

Article

Dynamics and Stabilization of Chaotic Monetary System Using Radial Basis Function Neural Network Control

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Abstract: In this paper, we investigated a three-dimensional chaotic system that models key aspects of a monetary system, including interest rates, investment demand, and price levels. The proposed system is described by a set of autonomous quadratic ordinary differential equations. We analyze the dynamic behavior of this system through equilibrium points and their stability, Lyapunov exponents (LEs), and bifurcation diagrams. The system demonstrates a variety of behaviors, including chaotic, periodic, and equilibrium states depending on parameter values. Additionally, we explore the multistability of the system and present a radial basis function neural network (RBFNN) controller design to stabilize the chaotic behavior. The effectiveness of the controller is validated through numerical simulations, highlighting its potential applications in economic and financial modeling.

Keywords: chaos; monetary system; multistability; radial basis function neural network

MSC: 65P20; 65P40; 34H10; 34H05; 37N35



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1. Introduction

In recent years, the study of chaotic systems has garnered significant interest across various fields, including economics and finance [1,2]. Traditional linear models often fail to account for the irregular and unpredictable fluctuations observed in real-world economic systems [3,4]. Chaotic models, characterized by sensitive dependence on initial conditions and non-linear interactions, offer a more accurate representation of these complex dynamics [5,6]. By studying Chaotic Monetary Systems (CMSs), economists can better understand the underlying mechanisms that drive economic instability, such as sudden market crashes or hyperinflation [7,8].

The integration of chaos theory into economic models enriches the theoretical framework of economics [7,8]. It challenges traditional assumptions and introduces new concepts that better reflect the complexities of modern economies [9,10]. This theoretical

advancement fosters a deeper understanding of economic phenomena and encourages the development of innovative models and analytical tools [11,12].

Some literature has discussed the influence of non-linear interactions on economic models using chaotic behavior, for example, Jhangeer et al. [13] examined a coupled non-linear volatility and option pricing model as an alternative to the Black–Scholes model. Also, they explored the model’s leverage effect, dynamics, and mathematical properties to understand market price fluctuations. Johansyah et al. [14] investigate the complex dynamics of a supply chain model that incorporates a quadratic interaction term between customer demand and distributor inventory levels. Also, they analyze the system’s stability, chaotic behaviors, and mathematical properties while proposing a numerical method and control strategy to stabilize chaotic dynamics. Hu and Wu [15] analyzed the interplay between government subsidy policies, market dynamic regulation capability, and enterprise revenue in the photovoltaic energy industry. By constructing a non-linear model, the study aims to explore how changes in parameters and initial values affect enterprise revenue and the relationship between government subsidies and corporate profits. Dorokhov et al. [16] developed a mathematical model of the “multiplier–accelerator” interaction in macroeconomic dynamics, as foundational to neo-Keynesian growth theory. They explored the non-linear dependencies between consumption, profit, and national income, incorporating factors like consumption saturation and delayed accelerator responses. By presenting the system as a set of first-order differential equations, the research seeks to identify conditions leading to self-oscillations, bifurcations, and critical states, offering insights for managing macroeconomic systems and preventing systemic instabilities.

Recently, a broad spectrum of sophisticated control algorithms has been proposed to manage and synchronize chaotic financial systems [17–21]. Lu [22] introduced an intermittent control method to address chaotic phenomena in financial systems, examining its practical implications for macrocontrol during financial crises. Shi et al. [23] analyzed the evolution of financial systems with incommensurate fractional order and delay, highlighting the system’s Hopf bifurcation and the dynamic behavior of financial markets. Yousri and Mirjalili [24] enhanced the global search capability of the cuckoo search optimizer, improving the balance between local and global search in hyperchaotic financial systems. Wang et al. [25] developed a novel method for controlling and synchronizing fractional-order financial time-delayed systems under time-varying disturbances, employing a type-2 fuzzy disturbance observer. Tian et al. [26] focused on synchronization and stabilization in financial systems using T3-FLSs. Chu et al. [27] examined the impact of market confidence on financial systems through the lens of fractional calculus. Zhou et al. [4] investigated the dynamic behavior of a discrete-time system and proposed a new fuzzy-based activation feedback controller for the synchronization of macroeconomic systems.

This work presents the development of a 3DCMS, contributing to the growing body of research that applies chaos theory to economic models. By exploring the intricate behaviors and complex dynamics inherent in chaotic systems, this study aims to provide deeper insights into monetary phenomena that traditional linear models might fail to capture.

The main contribution and novelty of this work as follows:

1. This work investigates the dynamic behavior of a three-dimensional chaotic monetary system using autonomous quadratic ordinary differential equations, examining equilibrium points, stability, Lyapunov exponents, and bifurcation diagrams to understand its chaotic, periodic, and equilibrium states.
2. The study explores the multistability of the system, revealing the presence of multiple stable states under varying conditions, which reflects the complex and unpredictable nature of real-world economic systems.
3. This work designs a RBFNN controller to stabilize the chaotic behavior of the system and validates its effectiveness through numerical simulations, highlighting its potential applications in economic and financial modeling.

The organization of this paper is as follows: Section 1 provides an introduction to the study, highlighting the significance of chaotic systems in monetary modeling and un-

undertaking a review of related literature. Section 2 describes the mathematical formulation of the proposed three-dimensional chaotic monetary system, including its equilibrium points and stability analysis. Section 3 explores the system's dynamical behaviors through Lyapunov exponents, bifurcation diagrams, and multistability analysis. Section 4 introduces the design and implementation of the RBFNN controller to stabilize the chaotic system and presents numerical simulation results to validate its effectiveness. Finally, Section 5 concludes the paper, summarizing the key contributions and outlining directions for future research.

2. Modelling of Chaotic Monetary System

The monetary system plays a vital role in chaotic system studies due to its intrinsic complexity and its critical influence on economic stability and global financial dynamics. Unlike traditional systems, the monetary system involves non-linear interactions among variables such as interest rates, investment demand, and price levels, which naturally lead to chaotic behaviors like unpredictability and sensitivity to initial conditions. These characteristics make the monetary system a prime candidate for exploring chaotic dynamics, as it helps uncover hidden patterns and instabilities that traditional linear models cannot address. Such insights are crucial for understanding real-world phenomena, such as economic cycles, market volatility, and sudden financial crises.

Moreover, a chaotic analysis of monetary systems provides valuable tools for policy-makers to predict and mitigate systemic risks. The presence of multistability—multiple stable states existing under similar conditions—highlights the potential for abrupt transitions between economic stability and instability. Understanding these dynamics allows researchers to design innovative control techniques, like neural network-based controllers, to stabilize chaotic behavior and ensure economic resilience. Given the global interconnectedness of modern economies, the insights gained from studying chaotic monetary systems are essential for crafting policies that enhance both national and international financial stability.

Lazureanu (2019) described the autonomous system of the quadratic ordinary differential equations model with the monetary system [28]:

$$\begin{cases} \dot{\beta}_1 = -a\beta_1 + \beta_3 + \beta_1\beta_2 + b\beta_2\beta_3, \\ \dot{\beta}_2 = 1 - c\beta_2 - \beta_1^2, \\ \dot{\beta}_3 = -\beta_1 - \beta_3 - b\beta_1\beta_2. \end{cases} \quad (1)$$

where the state variables $\beta_1, \beta_2, \beta_3 \in \mathbb{R}$ involve the interest rate, investment demand, and the price exponent, respectively. With $a = 0.5; b = -2; c = 0.1$, Lazureanu (2019) observed a chaotic attractor for the CMS (1) with the initial values $\beta_1(0) = 0.1, \beta_2(0) = 0.1$, and $\beta_3(0) = 0.1$.

The equilibrium points of the CMS (1) are found by solving the equations:

$$\begin{cases} \dot{\beta}_1 = -a\beta_1 + \beta_3 + \beta_1\beta_2 + b\beta_2\beta_3 = 0, \\ \dot{\beta}_2 = 1 - c\beta_2 - \beta_1^2 = 0, \\ \dot{\beta}_3 = -\beta_1 - \beta_3 - b\beta_1\beta_2 = 0. \end{cases} \quad (2)$$

For the chosen parameters $a = 0.5; b = -2; c = 0.1$. By solving the above system, we obtain;

$$\begin{cases} P_1 = (\beta_1 = 0, \beta_2 = 10, \beta_3 = 0), \\ P_{2,3} = (\beta_1 = \pm\sqrt{370}/20, \beta_2 = 3/4, \beta_3 = \pm\sqrt{370}/40), \\ P_{4,5} = (\beta_1 = \pm\sqrt{95}/10, \beta_2 = 1/2, \beta_3 = 0). \end{cases} \quad (3)$$

The eigenvalues of the linearization matrix of the CMS (1) at P_1 are found as follows:

$$\lambda_2 = -0.1, \lambda_{2,3} = 4.25 \pm 18.2603i \quad (4)$$

This calculation shows that P_1 is an unstable saddle-focus equilibrium point. The eigenvalues of the linearization matrix of the CMS (1) at $P_{2,3}$ are given as follows:

$$\lambda_2 = 0.98409, \lambda_{2,3} = -0.917045 \pm 1.01928i \quad (5)$$

This calculation shows that $P_{2,3}$ is an unstable saddle-focus equilibrium points. The eigenvalues of the linearization matrix of the CMS (1) at $P_{4,5}$ are given as follows:

$$\lambda_2 = -1, \lambda_{2,3} = -0.05 \pm 1.3775i \quad (6)$$

This calculation shows that $P_{4,5}$ are asymptotically stable equilibrium point.

