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EEG-Based Emotion Recognition with Combined Fuzzy Inference via Integrating Weighted Fuzzy Rule Inference and Interpolation

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Abstract: Emotions play a significant role in shaping psychological activities, behaviour, and interpersonal communication. Reflecting this importance, automated emotion classification has become a vital research area in artificial intelligence. Electroencephalogram (EEG)-based emotion recognition is particularly promising due to its high temporal resolution and resistance to manipulation. This study introduces an advanced fuzzy inference algorithm for EEG data-driven emotion recognition, effectively addressing the ambiguity of emotional states. By combining adaptive fuzzy rule generation, feature evaluation, and weighted fuzzy rule interpolation, the proposed approach achieves accurate emotion classification while handling incomplete knowledge. Experimental results demonstrate that the integrated fuzzy system outperforms state-of-the-art techniques, offering improved recognition accuracy and robustness under uncertainty.

Keywords: EEG data; emotion recognition; fuzzy inference; fuzzy rule interpolation; fuzzy systems integration

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1. Introduction

Emotional states play a very important role in stimulating people's psychological activities and behaviours, as well as in promoting interpersonal communication. Research on emotion recognition methods is closely related to and mutually reinforcing with psychology, cognitive science, neuroscience, and computer science. Such work has broad application prospects and significant application value, including but not limited to building a more intelligent and humane human–computer interaction interface; helping to identify high-risk groups with emotional disorders by detecting patients' emotional states; detecting crime and anti-social behaviour; offering joyful environments for games and entertainment; monitoring emotions in teaching and learning scenarios, etc. [1]. It has increasingly become one of the cutting-edge topics in the field of artificial intelligence.

A key to facilitating automated emotion recognition is to exploit electroencephalograms (EEGs) [2]. This is because an EEG is a form of electronic signal obtained by directly measuring neuron activity. It has a high time resolution and is difficult to disguise as compared with other modalities (e.g., facial expression and voice), thereby providing reliable information for identifying emotional states. In addition, EEG collection equipment is

non-invasive to the subject and is generally of a low cost. EEG-based emotion recognition methods can be roughly divided into two categories. One type is the classification method that uses machine learning by extracting features [3–11]. Representative methods include LDA [12], SVM [5], MLP [6], KNN [13], and AdaBoost [10]. The other type is based on the end-to-end deep learning technology [14–22] that has attracted widespread attention in recent years, such as deep network models based on LSTM-RNN [17,22], DBN [19], CNN [20,21], and other structures.

A main challenge of emotion detection is the ambiguity of its domain representation. On the one hand, there are individual subjective differences in emotional expression, and it is difficult to accurately distinguish the expressions of different emotions [23]; on the other hand, there is also subjective uncertainty in the division of boundaries between different emotional states [24]. Deeply understanding and correctly characterising the ambiguity or vagueness of the emotions in a given problem domain are issues that emotion recognition methods must deal with.

To handle and recognise the vague or ambiguous expressions of emotion, there have been EEG-based methods based on fuzzy systems [23–28], and most of them belong to the first type of the above classification. For example, three-dimensional fuzzy tensors are used to extract the subjective knowledge of emotional features from EEG signals [23], and an adaptive neuro-fuzzy inference system [29] is employed for feature learning to classify positive and negative emotions. A fuzzy clustering emotion classification algorithm [25] on EEG signals is proposed using Fuzzy C-Means [30]. Another method to extract emotions from EEG signals exploiting an incremental neuro-fuzzy inference system is presented in [26]. The classical Mamdani fuzzy systems are also explored to identify positive–negative emotions [24]. This method using traditional fuzzy rule models can achieve rapid classification and can be applied to real-time emotion classification, but its scalability is low, a common problem of such techniques.

Despite the developments as represented by the above techniques, important issues remain to be resolved around emotion recognition under ambiguities. First, the vagueness of emotion is inadequately portrayed. Most existing methods can only identify two types of emotional states: negative and positive. Second, most methods fail to clearly distinguish the differences amongst the significance levels of the multi-dimensional features in constructing fuzzy systems for emotion classification, leading to inaccurate results being derived. Furthermore, emotional EEG signals typically exhibit high-dimensional small sample characteristics, and the fuzzy rules for fuzzy reasoning learned from EEG data may be incomplete and cannot completely cover the entire domain of the problem. This latter observation inspires the present research, as no EEG emotion recognition technique has been reported in the literature so far.

This paper presents an EEG emotion recognition algorithm based on fuzzy inference. By characterising the fuzziness and fuzzy relationships in emotional states, data-driven methods are utilised to adaptively learn and generate a fuzzy rule base for emotion classification. During the process of selecting high-dimensional EEG features, the importance of different features is quantitatively evaluated and computed as their respective weights. Through this, an attribute-weighted fuzzy inference mechanism is constructed to obtain more accurate classification results. In particular, fuzzy rule interpolation (FRI) technology [31] is utilised, resulting in the introduction of a comprehensive weighted fuzzy inference model. This forms the first contribution in the literature that applies FRI techniques to address the challenging EEG data-based emotion detection and classification problems.

Depending on whether the input observation matches the fuzzy rules, it is implemented with two weighted fuzzy inference mechanisms with complementary functions [32]: one performing classical pattern-matching style classification and the other inferring the

class outcomes via advanced fuzzy rule interpolation techniques. The efficacy of the overall recognition system is attained thanks to its allowance for fast switch between the two operational modes. It characterises imprecise emotional expressions and resolves the ambiguity of emotion category boundaries. In so doing, the proposed approach successfully deals with problems involving incomplete approximate knowledge under the uncertain environment faced in EEG emotion recognition modelling, effectively improving the recognition accuracy.

The rest of this paper is organised as follows. Section 2 describes the generic architecture for the approach introduced herein. Section 3 specifies the details for implementing a system that realises the proposed approach. Section 4 reports on the results of the experimental investigation from running the implemented system over real-world data. Finally, Section 5 summarises the work presented and points out directions for further research.

2. Framework

The proposed EEG emotion recognition system works based on the utilisation of fuzzy inference, addressing the following three research questions or objectives:

(a) How to use data-driven methods to depict the ambiguity of emotional states from EEG data, to adaptively learn to generate a fuzzy rule base for emotional state inference.
(b) How to quantitatively distinguish the importance of different attributes in multi-attribute emotion recognition problems to construct a more accurate weighted inference mechanism.
(c) How to efficiently select an appropriate weighted inference mechanism based on the incomplete knowledge that may be brought about by high-dimensional small-sample EEG data, to obtain accurate recognitions.

Figure 1 shows the general framework of the system design to accomplish the aforementioned three objectives. Each of the three core sub-systems is proposed to conduct the following operations:

- Adaptive generation algorithm of fuzzy rule bases based on EEG emotional characteristics: This sub-system learns from EEG emotional features to generate a fuzzy rule base in preparation for subsequent reasoning, focusing on solving the problem of accurately portraying emotional ambiguity. It includes the implementations of the components that use fuzzy clustering algorithms to divide the space of emotional semantic features in a fuzzy manner, that adaptively learn the number of fuzzy semantic values and fuzzy set representations used to describe different features, and that utilise data-driven methods to extract features and corresponding fuzzy sets.
- Attribute-weighted fuzzy inference mechanism: This sub-system constructs a fuzzy classifier based on fuzzy rules, focusing on solving the problem of inaccurate fuzzy inference based on multi-dimensional input. This includes devising algorithms for quantitatively evaluating the impact of different EEG features on emotional state decision-making, as well as computational methods for calculating weights corresponding to the features and rules, and then applying attribute-weighted fuzzy inference mechanisms.
- Comprehensive weighted fuzzy inference model: This sub-system deals with the high-dimensional small sample characteristics of EEG signals, focusing on solving the decision-making problem based on incomplete knowledge bases, while utilising generated fuzzy rule bases and the attribute-weighted fuzzy inference mechanism. This includes employing the selection of a suitable weighted inference mechanism based on the situation of input observations, and constructing an accurate and efficient weighted fuzzy inference model that works on such observations.

