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Optimizing Traveler Behavior Between MADINA and JEDDA Using UPPAAL Stratego: A Stochastic Priced Timed Games Approach

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Abstract: This study looks at how travelers move between MADINA and JEDDA, using the UPPAAL Stratego tool to tackle the complexities of urban mobility. As cities grow, effective transportation planning becomes more challenging. Travelers have three options: car, bus, and train. The choices for car and bus travel are impacted by traffic conditions, which can vary between heavy and light, affecting both travel time and cost. We propose a detailed mathematical model that captures all possible scenarios related to these travel options, incorporating the uncertainties of real life. This allows us to simulate different traffic situations. By using UPPAAL Stratego, we evaluate three strategies: the Safe Strategy, which minimizes risk; the Fast Strategy, which aims to reduce travel time; and the Fast and Safe Strategy, which seeks a balance between speed and safety. This paper starts with an introduction to the Stochastic Priced Timed Games approach, highlighting its relevance in modeling dynamic travel environments. We then provide an overview of UPPAAL Stratego, showcasing its abilities in generating, optimizing, and comparing strategies. Next, we outline our mathematical model, explaining the assumptions, parameters, and data sources we used. Our simulation results illustrate how each strategy performs under different conditions, shedding light on traveler preferences and behaviors. The findings underscore the significance of accounting for traffic variability in travel planning and offer important insights for urban transportation policies aimed at improving the traveler experience and optimizing resource use. Additionally, we emphasize the theoretical contributions of our model by demonstrating its applicability to real-world scenarios and its potential to inform future research in urban mobility optimization. Ultimately, this research adds to the growing knowledge of smart transportation systems, demonstrating how formal mathematical modeling can address complex real-world challenges and inform future urban mobility strategies.

Keywords: traveler behavior; stochastic priced timed games; UPPAAL Stratego; mathematical modeling; traffic optimization; urban transportation

MSC: 14D15

check for updates

Citation: Krichen, M.; Harbaoui, A. Optimizing Traveler Behavior Between MADINA and JEDDA Using UPPAAL Stratego: A Stochastic Priced Timed Games Approach. *Mathematics* **2024**, *12*, 3421. https://doi.org/10.3390/math12213421

Academic Editor: Noureddine Lakouari

Received: 26 September 2024
Revised: 30 October 2024
Accepted: 30 October 2024
Published: 31 October 2024

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1. Introduction

In recent years, the optimization of urban transportation has become a critical area of research, particularly in rapidly developing regions like Saudi Arabia [1,2]. Efficient traveler behavior between major cities, such as MADINA and JEDDA, is essential for improving mobility, reducing travel times, and minimizing costs. With multiple transportation options available, including cars, buses, and trains, understanding the dynamics of traveler choices and the impact of varying conditions—such as traffic congestion—can significantly enhance transportation planning and policy-making [3]. This study employs a

mathematical modeling approach to analyze and optimize traveler behavior under different scenarios, providing valuable insights for urban planners and decision-makers.

Mathematical modeling plays a crucial role in representing complex systems and understanding their behavior through formalized frameworks. By abstracting real-world scenarios into mathematical constructs, researchers can simulate and analyze various outcomes, facilitating informed decision-making [4]. In the context of urban transportation, mathematical models can account for uncertainties, enabling the evaluation of different strategies and their implications on traveler behavior [5–9].

This research specifically utilizes the framework of Stochastic Priced Timed Games (SPTGs), which allows for the incorporation of both stochastic elements and timing constraints [10–12]. This approach is particularly relevant for modeling the unpredictable nature of traffic conditions and travel times, as well as the associated costs of different travel options. By leveraging this framework, we can capture the inherent uncertainties in travel scenarios, leading to more robust optimization strategies.

The UPPAAL Stratego tool is employed in this study for its capabilities in strategy generation and optimization within the context of timed games [13–18]. This tool enables the modeling of complex systems with real-time constraints and the evaluation of different strategies under various conditions (https://uppaal.org/casestudies/stratego/, accessed on 26 September 2024). Its user-friendly interface and powerful computational features make it an ideal choice for analyzing traveler behavior in urban settings.

The tool developed in this study is firmly grounded in robust theoretical principles, which are crucial for its effective application in real-world scenarios. A solid theoretical background is necessary for utilizing the UPPAAL platform correctly, as it informs the modeling and analysis processes. Moreover, modeling the entire system and interpreting the results demand not only expertise in formal methods but also creativity and substantial effort. This combination of theoretical knowledge and practical implementation ensures that our contributions are both relevant and significant within the context of urban transportation optimization.

The primary contributions of this research include the following:

1. A comprehensive mathematical model that integrates stochastic elements to capture the variability in travel conditions and choices.
2. The application of the UPPAAL Stratego tool to compute and evaluate multiple strategies for optimizing travel.
3. An analysis of the implications of traveler behavior in the context of urban transportation planning.
4. A demonstration of the theoretical contributions of our model, highlighting its applicability to real-world scenarios and its potential to inform future research in urban mobility optimization.

The remainder of this paper is structured as follows: Section 2 presents a review of the related work, highlighting previous studies on traveler behavior and optimization techniques. In Section 3, we introduce the foundational concepts of Stochastic Priced Timed Games and provide an overview of the UPPAAL Stratego tool. Section 4 details the implementation of our proposed model, including the setup for different transportation choices and traffic scenarios. In Section 5, we present the results of our simulations and discuss their implications for optimizing traveler behavior. Section 6 outlines the limitations of the current study, while Section 7 suggests avenues for future research. Finally, Section 8 summarizes the key findings and their significance for urban transportation planning.

2. Related Work

This section reviews various studies that tackle the complexities of transportation systems through formal modeling and verification methods. A notable contribution by Valente et al. [19] highlights the challenges associated with urban road traffic management. The authors propose a formal and adaptable approach that employs modeling techniques, such as cellular and timed automata, alongside analysis methods, like simulation and

formal verification. This comprehensive methodology aims to address the dynamic behaviors of urban traffic under varying conditions. A case study illustrates the practical application of this framework, demonstrating its effectiveness in real-world scenarios. In a related effort, Avram et al. [20] introduce the Traffic Cellular Automata (TCA) model, aimed at improving urban transport efficiency as vehicle numbers rise. The study underscores the importance of addressing traffic congestion and optimizing lane usage by integrating real-time data into navigation systems. The TCA model emphasizes flexibility, extensibility, and computational efficiency, with simulations conducted in the UPPAAL environment showcasing its potential to enhance urban traffic management. Expanding the discussion to railway systems, Avram et al. [21] present a new modeling framework that captures the intricate interactions within railway transport. Utilizing timed automata and the UPPAAL tool, the framework allows for the easy modification of simulated maps through simple variable adjustments. This two-layered model effectively represents both infrastructure and mobile components, with interaction rules implemented in C programming. A case study demonstrates its utility in simulating complex railway scenarios. Continuing in the realm of traffic management, Kamput et al. [22] explore smart traffic light control systems, utilizing UPPAAL for modeling and verification. Their study highlights the need for scalability in traffic scenarios, developing templates that ensure synchronization across intersections. By focusing on safety and performance, the authors reveal significant improvements in usability through formal model templates, showcasing the adaptability of their approach in dynamic environments.

Shifting focus back to railway systems, Lukacs et al. [23] introduce a formal model-based methodology aimed at enhancing the safety of electronic urban railway control systems. By addressing the limitations of traditional design methods, this methodology improves the specification and verification process, resulting in higher quality outputs. A case study on a tram-road level crossing protection system illustrates its practical applicability. Kang et al. [24] further extend the conversation by incorporating probabilistic extensions into the East-adl architectural language for safety-critical automotive systems. Their work translates energy-aware timed behaviors into models suitable for verification, validated through a case study on a cooperative automotive system. This demonstrates the framework's effectiveness in modeling nonfunctional requirements related to timing and energy constraints. Mehdipour et al. [25] provide a review of formal methods in the context of autonomous driving. They emphasize the application of formal languages, particularly temporal logics, to ensure safe driving behavior. The review focuses on system-level methods from an ego-centric perspective, addressing the challenges in the field and offering insights into verification, monitoring, and synthesis techniques crucial for autonomous vehicle operation.

In a broader context, Ferrari et al. [26] conduct a comprehensive mapping study on formal methods in railway systems, identifying key trends and gaps in the research. Their findings indicate a significant uptick in research activity, with model checking being the predominant technique. They advocate for empirical research methods, such as case studies, to enhance the application of formal methods in lower-level development phases. Lastly, Ter et al. [27] present empirical findings on state-based formal methods in the railway sector. Their analysis reveals challenges faced by engineers in integrating formal methods into practice, emphasizing the need for collaboration between researchers and practitioners. This work underscores the importance of usability in formal methods, aiming to improve their integration within the industry. Complementing these discussions, Hammad et al. [28] address the specification and verification of real-time systems in land transportation. Through a case study involving automated vehicles, the authors model system components with timed automata and verify key properties using the timed logic TCTL. This study emphasizes the significance of nonfunctional properties in ensuring reliability within transportation systems.

Table 1 provides a concise summary of the related works in the field of transportation systems, highlighting key references, their focus areas, and significant contributions. Our

study distinguishes itself from the existing literature by focusing on optimizing traveler behavior between MADINA and JEDDA using the UPPAAL Stratego tool within a Stochastic Priced Timed Games framework. While prior works, such as those by Valente et al. [19] and Avram et al. [20], primarily employ deterministic models for traffic simulation, our research integrates stochastic elements to more accurately capture real-world uncertainties affecting travel time and cost across multiple transport modes—car, bus, and train. Furthermore, unlike studies that concentrate on specific traffic management strategies, our approach evaluates three distinct strategies—Safe, Fast, and Fast and Safe—allowing for a comprehensive analysis of traveler preferences under varying traffic conditions. This emphasis on strategy optimization not only contributes to the theoretical framework of smart transportation systems but also offers practical insights for urban mobility policies aimed at enhancing traveler experience and resource allocation.

