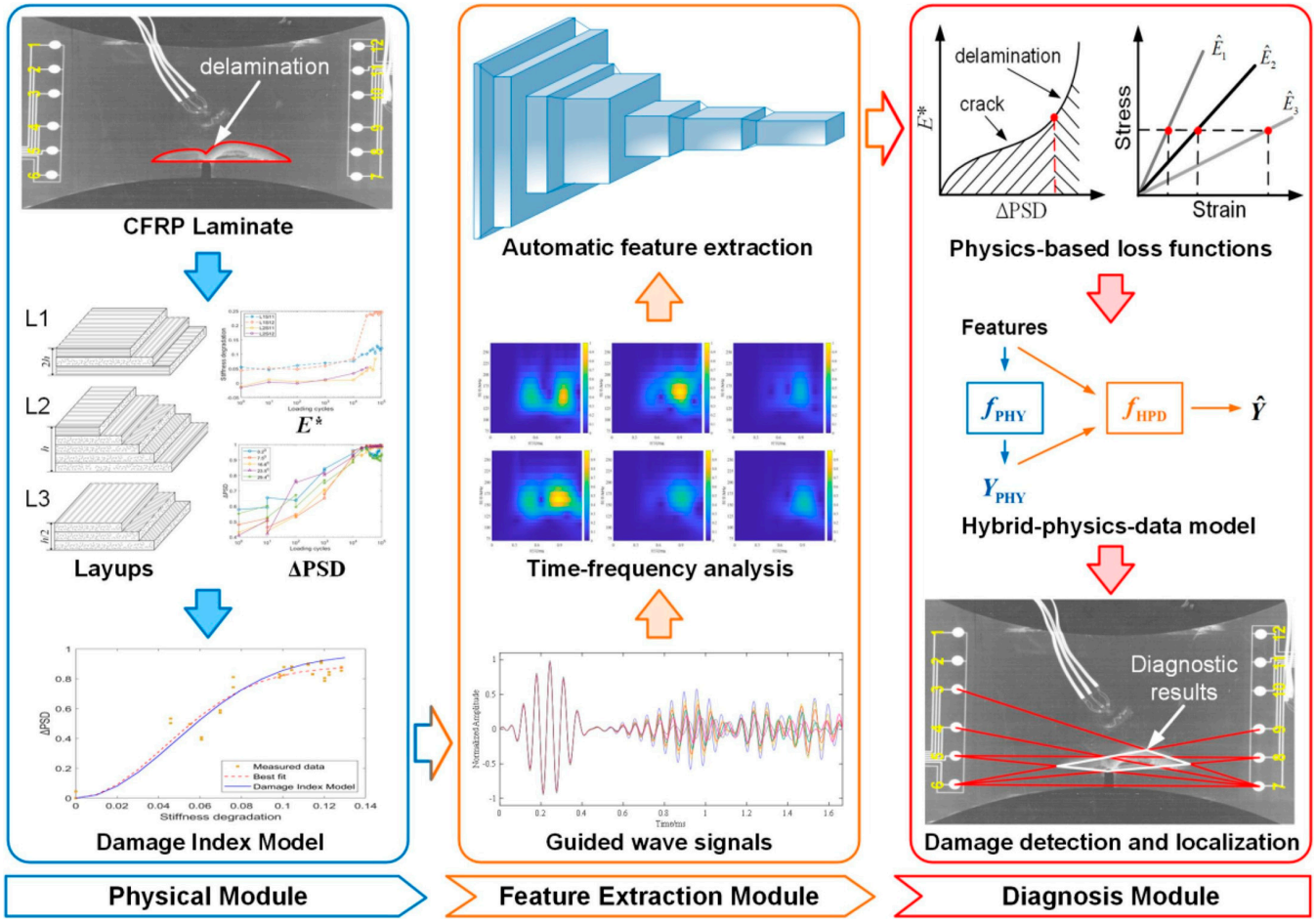


Figure 12. Hybrid framework for SHM of laminated composites using PINNs [24].