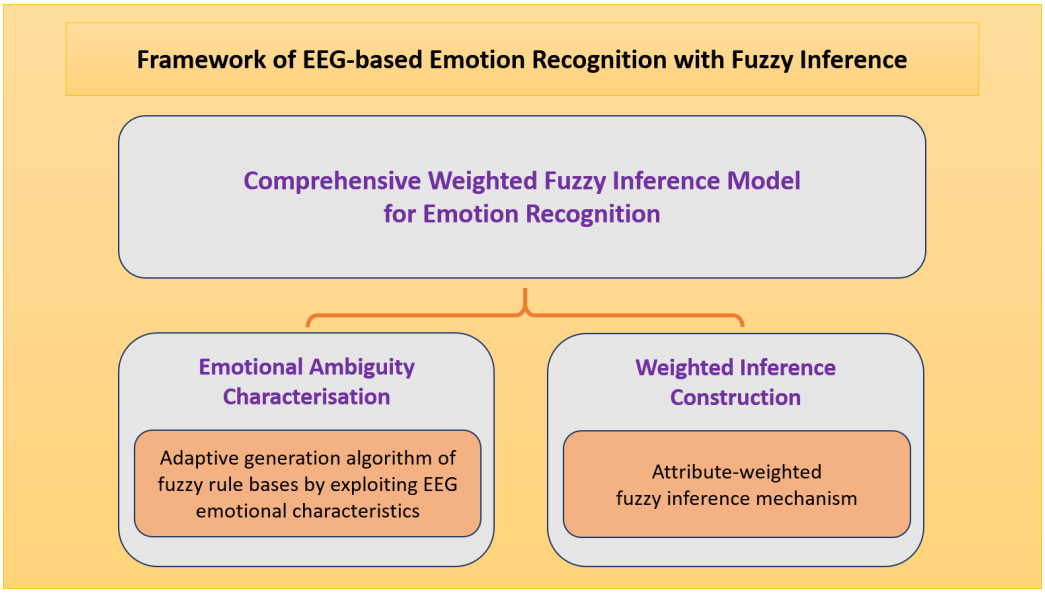


Figure 1. Framework of fuzzy inference system for EEG-based emotion recognition.

3. Implementation

The EEG emotion recognition approach outlined in Figure 1 is herein specified in terms of the computational process as shown in Figure 2. It includes the following modules: EEG emotional feature extraction, feature selection and weight evaluation, fuzzy rule base adaptive generation based on EEG features, attribute-weighted fuzzy inference, and comprehensive weighted fuzzy inference with rule interpolation. The implementational details of these modules are detailed as follows.

3.1. EEG Emotional Feature Extraction, Selection and Evaluation

3.1.1. EEG Emotional Feature Extraction

Amongst the large number of EEG features proposed in the literature for emotion recognition, features measured in the time domain, frequency domain, and time–frequency domain dominate in practical applications. Frequency domain features are particularly widely used. It is common recognition that extracting non-redundant features from different fields often leads to better classification results than using a single feature source. Therefore, based on the evaluation and analysis of various features in the literature, the following EEG emotional features are selected for use in the present work:

(a) Higher-Order Crossings (HOCs);
(b) Fractal Dimension (FD);
(c) Non-Stationary Index (NSI);
(d) Complexity of Hjorth Features;
(e) Fast Fourier Transform (FFT)-related characteristics.

The reasons for selecting these features are as follows: (1) These types of features are the most popular; their frequent use has been appraised for being capable of reflecting more representative characteristics of the EEG signals. (2) Features (a)–(d) are typical representatives of time domain ones, which are very important for EEG signals since time domain-based analysis can effectively describe the intuitive characteristics reflected by EEG signals. For example, NSI, as a measure of signal complexity, can capture changes in local average values over time. The higher the index value, the more inconsistent the average value. Also, HOC features reflect the oscillation patterns of the underlying EEG signals. (3) FFT-based features are extracted via frequency domain analysis, which are computed and analysed by estimating the power spectral density (PSD) to selectively represent the

EEG sample signals. (4) Generating time domain features generally has the strength of low computational complexity, while extracting FFT-based features also has high computational efficiency, so these features can be used in real-time applications.

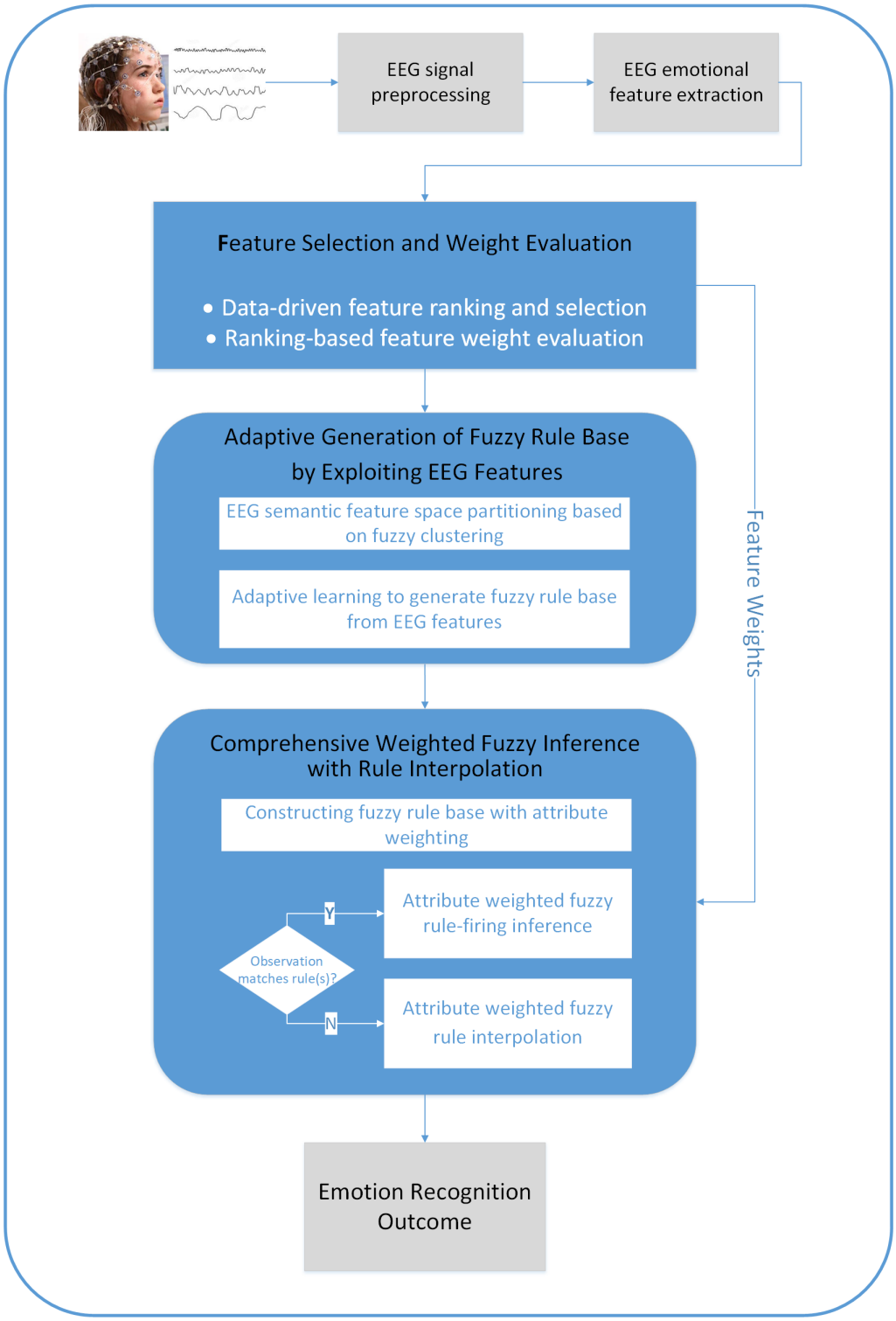


Figure 2. Flowchart of EEG emotion recognition via weighted fuzzy reasoning.

