mathematics

MDPI

Article

Fuzzy Radar Evaluation Chart for Improving Machining Quality of Components

Kuen-Suan Chen [1,2,3], Chun-Min Yu [1], Jin-Shyong Lin [4,*], Tsun-Hung Huang [1] and Yun-Syuan Zhong [1]

[1] Department of Industrial Engineering and Management, National Chin-Yi University of Technology, Taichung 411030, Taiwan; kschen@ncut.edu.tw (K.-S.C.); march@ncut.edu.tw (C.-M.Y.); toby@ncut.edu.tw (T.-H.H.); asd576188@gmail.com (Y.-S.Z.)
[2] Department of Business Administration, Chaoyang University of Technology, Taichung 413310, Taiwan
[3] Department of Business Administration, Asia University, Taichung 413305, Taiwan
[4] Department of Mechanical Engineering, National Chin-Yi University of Technology, Taichung 411030, Taiwan
* Correspondence: linjsh@ncut.edu.tw

Abstract: Some studies have shown that any part machined by an outsourcer usually has several basic quality characteristics. When the outsourcer's process capabilities are insufficient, the defective rate of various quality characteristics of the product will increase, thereby raising the rework rate and scrap rate. As a result, maintenance costs will go up, economic value will decrease, and even carbon emissions can increase during the production process. In addition, the process capability index and the radar chart are widely used in engineering management and other fields. Since process indicators often contain unknown parameters, sample data are needed for evaluation. With the rapid development of the Internet of Things and big data analysis, many companies regard rapid response as a basic requirement for timeliness and cost consideration. Therefore, companies often have to evaluate the process quality of ten small samples and decide whether to make some improvements. In order to solve the above problems, this study proposed a fuzzy radar chart evaluation model for the process quality of multi-quality characteristic parts based on the process capability index. Using this model can help all parts manufacturers continue to improve the quality of their machined parts as well as reduce their rework and scrap rates. Meanwhile, carbon emissions can be lessened during the production process, and companies can fulfill their social responsibilities. This fuzzy radar chart evaluation model is based on confidence intervals. As the company's past experience is incorporated, the evaluation accuracy can be maintained even with a smaller sample size. Furthermore, the fuzzy radar evaluation chart can simultaneously evaluate the process capabilities of all quality characteristics of the part. In addition to making it easier for manufacturers to master all quality characteristics, quality process capability can also help them seize improvement opportunities.

Keywords: process capability index; upper confidence limit; minimum value; fuzzy membership function; radar chart

MSC: 62C05; 62C86

check for updates

Citation: Chen, K.-S.; Yu, C.-M.; Lin, J.-S.; Huang, T.-H.; Zhong, Y.-S. Fuzzy Radar Evaluation Chart for Improving Machining Quality of Components. *Mathematics* **2024**, *12*, 732. https://doi.org/10.3390/math12050732

Academic Editor: Michael Voskoglou

Received: 21 January 2024
Revised: 26 February 2024
Accepted: 28 February 2024
Published: 29 February 2024

Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

A machine tool known as the "Mother of Machines" is applied in precision manufacturing, aerospace, national defense, biomedicine, electronics, and other related industries [1,2]. Many studies have pointed out that a machine tool is composed of many components, and the final product yield is affected by the quality of the processing materials and components [3,4].

According to the research conducted by Prahalad and Hamel [5], companies outsource non-core or auxiliary parts to professional manufacturers to improve the production efficiency and reduce operating costs, thereby enhancing their own value and competitiveness. Additionally, some studies have shown that any part machined by an outsourcer typically

has several basic quality characteristics. When the outsourcer's process capability is insufficient, the defective rate of various quality characteristics of the product will grow, thereby raising the rework rate and scrap rate as well as leading to an increase in maintenance costs, a decrease in economic value, and even a rise in carbon emissions during the production process [6–8].

The process capability index is one of the tools commonly used in the industry to evaluate process quality. It is not only a quality problem analysis tool for internal process engineers, but also an external communication tool for the sales department [9–11]. In particular, the process capability index proposed by Kane [12] is the most widely used process quality evaluation tool in the industry and has an unequal relationship with process yield. In addition, according to some related studies, the radar chart is a visual evaluation tool widely used in engineering, management, education, and other fields [13–15]. Since process indices often contain unknown parameters, sample data are needed for evaluation. With the rapid development of the Internet of Things and big data analysis, many companies regard rapid response as a basic requirement for timeliness and cost consideration. Therefore, companies often have to evaluate the process quality of ten small samples and decide whether to make some improvements.

In order to solve the above problems, this study proposes a fuzzy radar chart evaluation model for the process quality of multi-quality characteristic parts based on the process capability index. Using this model can help all parts manufacturers continue to improve the quality of their machined parts and reduce the rework and scrap rates of parts. In the meantime, carbon emissions can be eased during the production process, and companies can fulfill their social responsibilities as well. This fuzzy radar chart evaluation model is built on the basis of confidence intervals. Since the company's past experience is integrated, the accuracy of evaluation can still be retained, even with a small sample size [16–18]. Additionally, the fuzzy radar evaluation chart can simultaneously evaluate the process capabilities of all quality characteristics of the part. In addition to making it easier for manufacturers to master all quality characteristics, quality process capabilities can also help them seize improvement opportunities. This study uses the machine tool industry chain in the central region as the background to discuss the process quality of the parts processed by a machining company to illustrate the application of the fuzzy radar evaluation chart proposed in this paper.

The rest of this study is arranged in the following sections. In Section 2, we use the mathematical programming method (MP) to derive the upper confidence limits of the process capability index. Next, in Section 3, we build a fuzzy hypothesis testing model, using the index estimators and fuzzy critical values to establish fuzzy testing rules for evaluating process capabilities. Section 4 takes an impact wrench as an example to explain the application of the fuzzy radar evaluation chart proposed in this paper. Section 5 presents the conclusions.

2. Upper Confidence Limits of the Process Capability Index

As mentioned above, the process capability index C_{pk} proposed by Kane [15] is the most widely used process quality evaluation tool in the industry, as follows:

$$C_{pk} = \frac{d - |\mu - T|}{3\sigma}, \tag{1}$$

where $T = (USL + LSL)/2$ is the process target value; $d = (USL - LSL)/2$ is the half length of the specification interval; μ is the process mean; σ is the process standard deviation; USL is the upper specification limit; and LSL is the lower specification limit.