Table 1. Summary of related work.

Ref.	Focus	Key Contributions
[19]	Urban Traffic Management	Proposes a flexible approach using cellular and timed automata for dynamic traffic behavior modeling.
[20]	Traffic Cellular Automata Model	Enhances urban transport efficiency; integrates real-time data for congestion management.
[21]	Railway Transport Systems	Introduces a framework with timed automata for simulating railway interactions and infrastructure.
[22]	Smart Traffic Light Control	Utilizes UPPAAL for scalable traffic light systems; emphasizes safety and performance improvements.
[23]	Electronic Urban Railway Control	Develops a formal methodology for safe railway control systems, improving specification processes.
[24]	Safety-Critical Automotive Systems	Extends East-adl with probabilistic features; translates energy-aware behaviors for verification.
[25]	Autonomous Driving	Reviews formal methods for vehicle behavior specification and verification, focusing on safety.
[26]	Railway Systems	Conducts a mapping study on formal methods, advocating for empirical research in lower-level phases.
[27]	State-Based Formal Methods	Highlights usability challenges in railway engineering; calls for collaboration between researchers and practitioners.
[28]	Real-Time Systems in Land Transportation	Models automated vehicles with timed automata, emphasizing nonfunctional properties for reliability.

3. Preliminaries

Formal methods are mathematically-based techniques for the specification, development, and verification of software and hardware systems [29–32]. They provide a rigorous framework for modeling system behaviors and properties, allowing developers and researchers to reason about correctness, safety, and performance [33–36]. By employing formal methods, one can identify potential errors and inconsistencies early in the development process, thereby improving system reliability and robustness. One of the key areas within formal methods is the study of games in a stochastic setting. Stochastic games

extend traditional game theory by incorporating randomness, making them suitable for modeling real-world scenarios where uncertainty plays a critical role, such as in automated systems and decision-making processes.

Stochastic Priced Timed Games (SPTGs) are a formal framework that combines aspects of timed automata, stochastic processes, and pricing mechanisms. These games model systems where multiple agents interact under conditions of uncertainty, with costs associated with different actions and time constraints. Mathematically, a Stochastic Priced Timed Game can be defined as a tuple $(P, A, \mathcal{T}, C, \mathcal{R}, \mathcal{D})$, which are as follows:

- P is a finite set of states;
- A is a finite set of actions available to players;
- $\mathcal{T} \subseteq P \times A \times P$ is a transition relation that specifies how actions lead from one state to another;
- $C : \mathcal{T} \to \mathbb{R}^+$ is a cost function that assigns a non-negative cost to each transition;
- $\mathcal{R}$ is a stochastic mechanism that describes probabilities associated with transitions, reflecting the uncertainty in the system;
- $\mathcal{D}$ is a set of timing constraints that dictate the temporal aspects of the game.

In SPTGs, players must strategize not only to minimize costs but also to optimize their actions in the face of uncertainty and time constraints. This makes SPTGs a powerful tool for modeling complex systems where financial and temporal factors are critical.

UPPAAL Stratego is an extension of the UPPAAL tool, designed for the synthesis of strategies in timed systems modeled with timed automata. UPPAAL is a widely used tool for modeling, simulation, and verification of real-time systems, providing a framework for checking properties such as safety and liveness.

In the context of Stochastic Priced Timed Games, UPPAAL Stratego allows users to automatically generate strategies that dictate how a player should act in order to optimize a given objective, typically minimizing costs or maximizing rewards. The tool employs algorithms that analyze the game structure and use quantitative techniques to derive optimal strategies.

The integration of UPPAAL Stratego with SPTGs enables researchers and practitioners to tackle complex decision-making problems in environments characterized by uncertainty, dynamic changes, and resource constraints. It provides a robust platform for exploring the interplay between time, cost, and stochastic behavior in system design and analysis.

4. Model Implementation

As illustrated in Figure 1, the Stochastic Priced Timed Game (SPTG) model captures the decision-making process of travelers choosing among different transportation options from MADINA to JEDDA. The XML code corresponding to the UPPAAL model of this traveler decision-making framework is provided in Listing A1 in Appendix A. The journey begins at the initial state, MADINA, represented as a double circle, indicating it is the starting point of the traveler's experience. From MADINA, the traveler has three transportation options: bus, car, and train.

1. Car Option: Choosing the car presents two potential outcomes based on traffic conditions:
    - **Easy_Car State**: The traveler may experience an easy and fast trip, with a travel time ranging from 220 to 240 min if traffic permits. This scenario has a high likelihood of occurring, with a probability weight of 5 out of 7.
    - **Heavy_Car State**: However, there is a 2 in 7 risk of encountering heavy traffic, which can significantly prolong the trip. In this case, the travel time may extend to 280 to 300 min.
2. Train Option: Opting for the train involves a different set of dynamics:
    - **Waiting Time**: The traveler may experience a waiting time of up to 6 min before boarding.

- **Go_Train State**: The likelihood of successfully boarding the train is quite high, estimated at 9 out of 10. Once on board, the journey to JEDDA takes approximately 110 to 120 min.
- **Wait_Train State**: There is a small risk (1 out of 10) that the train may be canceled, leaving the traveler in a state of uncertainty. If this occurs, the traveler must decide whether to switch to another train or return home to MADINA, necessitating a reevaluation of their travel plans.

3. Bus Option: The bus option also involves periodic schedules and similar risk factors:
   - **Waiting Time**: Like the train, the bus may require a waiting time of up to 6 min before departure.
   - **Go_Bus State**: The probability of successfully boarding the bus is also high, at 9 out of 10. Once on the bus, the traveler faces two potential scenarios based on road conditions:
     - **Easy_Bus State**: If traffic permits, the bus journey could be easy and fast, taking 280 to 300 min, with a likelihood of 5 out of 7.
     - **Heavy_Bus State**: Conversely, there is a 2 in 7 risk of heavy traffic, which could extend the journey to 340 to 360 min.
   - **Wait_Bus State**: Similar to the train, there is a 1 in 10 risk of the bus being canceled. Should this happen, the traveler would once again need to decide whether to wait for another bus or return home to MADINA.

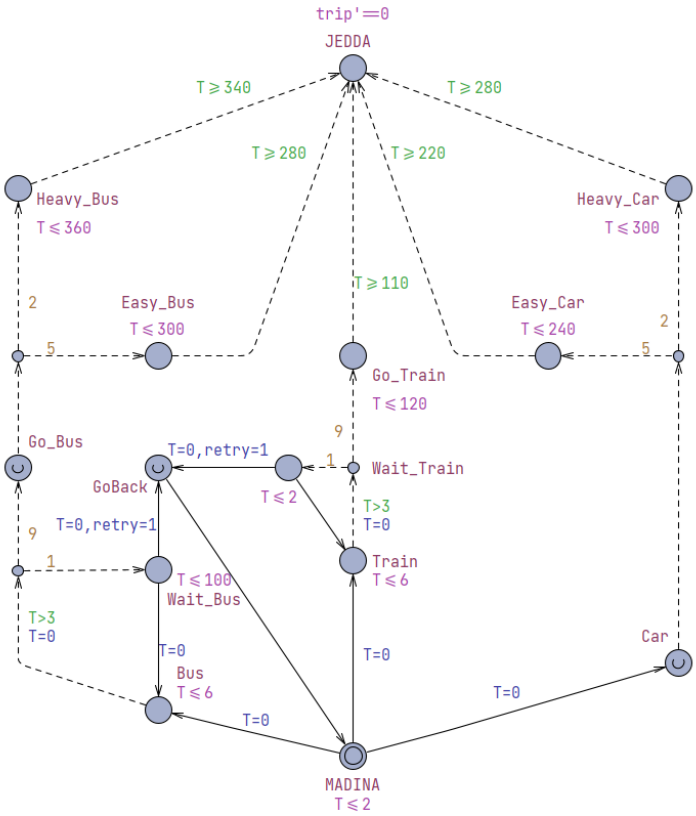
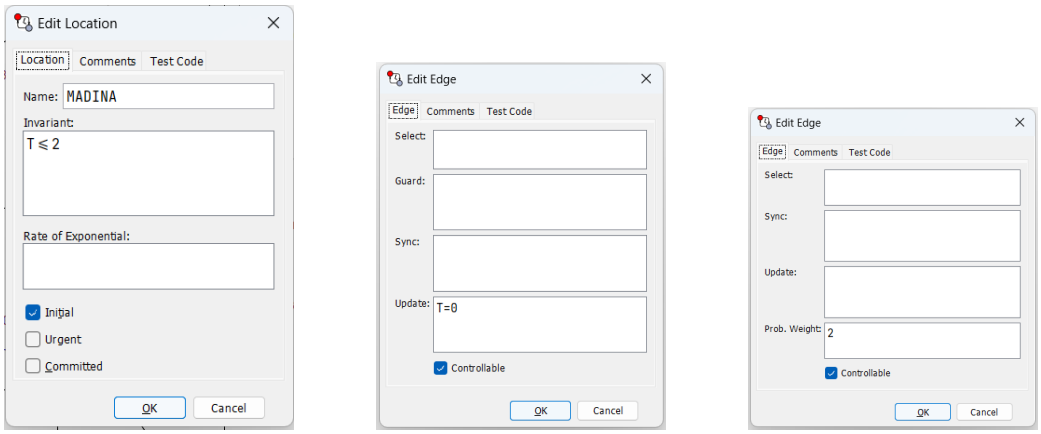


Figure 1. Stochastic Priced Timed Game (SPTG) model for traveler decision-making between MADINA and JEDDA drawn using the UPPAAL platform.

