mathematics

MDPI

Article

Dissipative Fuzzy Filtering for Nonlinear Networked Systems with Dynamic Quantization and Data Packet Dropouts

Shuxia Jing, Chengming Lu and Zhimin Li *

School of Electronic and Control Engineering, North China Institute of Aerospace Engineering, Langfang 065000, China; jsx01@nciae.edu.cn (S.J.); bhhtchengming@stumail.nciae.edu.cn (C.L.)
* Correspondence: lizhimin@nciae.edu.cn

Abstract: This paper discusses the dissipative filtering problem for discrete-time nonlinear networked systems with dynamic quantization and data packet dropouts. The Takagi–Sugeno (T–S) fuzzy model is employed to approximate the considered nonlinear plant. Both the measurement and performance outputs are assumed to be quantized by the dynamic quantizers before being transmitted. Moreover, the Bernoulli stochastic variables are utilized to characterize the effects of data packet dropouts on the measurement and performance outputs. The purpose of this paper is to design full- and reduced-order filters, such that the stochastic stability and dissipative filtering performance for the filtering error system can be guaranteed. The collaborative design conditions for the desired filter and the dynamic quantizers are expressed in the form of linear matrix inequalities. Finally, simulation results are used to illustrate the feasibility of the proposed filtering scheme.

Keywords: dissipative filtering; T–S fuzzy systems; dynamic quantization; data packet dropouts

MSC: 93C42

check for updates

Citation: Jing, S.; Lu, C.; Li, Z. Dissipative Fuzzy Filtering for Nonlinear Networked Systems with Dynamic Quantization and Data Packet Dropouts. *Mathematics* **2024**, *12*, 203. https://doi.org/10.3390/math12020203

Academic Editors: Lijuan Zha, Jian Liu and Jinliang Liu

Received: 4 December 2023
Revised: 4 January 2024
Accepted: 6 January 2024
Published: 8 January 2024

1. Introduction

In recent years, there has been a surge in academic interest in networked systems. The fundamental reason is that, due to their benefits of low cost, easy maintenance, and high reliability, networked systems are gradually replacing traditional control systems and taking center stage in the development of control systems [1]. Nowadays, networked systems are used in industries such as autonomous vehicles, industrial process control, smart homes, and others, with great success [2]. However, because of network restrictions, networked systems invariably generate some issues such as quantization, data packet dropouts, and so on [3]. These issues not only cause networked systems to run less efficiently, but they additionally possess the potential to cause instability. One of the primary sources of these issues is signal quantization inaccuracy and data packet dropouts. Among them, one of these causes of networked systems' poor operating efficiency and instability is quantization error. Therefore, it is crucial to deal with the analysis and design problems for networked systems subject to signal quantization and data packet dropouts. Over the past several years, a great number of achievements have been reported on these topics. The analysis and design problems for networked systems with quantization were addressed in [4–11]. The analysis and design problems for networked systems with data packet dropouts were studied in [8–12].

As is well known, nonlinearities exist in many practical physical systems [13]. Therefore, nonlinear control systems have attracted the attention of many scholars. As an effective means to deal with nonlinear systems, the Takagi–Sugeno (T–S) fuzzy model approach has received extensive attention from many international scholars and a series of important results have been published in the open literature (see, e.g., [14–16] and references therein). In recent years, based on the T–S fuzzy model approach, the study on networked systems has also attracted attention and some important results have been achieved (see, e.g., [17–20] and references therein). Particularly, based on the T–S fuzzy model approach, the control problem of nonlinear networked systems with quantization was studied in [21–26] and the control problem of nonlinear networked systems subject to data packet dropouts was addressed in [27–29].

In addition, the filtering problem is considered to be an important issue in the study of control theory because the state variables that can reflect the inside of the system are not always available in the vast majority of practical systems. Scholars at home and abroad have undertaken enormous research on the filtering problem and many significant results have been proposed. For linear networked systems, the filter design problem was researched in [30–32]. For nonlinear systems, the resilient mixed $\mathcal{H}_\infty$ and energy-to-peak filtering problem and the $\mathcal{H}_\infty$ filtering problem with D stability constraints were addressed based on the T–S fuzzy model approach in [33] and [34], respectively. For nonlinear networked systems, based on the T–S fuzzy model approach, the event-triggered $\mathcal{H}_\infty$ filtering problem was addressed with the effect of weighted try-once-discard protocol in [35]. Particularly, based on the T–S fuzzy model approach, the filtering problem for nonlinear networked systems with the effect of quantization was investigated in [36–41] and the filtering problem for nonlinear networked systems with the effects of data packet dropouts was considered in [39–43]. However, it should be noted that most of the above literature is about $\mathcal{H}_\infty$ filtering. As pointed out in [23,44], the dissipative performance is more general than the $\mathcal{H}_\infty$ performance. As a result, the study of the dissipative filtering problem is significant for nonlinear networked systems. As far as the author knows, there is no relevant research on the dissipative filtering problem for nonlinear discrete-time networked systems under the effects of dynamic quantization and data packet dropouts on the measurement output and the performance output, simultaneously, which motivated the current research.

This paper considered the quantized dissipative filtering problem of discrete-time nonlinear networked systems with data packet dropouts based on the T–S fuzzy model strategy. The primary contributions of this paper can be summarized as follows.

(1) According to the T–S fuzzy model approach, the dissipative filtering problem is investigated for discrete-time nonlinear networked systems subject to dynamic quantization and data packet dropouts.
(2) In this paper, both the effects of dynamic quantization and data packet dropouts on the measurement output and performance output are considered, simultaneously. Moreover, a more general adjusting strategy is proposed for the dynamic parameter of the dynamic quantizer.
(3) By introducing a dimension adjustment matrix, the design conditions for both the desired full- and reduced-order dissipative filters are proposed in the unified framework of linear matrix inequalities.

The rest of this paper is organized as follows. The filtering problem to be investigated is formulated in Section 2. In Section 3, the main results on the design of the dissipative filter with dynamic quantization and data packet dropouts are presented. In Section 4, an example is provided to demonstrate the effectiveness of the developed filtering strategy. Finally, the conclusion of this paper is provided in Section 5.

Notations: The notations used in this paper are standard. $\mathbf{R}^n$ and $\mathbf{R}^{m\times n}$ indicate the n-dimensional Euclidean space and the set of all real matrices of dimension $m \times n$, respectively. I is used to denote the identity matrix with compatible dimensions. $|\cdot|$ stands for Euclidean vector norm. The symbols $diag\{\cdots\}$ and $*$ are utilized to denote block-diagonal matrix and symmetric element in the matrix, respectively. A^T and A^{-1} represent the transpose matrix and inverse matrix of matrix A, respectively. $\lambda_{min}(A)$ stands for the smallest eigenvalue of the matrix A and $l_2[0,\ \infty)$ denotes the space of the square integrable vectors over $[0,\ \infty)$.

2. Problem Formulation

2.1. Nonlinear Plant

In this paper, a discrete-time T–S fuzzy model is used to approximate the nonlinear plant under consideration and ith is formulated as follows

Plant Rule i: IF $n_1(t)$ is M_{1i} and $n_2(t)$ is M_{2i} and …and $n_p(t)$ is M_{pi}, THEN

$$\begin{aligned} x(t+1) &= A_i x(t) + B_i w(t) \\ y(t) &= C_i x(t) + D_i w(t) \\ z(t) &= E_i x(t) + F_i w(t) \end{aligned} \tag{1}$$

where $M_{\tau i}$ with $i = 1, 2, \dots, s$ and $\tau = 1, 2, \dots, p$ are the fuzzy sets, s stands for the number of fuzzy rules, and $n(t) = [n_1(t), n_2(t), \dots, n_p(t)]$ stands for the premise variable. $x(t) \in \mathbf{R}^{n_x}$ and $y(t) \in \mathbf{R}^{n_y}$ stand for the system state and the measurement output, respectively, $z(t) \in \mathbf{R}^{n_z}$ stands for the performance output, and $w(t) \in \mathbf{R}^{n_w}$ stands for the noise signal belonging to $l_2[0, \infty)$. $A_i \in \mathbf{R}^{n_x \times n_x}$, $B_i \in \mathbf{R}^{n_x \times n_w}$, $C_i \in \mathbf{R}^{n_y \times n_x}$, $D_i \in \mathbf{R}^{n_y \times n_w}$, $E_i \in \mathbf{R}^{n_z \times n_x}$, and $F_i \in \mathbf{R}^{n_z \times n_w}$ are the system matrices.

Denote

$$b_i(n(t)) = \prod_{\tau=1}^{p} M_{\tau i}(n_\tau(t)), \ i = 1, 2, \dots, s \tag{2}$$

where $M_{\tau i}(n_\tau(t))$ is the grade of membership of $n_\tau(t)$ in $M_{\tau i}$.