According to Boyles [19], index C_{pk} has an unequal relationship with the process yield (Yield%), shown as follows:

$$Yield\% \geq 2\Phi\left(3C_{pk}\right) - 1. \tag{2}$$

As mentioned above, when the index value is large enough, it can be guaranteed that the process yield is high enough. For example, when $C_{pk} = 1$, it can be guaranteed that the process yield is $Yield\% \geq 2\Phi(3) - 1 = 99.730\%$; when $C_{pk} = 1.33$, it can be guaranteed that the process yield $Yield\% \geq 2\Phi(4) - 1 = 99.993\%$. In fact, as the index value is larger, the process yield also rises gradually. In addition, a product usually has multiple quality characteristics, and its tolerances are inconsistent. Therefore, in order to facilitate the evaluation of process capability, this paper, based on the research of Pearn and Chen [20], uses the transformation of variables to let

$$Y = \frac{X - T}{d}. \tag{3}$$

Then, the original specification tolerance (LSL, T, USL) is converted to $(-1, 0, 1)$. Also, we assume $X \sim N(\mu, \sigma^2)$, and then $Y \sim N(\delta, \gamma^2)$, where δ is the accuracy index and γ is the precision index, expressed as follows:

$$\text{Accuracy index : } \delta = \frac{\mu - T}{d} \tag{4}$$

and

$$\text{Precision index : } \gamma = \frac{\sigma}{d} \tag{5}$$

Then, the process capability index C_{pk} can be rewritten as follows:

$$C_{pk} = \frac{1 - |\delta|}{3\gamma} \tag{6}$$

Since the accuracy index δ and the precision index γ are unknown parameters, they must be estimated using samples. This paper assumes that $Y_1, Y_2, \ldots, Y_n$ is a set of random samples of $N(\delta, \gamma^2)$; then, the estimator of the accuracy index δ and the precision index γ can be expressed as follows:

$$\delta^* = \frac{1}{n}\sum_{i=1}^{n} Y_i \tag{7}$$

$$\gamma^* = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - \overline{Y})^2} \tag{8}$$

In this paper, we let $Z = \sqrt{n}(\delta^* - \delta)/\gamma$ and $W = n \times (\gamma^*)^2/\gamma^2$. Under the assumption of the normal process, the random variable Z obeys the standard normal distribution, denoted as $Z \sim N(0, 1)$; the random variable W obeys the chi-square distribution, where the degree of freedom is $n - 1$, denoted as $W \sim \chi^2_{n-1}$. The random variables Z and W are mutually independent. Let $\alpha' = 1 - \sqrt{1 - \alpha}$; then,

$$P\{-Z_{\alpha'/2} \leq Z \leq Z_{\alpha'/2}\} = \sqrt{1 - \alpha} \tag{9}$$

and

$$P\left\{\chi^2_{\alpha'/2;n-1} \leq W \leq \chi^2_{1-(\alpha'/2);n-1}\right\} = \sqrt{1 - \alpha} \tag{10}$$

where $Z_{\alpha'/2}$ is the upper $\alpha'/2$ quantile of $N(0, 1)$ and $\chi^2_{\alpha'/2;n-1}$ is the lower $\alpha'/2$ quantile of χ^2_{n-1}. Since the random variables Z and W are mutually independent, this paper lets event A and event B be as follows:

$$A = \{-Z_{\alpha'/2} \leq Z \leq Z_{\alpha'/2}\} \tag{11}$$

$$B = \left\{\chi^2_{\alpha'/2;n-1} \leq W \leq \chi^2_{1-(\alpha'/2);n-1}\right\} \tag{12}$$

The random variables Z and W are mutually independent, and so are event A and event B. Thus,

$$P\left\{-Z_{\alpha'/2} \le Z \le Z_{\alpha'/2}, \chi^2_{\alpha'/2;n-1} \le W \le \chi^2_{1-(\alpha'/2);n-1}\right\} = 1-\alpha \tag{13}$$

$$\Rightarrow P\left\{-Z_{\alpha'/2} \le \frac{\sqrt{n}(\delta^* - \delta)}{\gamma} \le Z_{\alpha'/2}, \sqrt{\chi^2_{\alpha'/2;n-1}} \le \sqrt{\frac{n \times \gamma^{*2}}{\gamma^2}} \le \sqrt{\chi^2_{1-(\alpha'/2);n-1}}\right\} = 1-\alpha \tag{14}$$

$$\Rightarrow P\left\{\delta^* - Z_{\alpha'/2}\left(\frac{\gamma}{\sqrt{n}}\right) \le \delta \le \delta^* + Z_{\alpha'/2}\left(\frac{\gamma}{\sqrt{n}}\right), \sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^* \le \gamma \le \sqrt{\frac{n}{\chi^2_{\alpha'/2;n-1}}}\gamma^*\right\} = 1-\alpha \tag{15}$$

According to the above description, the joint confidence region *(CR)* of (δ, γ) is expressed as follows:

$$CR = \left\{(\delta,\gamma)\middle| \begin{array}{l} \delta^* - Z_{\alpha'/2} \times \left(\frac{\gamma}{\sqrt{n}}\right) \le \delta \le \delta^* + Z_{\alpha'/2} \times \left(\frac{\gamma}{\sqrt{n}}\right), \\ \sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^* \le \gamma \le \sqrt{\frac{n}{\chi^2_{\alpha'/2;n-1}}}\gamma^* \end{array}\right\}. \tag{16}$$

It can be seen from the above that C_{pk} is the function of (δ, γ). Next, this paper uses Mathematical Programming (MP) to solve the upper confidence limit UC_{pk} of index C_{pk}. Also, $C_{pk}(\delta, \gamma)$ is the objective function, and the *CR* is regarded as a feasible solution region. Then, the MP model is expressed as follows:

$$\begin{cases} UC_{pk} = Max C_{pk}(\delta, \gamma) \\ subject\ to\ CR \end{cases} \tag{17}$$

Obviously, it can be known that the confidence region is a convex set that is wide at the top and narrow at the bottom and resembles a rectangle. This paper assumes that the two points where the horizontal line $\gamma = \gamma_L$ intersects the confidence region are $P_{LL}(\delta_L, \gamma_L)$ and $P_{RL}(\delta_L, \gamma_L)$ from the left to the right, and we also assume that the two points where the horizontal line $\gamma = \gamma_U$ intersects the confidence region are $P_{LU}(\delta'_L, \gamma_U)$ and $P_{RU}(\delta'_R, \gamma_U)$ from left to the right, where

$$\begin{cases} \delta_L = \delta^* - \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right) \times \gamma_L, \delta_R = \delta^* + \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right) \times \gamma_L \\ \delta'_L = \delta^* - \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right) \times \gamma_U, \delta'_R = \delta^* + \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right) \times \gamma_U \end{cases} \tag{18}$$

$$\gamma_L = \sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^*,\ \gamma_U = \sqrt{\frac{n}{\chi^2_{\alpha'/2;n-1}}}\gamma^*. \tag{19}$$

It can be learned from the above equation that if $(\delta, \gamma) \in CR$ and $\gamma \ne \gamma_L$, then $C_{pk}(\delta, \gamma) < C_{pk}(\delta, \gamma_L)$. Therefore, the MP model can be rewritten as:

$$\begin{cases} UC_{pk} = Max C_{pk}(\delta, \gamma_L) \\ s.t.\ \delta_L \le \delta \le \delta_R \end{cases} \tag{20}$$

Next, this paper will discuss three scenarios—Case 1: $0 \in [\delta_L, \delta_R]$, Case 2: $\delta_L > 0$, and Case 3 $\delta_R < 0$, which can be solved for UC_{pk} as follows:

Case 1: $0 \in [\delta_L, \delta_R]$

In this case, if $\delta \in [\delta_L, \delta_R]$, then $C_{pk}(\delta, \gamma_L) \leq C_{pk}(0, \gamma_L)$. Obviously, it can be inferred from $UC_{pk} = C_{pk}(0, \gamma_L)$ that $\delta = 0$ and $C_{pk} = \frac{1-|\delta|}{3\gamma} = \frac{1-0}{3\gamma} = \frac{1}{3\gamma}$. UC_{pk} can be expressed as follows:

$$\begin{aligned} UC_{pk} &= \frac{1}{3\gamma_L} \\ &= \frac{1}{3\gamma^*}\sqrt{\frac{\chi^2_{1-(\alpha'/2);n-1}}{n}} \\ &= C^*_{pk}\sqrt{\frac{\chi^2_{1-(\alpha'/2);n-1}}{n}}. \end{aligned} \tag{21}$$

Case 2: $\delta_L > 0$

In this case, it can be inferred that $\delta > 0$ and $C_{pk} = \frac{1-|\delta|}{3\gamma} = \frac{1-\delta}{3\gamma}$. If $\delta \in [\delta_L, \delta_R]$, then $C_{pk}(\delta, \gamma_L) \leq C_{pk}(\delta_L, \gamma_L)$. Thus, we can learn $UC_{pk} = C_{pk}(\delta_L, \gamma_L)$, as follows:

$$\begin{aligned} UC_{pk} &= \frac{1-\delta_L}{3\gamma_L} \\ &= \frac{1-\delta^* + \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right)\sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^*}{3\sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^*} \\ &= \sqrt{\frac{\chi^2_{1-(\alpha'/2);n-1}}{n}} \times \frac{1-\delta^*}{3\gamma^*} + \frac{\frac{Z_{\alpha'/2}}{\sqrt{n}}}{3} \\ &= \sqrt{\frac{\chi^2_{1-(\alpha'/2);n-1}}{n}} \times C^*_{pk} + \frac{Z_{\alpha'/2}}{3\sqrt{n}} \end{aligned} \tag{22}$$

where $C^*_{pk} = (1-\delta^*)/3\gamma^*$.

Case 3: $\delta_R < 0$

In this case, it can be inferred that $\delta < 0$ and $C_{pk} = \frac{1-|\delta|}{3\gamma} = \frac{1-(-\delta)}{3\gamma} = \frac{1+\delta}{3\gamma}$. If $\delta \in [\delta_L, \delta_R]$, then $C_{pk}(\delta, \gamma_L) \leq C_{pk}(\delta_R, \gamma_L)$. Thus, we can learn $UC_{pk} = C_{pk}(\delta_R, \gamma_L)$, as follows:

$$\begin{aligned} UC_{pk} &= \frac{1+\delta_R}{3\gamma_L} \\ &= \frac{1+\delta^* + \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right)\sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^*}{3 \times \sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n-1}}}\gamma^*} \\ &= \sqrt{\frac{\chi^2_{1-(\alpha'/2);n}}{n}} \times \frac{1+\delta^*}{3\gamma^*} + \frac{\frac{Z_{\alpha'/2}}{\sqrt{n}}}{3} \\ &= \sqrt{\frac{\chi^2_{1-(\alpha'/2);n}}{n}} \times C^*_{pk} + \frac{Z_{\alpha'/2}}{3\sqrt{n}} \end{aligned} \tag{23}$$

where $C^*_{pk} = (1+\delta^*)/3\gamma^*$. Based on the above three cases, the $100(1-\alpha)\%$ upper confidence limit UC_{pk} of C_{pk} and $\alpha' = 1 - \sqrt{1-\alpha}$ can be obtained. Thus, UC_{pk} can be rewritten as:

$$UC_{pk} = \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{n}} \times C^*_{pk} + I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}}, \tag{24}$$

where

$$I = \begin{cases} 0 \; if \; 0 \in (\delta_L, \delta_R) \\ 1 \; if \; 0 \notin (\delta_L, \delta_R) \end{cases}. \tag{25}$$

3. Constructing a Fuzzy Hypothesis Testing Model

Wu et al. [4] pointed out that a product usually has multiple quality characteristics. In order not to lose generality, this paper assumes that there are q quality characteristics. If the process capability of the product is required to be v, $\left(C_{pk} = v\right)$, then the process capability index of the individual quality characteristic h, $\left(C_{pkh} = v'\right)$ must be greater than v, so as to ensure that the process capability index of the product is $C_{pk} > v$. When all individual quality characteristics (q) meet the requirement, $\left(C_{pkh} > v',\ h = 1, 2, \ldots, q\right)$, and then it can be claimed as a qualified product or component.

According to some related research, fuzzy hypothesis testing can judge whether the process yield is an acceptable process evaluation method, and process index C_{pk} can effectively reflect the characteristics of the yield [12–14]. Consequently, in order to quickly judge the quality of the process, the industry uses C_{pk} as an evaluation method, and its judgment groups are divided as follows:

(1) If $C_{pk} < 1.00$, then the process capability level is insufficient.
(2) If $1.00 \leq C_{pk} < 1.33$, then the process capability level is sufficient.
(3) If $C_{pk} \geq 1.33$, then the process capability level is good.

According to Hsu et al. [21], to estimate the process capability index of the hth random sample, event E_h represents that its quality characteristic index value is greater than v', $\left(C_{pkh} > v'\right)$, and we let $E = \bigcap\limits_h E_h$. Based on the concept of Wu et al. [3], this study used Boole's inequality and De Morgan's theorem to rewrite the bilateral specification index, as follows:

$$p(E) = p\left(\bigcap_h E_h\right) \geq 2\Phi\left(3C_{pk}\right) - 1 \tag{26}$$

Besides, according to the relationship between the index value and the yield rate, $Yield\% = \Phi\left(3C_{pk}\right)$, the above-stated bilateral specification index can be re-expressed as

$$2\Phi(3v) - 1 = 1 - \sum_{h=1}^{q} \left[2 - 2\Phi\left(3v'\right)\right] \tag{27}$$

Thus,

$$v' = \frac{1}{3}\Phi^{-1}\left\{1 - \frac{1}{q}\left[1 - \Phi(3v)\right]\right\} \tag{28}$$

Assume that a product or a component contains a total of q corresponding bilateral specification evaluation indicators. If the process capability index of the product is required to be at least v, ($C_{pk} = 1.00$ and $C_{pk} = 1.33$), then the corresponding q bilateral specification evaluation indicators must be at least v'. The corresponding values for various quality levels are shown in Table 1.

This study developed a fuzzy membership function of process capability index C_{pk} for a product according to the fuzzy testing model proposed by Buckley [22]. If the process capability index of the product is required to be $C_{pk} \geq v$, then the process capability index of the quality characteristic h is $C_{pkh} \geq v'$, whose null hypothesis and alternative hypothesis are expressed as follows:

$$\begin{cases} H_0 : C_{pkh} \geq v' \\ H_1 : C_{pkh} < v' \end{cases} \tag{29}$$

Table 1. Corresponding values for various quality levels.

