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A Novel Centralized Allocation Data Envelopment Analysis Model for Carbon Emission Allocation Under a Heterogeneous Abatement Cost: Application Within the Chinese Industrial Sector

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Abstract: This paper presents a mathematical approach to analyzing carbon abatement costs and the allocation of carbon emission allowances in China's industrial sectors. We utilize input–output data from 30 Chinese provinces between 2009 and 2018 to estimate carbon abatement costs by applying the slack-based measure (SBM) efficiency model and its dual form. The SBM model captures inefficiencies and offers a rigorous framework for measuring abatement costs. Using these costs, we develop a centralized allocation data envelopment analysis (DEA) model, which maximizes sectoral benefits through optimal reallocation. This DEA model is formalized as a linear programming problem, with the aim of determining the efficient allocations of carbon allowances while maintaining the system's economic productivity. Furthermore, we construct intertemporal, interregional, and spatiotemporal allocation DEA models to examine the dynamics of carbon emission allowance allocation over time, space, and combined spatiotemporal dimensions. These models offer insights into the efficiency of carbon markets under varying conditions. Our proposed new mathematical formulations reveal optimal allocation strategies that can balance emission reductions with industrial productivity. This study also provides novel mathematical frameworks for analyzing the carbon allowance distribution and contributions to both the theory and application of mathematical optimization in environmental policy design. Our findings reveal that China's industrial carbon abatement costs exhibit significant interprovincial and regional differences. Developed provinces with higher levels of industrial development have higher carbon abatement costs, while provinces with less-developed industrial sectors have lower costs. Under the interregional allocation scenario of carbon emission allowances that consider abatement costs, developed provinces have smaller industrial carbon emission reductions, whereas less-developed provinces have larger reductions. In the intertemporal allocation scenario, provinces with larger industrial economies face greater emission reduction tasks. Under the combined interregional and intertemporal allocation scenario, industrial sectors in coastal developed provinces have lower carbon emission reductions, while those in inland less-developed provinces have higher reductions, mirroring the spatial allocation results of carbon emission allowances.

Keywords: centralized allocation DEA; marginal abatement cost; carbon emissions; carbon emission allowances

MSC: 90C05; 90C59; 91B76

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1. Introduction

As the leading sector of the national economy, the industrial sector is also the main source of carbon emissions. Achieving carbon emission reduction targets in this sector is of great significance for China's efforts in reducing carbon dioxide emissions. For a long time, industrial economic growth has been characterized by high energy consumption and high

pollution. This extensive growth model has generated a large amount of carbon dioxide emissions, posing severe challenges to sustainable economic development. According to statistics from the International Energy Agency (IEA) in 2018, the carbon emissions of China's industrial sector accounted for as much as 84.63% of the nation's total carbon emissions and 38.13% of global industrial carbon emissions. The industrial carbon intensity is 1.251 times the global average, far exceeding the levels of developed countries.

Currently, market mechanisms represented by carbon trading have become important means for achieving low-carbon development in the industrial economy. However, China's carbon trading market is still in its early stages; only a few regions and specific industrial sectors are included, and the existing carbon emission allowance allocation system is still imperfect. Issues such as low carbon trading prices, significant differences across pilot regions, large price fluctuations, and low market activity have resulted in the value of carbon emission allowances not being fully reflected through price mechanisms. Therefore, it is necessary to objectively examine the problems in the current allocation mechanisms and study effective ways of distributing carbon emission allowances.

As the core commodity in the carbon trading market, the price and allocation method of carbon emission allowances are among the most important aspects in designing carbon trading systems. According to the externality theory, the emission of carbon dioxide during industrial production generates negative externalities, and the essence of carbon emission allowance allocation is the initial distribution of emission rights, serving as an important tool for reducing these externalities [1]. The carbon price, embodying the value of emission rights, directly determines the abatement costs for emitters. In the context of an immature price mechanism, imperfect policies and allocation mechanisms, and the significant regional segmentation of the carbon market, government departments need to carry out the initial allocation of emission rights. This will help reduce transaction costs in the carbon market and mitigate the social welfare losses caused by carbon emissions.

So, how can government departments, in the absence of a perfect price mechanism, reasonably estimate the carbon abatement costs of different industrial sectors? How can they allocate emission reduction quotas based on these differentiated costs to maximize the total benefits after allocation? Moreover, how should the emission reduction pathways of provincial-level industrial sectors be planned? A systematic examination of these questions can provide practical support for formulating effective allocation strategies, thereby facilitating China's low-carbon industrial transition.

This paper conducts research on the allocation of carbon emission allowances for the industrial sector to explore reasonable strategies for distributing emission reduction tasks, minimizing abatement costs, and maximizing benefits. To this end, the slack-based measure (SBM) efficiency model and the centralized allocation data envelopment analysis (DEA) model are employed. The SBM model can capture inefficiencies by measuring slacks associated with undesirable outputs, such as carbon emissions, which allows for the precise estimation of sectoral abatement costs. Its dual form further facilitates differentiated cost evaluations, essential for designing effective allocation strategies. Meanwhile, the centralized allocation DEA model optimizes resource allocation by comparing the performance of different sectors with an aim to maximize overall benefits. Together, these models enable a comprehensive analysis of temporal and spatial factors in China's industrial sectors. The intertemporal, interregional, and spatiotemporal allocation models are then developed for carbon emission allowances from temporal, spatial, and combined dimensions to study the allocation in China's industrial sectors and the total emission reduction benefits after allocation.

The marginal contributions of this paper to the existing research are mainly reflected in two aspects: First, in terms of research content, we conduct intertemporal, interregional, and spatiotemporal allocations of industrial carbon emission allowances from the perspective of carbon emission efficiency and differentiated marginal abatement costs. We analyze the emission reduction benefits of different industrial sectors under these three scenarios, thereby expanding related research on carbon emission allowance allocation. Second, in

terms of methodology, we improve the centralized allocation DEA model by incorporating differentiated abatement costs into the objective function. We propose centralized allocation DEA models for three scenarios—intertemporal, interregional, and spatiotemporal allocations—thus providing theoretical and methodological references for related studies on carbon emission allowance allocation.

This paper is structured as follows: Section 2 reviews the related literature. Section 3 presents the DEA mathematical models and the associated Karush–Kuhn–Tucker (KKT) conditions. Section 4 conducts an empirical analysis of the allocation of carbon emission quotas for China's provincial industrial sectors. Section 5 concludes the paper and gives a summarization of the main findings of this paper.

2. Literature Review

Carbon emission trading is one of the important policy tools for reducing China's carbon dioxide emissions, and the issue of carbon emission allowance allocation is the core problem of the trading system, serving as the premise for implementing all institutional arrangements. The allocation determines the carbon emission permits at regional, industrial, and enterprise levels. Under the establishment of a unified national carbon trading market, these allowances will transform into the ecological environment endowments of various regions, industries, and enterprises [2]. Therefore, how to allocate carbon emission allowances and what the allocation basis should be have become hot topics among scholars and policymakers [3–5].

Allocation principles are a key focus in the study of carbon emission allowance allocation. The existing research mainly unfolds based on two principles: efficiency and fairness. Studies based on the efficiency principle argue that regions with high carbon productivity should be allocated higher allowances, while regions with low productivity should receive fewer, emphasizing achieving reduction targets at minimal cost and maximizing economic benefits [6,7]. The fairness principle holds that every economic entity has the right to develop economically and generate emissions. This principle often employs different indicators to measure fairness, such as sovereignty standards, grandfathering standards, per capita allocation, and ability-to-pay standards [2]. The choice between fairness and efficiency has always been a hot topic among scholars. Some believe that both should be combined in allocations [1,8]; others argue for efficiency-based allocation to ensure overall benefit maximization, with appropriate compensation for those whose benefits are diminished [6]. Moreover, some scholars believe that efficiency-based methods are themselves manifestations of fairness [7].

Based on different allocation principles, methods for distributing carbon emission allowances have become another research focus. Extensive studies have been conducted on methods suitable for China, mainly including index allocation methods and those based on linear or nonlinear programming. The index allocation method is the most commonly used; it allocates allowances or reduction tasks based on indicators such as population, total emissions, emission intensity, and per capita GDP [1,2]. This method is mainly divided into single-indicator and composite-indicator methods. Single-indicator methods were mostly applied in early allocations. For example, Grub (1990) [9] proposed allocating based on population size; Den Elzen et al. (2014) [10] introduced the per capita cumulative emission permit rule; Ciscar et al. (2013) [11] used indicators like per capita GDP and emissions per unit GDP to allocate allowances among developed countries and compared different results; and Pan et al. (2014) [12] used per capita cumulative emissions to allocate allowances among countries, considering historical emissions and future demands of developed and developing nations. However, single-indicator methods are often not comprehensive and may harm the interests of some entities, leading to rejection by emitters. Therefore, some studies have chosen multi-indicator methods, which can aggregate different indicators measuring fairness and efficiency, making results more reasonable. Qin et al. (2017) [13] used the MCDA method, considering four indicators—emission reduction capacity, potential, responsibility, and efficiency—to study allocation

among China's eastern coastal provinces. Tian et al. (2022) [1] proposed a comprehensive method based on historical, demand, contribution, and security fairness, analyzing fair and effective allocation among provinces. However, in multi-indicator methods, determining indicator weights and choosing reasonable composite methods remain controversial.

In addition to index methods, linear and nonlinear programming methods are also used, with data envelopment analysis (DEA) being a representative. Gomes and Lins (2008) [14] first proposed the Zero Sum Gains DEA model (ZSG-DEA) to study allocation among Kyoto Protocol signatories. Lozano et al. (2009) [15] proposed using a centralized allocation DEA model, allocating allowances by maximizing desirable outputs, reducing total emissions, and decreasing resource consumption. Early DEA models typically considered only efficiency, paying insufficient attention to fairness and abatement costs. In recent years, some studies have simultaneously considered fairness and abatement costs when using DEA models. Cui et al. (2021) [16] proposed a zero-sum game DEA model combined with the entropy method, aiming to incorporate fairness, efficiency, and sustainability into the allocation framework, studying allocation among 30 Chinese provinces. Guo et al. (2021) [17] constructed a bi-objective DEA model including fairness and efficiency to study inter-provincial particulate matter allocation, finding that it reduced abatement costs and fluctuations, improved overall performance, and made allocations more equitable. Kong et al. (2019) [8], based on fairness and efficiency, combined DEA models and the entropy method to analyze China's quota allocation from provincial and regional perspectives. Yu et al. (2019) [18] established a centralized allocation DEA model considering abatement costs, achieving allocation among provincial industrial sectors by maximizing revenue and minimizing costs. Additionally, some studies have used nonlinear methods. Wang et al. (2014) [19] established a hybrid nonlinear grey prediction and quota allocation model for inter-provincial allocation; Liu and Lin (2017) [20] proposed a new nonlinear programming method to obtain optimal allocations at minimal cost, simulating marginal abatement cost curves based on environmental performance.