Throughout this paper, it is assumed that

$$b_i(n(t)) > 0, \ \sum_{i=1}^{s} b_i(n(t)) > 0, \ i = 1, 2, \dots, s. \tag{3}$$

Let

$$p_i(n(t)) = \frac{b_i(n(t))}{\sum_{i=1}^{s} b_i(n(t))}, \ i = 1, 2, \dots, s. \tag{4}$$

Then

$$p_i(n(t)) \geq 0, \sum_{i=1}^{s} p_i(n(t)) = 1, \ i = 1, 2, \dots, s. \tag{5}$$

Moreover, the T–S fuzzy model can be further represented as

$$\begin{aligned} & x(t+1) = A(p)x(t) + B(p)w(t) \\ & y(t) = C(p)x(t) + D(p)w(t) \\ & z(t) = E(p)x(t) + F(p)w(t) \end{aligned} \tag{6}$$

where

$$\begin{aligned} A(p) &= \sum_{i=1}^{s} p_i(n(t))A_i, \ B(p) = \sum_{i=1}^{s} p_i(n(t))B_i, \\ C(p) &= \sum_{i=1}^{s} p_i(n(t))C_i, \ D(p) = \sum_{i=1}^{s} p_i(n(t))D_i, \\ E(p) &= \sum_{i=1}^{s} p_i(n(t))E_i, \ F(p) = \sum_{i=1}^{s} p_i(n(t))F_i. \end{aligned}$$

2.2. Dynamic Quantizers and Data Dropouts

In order to reduce the frequency of information exchange and the burden of communication, the measurement output $y(t)$ and the performance output $z(t)$ will be quantized by the dynamic quantizer developed in [6], respectively. According to [6], the quantized measurement output and the quantized performance output can be formulated as

$$g_{\alpha_\varsigma(t)}(\varsigma(t)) = \alpha_\varsigma(t) g_\varsigma\left(\frac{\varsigma(t)}{\alpha_\varsigma(t)}\right), \ \varsigma = y, z. \tag{7}$$

In (7), $\alpha_\varsigma(t) > 0$ stands for the dynamic parameter of the quantizer and $g_\varsigma(\varsigma(t)/\alpha_\varsigma(t))$ stands for a static quantizer satisfying

$$\left| g_\varsigma\left(\frac{\varsigma(t)}{\alpha_\varsigma(t)}\right) - \frac{\varsigma(t)}{\alpha_\varsigma(t)} \right| \leq \Delta_\varsigma, \ \ \text{IF} \left| \frac{\varsigma(t)}{\alpha_\varsigma(t)} \right| \leq \mathcal{R}_\varsigma \tag{8}$$

$$\left|g_{\varsigma}\left(\frac{\varsigma(t)}{\alpha_{\varsigma}(t)}\right)-\frac{\varsigma(t)}{\alpha_{\varsigma}(t)}\right|>\Delta_{\varsigma},\ \ \text{IF}\left|\frac{\varsigma(t)}{\alpha_{\varsigma}(t)}\right|>\mathcal{R}_{\varsigma} \tag{9}$$

where $\mathcal{R}_{\varsigma}$ stands for the range of the quantizer and Δ_{ς} denotes the bound of the quantization error.

As an important challenge in networked systems, the effects of data packet dropouts will also be considered in this paper. Two independent Bernoulli stochastic variables ε and ρ will be employed to characterize the effects of data packet dropouts on the quantized measurement output and quantized performance output. In this way, the measurement output and performance output signals received by the filter can be indicated as

$$\overline{y}(t)=\varepsilon\alpha_{y}(t)g_{y}\left(\frac{y(t)}{\alpha_{y}(t)}\right),\ \ \alpha_{y}(t)>0 \tag{10}$$

$$\overline{z}(t)=\rho\alpha_{z}(t)g_{z}\left(\frac{z(t)}{\alpha_{z}(t)}\right),\ \ \alpha_{z}(t)>0. \tag{11}$$

This implies that the quantized measurement output (quantized performance output) is successfully transmitted when $\varepsilon=1$ ($\rho=1$), and that the quantized measurement output (quantized performance output) is unsuccessfully transmitted when $\varepsilon=0$ ($\rho=0$). Moreover, we assume that ε and ρ satisfy

$$\begin{array}{l} Prob\{\varepsilon=1\}=\mathbf{E}\{\varepsilon\}=\bar{\varepsilon}\\ Prob\{\varepsilon=0\}=1-\bar{\varepsilon}\\ Prob\{\rho=1\}=\mathbf{E}\{\rho\}=\bar{\rho}\\ Prob\{\rho=0\}=1-\bar{\rho}\end{array} \tag{12}$$

with known constants $0\leq\bar{\varepsilon}\leq1$ and $0\leq\bar{\rho}\leq1$.

Remark 1. *As claimed in [38,43], in the study of the filtering problem for networked systems, both the measurement and performance outputs should be transmitted by an unreliable communication network. Therefore, the effects of both the dynamic quantization and data packet dropouts on the measurement and performance outputs are considered in this paper. In contrast with the results in [38] where only the effects of quantization are considered, and the results in [43] where only the effects of data packet dropouts are considered, the problem studied in this paper is more general for networked systems.*

2.3. Filtering Error Systems

In this paper, the structure of the employed filter is provided as

$$\begin{array}{l} x_f(t+1)=\widehat{A}x_f(t)+\widehat{B}\overline{y}(t)\\ z_f(t)=\widehat{E}x_f(t)\end{array} \tag{13}$$

where $x_f(t)\in\mathbf{R}^{n_{\overline{x}}}$ denotes the state of the filter and $z_f(t)\in\mathbf{R}^{n_z}$ stands for the output of the filter. $\widehat{A}\in\mathbf{R}^{n_{\overline{x}}\times n_{\overline{x}}}$, $\widehat{B}\in\mathbf{R}^{n_{\overline{x}}\times n_y}$, and $\widehat{E}\in\mathbf{R}^{n_z\times n_{\overline{x}}}$ stand for the parameters of the designed filter. The structure of the filter in (13) is general, which can be utilized to investigate the full-order filtering problem with $n_{\overline{x}}=n_x$ and the reduced-order filtering problem with $1\leq n_{\overline{x}}<n_x$.

Then, we can express the filtering error system as

$$\begin{array}{l} \phi(t+1)=(A_a+\tilde{\varepsilon}A_b)\phi(t)+(B_a+\tilde{\varepsilon}B_b)w(t)\\ \qquad\qquad +(H_a+\tilde{\varepsilon}H_b)r_y(t)\\ e(t)=(C_a+\tilde{\rho}C_b)\phi(t)+(D_a+\tilde{\rho}D_b)w(t)\\ \qquad\qquad +(\bar{\rho}+\tilde{\rho})r_z(t)\end{array} \tag{14}$$

where $\phi^T(t) = [x^T(t)\ x_f^T(t)]$, $e(t) = \bar{z}(t) - z_f(t)$, and

$$\begin{aligned}
&A_a = \begin{bmatrix} A(p) & 0 \\ \bar{\varepsilon}\widehat{B}C(p) & \widehat{A} \end{bmatrix}, A_b = \begin{bmatrix} 0 & 0 \\ \widehat{B}C(p) & 0 \end{bmatrix}, \\
&B_a = \begin{bmatrix} B(p) \\ \bar{\varepsilon}\widehat{B}D(p) \end{bmatrix}, B_b = \begin{bmatrix} 0 \\ \widehat{B}D(p) \end{bmatrix}, \\
&H_a = \begin{bmatrix} 0 \\ \bar{\varepsilon}\widehat{B} \end{bmatrix}, H_b = \begin{bmatrix} 0 \\ \widehat{B} \end{bmatrix}, \\
&C_a = \begin{bmatrix} \bar{\rho}E(p) & -\widehat{E} \end{bmatrix}, C_b = \begin{bmatrix} E(p) & 0 \end{bmatrix}, \\
&D_a = \bar{\rho}F(p), D_b = F(p), \\
&r_y(t) = \alpha_y(t)\left(g_y\left(\frac{y(t)}{\alpha_y(t)}\right) - \frac{y(t)}{\alpha_y(t)}\right), \\
&r_z(t) = \alpha_z(t)\left(g_z\left(\frac{z(t)}{\alpha_z(t)}\right) - \frac{z(t)}{\alpha_z(t)}\right), \\
&\tilde{\varepsilon} = \varepsilon - \bar{\varepsilon}, \tilde{\rho} = \rho - \bar{\rho}.
\end{aligned}$$

Next, we will provide the definitions on the dissipativity and stochastic stability of the filtering error system (14), which will be needed in the process of dissipative filtering performance analysis.

Definition 1 ([27,37,43])**.** *For any initial condition $\phi(0)$, if there exists a matrix $Y > 0$ such that*

$$\mathbf{E}\left\{\sum_{t=0}^{\infty} |\phi(t)|^2 \middle| \phi(0)\right\} < \phi^T(0)Y\phi(0) \tag{15}$$

holds. Then, the filtering error system in (14) is stochastically stable with $w(t) = 0$.

Definition 2 ([44])**.** *For zero initial condition, the filtering error system in (14) is strictly dissipative with the dissipativity performance bound $\gamma > 0$, such that*

$$\begin{aligned}
\sum_{t=0}^{\varrho} \mathbf{E}\big\{ \big(& e^T(t)J_1e(t) + e^T(t)J_2w(t) + w^T(t) \\
&\times J_2^T e(t) + w^T(t)(J_3 - \gamma I)w(t)\big)\big\} \geq 0
\end{aligned} \tag{16}$$

holds with $\varrho \geq 0$. In (16), $J_1 = J_1^T \in \mathbf{R}^{n_z \times n_z} \leq 0$, $J_2 \in \mathbf{R}^{n_z \times n_w}$, and $J_3 = J_3^T \in \mathbf{R}^{n_w \times n_w}$ are known matrices and $-J_1 = J_{11}^T J_{11}$ with $J_{11} \in \mathbf{R}^{n_z \times n_z} \geq 0$.

Finally, the purpose of this paper is to design the filter in the form of (13), such that the filtering error system in (14) is stochastically stable in the sense of Definition 1 and strictly dissipative in the sense of Definition 2.

3. Main Results

3.1. Filtering Performance Analysis

In this subsection, it is assumed that the filter (13) studied in this paper is known. Based on the Lyapunov approach, a significant dissipative filtering performance analysis criterion for the filtering error system (14) will be presented in the following theorem.