Using Matlab, we calculate the LEs of the CMS (1) and obtain their values as $L_1 = 0.2483$, $L_2 = 0.00$, and $L_3 = -0.4169$.

Also, the Kaplan dimension of the CMS (1) is found as

$$D_K = 2 + \frac{L_1 + L_2}{|L_3|} = 2.59558 \quad (7)$$

The above results show that the considered CMS (1) is chaotic and exhibits chaotic attractors. The phase plots of the CMS (1) are shown in Figure 1.

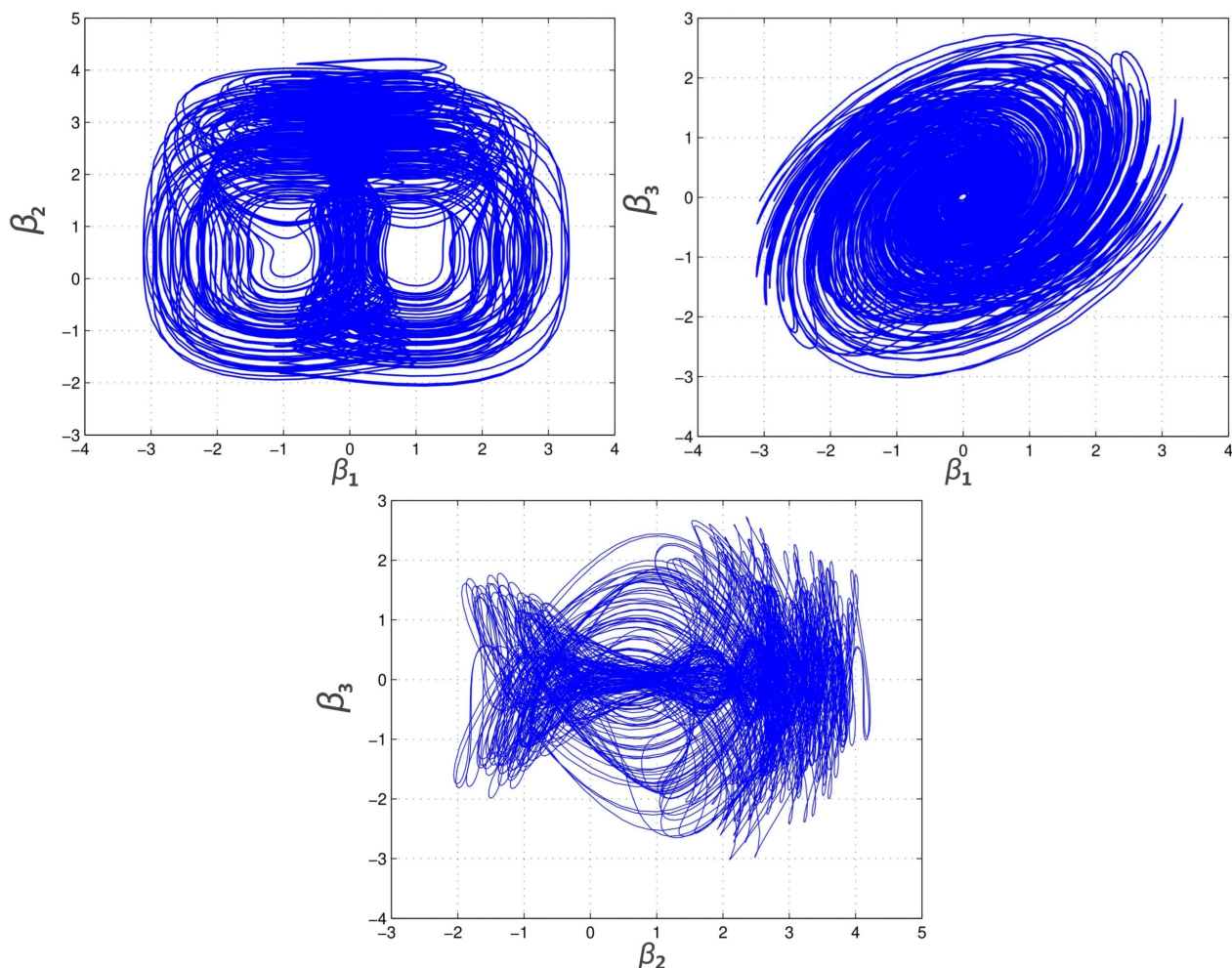


Figure 1. Phase plots of the CMS (1) in different plane.

3. Dynamics of the System

In this section, we numerically investigated the dynamical behaviors of the CMS (1) using the LE spectrum and bifurcation diagrams. Figures 2–4 show the LE spectrum and the bifurcation diagrams of the CMS (1) with respect to parameter a, b, c , respectively.

Obviously, when $a \in [0, 1], b \in [-3, -1], c \in [0, 1]$ the behavior of the CMS (1) is either chaotic or periodic.

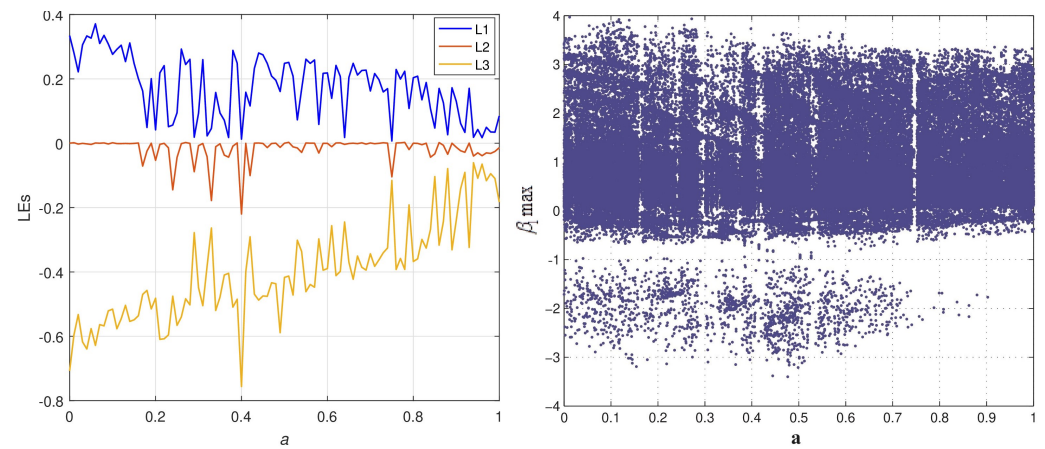


Figure 2. LE spectrum and bifurcation diagram for $a \in [0, 1]$.

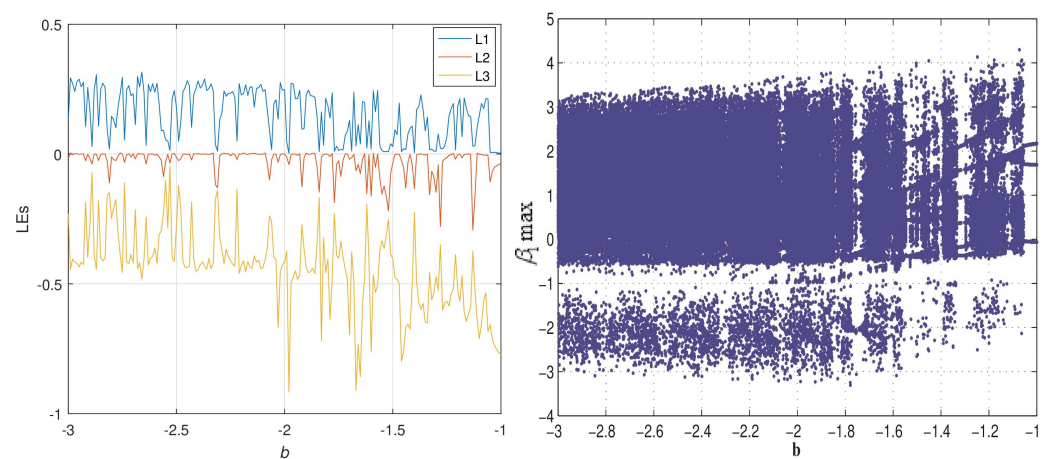


Figure 3. LE spectrum and bifurcation diagram for $b \in [-3, -1]$.

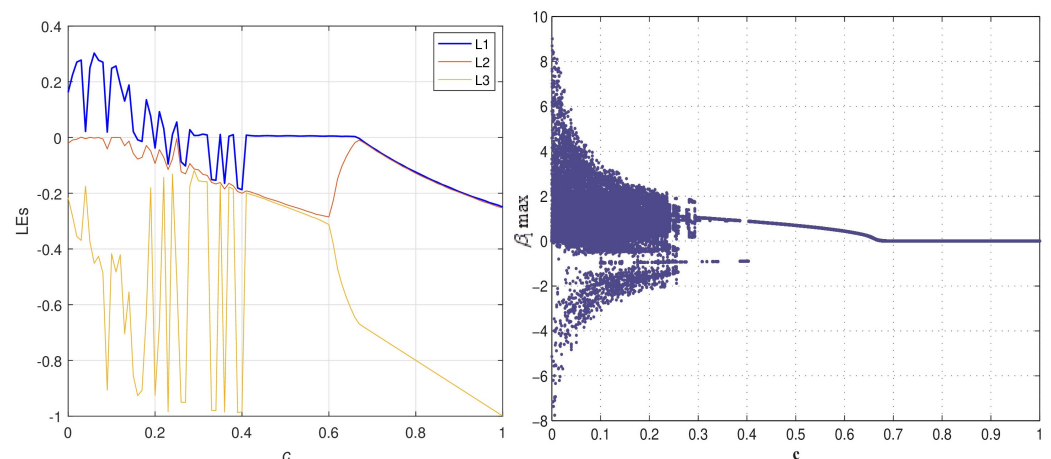


Figure 4. LE spectrum and bifurcation diagram for $c \in [0, 1]$.