Liu et al. [79] advanced the application of PINNs in SHM by proposing a hybrid PINN framework that combines physics-based constraints with traditional data-driven methods for analyzing dynamic behaviors in aerospace components. By embedding governing equations, boundary conditions, and physical constraints into the learning process, the framework achieved 98% accuracy in predicting stress distributions and vibration responses in laminated composites. This represents a significant improvement over purely data-driven methods while ensuring adherence to physical principles. Additionally, the framework reduced computational costs by up to 40% compared to traditional FEM-based approaches. The hybrid PINN's meshless nature further eliminates the need for complex preprocessing, enhancing its suitability for real-time SHM applications in aerospace systems, where accuracy and efficiency are paramount. Despite challenges like hyperparameter tuning and dependency on high-quality training data, Liu et al.'s work underscores the utility of PINNs for ensuring the structural integrity of aerospace systems while enhancing computational efficiency.

Kalimullah et al. [64] explored the use of multi-fidelity PINNs (mfPINNs) with transfer learning for acoustic emission (AE) source localization in anisotropic composites. By integrating low-fidelity physics-based models with high-fidelity experimental data, the framework effectively addressed challenges posed by noisy and sparse datasets. A probabilistic framework was incorporated to quantify uncertainties in Time-of-Flight (TOF) measurements, providing confidence intervals for AE localization. Validated on CFRP panels, the mfPINN framework demonstrated superior accuracy, achieving an average localization error of 0.5075 cm and a maximum error of 0.8062 cm, compared to 10.3217 cm and 19.9423 cm, respectively, for conventional data-driven ANN models. Additionally, the low-fidelity PINN achieved errors of 3.5529 cm (average) and 7.8926 cm (maximum), highlighting the incremental accuracy improvements achieved by mfPINNs. This represents a 95% reduction in average localization error over traditional data-driven approaches. Furthermore, the use of transfer learning significantly reduced computational costs, making mfPINNs highly suitable for real-time SHM applications and precise damage localization in aerospace and marine composites.

Despite their transformative potential, the application of PINNs in SHM of laminated composites faces several challenges. While PINNs reduce reliance on large datasets by embedding governing physics into their loss functions, the quality of available data becomes critical in applications like defect localization and stress analysis. Real-world SHM scenarios often involve noisy, sparse, or incomplete data, which can hinder the accuracy of PINN predictions when fine-scale features, such as delamination or matrix cracking, are significant. To address this, multi-fidelity approaches, such as those integrating low-fidelity physics-based models with high-fidelity experimental data, have been proposed to address this issue, but they require careful calibration and balancing of model complexity. Another critical challenge is the computational expense of hyperparameter tuning, which can significantly affect model performance and robustness, particularly in dynamic and real-time SHM applications. Additionally, the dependence on advanced numerical techniques to integrate physics-based constraints, such as wave propagation or damage evolution equations, into PINNs adds complexity to model development and implementation. Addressing these challenges requires further advancements in hybrid frameworks, transfer learning, and uncertainty quantification to ensure that PINNs remain computationally efficient, accurate, and adaptable to diverse operational conditions in SHM applications.

5.3. Stress-Strain and Failure Analysis in Composites

The stress-strain behavior and failure mechanisms of laminated composites are critical to ensuring the structural integrity and reliability of modern engineering applications, including aerospace, automotive, and marine industries [80]. Traditional approaches, such as FEM, while effective, often face significant challenges in handling complex geometries, nonlinear material behaviors, and localized phenomena such as delamination, crack propagation, and interlaminar stresses [81–83]. PINNs have emerged as a revolutionary computational framework to address these limitations. By embedding governing physical laws, constitutive relationships, and failure criteria directly into their neural network architecture, PINNs enable highly accurate and computationally efficient modeling of stress-strain responses and damage progression. Their meshless nature and ability to integrate data-driven insights with physics-based constraints position PINNs as an innovative solution for tackling the multifaceted challenges of stress-strain and failure analysis in composites, paving the way for enhanced predictive accuracy and reduced computational costs.