In summary, the existing literature has studied allocation among different provinces and industries in China based on fairness and efficiency, proposing various methods including index and optimization allocations, providing important support for policies and tasks. However, several areas remain for expansion and improvement: First, existing DEA models often aim to maximize overall desirable output, ignoring differences in abatement costs among entities and their impact on allocation. Second, studies using DEA models mainly focus on allocations among countries, regions, and provinces, paying less attention to industrial sectors. Third, although some of the literature allocates allowances in the industrial sector using DEA models, they often overlook the impact of input–output slacks on cost estimation accuracy.

3. Mathematical Models

In this section, we construct a new centralized allocation DEA model by considering carbon reduction costs. The proposed model studies the allocation of carbon emission quotas across periods, regions, and a combined spatio-temporal allocation for the industrial sector from an overall optimal perspective. Different from the existing centralized allocation DEA models proposed by Zhou et al. (2014) [7] and Yu et al. (2019) [18], the DEA model in this paper considers the impact of carbon reduction costs on the allocation of carbon emissions while maximizing the overall expected output.

3.1. Estimation of Carbon Reduction Costs Based on Efficiency Measurement Models

This subsection first presents the linear primal form of the slack-based measure (SBM) efficiency model and then derives its dual form based on the SBM model. The carbon reduction cost is estimated based on the shadow prices of the expected output and the undesirable output from the dual model. The linear primal form of the SBM model is given as follows:

$$
\begin{aligned}
&\min EF_v = \alpha - 1/((m+h)(\sum_{i=1}^{m} s_i^x / x_{ito} + \sum_{f=1}^{h} s_f^b / b_{fto})) \\
&\text{s.t.} \\
&\alpha + 1/\left(\sum_{r=1}^{s} s_r^y / y_{rto}\right) = 1, \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} z_{tj} x_{itj} + s_i^x = \alpha x_{ito},\ (i = 1, \ldots, m) \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} z_{tj} y_{rtj} - s_r^y = \alpha y_{rto},\ (r = 1, \ldots, s) \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} z_{tj} b_{ftj} + s_f^b = \alpha b_{fto},\ (f = 1, \ldots, h) \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} z_{tj} = \alpha, \\
&z_{tj} \geq 0,\ s_i^x \geq 0,\ s_r^y \geq 0,\ s_f^b \geq 0,\ \alpha > 0,\ t = 1, \ldots, T,\ j = 1, \ldots, J,
\end{aligned}
\tag{1}
$$

where the objective function represents the carbon emission efficiency of the evaluated decision-making unit o, and x_{ito}, b_{fto}, and y_{rto} are the inputs, expected output, and undesirable output of the decision unit o, respectively. s_i^x, s_r^y, and s_f^b are slack variables for inputs, outputs, and undesirable outputs, while z_{tj} is the intensity variable, and α is a positive real number. The subscript j represents the j-th decision-making unit, and i, r, f denote the types of inputs, expected outputs, and undesirable outputs, respectively.

The dual form of the above SBM linear form is as follows:

$$
\begin{aligned}
&\max \phi \\
&\text{s.t.} \\
&\phi + \sum_{i=1}^{m} v_i x_{ito} - \sum_{r=1}^{s} u_r y_{rto} + \sum_{f=1}^{h} w_f b_{fto} + \sigma \leq 1, \\
&\sum_{i=1}^{m} v_i x_{itj} - \sum_{r=1}^{s} u_r y_{rtj} + \sum_{f=1}^{h} w_f b_{ftj} + \sigma \geq 0, && (j = 1, \ldots, J) \\
&v_i \geq \frac{1}{x_{ito}(m+h)}, && (i = 1, \ldots, m) \\
&u_r \geq \frac{\phi}{s y_{rto}}, && (r = 1, \ldots, s) \\
&w_f \geq \frac{1}{b_{fto}(m+h)}, && (f = 1, \ldots, h)
\end{aligned}
\tag{2}
$$

In the above model, ϕ is the objective function that represents the carbon emission efficiency value of the decision-making unit. v_i, u_r and w_f are dual variables corresponding to the input, output, and undesirable output constraints in the primal model. Other variables have the same meaning as in model (1).

Following the methods of Wang and He (2017) [21] and Yu et al. (2019) [18], the carbon reduction cost of decision unit o (industrial sector) is defined as follows:

$$
p_{fo} = p_{ro} \frac{w_{fo}}{u_{ro}}, \tag{3}
$$

where w_{fo} and u_{ro} are the shadow prices of CO_2 and output for the decision unit o, obtained by solving model (2). p_{ro} is the actual price of output and is assumed to be 1. Then, p_{fo} represents the relative shadow price of carbon emissions, which indicates the carbon reduction cost and reflects the marginal rate of substitution between carbon emissions and output. This implies that decision units cannot reduce emissions at no cost, and carbon reduction has a cost effect (Wang and He, 2017) [21].

3.2. The Allocation of Industrial Carbon Emission Quotas Based on the Centralized Allocation DEA Model

3.2.1. Production Technology Set

Assume there is a central decision-maker (government department) and J decision-making units (provincial industrial sectors). Each decision-making unit uses m types of inputs $\boldsymbol{x}_j = (x_{j1}, \ldots, x_{jm})$ to produce s types of expected outputs $\boldsymbol{y}_j = (y_{j1}, \ldots, y_{js})$ and h types of undesirable outputs $\boldsymbol{b}_j = (b_{j1}, \ldots, b_{jh})$. The variable λ represents a weight or intensity variable that is used to construct a convex combination of different decision-making units (DMUs). The central decision-maker allocates carbon emission quotas by maximizing the total revenue of emission reductions for all decision-making units. Each decision unit determines the quantities of inputs, expected outputs, and undesirable outputs under the direction of the central decision-maker while satisfying the DEA production technology set. The specific form of the production technology set is as follows:

$$P_t(x_t) = \left\{ \begin{array}{lll} (y_t, b_t): & \sum\limits_{k=1}^{K} \lambda_{tk} x_{tki} \le x_{ti}, & (i = 1, \ldots, m) \\ & \sum\limits_{k=1}^{K} \lambda_{tk} y_{tk} \ge y_t, & (r = 1, \ldots, s) \\ & \sum\limits_{k=1}^{J} \lambda_{tk} b_{tk} = b_t, & (f = 1, \ldots, h) \\ & \sum\limits_{k=1}^{J} \lambda_{tk} = 1, & \\ & \lambda_{tk} \ge 0, & (k = 1, \ldots, J) \end{array} \right\}. \tag{4}$$

The production technology set (4) satisfies the following properties:

Property 1. *$P(0) = (0, 0)$. This means that when the input is 0, both the desired output and the undesired output will be 0.*

Property 2. *If $(y_t, b_t) \in P_t(x_t)$ and $b_t = 0$, then $y_t = 0$. This means that when a decision-making unit produces a desired output, it will definitely produce an undesired output, with the undesired output being a by-product of the desired output.*

Property 3. *If $(y_t, b_t) \in P_t(x_t)$ and $y'_t \le y_t$, then $(y'_t, b_t) \in P_t(x_t)$ implies that the desired output satisfies free disposability.*

Property 4. *If $x_t > x'_t$, then $P_t(x_t) \subseteq P_t(x'_t)$ implies that the input satisfies free disposability.*

Property 5. *If $(y_t, b_t) \in P_t(x_t)$, then $(\theta y'_t, \theta b'_t) \in P_t(x_t)$, with $0 \le \theta \le 1$. This reflects that the undesired output satisfies weak disposability.*

3.2.2. Spatial Allocation Model for Carbon Emissions

Based on the above production possibility set (4) and carbon reduction costs (3), we construct a spatial allocation model for carbon emissions. The spatial allocation model assumes that carbon emission quotas can be optimally allocated between different decision-making units with the aim of maximizing the total revenue of expected outputs after subtracting carbon reduction costs, while satisfying the DEA production technology set. The specific form of the spatial allocation model is given as follows:

$$
\begin{aligned}
&\max \sum_{r=1}^{s}\sum_{k=1}^{J} p_{rk}\hat{y}_{rk} - \sum_{f=1}^{h}\sum_{k=1}^{J} p_{fk}b_{fk} \\
&\text{s.t.} \\
&\sum_{j=1}^{J} \lambda_{jk}x_{ij} \le x_{ik}, && (i = 1,\ldots,m;\ k = 1,\ldots,J) \\
&\sum_{j=1}^{J} \lambda_{jk}y_{rj} \ge \hat{y}_{rk}, && (r = 1,\ldots,s;\ k = 1,\ldots,J) \\
&\sum_{j=1}^{J} \lambda_{jk}b_{fj} = b_{fk} - \Delta b_{fk}, && (f = 1,\ldots,h;\ k = 1,\ldots,J) \\
&\sum_{k=1}^{J} \Delta b_{fk} = \delta \sum_{k=1}^{J} b_{fk}, && (f = 1,\ldots,h) \\
&\lambda_{jk} \ge 0,\ \hat{y}_{rk} \ge 0,\ \Delta b_{fk} \ge 0.
\end{aligned}
\tag{5}
$$

Here, p_{rk} and p_{fk} are the prices of the expected outputs and the carbon reduction cost for the k-th decision unit, which are obtained from model (2) and Equation (3), respectively. $\hat{y}_{rk}$, Δb_{fk} are decision variables representing the optimal output and carbon reduction quantities for each decision unit, with λ_{jk} being the intensity variable. The model achieves the allocation of carbon emission quotas between decision units by maximizing the difference between the expected output and the total carbon reduction costs for all decision units under the production technology set (4). The total amount of CO_2 reduction is set as a multiple of the current CO_2 emissions for all decision-making units, where δ is the reduction coefficient with $0 \le \delta \le 1$. The model (5) needs to be solved t times. In each period t, the carbon emissions for J DMUs are computed.

Model (5) theoretically enriches the application of centralized allocation DEA in the field of carbon emission allocation. By incorporating carbon abatement costs and emission quota constraints, model (5) not only optimizes economic outputs in various regions but also achieves equitable emission reductions across regions. This framework provides a balanced approach to economic benefits and environmental sustainability, which extends the theory of multi-dimensional resource allocation and offers theoretical support for policymakers to find optimal solutions in multi-regional carbon management.

In terms of managerial implications, the model (5) provides central decision-makers with a scientific method for allocating carbon emission quotas, allowing them to maximize overall economic benefits while ensuring the reasonable distribution of emission reduction tasks across regions. By identifying regions with a higher potential for carbon reduction and prioritizing the allocation of reduction quotas to these areas, decision-makers can formulate policies that align with regional realities and balance economic and environmental objectives. This model assists the central government in achieving optimal resource allocation when designing cross-regional carbon policies, thereby promoting regional coordinated development and sustainable growth.

Figure 1 gives an intuitive explanation of model (5). As shown in the figure, there are three DMUs (i.e., A, B, and C) undertaking carbon reduction over the future T periods. In each period, the decision-making units A, B, and C form a production frontier F, with the frontier differing across periods. Model (5) assumes that the total CO_2 emissions are reallocated among different DMUs in the same period. As illustrated in the figure, the carbon reduction allocation for each DMU A, B, and C is determined solely by the total reduction quota of the current period (see the dashed lines and arrows in the figure) and is independent of the reduction quotas in other periods.

