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Generalized Weighted Mahalanobis Distance Improved VIKOR Model for Rockburst Classification Evaluation

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Abstract: Rockbursts are hazardous phenomena of sudden and violent rock failure in deep underground excavations under high geostress conditions, which poses a serious threat to geotechnical engineering. The occurrence of rockbursts is influenced by a combination of factors. Therefore, it is necessary to find an efficient method to assess rockburst grades. In this paper, we propose a novel method that enhances the VIKOR method using a novel combination of weight and generalized weighted Mahalanobis distance. The combination weights of the evaluation indicators were calculated using game theory by combining subjective experience and objective data statistical characteristics. By introducing the generalized weighted Mahalanobis distance, the VIKOR method is improved to address the issues of inconsistent dimensions, different importance, and inconsistent correlation among indicators. The proposed method can deal with the complexity of the impact factors of rockburst evaluation and classify the rockburst intensity level. The results show that the accuracy of the improved VIKOR method with the distance formula is higher than that of the unimproved VIKOR method; the evaluation accuracy of the improved VIKOR method with the generalized weighted Mahalanobis distance is 91.67%, which outperforms the improved VIKOR methods with the Euclidean and Canberra distances. This assessment method can be easily implemented and does not depend on the discussion of the rockburst occurrence mechanism, making it widely applicable for engineering rockburst evaluation.

Keywords: rockburst; combination weights; intensity classification; VIKOR; Mahalanobis distance

MSC: 65K99

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1. Introduction

Rockbursts are the rapid and intense fracturing of rock masses under high geostress in deep underground excavations [1,2]. The occurrence of rockbursts involves complex physical and mechanical processes, including stress concentration, energy accumulation and release, rock failure and fragmentation, dynamic wave propagation, and vibration. Rockbursts, characterized by its unpredictability and high intensity, are considered one of the most hazardous geological disasters, capable of causing casualties and equipment damage [3,4]. Given their potential hazards, predicting and evaluating rockbursts is of utmost importance in geotechnical engineering, particularly in hard rock mining, tunneling, and hydropower projects [5]. With the continuous development of deep resources and underground engineering construction, rockburst disasters have become more frequent and severe [6,7]. Therefore, accurate rockburst evaluation is crucial for reducing the probability of rockburst hazards and enhancing the safety of deep rock engineering during the preliminary design phase [3].

Evaluating rockbursts accurately is a challenging task that has attracted extensive research from numerous countries. Various models have been proposed to address this complex phenomenon. Early studies on rockbursts mainly focused on the influence of single factors on rockburst classification, such as Turchaninov criterion [8], Barton criterion [9], Russense criterion [10], Hoek criterion [11], Kidybinski criterion [12], and Potential stress failure [13]. However, later research revealed that rockburst occurrence is affected by multiple factors, not just one [14]. The relationship between rockbursts and these factors is highly nonlinear and interactive, which makes it hard to achieve high prediction accuracy using traditional empirical criteria [1,15]. Therefore, researchers have introduced new mathematical methods for rockburst evaluation, such as fuzzy comprehensive evaluation [16,17], rough set theory [18,19], cloud model [4,20], attribute recognition model [21], matter-extension theory [22], set pair analysis [23], unascertained measurement [24,25], distance discriminant analysis [26,27], support vector machine [28], XGboost [29], artificial neural networks [30,31], particle swarm optimization [32], K-nearest neighbor [15,33], decision tree [34], Bayesian network [35], and random forest [36]. Although many machine learning models have shown promising predictive performance for underground engineering projects, they rely heavily on sample data [37]. Mathematical models aim to establish analytical relationships between rockburst occurrences and influencing factors using mathematical equations. Some researchers have shown interest in applying multi-criteria decision-making (MCDM) methods to address classification problems. One such method is VlseKriterijumska Optimizacijia IKompromisno Resenje (VIKOR), which incorporates the concept of maximizing overall utility and minimizing individual regrets, ranking evaluated objects based on their distances from the ideal reference values [38]. Its practical applications span diverse fields, including manufacturing [39], management [40], environment [41], agriculture [38], healthcare [42], and finance [43]. However, traditional VIKOR methods, which rely on the Minkowski distance formula, suffer from limitations such as inconsistent indicator dimensions, variations in indicator importance, and correlations between indicators. These challenges necessitate the development of a new method to overcome these issues. In addition, the determination of evaluation criteria weights is a key issue in evaluating rockbursts using a multi-criteria evaluation method, as it directly impacts the accuracy of the evaluation results and rockburst evaluations. Common weighting methods include subjective weighting and objective weighting. Subjective weighting reflects the personal preferences of decision makers, while objective weighting utilizes calculations based on differences in objective data. Reducing the influence of subjective factors in the weight calculation process is crucial for ensuring the credibility of the final rockburst evaluation levels. To address this issue, this study introduces game theory and combines it with the AHP and the CRITIC methods to calculate a combination weight that achieves a relative balance between subjective and objective considerations.