Theorem 1. *Suppose that the quantization ranges $\mathcal{R}_y$ and $\mathcal{R}_z$, the quantization error bounds Δ_y and Δ_z, and the constants $\bar{\rho}$, $\bar{\varepsilon}$, $\gamma > 0$, $0 < c_{1y} \leq c_{2y}$, $0 < d_{1y} \leq d_{2y}$, $0 < c_{1z} \leq c_{2z}$, $0 < d_{1z} \leq d_{2z}$, satisfying $c_{1y}d_{1y} \geq 1$ and $c_{1z}d_{1z} \geq 1$ are provided. The filtering error system in (14) is stochastically stable with the provided dissipative filtering performance γ, if there exist matrix $P > 0$, positive scalars o_y, o_z, ζ_y, and ζ_z satisfying*

$$\frac{c_{1\varsigma}}{\mathcal{R}_\varsigma} \leq o_\varsigma \leq \frac{c_{2\varsigma}}{\mathcal{R}_\varsigma},\ \varsigma = y,\ z \tag{17}$$

$$\begin{bmatrix} \Upsilon_{11} & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * \\ \Upsilon_{31} & 0 & -I & * \\ \Upsilon_{41} & 0 & 0 & \Upsilon_{44} \end{bmatrix} < 0 \tag{18}$$

where

$$\begin{aligned}
\Upsilon_{11} &= \begin{bmatrix} -P & * & * & * \\ -J_2^T C_a & \Omega_{22} & * & * \\ 0 & 0 & -\zeta_y I & * \\ 0 & -\bar{\rho} J_2 & 0 & -\zeta_z I \end{bmatrix}, \\
\Upsilon_{21} &= \begin{bmatrix} A_a & B_a & H_a & 0 \\ \hat{\varepsilon} A_b & \hat{\varepsilon} B_b & \hat{\varepsilon} H_b & 0 \end{bmatrix}, \\
\Upsilon_{31} &= \begin{bmatrix} J_{11} C_a & J_{11} D_a & 0 & \bar{\rho} J_{11} \\ \hat{\rho} J_{11} C_b & \hat{\rho} J_{11} D_b & 0 & \hat{\rho} J_{11} \end{bmatrix}, \\
\Upsilon_{41} &= \begin{bmatrix} \sigma_y \overline{C} & \sigma_y \overline{D} & 0 & 0 \\ \sigma_z \overline{E} & \sigma_z \overline{F} & 0 & 0 \end{bmatrix}, \\
\Omega_{22} &= -J_2^T D_a - D_a^T J_2 - (J_3 - \gamma I),
\end{aligned}$$

$\Upsilon_{22} = -diag\{P^{-1},\ P^{-1}\}$, $\Upsilon_{44} = -diag\{\zeta_y{}^{-1} I,\ \zeta_z{}^{-1} I\}$, $\hat{\varepsilon} = (\bar{\varepsilon}(1-\bar{\varepsilon}))^{1/2}$, $\hat{\rho} = (\bar{\rho}(1-\bar{\rho}))^{1/2}$, $\overline{C} = [\,C(p)\ \ 0\,]$, $\overline{D} = D(p)$, $\overline{E} = [\,E(p)\ \ 0\,]$, $\overline{F} = F(p)$, $\sigma_y = (c_{2y} d_{2y} \Delta_y)/\mathcal{R}_y$, $\sigma_z = (c_{2z} d_{2z} \Delta_z)/\mathcal{R}_z$, *and the adjusting strategy for the dynamic parameters* $\alpha_y(t)$ *and* $\alpha_z(t)$ *are provided as:*

$$d_{1\varsigma} o_\varsigma |\varsigma(t)| \le \alpha_\varsigma(t) \le d_{2\varsigma} o_\varsigma |\varsigma(t)|,\ \varsigma = y,\ z. \tag{19}$$

Proof. For the filtering error system (14), the Lyapunov function is established as

$$V(\phi(t)) = \phi^T(t) P \phi(t),\ P > 0. \tag{20}$$

Then, one can be obtain that

$$\begin{aligned}
&\mathbf{E}\{V(\phi(t+1))\} - V(\phi(t)) - \mathbf{E}\big\{\big(e^T(t) J_1 e(t) + e^T(t) \\
&\quad \times J_2 w(t) + w^T(t) J_2^T e(t) + w^T(t)(J_3 - \gamma I) w(t)\big)\big\} \\
&= \mathbf{E}\{((A_a + \tilde{\varepsilon} A_b)\phi(t) + (B_a + \tilde{\varepsilon} B_b) w(t) \\
&+ (H_a + \tilde{\varepsilon} H_b) r_y(t))^T P((A_a + \tilde{\varepsilon} A_b)\phi(t) \\
&+ (B_a + \tilde{\varepsilon} B_b) w(t) + (H_a + \tilde{\varepsilon} H_b) r_y(t))\} \\
&- \phi^T(t) P \phi(t) - \mathbf{E}\{((C_a + \tilde{\rho} C_b)\phi(t) \\
&+ (D_a + \tilde{\rho} D_b) w(t) + (\tilde{\rho} + \bar{\rho}) r_z(t))^T J_1 \\
&((C_a + \tilde{\rho} C_b)\phi(t) + (D_a + \tilde{\rho} D_b) w(t) \\
&+ (\tilde{\rho} + \bar{\rho}) r_z(t)) + ((C_a + \tilde{\rho} C_b)\phi(t) \\
&+ (D_a + \tilde{\rho} D_b) w(t) + (\tilde{\rho} + \bar{\rho}) r_z(t))^T J_2 w(t) \\
&+ w^T(t) J_2^T ((C_a + \tilde{\rho} C_b)\phi(t) \\
&+ (D_a + \tilde{\rho} D_b) w(t) + (\tilde{\rho} + \bar{\rho}) r_z(t)) \\
&+ w^T(t)(J_3 - \gamma I) w(t)\} \\
&= \eta^T(t)\Big(\mathbf{E}\Big\{([\,A_a\ B_a\ H_a\ 0\,] + \tilde{\varepsilon}[\,A_b\ B_b\ H_b\ 0\,])^T \\
&P([\,A_a\ B_a\ H_a\ 0\,] + \tilde{\varepsilon}[\,A_b\ B_b\ H_b\ 0\,]) \\
&-([\,C_a\ D_a\ 0\ \bar{\rho} I\,] + \tilde{\rho}[\,C_b\ D_b\ 0\ I\,])^T J_1 \\
&([\,C_a\ D_a\ 0\ \bar{\rho} I\,] + \tilde{\rho}[\,C_b\ D_b\ 0\ I\,]) \\
&-([\,C_a\ D_a\ 0\ \bar{\rho} I\,] + \tilde{\rho}[\,C_b\ D_b\ 0\ I\,])^T \\
&J_2[\,0\ I\ 0\ 0\,] - [\,0\ I\ 0\ 0\,]^T J_2^T([\,C_a\ D_a\ 0\ \bar{\rho} I\,] \\
&+ \tilde{\rho}[\,C_b\ D_b\ 0\ I\,])\Big\} - diag\{P,\ J_3 - \gamma I,\ 0,\ 0\}\Big)\eta(t) \\
&= \eta^T(t) \Phi_0 \eta(t)
\end{aligned} \tag{21}$$

where $\eta^T(t) = [\,\phi^T(t)\ w^T(t)\ r_y^T(t)\ r_z^T(t)\,]$ and

$$\begin{aligned}\Phi_0 = &[\,A_a\ B_a\ H_a\ 0\,]^T P [\,A_a\ B_a\ H_a\ 0\,]\\ &+\hat{\varepsilon}^2[\,A_b\ B_b\ H_b\ 0\,]^T P[\,A_b\ B_b\ H_b\ 0\,]\\ &-[\,C_a\ D_a\ 0\ \bar{\rho}I\,]^T J_1[\,C_a\ D_a\ 0\ \bar{\rho}I\,]\\ &-\hat{\rho}^2[\,C_b\ D_b\ 0\ I\,]^T J_1[\,C_b\ D_b\ 0\ I\,]\\ &-[\,C_a\ D_a\ 0\ \bar{\rho}I\,]^T J_2[\,0\ I\ 0\ 0\,]\\ &-[\,0\ I\ 0\ 0\,]^T J_2^T[\,C_a\ D_a\ 0\ \bar{\rho}I\,]\\ &-diag\{P,\ J_3-\gamma I,\ 0,\ 0\}.\end{aligned}$$

As in [4], based on the online adjusting strategy in (19) and the conditions in (8) and (17), we have

$$\begin{aligned} r_y^T(t)r_y(t) \le \sigma_y^2 y^T(t)y(t)\\ r_z^T(t)r_z(t) \le \sigma_z^2 z^T(t)z(t)\end{aligned} \tag{22}$$

which can be further expressed as

$$\begin{aligned}\eta^T(t)\Phi_1\eta(t) \ge 0\\ \eta^T(t)\Phi_2\eta(t)\ge 0\end{aligned} \tag{23}$$

with

$$\begin{aligned}\Phi_1 &= [\,\sigma_y\overline{C}\ \sigma_y\overline{D}\ 0\ 0\,]^T[\,\sigma_y\overline{C}\ \sigma_y\overline{D}\ 0\ 0\,]-diag\{0,\ 0,\ I,\ 0\},\\ \Phi_2 &= [\,\sigma_z\overline{E}\ \sigma_z\overline{F}\ 0\ 0\,]^T[\,\sigma_z\overline{E}\ \sigma_z\overline{F}\ 0\ 0\,]-diag\{0,\ 0,\ 0,\ I\}.\end{aligned}$$