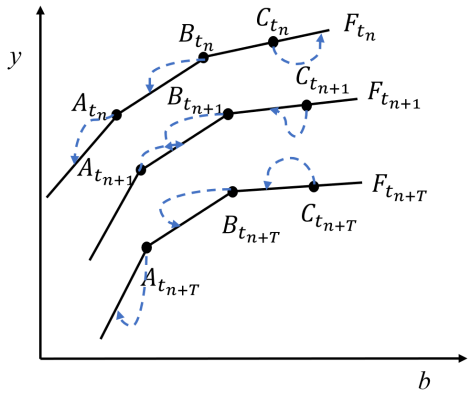


Figure 1. Intuitive illustration of model (5).

3.2.3. Temporal Allocation Model for Carbon Emissions

The temporal allocation model assumes that the carbon reduction quota for a given decision-making unit can be distributed over different time periods, maximizing the difference between the total expected output and the carbon reduction costs for the unit across all periods. The specific form of the temporal allocation model is as follows:

$$
\begin{aligned}
&\max \sum_{r=1}^{s}\sum_{t=1}^{T} p_{rt}\hat{y}_{rt} - \sum_{f=1}^{h}\sum_{t=1}^{T} p_{ft}b_{ft} \\
&\text{s.t.} \\
&\sum_{k=1}^{J} \lambda_{tk}x_{itk} \le x_{it}, && (i = 1,\ldots,m;\ t = 1,\ldots,T) \\
&\sum_{k=1}^{J} \lambda_{tk}y_{rtk} \ge \hat{y}_{rt}, && (r = 1,\ldots,s;\ t = 1,\ldots,T) \\
&\sum_{k=1}^{J} \lambda_{tk}b_{ftk} = b_{ft} - \Delta b_{ft}, && (f = 1,\ldots,h;\ t = 1,\ldots,T) \\
&\sum_{t=1}^{T} \Delta b_{ft} = \delta \sum_{t=1}^{T} b_{ft}, && (f = 1,\ldots,h) \\
&\lambda_{tk} \ge 0,\ \hat{y}_{rt} \ge 0,\ \Delta b_{ft} \ge 0,
\end{aligned}
\tag{6}
$$

where p_{rt} and p_{ft} are the prices of expected outputs and carbon reduction costs for the given decision unit, respectively. The other variables are the same as those in model (5). Model (6) maximizes the difference between the total expected output revenue and the total carbon reduction costs for the decision unit across all periods, with inputs, outputs, and carbon emission constraints under the production technology set (4). The total carbon reduction amount is also set as a multiple of the reduction amount for the decision-making unit in all periods, with δ being the reduction coefficient. Model (6) needs to be solved J times. For each time, the carbon emissions of each DMU in different times can be computed.

Model (6) expands the theoretical framework of centralized allocation DEA by introducing the dimension of time in resource and emission allocation. It accounts for the temporal variability in both carbon reduction efforts and output production and enhances the flexibility of the model to accommodate dynamic efficiency evaluations. Model (6) also introduces a proportional reduction constraint, which ensures that emission reductions are achieved uniformly across different time periods, and contributes to the theoretical understanding of how efficiency and environmental sustainability can be balanced over time.

From a managerial perspective, model (6) provides a practical tool for distributing carbon emission reduction quotas across multiple time periods, allowing decision-making units to align their long-term strategic planning with regulatory emission reduction targets. It supports efficient resource allocation by helping decision-makers prioritize periods where emission reductions are most cost-effective. Managers can use this model to implement

cost-effective, phased emission reduction strategies and ensure compliance with carbon regulations while maximizing economic output over time.

Figure 2 gives the intuitive explanation of model (6). As shown in the figure, there are three DMUs (i.e., *A*, *B*, and *C*) undertaking carbon reduction over the future *T* periods. In each period, the decision-making units *A*, *B*, and *C* form a production frontier *F*, with the frontier differing across periods. Model (6) allows each DMU to reallocate its total CO_2 emissions, given by regulators, across different periods to optimize its cumulative output over the designated timeframe. Thus, model (6) calculates the carbon reduction amounts for each DMU across different future periods and generates a reduction path for each unit. As illustrated, the reduction path for DMU *A* over the future periods is labeled as *Path A* (see dashed lines and arrows). Similarly, the reduction path for DMU B is labeled as *Path B*, and for DMU C as *Path C*.

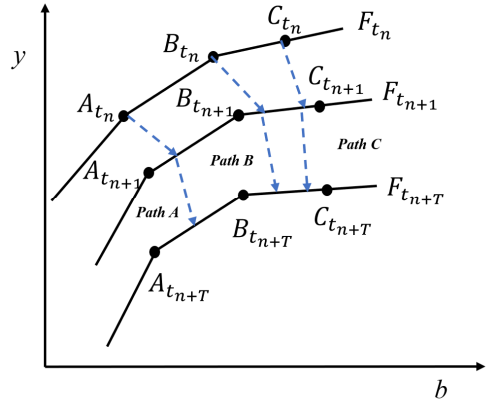


Figure 2. Intuitive illustration of model (6).

3.2.4. Spatio-Temporal Allocation Model for Carbon Emissions

Model (5) distributes carbon reduction targets across regions from a static perspective. Meanwhile, model (6) distributes carbon reduction targets over time for a given decision unit. The spatio-temporal allocation model combines these two by distributing carbon reduction targets across both regions and time. The model assumes that the central decision-maker maximizes the difference between the total expected output and the carbon reduction costs across all periods and regions. The specific form of the model is as follows:

$$
\begin{aligned}
&\max \sum_{r=1}^{s}\sum_{t'=1}^{T}\sum_{k=1}^{J} p_{rkt'}\hat{y}_{rkt'} - \sum_{f=1}^{h}\sum_{t'=1}^{T}\sum_{k=1}^{J} p_{fkt'}b_{fkt'} \\
&\text{s.t.} \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}x_{ijt} \le x_{ikt'}, && (i=1,\ldots,m;\ k=1,\ldots,J;\ t'=1,\ldots,T) \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}y_{rjt} \ge \hat{y}_{rkt'}, && (r=1,\ldots,s;\ k=1,\ldots,J;\ t'=1,\ldots,T) \\
&\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}b_{fjt} = b_{fkt'} - \Delta b_{fkt'}, && (f=1,\ldots,h;\ k=1,\ldots,J;\ t'=1,\ldots,T) \\
&\sum_{t=1}^{T}\sum_{k=1}^{K} \Delta b_{fkt'} = \delta\sum_{t=1}^{T}\sum_{k=1}^{K} b_{fkt'}, && (f=1,\ldots,h) \\
&\lambda_{jtkt'} \ge 0, \\
&\hat{y}_{rkt'} \ge 0, \\
&\Delta b_{fkt'} \ge 0,
\end{aligned}
\tag{7}
$$

where $p_{rkt'}$ and $p_{fkt'}$ represent the prices of the expected output and the carbon reduction costs for different decision units at different times. Model (7) differs from models (5) and (6) in two ways: one is that the total carbon reduction amount is set as a multiple of the

total carbon emissions for different decision units across all periods; the other is that the efficiency frontier in model (7) is formed by the convex combination of all effective decision units across different periods, where the frontier is only determined by decision units within the same period or given decision units across periods for models (5) and (6). Model (7) needs to be solved one time. The carbon emission of each DMU at different times can be computed simultaneously.

Model (7) advances the theoretical frameworks of models (5) and (6) by integrating both temporal and spatial dimensions into carbon allocation. It demonstrates the complexity of optimizing resource allocation when decision units are evaluated across both time and space and provides a more dynamic and realistic representation of carbon reduction strategies. Model (7) contributes to the broader eco-efficiency literature by showing how regional and temporal factors can influence carbon reduction efforts and offers a refined approach to understanding multi-dimensional efficiency in production systems.

For managers and policymakers, model (7) offers a comprehensive tool to strategically allocate carbon reduction quotas across regions and time periods, allowing for flexibility in both geographic and temporal planning. It helps identify cost-effective emission reduction strategies by considering both regional capabilities and temporal constraints. Model (7) is particularly useful for national or multi-regional authorities, and enables them to balance the economic output and environmental targets of various regions while accounting for future emission reductions. Managers can use this approach to ensure that both regional policies and long-term carbon reduction goals are aligned efficiently.

Figure 3 gives the intuitive explanation of model (7). As shown in the figure, there are three DMUs (i.e., *A*, *B*, and *C*) undertaking carbon reduction over the future *T* periods. The decision-making units *A*, *B*, and *C* in all periods form a meta-frontier *F*. Model (7) assumes that the total CO_2 emissions are reallocated among different DMUs in different periods. As illustrated in the figure, the carbon reduction path (see dashed lines and arrows) for each DMU *A*, *B*, and *C* is determined by the total reduction quota across all periods and all DMUs.

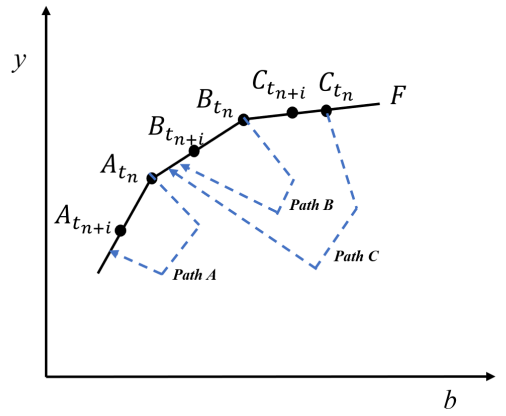


Figure 3. Intuitive illustration of model (7).

3.2.5. Karush–Kuhn–Tucker (KKT) Conditions

The KKT conditions are a set of necessary conditions for optimality in a constrained optimization model. They generalize the Lagrange multiplier method to handle inequality and equality constraints. To obtain the KKT conditions of proposed models (5)–(7), we employ the techniques of the Lagrangian duality. The basic idea in Lagrangian duality is to take the inequality constraints and equality constraints into account by augmenting the objective functions. In this subsection, we only give the detailed algorithm for solving model (7); then, the corresponding algorithms for models (5) and (6) can be correspondingly obtained. The Lagrangian functional is actually a combination of the original objective function and the constraints, each multiplied by their corresponding Lagrange multipliers, that is briefly written as follows:

$$\begin{aligned}\mathcal{L} = & -\left(\sum_{r=1}^{s}\sum_{t'=1}^{T}\sum_{k=1}^{J} p_{rkt'}\hat{y}_{rkt'} - \sum_{f=1}^{h}\sum_{t'=1}^{T}\sum_{k=1}^{J} p_{fkt'}b_{fkt'}\right) \\ & + \sum_{i=1}^{m}\sum_{k=1}^{J}\sum_{t'=1}^{T} \mu_{ikt'}\left(\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}x_{ijt} - x_{ikt'}\right) \\ & + \sum_{r=1}^{s}\sum_{k=1}^{J}\sum_{t'=1}^{T} \nu_{rkt'}\left(\hat{y}_{rkt'} - \sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}y_{rjt}\right) \\ & + \sum_{f=1}^{h}\sum_{k=1}^{J}\sum_{t'=1}^{T} \xi_{fkt'}\left(\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}b_{fjt} - \left(b_{fkt'} - \Delta b_{fkt'}\right)\right) \\ & + \sum_{f=1}^{h} \gamma_f\left(\sum_{t=1}^{T}\sum_{k=1}^{K} \Delta b_{fkt'} - \delta\sum_{t=1}^{T}\sum_{k=1}^{K} b_{fkt'}\right) \\ & - \sum_{t=1}^{T}\sum_{j=1}^{J}\sum_{k=1}^{J}\sum_{t'=1}^{T} \alpha_{jtkt'}\lambda_{jtkt'} - \sum_{r=1}^{s}\sum_{k=1}^{J}\sum_{t'=1}^{T} \beta_{rkt'}\hat{y}_{rkt'} - \sum_{f=1}^{h}\sum_{k=1}^{J}\sum_{t'=1}^{T} \theta_{fkt'}\Delta b_{fkt'}\end{aligned} \tag{8}$$

Here, we refer to $\mu_{ikt'}$ and $\nu_{rkt'}$ as the Lagrange multiplier associated with the first two inequality constraints in model (7); similarly, we refer to $\xi_{fkt'}$ and γ_f as the Lagrange multiplier associated with the third and fourth equality constraints in model (7); and $\alpha_{jtkt'}$, $\beta_{rkt'}$, and $\theta_{fkt'}$ are the Lagrange multipliers associated with the last three constraints, respectively.