Number of Quality Characteristics	C_{pk}=v=1.00	C_{pk}=v=1.33
q = 1	$C_{pkh} = v' = 1.000$	$C_{pkh} = v' = 1.330$
q = 2	$C_{pkh} = v' = 1.068$	$C_{pkh} = v' = 1.384$
q = 3	$C_{pkh} = v' = 1.107$	$C_{pkh} = v' = 1.414$
q = 4	$C_{pkh} = v' = 1.133$	$C_{pkh} = v' = 1.436$
q = 5	$C_{pkh} = v' = 1.153$	$C_{pkh} = v' = 1.452$
q = 6	$C_{pkh} = v' = 1.170$	$C_{pkh} = v' = 1.465$
q = 7	$C_{pkh} = v' = 1.183$	$C_{pkh} = v' = 1.477$
q = 8	$C_{pkh} = v' = 1.195$	$C_{pkh} = v' = 1.486$
q = 9	$C_{pkh} = v' = 1.205$	$C_{pkh} = v' = 1.495$
q = 10	$C_{pkh} = v' = 1.214$	$C_{pkh} = v' = 1.502$

This study also took the upper confidence limit UC_{pkh} of process capability index C_{pkh} as a statistic. Obviously, when $UC_{pkh} \geq v'$, then $C^*_{pkh} \geq MV$, and the index estimate C^*_{pkh0} is used to make statistics, where the required minimum value (MV) is expressed as follows:

$$\sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n}}{n}} \times C^*_{pkh0} + I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}} \geq v'$$

$$\Rightarrow C^*_{pkh0} \geq \left(v' - I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}}\right) \sqrt{\frac{n}{\chi^2_{0.5+\sqrt{1-\alpha}/2;n}}} \tag{30}$$

Then,

$$MV = \left(v' - I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}}\right) \times \sqrt{\frac{n}{\chi^2_{0.5+\sqrt{1-\alpha}/2;n}}} \tag{31}$$

According to the above, the statistical testing rules are illustrated as follows:

(1) If $C^*_{pkh} \geq MV$, then do not reject H_0.

(2) If $C^*_{pkh} < MV$, then reject H_0.

In fact, MV is a function of n. When n is larger, the value of $MV(n)$ is also larger. When $n_1 < n_2$, then $MV(n_1) < C^*_{pkh} < MV(n_2)$, as shown in Figure 1. When $n = n_1$, then $C^*_{pkh} > MV(n_1)$, so do not reject H_0 However, when $n = n_2$, then $C^*_{pkh} < MV(n_2)$, so reject H_0.

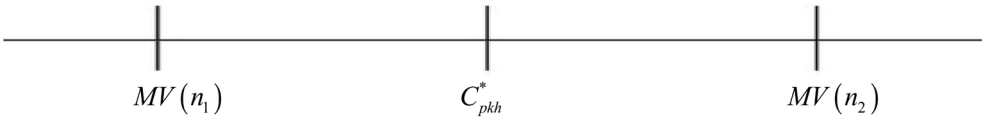

Figure 1. Schematic diagram of MV values.

Therefore, when the sample size is different, the decision-making of the statistical test is also different. Therefore, according to the aforementioned statistical testing rules and the fuzzy hypothesis test proposed by Chen et al. [23], we assume $v' = 1$ and $v' = 1.33$, respectively. The MV values are displayed in Figure 2.

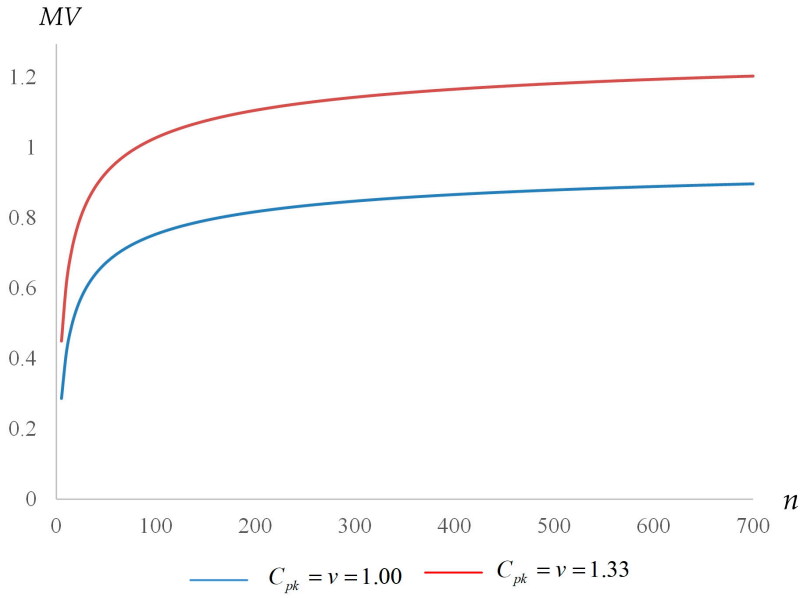

Figure 2. Required minimum values (MV) corresponding to various values of n.

Based on the abovementioned results, it can be found that the number of samples, n, will affect the final statistical inference. Also, let y_i be the observation value of Y_i; the observation values of accuracy index δ^* and precision index γ^* are expressed as follows:

$$\delta^*_{h0} = \frac{1}{n}\sum_{i=1}^{n} y_{hi} \tag{32}$$

$$\gamma^*_{h0} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(y_{hi} - \delta^*_{h0}\right)^2} \tag{33}$$

Therefore, the observation value of UC_{pkh} can be expressed as:

$$UC_{pkh0} = \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{n}} \times C^*_{pkh0} + I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}} \tag{34}$$

where

$$C^*_{pkh0} = \frac{1 - \left|\delta^*_{h0}\right|}{3\gamma^*_{h0}} \tag{35}$$

According to the upper confidence limit of UC_{pkh0} and Buckley's approach [22], the fuzzy membership function, $\widetilde{C}^*_{pkh0}$, which resembles a semi-triangular shape formed by $\alpha - cuts$, can be expressed as:

$$\widetilde{C}^*_{pkh0}[\alpha] = \begin{cases} \left[C^*_{pkh0}(1), C^*_{pkh0}(\alpha)\right], \text{ for } 0.01 < \alpha \leq 1 \\ \left[C^*_{pkh0}(1), C^*_{pkh0}(0.01)\right], \text{ for } 0 \leq \alpha \leq 0.01 \end{cases} \tag{36}$$

where

$$C^*_{pkh0}(1) = \sqrt{\frac{\chi^2_{0.5;n-1}}{n}} \times C^*_{pkh0} \tag{37}$$

$$C^*_{pkh0}(\alpha) = \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{n}} \times C^*_{pkh0} + I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}} \tag{38}$$

Obviously, $C^*_{pkh0}(1) \neq C^*_{pkh0}$. Let

$$C'_{pkh0}(\alpha) = C^*_{pkh0}(\alpha) \times \sqrt{\frac{n}{\chi^2_{0.5;n-1}}} \tag{39}$$

Thus,

$$C'_{pkh0}(\alpha) = \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{\chi^2_{0.5;n-1}}} \times C'_{pkh0} + I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}}. \tag{40}$$

Clearly, $C'_{pkh0}(1) = C'_{pkh0}$, and a new fuzzy membership function, $\widetilde{C}^*_{pkh0}$, similar to a semi-triangular shape formed by $\alpha - cuts$, can be expressed as:

$$\widetilde{C}'_{pkh0}[\alpha] = \begin{cases} \left[C^*_{pkh0}, C'_{pkh0}(\alpha)\right], \text{ for } 0.01 < \alpha \leq 1 \\ \left[C^*_{pkh0}, C'_{pkh0}(0.01)\right], \text{ for } 0 \leq \alpha \leq 0.01 \end{cases}, \tag{41}$$

where

$$C'_{pkh0}(0.01) = \sqrt{\frac{\chi^2_{0.9975;n-1}}{\chi^2_{0.5;n-1}}} \times C'_{pkh0} + I \times \frac{\sqrt{Z_{0.0025}}}{3\sqrt{n}}. \tag{42}$$