Li et al. [84] introduced a Physics-Informed Neural Network (PINN) framework for the static analysis of two-sided supported, 2-ply laminated glass panes. This PINN

model integrates governing physical equations into its learning architecture, enabling the efficient prediction of stress-strain distributions and structural behaviors under various loading conditions. Unlike traditional methods that rely heavily on iterative finite element simulations or large datasets, the PINN framework eliminates such requirements by embedding physics-based principles like PDEs and boundary conditions directly into the loss function. The methodology demonstrated exceptional computational efficiency and accuracy in evaluating composite action, temperature sensitivity, and uncertainty in glass failure, significantly reducing computational time. For instance, its capability in uncertainty evaluation via Monte Carlo Simulations achieved a 48-fold improvement in computational speed compared to traditional methods, such as finite difference models. This is particularly beneficial for addressing challenges in brittle failure mechanisms and viscoelasticity of laminated glass, which are crucial for their design and structural integrity. Figure 13 illustrates the comparison of deflection profiles for laminated glass panes under various support and loading conditions. The PINN framework effectively aligns with FEM simulations, showcasing deformation contours and structural responses under applied loads, further highlighting its accuracy and applicability in structural and failure analysis of laminated glass systems. Abouali et al. [65] introduced a PINN framework to efficiently characterize and calibrate damage-plasticity constitutive models in laminated composites. Leveraging Digital Image Correlation (DIC) datasets for displacement fields, the framework integrates damage mechanics principles to predict stress-strain responses and damage evolution under various loading conditions. Validated on quasi-isotropic laminates, the model demonstrated its capability in capturing delamination and irreversible deformation, significantly reducing computational costs compared to traditional FEM while maintaining high accuracy. Despite challenges in addressing highly nonlinear material behaviors and scaling, this study highlights the practical applications of PINNs for damage modeling in composite structures.

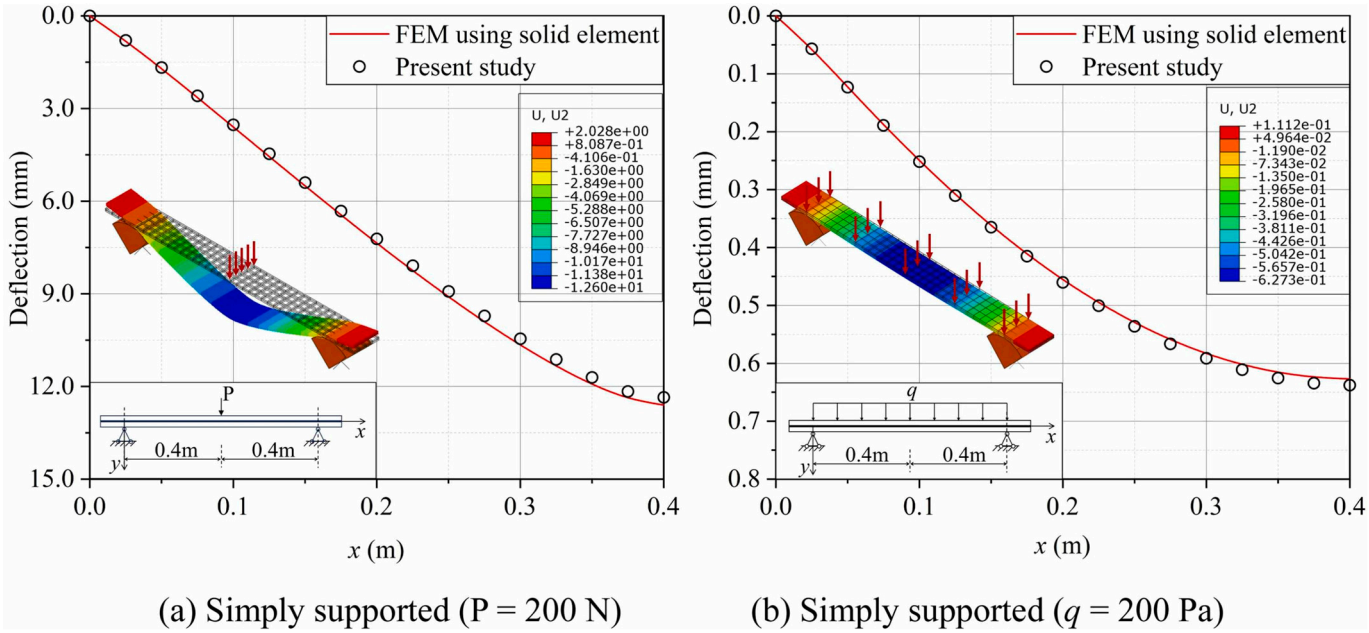


Figure 13. Comparison of deflection profiles for 2-ply laminated glass panes under simply supported conditions: (**a**) Point load (P = 200 N) and (**b**) Uniform distributed load (q = 200 Pa), illustrating the agreement between FEM results and the present PINN framework [84].