The KKT conditions of model (7) usually consist of four essential components, which are primal feasibility, dual feasibility, stationarity, and complementary slackness:

(1) **Primal feasibility:** The primal feasibility condition in the KKT optimality conditions ensures that the primal variables $\lambda_{jtkt'}$, $\hat{y}_{rkt'}$, $\Delta b_{fkt'}$ satisfy the original constraints of the optimization problem (7). The primal feasibility is directly tied to the constraints provided in the original model (7) as follows:

$$\begin{aligned}&\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}x_{ijt} \le x_{ikt'}, && (i = 1,\ldots,m;\ k = 1,\ldots,J;\ t' = 1,\ldots,T) \\ &\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}y_{rjt} \ge \hat{y}_{rkt'}, && (r = 1,\ldots,s;\ k = 1,\ldots,J;\ t' = 1,\ldots,T) \\ &\sum_{t=1}^{T}\sum_{j=1}^{J} \lambda_{jtkt'}b_{fjt} = b_{fkt'} - \Delta b_{fkt'}, && (f = 1,\ldots,h;\ k = 1,\ldots,J;\ t' = 1,\ldots,T) \quad (F_{\text{primal}}) \\ &\sum_{t=1}^{T}\sum_{k=1}^{K} \Delta b_{fkt'} = \delta\sum_{t=1}^{T}\sum_{k=1}^{K} b_{fkt'}, && (f = 1,\ldots,h) \\ &\lambda_{jtkt'} \ge 0, \\ &\hat{y}_{rkt'} \ge 0, \\ &\Delta b_{fkt'} \ge 0.\end{aligned}$$

(2) **Stationarity:** The stationarity condition ensures that the gradient of the Lagrangian function with respect to the primal variables is zero, which makes a balance between the objective function and the active constraints. Mathematically, the stationarity condition of model (7) is given as follows:

$$\begin{aligned}&\frac{\partial\mathcal{L}}{\partial\lambda_{jtkt'}} = -\sum_{i=1}^{m} \mu_{ikt'}x_{ijt} + \sum_{r=1}^{s} \nu_{rkt'}y_{rjt} + \sum_{f=1}^{h} \xi_{fkt'}b_{fjt} - \alpha_{jtkt'} = 0, \\ &\frac{\partial\mathcal{L}}{\partial\hat{y}_{rkt'}} = -p_{rkt'} - \nu_{rkt'} - \beta_{rkt'} = 0, \qquad (F_{\text{stationarity}}) \\ &\frac{\partial\mathcal{L}}{\partial\Delta b_{fkt'}} = -\xi_{fkt'} + \gamma_f - \theta_{fkt'} = 0.\end{aligned}$$

(3) **Dual feasibility:** This condition ensures that the Lagrange multipliers associated with inequality constraints are non-negative. For each inequality constraint in model (7), we have the following:

$$F_{\text{dual}} = \left\{ \mu_{ikt'} \geq 0, \nu_{rkt'} \geq 0, \alpha_{jtkt'} \geq 0, \beta_{rkt'} \geq 0, \theta_{fkt'} \geq 0 \right\}.$$

(4) **Complementary slackness:** The complementary slackness condition ensures that for each inequality constraint, either the constraint is active (binding), or the corresponding Lagrange multiplier is zero.

$$\begin{aligned} &\mu_{ikt'} \cdot \left(x_{ikt'} - \sum_{t=1}^{T} \sum_{j=1}^{J} \lambda_{jtkt'} x_{ijt} \right) = 0, \\ &\nu_{rkt'} \cdot \left(\sum_{t=1}^{T} \sum_{j=1}^{J} \lambda_{jtkt'} y_{rjt} - \hat{y}_{rkt'} \right) = 0, \quad (F_{\text{complementarity}}) \\ &\alpha_{jtkt'} \cdot \lambda_{jtkt'} = 0, \\ &\beta_{rkt'} \cdot \hat{y}_{rkt'} = 0, \\ &\theta_{fkt'} \cdot \Delta b_{fkt'} = 0. \end{aligned}$$

In practice, the primal–dual interior point method is an efficient method for solving the KKT system. It works by solving a perturbed version of the KKT conditions and iteratively converging to the exact solution while maintaining both primal and dual feasibility.

4. Empirical Analysis

4.1. Indicator Selection and Data Sources

This paper studies the allocation of carbon emission quotas for China's provincial industrial sectors based on the dual form of the SBM efficiency model and the centralized allocation DEA model. Each provincial industrial sector in different years is treated as a different decision unit. Due to data availability, this paper uses input–output data from 30 provincial industrial sectors in China, excluding Tibet, Taiwan, Hong Kong, and Macau, from 2009 to 2018. The inputs for the industrial sector include labor, energy, and capital; the expected output is industrial value-added; and the undesirable output is carbon emissions. The specific selection methods and data sources for input–output indicators are as follows:

Labor: Following Ouyang et al. (2021) [22], the labor input is calculated as the total number of employees in the private and non-private sectors in mining, manufacturing, electricity, gas, and water production and supply industries. Labor data for these industries are sourced from the *China Statistical Yearbook* (2010–2019).

Energy: The energy input is calculated by multiplying the consumption of nine types of energy (coal, coke, liquefied petroleum gas, kerosene, crude oil, fuel oil, gasoline, natural gas, and diesel) by their respective standard coal conversion coefficients, then summing the results. The data on energy consumption and standard coal conversion coefficients are from the *China Energy Statistical Yearbook* (2010–2019).

Capital: The capital input is represented by the total assets of industries above a designated size, with data from the *China Statistical Yearbook* (2010–2019).

Industrial Value-Added: The industrial value-added is selected as the expected output. To account for price factors, the GDP deflator is used to adjust industrial value-added to constant 2009 prices. Data on GDP indices and industrial value-added are sourced from the *China Statistical Yearbook* (2010–2019).

CO_2 Emissions: Following the methods of Sueyoshi et al. (2019) [23] and Liu et al. (2022) [24], CO_2 emissions are indirectly calculated based on energy consumption in the industrial sector. The calculation is as follows:

$$b_{tj} = \sum_{n=1}^{N} x_{ntj}^{E} \times \alpha_n,$$

where b_{tj} is the CO_2 emissions of the j-th provincial industrial sector in period t, and x^E_{ntj} is the consumption of the n-th type of energy by the j-th sector in period t. α_n is the carbon emission coefficient for the n-th type of energy. The total CO_2 emissions for the provincial industrial sector are obtained by multiplying the consumption of various types of energy by their carbon emission coefficients and summing the results. The types of energy used in this calculation are the same as those in the calculation of the total energy consumption, as mentioned earlier. The energy consumption data by type and the carbon emission coefficients are sourced from the *China Energy Statistical Yearbook* (2010–2019), the 2006 IPCC Guidelines for National Greenhouse Gas Inventories, and the Guidelines for Compiling Provincial Greenhouse Gas Inventories (NDRC Climate [2011]1041).

4.2. Industrial Sector Carbon Reduction Cost Estimates

This subsection estimates the marginal cost of carbon dioxide reduction in China's provincial industrial sectors from 2009 to 2018 based on the dual SBM model. The specific results are shown in Figure 4 and Table 1.

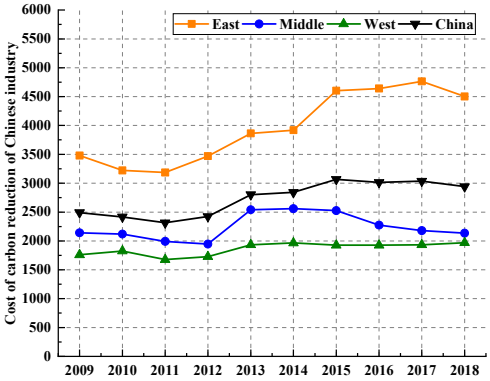


Figure 4. Trends of China's industrial carbon reduction costs from 2009 to 2018.

From a national and regional perspective, there are significant regional differences in the carbon reduction costs of the industrial sector. The carbon reduction costs are the highest in the eastern region, followed by the central region, and lowest in the western region. Both the central and western regions have reduction costs lower than the national average. Specifically, from 2009 to 2018, the average carbon reduction cost in the eastern region was 3964.261 CNY/ton, while the central and western regions had costs of 2236.123 CNY/ton and 1863.284 CNY/ton, respectively. The national average reduction cost was 2740.266 CNY/ton. This indicates that the industrial structure in the eastern region is more advanced, with higher industrial carbon emission efficiency and less room for further reductions, leading to higher marginal costs of carbon reduction. In contrast, the central and western regions have relatively lower industrial development levels and carbon emission efficiency, leaving more room for reductions and thus lower costs.

From the perspective of changes over time, as shown in Figure 4, carbon reduction costs increased in the eastern, western, and national averages, while the costs in the central region first rose and then declined. The regional differences in reduction costs have been expanding over time. Specifically, the eastern region saw the most significant growth in reduction costs, rising from 3478.204 CNY/ton in 2009 to 4502.262 CNY/ton in 2018, with an average annual growth rate of 3.18%. In the central region, the costs increased from 2140.171 CNY/ton in 2009 to 2559.138 CNY/ton in 2014, then dropped to 2360.482 CNY/ton in 2018. In the western region, the costs rose from 1759.748 CNY/ton in 2009 to 1967.860 CNY/ton in 2018, with an average annual growth rate of 1.37%. The national average carbon reduction cost increased from 2491.295 CNY/ton in 2009 to 3001.840 CNY/ton in 2018, with an average annual growth rate of 2.26%.

Table 1. Carbon emission reduction costs for provincial industrial sectors in China from 2009 to 2018 (CNY/ton).