Figure 2a illustrates that by clicking twice on a location, a configuration window appears, allowing users to update various parameters of the location, including its name, invariant, rate of exponential decay, and type (Initial, Urgent, or Committed). Additionally, small nodes within the model are referred to as branching states. Figure 2b demonstrates that by clicking twice on a transition, a configuration window allows for

the definition of the transition's guard conditions and the specification of whether the transition is controllable. Controllable transitions are depicted with continuous lines, while non-controllable transitions are represented with dotted lines, signifying their different roles in the model's behavior. Finally, Figure 2c focuses on probabilistic transitions, where users can assign weights to each transition. These probabilistic transitions emanate from branching states, indicating the likelihood of taking specific paths within the model based on defined probabilities.



(**a**) Locations. (**b**) Transitions. (**c**) Probabilistic Transitions.

Figure 2. Configuration windows for locations, transitions, and probabilistic transitions.

The model employs several key variables to accurately represent the travel dynamics. The clock variable 'time' serves as a global time reference, measured in minutes, ensuring consistent tracking of the entire journey. The clock variable 'T' is specifically used to monitor the duration of the transportation phase, allowing for precise timing of travel events. Additionally, the hybrid clock variable 'trip' functions as a stopwatch to measure the trip time, enabling the model to differentiate between various phases of the travel process and account for delays or waiting periods effectively.

In summary, this SPTG model captures the traveler's decision-making process and the uncertainties they face when choosing a mode of transportation from MADINA to JEDDA. Each option—car, train, and bus—presents unique probabilities and potential outcomes, emphasizing the complex interplay between time, cost, and the likelihood of success. The model not only highlights the risks associated with each mode of transport but also the need for the traveler to adapt their strategy based on real-time conditions and decisions.

5. Results and Discussion

In this study, the computational experiments and simulations were conducted on a personal laptop equipped with the following specifications: an Intel(R) Core(TM) i7-10510U CPU running at 1.80 GHz, with a maximum boost frequency of 2.30 GHz, and 8.00 GB of RAM. The system operates on a 64-bit architecture, utilizing an x64-based processor, with Windows 11 Home as the operating system. For modeling and analysis, we employed UPPAAL version 5.0.0, which facilitated the development and verification of the SPTG model.

5.1. Symbolic and Concrete Simulations

In our analysis, we utilized both the symbolic and concrete simulators provided by UPPAAL to explore the behavior of the SPTG model. The symbolic simulator allows for the efficient exploration of the state space by representing states and transitions symbolically, thus enabling the analysis of larger models without exhaustive enumeration. This mode is particularly useful for understanding the overall system dynamics and verifying properties

over a wide range of scenarios. In contrast, the concrete simulator focuses on executing specific instances of the model, providing detailed insights into individual paths and real-time behaviors. This mode is essential for validating the model against expected outcomes and for observing the effects of different variables in concrete situations. Together, these simulators offer a comprehensive toolkit for analyzing the complex interactions within the transportation model.

In both simulation modes, the symbolic and concrete, we can adopt either an interactive or random mode, providing flexibility in exploring the model's behavior. Additionally, the obtained trace can be exported for later analysis and replayed to review specific scenarios. During simulations, we have access to the values of global variables, allowing for real-time monitoring of the system's state. In the concrete simulation mode, actions occur after specific delays, as illustrated in Figure 3. During the simulation, the current state is highlighted in red, making it easy to identify the active state of the model at any given moment. Figure 4 illustrates the symbolic simulation process, showcasing how the state space is explored efficiently.

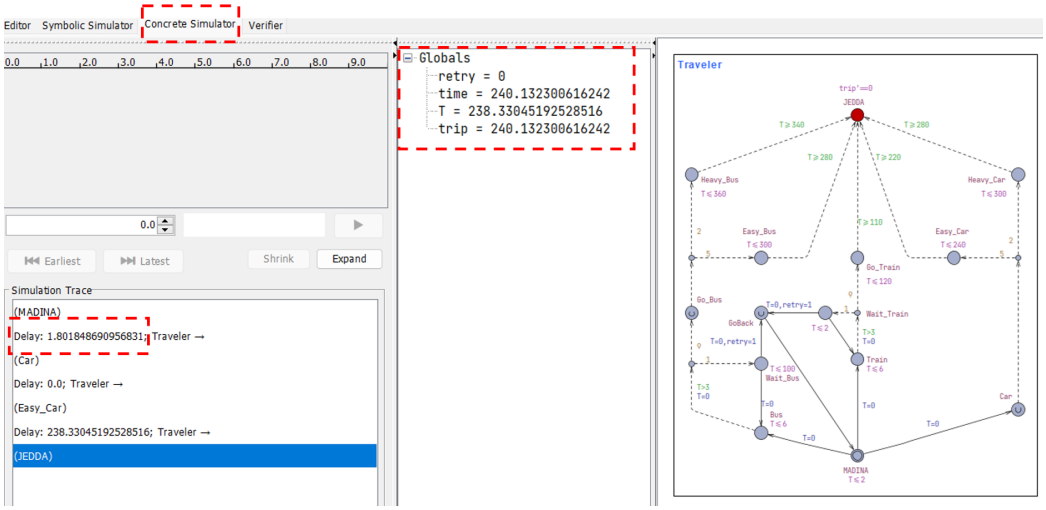


Figure 3. An example of the concrete simulation of the SPTG model.

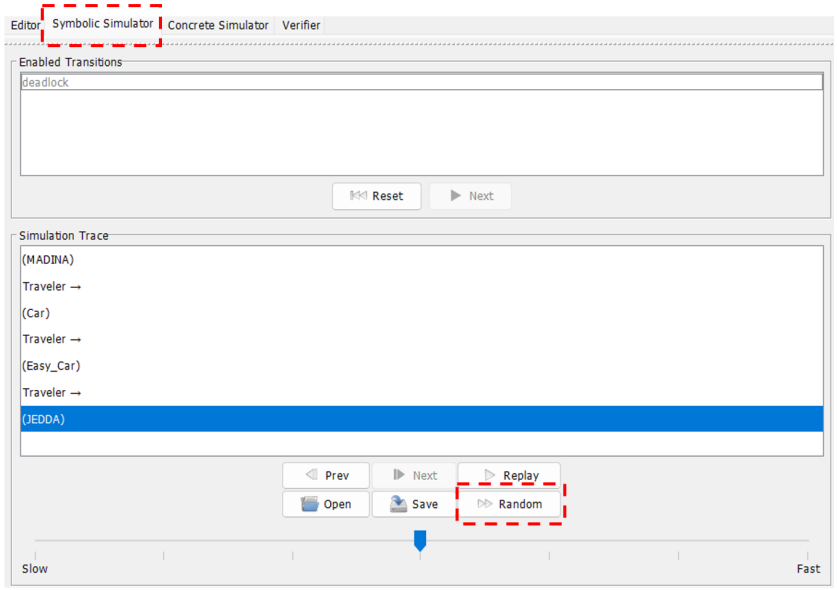


Figure 4. An example of the symbolic simulation of the SPTG model.

5.2. Verification and Strategy Generation

In the verification and strategy generation phase, users can initiate the process by clicking the verifier button in the UPPAAL tool, as illustrated in Figure 5. This interface allows for the manipulation of a set of properties and queries, which include validation queries, safe strategy queries, fast strategy queries, and safe and fast queries. Each query is accompanied by an editing zone for inputting the query itself and an additional zone for adding comments.

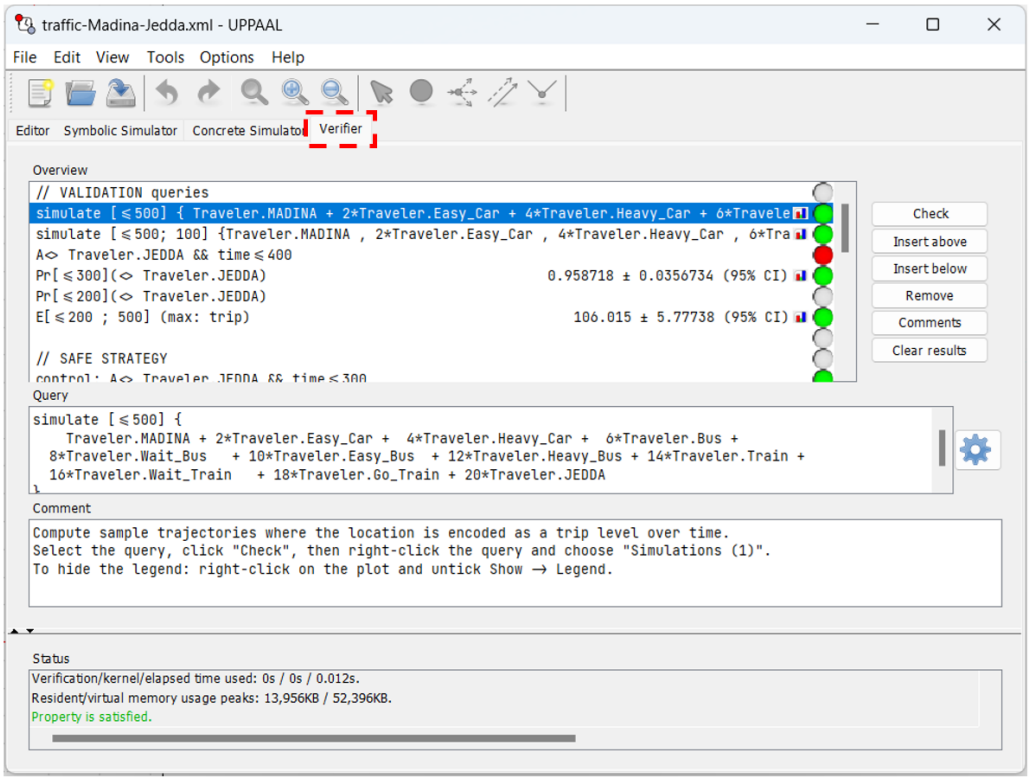


Figure 5. Verification and strategy generation in UPPAAL.