Hamed Bolandi et al. [85] proposed the PINN–Stress model for dynamic stress analysis under extreme conditions, such as earthquakes and explosions. Incorporating PDEs

into the loss function, the model predicted von Mises stress and other dynamic stress distributions with remarkable accuracy and efficiency. This approach eliminated the need for extensive meshing while achieving real-time analysis of stress patterns across complex geometries. The PINN–Stress model demonstrated robust generalization capabilities for unseen load sequences and boundary conditions, underscoring its potential for real-world structural engineering applications. Wang et al. [86] developed a data-assisted PINN (DA-PINN) framework for predicting fretting fatigue lifetimes in laminated composites. By integrating the Findley parameter (FP) as a damage indicator, the framework employed a two-step approach: using ANN models for FP predictions and incorporating FP into the PINN framework for fatigue lifetime analysis. This hybrid method significantly improved precision and reduced computational costs, outperforming traditional ANN models and providing a robust tool for fretting fatigue analysis in composite materials. Hu et al. [38] developed a Physics-Informed Neural Network (PINN) framework to enhance stress-strain and failure analysis in laminated composites, focusing on constitutive modeling and damage prediction. The methodology integrates governing equations, such as linear elasticity, hyperelasticity, and elastoplasticity, directly into the PINN's loss function, ensuring physically consistent predictions of stress-strain relationships. The framework also incorporates boundary conditions and material parameters, enabling accurate modeling of multi-axial loading conditions. The application of this PINN framework is particularly significant for predicting critical phenomena such as delamination, crack propagation, and fatigue life in laminated composites. The meshless nature of the framework allows for the coupling of mechanical and thermal effects, which is essential for comprehensive failure analysis under operational stresses. Ziqi Li et al. [87] presented a mechanics-informed machine learning model to capture elastoplastic behavior in unidirectional fiber-reinforced composites. By decomposing stress and strain into mechanics-based components and integrating them into neural networks, the study reduced data requirements and computational costs. Using direct numerical simulations (DNS) for validation, the model demonstrated high accuracy in predicting elastoplastic responses under diverse loading conditions, offering a powerful alternative for stress-strain modeling in anisotropic composites. Pinho et al. [88] developed a Physics-Informed Neural Network (PINN) framework for predicting focused on predicting stress distributions in composite structures with open holes, accounting for finite-width effects. By embedding analytical solutions into the neural network as input features, the study achieved higher accuracy with fewer training datasets compared to traditional heuristic scaling methods. Validated under uniaxial and biaxial loading conditions, the model excelled in global–local submodeling contexts common in airframe designs, reducing computational time six-fold compared to FEM. This study highlights the scalability and efficiency of PINNs for complex stress analyses in structural applications.

In summary, the application of PINNs in stress-strain and failure analysis of laminated composites demonstrates significant advancements in computational efficiency, accuracy, and adaptability. By embedding constitutive laws, governing equations, and material behaviors directly into their loss functions, PINNs ensure physically consistent predictions while effectively modeling critical phenomena such as interlaminar stresses, delamination, and crack propagation. However, challenges remain in handling highly nonlinear material behaviors, scaling to complex systems, and balancing computational demands with data requirements. The integration of physics-informed frameworks with advanced machine learning techniques, such as data-assisted models and transfer learning, highlights the potential of PINNs to revolutionize composite material analysis. These frameworks offer robust tools for addressing real-world engineering challenges, particularly in applications where computational efficiency and precise failure prediction are crucial, such as in aerospace and structural engineering domains. Moving forward, further research is needed

to overcome scalability challenges and enhance the generalization capabilities of PINNs for diverse composite material systems.

5.4. Multi-Scale Modeling of Composites

Multi-scale modeling of laminated composites represents a pivotal approach for capturing the intricate interactions between microstructural features and macroscopic behaviors [89,90]. Traditional multi-scale techniques, such as the FEM and the generalized method of cells, often encounter significant challenges, including high computational demands and difficulties in addressing discontinuities or complex geometries. PINNs emerge as a transformative alternative by embedding governing physical equations directly into the neural network architecture. This integration facilitates efficient and accurate multi-scale analysis without requiring extensive datasets or meshing procedures. PINNs are particularly adept at linking macroscopic homogenization theories with microscopic material descriptions, encompassing stress-strain responses, damage mechanics, and thermal properties.