3.1.2. Feature Selection and Weight Evaluation

Features extracted from EEG data generally have high dimensions. High-dimensional features may not only contain information redundancy and cause unnecessary computation in subsequent application but also reduce the readability of rules and the interpretability of emotion recognition systems. Therefore, feature selection technology is herein utilised to process the original EEG features to reduce their dimensionality while retaining the physical meaning of the original features. In subsequent steps, dimension-reduced EEG features are directly used to learn to generate a fuzzy rule base, which is of great significance for maintaining the original meaning and readability of fuzzy rules.

In implementation, the popular Relief-F feature selection algorithm [33] is adopted to evaluate individual features, reduce feature dimensionality, and complete the weight evaluation of the selected features. That is, first, Relief-F is used to estimate the significance of each feature one by one, producing a set of scores corresponding to each feature. Then, features with higher scores are selected on the basis of a certain threshold. After this, the scores are utilised to construct a weight computation mechanism with respect to each individual feature.

Of course, in utilising Relief-F for the present work, the key is to obtain the weighting scheme over the features, supported with the setting of an appropriate threshold for selecting initial features. The weights are calculated as follows:

$$Weight_i = \frac{RS_i}{\sum_{t=1,\ldots,m} RS_t} \tag{1}$$

where RS_i is the ranking score of the selected feature i, and m is the total number of features being selected.

3.2. Adaptive Generation of Fuzzy Rule Base by Exploiting EEG Features

The training data used to learn the fuzzy rule base consist of EEG features and corresponding emotion categories that have been extracted and selected. Without losing generality, suppose that there are m EEG features $a_1, a_2, \ldots, a_m$ and denote the output variable, i.e., emotion category as $class_e$. Then, the outcome of this module can be generally expressed as a rule base R, consisting of if–then fuzzy production rules, $R = \{r^1, r^2, \ldots, r^n\}$:

$$\begin{aligned} r^i \ &: if\ a_1\ is\ A_1^i\ and\ a_2\ is\ A_2^i\ and\ \cdots\ and\ a_m\ is\ A_m^i, \\ &\ then\ class_e\ is\ B^i \end{aligned} \tag{2}$$

where A_j^i represents the fuzzy value taken by feature a_j, and B^i represents the fuzzy value of the rule consequent, indicating a certain emotion category.

The basic process of adaptively generating a fuzzy rule base involves the following: First, each feature or emotional category attribute is fuzzified in their underlying value space. This leads to the fuzzy values required to describe each feature and the corresponding fuzzy emotion category. It provides a powerful mechanism to capture and reflect the inherent ambiguity of emotions. Figure 3 shows an example of fuzzifying the emotional state $class_e$, where the emotion expressions are divided into multiple categories each with a certain degree of membership. Such a vague characterisation of the emotional domain is more in line with real-world situations than Boolean division, as, often, emotion is not strictly black and white in between two adjacent classes.

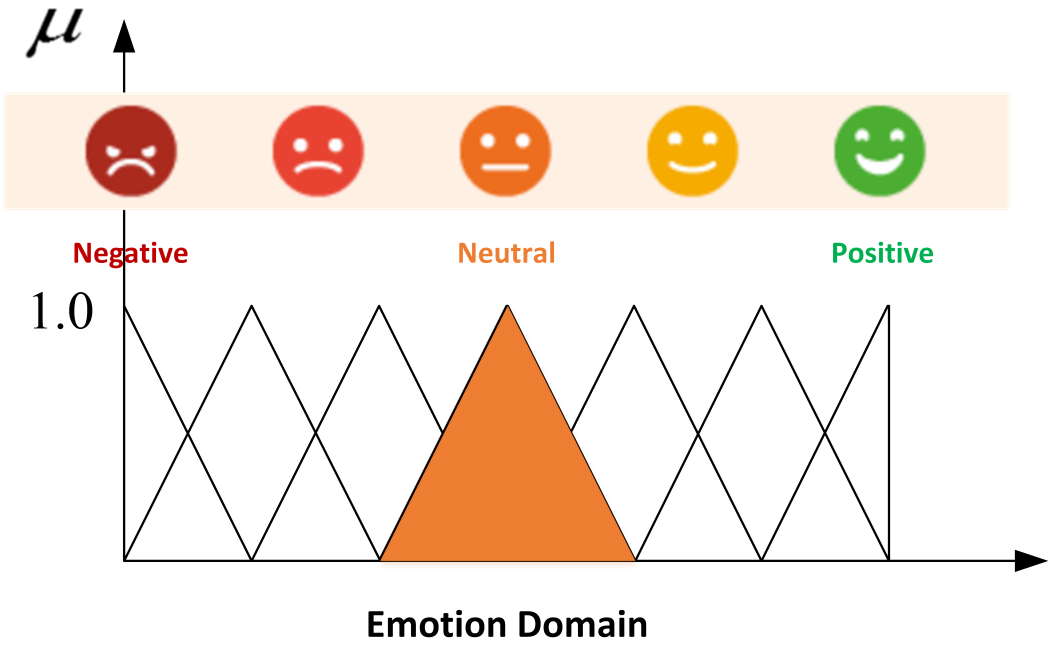


Figure 3. Example of fuzzy partition of emotional dimension.

To fully explore the data distribution embedded in the original EEG signals, data-driven fuzzy clustering is utilised herein. Particularly, clustering results learned from the data help reveal the membership distributions of the attributes, from which a set of continuously distributed semantic labels (such as "Small, . . . , Medium, . . . , Large") can be introduced according to the numerical size of specific features. In so doing, the resulting fuzzification of the underlying feature space is learned using the distribution characteristics of the data itself. This can not only accurately reflect the nature of data distribution but also avoid the manual division of qualitative feature values. Meanwhile, the number and distribution of learned membership functions can be adjusted according to changes in the dataset. As such, this approach has a stronger generalisation ability than fixedly used fuzzy sets and hardwired partition by human (that would assume the availability of significant domain expertise).

Based on what is learned from fuzzy clustering, in terms of fuzzy feature values and their corresponding fuzzy categories, a fuzzy rule base can be generated, following the process as illustrated in Figure 4. Running through this process using EEG data results in a working emotion classification rule base.

In the process of constructing a required fuzzy rule base, as shown in Figure 4, fuzzy rules are created from the grid of fuzzy region space (FRS) using a simple maximal data-hitting procedure. As indicated above, a threshold is used to determine whether the hypergrid with the highest number of hits can form a rule and therefore be added into the emerging rule base. If such a number is larger than the threshold, a rule is extracted from this hypergrid. The rule antecedent values returned by one iteration are those fuzzy values associated with the corresponding hypergrid. The rule consequent adopts the fuzzy value which corresponds to the one at which the instances have the highest number of hits. Note that the setting of the threshold used in this process is to consider the situations where the lower data-hitting rate may well be due to the presence of noise in the EEG signals. Those rules generated by such data can be deemed to be not sufficiently general to cover the relevant grids within the FRS, and therefore, the respective otherwise learned rules are not retained.