Upon selecting a query, several options become available on the right side of the window, including Check, Insert Above, Insert Below, Remove, Comments, and Clear Results. When a query is checked, a visual indicator appears in the form of a colored disk on the right: a green disk signifies that the query has been satisfied, while a red disk indicates that it has not. This visual feedback is crucial for understanding the status of the model's properties and guiding subsequent decisions in the verification process.

As shown in Figure 6, we consider six validation queries: Q1, Q2, Q3, Q4, Q5, and Q6. Table 2 summarizes the meaning of each validation query used in our analysis.

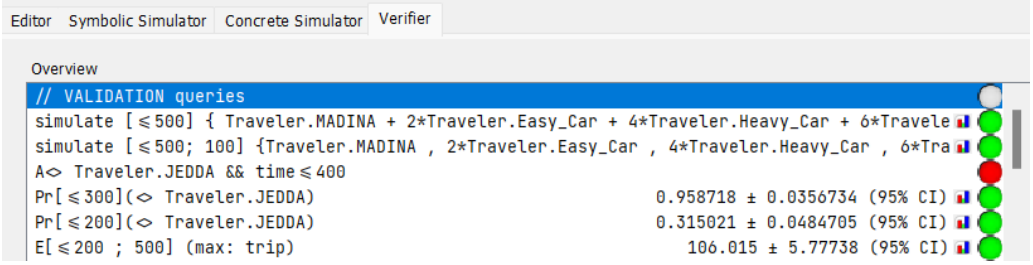


Figure 6. Validation queries in UPPAAL.

Table 2. Summary of validation queries.

Query	Description	Validation Status
Q1	Compute a sample trajectory where the location is encoded as a trip level over time.	Validated
Q2	Show an overview of when and where most of the time is being spent.	Validated
Q3	Is it possible that we can always reach JEDDA within 400 min?	Not Validated
Q4	What is the probability of reaching JEDDA within 300 min?	Validated
Q5	Estimate the average trip time to JEDDA by inspecting the probability distribution over time.	Validated
Q6	Estimate the maximum time needed to reach JEDDA, where the clock time is stopped in JEDDA.	Validated

All the validation queries were successful except for Q3, which indicates that while it is possible for the traveler to reach JEDDA within 400 min, this is not guaranteed in all cases. For instance, the traveler may experience prolonged waiting times for a train or bus, which could prevent him from starting his trip early enough to arrive within that time frame. For each validated query, the tool offers additional results in the form of graphs, including the following:

- **Simulation Graphs**: Visual representations showing the occupancy of different nodes over time, illustrating how the position of the traveler changes throughout the simulation. In the considered model, only one node can be occupied at any given time. When a transition is firable, an action will be taken, and the traveler will move from the current node to the destination node of the considered transition. This contrasts with models like timed Petri nets, where multiple nodes can be occupied simultaneously by tokens, allowing for a more flexible representation of states and transitions [37–40]. For more complex scenarios, the system may consist of a product of timed automata. In this case, the current state of the system is represented by the combination of the currently occupied nodes in each individual automaton. This means that the overall state reflects not just the position in one automaton but rather the collective state across all automata involved, providing a comprehensive view of the system's dynamics. For instance, Figures 7 and 8 are two possible simulation graphs for the query Q1. In the first case, the traveler reaches the final destination in approximately 185 min, while in the second situation, the trip takes approximately 370 min. Figure 9 corresponds to one hundred possible simulations, satisfying the query Q2. The horizontal axis corresponds to the progress of time, while the vertical axis indicates which node is currently occupied by the traveler. We used a different weight for each node in the model; for instance, the MADINA and JEDDA nodes take the lowest (0) and highest (20) weights, respectively. The other nodes take values of 2, 4, 6, ..., 18. A step of 2 is taken between each two consecutive nodes to enhance clarity. Additionally, a different color is used for each node, and the sudden switch from one level/color to another corresponds to the move from one node to another. The horizontal lines represent the situation when the system is at the same node. For example, the blue vertical lines indicate the transition of the traveler from the MADINA node to the train node, while the horizontal blue lines denote the durations spent at this node. Next, the purple vertical lines represent the transition to the Go_Train node, with the horizontal purple lines indicating the time spent there. Subsequently, the pink vertical lines correspond to the arrival at the JEDDA node, and the horizontal pink lines show the duration spent at this node. It is also important to note that certain strategies do not allow for reaching the JEDDA destination. For instance, the green horizontal lines associated with the Wait_Bus node lack continuity, indicating that it is not possible to reach the destination from this node given the considered constraints. The figure

shows that, in most cases, the time for reaching the destination is between 270 and 324 min. This range is likely associated with the use of a car for travel, as this mode of transportation minimizes the risk of waiting for public transport and eliminates unnecessary delays. Consequently, the traveler experiences a more direct and efficient journey, resulting in the observed time frame.

- **Probability Density Distribution**: A function that describes the likelihood of a random variable falling within a particular range of values, providing insight into the distribution of outcomes. In the context of UPPAAL Stratego, this distribution plays a crucial role in analyzing the performance of stochastic systems. It allows for the evaluation of various strategies by quantifying the probabilities associated with different outcomes, such as the time to reach a destination or the likelihood of encountering specific events. By leveraging the probability density distribution, researchers can make informed decisions about optimizing strategies in uncertain environments, thus enhancing the overall efficiency and effectiveness of the modeled system. For instance, the probability density distribution for the query Q4 is shown in Figure 10. We can observe that the probability density reaches its maximum during the duration between 33 and 108 min. Following this peak, it decreases gradually within the interval from 108 to 183 min. Afterward, it experiences another increase before declining once more.
- **Probability Density Confidence Intervals**: Ranges within which the true probability density function is expected to lie, reflecting the uncertainty in the estimate. In the framework of UPPAAL Stratego, these confidence intervals serve as a critical tool for understanding the reliability of the probabilistic outcomes generated by the model. They encapsulate the uncertainty inherent in the estimates, allowing researchers to gauge how robust different strategies are under varying conditions. By analyzing these intervals, one can discern the range in which actual performance metrics are likely to reside, thereby empowering decision-makers to optimize system behaviors effectively, even in the face of uncertainty. For example, Figure 11 illustrates probability density confidence intervals for the query Q4. We can observe that the upper and lower limits have nearly identical shapes, both reaching their maximum values within the same interval (between 33 and 108 min) and increasing or decreasing simultaneously at the same instants.
- **Cumulative Probability Distribution**: A function that indicates the probability that a random variable takes on a value less than or equal to a specific value, useful for understanding the likelihood of outcomes up to a certain point. In the context of UPPAAL Stratego, the cumulative probability distribution is instrumental in analyzing the overall performance of stochastic systems. It provides a comprehensive view of the probabilities associated with various outcomes, allowing researchers to assess how likely it is for a traveler to reach a destination within a certain time frame. This distribution not only aids in evaluating the effectiveness of different strategies but also highlights critical thresholds that can inform decision-making processes, thereby enhancing the strategic planning in uncertain environments. The cumulative probability distribution for the query Q4 is shown in Figure 12. We can observe that the slope of the curve representing the cumulative probability density distribution remains constant until 123 min. It then decreases and stays stable until 192 min. At this point, it increases again before experiencing a final decrease between 238 and 261 min.
- **Cumulative Probability Confidence Intervals**: Ranges that provide an estimate of uncertainty around the cumulative probability distribution, indicating where the true cumulative probabilities are likely to fall. Within the framework of UPPAAL Stratego, these confidence intervals are essential for quantifying the reliability of cumulative probability estimates. They allow researchers to understand the degree of uncertainty associated with the predicted cumulative outcomes, offering insights into how robust different strategies may be. By examining these intervals, decision-makers can better assess the likelihood of achieving desired results, thus facilitating more informed planning and optimization of systems under uncertain conditions. An

example for the query Q4 is given in Figure 13. We can observe that the upper limit, lower limit, and cumulative probability density confidence intervals exhibit nearly identical shapes.

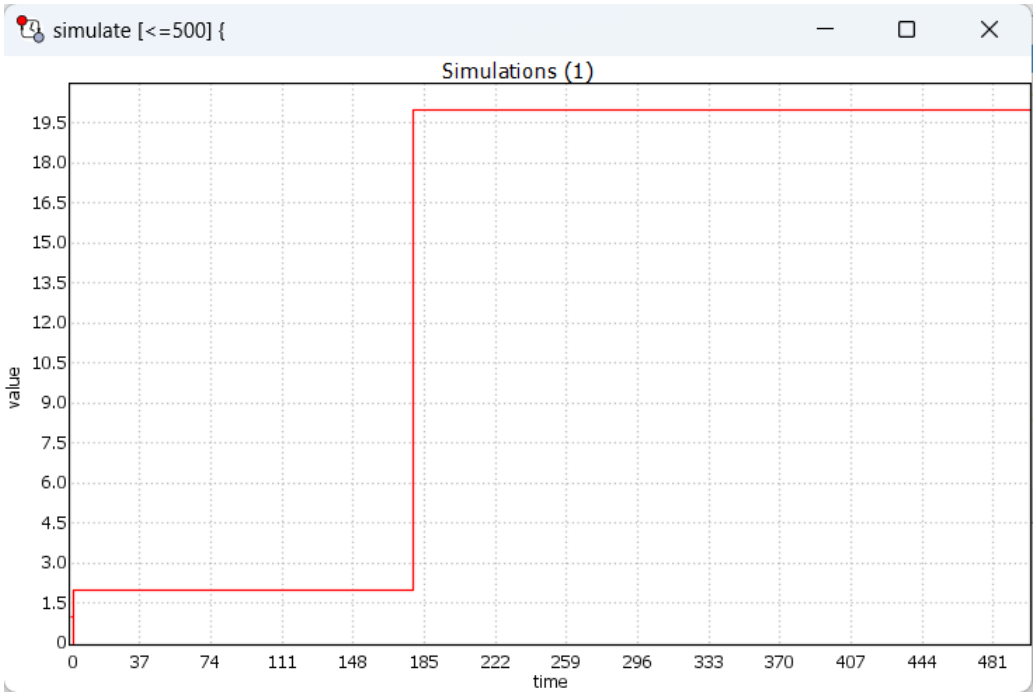


Figure 7. A strategy satisfying query Q1.