Linghu et al. [91] introduced a Higher-Order Multi-Scale Physics-Informed Neural Network (HOMS–PINN) framework to analyze the elastic properties of composite materials with high-contrast features. This innovative method bridges macroscopic homogenization solutions and microscopic unit cell (UC) functions within the PINN framework, leveraging transfer learning to accelerate convergence and precisely capture microscopic oscillations in materials with high contrasts. Validated on multi-inclusion and three-dimensional composites, this approach demonstrated exceptional computational efficiency and accuracy. While the article lacks direct experimental validation, the robustness of the HOMS–PINN framework in replicating simulated microstructural behavior suggests its strong potential for capturing complex material responses in engineering applications. Figure 14 illustrates the HOMS–PINN framework, emphasizing its ability to connect macroscopic and microscopic scales, making it highly relevant for applications in aerospace and mechanical engineering.

Borkowski et al. [92] proposed a recurrent neural network (RNN)-based surrogate model enhanced with physics-informed constraints to simulate the nonlinear behavior of woven ceramic matrix composites (CMCs) across multiple scales. By utilizing the Multi-scale Generalized Method of Cells (MSGMC) for generating training data, the model accurately predicted stress-strain responses under various microstructural and loading conditions. The incorporation of physics-informed constraints, such as maintaining the positive semi-definiteness of the tangent stiffness matrix, minimized data requirements while enhancing predictive robustness. This framework provides an efficient solution for large-scale simulations of complex composite systems. Jiang et al. [93] developed a PINN framework tailored for multi-scale modeling of laminated composites, addressing the challenge of seamlessly linking micro- and macro-scale behaviors. Their approach integrates periodic homogenization theory into the PINN architecture, embedding governing equations like the two-scale expansion for temperature and stress fields. This facilitates accurate predictions of thermal and mechanical properties across scales without relying on mesh-based discretization. The methodology effectively modeled thermal conductivity and stress distributions in porous and fiber-reinforced composites, showcasing adaptability to irregular microstructures through transfer learning. It achieved high computational efficiency and fidelity, making it an indispensable tool for diverse engineering applications.

The application of PINNs in multi-scale modeling offers several significant advantages, including reduced computational costs, the ability to handle complex geometries and discontinuities, and seamless integration of micro- and macro-scale behaviors. These methodologies leverage advanced machine learning techniques like transfer learning and physics-informed constraints to ensure accurate predictions while minimizing the need for extensive datasets. However, challenges persist, such as tuning hyperparameters for

complex material behaviors, managing the trade-off between computational efficiency and model fidelity, and ensuring robustness under varying operational conditions. Future developments should focus on addressing these limitations to unlock the full potential of PINNs in multi-scale composite modeling.