By utilizing the Schur complement to (18), we obtain

$$\Phi_0+\zeta_y\Phi_1+\zeta_z\Phi_2<0. \tag{24}$$

According to the S-Procedure in [6,36], we have that $\eta^T(t)\Phi_0\eta(t)<0$ based on (21), (23), and (24), i.e.,

$$\begin{aligned}&\mathbf{E}\{V(\phi(t+1))\}-V(\phi(t))-\mathbf{E}\big\{\big(e^T(t)J_1e(t)+e^T(t)\\ &\times J_2w(t)+w(t)^TJ_2^Te(t)+w^T(t)(J_3-\gamma I)w(t)\big)\big\}<0\end{aligned} \tag{25}$$

Then, by summing up (25) from $t=0$ to $t=\varrho$ with $\varrho\ge 1$, one can obtain

$$\begin{aligned}&\mathbf{E}\{V(\phi(\varrho+1))\}-V(\phi(0))-\textstyle\sum_{t=0}^{\varrho}\mathbf{E}\big\{\big(e^T(t)J_1e(t)+e^T(t)\\ &\times J_2w(t)+w(t)^TJ_2^Te(t)+w^T(t)(J_3-\gamma I)w(t)\big)\big\}<0\end{aligned} \tag{26}$$

By considering $\mathbf{E}\{V(\phi(\varrho+1))\}\ge 0$ and $V(\phi(0))=0$, we have

$$\begin{aligned}&\textstyle\sum_{t=0}^{\varrho}\mathbf{E}\{(e^T(t)J_1e(t)+e^T(t)J_2w(t)+w(t)^T\\ &\times J_2^Te(t)+w^T(t)(J_3-\gamma I)w(t))\}\ge 0\end{aligned} \tag{27}$$

Therefore, according to Definition 2, one can obtain that the given dissipative filtering performance bound $\gamma>0$ of the filtering error system in (14) can be guaranteed.

Next, for $w(t)=0$, the stochastic stability of the filtering error system in (14) will be discussed.

For $w(k)=0$, the inequality in (25) reduces to

$$\mathbf{E}\{V(\phi(t+1))\}-V(\phi(t))<\mathbf{E}\{e^T(t)J_1e(t)\}. \tag{28}$$

By considering the fact that $J_1\le 0$, we have that

$$\mathbf{E}\{V(\phi(t+1))\}-V(\phi(t))=\overline{\eta}^T(t)\widehat{\Phi}_0\overline{\eta}(t)<0 \tag{29}$$

where $\overline{\eta}^T(t) = [\,\phi^T(t)\ r_y^T(t)\ r_z^T(t)\,]$ and

$$\begin{aligned}\widehat{\Phi}_0 = {} & [\,A_a\ H_a\ 0\,]^T P[\,A_a\ H_a\ 0\,] + \hat{\varepsilon}^2[\,A_b\ H_b\ 0\,]^T \\ & \times P[\,A_b\ H_b\ 0\,] - [\,C_a\ 0\ \bar{\rho}I\,]^T J_1[\,C_a\ 0\ \bar{\rho}I\,] \\ & - \hat{\rho}^2[\,C_b\ 0\ I\,]^T J_1[\,C_b\ 0\ I\,] - diag\{P,\ 0,\ 0\}.\end{aligned}$$

Based on (29), it can be obtained that

$$\mathbf{E}\{V(\phi(t+1))\} - V(\phi(t)) \leq -\lambda_{\min}(-\widehat{\Phi}_0)\overline{\eta}^T(t)\overline{\eta}(t). \tag{30}$$

By calculating the mathematical expectation of (30) on both sides and summing up both sides of (30) from $t = 0$ to $t = \varrho$ with $\varrho \geq 1$, one can obtain that

$$\begin{aligned}& \mathbf{E}\{\phi^T(\varrho+1)P\phi(\varrho+1)\} - \phi^T(0)P\phi(0) \\ & \leq -\lambda_{\min}(-\widehat{\Phi}_0)\mathbf{E}\left\{\textstyle\sum_{t=0}^{\varrho} |\overline{\eta}(t)|^2\right\},\end{aligned} \tag{31}$$

which is equivalent to

$$\begin{aligned}& \mathbf{E}\left\{\textstyle\sum_{t=0}^{\varrho} |\overline{\eta}(t)|^2\right\} \\ & \leq (\lambda_{\min}(-\widehat{\Phi}_0))^{-1}\big(\phi^T(0)P\phi(0) - \mathbf{E}\{\phi^T(\varrho+1)P\phi(\varrho+1)\}\big).\end{aligned} \tag{32}$$

For $\varrho \longrightarrow \infty$, we have that $\mathbf{E}\{\phi^T(\infty)P\phi(\infty)\} \geq 0$ and $\mathbf{E}\{\sum_{t=0}^{\infty} |\overline{\eta}(t)|^2\} \geq \mathbf{E}\{\sum_{t=0}^{\infty} |\phi(t)|^2\}$. Then, based on inequality in (32), it can be obtained that

$$\begin{aligned}& \mathbf{E}\{\textstyle\sum_{t=0}^{\infty} |\phi(t)|^2\} \\ & \leq (\lambda_{\min}(-\widehat{\Phi}_0))^{-1}(\phi^T(0)P\phi(0)) \\ & = \phi^T(0)(\lambda_{\min}(-\widehat{\Phi}_0))^{-1}P\phi(0) = \phi^T(0)Y\phi(0)\end{aligned} \tag{33}$$

with $Y = (\lambda_{\min}(-\widehat{\Phi}_0))^{-1}P$.

According to $\overline{\eta}^T(t)\widehat{\Phi}_0\overline{\eta}(t) < 0$, it can be deduced that $\widehat{\Phi}_0 < 0$, which implies that $\lambda_{\min}(-\widehat{\Phi}_0) > 0$. Based on the above discussions, we have that $Y = (\lambda_{\min}(-\widehat{\Phi}_0))^{-1}P > 0$. Therefore, for $w(t) = 0$, one can obtain that the filtering error system in (14) is stochastically stable in accordance with Definition 1. □

Remark 2. *As pointed out in [4], the adjusting strategy for the dynamic parameters $\alpha_y(t)$ and $\alpha_z(t)$ proposed in (19) is more general than the one in [6,11,36] and the one in [21,37]. The adjusting strategy in [6,11,36] can be obtained from the one in (19) by choosing $d_{1\varsigma} = d_{2\varsigma}$ and the adjusting strategy in [21,37] can be obtained from the one in (19) by choosing $c_{1\varsigma} = 1$, $d_{1\varsigma} = 1$, and $d_{2\varsigma} = 2$. Moreover, another advantage of the adjusting strategy in (19) is that the constant o_ς is independent of the matrix inequality (18).*

3.2. Filter Design

Based on the results developed in Theorem 1, the design results characterized by linear matrix inequalities for the desired filter in (13) will be proposed in the following theorem.

Theorem 2. *Suppose that the quantization ranges $\mathcal{R}_y$ and $\mathcal{R}_z$, the quantization error bounds Δ_y and Δ_z, the dimension adjustment matrix K, and the constants $\bar{\rho}$, $\bar{\varepsilon}$, $\gamma > 0$, $0 < c_{1y} \leq c_{2y}$, $0 < d_{1y} \leq d_{2y}$, $0 < c_{1z} \leq c_{2z}$, $0 < d_{1z} \leq d_{2z}$, satisfying $c_{1y}d_{1y} \geq 1$ and $c_{1z}d_{1z} \geq 1$ are provided. In the presence of the adjusting strategy for the dynamic parameters $\alpha_y(t)$ and $\alpha_z(t)$ provided in (19) with the inequality in (17), the filtering error system in (14) is stochastically stable with the provided dissipative filtering performance γ, if there exist matrices $P_1 > 0$, P_2, $P_3 > 0$, G_1, G_2, $\widetilde{A}$, $\widetilde{B}$, $\widetilde{E}$, nonsingular matrix G_3, and positive scalars ζ_y, ζ_z satisfying*

$$\Psi_i < 0,\ i = 1, 2, \ldots, s. \tag{34}$$

where

$$\Psi_i = \begin{bmatrix} \Theta_{11_i} & * & * & * & * \\ \Theta_{21_i} & \Theta_{22} & * & * & * \\ \Theta_{31_i} & 0 & \Theta_{22} & * & * \\ \Theta_{41_i} & 0 & 0 & -I & 0 \\ \Theta_{51_i} & 0 & 0 & 0 & \Theta_{55} \end{bmatrix},$$

$$\Theta_{11_i} = \begin{bmatrix} -P_1 & * & * & * & * \\ -P_2 & -P_3 & * & * & * \\ -\bar{\rho}J_2^T E_i & J_2^T \widetilde{E} & \Lambda_{33} & * & * \\ 0 & 0 & 0 & -\zeta_y I & * \\ 0 & 0 & -\bar{\rho}J_2 & 0 & -\zeta_z I \end{bmatrix},$$

$$\Lambda_{33} = -\bar{\rho}F_i^T J_2 - \bar{\rho}J_2^T F_i - (J_3 - \gamma I),$$

$$\Theta_{21_i} = \begin{bmatrix} \Delta_{11} & K\widetilde{A} & \Delta_{13} & \bar{\varepsilon}K\widetilde{B} & 0 \\ \Delta_{21} & \widetilde{A} & \Delta_{23} & \bar{\varepsilon}\widetilde{B} & 0 \end{bmatrix},$$