Province	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
BJ	6032.385	5933.161	5762.052	8143.525	6292.574	6611.138	12,945.192	13,858.922	15,646.561	15,903.480	9712.899
TJ	2915.061	3224.887	3600.160	3505.765	3504.609	3509.922	3634.033	3564.893	3364.431	2128.672	3295.243
HEB	1333.357	1319.507	1246.934	1096.147	1188.117	1184.793	1286.139	1339.142	1221.089	1220.455	1243.568
LN	2647.979	2393.154	2269.248	2280.826	2584.412	2563.209	2711.716	2656.567	2619.860	2510.488	2523.746
SH	4908.387	4672.978	4801.668	5176.385	5210.892	5090.585	5265.446	5456.083	5673.653	5465.866	5172.194
JS	4200.827	4371.178	4012.351	4290.012	4901.423	5051.207	4938.663	4897.972	4858.793	5031.835	4655.426
ZJ	5494.634	5818.839	5756.586	5944.916	6213.605	6358.773	6486.915	6912.586	7291.996	7438.525	6371.737
FJ	3055.741	2152.305	2409.953	2400.342	2370.806	3582.028	3913.673	2699.666	2900.151	887.780	2637.244
SD	2438.853	2491.372	2445.485	2479.536	2766.945	1850.455	1948.212	2010.936	2016.447	2060.927	2250.917
GD	4268.454	2052.234	1993.612	2018.974	6222.717	6183.302	6301.003	6570.477	6547.670	6605.973	4876.442
HI	964.568	1007.136	735.248	810.520	1223.148	1126.611	1191.491	1074.893	270.064	270.884	867.456
SX	1722.775	1756.527	1737.933	1779.321	2016.887	2080.154	2147.232	2109.784	2127.036	2228.982	1970.663
JL	1979.330	1855.565	1710.761	1850.365	2387.582	2569.083	2980.527	3067.576	3186.686	3245.696	2483.317
HL	2657.199	2604.984	2586.866	2561.337	2711.886	2626.312	2483.021	2297.249	1930.044	2148.948	2460.785
AH	2046.470	2299.614	2330.781	2566.753	2970.318	2998.434	3092.831	3084.095	3099.403	3349.906	2783.860
JX	2720.177	2702.705	1722.135	1725.744	2886.209	3027.775	3260.626	3437.440	3181.501	3478.559	2814.287
HEN	1791.563	1516.620	1557.435	1730.128	2810.441	2867.302	1918.943	1935.529	1978.726	143.699	1825.039
HB	2355.498	2169.999	2099.312	1742.380	2739.598	2848.991	2882.633	1774.817	1434.835	963.257	2101.132
HN	1848.359	2044.167	2184.637	1608.058	1797.578	1455.051	1433.596	478.369	484.421	1511.381	1484.562
IM	1379.471	1615.853	1474.249	1773.816	1731.387	1859.452	1662.474	1537.554	1513.202	1272.351	1581.981
GX	1997.166	2092.091	1960.682	2010.901	2122.559	2123.019	2216.817	2074.468	2046.288	2238.692	2088.268
CQ	1656.023	1879.342	1006.380	1935.613	2912.255	2633.392	2799.518	2897.113	2776.417	3301.180	2379.723
SC	2264.212	2263.138	2358.579	2254.142	2213.628	2363.005	1522.768	1570.596	1553.560	1620.601	1998.423
GZ	1602.031	1605.529	1359.793	1509.839	2078.134	2089.766	2162.240	2223.266	2563.186	2368.851	1956.263
YN	1802.770	1912.577	1921.368	1971.719	2170.400	2331.658	2456.833	2354.881	2347.496	2283.642	2155.335
SN	2672.541	2462.765	2374.698	1815.947	1743.568	1903.610	1958.439	2073.921	2028.892	2087.606	2112.199
GS	2069.256	2048.144	1879.089	1789.640	2100.911	2167.374	2337.368	2466.611	2565.582	2645.558	2206.953
QH	1273.534	1519.062	1608.937	1416.404	1439.307	1338.392	1170.452	1099.269	1239.545	1206.291	1331.119
NX	1066.865	1115.688	1060.559	1158.518	1116.753	1132.375	1059.444	1012.815	868.020	757.336	1034.837
XJ	1573.360	1555.232	1435.965	1354.011	1615.181	1665.813	1835.907	1859.800	1750.650	1864.346	1651.027
East	3478.204	3221.523	3184.845	3467.904	3861.750	3919.275	4602.044	4640.194	4764.610	4502.262	3964.261
Middle	2140.171	2118.773	1991.232	1945.511	2540.063	2559.138	2524.926	2273.107	2177.831	2133.804	2240.456
West	1759.748	1824.493	1676.391	1726.414	1931.280	1964.351	1925.660	1924.572	1932.076	1967.860	1863.284
China	2491.295	2415.212	2313.448	2423.386	2801.461	2839.766	3066.805	3013.243	3036.207	2941.392	2734.222

Note: BJ: Beijing; TJ: Tianjin; HEB: Hebei; LN: Liaoning; SH: Shanghai; JS: Jiangsu; ZJ: Zhejiang; FJ: Fujian; SD: Shandong; GD: Guangdong; HI: Hainan; SX: Shanxi; JL: Jilin; HL: Heilongjiang; AH: Anhui; JX: Jiangxi; HEN: Henan; HB: Hubei; HN: Hunan; IM: Inner Mongolia; GX: Guangxi; CQ: Chongqing; SC: Sichuan; GZ: Guizhou; YN: Yunnan; SN: Shaanxi; GS: Gansu; QH: Qinghai; NX: Ningxia; XJ: Xinjiang.

Looking at the provincial differences in industrial reduction costs, as shown in Table 1, provinces with higher levels of industrial development have higher reduction costs, while provinces with less developed industries have lower costs. The top five provinces with the highest reduction costs are Beijing (9712.899 CNY/ton), Zhejiang (6371.737 CNY/ton), Shanghai (5172.194 CNY/ton), Guangdong (4876.442 CNY/ton), and Jiangsu (4655.426 CNY/ton). In contrast, Hainan (867.456 CNY/ton), Ningxia (1034.837 CNY/ton), Hebei (1243.568 CNY/ton), and Qinghai (1331.119 CNY/ton) have the lowest reduction costs. The higher reduction costs in more developed provinces may be due to their more advanced energy-saving and emission-reduction technologies, strong innovation capabilities, and higher proportions of high-tech and emerging industries, which result in lower carbon emissions per unit of industrial value-added. Consequently, further reducing emissions is more difficult, leading to higher reduction costs. In contrast, less developed regions have less advanced energy-saving and emission-reduction technologies, a higher proportion of traditional high-pollution industries, and relatively inefficient industrial development, leaving more room to reduce emissions by introducing technologies or adjusting the industrial structure, resulting in lower carbon reduction costs. The relatively low carbon reduction costs in

underdeveloped provinces also provide a reasonable explanation for the phenomenon of transferring carbon emissions from developed to underdeveloped regions.

From the perspective of changes in provincial industrial carbon reduction costs over time, 15 provinces experienced an overall decline in carbon reduction costs, while another 15 saw an overall increase. Among them, Beijing, Guangdong, and Zhejiang experienced the largest increases, with increases of 9871.095 CNY/ton, 2337.520 CNY/ton, and 1943.891 CNY/ton, respectively. On the other hand, Fujian, Hubei, and Tianjin saw the largest reductions in carbon reduction costs, decreasing by 2167.961 CNY/ton, 1392.241 CNY/ton, and 786.389 CNY/ton, respectively.

4.3. Research on the Cross-Regional Allocation of Carbon Emissions in the Industrial Sector

Model (5) assumes that a central decision-maker allocates carbon reduction quotas across provinces to maximize the expected output of all industrial sectors after subtracting carbon reduction costs. Additionally, following the approach of Wu et al. (2016) [2] and Yu et al. (2019) [18], the reduction coefficient is set at 30%, meaning that the carbon reduction target for China's industrial sector is set at 30% of the current emission levels. The spatial allocation results for carbon emissions are shown in Table 2 and Figure 5.

Table 2. Inter-regional distribution results and distribution income of industrial carbon emissions from 2009 to 2018 (annual average).

Province	Carbon Quota	Carbon Reduction	Industrial Added Value	Distribution Income
BJ	1032.617	0.288	2795.773	29.015
TJ	4713.612	19.968	4254.590	932.430
HEB	5653.427	31,263.523	8524.423	−177.737
LN	15,339.917	2277.738	10,808.011	3813.347
SH	4511.239	0.000	7311.999	89.072
JS	19,955.948	1184.351	29,997.996	7005.036
ZJ	8086.963	95.174	15,783.926	1562.549
FJ	5876.626	2652.818	8173.755	−353.298
SD	28,583.145	648.560	24,668.007	6694.284
GD	13,429.097	0.000	25,443.536	0.001
HI	286.810	888.837	446.209	73.629
SX	9918.080	6239.247	7607.917	2837.093
JL	5882.305	1618.932	4615.835	2186.343
HL	6810.475	371.083	4890.879	1111.850
AH	10,030.044	1456.238	8318.291	1819.241
JX	4821.354	2975.476	6178.621	810.221
HEN	14,083.875	5256.916	14,325.112	1805.838
HB	11,074.037	3512.481	9488.628	829.287
HN	7063.939	4571.069	6099.945	−1486.67
IM	3647.902	9768.788	3296.808	−400.482
GX	6032.616	2075.310	4545.572	1287.841
CQ	5231.684	2337.634	5081.324	864.470
SC	13,324.657	2874.138	9427.524	959.609
GZ	3224.305	2782.812	3114.581	1030.352
YN	7119.864	2004.297	5303.965	2213.264
SN	8611.724	1009.469	6092.825	576.164
GS	3894.247	588.536	2708.953	929.756
QH	503.339	1569.237	614.493	181.453
NX	506.957	3960.103	767.270	47.027
XJ	3161.513	5602.254	2873.216	601.408
East	9769.945	3548.296	12,564.384	1788.030
Middle	8710.514	3250.180	7690.654	1239.151
West	5023.528	3142.962	3984.230	753.715
China	7747.077	3320.176	8118.666	1262.413

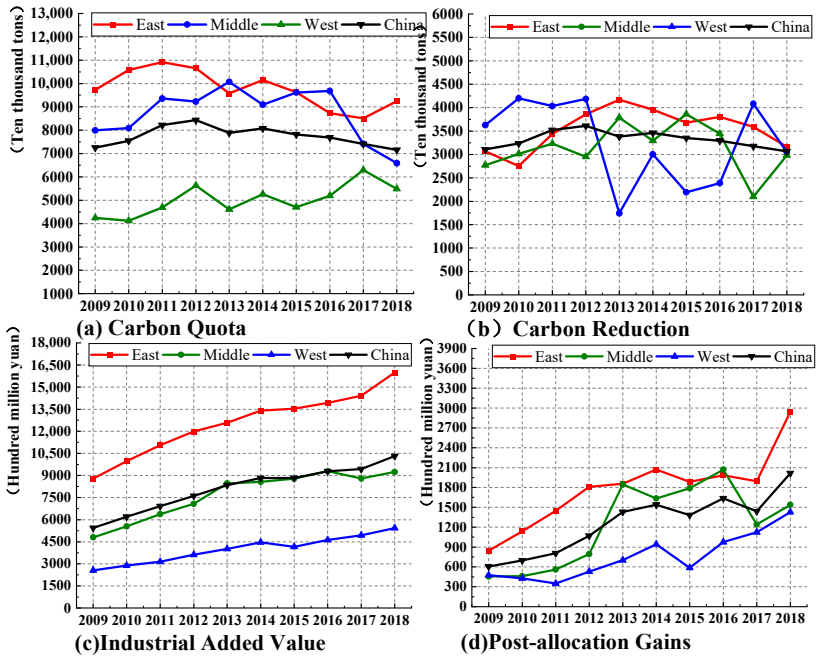


Figure 5. Trends in cross-regional allocation results and distribution gains of carbon emissions from 2009 to 2018.