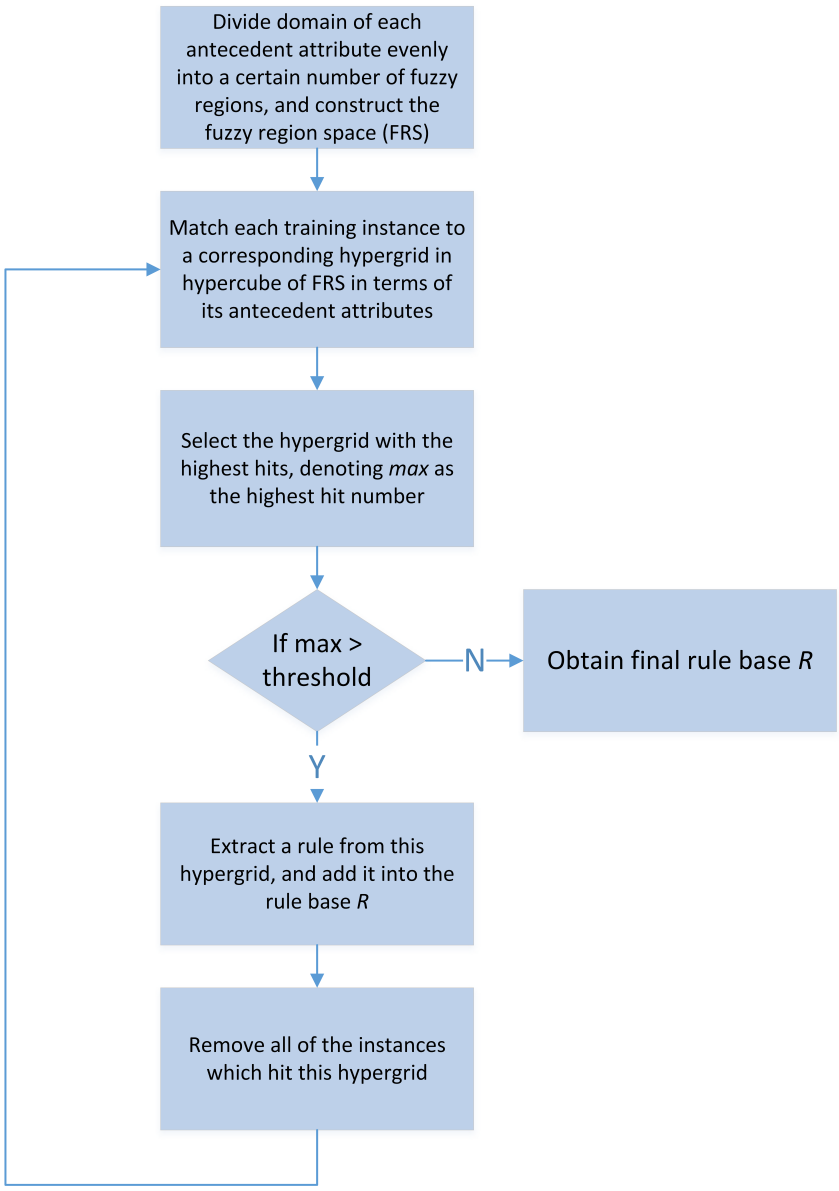


Figure 4. Emotional rule base generation method via fuzzy space partitioning.

3.3. Feature-Weighted Fuzzy Inference

Given the fuzzy rule base learned from EEG emotional data, this subsection focusses on solving the feature-weighted inference problem of multi-dimensional inputs, developing an accurate fuzzy inference mechanism that handles fuzzified EEG signals.

In general, a fuzzy inference system works by pattern matching a given observation against the conditions set in the rules within a rule base to derive or decide on the consequent. The observations are typically represented in a similar format to the premise of the rules. Without losing generality, an observation o^* is hereafter denoted in the following form:

$$o^* : a_1 \text{ is } A_1^* \text{ and } a_2 \text{ is } A_2^* \text{ and } \cdots \text{ and } a_m \text{ is } A_m^* \tag{3}$$

where A_j^* denotes the fuzzy value of feature a_j. For consistency in the use of notations, the value of the consequent attribute $class_e$ that needs to be inferred, corresponding to the observation value, is denoted by B^*.

To reduce the potential accuracy loss caused by simply treating multiple different features equally, it is necessary to introduce the weight of features to reinforce reasoning. Recall that such weights can be obtained by using Relief-F to assign scores to individual

raw features directly extracted from EEG data. Denote such weights as $AW_j, j \in 1, \ldots, m$. Now, the key question is how to make justified use of the resulting weights to perform weighted fuzzy inference.

In traditional fuzzy rule-based reasoning systems, fuzzy rules are usually described by Equation (2), where the conditional features and consequent category are not given any weight, with all features treated equally. Now that different features have been assigned corresponding weights, the missing information in the original rules can be supplemented, so the form of expression of fuzzy rules must be redefined. Fortunately, this can be straightforwardly introduced by generalising the rule base previously learned to become one that contains the following weighted fuzzy rules:

$$\begin{aligned} \tilde{r^i} \ &: if\ a_1\ is\ AW_1 A_1^i\ and\ a_2\ is\ AW_2 A_2^i\ and\ \cdots\ and\ a_p\ is\ AW_p A_p^i, \\ &\quad then\ class_e\ is\ D_i B^i \end{aligned} \tag{4}$$

where n is the cardinality of the rule base (weighted or not), m is the number of all possible features considered in the problem case, and $p \leq m$, expressing that a different number p of conditional features may appear in a given rule. Different rules may involve a different number of premise features, although the maximum number of conditional features appearing in any rule can only be m, obviously.

Note that in Equation (4), D_i is the weight introduced to the consequent category to balance the influence caused by the assignments of weights to the premise features within each rule. In this work, D_i is defined as follows, which has a logical and intuitive appeal in terms of how the modified rule conditions are contributing towards the consequent across all rules in the modified rule base:

$$D_i = \frac{\sum\limits_{\substack{j=1, \\ a_j^i\ in\ \tilde{r}^i}}^{m} AW_j}{\sum\limits_{\substack{j=1, \\ a_j^i\ in\ \tilde{r}^i}}^{m} AW_j^2} \tag{5}$$

The purpose of assignment of such a weight to the consequent part of each rule is to balance against the weights added to the antecedent part so that each weighted rule (in the form of Equation (4)) will consistently degenerate to its original (in the form of Equation (2)) if the weights are removed (or not created). Note that whilst this work utilises the present definition for specifying D_i, alternative forms may be introduced. Indeed, other alternatives can be found in the relevant research work as per [32].

Given the rule base where the features appearing in each rule are weighted, $\tilde{R} = \{\tilde{r^1}, \tilde{r^2}, \ldots, \tilde{r^n}\}$, classical compositional rule of inference [34] can be applied to establish a mapping from the fuzzified input feature values onto a fuzzy value of the consequent category such that $F : \mathscr{F}(A^*) \rightarrow \mathscr{F}(class_e)$ so that $class_e = F(A^*) = A^* \circ \tilde{R}$, where $\circ : \mathscr{F}(A^*) \times \mathscr{F}(A^* \times class_e) \rightarrow \mathscr{F}(class_e)$ represents a mathematical interpretation of multi-dimensional fuzzy relations between the (weighted) conditional features and the (fuzzy-valued) consequent.

As with the previous introduction of the corresponding weights for each feature in the fuzzy rule base, these weights are also needed as a guide for weighted inference in performing weight-reinforced fuzzy reasoning. Therefore, the corresponding characteristics of the observation should be given the same weight as the properties of the premise features involved within the rule. From this, the reinforced inference procedure is then implemented using the following:

$$B^*(class_e) = (AW_1A_1^* \wedge AW_2A_2^* \wedge \cdots \wedge AW_pA_p^*) \circ \tilde{R} = \bigvee_{\{a_1,a_2,\ldots,a_p\} \in A} AW_1A_1^*(a_1) \wedge AW_2A_2^*(a_2) \wedge \cdots \wedge AW_pA_p^*(a_p) \wedge \tilde{R} \quad (6)$$

where the derivation of the consequent category value B^* is obtained by computing the conjunctive combination of the weighted observation and those rules within the rule base that are instantiated by the very same observation. In particular, the rule base $\tilde{R} = \{\tilde{r^1}, \tilde{r^2}, \ldots, \tilde{r^n}\}$ can be depicted by the union of individual fuzzy relations held between the premise feature values and their corresponding consequent fuzzy category value such that

$$\tilde{R} = \bigvee_{i=1}^{n} AW_1A_1^i(a_1) \wedge AW_2A_2^i(a_2) \wedge \cdots \wedge AW_pA_p^i(a_p) \wedge D_iB^i(class_e) \quad (7)$$

From the above, the logical expression of feature-weighted fuzzy inference can be attained, reflecting the fuzzy extension of classical logic-based hypothetical reasoning. That is, given the general premise as expressed by the rule conditions and the specific condition as stated by the observation, the consequent value can be computed by the application of the compositional operation over them.