In this paper, we propose a novel method for predicting rockburst classification based on the VIKOR model and generalized weighted Mahalanobis distance. This method aims to improve the accuracy and objectivity of rockburst evaluation by comprehensively considering the subjective and objective evaluation methods, the correlation and importance difference between the attribute indicators, the elimination of the attribute magnitude influence, and the solution of the covariance matrix irreversibility problem. This method can provide a new perspective and research idea for rockburst propensity evaluation.

2. Materials and Methods

In this study, the game theory comprehensive weighting method and the VIKOR model are used to assess the rockburst intensity grade, and the specific algorithm flowchart is shown in Figure 1.

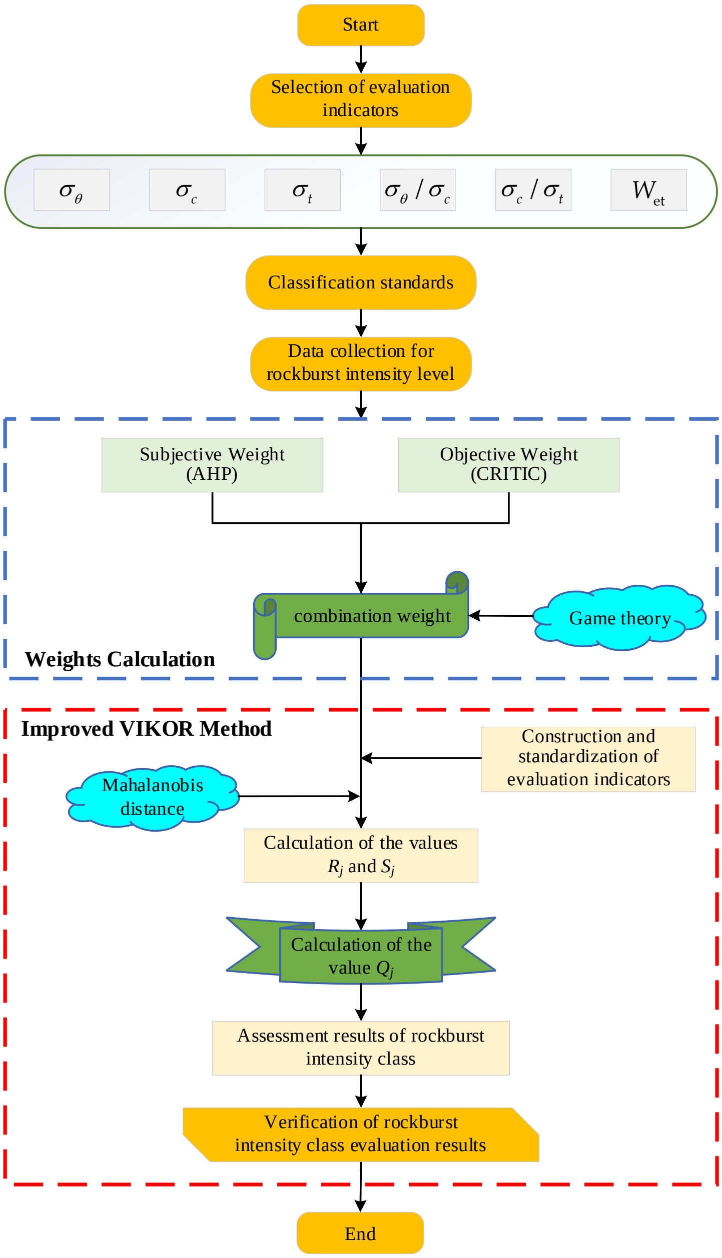

Figure 1. Flowchart of the proposed method.