3.1. Varying the Parameter a

Figure 2 shows the LE spectrum and the bifurcation diagram of the CMS (1) with respect to parameter a . We fix the values of the parameters b, c as $(b, c) = (-2, 0.1)$. We vary parameter a in the range $[0, 1]$. When $a \in [0, 1]$, we can see from Figure 2 that the CMS (1) has only one positive LEs ($L_1 > 0$) and two negative LEs ($L_{2,3} \leq 0$). Thus, the CMS (1) is chaotic and generates a chaotic attractor. The values of LEs when $a = 0.1$ are:

$$L_1 = 0.2762, L_2 = 0, L_3 = -0.5156 \quad (8)$$

and the values of LEs when $a = 0.44$ are:

$$L_1 = 0.2804, L_2 = 0, L_3 = -0.4875 \quad (9)$$

and the values of LEs when $a = 0.66$ are:

$$L_1 = 0.2483, L_2 = 0, L_3 = -0.4226 \quad (10)$$

and the values of LEs when $a = 0.8$ are:

$$L_1 = 0.2037, L_2 = 0, L_3 = -0.3676 \quad (11)$$

and the values of LEs when $a = 0.9$ are:

$$L_1 = 0.13, L_2 = -0.0124, L_3 = -0.239 \quad (12)$$

when the saving amount (parameter a) exhibits chaotic behavior, it introduces unpredictability into the monetary system, as savings levels fluctuate irregularly and are highly sensitive to initial conditions or external influences. This instability can disrupt economic variables such as investment demand, interest rates, and price levels, making it challenging for policymakers to implement effective monetary strategies. Chaotic savings behavior can hinder the predictability of capital availability for investments, exacerbating market volatility and increasing the risk of financial crises. In practical terms, this could occur when economic agents alternate unpredictably between high and low savings due to factors like fluctuating economic confidence, policy changes, or global financial shocks.

On the other hand, the periodic behavior of savings (parameter a) implies a stable, cyclical pattern that aligns with natural economic rhythms. This predictability fosters a more stable monetary system, as consistent savings levels ensure reliable capital availability for investments and smoother economic interactions. Periodic savings behavior supports balanced economic growth and allows central banks to design targeted policies that correspond to these regular cycles. For example, seasonal savings patterns, such as higher savings during specific times of the year due to cultural or economic factors, help maintain stable investment demand and price levels, reducing the likelihood of abrupt economic disruptions.

3.2. Varying the Parameter b

Figure 3 shows the LE spectrum and the bifurcation diagram of the CMS (1) with respect to parameter b . We fix the values of the parameters a, c as $(a, c) = (0.5, 0.1)$. We vary parameter b in the range $[-3, -1]$. We can identify the dynamic behavior of the CMS (1) when parameter b varies in the range $[-3, -1]$ as follows: let $b \in [-3, -1]$ and we define:

$$A = [-3, -1.54) \cup (-1.52, -1.05)$$

$$B = [-1.54, -1.52] \cup [-1.05, -1]$$

When $b \in A$, we can see from Figure 3 that the CMS (1) has only one positive LE ($L_1 > 0$) and two negative LEs ($L_{2,3} \leq 0$). Thus, the CMS (1) is chaotic and generates a chaotic attractor in this range of parameter. The values of LEs when $b = -2.6$ are:

$$L_1 = 0.2385, L_2 = 0, L_3 = -0.3615 \quad (13)$$

and the values of LEs when $b = -2.4$ are:

$$L_1 = 0.2817, L_2 = 0, L_3 = -0.4461 \quad (14)$$

and the values of LEs when $b = -1.89$ are:

$$L_1 = 0.2467, L_2 = 0, L_3 = -0.4579 \quad (15)$$

and the values of LEs when $b = -2.1$ are:

$$L_1 = 0.2417, L_2 = 0, L_3 = -0.4378 \quad (16)$$

when $b \in B$, we can see from Figure 3 that the CMS (1) has only one zero LEs ($L_1 = 0$) and two negative Lyapunov exponents ($L_{2,3} < 0$). Thus, the CMS (1) is periodic and generates a periodic attractor. The values of LEs when $b = -1.54$ are:

$$L_1 = 0, L_2 = -0.1224, L_3 = -0.3633 \quad (17)$$

and the values of LEs when $b = -1.52$ are:

$$L_1 = 0, L_2 = -0.2185, L_3 = -0.2517 \quad (18)$$

and the values of LEs when $b = -1.04$ are:

$$L_1 = 0, L_2 = -0.06781, L_3 = -0.7111 \quad (19)$$

and the values of LEs when $b = -1$ are:

$$L_1 = 0, L_2 = -0.03768, L_3 = -0.7733 \quad (20)$$

when the per-investment cost (parameter b) exhibits chaotic behavior, it creates an unpredictable and erratic economic environment. Such instability leads to irregular fluctuations in investment levels, as investors face uncertainty about the costs associated with their activities. This unpredictability can reduce overall investment demand, as businesses and individuals become cautious about allocating resources to projects with uncertain costs. In turn, this can destabilize key monetary variables like interest rates and price levels, amplifying market volatility and increasing the risk of economic downturns. For instance, chaotic investment costs can lead to abrupt changes in economic productivity, making it challenging for policymakers to stabilize the monetary system through conventional interventions.

In contrast, the periodic behavior of per-investment costs (parameter b) indicates a regular and predictable pattern, which fosters a more stable economic environment. Predictable investment costs enable businesses to plan their financial activities more effectively, encouraging consistent investment demand and promoting economic growth. A periodic cost structure supports stability in monetary variables, as the steady demand for capital ensures smoother adjustments in interest rates and price levels. For example, if investment costs fluctuate seasonally in a predictable manner, investors can align their strategies with these cycles, leading to enhanced economic resilience and balanced growth in the monetary system.

3.3. Varying the Parameter c

Figure 4 shows the LE spectrum and the bifurcation diagram of the CMS (1) with respect to parameter c . We fix the values of the parameters a, b as $(a, b) = (0.5, -2)$. We vary the parameter c in the range $[0, 1]$. We can identify the dynamic behavior of the CMS (1) when parameter c varies in the range $[0, 1]$ as follows: let $c \in [0, 1]$ and define:

$$\begin{aligned} A &= [0, 0.157) \cup (0.171, 0.1967) \\ B &= [0.2029, 0.2225) \cup (0.24, 0.254) \\ C &= [0.157, 0.171) \cup (0.1967, 0.2029) \cup [0.2225, 0.24) \\ D &= (0.254, 0.278) \cup (0.32, 0.37) \cup (0.38, 0.41) \cup [0.67, 1] \\ E &= (0.29, 0.32] \cup [0.37, 0.38] \cup (0.41, 0.67) \end{aligned}$$

When $c \in A \cup B$, we can see from Figure 4 that the CMS (1) has only one positive LEs ($L_1 > 0$) and two negative LEs ($L_{2,3} \leq 0$). Thus, the CMS (1) is chaotic and generates a chaotic attractor in this range of parameter. The values of LEs when $c = 0.05$ are:

$$L_1 = 0.2497, L_2 = 0, L_3 = -0.3732 \quad (21)$$

and the values of LEs when $c = 0.18$ are:

$$L_1 = 0.1356, L_2 = -0.0287, L_3 = -0.6314 \quad (22)$$

and the values of LEs when $c = 0.25$ are:

$$L_1 = 0.05564, L_2 = 0, L_3 = -0.5094 \quad (23)$$

when $c \in C \cup D$, we can see from Figure 4 that all the LEs of the CMS (1) are negative ($L_{1,2,3} < 0$). Thus, the CMS (1) converges to an equilibrium point. The values of LEs when $c = 0.27$ are:

$$L_1 = -0.102, L_2 = -0.1305, L_3 = -0.9509 \quad (24)$$

and the values of LEs when $c = 0.8$ are:

$$L_1 = -0.1286, L_2 = -0.1286, L_3 = -0.7997 \quad (25)$$

when $c \in E$, we can see from Figure 4 that the CMS (1) has only one zero LEs ($L_1 = 0$) and two negative LEs ($L_{2,3} < 0$). Thus, the CMS (1) is periodic and generates periodic attractor. The values of LEs when $c = 0.30$ are:

$$L_1 = 0, L_2 = -0.1161, L_3 = -0.1555 \quad (26)$$

and the values of LEs when $c = 0.45$ are:

$$L_1 = 0, L_2 = -0.2135, L_3 = -0.219 \quad (27)$$

and the values of LEs when $c = 0.6$ are:

$$L_1 = 0, L_2 = -0.2852, L_3 = -0.31231 \quad (28)$$

and the values of LEs when $c = 0.65$ are:

$$L_1 = 0, L_2 = -0.04096, L_3 = -0.6118 \quad (29)$$

Figures 5–7 show the different attractors of CMS (1) for different values of the system parameters a, b, c , respectively.