In realisation of this computational method, in this work, logical conjunctions and implications are both implemented with the minimum operation, whilst logical disjunction is implemented by the maximum operation, over the fuzzy values involved. Importantly, however, the feasibility of the entire input–output mapping process as described above assumes that the data-driven learned rule base is dense, completely covering the problem space. Otherwise, certain observations may fail to find any matching rules to fire (that is, no rules are instantiated), which is the situation that will be dealt with using a different reasoning approach. This is explained next.

3.4. Comprehensive Weighted Fuzzy Inference with Rule Interpolation

3.4.1. Justification for Integrated Approach

The feature-weighted fuzzy inference mechanism as described above can obtain accurate conclusions once a novel observation matches certain rules within the rule base. Nonetheless, due to the high-dimensional small-sample characteristics of emotional EEG signals, the fuzzy rules learned from EEG data may be incomplete and cannot completely cover the entire universe of discourse for the given problem. Thus, the pattern-matching based approach collapses. Unfortunately, this is a quite common case for emotion recognition, especially when handling novel situations where the data available for learning are limited. To overcome this important limitation, a comprehensive weighted fuzzy inference model is proposed here.

The basic idea is that when a new observation o^* is present, in deducing the consequent through the application of the fuzzy rule base, $\tilde{R} = \{\tilde{r^1}, \tilde{r^2}, \ldots, \tilde{r^n}\}$, a choice is first made between two different reasoning mechanisms. The selection of which mechanism to apply is determined by the coverage level of the rule base over the observation. If the rule base is complete, being capable to cover the entire universe of discourse, any new observation can find at least one fuzzy rule to fire. Yet, if the rule base is incomplete, typically referred to as being sparse in the literature, certain novel observations cannot be matched with any existing rules in the rule base. The aforementioned compositional rule of inference works for the former case. For the latter case, where a sparse rule base is present, for those observations which match no rules, fuzzy rule interpolation [35,36] is

then employed to perform the required computational inference, yielding an approximate consequent outcome.

To obtain more accurate inference results, similar to pattern-matching based techniques, weighting factors associated with different features need to be introduced in the fuzzy rule interpolation mechanism. In the recent literature, a variety of weighted interpolation reasoning methods have been reported. Amongst them is the weighted interpolation algorithm based on attribute score ranking [37], which has, since its inception, gained significant attention. This algorithm not only effectively improves the reasoning accuracy because of the introduction of weight information on the rule attributes into the rule interpolation process but also makes the weighted interpolation reasoning significantly more efficient. The efficiency is gained by requiring only the minimum number (i.e., two) of the fuzzy rules to achieve optimal interpolative results for many real-world problems.

3.4.2. Outline of Weighted Fuzzy Rule Interpolation

In cognition of its efficacy as well as its popularity in the literature, the aforementioned weighted fuzzy rule interpolation method is herein adopted to deal with the inference problems when the observed values cannot match any existing rules. To be self-contained, a summary of this method is given in Algorithm 1. More details can be referred to in [37].

Algorithm 1 Attribute weighted scale and move transformation-based FRI.

Input: • Sparse rule base $R = \{r^1, \ldots, r^N\}$ consisting of N rules in form of Equation (2);
- Observation $o^* = \{A_1^*, \ldots, A_m^*\}$, over m conditional attributes;
- Number of closest rules n;
- Attribute weights $AW = (AW_1, \ldots, AW_m)$.

Output: • Interpolated consequent B^*

– Step 1: Closest Rules Selection Guided by Attribute Weights:

1: **for** $i = 1$ to $i = N$ **do**

2: Calculate weighted distance $\tilde{d}(o^*, r^i, AW)$ between o^* and r^i, such that

$$\tilde{d}(o^*, r^i, AW) = \frac{1}{\sqrt{\sum_{t=1}^{m}(1 - AW_t)^2}} \sqrt{\sum_{j=1}^{m} \Big((1 - AW_j) d(A_j^i, A_j^*) \Big)^2}$$

3: **end for**

4: Select n rules of shortest distance(s);

– Step 2: Intermediate Rule (r') Construction Guided by Attribute Weights:

5: Obtain weights $w_j^i, i = 1, \ldots, n, j = 1, \ldots, m$, as computed by original T-FRI to jth antecedent attribute of ith selected rule;

6: Compute antecedent attribute values of intermediate rule $A_j', j = 1, 2, \ldots, m$, by linearly aggregating corresponding weighted antecedent values over selected n rules using normalised weights $\hat{w}_j^i$;

7: Calculate weight $\tilde{w}_z^i$ for each consequent per selected closest rule by accumulating the normalised weights contributed by $\hat{w}_j^i$ such that

$$\tilde{w}_z^i = \sum_{j=1}^{m} AW_j \hat{w}_j^i$$

8: Construct fuzzy set B' for the consequent attribute of intermediate rule by aggregating the consequent values of n closest rules $B^i, i = 1, \ldots, n$, which are respectively weighted by $\tilde{w}_z^i$;

Algorithm 1 *Cont.*

– Step 3: Scale and Move Factors Calculation:

for each rule antecedent attribute **do**

Obtain scale rate s_{A_j} that modifies A'_j into $\hat{A}'_j$ such that it maintains same scale as corresponding component in o^*;

Obtain move ratio m_{A_j} that modifies $\hat{A}'_j$ for it to maintain the same position as the corresponding component in o^*;

end for

– Step 4: Scale and Move Transformations with Weighted Factors:

Calculate the overall transformation factors for B' to ensure analogy, by aggregating the corresponding weighted scale and move factors such that

$$\tilde{s}_z = \sum_{j=1}^{m} AW_j s_{A_j} \qquad \tilde{m}_z = \sum_{j=1}^{m} AW_j m_{A_j}$$

Compute the final interpolated outcome B^* by applying scale and move factors to B' such that $B^* = T(B', \tilde{s}_z, \tilde{m}_z)$;

Return B^*

3.4.3. W-Infer-Polation for EEG-Based Emotion Recognition

This study intends to organically combine the two weighted fuzzy inference mechanisms, the compositional rule of inference and fuzzy rule interpolation, to provide an effective approach to emotion recognition. It proposes a comprehensive weighted fuzzy inference model to cope with the recognition problem in response to two different types of novel observed values, or under two different matching conditions.

The specific implementation is illustrated in the last module on the bottom of Figure 2. This model realises rapid switching between two weighted fuzzy inference modes based on whether the input observations match the fuzzy rules. It exploits the functional complementarity of the two modes, not only organically integrating the two types of fuzzy inference mechanism but also utilising feature-weighted inference methods respectively appropriate under different circumstances, in order to obtain more accurate and efficient classification results.

In judging whether a new observation matches any rule, the firing strength between the two is calculated, and a certain threshold is preset to determine whether they match or not. The calculation of the rule firing strength requires the participation of each premise feature value and the respective observed feature value per rule. Therefore, for EEG emotion recognition, the selection of an appropriate threshold to judge the matching degree or firing strength is of significance. This needs to be considered in conjunction with the feature weighting scheme and the calculation method of the firing strength.

To accurately determine which feature-weighted inference method to use for a given observation, the following mechanism is employed in the present study, unless otherwise stated. The decision of whether an input observation matching any rule(s) or not is made by checking the weighted firing strength. Thus, the process of rule-firing inference is herein recalled to mathematically determine the formation of the matching process.

Given the observation as presented in the form of (3), the input data are treated as crisp values without fuzzification; such data are denoted as singleton fuzzy sets to retain real-world semantics in the application of EEG-based emotion recognition.