At the provincial level, Table 2 shows that more developed provinces have smaller industrial carbon reduction quotas, while less developed provinces have larger quotas. Guangdong, Shanghai, Beijing, Tianjin, and Zhejiang have the lowest carbon reduction amounts, with the allocated carbon emission quotas closely matching their actual emissions, indicating that these provinces have already achieved high carbon emission efficiency and face less pressure to reduce emissions. On the other hand, Hebei, Inner Mongolia, and Shanxi have the largest carbon reduction amounts, with reductions of 312.635 million tons, 97.688 million tons, and 62.392 million tons, respectively, accounting for 84.686%, 72.811%, and 38.616% of their current emissions, indicating significant pressure to reduce emissions. Hebei's industrial sector is dominated by coal and steel industries, while Shanxi and Inner Mongolia have coal-based industrial structures with high energy consumption and low energy efficiency, leading to large amounts of carbon dioxide emissions, which explains their high carbon reduction quotas.

Regarding carbon emission quotas, after redistribution, Shandong, Jiangsu, Liaoning, Henan, and Guangdong have the highest quotas, ranking in the top five nationwide. These provinces have large industrial economies and high carbon emission baselines, resulting in relatively high allocated carbon emission quotas. Hainan, Qinghai, and Ningxia have the lowest quotas, ranking in the bottom three, as their carbon emissions are relatively low, and their carbon reduction targets are a high proportion of their current emissions (75.604%, 75.714%, and 88.651%, respectively), leading to a reduction in their allocated carbon emission quotas. Additionally, the reallocation of carbon emission quotas has promoted industrial economic growth in most provinces. After reallocation, Jiangsu, Shandong, Liaoning, and Shanxi experienced the largest increases in industrial value-added, with increases of CNY 700.503 billion, CNY 669.428 billion, CNY 381.334 billion, and CNY 283.709 billion, respectively. Meanwhile, Jilin, Yunnan, Shanxi, and Liaoning saw the largest percentage increases in industrial value-added, with growth rates of 89.992%, 71.610%, 59.468%, and 54.518%. However, carbon reduction has restrained industrial economic growth in a few provinces. For example, industrial carbon reduction caused a decrease in industrial value-added in Hunan by CNY 148.666 billion (19.596% of its current industrial value-added), Inner Mongolia by CNY 40.048 billion (10.832%), Fujian by CNY 35.330 billion (4.143%), and Hebei by CNY 17.737 billion (2.042%). Overall, the reallocation

of industrial carbon emission quotas has helped most provinces reduce carbon emissions while enhancing their industrial economic development.

At the regional level, Table 2 shows that to achieve the 30% reduction target, the eastern region needs to reduce emissions by 35.483 million tons (26.642% of its current emissions), the central region by 32.501 million tons (27.174%), and the western region by 31.429 million tons (38.486%). In terms of absolute reduction amounts, the three regions have similar reduction targets, but as a percentage of current emissions, the western region has the highest reduction proportion, followed by the central region, and the eastern region has the lowest. This indicates that the western and central regions have more potential for carbon reduction and are the key areas for achieving emission reductions. After reallocation, the eastern region has the highest allocated carbon emission quota, followed by the central region, with the western region receiving the lowest. The post-allocation industrial carbon emission quotas for the eastern, central, and western regions are 97.699 million tons, 87.105 million tons, and 50.235 million tons, respectively. This indicates that the more efficient eastern region received a higher carbon emission quota, while the less efficient central and western regions received lower quotas. Additionally, the reallocation of carbon emission quotas has promoted industrial value-added growth in all three regions, with the central and western regions benefiting the most. After reallocation, the industrial value-added in the eastern region reached CNY 1.256 trillion, an increase of CNY 178.803 billion (a growth rate of 16.592%); in the central region, the industrial value-added reached CNY 769.065 billion, an increase of 123.915 billion yuan (a growth rate of 19.207%); and in the western region, the industrial value-added reached CNY 398.423 billion, an increase of CNY 75.371 billion (a growth rate of 23.331%).

The trends in carbon reduction, carbon emission quotas, post-allocation industrial value-added, and industrial value-added growth in the three regions from 2009 to 2018 are shown in Figure 5. The national carbon emission quotas showed a stable trend, first rising and then falling, with similar levels at the beginning and end of the period (72.516 million tons in 2009 and 71.575 million tons in 2018). In the eastern region, the carbon emission quota showed an overall downward trend, decreasing from 97.229 million tons in 2009 to 92.438 million tons in 2018. In the central region, the quota first increased and then decreased, rising from 79.899 million tons in 2009 to a peak of 100.690 million tons in 2013, and then falling to 65.857 million tons in 2018. The western region's industrial carbon emission quota showed an upward trend, rising from 42.434 million tons to 54.871 million tons. The gap between the carbon emission quotas of the three regions narrowed over time. In terms of carbon reduction trends, before 2012, the central region had higher reduction amounts than the eastern and western regions, making it the primary area for industrial carbon reduction. From 2012 to 2016, the eastern region became the main area for carbon reduction, followed by the western region. After 2016, the central region once again took the lead in carbon reduction, becoming the main area for industrial carbon reduction. Nationally, the carbon reduction amount remained stable between 30 million and 35 million tons from 2009 to 2018. In terms of the change in industrial value-added after reallocation, the industrial value-added in the three regions and nationwide increased year by year, with the eastern region growing the fastest, followed by the central region, and the western region growing the slowest. The growth in industrial value-added before and after reallocation showed an overall upward trend across the three regions and nationally, indicating that carbon reduction increasingly promoted industrial economic growth year by year.

4.4. Research on Cross-Temporal Allocation of Carbon Emissions in the Industrial Sector

This subsection utilizes model (6) to calculate the cross-temporal allocation of industrial carbon emissions, assuming that the carbon emission quotas for a given industrial sector can be distributed across different periods. The results of the cross-temporal allocation of carbon emission quotas are shown in Table 3 and Figure 6.

Table 3. Results of the intertemporal allocation of industrial carbon emissions and post-allocation gains from 2009 to 2018 (annual average).

Province	Carbon Quota	Carbon Reduction	Industrial Added Value	Distribution Income
BJ	723.033	309.871	2396.691	−370.066
TJ	3313.506	1420.074	3937.373	615.212
HEB	25,841.865	11,075.085	11,551.407	2849.247
LN	12,332.359	5285.297	10,553.062	3558.399
SH	3157.867	1353.372	6355.271	−867.656
JS	14,798.209	6342.090	28,148.860	5155.901
ZJ	5727.496	2454.641	12,792.244	−1429.13
FJ	5970.611	2558.833	8290.609	−236.445
SD	20,462.194	8769.512	23,513.296	5539.573
GD	9400.368	4028.729	20,213.256	−5230.28
HI	822.953	352.694	621.099	248.520
SX	11,310.129	4847.198	7712.576	2941.752
JL	5250.866	2250.371	4543.720	2114.228
HL	5027.091	2154.468	4693.049	914.021
AH	8040.398	3445.885	8149.026	1649.975
JX	5457.781	2339.049	6277.652	909.253
HEN	13,538.554	5802.237	14,452.090	1932.816
HB	10,210.563	4375.955	9652.294	992.953
HN	8144.506	3490.503	7250.079	−336.532
IM	9391.683	4025.007	3844.681	147.391
GX	5675.548	2432.378	4558.338	1300.607
CQ	5298.522	2270.795	5104.566	887.712
SC	11,339.157	4859.639	9764.903	1296.988
GZ	4204.981	1802.135	3247.366	1163.137
YN	6386.913	2737.248	5229.011	2138.310
SN	6734.835	2886.358	5882.355	365.693
GS	3137.948	1344.835	2639.242	860.045
QH	1450.803	621.773	838.523	405.484
NX	3126.942	1340.118	1048.181	327.938
XJ	6134.637	2629.130	3111.965	840.157
East	9322.769	3995.473	11,670.288	893.934
Middle	8372.486	3588.208	7841.311	1389.808
West	5716.543	2449.947	4115.376	884.860
China	7747.077	3320.176	7879.093	1022.840

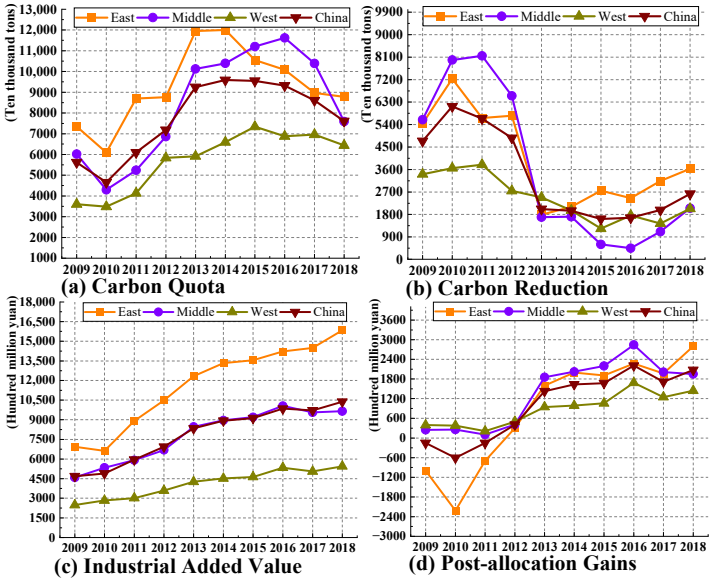


Figure 6. Trends in the results of intertemporal allocation of industrial carbon emissions and post-allocation gains (2009–2018).

As indicated by Table 3, Hebei, Shandong, Jiangsu, Henan, and Liaoning rank in the top five nationwide in terms of average annual carbon reduction. These provinces need to reduce their carbon emissions by 110.751 million tons, 87.695 million tons, 63.421 million tons, 58.022 million tons, and 52.853 million tons annually, respectively. Correspondingly, the industrial value-added growth for these provinces is CNY 284.9247 billion, CNY 553.9573 billion, CNY 515.5901 billion, CNY 193.2808 billion, and CNY 355.8399 billion.

Among these, Hebei, Shandong, Liaoning, and Jiangsu do not only lead in carbon reduction but also in industrial value-added growth, showing significant economic benefits from carbon reduction. This is likely because these provinces are dominated by energy-intensive heavy industries with a large carbon emission baseline, thus receiving large carbon reduction quotas. The pressure to reduce emissions can promote energy efficiency improvements and industrial upgrades, boosting local industrial economic growth and significantly increasing value-added.