$$\Theta_{31_i} = \begin{bmatrix} \hat{\varepsilon}K\widetilde{B}C_i & 0 & \hat{\varepsilon}K\widetilde{B}D_i & \hat{\varepsilon}K\widetilde{B} & 0 \\ \hat{\varepsilon}\widetilde{B}C_i & 0 & \hat{\varepsilon}\widetilde{B}D_i & \hat{\varepsilon}\widetilde{B} & 0 \end{bmatrix},$$

$$\Theta_{41_i} = \begin{bmatrix} \bar{\rho}J_{11}E_i & -J_{11}\widetilde{E} & \bar{\rho}J_{11}F_i & 0 & \bar{\rho}J_{11} \\ \hat{\rho}J_{11}E_i & 0 & \hat{\rho}J_{11}F_i & 0 & \hat{\rho}J_{11} \end{bmatrix},$$

$$\Theta_{51_i} = \begin{bmatrix} \zeta_y\sigma_y C_i & 0 & \zeta_y\sigma_y D_i & 0 & 0 \\ \zeta_z\sigma_z E_i & 0 & \zeta_z\sigma_z F_i & 0 & 0 \end{bmatrix},$$

$$\Theta_{22} = \begin{bmatrix} P_1 - G_1 - G_1^T & * \\ P_2 - G_2 - G_3^T K^T & P_3 - G_3 - G_3^T \end{bmatrix},$$

$$\Theta_{55} = -diag\{\zeta_y I,\ \zeta_z I\},$$

$$\Delta_{11} = G_1 A_i + \bar{\varepsilon}K\widetilde{B}C_i, \Delta_{21} = G_2 A_i + \bar{\varepsilon}\widetilde{B}C_i,$$

$$\Delta_{13} = G_1 B_i + \bar{\varepsilon}K\widetilde{B}D_i, \Delta_{23} = G_2 B_i + \bar{\varepsilon}\widetilde{B}D_i.$$

Moreover, the parameters for the filter (13) can be obtained by

$$\widehat{A} = G_3^{-1}\widetilde{A},\ \widehat{B} = G_3^{-1}\widetilde{B},\ \widehat{E} = \widetilde{E}. \tag{35}$$

Proof. For the nonsingular matrix G, based on $-(P-G)^T P^{-1}(P-G) \le 0$ and $P > 0$, we have that

$$-G^T P^{-1} G \le -G - G^T + P \tag{36}$$

By considering (36) and performing congruence transformation to (18) by $diag\{I,\ \widehat{G}^T,\ I,\ \hat{\zeta}\}$ with $\widehat{G} = diag\{G,\ G\}$ and $\hat{\zeta} = diag\{\zeta_y I,\ \zeta_z I\}$, it can be obtained that

$$\begin{bmatrix} \Upsilon_{11} & * & * & * \\ \overline{\Upsilon}_{21} & \overline{\Upsilon}_{22} & * & * \\ \Upsilon_{31} & 0 & -I & * \\ \overline{\Upsilon}_{41} & 0 & 0 & \overline{\Upsilon}_{44} \end{bmatrix} < 0 \tag{37}$$

where

$$\overline{\Upsilon}_{21} = \begin{bmatrix} G^T A_a & G^T B_a & G^T H_a & 0 \\ \hat{\varepsilon}G^T A_b & \hat{\varepsilon}G^T B_b & \hat{\varepsilon}G^T H_b & 0 \end{bmatrix},$$

$$\overline{\Upsilon}_{22} = diag\{\ -G - G^T + P, -G - G^T + P\}\ ,$$

$$\overline{\Upsilon}_{41} = \begin{bmatrix} \zeta_y\sigma_y\overline{C} & \zeta_y\sigma_y\overline{D} & 0 & 0 \\ \zeta_z\sigma_z\overline{E} & \zeta_z\sigma_z\overline{F} & 0 & 0 \end{bmatrix},$$

$$\overline{\Upsilon}_{44} = -diag\{\zeta_y I,\ \zeta_z I\}.$$

We assume $P = \begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix}$, $G^T = \begin{bmatrix} G_1 & KG_3 \\ G_2 & G_3 \end{bmatrix}$ with G_3 is nonsingular and define $\widetilde{A} = G_3\widehat{A}$, $\widetilde{B} = G_3\widehat{B}$, and $\widetilde{E} = \widehat{E}$, the inequality in (37) can be expressed as

$$\sum_{i=1}^{s} p_i(n(t))\Psi_i < 0 \tag{38}$$

Finally, by considering $p_i(n(t)) \geq 0$ stated in (5), one can deduce that if the inequality in (34) is satisfied, then the inequality in (38) holds, which completes the proof. $\square$

Next, some discussions on the main results in this paper will be provided.

Remark 3. *In Theorem 2, for the provided dimension adjustment matrix K, both the full-order dissipative filter and the reduced-order dissipative filter design results are presented in a unified framework characterized by linear matrix inequalities, which can be effectively solved by the LMI toolbox. In general, the dimension adjustment matrix K can be chosen as* $K = I_{n_x \times n_x}$ *for full-order dissipative filter and* $K = \begin{bmatrix} I_{n_{\bar{x}} \times n_{\bar{x}}} & 0_{n_{\bar{x}} \times (n_x - n_{\bar{x}})} \end{bmatrix}^T$ *for reduced-order dissipative filter.*

Remark 4. *The deign results proposed in Theorem 2 on the dissipative filter for nonlinear networked systems with dynamic quantization and data packet dropouts are general. By selecting* $J_{11} = I$*,* $J_2 = 0$*, and* $J_3 = (\gamma^2 + \gamma)I$*, the deign results proposed in Theorem 2 can be utilized to design the* $\mathcal{H}_\infty$ *filter. By selecting* $J_{11} = 0$*,* $J_2 = I$*, and* $J_3 = 2\gamma I$*, the deign results proposed in Theorem 2 can be utilized to design the passive filter. By selecting* $J_{11} = \sqrt{\kappa} I$*,* $J_2 = (1-\kappa)I$*, and* $J_3 = (\kappa(\gamma^2 - \gamma) + 2\gamma)I$ *with* $0 \leq \kappa \leq 1$*, the deign results proposed in Theorem 2 can be utilized to design the mixed passive/*$\mathcal{H}_\infty$ *filter.*

Remark 5. *Based on the results in [8,21], we know that a feasible adjusting rule is necessary for the dynamic parameter* $\alpha_\varsigma(t)$ *due to the use of the unreliable transmission communication network. As in [4], the adjusting rule for the dynamic parameter* $\alpha_\varsigma(t)$ *in this paper is proposed as*

$$\alpha_\varsigma(t) = \mathit{floor}(d_{2\varsigma} o_\varsigma |\varsigma(t)| \times 10^{-\jmath})$$

where $\jmath = \min\{\jmath \in \mathcal{N}^+ \big| \big(d_{2\varsigma} o_\varsigma |\varsigma(t)| \times 10^{\jmath}\big) > 1\}$ *and the function floor*$(\hbar)$ *denotes the maximum integer that is not bigger than* $\hbar$*.*

Remark 6. *According to the conclusions in [23], we have that the numerical complexity of the design results proposed in Theorem 2 is closely related to the number of variables* V *and the number of rows* L*. Moreover, the deign conditions in Theorem 2 can be solved in polynomial time with complexity proportional to* $\mathrm{C} = \mathrm{V}^3\mathrm{L}$*, where* $\mathrm{V} = 2 + 2n_{\bar{x}}n_{\bar{x}} + 2n_{\bar{x}}n_x + \frac{1}{2}n_x(n_x+1) + \frac{1}{2}n_{\bar{x}}(n_{\bar{x}} + 1) + n_x n_x + n_{\bar{x}} n_y + n_{\bar{x}} n_z$ *and* $\mathrm{L} = (3n_x + 3n_{\bar{x}} + 2n_y + 4n_z + n_w)s$*.*

Remark 7. *In general,* $\mathcal{R}_y$*,* $\mathcal{R}_z$*,* Δ_y*,* Δ_z *are provided parameters for dynamic quantizers and* $\bar{\varepsilon}$*,* $\bar{\rho}$ *are provided parameters for data packet dropouts. However, how to deal with the dissipative filtering problem with the unknown parameters* $\mathcal{R}_y$*,* $\mathcal{R}_z$*,* Δ_y*,* Δ_z*,* $\bar{\varepsilon}$*, and* $\bar{\rho}$ *is still a open problem, which needs further study. Moreover, it should be noted that the conservatism of the results proposed in Theorem 2 can be further reduced by employing the fuzzy Lyapunov function strategy in [15] and introducing slack matrix variables via Lemma 4 in [45].*

4. Simulation Example

In this section, we will show that the proposed dissipative filtering strategy is effective via a practical example.

Consider the tunnel diode circuit depicted in Figure 1, which is also employed to study the l_2–l_∞ fuzzy filtering problem for nonlinear networked systems with dynamic quantization in [36]. As in [36], by choosing $x_1(t) = v_C(t)$, $x_2(t) = i_{L1}(t)$, and $x_3(t) = i_{L2}(t)$, state equations for the tunnel diode circuit can be represented as

$$\begin{aligned} &\mathbb{C}\dot{x}_1(t) = -\mathbb{W}x_1(t) - \mathbb{N}x_1^3(t) + x_2(t) + x_3(t) \\ &\mathbb{L}_1\dot{x}_2(t) = -x_1(t) - \mathbb{R}_1 x_2(t) + \mathbb{V}w(t) \\ &\mathbb{L}_2\dot{x}_3(t) = -x_1(t) - \mathbb{R}_2 x_3(t) \end{aligned} \tag{39}$$

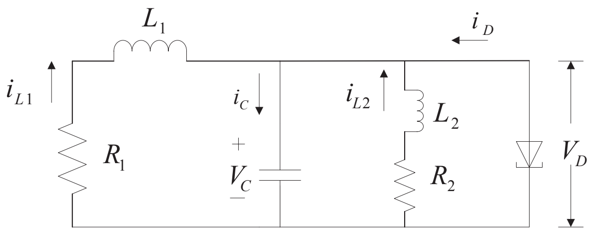

Figure 1. Tunnel diode circuit.