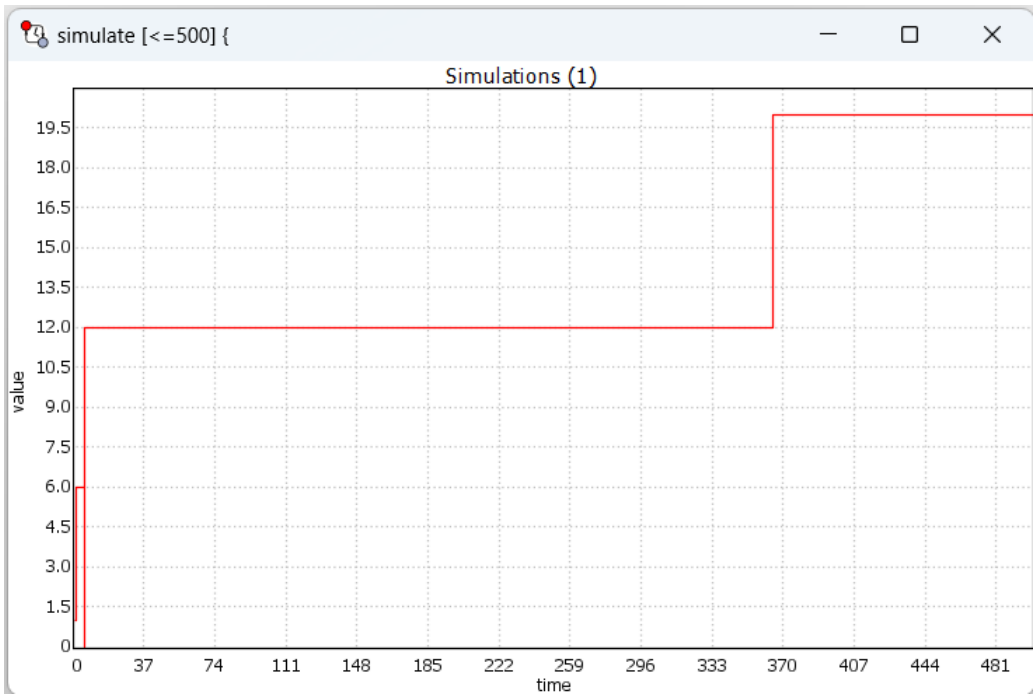


Figure 8. A second possible strategy satisfying query Q1.

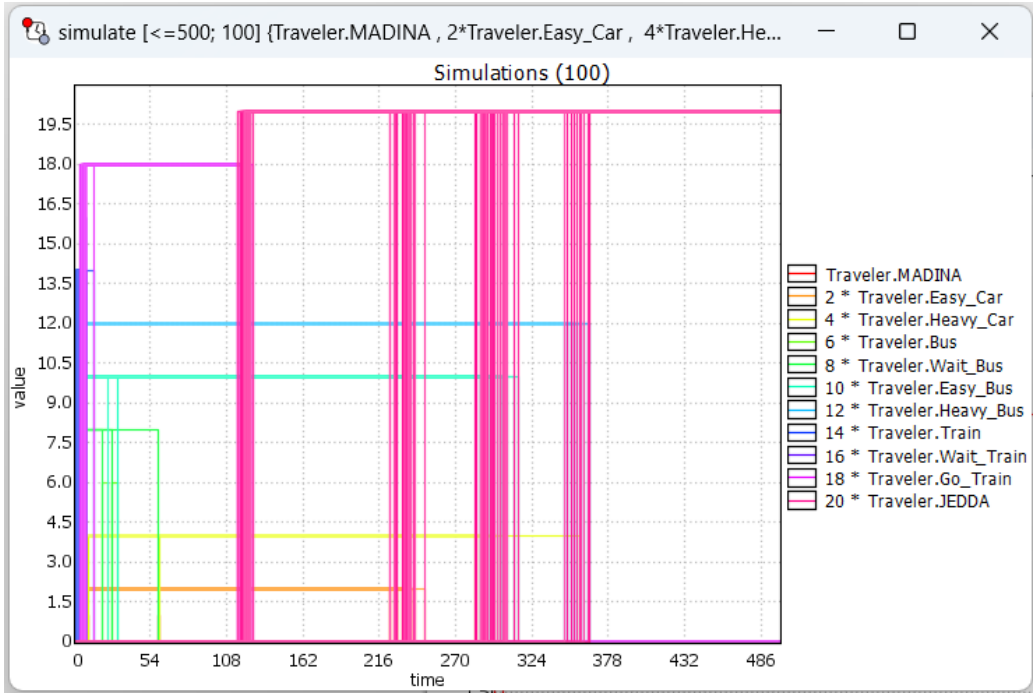


Figure 9. A set of possible strategies satisfying query Q2.

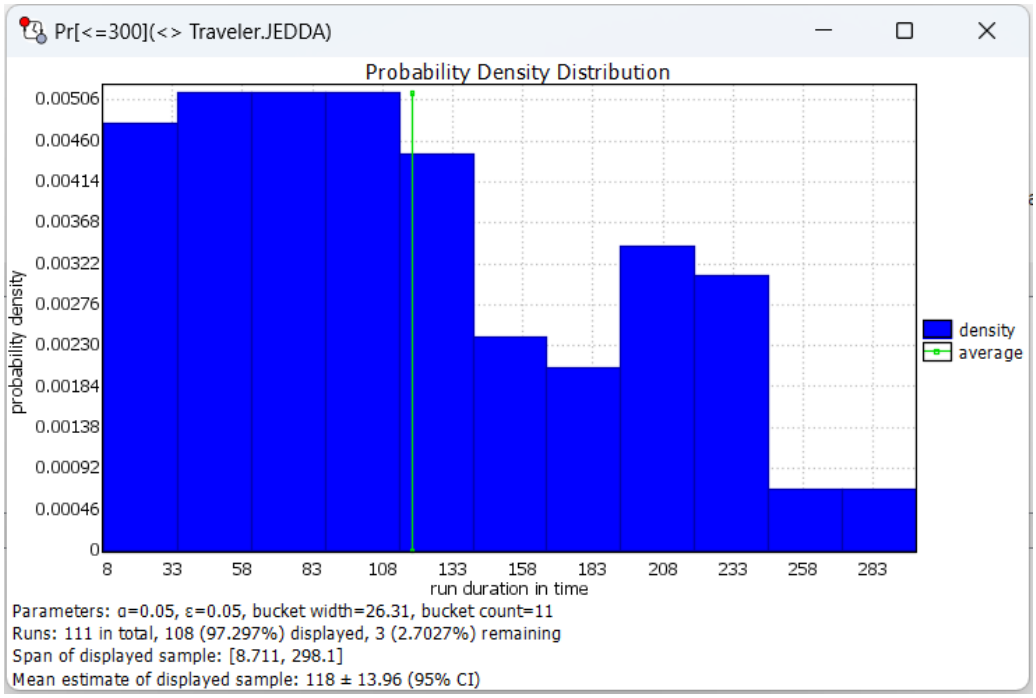


Figure 10. Probability density distribution for query Q4.

The queries and simulation results corresponding to the Safe, Fast, and Safe and Fast Strategies are illustrated in the Appendix A as follows:

- The queries for the Safe Strategy are shown in Figure A1. The analysis begins by defining a control property that evaluates whether the traveler can select modes of transport to reach JEDDA within 400 min, irrespective of traffic conditions. Next, the tool is utilized to produce the GoSafe strategy, a fully permissive approach that meets this control property. Following this, the feasibility of reaching JEDDA within 400 min under the GoSafe strategy is assessed. This is accompanied by an estimation of the time required for the journey, with the clock time paused during this evaluation.

Finally, an overview is generated to illustrate the allocation of time and the specific points at which time is spent while employing the GoSafe strategy.

- The simulation results for the Fast Strategy are illustrated in Figure A2. The results indicate that the probability of reaching JEDDA within 400 min while following the GoSafe strategy is 0.95. Additionally, the estimated time required to reach JEDDA is approximately 227.96 min.
- The queries for the Fast Strategy are shown in Figure A3. The GoFast strategy is introduced with the objective of reducing the travel time to JEDDA. Following this, the probabilities of arriving at JEDDA within 300 min and 400 min under the GoFast strategy are calculated. Additionally, the maximum time needed to reach JEDDA is estimated, with the clock time halted during this evaluation. Lastly, an overview is created to show the allocation of time and the specific points at which time is spent while using the GoFast strategy.
- The simulation results for the Fast Strategy are shown in Figure A4. The results show that the probabilities of arriving at JEDDA within both 300 min and 400 min are 0.95, with the maximum time needed to reach JEDDA estimated at 119.5 min.
- The queries for the Safe and Fast Strategy are illustrated in Figure A5. The GoFastSafe strategy is developed by optimizing the GoSafe strategy to reduce the travel time to JEDDA. This new strategy is evaluated by estimating the probabilities of reaching JEDDA within 200 and 400 min, as well as determining the time needed to reach JEDDA under the GoFastSafe strategy. Additionally, an overview is generated to illustrate how and when time is allocated while using this strategy.
- The simulation results for the Fast Strategy are given in Figure A6. The results obtained are similar to those achieved with the GoFast strategy, with the probabilities of reaching JEDDA within 200 and 400 min both measuring 0.95. Additionally, the time required to reach JEDDA is approximately 119.94 min.