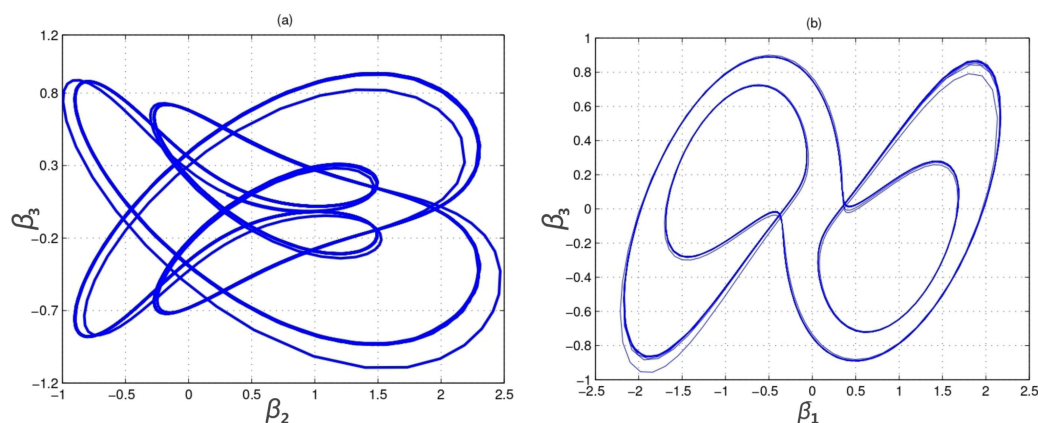


Figure 5. Cont.

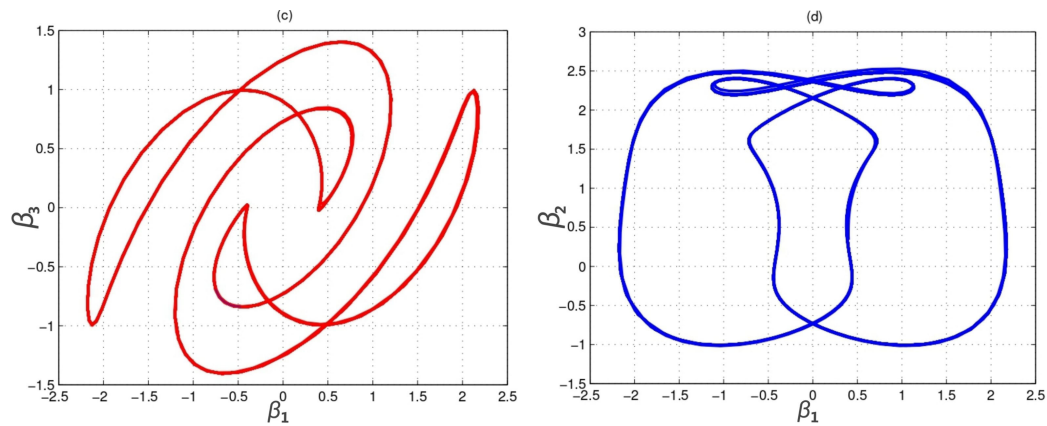


Figure 5. Behaviors of CMS (1): (a) periodic for $b = -1.02$, (b) periodic for $b = -1.52$, (c) periodic for $b = -1$, and (d) periodic for $b = -1.54$.

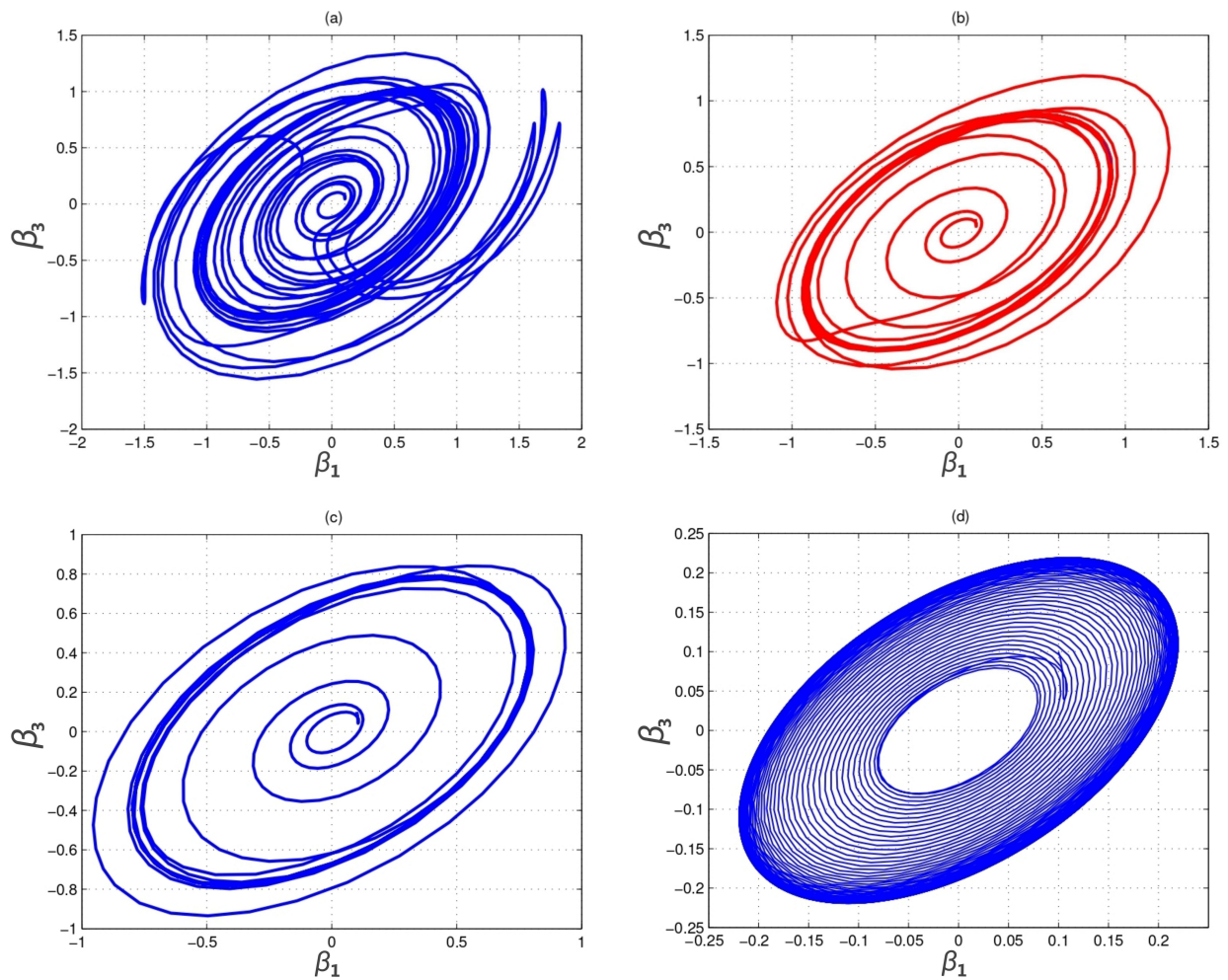


Figure 6. Behaviors of CMS (1): (a) periodic for $c = 0.30$, (b) periodic for $c = 0.38$, (c) periodic for $c = 0.45$, and (d) periodic for $c = 0.65$.

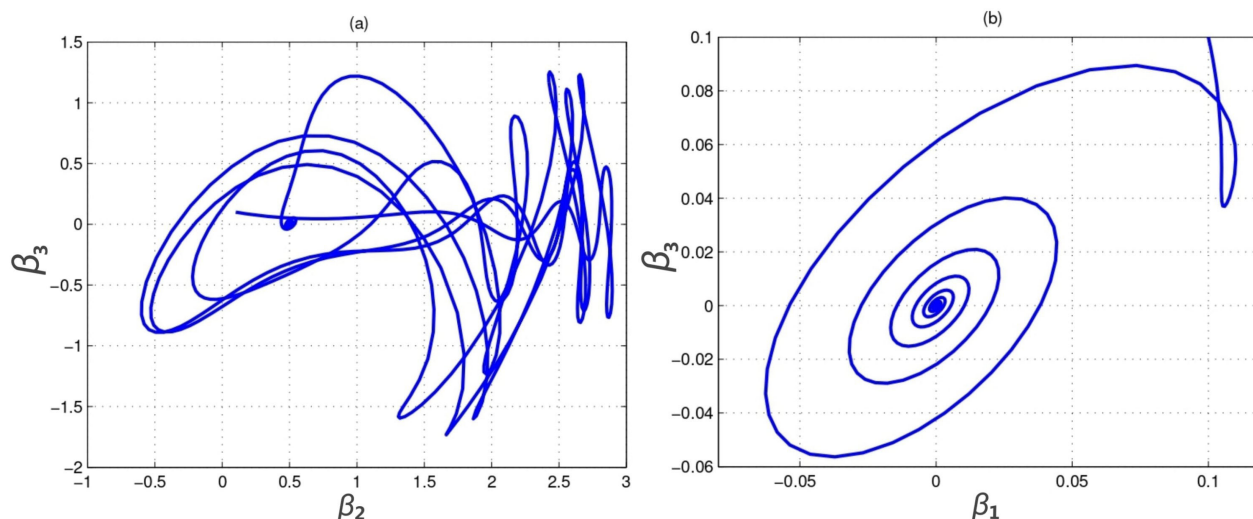


Figure 7. Behaviors of CMS (1) converge to an equilibrium point: (a) for $c = 0.27$, (b) periodic for $c = 0.80$.