Therefore, the fuzzy number of this semi-triangular-like fuzzy membership function C^*_{pkh0} is $C^*_{pkh0} = \Delta(C_M, C_R)$, where $C_M = C^*_{pkh0}$ and $C_R = C'_{pkh0}(0.01)$, so the fuzzy membership function $\widetilde{C}^*_{pkh0}$ can be expressed as follows:

$$\eta(x) = \begin{cases} 0 \; if \; x < \; C_M \\ 1 \; if \; x = C_M \\ \alpha \; if \; C_M < x < \; C_R \\ 0 \; if \; C_R < x \end{cases}, \tag{43}$$

where α is observed from $C'_{pkh0}(\alpha) = x$, illustrated as follows:

$$\sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{\chi^2_{0.5;n-1}}} \times C'_{pkh0} + I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{n}} = x. \tag{44}$$

According to Buckley [22] and $\widetilde{C}^*_{pkh0}$, the fuzzy number of $\widetilde{MV}$, similar to the semi-triangular shape formed by $\alpha - cuts$, is as follows:

$$\widetilde{MV}[\alpha] = \begin{cases} [MV, MV(\alpha)], \text{ for } 0.01 < \alpha \leq 1 \\ [MV, MV(0.01)], \text{ for } 0 \leq \alpha \leq 0.01 \end{cases}, \tag{45}$$

where

$$MV(\alpha) = \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{\chi^2_{0.5;n-1}}} \times MV + I \times \frac{\sqrt{Z_{0.5-\sqrt{1-\alpha}/2}}}{3\sqrt{n}}. \tag{46}$$

Obviously, it can be observed that $MV(1) = \widetilde{MV}$, and the semi-triangular-like fuzzy number of MV is $\widetilde{MV} = \Delta(MV_M, MV_R)$, where

$$MV_M = MV(1) = MV; \tag{47}$$

$$MV_R = MV(0.01) = \sqrt{\frac{\chi^2_{0.9975;n-1}}{\chi^2_{0.5;n-1}}} \times MV + I \times \frac{\sqrt{Z_{0.0025}}}{3\sqrt{n}} \tag{48}$$

The membership function $\widetilde{MV}$ of a fuzzy number similar to a triangle is:

$$\eta_0(x) = \begin{cases} 0 \; if \; x < MV \\ 1 \; if \; x = MV \\ \alpha \; if \; MV < x < MV_R \\ 0 \; if \; MV_R < x \end{cases}, \tag{49}$$

where α is observed from $MV(\alpha) = x$, expressed as follows:

$$\sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{\chi^2_{0.5;n-1}}} \times MV + I \times \frac{\sqrt{Z_{0.5-\sqrt{1-\alpha}/2}}}{3\sqrt{n}} = x. \tag{50}$$

Figure 3 is a schematic diagram of the fuzzy membership function $\eta_0(x)$ and $x = C^*_{pkh0}$.

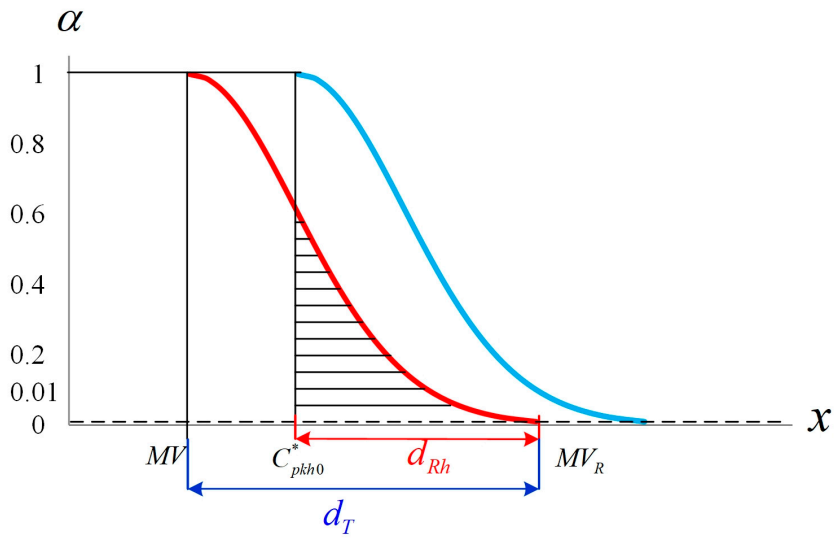

Figure 3. Schematic diagram of the fuzzy membership function $\eta_0(x)$ and $x = C^*_{pkh0}$.

According to Chen et al. [23], let A_{Th} be the area between the fuzzy membership function $\eta_0(x)$ of quality characteristic h and the x-axis, as follows:

$$A_{Th} = \{(x, \alpha) | MV \leq x < MV_R(\alpha), 0 \leq \alpha \leq 1\} \tag{51}$$

Similarly, let A_{Rh} be the area between the vertical line $x = C^*_{pkh0}$, the fuzzy membership function $\eta_0(x)$ of quality characteristic h, and the x-axis, as follows:

$$A_{Rh} = \left\{(x, \alpha) \middle| C^*_{pkh0} \leq x < MV_R(\alpha), 0 \leq \alpha \leq 1\right\}. \tag{52}$$

According to the notation of Figure 3, this paper let $d_{Rh} = MV_R - C^*_{pkh0}$ and $d_T = MV_R - MV$. Then, we have

$$\frac{d_{Rh}}{2d_T} = \frac{MV_R - C^*_{pkh0}}{2(MV_R - MV)}. \tag{53}$$

Following Chen et al. [23], we let $0 < \phi \leq 0.5$, and the decision value (dv_h) of the quality characteristic h can satisfy the following equation.

$$\frac{MV_R - dv_h}{2d_T} = \frac{MV_R - dv_h}{2(MV_R - MV)} = \phi. \tag{54}$$

Thus, the decision value can be expressed as follows:

$$dv_h = \left[(1-2\phi) \sqrt{\frac{\chi^2_{0.9975;n-1}}{\chi^2_{0.5;n-1}}} + 2\phi \right] \times MV + I \times \frac{\sqrt{Z_{0.0025}}}{3\sqrt{n}}. \quad (55)$$

According to Chen et al. [23] and Yu et al. [24], the fuzzy testing rules are illustrated as follows:

(1) If $C^*_{pkh0} \geq dv_h$, which means $d_{Rh}/2d_T \leq \phi$, then do not reject H_0 and conclude $MV > dv_h$.
(2) If $C^*_{pkh0} < dv_h$, which means $d_{Rh}/2d_T > \phi$, then reject H_0 and conclude $MV \leq dv_h$.