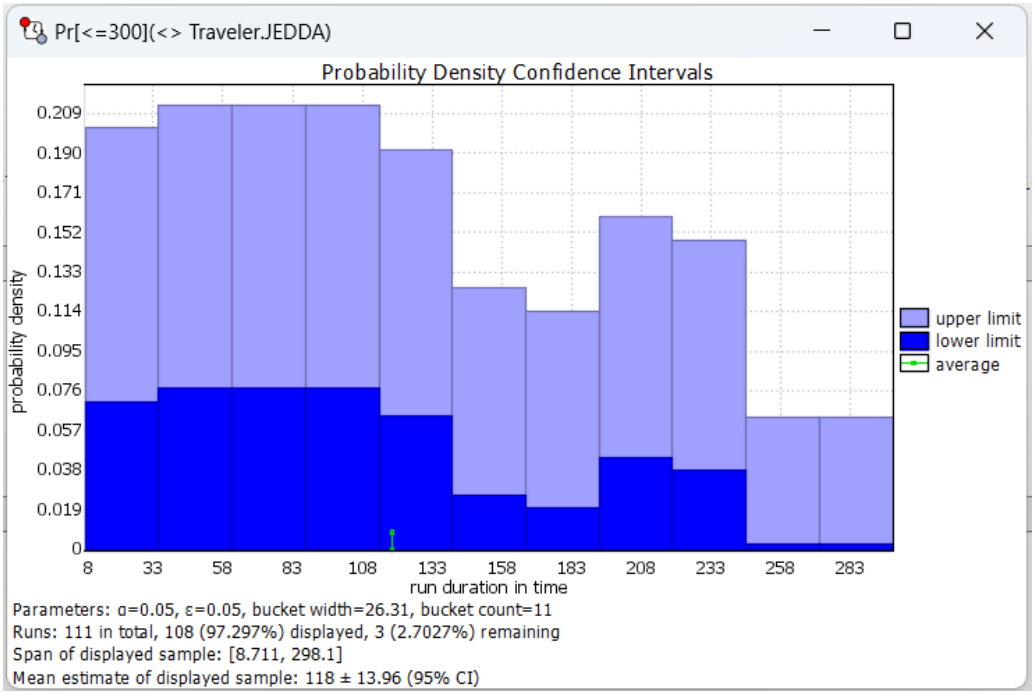


Figure 11. Probability density confidence intervals for query Q4.

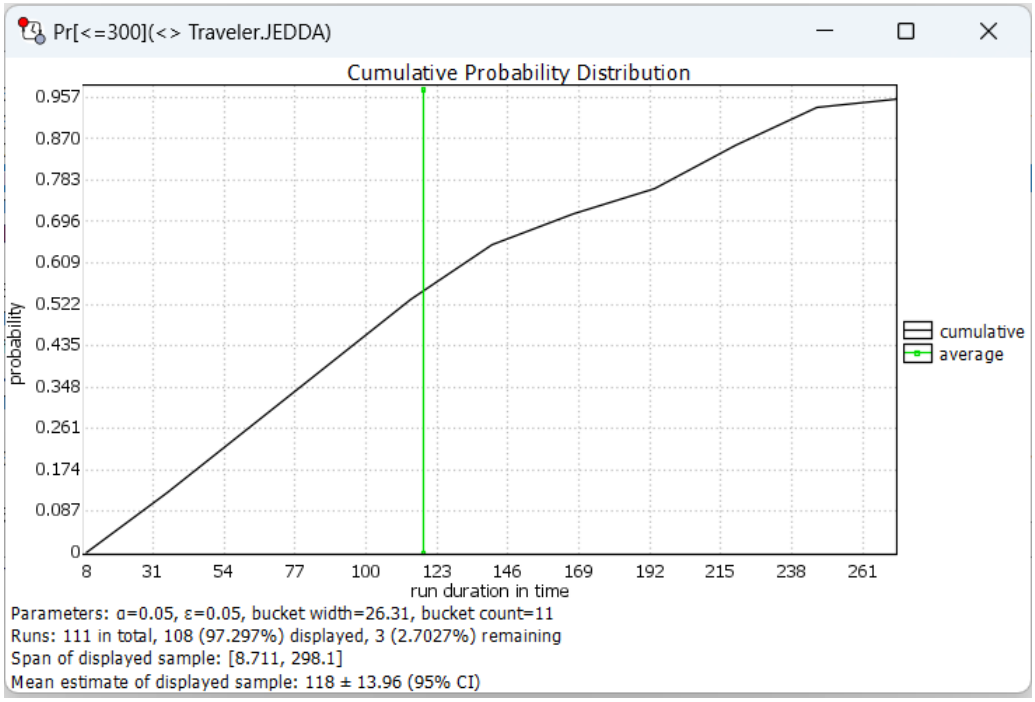


Figure 12. Cumulative probability density distribution for query Q4.

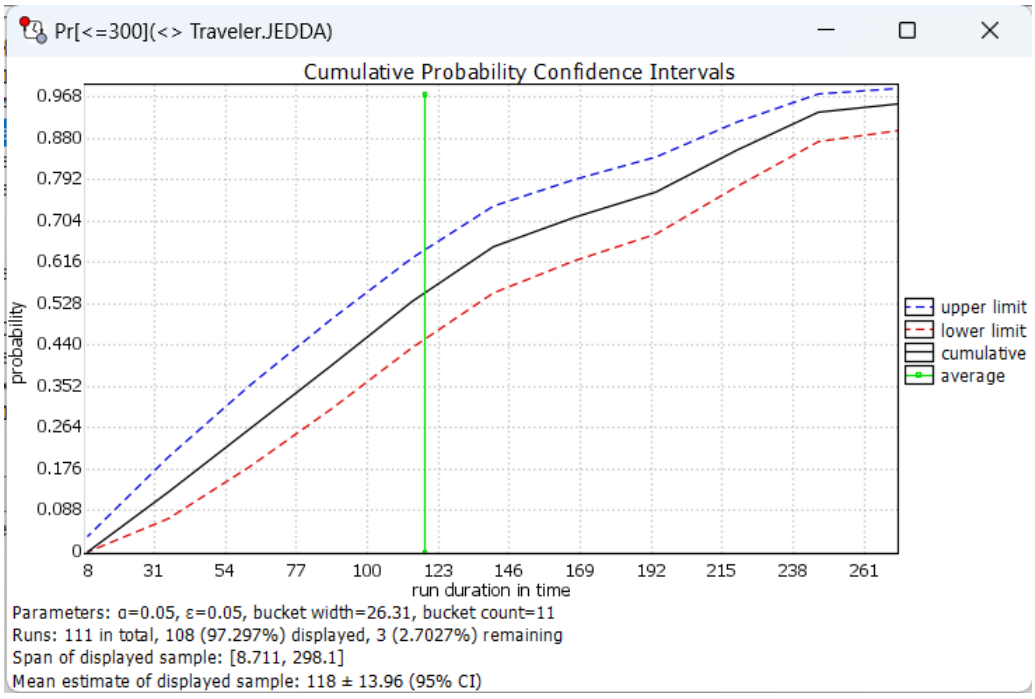


Figure 13. Cumulative probability density confidence intervals for query Q4.

6. Limitations

Despite the strengths and applicability of SPTGs and tools like UPPAAL Stratego, several limitations must be acknowledged. Firstly, the complexity of modeling in SPTGs can be a significant challenge. As the number of states, actions, and transitions increases, the state space can grow exponentially. This complexity can lead to difficulties in both the formulation of models and the computational feasibility of analyzing them. Consequently, larger systems may become intractable, limiting the applicability of SPTGs to smaller or more manageable scenarios. Secondly, while SPTGs effectively represent stochastic behaviors, they often rely on assumptions that may not hold in real-world systems. For instance,

the transition probabilities and costs may not be accurately known or may fluctuate over time, leading to potential discrepancies between the model and the actual system behavior. This uncertainty can impact the reliability of the strategies generated by tools like UPPAAL Stratego.

Additionally, the expressiveness of SPTGs can be a limiting factor. Certain complex behaviors or interactions may be difficult to capture within the framework of SPTGs. For example, scenarios involving intricate dependencies among multiple agents or dynamic changes in the system structure may not be easily represented, potentially reducing the effectiveness of the modeling approach. Moreover, while UPPAAL Stratego provides powerful synthesis capabilities, it may not fully account for all relevant constraints in a given application context. Limitations in the underlying algorithm can lead to suboptimal strategies, particularly in highly dynamic environments where conditions change rapidly. Finally, there is a learning curve associated with using formal methods and tools, like UPPAAL Stratego. Practitioners may require significant training and experience to effectively model systems, interpret results, and implement strategies. This barrier can hinder broader adoption in industry settings, where rapid development and flexibility are often prioritized.