When the demand elasticity of commercials (parameter c) exhibits chaotic behavior, it introduces significant unpredictability into how consumers respond to changes in prices for goods and services. Such erratic elasticity can destabilize the monetary system, as fluctuating consumer demand disrupts the balance of supply and demand in the market. This unpredictability makes it challenging for businesses to set optimal pricing strategies, potentially leading to overproduction or underproduction. Consequently, the instability can ripple through the economy, affecting investment decisions, price levels, and inflation rates. For instance, chaotic elasticity may cause sudden and unexpected shifts in consumer behavior, leading to volatile market conditions and complicating monetary policy planning.

On the other hand, the periodic behavior of demand elasticity (parameter c) implies a predictable, cyclical pattern in consumer responsiveness to price changes. This stability allows businesses to anticipate and adapt their pricing and production strategies in line with consistent demand cycles. A periodic elasticity pattern supports the monetary system by maintaining smoother market dynamics, stabilizing price levels, and ensuring balanced economic activity. For example, predictable shifts in consumer demand during specific seasons or economic phases enable policymakers to design monetary strategies that align with these cycles, fostering a more resilient and sustainable economic environment.

3.4. Multistability in the CMS (1)

To enhance the examination of coexistence attractors and other system characteristics, introducing a disturbance to the initial conditions is essential, while maintaining constant system parameters [29,30]. The CMS (1) is invariant under the change of coordinates (coordinate transformation).

$$(\beta_1, \beta_2, \beta_3) \longrightarrow (-\beta_1, \beta_2, -\beta_3) \quad (30)$$

Therefore, CMS (1) is symmetric. The symmetrical property of CMS (1) leads to the appearance of multistability. Different coexisting attractors are reported in Figures 8–11.

Multistability in monetary models represents the coexistence of multiple stable states under identical parameter conditions, highlighting the system's sensitivity to initial conditions and external perturbations. This phenomenon has profound implications for the monetary system, as it can lead to abrupt transitions between different economic states, such as moving from stability to instability or vice versa. For instance, a monetary system with multistability may oscillate between steady growth and recession depending on minor shifts in savings, investments, or policy decisions. This behavior underscores the challenges policymakers face in steering the economy, as slight changes in economic variables or

external shocks can push the system into entirely different trajectories. Understanding and managing multistability is crucial for designing robust monetary policies that prevent undesirable economic outcomes and enhance the resilience of the financial system.

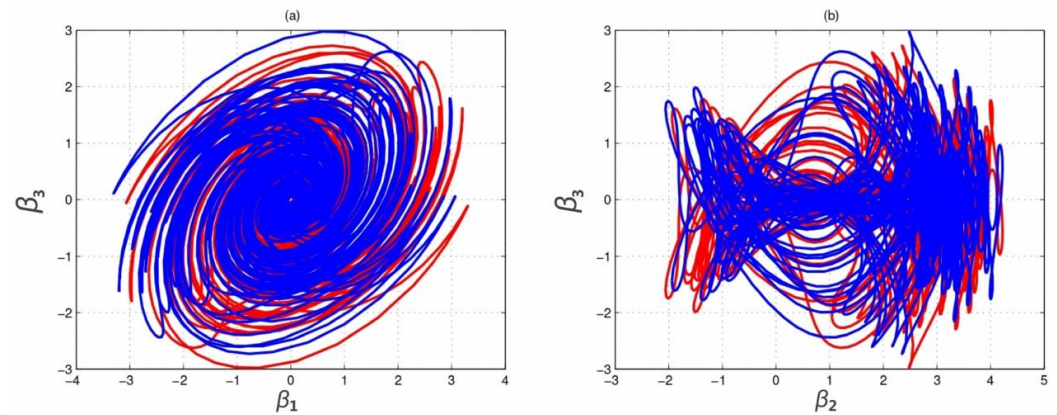


Figure 8. Coexistence of two chaotic attractors with different initial values, where $a = 0.5$, $b = -2$, $c = 0.1$: (a) phase diagram in $\beta_1 - \beta_3$ plane and (b) phase diagram in $\beta_2 - \beta_3$ plane, where the initial states are chosen as $\beta_1(0) = 0.4$, $\beta_2(0) = 0.3$, $\beta_3(0) = 0.1$ (red orbit) and $\beta_1(0) = -0.1$, $\beta_2(0) = 0.1$, $\beta_3(0) = -0.1$ (blue orbit).

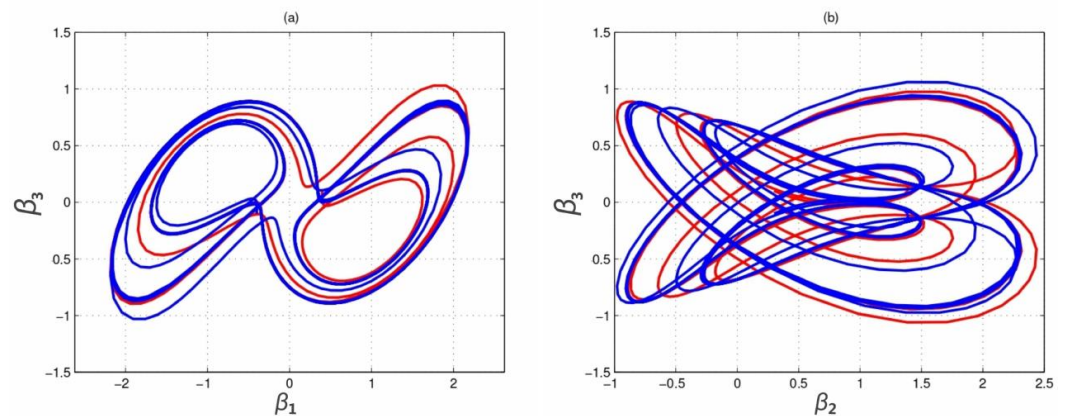


Figure 9. Coexistence of two periodic attractors with different initial values for (a) $b = -1$, (b) $b = -1.02$, where the initial states are chosen as $\beta_1(0) = 0.4$, $\beta_2(0) = 0.3$, $\beta_3(0) = 0.1$ (red orbit) and $\beta_1(0) = -0.4$, $\beta_2(0) = 0.3$, $\beta_3(0) = -0.1$ (blue orbit).

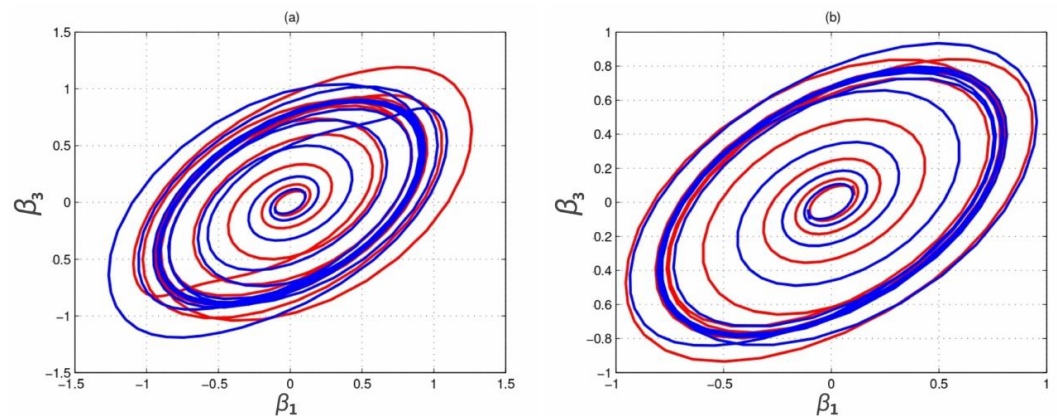


Figure 10. Coexistence of two periodic attractors with different initial values for (a) $c = 0.38$, (b) $c = 0.45$, where the initial states are chosen as $\beta_1(0) = 0.1$, $\beta_2(0) = 0.1$, $\beta_3(0) = 0.1$ (red orbit) and $\beta_1(0) = -0.1$, $\beta_2(0) = 0.1$, $\beta_3(0) = -0.1$ (blue orbit).

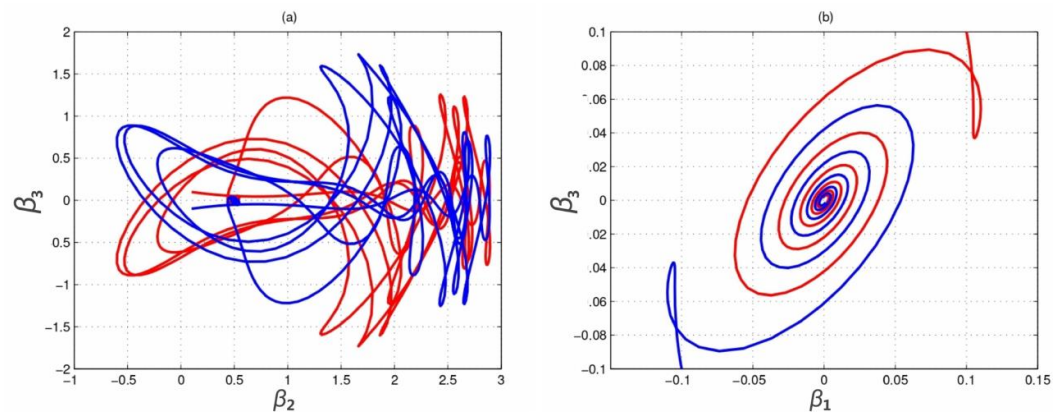


Figure 11. Coexistence of two attractors (converge to an equilibria) with different initial values for (a) $c = 0.27$, (b) $c = 0.80$, where the initial states are chosen as $\beta_1(0) = 0.4, \beta_2(0) = 0.3, \beta_3(0) = 0.1$ (red orbit) and $\beta_1(0) = -0.1, \beta_2(0) = 0.1, \beta_3(0) = -0.1$ (blue orbit).