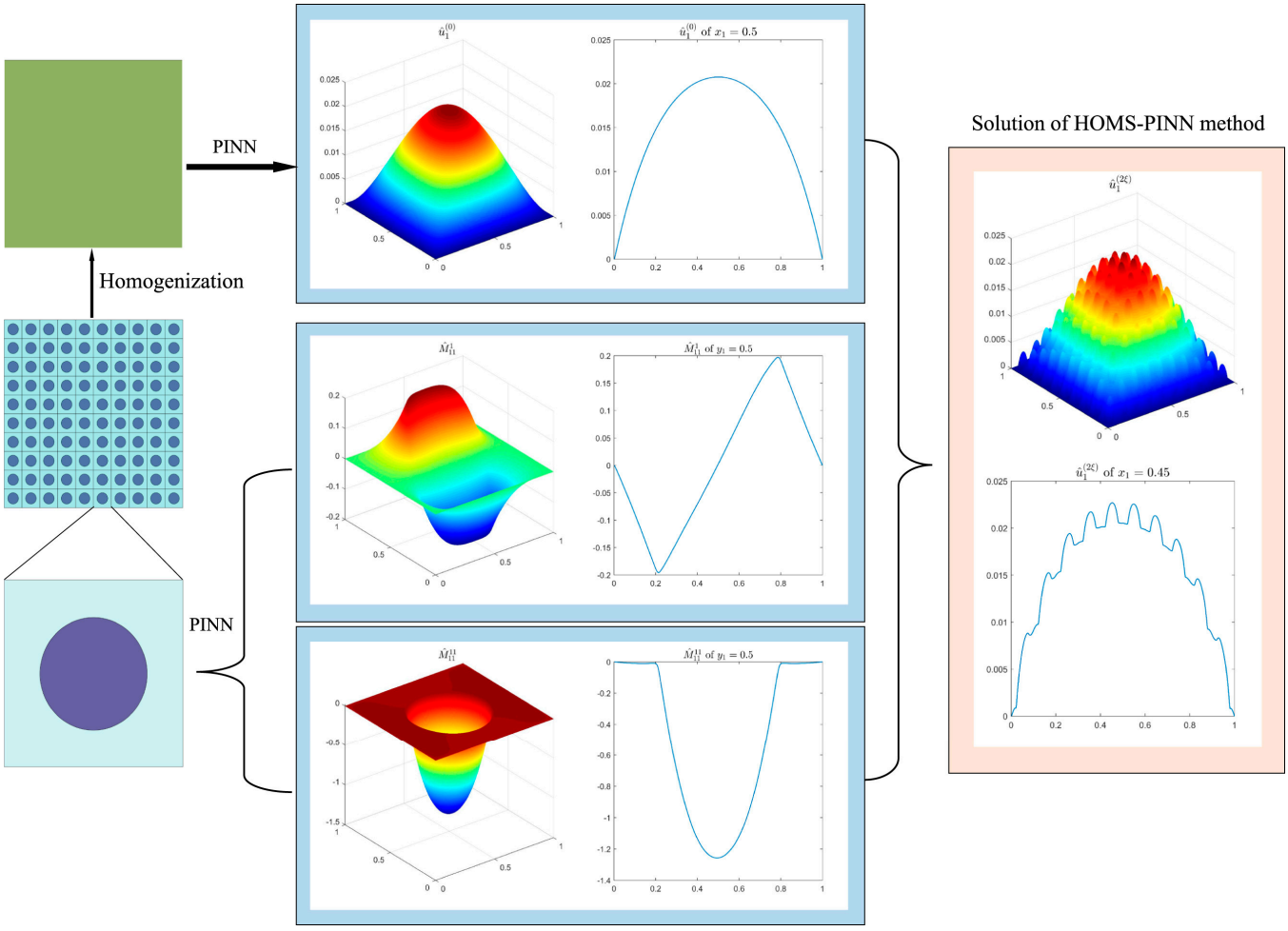


Figure 14. Illustration of the HOMS–PINN framework for composite material analysis [91].

Table 3 provides a comprehensive summary of the diverse applications of PINNs in the field of composite analysis. The table categorizes these applications into four major areas: structural analysis of plates and shells, SHM, stress-strain and failure analysis, and multi-scale modeling.

Table 3. Overview of PINNs applications in composite analysis.

Application	Key Contribution	Accuracy	Computational Efficiency	Challenges	References
Structural Analysis	Solves direct problems in linear elasticity for plates and shells using PINNs combined with ELM.	Error: L2 < 0.01 (deflection), RMSE~0.0039%	Training time: ~0.9 s (up to 4× faster than FEM).	Hyperparameter tuning; sensitivity to gradients.	[20]
SHM with Lamb Wave PINNs	Detects delamination and matrix cracking using guided Lamb wave analysis.	Detection accuracy: >90% across CFRP layups	Improved training efficiency with limited datasets; reduced computational costs.	Dependency on high-quality data; handling waveform noise.	[24]

Table 3. *Cont.*

Application	Key Contribution	Accuracy	Computational Efficiency	Challenges	References
SHM with Multi-Fidelity PINNs	Localizes acoustic emission (AE) sources in anisotropic composites using multi-fidelity models.	Average localization error: 0.5075 cm; maximum error: 0.8062 cm	Training time reduced to 10,000 epochs with transfer learning; significant improvement over ANN (10.32 cm).	Sparse datasets; noise handling; complexity of multi-fidelity integration.	[64]
SHM with Hybrid PINNs	Analyzes dynamic behaviors such as stress distributions and vibration responses in aerospace components.	Accuracy improved to 98% compared to purely data-driven methods.	Computational cost reduced by up to 40%; faster training convergence.	Requires careful tuning of hyperparameters and quality data.	[79]
Stress-Strain Analysis	Predicts deflections and stress distributions in laminated glass panes.	Relative error: <5% compared to FEM and experimental results.	Computational time reduced from 6+ h (FEM) to a few minutes (~48× speed-up).	Handling temperature effects and uncertainty in complex systems.	[84]
Multi-Scale Modeling	Captures sharp oscillatory behavior and homogenization in multi-scale composites using HOMS–PINNs.	Accuracy error: 1–3% compared to DNS–FEM benchmarks.	Computational time reduced by 50% compared to traditional PINNs.	Handling high-contrast properties and discontinuous solutions.	[91]