As mentioned above, the radar chart is a visual evaluation tool and is widely used in various fields. Therefore, this paper drew the fuzzy radar evaluation chart for the process capability based on the above two fuzzy testing rules. Not only can this chart simultaneously evaluate all of the quality characteristics of the product, but it also can help process engineers to judge the quality characteristics of components that need to be improved in a timely manner. Consequently, this paper used the radar chart to evaluate the process capability. For example, if a product contains five quality characteristics, there will be five radar lines on the fuzzy radar evaluation chart for the process capability; a regular pentagon with an included angle of 72 degrees will be formed; then, the endpoint value on each radar line with the distance from the center point will be connected to form a regular pentagon control block. Finally, the index observation value C^*_{pkh0} $(h = 1, 2 \ldots, 5)$ will be marked on each radar line, as shown in Figure 4.

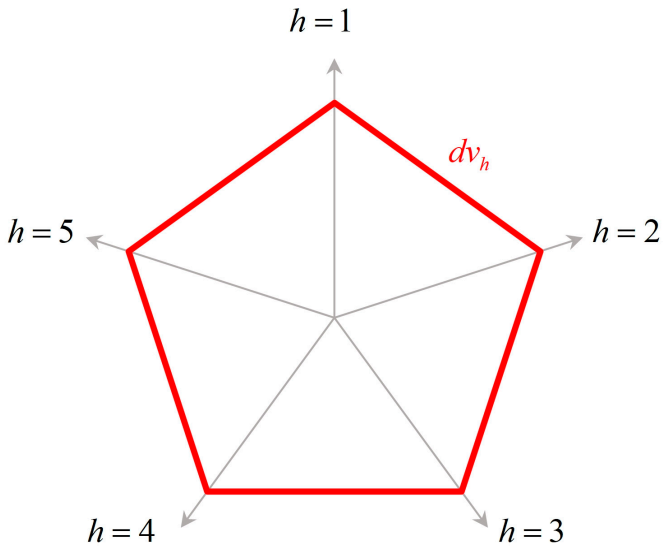

Figure 4. Fuzzy radar evaluation chart.

Through this fuzzy radar evaluation chart, whether various indicators are located in the control block can be immediately spotted. When the index observation value falls within the block, which means $C^*_{pkh0} < dv_h$, then the quality characteristic needs to make some improvements on its process. When the index observation value falls outside the block or the value of dv_h is on the line, which means $C^*_{pkh0} \geq dv_h$, then the process capability meets the required quality level. According to the above, this study established evaluation rules for the fuzzy radar chart, as follows:

(1) When $C^*_{pkh0} < dv_h$, it means that this quality characteristic has not reached the required level, so that process improvement needs to be caried out.
(2) When $C^*_{pkh0} \geq dv_h$, it means that this quality characteristic has reached the required quality level.

4. A Practical Application

In order to help engineers and practitioners understand this evaluation model, this section will explain its application with data through a case study. As mentioned earlier, Taiwan has been dedicated to developing the machine tool industry in recent years, and its export values have been ranked among the best. In particular, the central region is a major industrial cluster. In practice, the impact cylinder, a component of the impact wrench, was taken as an example, and it was discovered that the problem of poor quality is usually caused by differences in size or tolerance, or a processing concentricity error. Therefore, this paper took the impact wrench manufactured by a company located in the central region as an example. In order to ensure the quality of subsequent processing and assembly, the impact cylinder, a component of the impact wrench, was explored through the bilateral specification index C_{pk}, and its specification indicators, including inside diameter (h= 1, 2), length (h = 3), thickness (h= 4), and axis distance (h= 5), were analyzed as well. In an attempt to facilitate the management of quality engineers, this study listed five quality characteristics and their tolerances (Table 2).

Table 2. Five quality characteristics and their tolerances.

h	Quality Characteristic	Specification Size
1	Length (inside diameter)	10.77 ± 0.02
2	Length (inside diameter)	21.44 ± 0.01
3	Length	34.60 ± 0.05
4	Thickness	24.70 ± 0.05
5	Axis distance	34.00 ± 0.10

Note: Unit—mm.

This study analyzed five quality specifications of the impact cylinder produced by an impact wrench factory in central Taiwan. Therefore, there were five groups of sample data ($h = 1, 2 \ldots, 5$), and each randomly selected 36 processed components. Based on these five groups of sample data and the aforementioned evaluation model, we conducted tests and established evaluation steps, as follows:

Step 1: To facilitate the management of quality engineers, we applied the sample data and calculated the accuracy index δ^*_{h0}, precision index (γ^*_{h0}), five observation values of C^*_{pkh0}, and five upper limit observation values of UC^*_{pkh0} based on Equations (32)–(35), as shown in Table 3.

Table 3. Observation values of δ^*_{h0}, γ^*_{h0}, C^*_{pkh0}, and UC^*_{pkh0}.

h	δ^*_{h0}	γ^*_{h0}	C^*_{pkh0}	UC^*_{pkh0}
1	0.0499	0.2701	1.1725	1.7242
2	−0.0744	0.4122	0.7484	1.1570
3	0.0766	0.2603	1.1825	1.7376
4	−0.0105	0.2750	1.1995	1.7603
5	0.0916	0.4776	0.6340	1.0040

Step 2: According to Equation (28), $q = 5$, and the process capability index value $v = 1.00$ required for the impact cylinder, the MV was calculated using the evaluation index value (v'), Equation (31), $n = 36$, and $\alpha = 0.01$ corresponding to the five bilateral specifications, expressed as follows:

$$v = 1.00;$$

$$v' = \frac{1}{3}\Phi^{-1}\left\{1 - \frac{1}{5}[1 - \Phi(3 \times 1.00)]\right\} = 1.1534; \quad (56)$$

$$MV = \left(1.1534 - I \times \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{3\sqrt{36}}\right) \times \sqrt{\frac{36}{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}} = 0.7457. \quad (57)$$

Step 3: According to Equation (55), $n = 36$, $\phi = 0.3$, and the MV, we calculated the decision value for the bilateral specification evaluation index of the random sample, as follows:

$$dv_h = \left[(1-2\phi)\sqrt{\frac{\chi^2_{0.9975;n-1}}{\chi^2_{0.5;n-1}}} + 2\phi\right] \times MV + I \times \frac{\sqrt{Z_{0.0025}}}{3\sqrt{n}} = 0.9441. \quad (58)$$

Step 4: We drew five axes with an included angle of 72 degrees, marked the dv_h value ($dv_h = 0.9441$) on the axis of the fuzzy radar chart, and connected the critical points to form a critical area in the shape of a regular pentagon. Next, we marked the index observation values of C^*_{pkh0} (h= 1, 2 . . . , 5) in Table 3 on the axes, as shown in Figure 5.

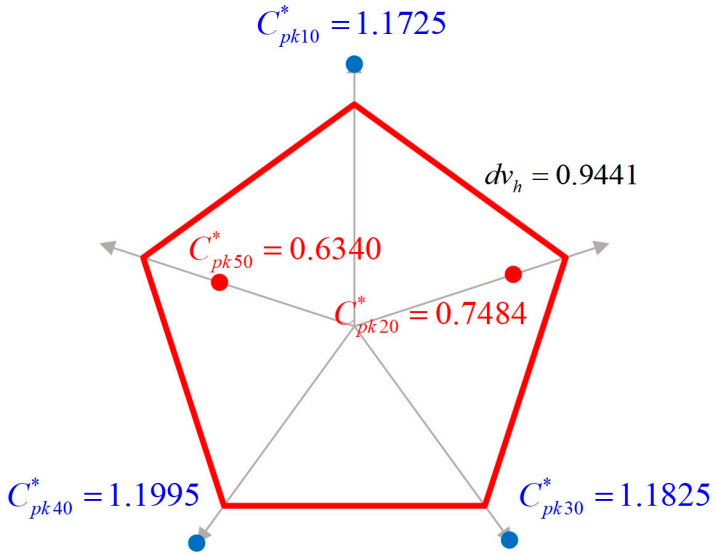

Figure 5. Fuzzy radar evaluation chart of the impact cylinder.