4. RBFNN Controller Design

In this section, the RBFNN controller design method is introduced. Training data must be available for neural network design. Therefore, the training data are obtained according to a non-linear controller design method.

4.1. Non-Linear Controller Design Method

Consider the equation model of CMS (1).

$$\begin{cases} \dot{\beta}_1 = -a\beta_1 + \beta_3 + \beta_1\beta_2 + b\beta_2\beta_3 + u_1 \\ \dot{\beta}_2 = -c\beta_2 + 1 - \beta_1^2 + u_2 \\ \dot{\beta}_3 = -\beta_1 - \beta_3 - b\beta_1\beta_2 + u_3 \end{cases} \quad (31)$$

where u_1, u_2, u_3 are non-linear controllers. Error calculation is required for design:

$$\begin{cases} e_1 = \beta_1 - \beta_1^* \\ e_2 = \beta_2 - \beta_2^* \\ e_3 = \beta_3 - \beta_3^* \end{cases} \quad (32)$$

where $\beta_1^*, \beta_2^*, \beta_3^*$ are the desired values. If derivative from Equation (32):

$$\begin{cases} \dot{e}_1 = \beta_1 - \beta_1^* \\ \dot{e}_2 = \beta_2 - \beta_2^* \\ \dot{e}_3 = \beta_3 - \beta_3^* \end{cases} \Rightarrow \begin{cases} \dot{e}_1 = -a\beta_1 + \beta_3 + \beta_1\beta_2 + b\beta_2\beta_3 + u_1 - \dot{\beta}_1^* \\ \dot{e}_2 = -c\beta_2 + 1 - \beta_1^2 + u_2 - \dot{\beta}_2^* \\ \dot{e}_3 = -\beta_1 - \beta_3 - b\beta_1\beta_2 + u_3 - \dot{\beta}_3^* \end{cases} \quad (33)$$

Theorem 1. The equations of the dynamic system (32) with arbitrary initial conditions move asymptotically to the desired points if the controllers of the system are as follows.

$$\begin{cases} u_1 = +a\beta_1 - \beta_3 - \beta_1\beta_2 - b\beta_2\beta_3 + \beta_1^* + \alpha_1 e_1 \\ u_2 = +c\beta_2 - 1 + \beta_1^2 + \beta_2^* + \alpha_2 e_2 \\ u_3 = +\beta_1 + \beta_3 + b\beta_1\beta_2 + \beta_3^* + \alpha_3 e_3 \end{cases} \quad (34)$$

where $\alpha_1, \alpha_2, \alpha_3$ are the control gain of Equation (34).

Proof. Consider the candidate Lyapunov function as follows:

$$V(e) = 0.5(e_1^2 + e_2^2 + e_3^2) > 0 \quad (35)$$

By derivative from the candidate Lyapunov Equation (35)

$$\begin{aligned}
 \dot{V} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 \\
 &\Rightarrow e_1(-a\beta_1 + \beta_3 + \beta_1\beta_2 + b\beta_2\beta_3 + u_1 - \dot{\beta}_1^*) \\
 &\quad + e_2(-c\beta_2 + 1 - \beta_1^2 + u_2 - \dot{\beta}_2^*) \\
 &\quad + e_3(-\beta_1 - \beta_3 - b\beta_1\beta_2 + u_3 - \dot{\beta}_3^*)
 \end{aligned} \tag{36}$$

by substituting Equation (34) in Equation (35)

$$\begin{aligned}
 \dot{V} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 \\
 &\Rightarrow e_1(\alpha_1 e_1 - \dot{\beta}_1^*) + e_2(\alpha_2 e_2 - \dot{\beta}_2^*) + e_3(\alpha_3 e_3 - \dot{\beta}_3^*)
 \end{aligned} \tag{37}$$

The values $\beta_{1,2,3}^* = 0$ are desirable, so they become $\dot{\beta}_{1,2,3}^* = 0$. Also, if the values are $\alpha_{1,2,3} < 0$, then the proof is complete.

$$\dot{V} = \alpha_1 e_1^2 + \alpha_2 e_2^2 + \alpha_3 e_3^2 < 0 (\alpha_{1,2,3} < 0) \tag{38}$$

□

4.2. Numerical Simulation

MATLAB software 2014a is used in the numerical simulation. Chaotic dynamic equations of CMS (1) with the initial conditions $\beta_1 = 0.1, \beta_2 = 0.1, \beta_3 = 0.1$, with controller gain equal to $\alpha_1 = -4, \alpha_2 = -4, \alpha_3 = -4$, have been selected. Figure 12 shows the system's state behavior both before and after the non-linear controller is activated. Initially, the system exhibits chaotic behavior, characterized by irregular, unpredictable dynamics. After the activation of the controller (at $t = 15$), the chaotic dynamics are stabilized, and the system converges to the desired state. This demonstrates the effectiveness of the designed non-linear controller in managing and mitigating chaos in the system. Figure 13 displays the control signals corresponding to the non-linear controller. The signals illustrate the controller's input before and after activation. The graph indicates when the controller begins to influence the system (post $t = 15$), showing how the control inputs change to stabilize the chaotic behavior.

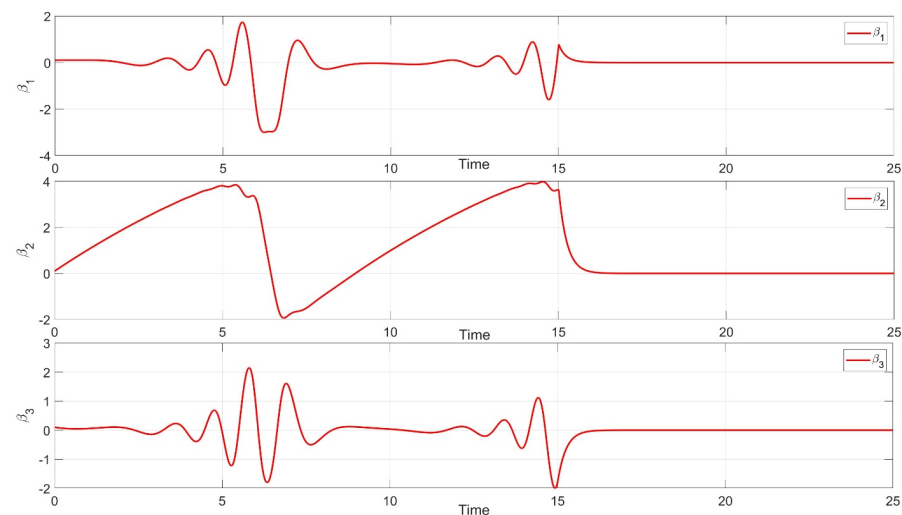


Figure 12. The behavior of the chaotic dynamic Equation (31) before and after the activation of the non-linear controller.

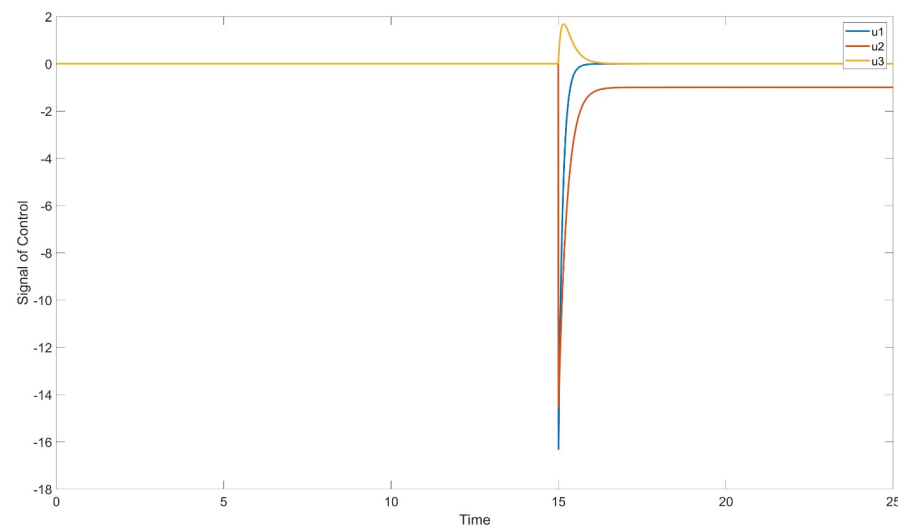


Figure 13. Signal of controller for before and after activation.

4.3. Implementation of RBFNN Controller

According to the block diagram of Figure 14, the design of the RBFNN controller will be executed with the help of the non-linear controller. The training data are based on system errors and controller signals. But network architecture is also very important [31–33]. Considering the complexity of chaotic dynamic systems, it is better to use the data of three error vectors and the non-linear controller signal for training data of RBFNN (See Figure 15).