6. Challenges, Limitations and Future Directions

PINNs have demonstrated transformative potential in analyzing laminated composites, offering significant advantages in terms of computational efficiency, accuracy, and adaptability. However, their application in real-world scenarios is not without challenges. This section outlines the key obstacles and proposes future directions to address these limitations, as summarized in the accompanying table.

PINNs face several critical challenges when applied to laminated composites. Data quality and scarcity are prominent issues, as access to high-quality, diverse datasets is often limited. Real-world scenarios frequently involve noisy, sparse, or incomplete data, which complicates model training and prediction accuracy. Additionally, PINNs are computationally demanding, especially when handling complex loss functions or scaling to large, multi-scale problems. Incorporating complex physics, such as coupled thermal, mechanical, and chemical behaviors, adds another layer of difficulty, particularly when addressing nonlinear or anisotropic material properties. Scalability and generalization remain significant hurdles, particularly for real-time applications in SHM and multi-physics scenarios. PINNs struggle to adapt to heterogeneous materials and diverse geometries without sacrificing accuracy. Optimization challenges, including hyperparameter tuning and the risk of overfitting on small datasets, further hinder their robustness. Another key limitation is the lack of robust uncertainty quantification in PINN predictions, which reduces their reliability in operational conditions. Moreover, the integration of multi-physics capabilities and the scalability of current computational methods for complex systems are still evolving.

Despite their promising capabilities, PINNs also face several limitations that hinder their broader adoption in laminated composites. Sensitivity to hyperparameters, such as learning rates, network architectures, and loss weighting, is particularly problematic when addressing the multi-scale and anisotropic behaviors common in composites using PINNs [94]. This sensitivity often necessitates computationally expensive tuning processes, which are impractical for large or highly heterogeneous systems. Additionally, convergence issues arise when solving problems with steep gradients, such as the sharp transitions at interfaces between distinct material phases in laminated composites, where PINNs may fail to capture these discontinuities accurately [30]. The requirement for smooth approximations of material properties further compounds these challenges, potentially introducing errors when modeling real-world composite systems with abrupt property variations. PINNs also rely heavily on dense residual point sampling to accurately resolve the fine-scale features of composites, which can significantly increase computational costs, especially for problems involving layered geometries or coupled multi-physics phenomena.

These limitations emphasize the need for further methodological advancements tailored specifically to the complex behaviors of laminated composites.

To overcome these challenges and limitations, several future directions are proposed. The integration of hybrid models combining PINNs with traditional numerical methods or data-driven approaches can address data scarcity and improve prediction reliability. Synthetic data generation techniques can supplement real-world datasets, enhancing model robustness. Employing adaptive optimization algorithms and dimensionality reduction techniques can mitigate computational demands and accelerate training times. PINNs must expand to handle coupled multi-physics phenomena through modular and tailored architectures. Transfer learning and pre-trained models for common configurations can improve scalability and generalization, especially for SHM applications. Automated hyperparameter tuning and multi-fidelity frameworks can reduce the reliance on high-fidelity datasets while enhancing accuracy. Additionally, integrating probabilistic frameworks for uncertainty quantification will enable robust predictions in uncertain or dynamic environments. Looking further ahead, exploring quantum computing and quantum-inspired techniques could offer breakthroughs in scalability and efficiency for complex, multi-scale systems. These advancements hold the potential to unlock the full capabilities of PINNs, making them indispensable tools for real-world engineering challenges in laminated composites. Table 4 highlights the primary challenges and proposed future directions in applying PINNs to laminated composite.

Table 4. Challenges and future directions for PINN applications in laminated composites.