Step 5: The evaluation rules, case study results, and discussions of the impact cylinder fuzzy radar chart are displayed as follows:

(1) When $C^*_{pkh0} \geq 0.9441$, then it can be concluded that $C_{pkh} \geq 1.1534$, so do not reject H_0.
(2) When $C^*_{pkh0} < 0.9441$, then it can be concluded that $C_{pkh} < 1.1534$, so reject H_0.

According to the above testing rules, the results and discussions of the case study are stated as follows:

(1) The quality evaluation observation values C^*_{pk10}, C^*_{pk30}, and C^*_{pk40} all fall outside the improvement block, so it means that C^*_{pk10}, C^*_{pk30} and C^*_{pk40} have all reached the required quality level.
(2) The quality evaluation observation values C^*_{pk20} and C^*_{pk50} both fall within the improvement block, so H_0 must be rejected according to the evaluation rules of the fuzzy radar chart. Therefore, these two quality characteristics both require process improvement.
(3) Since $UC^*_{pk20} = 1.1570$ of quality characteristic 2 (inside diameter) is greater than $v' = 1.1534$, H_0 cannot be rejected according to the statistical testing rules. Based on the fuzzy testing rules, since C^*_{pk20} is smaller than $dv_h = 0.9441$, H_0 must be rejected. In fact, the value of C^*_{pk20} is only 0.7484, which is much smaller than 1.1534. Obviously, the fuzzy testing model provided by this paper is more reasonable than the statistical testing model, and it can grasp the opportunity for improvement at the same time.

5. Conclusions

A machine tool is composed of many components. After the components are processed, they usually have multiple important quality characteristics. When the process quality of all quality characteristics meets the required quality level, the process quality of the components can be guaranteed to meet customer demand. Based on process capability index C_{pk}, this paper proposed a fuzzy radar evaluation chart to evaluate the process capabilities for all quality characteristics of components. In addition, not only can this chart make it easier for manufacturers to master the quality process capabilities for all quality characteristics, but it also can help them grasp the opportunity for improvement. First, the upper confidence limit UC_{pk} of the index was deduced according to the MP method, and the MV for each quality characteristic was deduced by UC_{pk}. Next, the fuzzy hypothesis testing model was established based on the MV. Finally, a visual fuzzy radar evaluation chart was developed. The advantages of this evaluation model are described as follows:

(1) C_{pk} is not only the most widely used process quality evaluation tool in the industry, but also has an unequal relationship with the process yield. When the index value is large enough, the process yield is also high enough.
(2) The fuzzy evaluation model proposed in this paper is built on the basis of confidence interval. Since the accumulated data or expert experience in the past can be incorporated, the accuracy and precision of the evaluation can still be maintained in the case of small samples.
(3) Without losing generality, this paper assumes that when all individual quality characteristics (q) of a product can meet the requirements $\left(C_{pkh} > v',\ h = 1, 2, \ldots, q\right)$, then the product can be claimed as being a qualified product.
(4) The MV is an index derived from the upper confidence limit UC_{pk}, so it can reduce the risk of misjudgment caused by sampling error.
(5) The estimated value of the index can be directly compared with the MV, which can ensure whether the process capability reaches the required level and conforms to the application in practice.
(6) The approach can directly compare the observation value of the index with the value of dv_h, which is not only convenient, but also quickly judges whether the process needs to be improved.
(7) The fuzzy radar evaluation chart shows the complete quality characteristics at the same time, and it can be seen at a glance whether each observation value falls within the evaluation chart. This chart not only facilitates the quality control process, but also greatly improves the management and control efficiency.

Furthermore, the fuzzy radar evaluation chart can construct important quality characteristics related to the whole picture of the product to ensure that each index value meets the requirements. In practical application, not only does it help the industry chain improve the process yield and reduce the occurrence of rework and scrap, but it also contributes to industry management and establishing long-term cooperation models and partnerships with suppliers in the era of rapid technological development.

The abovementioned points are the advantages of this paper. However, since the fuzzy evaluation model proposed in this study must incorporate sufficient experts, data, and experience and will vary from industry to industry, it needs to take a longer time and accumulate more data to make more accurate inferences. Consequently, the future research can focus on how to integrate the accumulated data and experience into decision-making rules.

Author Contributions: Conceptualization, K.-S.C. and C.-M.Y.; methodology, K.-S.C., C.-M.Y. and J.-S.L.; software, Y.-S.Z.; validation, Y.-S.Z. and T.-H.H.; formal analysis, K.-S.C., C.-M.Y. and J.-S.L.; resources, J.-S.L.; data curation, T.-H.H.; writing—original draft preparation, K.-S.C., C.-M.Y., J.-S.L., T.-H.H. and Y.-S.Z.; writing—review and editing, K.-S.C., C.-M.Y. and J.-S.L.; visualization, C.-M.Y.; supervision, K.-S.C.; project administration, J.-S.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

MP	mathematical programming
C_{pk}	process capability index
T	process target value
d	half length of the specification interval
μ	process mean
σ	process standard deviation
USL	upper specification limit
LSL	lower specification limit
Yield%	process yield
X	random sample
N	normal distribution
σ^2	variance
δ	accuracy index
γ	precision index
$Y_1, Y_2, \ldots, Y_n$	set of random samples of $N(\delta, \gamma^2)$
δ^*	estimator of the accuracy index δ
γ^*	estimator of the precision index γ
n, n_1, n_2	sample size
Z	standard normal distribution
W	chi-square distribution with $n-1$ degree of freedom
P	probability
α	$\alpha - cuts$ of fuzzy numbers
α'	$1-\sqrt{1-\alpha}$
$Z_{\alpha'/2}$	upper $\alpha'/2$ quantile of $N(0,1)$
$\chi^2_{\alpha'/2;n}$	lower $\alpha'/2$ quantile of χ^2_n
A, B	event
CR	confidence region
UC_{pk}	upper confidence limit of index C_{pk}
$C_{pk}(\delta, \gamma)$	objective function
$P_{LL}(\delta_L, \gamma_L)$	left point of the horizontal line $\gamma = \gamma_L$ intersects the confidence region
$P_{RL}(\delta_L, \gamma_L)$	right point of the horizontal line $\gamma = \gamma_L$ intersects the confidence region
$P_{LU}(\delta'_L, \gamma_U)$	left point of the horizontal line $\gamma = \gamma_U$ intersects the confidence region
$P_{RU}(\delta'_R, \gamma_U)$	right point of the horizontal line $\gamma = \gamma_U$ intersects the confidence region
δ_L	$\delta^* - \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right) \times \gamma_L$
δ_R	$\delta^* + \left(\frac{Z_{\alpha'/2}}{\sqrt{n}}\right) \times \gamma_U$
γ_L	$\sqrt{\frac{n}{\chi^2_{1-(\alpha'/2);n}}}\gamma^*$
γ_U	$\sqrt{\frac{n}{\chi^2_{\alpha'/2;n}}}\gamma^*$
$C_{pk} = v$	value of the process capability of the product
$C_{pkh} = v'$	value of the process capability index of the individual quality characteristic
q	individual quality characteristics
event E_h	represents that its quality characteristic index value is greater than v'
E	$\bigcap_h E_h$
UC_{pkh}	upper confidence limit of process capability index C_{pkh}
MV	required minimum value
H_0	null hypothesis
H_1	alternative hypothesis
C^*_{pkh0}	observation value of C_{pkh}
y_i	observation value of Y_i