7. Future Works

The exploration of SPTGs in the context of transportation presents numerous opportunities for future research and development. This section outlines several promising directions that could enhance the application of SPTG methodologies in transportation systems. Firstly, enhancing model expressiveness is essential for capturing the complexities of modern transportation networks. Future research could focus on incorporating additional factors such as traffic dynamics, vehicle interactions, and environmental conditions into SPTG models. This could enable more accurate representations of scenarios like multi-modal transport systems, where different modes of transportation interact, and help in understanding their impacts on efficiency and cost. Secondly, improving algorithmic efficiency for real-time decision-making in transportation is crucial. Current strategies for route optimization and traffic management can be computationally intensive, particularly during peak hours. Research into more efficient algorithms that leverage real-time traffic data and historical patterns could enhance the scalability of tools like UPPAAL Stratego, enabling quicker and more effective responses to changing conditions.

Another promising direction is the integration of smart technologies and data analytics into SPTG frameworks. By utilizing real-time data from connected vehicles, traffic sensors, and GPS systems, transportation models can be dynamically updated to reflect current conditions. This integration could lead to more responsive traffic management strategies, reducing congestion and improving overall system efficiency. Moreover, the development of user-friendly tools and interfaces tailored for transportation planners and operators is vital. Simplifying the modeling and analysis processes can facilitate the adoption of formal methods in transportation management. Future work could focus on creating intuitive graphical user interfaces that allow users to easily simulate various scenarios, assess strategies, and visualize outcomes without requiring extensive training in formal methods.

Finally, expanding the application of SPTGs to emerging transportation challenges such as autonomous vehicles, electric vehicle (EV) integration, and smart city initiatives presents exciting opportunities. Research could explore how SPTG methodologies can be applied to optimize fleet management, charging station placement, and traffic flow in smart environments, ultimately contributing to more sustainable and efficient transportation systems. In conclusion, the future of Stochastic Priced Timed Games in the transportation field is rich with potential. By addressing the challenges outlined above and exploring new research directions, the field can provide powerful tools for improving decision-making and operational efficiency in transportation networks.

8. Conclusions

In this work, we explored the application of SPTGs as a robust framework for modeling and optimizing decision-making processes in transportation systems. By integrating aspects of timing, cost, and stochastic behavior, an SPTG provides a comprehensive approach to address the inherent uncertainties and complexities of modern transportation networks. We demonstrated that an SPTG effectively captures the dynamic interactions between various agents in the transportation domain, such as vehicles, traffic control systems, and infrastructure. The mathematical formulation of an SPTG enables transportation planners to analyze different scenarios, evaluate strategies, and make informed decisions that enhance operational efficiency while minimizing costs. Moreover, we highlighted the potential of tools like UPPAAL Stratego in synthesizing optimal strategies within the SPTG framework. These tools facilitate the analysis of large and complex systems, allowing for real-time adaptation to changing conditions, which is crucial in an environment characterized by fluctuating traffic patterns and diverse transportation modes. However, our study also acknowledged the limitations of the current methodologies and tools, particularly in terms of model expressiveness and computational efficiency. Addressing these limitations will be essential for further advancements in the application of SPTGs to real-world transportation challenges. Looking ahead, future research should focus on enhancing the expressiveness of SPTG models by incorporating more detailed representations of transportation dynamics and integrating real-time data analytics. Additionally, the development of user-friendly interfaces will promote wider adoption of formal methods among transportation professionals, leading to improved traffic management and planning.

Author Contributions: Conceptualization, M.K.; formal analysis, M.K.; funding acquisition, A.H.; methodology, M.K.; project administration, A.H.; software, M.K.; validation, M.K.; writing—original draft, M.K. and A.H.; writing—review and editing, M.K. and A.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research work was funded by Institutional Fund Projects under grant no. (IFPIP:1566-611-1443). The authors gratefully acknowledge technical and financial support provided by the Ministry of Education and King Abdulaziz University, DSR, Jeddah, Saudi Arabia.

Data Availability Statement: No new data were created or analyzed in this study.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Listing A1. The UPAALL XML code of the considered system.

```
<template>
<name x="9" y="9">Traveler</name>
<declaration>// Place local declarations here. </declaration>
<location id="id0" x="−263" y="119">
<name x="−323" y="127">GoBack</name>
<urgent/>
</location>
<location id="id1" x="−85" y="17">
<name x="−68" y="25">Go_Train</name>
<label kind="invariant" x="−68" y="51">T<=120</label>
</location>
<location id="id2" x="212" y="−136">
<name x="119" y="−136">Heavy_Car</name>
<label kind="invariant" x="144" y="−110">T<=300</label>
</location>
<location id="id3" x="93" y="17">
<name x="59" y="−42">Easy_Car</name>
<label kind="invariant" x="68" y="−17">T<=240</label>
</location>
<location id="id4" x="−85" y="−246">
<name x="−111" y="−280">JEDDA</name>
<label kind="invariant" x="−119" y="−305">trip==0</label>
```

```
</location>
<location id="id5" x="−144" y="119">
<name x="−68" y="110">Wait_Train</name>
<label kind="invariant" x="−170" y="136">T<=2</label>
</location>
<location id="id6" x="−85" y="204">
<name x="−68" y="195">Train</name>
<label kind="invariant" x="−68" y="212">T<=6</label>
</location>
<location id="id7" x="212" y="297">
<name x="178" y="263">Car</name>
<urgent/>
</location>
<location id="id8" x="−85" y="382">
<name x="−111" y="399">MADINA</name>
<label kind="invariant" x="−102" y="416">T<=2</label>
</location>
<location id="id9" x="−263" y="340">
<name x="−246" y="297">Bus</name>
<label kind="invariant" x="−246" y="314">T<=6</label>
</location>
<location id="id10" x="−391" y="119">
<name x="−382" y="85">Go_Bus</name>
<urgent/>
</location>
<location id="id11" x="−263" y="212">
<name x="−255" y="229">Wait_Bus</name>
<label kind="invariant" x="−246" y="212">T<=100</label>
</location>
<location id="id12" x="−263" y="17">
<name x="−297" y="−42">Easy_Bus</name>
<label kind="invariant" x="−289" y="−17">T<=300</label>
</location>
<location id="id13" x="−391" y="−136">
<name x="−374" y="−136">Heavy_Bus</name>
<label kind="invariant" x="−374" y="−110">T<=360</label>
</location>
<branchpoint id="id14" x="−391" y="213"/>
<branchpoint id="id15" x="−391" y="17"/>
<branchpoint id="id16" x="−85" y="119"/>
<branchpoint id="id17" x="212" y="17"/>
<init ref="id8"/>
<transition id="id18" controllable="false">
<source ref="id13"/>
<target ref="id4"/>
<label kind="guard" x="−229" y="−238">T>=340</label>
</transition>
<transition id="id19" controllable="false">
<source ref="id12"/>
<target ref="id4"/>
<label kind="guard" x="−178" y="−178">T>=280</label>
<nail x="−178" y="17"/>
</transition>
<transition id="id20" controllable="false">
<source ref="id3"/>
<target ref="id4"/>
<label kind="guard" x="−51" y="−178">T>=220</label>
<nail x="8" y="17"/>
</transition>
<transition id="id21">
<source ref="id5"/>
<target ref="id6"/>
</transition>
<transition id="id22">
<source ref="id0"/>
<target ref="id8"/>
</transition>
<transition id="id23" controllable="false">
<source ref="id1"/>
```

```
<target ref="id4"/>
<label kind="guard" x="−85" y="−59">T>=110</label>
</transition>
<transition id="id24">
<source ref="id16"/>
<target ref="id1"/>
<label kind="probability" x="−102" y="76">9</label>
</transition>
<transition id="id25">
<source ref="id17"/>
<target ref="id2"/>
<label kind="probability" x="195" y="−25">2</label>
</transition>
<transition id="id26" controllable="false">
<source ref="id7"/>
<target ref="id17"/>
<nail x="212" y="42"/>
</transition>
<transition id="id27" controllable="false">
<source ref="id9"/>
<target ref="id14"/>
<label kind="guard" x="−382" y="246">T>3</label>
<label kind="assignment" x="−382" y="263">T=0</label>
<nail x="−391" y="298"/>
<nail x="−391" y="289"/>
<nail x="−391" y="289"/>
</transition>
<transition id="id28">
<source ref="id14"/>
<target ref="id11"/>
<label kind="probability" x="−365" y="195">1</label>
</transition>
<transition id="id29">
<source ref="id14"/>
<target ref="id10"/>
<label kind="probability" x="−382" y="170">9</label>
</transition>
<transition id="id30">
<source ref="id17"/>
<target ref="id3"/>
<label kind="probability" x="178" y="0">5</label>
</transition>
<transition id="id31">
<source ref="id11"/>
<target ref="id9"/>
<label kind="assignment" x="−263" y="276">T=0</label>
</transition>
<transition id="id32">
<source ref="id11"/>
<target ref="id0"/>
<label kind="assignment" x="−357" y="161">T=0,retry=1</label>
</transition>
<transition id="id33">
<source ref="id15"/>
<target ref="id13"/>
<label kind="probability" x="−382" y="−42">2</label>
</transition>
<transition id="id34">
<source ref="id15"/>
<target ref="id12"/>
<label kind="probability" x="−365" y="0">5</label>
<nail x="−323" y="17"/>
</transition>
<transition id="id35" controllable="false">
<source ref="id10"/>
<target ref="id15"/>
</transition>
<transition id="id36">
<source ref="id8"/>
```

```
<target ref="id9"/>
<label kind="assignment" x="−187" y="340">T=0</label>
<nail x="−195" y="357"/>
</transition>
<transition id="id37">
<source ref="id16"/>
<target ref="id5"/>
<label kind="probability" x="−110" y="102">1</label>
</transition>
<transition id="id38" controllable="false">
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<target ref="id16"/>
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<label kind="assignment" x="−76" y="161">T=0</label>
<nail x="−85" y="161"/>
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<target ref="id4"/>
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<nail x="8" y="−212"/>
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<transition id="id40">
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<nail x="−229" y="119"/>
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<transition id="id41">
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<label kind="assignment" x="−76" y="280">T=0</label>
<nail x="−85" y="238"/>
<nail x="−85" y="238"/>
</transition>
<transition id="id42">
<source ref="id8"/>
<target ref="id7"/>
<label kind="assignment" x="42" y="314">T=0</label>
<nail x="8" y="357"/>
</transition>
</template>
```

Overview

```
// SAFE STRATEGY
control: A<> Traveler.JEDDA && time ⩽ 400
strategy GoSafe = control: A<> Traveler.JEDDA && time ⩽ 400
A<> (Traveler.JEDDA && time ⩽ 400) under GoSafe
Pr[⩽400](<> Traveler.JEDDA) under GoSafe                    ≥ 0.950056 (95% CI)
E[⩽400; 500] (max: trip) under GoSafe                  227.968 ± 7.14626 (95% CI)
simulate [⩽400; 100] { Traveler.MADINA , 2*Traveler.Easy_Car , 4*Traveler.Heavy_Car , 6*Tr
```

Figure A1. Queries related to the Safe Strategy and the corresponding results.