In this paper, we assume that $\mathbb{C} = 20\,\text{mF}$, $\mathbb{W} = 0.002\,\text{s}$, $\mathbb{N} = 0.01\,\text{s}$, $\mathbb{L}_1 = 1000\,\text{mH}$, $\mathbb{R}_1 = 10\,\Omega$, $\mathbb{V} = 1$, $\mathbb{L}_2 = 100\,\text{mH}$, $\mathbb{R}_2 = 1\,\Omega$, and $|x_1(t)| \leq 3$, i.e., $0 \leq x_1^2(t) \leq 9$. Then, the nonlinear tunnel diode circuit in (39) can be approximated by the following continuous-time T–S fuzzy model:

$$\begin{aligned} &\text{Plant Rule 1 : IF } x_1^2(t) \text{ is } 0, \text{ THEN} \\ &\qquad \dot{x}(t) = A_1 x(t) + B_1 u(t) \\ &\text{Plant Rule 2 : IF } x_1^2(t) \text{ is } 9, \text{ THEN} \\ &\qquad \dot{x}(t) = A_2 x(t) + B_2 u(t) \end{aligned} \tag{40}$$

where

$$A_1 = \begin{bmatrix} -0.1 & 50 & 50 \\ -1 & -10 & 0 \\ -10 & 0 & -10 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -4.6 & 50 & 50 \\ -1 & -10 & 0 \\ -10 & 0 & -10 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

Moreover, the membership functions can be provided as

$$p_1(x_1(t)) = \begin{cases} 1 - \frac{x_1^2(t)}{9}, & -3 \leq x_1(t) \leq 3 \\ 0, & \text{otherwise} \end{cases}$$
$$p_2(x_1(t)) = 1 - p_1(x_1(t)).$$

By setting the sampling period $T = 0.02\,\text{s}$, we have that

$$A_1 = \begin{bmatrix} 0.8970 & 0.8726 & 0.8726 \\ -0.0175 & 0.8101 & -0.0086 \\ -0.1745 & -0.0859 & 0.7328 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.8170 & 0.8332 & 0.8332 \\ -0.0167 & 0.8104 & -0.0083 \\ -0.1666 & -0.0833 & 0.7354 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.0092 \\ 0.0181 \\ -0.0006 \end{bmatrix}, B_2 = \begin{bmatrix} 0.0089 \\ 0.0181 \\ -0.0006 \end{bmatrix},$$

and other relative matrices are supposed to be

$$C_1 = C_2 = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}, D_1 = D_2 = 0.4,$$
$$E_1 = E_2 = \begin{bmatrix} -2 & -2 & -4 \end{bmatrix}, F_1 = F_2 = 0.1.$$

By applying Theorem 2 with $K = I_{3\times3}$, $J_1 = -2$, $J_2 = 2$, $J_3 = 2$, $\mathcal{R}_y = \mathcal{R}_z = 50$, $\Delta_y = \Delta_z = 0.5$, $\bar{\rho} = \bar{\varepsilon} = 0.8$, $c_{1y} = c_{1z} = 1$, $d_{1y} = d_{1z} = 1$, $c_{2y} = c_{2z} = 2$, $d_{2y} = d_{2z} = 2$, and $\gamma = 0.55$, the related parameters for the desired full-order dissipative filter can be obtained as

$$\widehat{A} = \begin{bmatrix} 0.7820 & 0.4766 & 0.5534 \\ 0.0017 & 0.4906 & -0.0466 \\ -0.2126 & -0.0845 & 0.6139 \end{bmatrix}, \widehat{B} = \begin{bmatrix} -0.0809 \\ -0.0552 \\ -0.0111 \end{bmatrix},$$
$$\widehat{E} = \begin{bmatrix} 1.1356 & 6.0793 & 5.2254 \end{bmatrix}.$$

For the simulation, we assume that $x(0) = x_f(0) = [0\ \ 0\ \ 0]^T$ and $w(t) = 5\cos(0.25t)e^{-0.2t}$. The simulation results are presented in Figures 2–7, where the responses of $x(t)$ and $x_f(t)$ are indicated in Figure 2 and Figure 3, respectively, Figure 4 plots the responses of $z(t)$ and $z_f(t)$, Figure 5 shows the trajectory of $e(t)$, and the trajectories of the dynamic parameters $\alpha_y(t)$ and $\alpha_z(t)$ are shown in Figure 6 and Figure 7, respectively. The simulation results presented in Figures 2–7 demonstrate that the proposed dissipative filter design approach in this paper is effective.

Next, the tunnel diode circuit system (39) will be utilized to investigate the $\mathcal{H}_\infty$ filter design problem according to the results developed in Theorem 2, and the other parameters without detailed definition are same as the first case. Firstly, the effects of quantization error bound $\Delta_y(\Delta_z)$ and quantization range $\mathcal{R}_y(\mathcal{R}_z)$ on the optimized $\mathcal{H}_\infty$ filtering performance $\gamma_{\min}$ will be studied with $J_1 = -1$, $J_2 = 0$, and $J_3 = \gamma + \gamma^2$. The optimized $\mathcal{H}_\infty$ filtering performances $\gamma_{\min}$ computed by Theorem 2 with different quantization error bound $\Delta_y(\Delta_z)$ and quantization range $\mathcal{R}_y(\mathcal{R}_z)$ are shown in Figure 8 and Figure 9, respectively. As expected, one can observe that $\gamma_{\min}$ increases as the quantization range $\mathcal{R}_y(\mathcal{R}_z)$ decreases and $\gamma_{\min}$ increases as the quantization error bound $\Delta_y(\Delta_z)$ increases. Moreover, it is well known that a higher filter order $n_{\bar{x}}$ will lead to less design conservatism, i.e., a smaller optimized $\mathcal{H}_\infty$ filtering performance $\gamma_{\min}$. Then, we demonstrate this proposition. In the presence of different filter order $n_{\bar{x}}$, the optimized $\mathcal{H}_\infty$ filtering performances $\gamma_{\min}$ computed by Theorem 2 with different quantization error bounds and quantization ranges are shown in Tables 1 and 2, respectively.

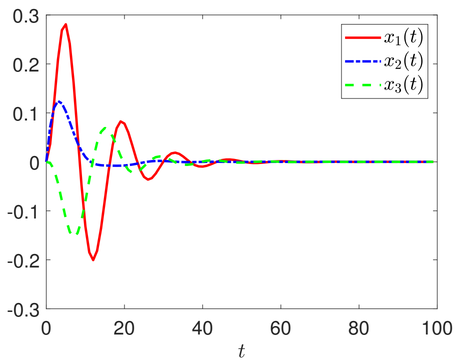

Figure 2. The response of $x(t)$.

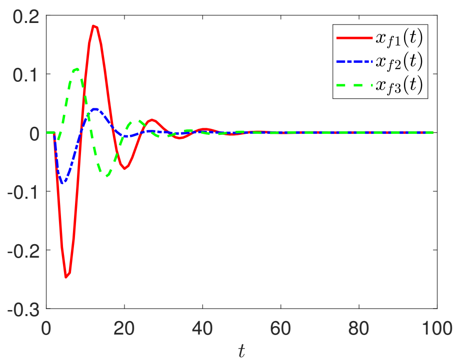

Figure 3. The response of $x_f(t)$.

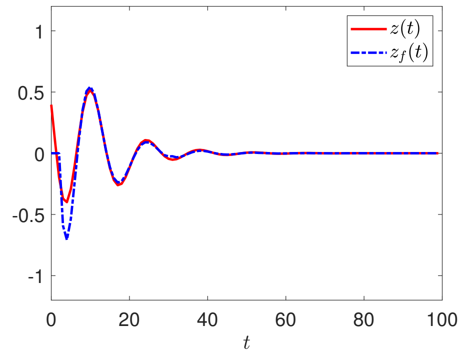

Figure 4. The responses of $z(t)$ and $z_f(t)$.

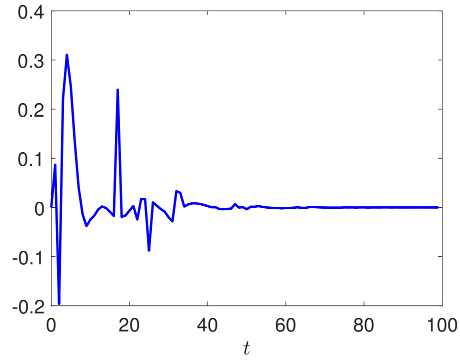

Figure 5. The trajectory of $e(t)$.

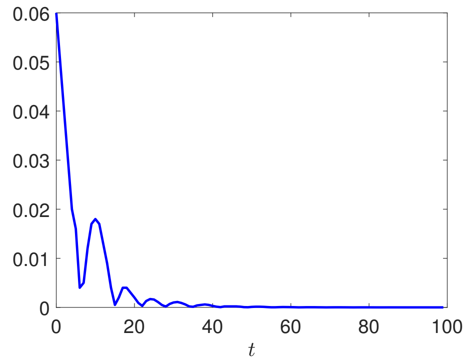

Figure 6. The trajectory of the dynamic parameter $\alpha_y(t)$.