In contrast, Beijing, Hainan, Qinghai, Ningxia, and Gansu have the smallest annual average carbon reduction, ranked at the bottom nationwide. Their reductions are 3.09871 million tons, 3.52694 million tons, 6.31773 million tons, 13.40118 million tons, and 13.44835 million tons, respectively, with corresponding industrial value-added growth of CNY −37.0066 billion, CNY 24.8520 billion, CNY 40.5484 billion, CNY 37.2938 billion, and CNY 86.0157 billion. Except for Beijing, the other four provinces experienced growth in industrial economies due to carbon reduction. This may be because Beijing's industrial economy makes up a smaller portion of its total economy, dominated by high-end manufacturing with high carbon emission efficiency and limited room for further reductions. In such cases, high-quality industrial development requires higher carbon reduction costs, putting pressure on industrial growth. The phenomenon of carbon reduction hindering industrial growth is also present in other developed provinces like Guangdong, Shanghai, Zhejiang, Fujian, and Hunan, where industrial development quality and carbon emission efficiency are high.

The findings suggest that provinces with lower industrial development quality and lower carbon emission efficiency can promote industrial growth through cross-temporal carbon emission allocations, whereas provinces with higher development quality and emission efficiency may find it difficult to use such allocations to boost their industrial economies.

At the regional level, the eastern region has the highest industrial carbon reduction, followed by the central region, while the western region has the lowest. However, the western region shows the strongest impact of carbon reduction on industrial economic growth, followed by the central region, with the eastern region showing the least. The industrial carbon reduction in the eastern region is 39.95473 million tons, corresponding to an industrial value-added growth of CNY 89.3934 billion. The central region's reduction is 35.88208 million tons, with a growth of CNY 138.9808 billion. In the western region, the reduction is 24.49947 million tons, corresponding to a value-added growth of CNY 88.486 billion. The national average industrial carbon reduction is 33.20176 million tons, with a value-added growth of CNY 102.284 billion.

This indicates that the eastern region, with its more advanced industrial economy and larger carbon emission baseline, receives higher carbon reduction quotas. The advanced industrial structure and high carbon efficiency in the eastern region mean that the effect of carbon reduction on economic growth is relatively small. Meanwhile, the central and western regions, with smaller industrial economies and lower total carbon emissions, receive lower carbon reduction quotas. However, their lower energy and carbon efficiency, along with industries dominated by high-pollution sectors, make carbon reduction tasks crucial. These tasks can stimulate the adoption of energy-saving technologies and promote industrial transformation, leading to new economic growth drivers and industrial growth.

The temporal variation trend of carbon emission quota allocation is illustrated in Figure 6. According to Figure 6a, after allocation, carbon emission quotas in the three regions and the national average follow a pattern of initial growth followed by a decline, with different regions reaching their peak quotas at different times and levels. In the eastern

region, carbon emission quotas showed an upward trend from 2009 to 2014, reaching a peak of 120.01803 million tons in 2014, after which the quotas gradually decreased year by year. The central region's quotas increased from 2009 to 2016, peaking at 116.16259 million tons in 2016, and then began to decline. The western region had the lowest quotas, reaching a peak of 73.35869 million tons in 2015. The national average carbon emission quota peaked at 95.88738 million tons in 2014, which is lower than the peak quotas of the eastern and central regions.

Figure 6b shows that the total industrial carbon reduction in the three regions and nationwide followed a pattern of initial decline followed by an increase. All regions should have made more reductions before 2013. Regarding regional differences, the central region had the highest carbon reduction before 2013, followed by the eastern region, and the western region had the lowest. The central region bore a heavier burden for carbon reduction. After 2013, the eastern region took the lead in industrial carbon reduction, followed by the western region, and the central region had the lowest reductions. The eastern region became the main area for carbon reduction, but its reduction burden was much lower than before 2013.

As shown in Figure 6c, after the cross-temporal allocation of carbon quotas, the industrial value-added of the three regions and the national average increased annually. The eastern region had the highest industrial value-added and the fastest growth rate. The western region had the lowest industrial value-added and the slowest growth rate. The central region's industrial value-added was close to the national average, with a growth rate higher than that of the western region. Over time, the differences in industrial value-added between the three regions increased.

Figure 6d reveals that the growth in industrial value-added before and after carbon reduction showed variability over different periods. Before 2011, the growth in industrial value-added was relatively small, indicating that carbon reduction had little effect on increasing industrial value-added, and even hindered industrial economic growth in the eastern region and nationwide. After 2012, the growth in industrial value-added became positive across the three regions, showing a consistent upward trend, indicating that carbon reduction had begun to promote industrial economic growth in all regions.

4.5. Research on the Cross-Temporal and Cross-Regional Allocation of Carbon Emissions in the Industrial Sector

This subsection studies the cross-temporal and cross-regional allocation of industrial carbon emission quotas using model (7). The allocation results are presented in Table 4 and Figure 7.

At the provincial level, coastal developed provinces have lower carbon reduction targets, while underdeveloped inland provinces have higher targets, consistent with the spatial allocation of carbon quotas. Hebei, Inner Mongolia, and Shanxi have the highest industrial carbon reductions, with reductions of 143.03950 million tons, 57.59779 million tons, and 91.46228 million tons, accounting for 61.245%, 57.070%, and 43.393% of their current emissions, respectively. In contrast, Tianjin, Beijing, Shanghai, Zhejiang, and Guangdong have the lowest reductions, at 2.6902 million tons, 4.4020 million tons, 4.7972 million tons, 16.7313 million tons, and 39.2579 million tons, respectively, which account for 0.568%, 4.262%, 1.063%, 2.045%, and 2.923% of their current emissions.

Table 4. Intertemporal and interregional allocation results of industrial carbon emissions and post-allocation gains (annual average for 2009–2018).

Province	Carbon Quota	Carbon Reduction	Industrial Added Value	Distribution Income
BJ	988.885	44.020	3597.997	831.239
TJ	4706.678	26.902	5590.758	2268.597
HEB	14,303.950	22,613.000	14,614.020	5911.860
LN	16,216.549	1401.106	13,429.963	6435.299
SH	4463.267	47.972	8804.985	1582.058
JS	19,857.883	1282.415	34,148.837	11,155.877
ZJ	8014.825	167.313	18,963.459	4742.083
FJ	5238.561	3290.883	9328.402	801.348
SD	26,046.134	3185.572	29,753.470	11,779.746
GD	13,036.518	392.579	27,645.762	2202.228
HI	688.906	486.741	747.165	374.585
SX	9146.228	7011.099	9428.771	4657.948
JL	5356.433	2144.804	5685.005	3255.513
HL	6165.646	1015.913	5856.534	2077.506
AH	9071.664	2414.619	9506.850	3007.800
JX	5107.546	2689.284	6904.742	1536.343
HEN	13,855.323	5485.468	17,679.278	5160.004
HB	8644.037	5942.481	11,046.671	2387.329
HN	5808.261	5826.747	7868.579	281.968
IM	5759.779	7656.912	5239.782	1542.492
GX	3724.299	4383.628	4879.233	1621.501
CQ	4830.546	2738.772	5508.016	1291.161
SC	12,724.519	3474.276	12,190.854	3722.939
GZ	3168.929	2838.187	3629.693	1545.464
YN	6019.638	3104.522	6024.326	2933.625
SN	8225.107	1396.085	7779.730	2263.068
GS	3785.011	697.772	3656.970	1877.773
QH	1283.007	789.569	1167.184	734.144
NX	1558.875	2908.185	1452.687	732.444
XJ	4615.315	4148.452	4208.986	1937.178
East	10,323.832	2994.409	15,147.711	4371.357
Middle	7894.392	4066.302	9247.054	2795.551
West	5063.184	3103.305	5067.042	1836.526
China	7747.077	3320.176	9877.957	3021.704

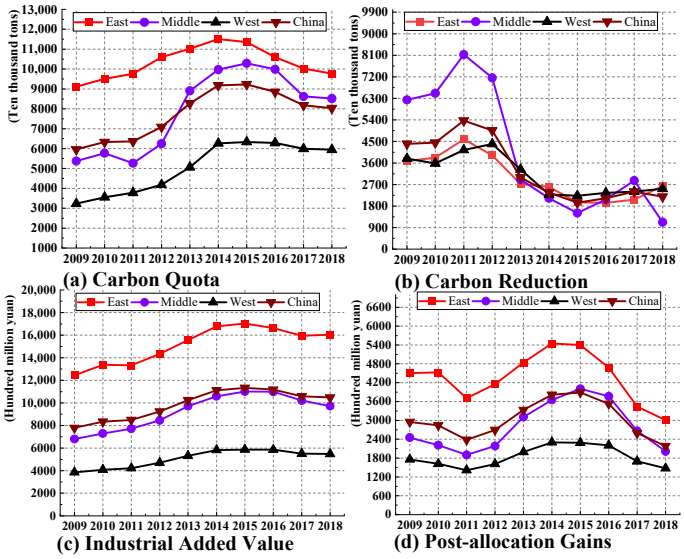


Figure 7. The trend of changes in intertemporal and interregional allocation results of industrial carbon emissions and post-allocation gains (2009–2018).

These results indicate that underdeveloped provinces face greater reduction pressure than developed provinces. Developed provinces typically have higher proportions of low-energy, low-pollution, high-value-added industries, and higher carbon emission efficiency, leaving less room for further reductions. In contrast, underdeveloped provinces have a higher share of energy-intensive, high-pollution industries such as coal and electricity generation, with lower energy efficiency and higher carbon emissions, resulting in higher assigned reduction quotas. To improve the performance of less developed regions, they can invest in energy efficiency and cleaner technologies. By modernizing their industrial base and adopting energy-efficient practices, they can reduce carbon emissions and maintain productivity. These regions should also focus on diversifying their economies by investing in less energy-intensive sectors such as renewable energy, high-tech manufacturing, and services, which can reduce their dependency on high-emission industries like coal and steel, etc. In addition, leveraging government incentives and engaging in carbon trading markets can provide financial support and additional revenue for transitioning to greener practices. Moreover, strengthening collaboration with more developed regions and enhancing innovation in green technologies are key actions. By learning from advanced provinces and investing in research and development, less-developed regions can implement better carbon reduction strategies and improve industrial efficiency. Public–private partnerships and green investments will also be essential to fund clean technology and infrastructure projects. Combined with the stricter enforcement of environmental regulations, these actions can help less-developed regions meet carbon reduction targets, improve their economic performance, and raise their rank in terms of carbon abatement efficiency.

After the reallocation of carbon quotas, all industrial sectors experience an increase in value-added. The largest industrial value-added growth is seen in Shandong, Jiangsu, Liaoning, Hebei, and Henan, with growth of CNY 1.1779746 trillion, CNY 1.1155877 trillion, CNY 643.5299 billion, CNY 591.1860 billion, and CNY 516.0004 billion, respectively. The corresponding growth rates are 65.539%, 48.519%, 92.003%, 67.936%, and 41.216%. This suggests that these provinces, which currently have relatively low energy efficiency and high carbon emissions, can significantly increase their industrial value-added through carbon reallocation.