Suppose that a multi-dimensional real-valued input observation is provided such that $[A_1^*, A_2^*, \ldots, A_m^*] = [a_{10}, a_{20}, \ldots, a_{m0}]$ ($a_{j0} \in \mathbb{R}, j = 1, 2, \ldots, m$), which is defined by the use of the following singleton fuzzy sets:

$$A_j^*(a_j) = \begin{cases} 1 & if \quad a_j = a_{j0} \\ 0 & if \quad a_j \neq a_{j0} \end{cases} \quad j = 1, 2, \ldots, m \tag{8}$$

This leads to the inference outcome $B^*(class_e)$ as per Equation (6) being computed as follows:

$$\begin{aligned} B^*(class_e) &= \tilde{R}(\{a_{10}, a_{20}, \ldots, a_{p0}\}, class_e) \\ &= \bigvee_{i=1}^{n} AW_1 A_1^i(a_{10}) \wedge AW_2 A_2^i(a_{20}) \wedge \cdots \wedge AW_p A_p^i(a_{p0}) \wedge D_i B^i(class_e) \\ &= \bigvee_{i=1}^{n} \tilde{\alpha_i} \wedge D_i B^i(class_e) \end{aligned} \tag{9}$$

From this, the weighted firing strength of rule $\tilde{r^i}$ against observation A^* can be represented by the term

$$\tilde{\alpha_i} = AW_1 A_1^i(a_{10}) \wedge AW_2 A_2^i(a_{20}) \wedge \cdots \wedge AW_p A_p^i(a_{p0}) \tag{10}$$

For simplicity, the threshold ($\geq$0) can be set to zero to facilitate making a decision in choosing the fuzzy inference mechanisms. That is, if $\tilde{\alpha_i} > 0$ for at least one fuzzy rule $\tilde{r^i}, i \in \{1, 2, \ldots, n\}$ in the rule base $\tilde{R}$, then the feature-weighted fuzzy inference as illustrated in Section 3.3 is adopted to carry out the inference. If however, $\tilde{\alpha_i} \leq 0$ for any rules in the rule base, the weighted fuzzy interpolative reasoning as shown in Algorithm 1 is conducted.

4. Experimentation

This section systematically evaluates the proposed fuzzy inference model for achieving EEG-based emotion recognition, supported with comparisons against the state-of-the-art (SOTA) methods.

4.1. Experimental Setup

4.1.1. Database

The DEAP database [38] is a collection of EEG data from 32 healthy participants (16 males and 16 females). The participants in the experimental processes for collecting the data were in a healthy physical and mental states, and EEG signals were collected using a 32-lead electrode cap according to the international "10–20" system. Volunteers were asked to watch 40 music videos, each of which was 60 s in duration. EEG signals were collected at the same time from the subjects at a sampling frequency of 512 Hz. All participants were asked to rate different emotion dimensions (i.e., valence, arousal, dominance, and liking) on a scale of 1 to 9 after watching each music video. The database includes not only 32 EEG channels but also 16 other physiological signal channels, including common signals such as eye electricity and electrocardiogram. In order to make fair comparisons with other methods in the relevant literature, arousal and valence emotion dimensions are selected as the goals in this work for emotion recognition task.

4.1.2. Experimental Environment

In general, an FIS required to perform emotion recognition and classification involves five key modules as specified below:

- Fuzzification: Triangular membership functions are employed to represent the fuzzy sets involved because of their simplicity and popularity. For fair comparison, all antecedent variables are first normalised into the range of [0,1]. The partition of each antecedent attribute domain into triangular membership fuzzy values is achieved by approximating what is learned by the use of Fuzzy C-Means (FCM) [30]. The number of triangular membership functions (representing individual clusters, respectively) for each attribute tuned by FCM is set to six throughout the experimentation for easy comparison. This is to have a common ground for fair comparison. The consequent attributes (i.e., valence and arousal) are presumed to employ continuous rating values. Without relying upon any prior knowledge, these crisp variables are fuzzified using isosceles triangular fuzzy sets with each covering $1/5$ of the unit range (as commonly set in the relevant literature [31]).
- Fuzzy Rule Bases (each consisting of originally unweighted rules): The fuzzy rules that are original and that are not associated with any attribute weights are learned from the raw data, after fuzzification, using the classical method as described in [39]. Although any other advanced data-driven fuzzy rule induction algorithm (e.g., [40]) may be utilised as an alternative if preferred, herein, for consistency and fairness of performance comparison, neither the optimised selection of a learning method nor the optimisation of the learning algorithm taken is carried out.
- Inference Mechanisms: The core of the implemented FIS for achieving EEG-based emotion recognition is realised by the use of the comprehensive weighted fuzzy inference with rule interpolation (namely, *W-Infer-polation*) as presented in Section 3.4.
- Defuzzification: The classical defuzzification method that uses the centroid of the area of a fuzzy set is employed. When applied to a fuzzy set, it results in a defuzzified crisp value calculated by

$$z_0 = \frac{\int \mu_A(z) \cdot z \, dz}{\int \mu_A(z) \, dz} \tag{11}$$

  where $\mu_A(z)$ denotes the membership degree of z that belongs to the fuzzy set A. As with fuzzification, any other alternative defuzzifier may be utilised provided just one is used throughout for consistency. The one of Equation (11) is employed owing to its popularity in the literature and simplicity in computational implementation.

For the DEAP emotion database, the processed data existing in the literature by the original creators are utilised. These include 32×40 trials of EEG data, where each trial is collected from one particular subject being watching one music video. As shown in Figure 2, before extracting features from the EEG raw data, further preprocessing steps are required. First, the 3 s pretrial baseline is removed from each trial, and then the data are downsampled to 128 Hz. Also, a band-pass filter from 4 to 45 Hz is applied. As indicated above, the goals of emotion recognition in this work are depicted in two emotion dimensions (i.e., arousal and valence), both of which are rated on a continuous nine-point scale. For emotion classification, most works in the literature divide each dimension into high/low classes to conduct a binary classification by abruptly setting five as the threshold to project the continuous nine-point scale onto two discrete classes. This is rather artificial, making little sense, as the emotion domain is continuous and vague. In this work, the partition of emotion domain into the arousal and valence dimensions for generating two emotion categories is addressed in a fuzzy manner, which is of a natural appeal and will be elaborated next. Also different from most recent emotion recognition

methods that are based on deep learning, this work does not split trials for generating more and shorter segments to meet the demand of feeding massive data for training neural networks. Instead, it treats every complete trial of collected EEG data as one single instance being processed while extracting EEG features. The results are then used as the training or testing sample.

To minimise the potential influence of noise in judging the classification quality, the experimental results shown are those obtained by averaging the outcomes of 10-fold cross validation (CV). During the training phase, for each fold, the unweighted fuzzy rule base per classification problem is learned independently from the training samples. Also generated at this stage are the weights of individual rule attributes using the proposed weight learning scheme. In testing, the inference mechanism of the implemented FIS with *W-Infer-polation* is utilised to make inference for each testing sample, deriving an emotion category per the arousal and valence dimensions. Note that 10-fold CV is adopted trial-wise for the subject-dependent experiments. In other words, the training and test data are all from the same subject across subject-dependent experiments. The trial-wise shuffling ensures that the highly correlated segments within a trial do not appear in both the train and test data in any single CV fold. This is mainly to avoid potential data leakage issues caused by improper random shuffling in the subject-dependent experiments.

4.1.3. Performance Metrics

To comprehensively evaluate the classification performance, the average accuracies and F1 scores (which are popularly used in the literature of emotion recognition) of all 32 subjects are reported as the final evaluation metrics. These performance indices are computed as follows:

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN} \tag{12}$$

$$F1 = \frac{2 \times precision \times recall}{precision + recall} \tag{13}$$

where $precision = \frac{TP}{TP+FP}$ and $recall = \frac{TP}{TP+FN}$, with TP, FP, TN, and FN standing for the number of true positives, false positives, true negatives, and false negatives, respectively. Both evaluation metrics take values between 0 and 1, and higher values indicate better emotion recognition performance.