δ^*_{h0}	observation values of accuracy index δ^*
γ^*_{h0}	observation values of precision index γ^*
UC_{pkh0}	observation value of UC_{pkh}
$\widetilde{C}^*_{pkh0}[\alpha]$	$\alpha - cuts$ of $\widetilde{C}^*_{pkh0}$
$\widetilde{C}'_{pkh0}[\alpha]$	$\alpha - cuts$ of $\widetilde{C}'_{pkh0}$
$C^*_{pkh0} = \Delta(C_M, C_R)$	semi-triangular-like fuzzy number
$\eta(x)$, $\eta_0(x)$	fuzzy membership function $\widetilde{C}^*_{pkh0}$
$\widetilde{MV}[\alpha]$	$\alpha - cuts$ of $\widetilde{MV}$
$\widetilde{MV} = \Delta(MV_M, MV_R)$	semi-triangular-like fuzzy number of MV
A_{Th}	range of the fuzzy membership function $\eta_0(x)$
A_{Rh}	range between the fuzzy membership function $\eta_0(x)$ and $x = C^*_{pkh0}$ of quality characteristic h
d_{Rh}	$MV_R - C^*_{pkh0}$
d_T	$MV_R - MV$
dv	decision value
ϕ	decision-making value

References

1. Chien, C.F.; Hong, T.Y.; Guo, H.Z. An empirical study for smart production for TFT-LCD to empower Industry 3.5. *J. Chin. Inst. Eng.* **2017**, *40*, 552–561. [CrossRef]
2. Liao, M.Y.; Pearn, W.L. Modified weighted standard deviation index for adequately interpreting a supplier's lognormal process capability. *P. I. Mech. Eng. B-J. Eng.* **2019**, *233*, 999–1008. [CrossRef]
3. Wu, M.F.; Chen, H.Y.; Chang, T.C.; Wu, C.F. Quality evaluation of internal cylindrical grinding process with multiple quality characteristics for gear products. *Int. J. Prod. Res.* **2019**, *57*, 6687–6701. [CrossRef]
4. Pearn, W.L.; Cheng, Y.C. Measuring production yield for processes with multiple characteristics. *Int. J. Prod. Res.* **2010**, *48*, 4519–4536. [CrossRef]
5. Prahalad, C.K.; Hamel, G. The Core Competence of the Corporation. *Harv. Bus. Rev.* **1990**, *68*, 79–91.
6. Wu, C.W.; Pearn, W.L. A variables sampling plan based on Cpmk for product acceptance determination. *Eur. J. Oper. Res.* **2008**, *184*, 549–560. [CrossRef]
7. Wu, C.W.; Liao, M.Y.; Yang, T.T. Efficient methods for comparing two process yields—Strategies on supplier selection. *Int. J. Prod. Res.* **2013**, *51*, 1587–1602. [CrossRef]
8. Wu, C.W.; Liao, M.Y.; Lin, C.W.; Lin, T.L. Testing and ranking multiple wafer manufacturing processes with fuzzy quality data. *J. Test. Eval.* **2016**, *44*, 1970–1977. [CrossRef]
9. Wang, S.; Chiang, J.Y.; Tsai, T.R.; Qin, Y. Robust process capability indices and statistical inference based on model selection. *Comput. Ind. Eng.* **2021**, *156*, 107265. [CrossRef]
10. Li, W.; Liu, G. Dynamic failure mode analysis approach based on an improved Taguchi process capability index. *Reliab. Eng. Syst. Saf.* **2022**, *218*, 108152. [CrossRef]
11. Shu, M.H.; Wang, T.C.; Hsu, B.M. Generalized quick-switch sampling systems indexed by Taguchi capability with record traceability. *Comput. Ind. Eng.* **2022**, *172*, 108577. [CrossRef]
12. Kane, V.E. Process capability indices. *J. Qual. Technol.* **1986**, *18*, 41–52. [CrossRef]
13. Claessen, J.H.T.; Van Wijk, J.J. Flexible linked axes for multivariate data visualization. *IEEE Trans. Vis. Comput. Graph.* **2011**, *17*, 2310–2316. [CrossRef] [PubMed]
14. Wan, Y.; Cheng, K.; Liu, Z.; Ye, H. An investigation on machinability assessment of difficult-to-cut materials based on radar charts. *P. I. Mech. Eng. B-J. Eng.* **2013**, *227*, 1916–1920. [CrossRef]
15. Zhang, H.; Hou, Y.; Zhang, J.; Qi, X.; Wang, F. A new method for nondestructive quality evaluation of the resistance spot welding based on the radar chart method and the decision tree classifier. *Int. J. Adv. Manuf. Technol.* **2015**, *78*, 841–851. [CrossRef]
16. Chen, H.Y.; Lin, K.P. Fuzzy supplier selection model based on lifetime performance index. *Expert Syst. Appl.* **2022**, *208*, 118135. [CrossRef]
17. Huang, C.C.; Chang, T.C.; Chen, B.L. Fuzzy assessment model to judge quality level of machining processes involving bilateral tolerance using crisp data. *J. Chin. Inst. Eng.* **2021**, *44*, 1–10. [CrossRef]
18. Lo, W.; Yang, C.M.; Lai, K.K.; Li, S.Y.; Chen, C.H. Developing a novel fuzzy evaluation model by one-sided specification capability indices. *Mathematics* **2021**, *9*, 1076. [CrossRef]
19. Boyles, R.A. Process capability with asymmetric tolerances. *Commun. Stat.-Simul. Comput.* **1994**, *23*, 615–635. [CrossRef]
20. Pearn, W.L.; Chen, K.S. New generalization of the process capability index Cpk. *J. Appl. Stat.* **1998**, *25*, 801–810. [CrossRef]
21. Hsu, C.H.; Chen, K.S.; Yang, C.M. Construction of closed interval for process capability indices Cpu, Cpl, and Spk based on Boole's inequality and de Morgan's laws. *J. Stat. Comput. Simul.* **2016**, *86*, 3701–3714. [CrossRef]
22. Buckley, J.J. Fuzzy statistics: Hypothesis testing. *Soft Comput.* **2005**, *9*, 512–518. [CrossRef]

23. Chen, K.S.; Wang, C.H.; Tan, K.H. Developing a fuzzy green supplier selection model using Six Sigma quality indices. *Int. J. Prod. Econ.* **2019**, *212*, 1–7. [CrossRef]
24. Yu, C.M.; Chen, K.S.; Lai, K.K.; Hsu, C.H. Fuzzy supplier selection method based on smaller-the-better quality characteristic. *Appl. Sci.* **2020**, *10*, 3635. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.