Aspect	Challenges	Future Directions
Data Quality and Scarcity	- Limited access to high-quality, diverse datasets [4] - Challenges in handling noisy, sparse, or incomplete data [93] - Difficulty ensuring adequate boundary conditions and constraints.	- Develop hybrid models combining PINNs with classical numerical methods or surrogate models to fill data gaps. - Generate high-fidelity synthetic datasets through physics-based simulations or generative models. - Create benchmark datasets specifically for laminated composites to standardize research efforts.
Computational Demands	- Training PINNs is computationally intensive due to complex loss functions [19] - Balancing efficiency with robustness in large-scale or multi-scale problems is difficult [91]	- Optimize loss function design by incorporating adaptive weights for better convergence. - Explore advanced computational techniques, like Fourier Neural Operators (FNOs) or randomized neural networks, to improve efficiency. - Benchmark computational times and accuracy for common applications to establish performance baselines.
Integration of Complex Physics	- Modeling multi-physics interactions (thermal, mechanical, chemical) requires advanced frameworks [26] - Incorporating nonlinear and anisotropic behaviors is challenging [20]	- Expand PINNs to handle coupled multi-physics phenomena with modular architectures. - Conduct theoretical studies to establish bounds on convergence for coupled physics scenarios. - Design tailored architectures for anisotropic and nonlinear behaviours, such as PINNs with custom activation functions or loss formulations.
Scalability and Generalization	- Adapting PINNs to heterogeneous materials and diverse geometries without losing accuracy is complex [88] - Real-time SHM demands scalability and dynamic adaptability [24]	- Develop pre-trained PINN libraries for common composite material configurations to reduce setup time. - Develop pretrained models for common laminated composite configurations. - Introduce meta-learning approaches to allow PINNs to learn generalizable patterns across multiple tasks.

Table 4. *Cont.*

Aspect	Challenges	Future Directions
Optimization Challenges	- Hyperparameter tuning (e.g., loss weights, learning rates) is tedious and requires trial and error [74] - Overfitting on small datasets remains a persistent issue.	- Automate hyperparameter tuning with techniques like Bayesian optimization or genetic algorithms. - Use multi-fidelity frameworks to enhance accuracy without requiring extensive high-fidelity data.
Uncertainty Quantification	- Lack of robust uncertainty quantification in PINN predictions limits their reliability in operational conditions [63]	- Embed Bayesian neural networks or Gaussian processes into PINN architectures for probabilistic predictions to quantify uncertainities. - Develop metrics and visualization tools to quantify and present uncertainties in predictions effectively.
Multi-Physics Capabilities	- Difficulties in integrating coupled phenomena (e.g., thermal-stress fields) [93]	- Expand multi-physics capabilities through modular and disjointed PINN frameworks. - Conduct validation studies using coupled experimental and simulation data to ensure realistic multi-physics predictions. - Build hierarchical PINN frameworks where each layer addresses a specific physical phenomenon but communicates with others.
Quantum Computing	- Current computational approaches limit scalability for complex multi-scale systems [43]	- Explore quantum-inspired classical algorithms to enhance PINN scalability for larger systems. - Investigate the potential of quantum neural networks (QNNs) in solving PINN-related optimization problems.

7. Conclusions

This review has highlighted the transformative potential of PINNs in advancing the analysis of laminated composites. By integrating physical laws directly into neural network architectures, PINNs bridge the gap between traditional numerical methods and data-driven approaches, offering significant advantages in computational efficiency, adaptability, and predictive accuracy. The exploration of various PINN methodologies, including hybrid, Theory-Constrained, k-space, and disjointed frameworks, demonstrates their versatility in addressing complex challenges such as nonlinear material behavior, multi-physics interactions, and multi-scale modeling.

Despite these advancements, challenges remain, including computational demands, data scarcity, and scalability issues for large-scale or heterogeneous systems. The integration of multi-physics capabilities and uncertainty quantification into PINNs is crucial for enhancing their reliability in real-world applications. Additionally, advancements in transfer learning, adaptive optimization techniques, and hybrid modeling approaches will be key to overcoming these limitations.

Future research should focus on developing more robust and scalable PINN frameworks capable of handling diverse operational conditions while exploring emerging technologies like quantum-inspired computing to further enhance their computational efficiency. By addressing these challenges, PINNs hold the promise of revolutionizing the modeling and analysis of laminated composites, enabling innovative solutions for applications in aerospace, automotive, marine, and structural engineering.

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