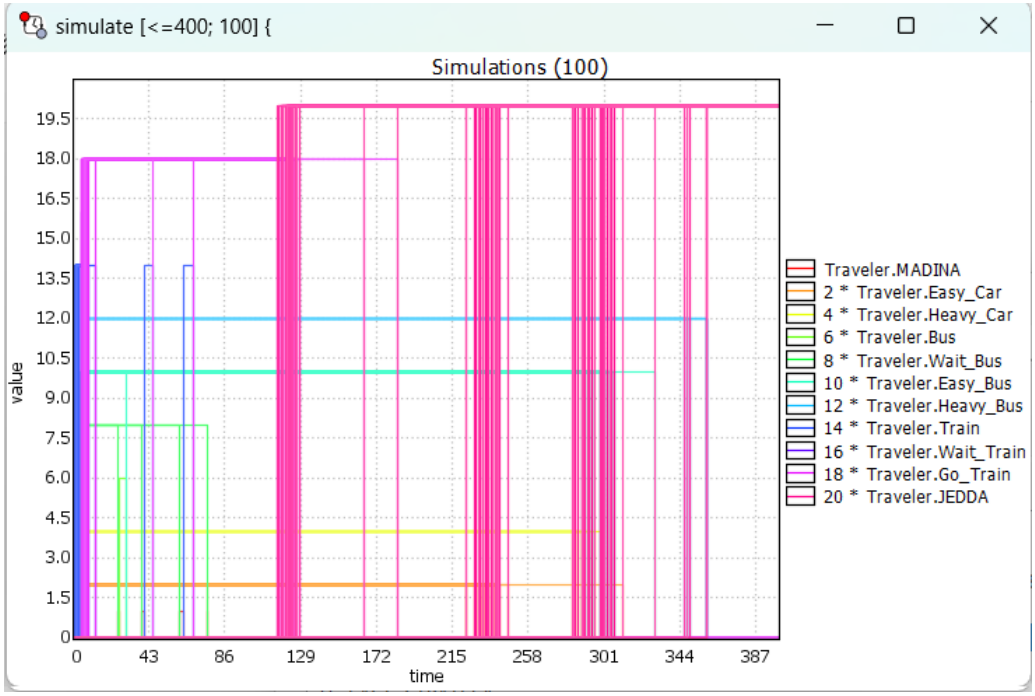


Figure A2. Results of the simulation of the Safe Strategies.

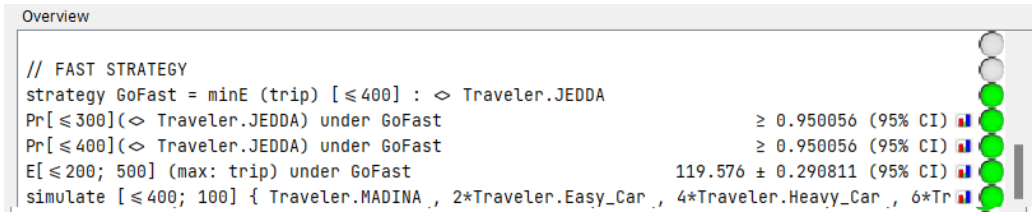


Figure A3. Queries related to the Fast Strategy and the corresponding results.

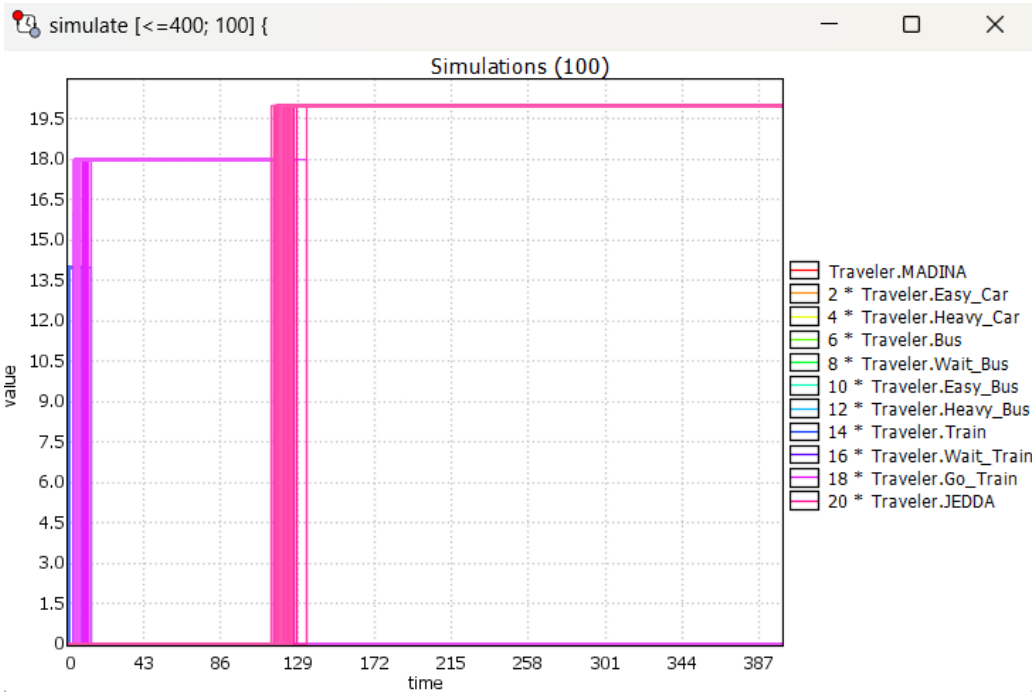


Figure A4. Results of the simulation of the Fast Strategies.

Overview

```
// SAFE & FAST STRATEGY
strategy GoFastSafe = minE (trip) [⩽400] : <> Traveler.JEDDA under GoSafe
Pr[⩽200](<> Traveler.JEDDA) under GoFastSafe                    ≥ 0.950056 (95% CI)
Pr[⩽400](<> Traveler.JEDDA) under GoFastSafe                    ≥ 0.950056 (95% CI)
E[⩽400 ; 500] (max: trip) under GoFastSafe                 119.94 ± 0.294294 (95% CI)
simulate [⩽400; 100] { Traveler.MADINA , 2*Traveler.Easy_Car , 4*Traveler.Heavy_Car , 6*Tr
```

Figure A5. Queries related to the Safe and Fast Strategy and the corresponding results.

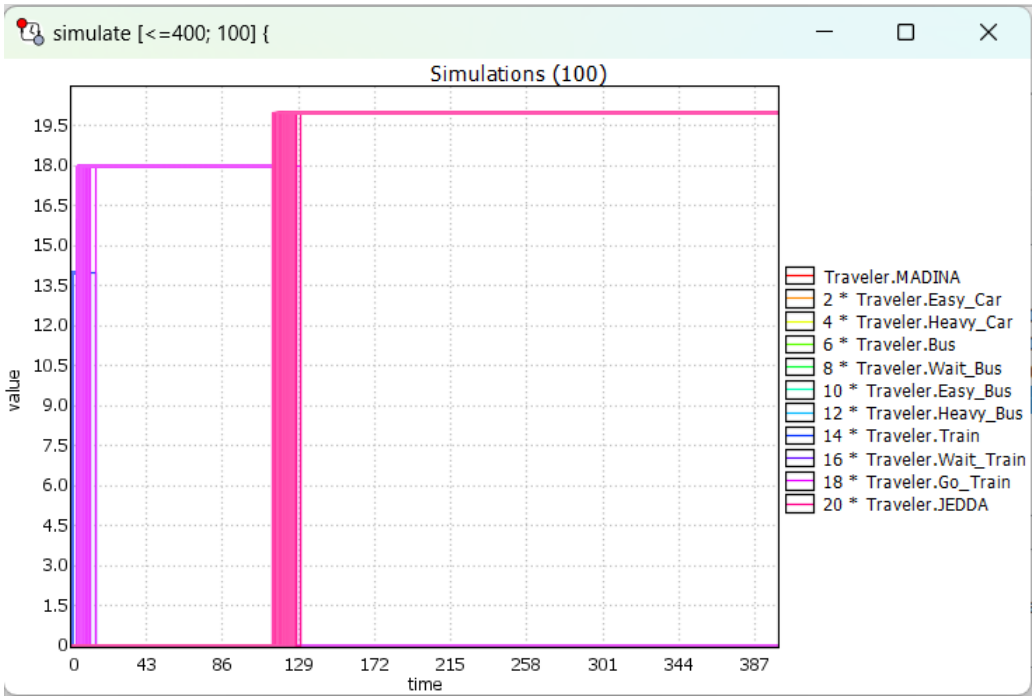


Figure A6. Results of the simulation of the Safe and Fast Strategies.

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