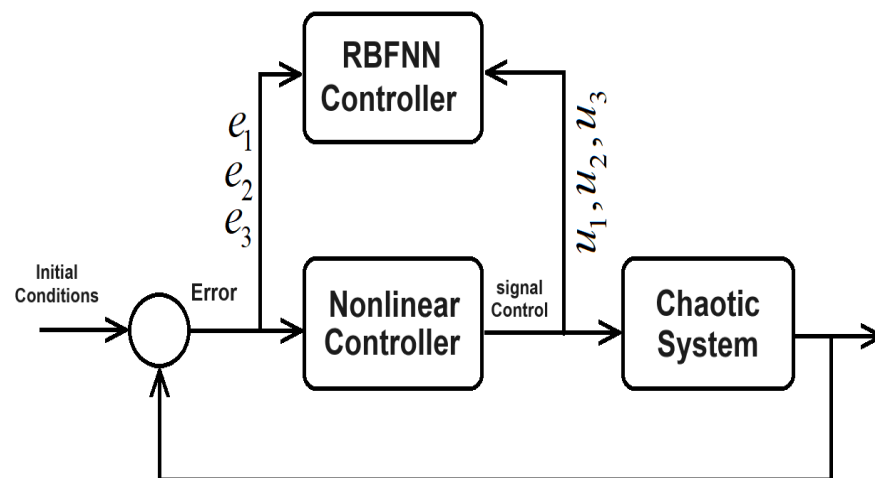


Figure 14. Block diagram of the implementation of the RBFNN controller.

As seen in Figure 15, the global RBFNN controller is composed of three sub-RBFNNs. Control vectors and three error vectors enter the neural network training architecture. This method gives better answers compared to the method of training an error vector and a control vector. The parameters P1 to P5 in the neural network can be adjusted to achieve an acceptable training error. The architecture of the proposed radial basis function neural network is described in Table 1.

Figure 16 shows the output results of neural network training. It can be seen that zero error is not obtained, but its value is very small. The test data are used to check the performance of the neural network. Figure 17 shows the testing phase of the controller, where it demonstrates a low error rate, indicating that the training process successfully aligned the controller's behavior with the desired system output. Although the error

is not entirely zero, its negligible magnitude confirms the controller's effectiveness in approximating the target control.

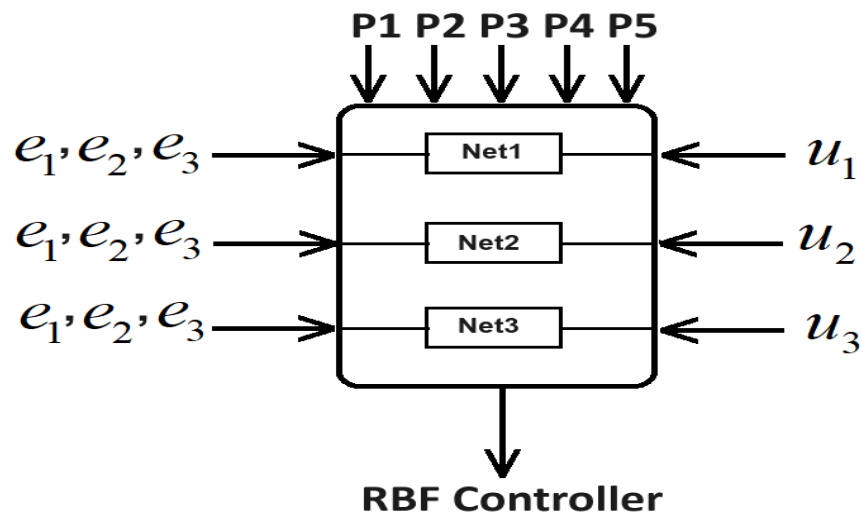


Figure 15. RBFNN controller training architecture.

Table 1. Description of RBFNN architecture.

Number	RBF Parameters
250	Total number of trainings
125	Number of training pairs
125	Number of test pairs
0	Mean squared error goal (P1)
0.2	Spread of radial basis functions (P2)
150	Maximum number of neurons (P3)
30	Number of neurons to add between displays (P4)
120	Epoch (5)

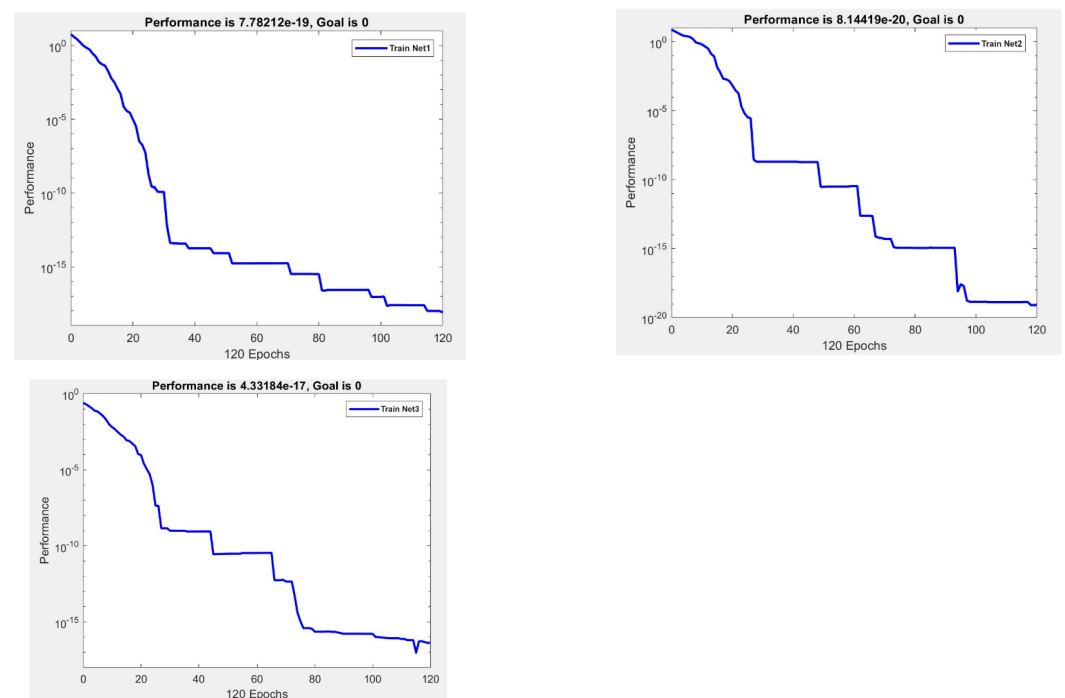


Figure 16. Performance results of the training of the proposed RBFNN controller.

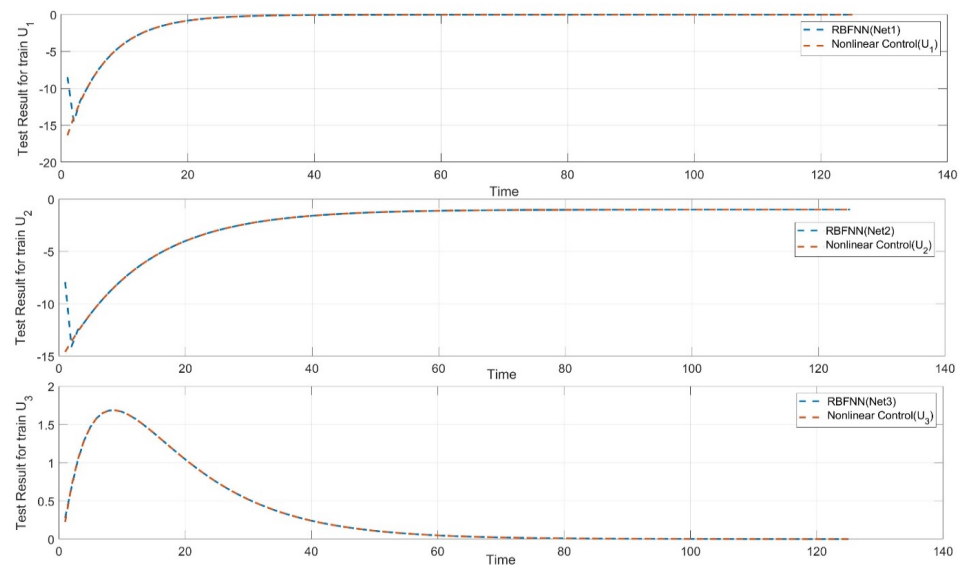


Figure 17. Test results of the RBFNN controller with the proposed architecture.

To check the performance of the controller, the initial conditions and time of the controller will be changed. So, the initial conditions are $\beta_{1,0} = 0.13$, $\beta_{2,0} = 0.14$, $\beta_{3,0} = 0.15$. As seen in Figure 18, the RBFNN controller is activated at time $t = 8$. Figure 18 shows the neural network controller signal. Due to the well-formed behavior of the RBFNN controller, it is possible to implement it in the real world.

Figure 19 illustrates the control signals generated by the RBFNN during operation. The smooth and well-regulated signal profile demonstrates the efficiency of the controller, avoiding excessive oscillations or abrupt spikes. Such characteristics are crucial for real-world implementation, as they ensure the system operates within manageable parameters without overwhelming computational or mechanical resources. Together, these figures validate the effectiveness of the RBFNN in managing chaotic dynamics and stabilizing the monetary system.