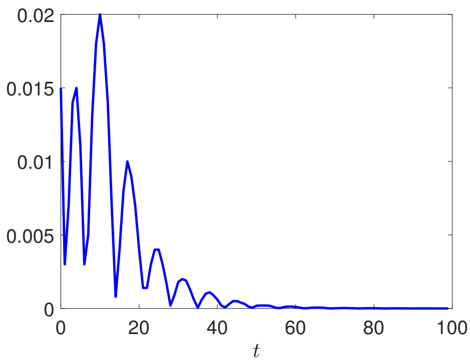

Figure 7. The trajectory of the dynamic parameter $\alpha_z(t)$.

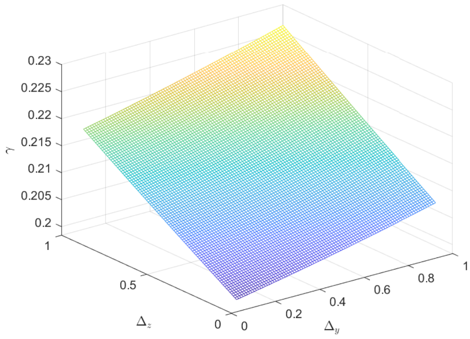

Figure 8. Optimized $\mathcal{H}_\infty$ filtering performance $\gamma_{\min}$ with different quantization error bound $\Delta_y(\Delta_z)$.

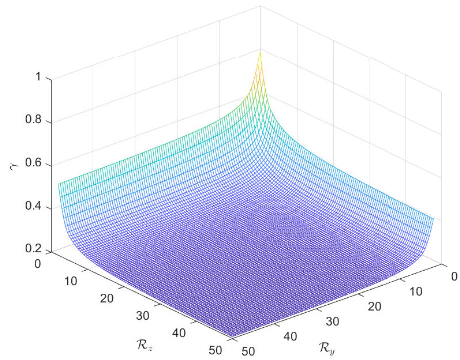

Figure 9. Optimized $\mathcal{H}_\infty$ filtering performance $\gamma_{\min}$ with different quantization error $\mathcal{R}_y(\mathcal{R}_z)$.

Table 1. Optimized $\mathcal{H}_\infty$ filtering performance $\gamma_{\min}$ with different quantization error bounds.

$\boldsymbol{\Delta_y = \Delta_z}$	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**
$\gamma_{\min}(n_{\overline{x}} = 3)$	0.1984	0.2040	0.2100	0.2163	0.2228
$\gamma_{\min}(n_{\overline{x}} = 2)$	0.3189	0.3257	0.3325	0.3393	0.3461
$\gamma_{\min}(n_{\overline{x}} = 1)$	0.3425	0.3504	0.3583	0.3664	0.3744

Table 2. Optimized $\mathcal{H}_\infty$ filtering performance $\gamma_{\min}$ with different quantization ranges.

$\boldsymbol{\mathcal{R}_y = \mathcal{R}_z}$	**10**	**30**	**50**	**70**	**90**
$\gamma_{\min}(n_{\overline{x}} = 3)$	0.2850	0.2206	0.2100	0.2057	0.2034
$\gamma_{\min}(n_{\overline{x}} = 2)$	0.4021	0.3438	0.3325	0.3276	0.3249
$\gamma_{\min}(n_{\overline{x}} = 1)$	0.4405	0.3717	0.3583	0.3526	0.3495

Comparative Explanations: In this paper, the developed filtering strategy can effectively solve both the full- and reduced-order dissipative filtering problems for the nonlinear tunnel diode circuit system in (39) with the effects of dynamic quantization and data packet dropouts based on the T–S fuzzy model strategy. In contrast with the existing results, the main advantages of the proposed filtering strategy can be summarized in the following three aspects.

(1) The proposed dissipative filtering strategy in this paper is more general than the existing results on fuzzy $\mathcal{H}_\infty$ filtering for nonlinear networked systems in [34,35,37,39–42], because it can also be utilized to deal with several kinds of filtering problems, including passive, $\mathcal{H}_\infty$, and mixed passive/$\mathcal{H}_\infty$ filtering problems for the nonlinear tunnel diode circuit system (39). Particularly, both the effects of dynamic quantization and data packet dropouts on the measurement output and the performance output have been considered

simultaneously; it implies that the problem addressed in this paper is more in agreement with practical circumstances than the ones considered in [36,38,41–43].

(2) In contrast with the quantized filtering problem considered in [38,41], the dynamical quantization methodology employed herein is more general. This is mainly because the stochastic stability of the filtering error system can be ensured under a finite number of quantization levels. By choosing the relevant parameters, the online adjusting strategies in [36,39] can be obtained from the one developed in (17) and (19), which implies that the adjusting strategy for the dynamic parameters $\alpha_\varsigma(t)$ ($\varsigma = y, z$) provided in this paper is more general. Moreover, simulation results in Figure 6 and Figure 7 show that the adjustment of the dynamic parameters $\alpha_\varsigma(t)$ can be realized based on the online adjusting strategy developed in this paper.

(3) In contrast with the existing results of the filtering problem for networked systems where only full-order filtering problems [34,35,38,39,41] or reduced-order filtering problems [46] were considered, the developed filtering strategy can effectively solve both the full- and reduced-order filtering problems, which is more general. Moreover, different from the results in [36], this example illustrates that both full- and reduced-order filtering problems have been solved in the unified framework of linear matrix inequalities by introducing a dimension adjustment matrix K.

5. Conclusions

In this paper, the dissipative filtering problem has been addressed for discrete-time nonlinear networked systems with dynamic quantization and data packet dropouts based on the T–S fuzzy strategy. Both the effects of dynamic quantization and data packet dropouts have been taken into consideration in both communication channels from the plant to the filter and from the filter to the plant. The sufficient design conditions for both the desired full- and reduced-order dissipative filters have been established in the unified framework of linear matrix inequalities, which guarantees the stochastic stability and the predefined dissipative filtering performance for the filtering error system subject to dynamic quantization and data packet dropouts. In addition, a practical simulation example has been employed to show the effectiveness of the proposed dissipative filtering approach.

However, it is well known that communication delays and cyber attacks, as important challenges in networked systems, are also considered to be unavoidable in practical cases. In this paper, we have only addressed dynamic quantization and data packet dropouts, and the study of the dissipative fuzzy filtering problem for nonlinear networked systems with the simultaneous consideration of dynamic quantization, data packet dropouts, communication delays, and cyber attacks deserves further investigation.