4.2. Results and Discussions

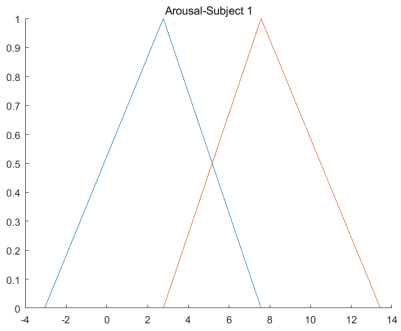
4.2.1. Fuzzification of Emotion Domain for Classification

The DEAP database allows continuous rating scores in the range of [1, 9] for both arousal and valence emotion dimensions. Hence, it is necessary to partition the emotion domain into discrete categories for the sake of emotion classification. As indicated previously, in the literature, most work divides each dimension into two classes (e.g., high/low valence or high/low arousal) by selecting five as the threshold to map the continuous nine-point scale into the two categories. Such implementation is mainly for simplicity to obtain a general emotion classification, but this may make less sense from the viewpoint of human cognition. Taking arousal for example, it is hard to distinguish the rating score of 4.99 as low arousal and that of 5.01 as high arousal from the human-being point of view. This is because the emotion domain is vague and the variation in emotion state is continuous; assigning discrete labels for emotion categories is, therefore, generally less reliable or accurate.

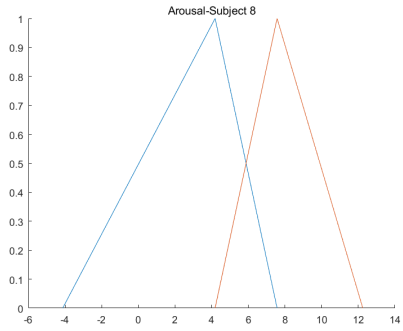
To reflect the above observation, this work partitions the arousal and valence emotion domains in a fuzzy manner. Whilst still two categories are constructed for each emotion

dimension, they are characterised by two fuzzy sets with soft boundaries between them (of a varying degree). The required fuzzy sets can be learned by fuzzy clustering techniques. Fuzzy C-Means (FCM) [30] is presently employed, which is one of the most widely used fuzzy clustering algorithms. It works by assigning a membership degree to each data sample corresponding to a certain cluster centre, based on the relative distance between the cluster centre and that sample. The closer to the cluster centre, the higher the membership degree to which the sample is deemed to belong to the corresponding cluster. Thus, the clustering outcome on a given emotion dimension reveals the distribution of the membership functions for the underlying emotion preference of one certain subject.

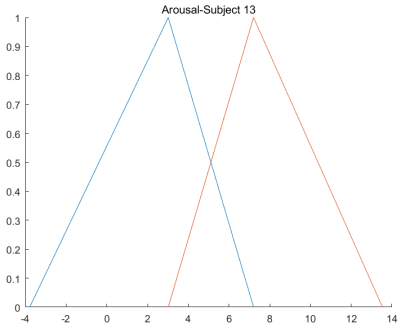
The fuzzy partitions of four (randomly selected) subjects (to save space) for the arousal and valence emotion dimensions are shown in Figure 5 and Figure 6, respectively. Triangular membership functions are employed due to their simplicity and popularity. In each subfigure, the horizontal ordinate presents the range of continuous rating scores. The left-hand side triangular fuzzy set represents the low category (be it arousal or valence), while the right-hand one represents the high class. These two fuzzy sets (each shown in a different colour) indicate the membership degree of one certain observation belonging to either corresponding emotion category (e.g., high/low valence or high/low arousal). Those classes with higher membership degrees are assigned to be the classification results while collectively calculating the final emotion recognition accuracy. As can be seen from these figures, different pattens of emotional fuzzy partition are obtained for different subjects. This is reasonable, as the emotional expression can vary for different human beings. Furthermore, from such a fuzzy partition, more information is attained other than just the classification result. That is, the membership degrees for each emotion category can be revealed, providing more precise knowledge over the emotional state.



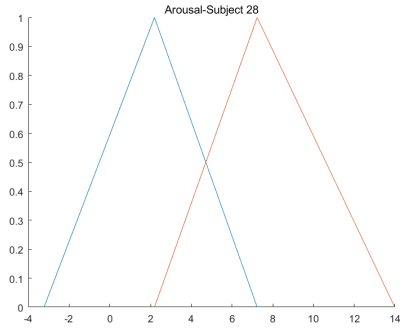
(**a**) Subject 1



(**b**) Subject 8

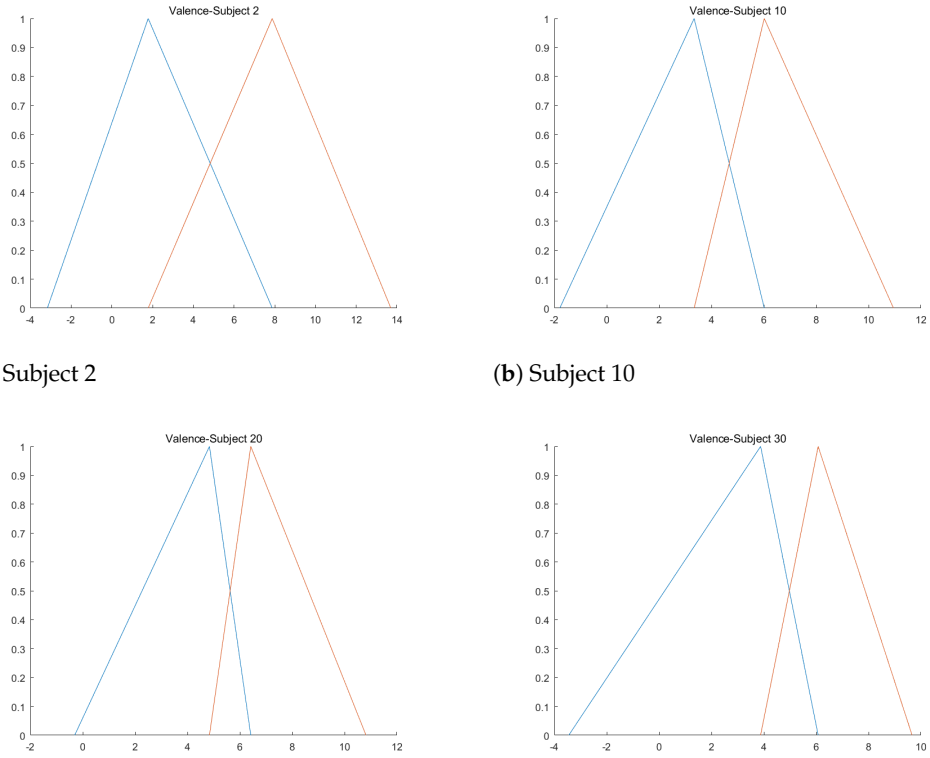


(**c**) Subject 13



(**d**) Subject 28

Figure 5. Fuzzy partitions of four subjects for arousal emotion dimension.



(**a**) Subject 2 (**b**) Subject 10

(**c**) Subject 20 (**d**) Subject 30

Figure 6. Fuzzy partitions of four subjects for valence emotion dimension.

4.2.2. Classification Performance over Individual Subjects

The experimental evaluation for emotion recognition is implemented in a subject-dependent way, presenting the classification performance over each individual subject. Figures 7 and 8 show the average accuracies and F1 scores obtained for each subject (out of 32 subjects in total) when the predicting emotion dimensions are arousal and valence, respectively.