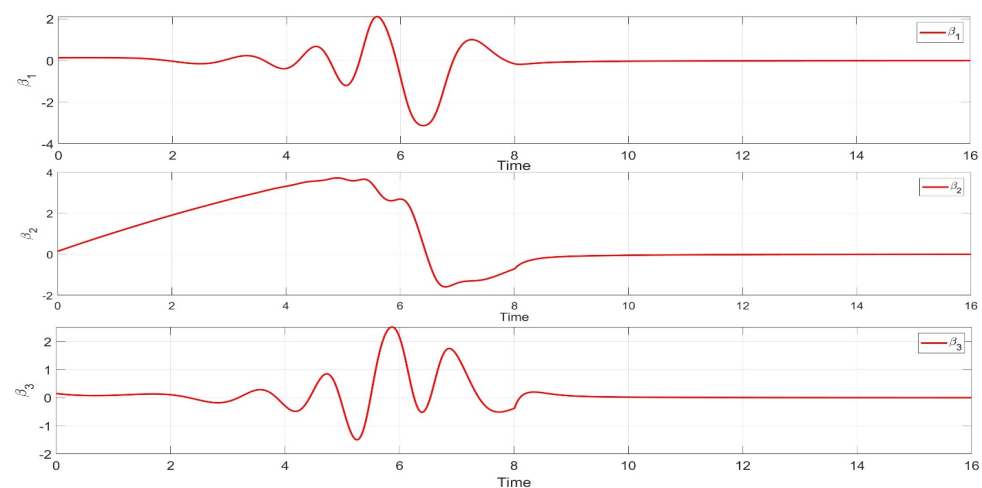


Figure 18. The behavior of chaotic dynamic system states before and after the RBFNN controller.

In the other part, the performance simulation of the proposed method will be investigated by reducing the number of RBFNN controllers. Consider the equations of chaotic dynamic system (31) as below. The RBFNN controller is only applied to the second-row equations.

$$\begin{cases} \dot{\beta}_1 = -a\beta_1 + \beta_3 + \beta_1\beta_2 + b\beta_2\beta_3 \\ \dot{\beta}_2 = -c\beta_2 + 1 - \beta_1^2 + u_2 \\ \dot{\beta}_3 = -\beta_1 - \beta_3 - b\beta_1\beta_2 \end{cases} \quad (39)$$

The initial conditions in this part of the simulation are equal to $\beta_{1,0} = 0.21$, $\beta_{2,0} = 0.31$, $\beta_{3,0} = 0.41$, and the controller structure will be unchanged. In Figure 20, it can be seen that the neural network controller has been applied to the system since time $t = 34$. Figure 21 shows the single RBFNN controller signal. In the behavior of the RBFNN controller, the radial basis function does not have sharp points and high amplitude, so this controller can be implemented in processors.

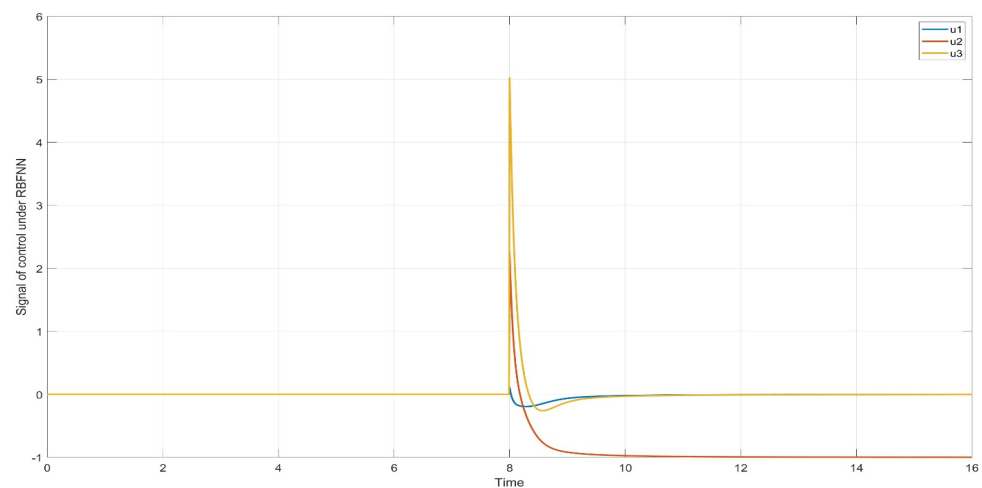


Figure 19. The signal of the RBFNN controller.

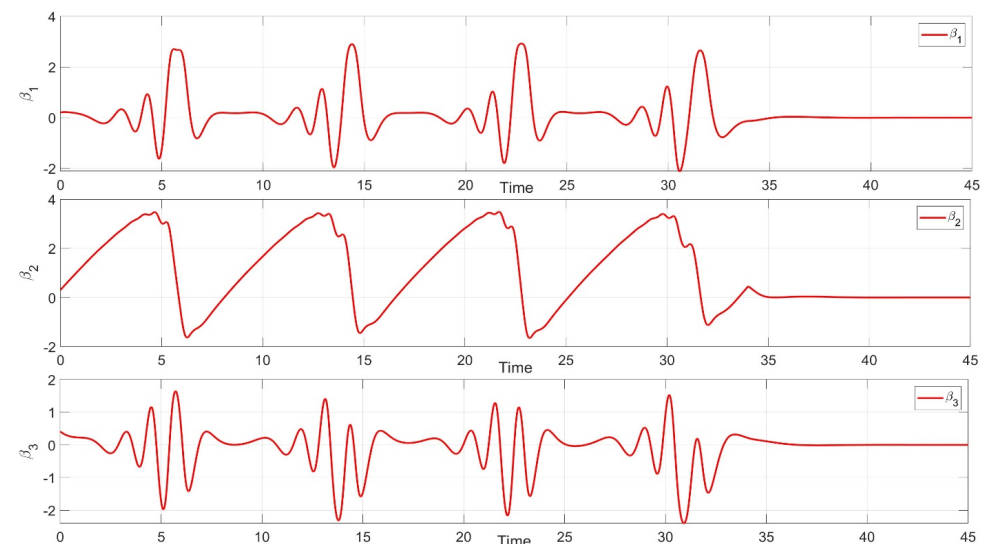


Figure 20. The behavior of the chaotic dynamic Equation (32) before and after the single RBFNN controller is activated.

The RBFNN controller effectively stabilizes such chaotic behaviors by utilizing a well-trained neural architecture capable of learning non-linear patterns [34,35]. This adaptability makes it suitable for implementation in practical scenarios where monetary systems are subject to unpredictable external influences, such as policy shifts or global economic shocks.

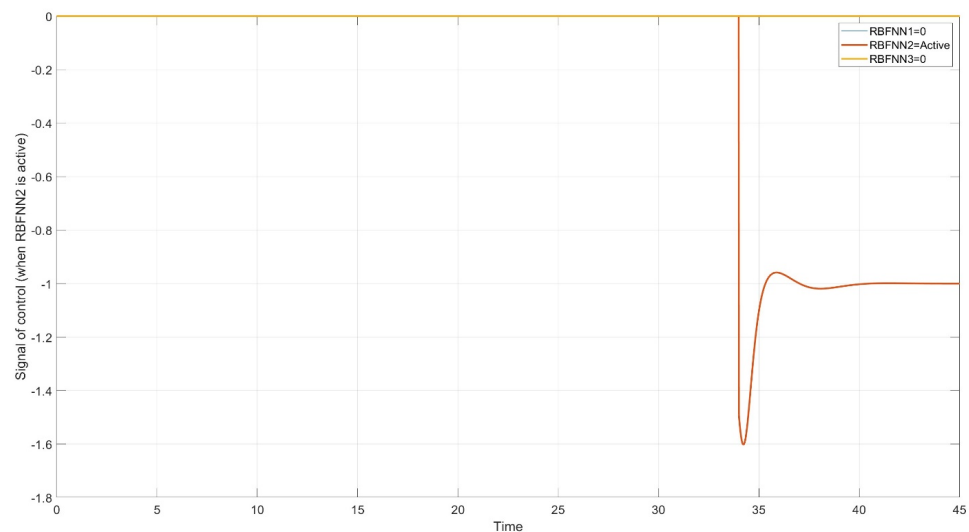


Figure 21. The signal of the RBFNN2 controller while it is nonactive RBFNN1 = 0 and RBFNN3 = 0.

Additionally, the architecture of the RBFNN controller allows for easy integration with existing financial monitoring systems, where real-time data can be utilized to adjust the controller's parameters dynamically. This adaptability ensures that the monetary system remains stable under varying conditions, making it a valuable tool for policymakers aiming to maintain economic equilibrium.

5. Conclusions

This work analyzes the dynamic behavior of the proposed 3D chaotic monetary system by examining equilibrium points and their stability, calculating Lyapunov exponents, and creating bifurcation diagrams. The system exhibits various behaviors such as chaotic, periodic, and equilibrium states, influenced by different parameter values. Furthermore, a RBFNN controller design method was designed. With the help of a non-linear controller, the training data for the neural network was made. The proposed architecture was to use three error vectors and one control vector of a RBFNN. In fact, to train the neural network, you should prepare questions and answers and use them for training. In this way, three error vectors have been selected as questions and one control vector as the correct answer. After training the neural network, the controller with different initial conditions and a different activation time was used. In this section, the initial conditions and activation time of the controller were changed again. For instance, with initial conditions $\beta_1 = 0.1, \beta_2 = 0.1, \beta_3 = 0.1$, and control gains $\alpha_1 = -4, \alpha_2 = -4, \alpha_3 = -4$, the system transitions to stability after $t = 15$. These results validate the effectiveness of the RBFNN controller and its potential application in real-world economic and financial modeling. Finally, the numerical simulation results showed that the proposed architecture for controller design has good performance. Future studies could enhance the controller by incorporating derivatives of error terms, potentially improving its precision and adaptability.

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