The smallest increases in industrial value-added occur in Hunan, Hainan, Ningxia, Qinghai, and Fujian, with increases of CNY 28.1968 billion, CNY 37.4585 billion, CNY 73.2444 billion, CNY 73.4144 billion, and CNY 80.1348 billion, respectively. Although the value-added growth in Hainan, Ningxia, and Qinghai is relatively small, these provinces rank in the top five nationwide for growth rate after allocation, at 100.538%, 101.694%, and 169.533%, respectively. This indicates that the industrial sectors in Hainan, Ningxia, and Qinghai can achieve substantial growth after carbon reallocation and the fulfillment of reduction targets.

The cross-temporal and cross-regional allocation of carbon emissions enables industrial sectors to reduce carbon emissions while simultaneously boosting economic growth, achieving a win–win scenario. At the regional level, the central region has the highest industrial carbon reduction, followed by the western region, with the eastern region having the lowest reduction. The central region's reduction is 40.66302 million tons, accounting for 33.997% of its current emissions; the western region's reduction is 31.03305 million tons, accounting for 38.000% of its current emissions; and the eastern region's reduction is 29.94409 million tons, accounting for 22.484% of its current emissions. The national average reduction is 33.20176 million tons, which is higher than the reductions in the western and eastern regions. This is consistent with the cross-temporal allocation results, where the industrial sectors in the central and western regions have greater carbon reduction potential and are assigned higher reduction quotas. The eastern region, with advanced emission reduction technology, energy efficiency, and carbon emission efficiency, receives a smaller reduction quota.

After the cross-temporal and cross-regional reallocation of carbon quotas, the industrial value-added increases across the board for the three regions and nationwide. The eastern

region experiences the largest value-added increase, followed by the central region, with the western region having the smallest increase. The value-added difference before and after allocation in the eastern region is CNY 4.317357 trillion, with a growth rate of 40.564%. The central region sees an increase of CNY 2.795551 trillion, with a growth rate of 43.332%. The western region has a CNY 1.836526 trillion increase, with a growth rate of 56.849%. The national average value-added increase is CNY 3.021704 trillion, higher than both the central and western regions. These results indicate that despite the high carbon reduction pressure in the central and western regions, industrial carbon reduction has the most significant effect on industrial economic growth in these two regions.

The temporal trends of cross-temporal and cross-regional carbon allocation for industrial sectors in the three regions and nationwide are shown in Figure 7. According to Figure 7a, after reallocation, the industrial carbon quotas in the three regions and nationwide initially increase and then decrease, with all regions reaching their peak quotas at approximately the same time. The eastern region reaches the highest quota in 2014, with a peak of 115.04029 million tons. The central, western, and national average carbon quotas peak in 2015, at 102.88379 million tons, 63.33886 million tons, and 92.27598 million tons, respectively.

As shown in Figure 7b, industrial carbon reductions in the three regions and nationwide follow a pattern of an initial increase followed by a decrease, with higher reductions before 2013 and significantly lower reductions after 2013. This pattern is consistent with the results of cross-temporal allocation. Regarding regional differences, before 2013, the central region had the highest carbon reductions, while the eastern and western regions had roughly similar levels of reductions, both lower than the national average. After 2013, the differences between the three regions' carbon reductions diminished. Figure 7c shows that after carbon quota reallocation, the industrial value-added of the three regions and nationwide continues to increase year by year. The eastern region has the highest industrial value-added, while the western region has the lowest. The industrial value-added of the central region is roughly equal to the national average. Figure 7d indicates that the growth in industrial value-added before and after carbon quota reallocation is positive in all cases, demonstrating that carbon reallocation promotes industrial economic growth. The eastern region experiences the greatest growth in industrial value-added, followed by the central region, with the western region experiencing the least growth. Specifically, before 2011, the growth in industrial value-added decreased year by year, suggesting that carbon reduction had a diminishing effect on industrial economic growth. From 2011 to 2015, industrial value-added growth increased year by year, indicating that carbon reduction progressively boosted economic growth. After 2015, as the industrial value-added baseline became larger, the positive impact of carbon reduction on industrial growth began to slow down.

4.6. Sensitive Analysis

To analyze the sensitivity of carbon emission rights allocation outcomes to the allocation coefficient, this study adjusts the allocation coefficients for carbon emission rights across 30 provinces by 50% in sequence, specifically changing δ in models (5)–(7) from 0.3 to 0.15. A Pearson correlation analysis is then employed to verify the consistency between the new allocation results and the original allocation results. The specific findings are presented in Table 5.

Table 5. The results of sensitive analysis.

Models	Correlation Coefficient	Carbon Reduction δ = 0.30	Industrial Added Value δ = 0.30
Cross-Regional Allocation	Carbon Reduction δ = 0.15	0.884 ***	/
	Industrial Added Value δ = 0.15	/	0.993 ***
Cross-Temporal Allocation	Carbon Reduction δ = 0.15	0.338 ***	/
	Industrial Added Value δ = 0.15	/	0.430 ***
Spatio-Temporal Allocation	Carbon Reduction δ = 0.15	0.901 ***	/
	Industrial Added Value δ = 0.15	/	0.999 ***

Note: *** is significant at the significance level of 1%.

It is found that the carbon reductions and post-allocation industrial added value in all three models show significant correlations when the delta parameter changes from 0.3 to 0.15, with a significance level reaching 1%. In particular, the correlation coefficient of carbon reduction and post-allocation industrial added value in the spatio-temporal allocation model is as high as 0.90. These results indicate that the carbon emission rights allocation outcomes are relatively robust.

4.7. Summary of Results of the Models by Regions

This subsection summarizes the results of the spatial, temporal, and spatio-temporal allocation models based on carbon reduction and industrial economic outcomes across different regions of China. The findings are presented for the eastern, central, and western regions, which offer insights into regional disparities in carbon abatement and industrial growth.

The eastern region, a region that encompasses developed provinces such as Beijing, Shanghai, and Guangdong, consistently exhibited the lowest carbon reduction targets across all models. This outcome is attributed to the region's higher carbon emission efficiency and advanced industrial structure, which limits its further reduction potential. Despite these lower reduction targets, the eastern region experienced the largest absolute industrial value-added growth following the reallocation of carbon quotas. This growth is primarily driven by the region's robust industrial base and its focus on high-value-added industries. In the spatio-temporal allocation model, the industrial value-added growth rate reached 40.56%, which underscores the region's capacity to sustain economic expansion while meeting its carbon reduction obligations.

The central region, which includes provinces such as Hebei, Henan, and Shanxi, faced higher carbon reduction targets compared to the eastern region, as observed in both the cross-regional and temporal models. The central region's reliance on energy-intensive industries accounts for its higher emission levels and the greater pressure to reduce emissions. Following the reallocation of carbon quotas, the central region recorded significant improvements in industrial value-added, with a growth rate of 43.33%. This result highlights the role of energy efficiency improvements and industrial restructuring in fostering economic growth while achieving carbon reduction goals.

The western region, which includes less-developed provinces such as Inner Mongolia, Gansu, and Xinjiang, encountered the highest carbon reduction pressures, with targets often exceeding 38% of current emissions. The region's industrial profile, dominated by high-emission sectors, explains the substantial reduction requirements. Despite this, the western region demonstrated the highest relative growth rate in industrial value-added, reaching 56.85% after carbon reallocation. This finding suggests that the western region has significant potential to enhance its carbon efficiency, which can, in turn, stimulate industrial growth and economic development.

Summarily, across all regions, the reallocation of carbon emission quotas resulted in meaningful reductions in carbon emissions. Developed provinces in the eastern region experienced lower reduction pressure, while less-developed provinces in the central and western regions faced greater reduction targets. Nevertheless, all regions benefited economically from reallocation, with industrial value-added growth ranging from 40% to 56%. These results indicate that carbon reduction measures can coexist with industrial economic growth, which further demonstrates that improving carbon efficiency contributes positively to regional economic development.

The empirical analysis reveals key theoretical implications regarding regional differences in carbon abatement costs and industrial efficiency within China's industrial sector. Specifically, developed eastern regions face higher abatement costs due to advanced industrial structures with limited reduction potential, whereas central and western regions show lower costs and greater room for emission reductions. These findings emphasize the importance of cost-efficiency and differentiation in regional carbon allowance policies, supporting a model that aligns carbon quotas with the specific economic and industrial characteristics of each region. Moreover, the study highlights a "win-win" potential for carbon reduction policies that not only achieve emission goals but also foster industrial growth, particularly through targeted support for less-developed areas.

From a managerial perspective, the findings suggest that carbon quotas be allocated flexibly to account for regional differences in reduction capacity and economic impact. This could involve granting more quotas to central and western regions where the reduction potential is higher, while eastern regions could focus on low-carbon, high-value industries. Phased, cross-regional, and cross-temporal allocation strategies would enhance the adaptability and effectiveness of policies, supporting economic stability while achieving environmental targets. Tailored technical and financial support for underdeveloped regions could further enhance their reduction capacity and economic growth, promoting balanced regional development and aiding China's overall carbon reduction goals.

5. Conclusions

This paper, based on the SBM model and its dual form, estimates the carbon abatement costs of industrial sectors in 30 provinces of China from 2009 to 2018. It incorporates differentiated abatement costs into a centralized allocation DEA model to examine the allocation and benefits of carbon emission quotas under spatial, temporal, and spatio-temporal distribution scenarios. The main findings are as follows:

China's industrial carbon abatement costs show significant regional and provincial differences. Developed provinces with higher industrial development levels incur higher abatement costs, while less-developed provinces face lower costs. The eastern region has the highest abatement costs, followed by the central region, with the western region having the lowest. In cross-regional allocation, developed provinces like Guangdong, Shanghai, Beijing, Tianjin, and Zhejiang have smaller carbon abatement amounts, while less-developed provinces have larger reductions.

Provinces with large industrial economies face greater abatement tasks. Hebei, Shandong, Jiangsu, Henan, and Liaoning lead in carbon reduction. In both cross-regional and temporal allocation scenarios, coastal provinces experience lower abatement pressures, while inland provinces face higher burdens, mirroring the spatial allocation results. After reallocating emission quotas, all industrial sectors benefit economically.

The primary limitations of this study include the exclusion of certain regions, such as Tibet, Taiwan, Hong Kong, and Macau, due to data unavailability, which may limit the comprehensiveness of the findings. Additionally, the study assumes homogeneous carbon abatement costs across sectors within each region, whereas actual costs may vary significantly based on industry-specific factors such as technology and resource availability. Furthermore, the models focus on carbon emission allocation without fully incorporating other environmental or social factors, which could offer a more holistic view of the implications of carbon reduction policies.

Future research can expand on this study in several ways. First, incorporating sector-specific carbon abatement costs rather than assuming uniform costs across sectors would provide more precise allocation strategies and better reflect the diverse industrial structures. Second, future studies could include additional regions, such as Tibet, Taiwan, Hong Kong, and Macau, to give a more comprehensive picture of carbon emission allocation across China. Moreover, integrating other environmental and social factors, such as water usage, air quality, and social equity, would offer a more holistic approach to carbon reduction and sustainable development. Finally, future research could explore the dynamic interactions between carbon trading markets and industrial growth to understand how market mechanisms influence long-term emission reduction and economic outcomes across different regions.

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