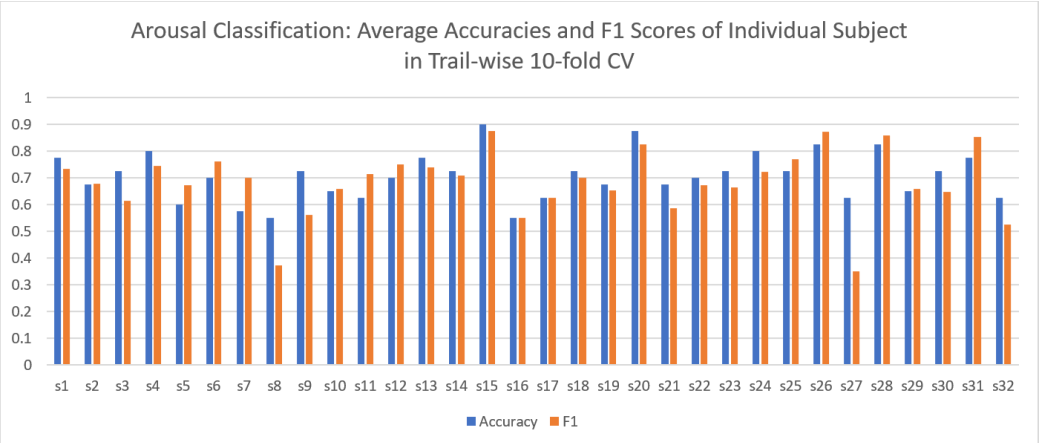


Figure 7. Classification performance over individual subject for arousal.

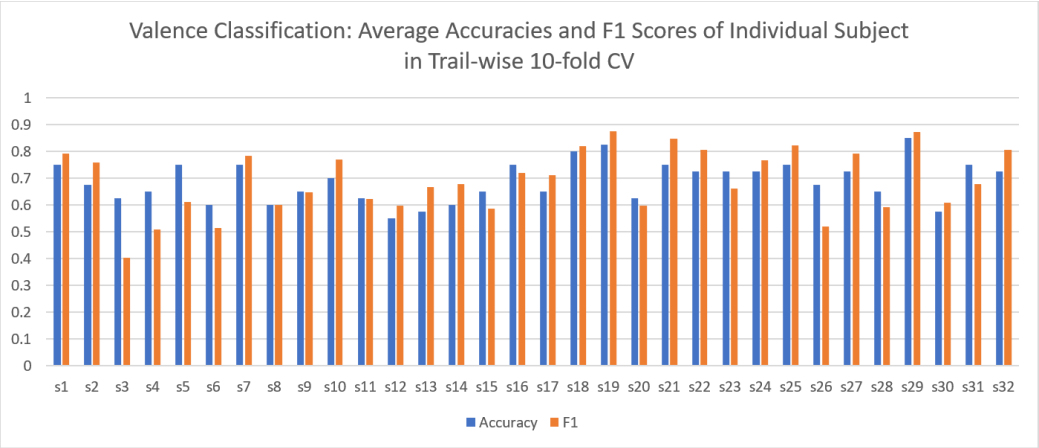


Figure 8. Classification performance over individual subject for valence.

Generally, the proposed FIS performs well across all tested subjects, with better results seen for the arousal emotion dimension. Also, variations over different subjects are detected in both dimensions, which can be readily explained since the emotional expression varies amongst different humans. More specifically, the overall performance (Average Accuracy/F1 $\pm$ Standard Deviation) of 32 subjects shown in the two figures are calculated as listed in Table 1. The fuzzy model exhibits slightly better performance in predicting the arousal dimension than in the valence dimension, regarding both metrics of accuracy and the F1 score. However, a lower F1 score is obtained compared to accuracy for arousal, while there is not much difference in the situation of the valence dimension. Such results may indicate that the problem of imbalanced data may be of a much more severe impact upon the classification along the arousal emotion dimension. This, in turn, implies that to have a better reliability for the practical application of this model, more in-depth fine-tuning on it may be required.

Table 1. Classification performance of 32 subjects over two emotion dimensions.

	Averaged Classification Performance (%)	
	Accuracy	**F1**
Arousal	70.70 $\pm$ 8.68	68.22 $\pm$ 12.12
Valence	68.83 $\pm$ 7.47	68.94 $\pm$ 11.76

4.2.3. Performance Comparison with SOTA Methods

The performance of the proposed fuzzy inference approach is compared with the state-of-the-art methods, in terms of achieving the binary classification on both arousal and valence emotion dimensions. The methods used for comparison are chiefly implemented with deep neural networks, which are very popular and have been widely studied recently in the domain of EEG-based emotion recognition. For fair comparison, all the baseline methods are implemented by the training codes provided in the respective original algorithmic description with the relevant parameter optimisation included. Also, all the methods are run under the same generalised evaluation settings, to avoid any leakage issue. That is, trial-wise 10-fold CV is used to ensure that a common ground is utilised for comparison.

Table 2 shows the classification results obtained by the different methods compared. Clearly, the FIS introduced in this work performs significantly better on classification accuracies, in both the arousal and valence emotion dimensions. In particular, along the two emotion dimensions, running the FIS results in 9.13% (70.70–61.57%) and 8.91% (68.83–59.92%) improvements when compared with the best SOTA methods, respectively. Although the F1 scores do not exceed the levels of accuracy in great amount, the proposed

approach outperforms the rest. Interestingly, the F1 scores are shown to be increased over the accuracies for most NN-based methods, while the opposite situation occurs for the proposed fuzzy model. As indicated previously, further work is required to address the problem of imbalanced data in order to achieve better reliability for the practical application of this model. Nevertheless, such an improvement is surprising, as those NN-based methods have frequently demonstrated of their remarkable successes in performing many pattern recognition tasks. Yet, the fuzzy method can offer an improvement over them in realising emotion classification.

Table 2. Performance comparison with SOTA methods over two emotion dimensions.

	Arousal		Valence	
Method	**Accuracy (%)**	**F1 (%)**	**Accuracy (%)**	**F1 (%)**
EEGNet [41]	58.29	60.60	54.56	57.61
DeepConvNet [42]	61.03	62.58	59.92	62.04
TSception [43]	61.57	63.24	59.14	62.33
LGGNet [44]	61.20	63.90	59.19	64.51
Proposed	70.70	68.22	68.83	68.94

As demonstrated previously, one of the key properties of the emotion domain is the vague expression of observations. This helps explain why the proposed fuzzy inference method is able to outperform the otherwise powerful neural network-based techniques, because fuzzy representation can better characterise such a property naturally. In addition, FIS also has the advantage in achieving EEG-based emotion recognition more efficiently, as, in general, a fuzzy system does not require massive data to train, whilst a relatively deep neural network does, to attain optimal performance. This potential of FIS is reinforced in the proposed comprehensive weighted fuzzy inference framework since it can deal with situations where knowledge is sparse and incomplete.

5. Conclusions

This paper has presented a comprehensive approach to integrating mechanisms for fuzzy compositional inference and fuzzy rule interpolation in order to perform EEG-based emotion recognition. Systematic experimental comparison has shown that the proposed approach outperforms popular and state-of-the-art classifiers. By observing the potential of what the implemented fuzzy system is able to perform when knowledge or domain data are imprecise and incomplete, it may be claimed that the efficiency of the fuzzy approach would also offer a significant improvement over neural network-based deep learning classifiers which are tailored to handling EEG data. However, such a general claim requires further investigations to verify, both empirically and theoretically. This forms an interesting piece of further research. Also, there are no obvious reasons why alternative fuzzification and defuzzification methods cannot be utilised for the implementation of the present approach, and similarly, different fuzzy rule induction algorithms may be employed to derive rules from raw EEG data. Such replacement of the implementational methods may further strengthen the overall efficacy of the present approach for EEG-based emotion classification. Last but not least, the utility of fuzzy representation has an advantage over neural network-based techniques, in support of producing explanations for the system's own reasoning outcomes. How the interpretability of fuzzy systems may be best exploited for EEG data analysis is an on-going research activity.

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Data Availability Statement: The data that support the findings of this study are openly available at http://www.eecs.qmul.ac.uk/mmv/datasets/deap/index.html (accessed on 1 December 2023), reference number [38].

Conflicts of Interest: The authors declare no conflicts of interest.

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