Author Contributions: Conceptualization, S.J. and Z.L.; formal analysis, S.J. and Z.L.; methodology, S.J. and Z.L.; funding acquisition, Z.L.; investigation, writing—original draft preparation and editing, and writing—review and editing, S.J., C.L. and Z.L.; software, S.J. and C.L. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported in part by the National Natural Science Foundation of China under Grant 62003006, in part by the Science and Technology Project of Hebei Education Department under Grant BJK2022053, in part by the Langfang Youth Talent Support Program under Grant LFBJ202202, in part by the Graduate Innovation Support Program in Hebei Province (CXZ ZSS2024142), and in part by the Graduate Innovation Support Program in North China Institute of Aerospace Engineering under Grant YKY-2023-25.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data sharing not applicable.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Liu, J.; Dong, Y.; Zha, L.; Tian, E.; Xie, X. Event-based security tracking control for networked control systems against stochastic cyber-attacks. *Inf. Sci.* **2022**, *612*, 306–321. [CrossRef]
2. Zha, L.; Liao, R.; Liu, J.; Xie, X.; Tian, E.; Cao, J. Dynamic event-triggered output feedback control for networked systems subject to multiple cyber attacks. *IEEE Trans. Cybern.* **2022**, *52*, 13800–13808. [CrossRef] [PubMed]
3. Zhang, X.M.; Han, Q.L.; Ge, X.; Ding, D.; Ding, L.; Yue, D.; Peng, C. Networked control systems: A survey of trends and techniques. *IEEE CAA J. Autom. Sin.* **2020**, *7*, 1–17. [CrossRef]
4. Xiong, J.; Chang, X.H.; Park, J.H.; Li, Z.M. Nonfragile fault-tolerant control of suspension systems subject to input quantization and actuator fault. *Int. J. Robust Nonlinear Control* **2020**, *30*, 6720–6743. [CrossRef]
5. Liberzon, D. Hybrid feedback stabilization of systems with quantized signals. *Automatica* **2003**, *39*, 1543–1554. [CrossRef]
6. Chang, X.H.; Xiong, J.; Li, Z.M.; Park, J.H. Quantized static output feedback control for discrete-time systems. *IEEE Trans. Industr. Inf.* **2018**, *14*, 3426–3435. [CrossRef]
7. Yu, K.; Chang, X.H. Quantized output feedback resilient control of uncertain systems under hybrid cyber attacks. *Int. J. Adapt. Control Signal Process.* **2022**, *36*, 2954–2970. [CrossRef]
8. Niu, Y.; Ho, D.W.C. Control strategy with adaptive quantizer's parameters under digital communication channels. *Automatica* **2014**, *50*, 2665–2671. [CrossRef]
9. Su, L.; Chesi, G. Robust stability of uncertain linear systems with input and output quantization and packet loss. *Automatica* **2018**, *87*, 267–273. [CrossRef]
10. Wu, C.; Zhao, X.; Xia, W.; Liu, J.; Başar, T. $\mathcal{L}_2$-gain analysis for dynamic event-triggered networked control systems with packet losses and quantization. *Automatica* **2021**, *129*, 109587. [CrossRef]
11. Li, Z.M.; Chang, X.H. Robust $\mathcal{H}_\infty$ control for networked control systems with randomly occurring uncertainties: Observer-based case. *ISA Trans.* **2018**, *83*, 13–24. [CrossRef] [PubMed]
12. Wang, Z.; Yang, F.; Ho, D.W.C.; Liu, X. Robust $\mathcal{H}_\infty$ control for networked systems with random packet losses. *IEEE Trans. Syst. Man Cybern. B Cybern.* **2007**, *37*, 916–924. [CrossRef] [PubMed]
13. Shen, H.; Hu, X.; Wang, J.; Cao, J.; Qian, W. Non-fragile $\mathcal{H}_\infty$ synchronization for Markov jump singularly perturbed coupled neural networks subject to double-layer switching regulation. *IEEE Trans. Neural Netw. Learn. Syst.* **2023**, *34*, 2682–2692. [CrossRef] [PubMed]
14. Nguyen, T.B.; Song, H.K. Further results on robust output-feedback dissipative control of Markovian jump fuzzy systems with model uncertainties. *Mathematics* **2022**, *10*, 3620. [CrossRef]
15. Tanaka, K.; Wang, H.O. *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*; Wiley: New York, NY, USA, 2001.
16. Chang, X.H.; Jing, Y.W.; Gao, X.Y.; Liu, X.P. $\mathcal{H}_\infty$ tracking control design of T–S fuzzy systems. *Control Decis.* **2008**, *23*, 329–332. (In Chinese)
17. Liu, J.; Gong, E.; Zha, L.; Tian, E.; Xie, X. Observer-based security fuzzy control for nonlinear networked systems under weighted try-once-discard protocol. *IEEE Trans. Fuzzy Syst.* **2023**, *31*, 3853–3865. [CrossRef]
18. Yao, H.; Gao, F. Design of observer and dynamic output feedback control for fuzzy networked systems. *Mathematics* **2022**, *11*, 148. [CrossRef]
19. Liu, J.; Ke, J.; Liu, J.; Xie, X.; Tian, E. Secure event-triggered control for IT-2 fuzzy networked systems with stochastic communication protocol and FDI attacks. *IEEE Trans. Fuzzy Syst.* 2023, *in press*. Available online: https://ieeexplore.ieee.org/abstract/document/10265199 (accessed on 27 September 2023).
20. Qiu, J.; Gao, H.; Ding, S.X. Recent advances on fuzzy-model-based nonlinear networked control systems: A survey. *IEEE Trans. Ind. Electron.* **2016**, *63*, 1207–1217. [CrossRef]
21. Chang, X.H.; Yang, C.; Xiong, J. Quantized fuzzy output feedback $\mathcal{H}_\infty$ control for nonlinear systems with adjustment of dynamic parameters. *IEEE Trans. Syst. Man Cybern. Syst.* **2019**, *49*, 2005–2015. [CrossRef]
22. Zha, L.; Huang, T.; Liu, J.; Xie, X.; Tian, E. Outlier-resistant quantized control for T–S fuzzy systems under multi-channel-enabled round-robin protocol and deception attacks. *Int. J. Robust Nonlinear Control* **2023**, *33*, 10916–10931. [CrossRef]
23. Li, Z.M.; Park, J.H. Dissipative fuzzy tracking control for nonlinear networked systems with quantization. *IEEE Trans. Syst. Man Cybern. Syst.* **2020**, *50*, 5130–5141. [CrossRef]
24. Wang, J.; Yang, C.; Xia, J.; Wu, Z.G.; Shen, H. Observer-based sliding mode control for networked fuzzy singularly perturbed systems under weighted try-once-discard protocol. *IEEE Trans. Fuzzy Syst.* **2022**, *30*, 1889–1899. [CrossRef]
25. Liu, J.; Wei, L.; Xie, X.; Tian, E.; Fei, S. Quantized stabilization for T–S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks. *IEEE Trans. Fuzzy Syst.* **2018**, *26*, 3820–3834. [CrossRef]
26. Zheng, Q.; Xu, S.; Du, B. Quantized guaranteed cost output feedback control for nonlinear networked control systems and its applications. *IEEE Trans. Fuzzy Syst.* **2022**, *30*, 2402–2411. [CrossRef]
27. Gao, H.; Zhao, Y.; Chen, T. $\mathcal{H}_\infty$ fuzzy control of nonlinear systems under unreliable communication links. *IEEE Trans. Fuzzy Syst.* **2009**, *17*, 265–278.
28. Qiu, J.; Feng, G.; Gao, H. Fuzzy-model-based piecewise $\mathcal{H}_\infty$ static-output-feedback controller design for networked nonlinear systems. *IEEE Trans. Fuzzy Syst.* **2010**, *18*, 919–934. [CrossRef]

29. Li, H.; Wu, C.; Jing, X.; Wu, L. Fuzzy tracking control for nonlinear networked systems. *IEEE Trans. Cybern.* **2017**, *47*, 2020–2031. [CrossRef]
30. Liu, J.; Wang, Y.; Cao, J.; Yue, D.; Xie, X. Secure adaptive-event-triggered filter design with input constraint and hybrid cyber attack. *IEEE Trans. Cybern.* **2021**, *51*, 4000–4010. [CrossRef]
31. Gao, H.; Chen, T. $\mathcal{H}_\infty$ estimation for uncertain systems with limited communication capacity. *IEEE Trans. Autom. Control* **2007**, *52*, 2070–2084. [CrossRef]
32. Liu, J.; Gong, E.; Zha, L.; Xie, X.; Tian, E. Outlier-resistant recursive security filtering for multirate networked systems under fading measurements and round-robin protocol. *IEEE Trans. Control Netw. Syst.* 2023, *in press*. Available online: https://ieeexplore.ieee.org/abstract/document/10068263 (accessed on 13 March 2023).
33. Zheng, Q.; Xu, S.; Du, B. Asynchronous resilent state estimation of switched fuzzy systems with multiple state impulsive jumps. *IEEE Trans. Cybern.* **2023**, *53*, 7966–7979. [CrossRef]
34. Chang, X.H. $\mathcal{H}_\infty$ filter design for T–S fuzzy systems with D stability constraints. *Control. Decis.* **2011**, *26*, 1051–1055. (In Chinese)
35. Liu, J.; Zha, L.; Tian, E.; Xie, X. Interval type-2 fuzzy-model-based filtering for nonlinear systems with event-triggering weighted try-once-discard protocol and cyber-attacks. *IEEE Trans. Fuzzy Syst.* 2023, *in press*. Available online: https://ieeexplore.ieee.org/abstract/document/10221224 (accessed on 16 August 2023).
36. Chang, X.H.; Li, Z.M.; Park, J.H. Fuzzy generalized $\mathcal{H}_2$ filtering for nonlinear discrete-time systems with measurement quantization. *IEEE Trans. Syst. Man Cybern. Syst.* **2018**, *48*, 2419–2430. [CrossRef]
37. Li, Z.M.; Xiong, J. Event-triggered fuzzy filtering for nonlinear networked systems with dynamic quantization and stochastic cyber attacks. *ISA Trans.* **2022**, *121*, 53–62. [CrossRef] [PubMed]
38. Chang, X.H.; Wang, Y.M. Peak-to-peak filtering for networked nonlinear DC motor systems with quantization. *IEEE Trans. Ind. Inf.* **2018**, *14*, 5378–5388. [CrossRef]
39. Chang, X.H.; Liu, Y. Robust $\mathcal{H}_\infty$ filtering for vehicle sideslip angle with quantization and data dropouts. *IEEE Trans. Veh. Technol.* **2020**, *69*, 10435–10445. [CrossRef]
40. Zhao, X.Y.; Chang, X.H. $\mathcal{H}_\infty$ filtering for nonlinear discrete-time singular systems in encrypted state. *Neural Process. Lett.* **2023**, *55*, 2843–2866. [CrossRef]
41. Zhang, C.; Feng, G.; Gao, H.; Qiu, J. $\mathcal{H}_\infty$ filtering for nonlinear discrete-time systems subject to quantization and packet dropouts. *IEEE Trans. Fuzzy Syst.* **2011**, *19*, 353–365. [CrossRef]
42. Gao, H.; Zhao, Y.; Lam, J.; Chen, K.E. $\mathcal{H}_\infty$ fuzzy filtering of nonlinear systems with intermittent measurements. *IEEE Trans. Fuzzy Syst.* **2009**, *17*, 291–300.
43. Chang, X.H.; Liu, Q.; Wang, Y.M.; Xiong, J. Fuzzy peak-to-peak filtering for networked nonlinear systems with multipath data packet dropouts. *IEEE Trans. Fuzzy Syst.* **2018**, *27*, 436–446. [CrossRef]
44. Liu, Y.; Guo, B.Z.; Park, J.H.; Lee, S. Event-based reliable dissipative filtering for T–S fuzzy systems with asynchronous constraints. *IEEE Trans. Fuzzy Syst.* **2017**, *26*, 2089–2098. [CrossRef]
45. Chang, X.H.; Park, J.H.; Zhou, J. Robust static output feedback $\mathcal{H}_\infty$ control design for linear systems with polytopic uncertainties. *Syst. Control Lett.* **2015**, *85*, 23–32. [CrossRef]
46. Cai, L.J.; Chang, X.H. Reduced-order filtering for discrete-time singular systems under fading channels. *Int. J. Syst. Sci.* **2023,** *54*, 99–112. [CrossRef]