2.1. *Weighting Calculation Method*

2.1.1. Subjective Weights Method

The subjective weighting method used in this paper is the Analytic Hierarchy Process (AHP), which was proposed by the famous operations researcher Saaty [44] in the 1970s. AHP is a typical subjective weighting method that simulates the thinking and judging process of the human brain for a complex decision problem. Based on a deep analysis of the essence and intrinsic relationships of the problem, it decomposes the complex and fuzzy system problem into layers, refines it layer by layer, and establishes a clear hierarchical structure from top to bottom, modeling and mathematizing the decision makers' thinking process, thus providing a simple, flexible, and practical decision-making method for complex problems. Due to its advantages of simplicity, flexibility, and ease of analysis and calculation, the analytical hierarchy process is widely used to determine the weights of different criteria in complex decision problems with uncertainty. The influencing factors of the rockburst evaluation problem are interrelated and interdependent.

For the rockburst evaluation problem, the influencing factors are interrelated and interdependent. The AHP can play an important role in rockburst evaluation when quantitative data are lacking. The AHP modeling can be divided into four steps:

Step 1. Establishing a hierarchical structure.

σ_θ, σ_c, σ_t, σ_θ/σ_c, σ_c/σ_t, and W_{et} are used as evaluation indicators in this paper. The hierarchical structure of rockburst evaluation constructed by them is shown in Figure 2.

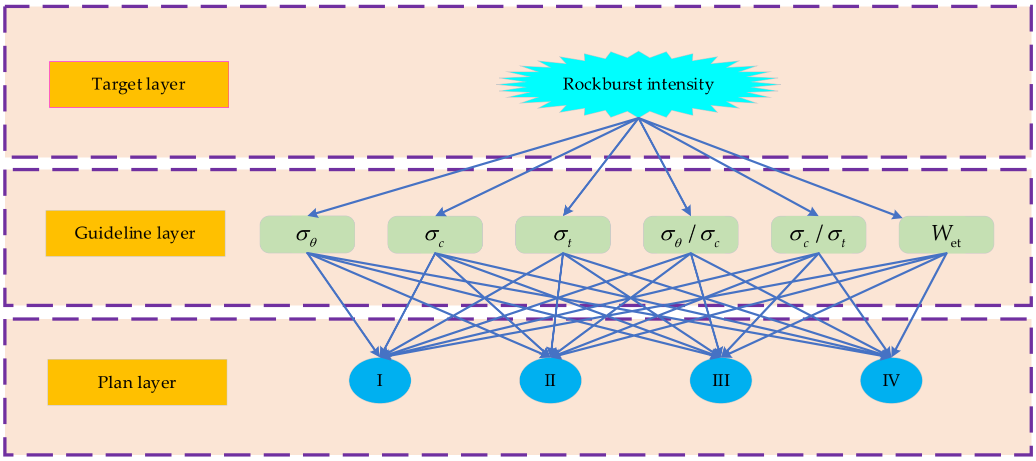

Figure 2. The diagram of grade evaluation of rockburst hierarchy.

Step 2. Constructing a judgment matrix.

The elements of the judgment matrix represent the relative importance of the six indicators, and the 1~9 scale method is commonly used to determine them [45]. The judgment matrix is shown in Equation (1), and the specific assignment method is shown in Table 1.

$$A = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ a_{21} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & \cdots & a_{1n} \\ 1/a_{12} & 1 & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/a_{1n} & 1/a_{2n} & \cdots & 1 \end{bmatrix} \quad (1)$$

Here, a_{ij} ($i = 1, 2, \ldots, n; j = 1, 2, \ldots, n$) is the level of importance of the comparison of 2 adjacent indicators; n is the number of indicators, which is taken as 6 in this paper.

Table 1. Scale of Analytic Hierarchy Process (AHP) [46].

Value	Definition	Explanation
1	Equally Important	Both criteria are equally important or both the indicators have same effect on occurrence of landslides
3	Moderately Important	One indicator is more effective as compared to the other indicator
5	Highly Important	One indicator affects highly as compared to the other indicator
7	Very Highly Important	One indicator is highly dominated over the other indicator
9	Extremely Important	One indicator has highest possibility of affecting the occurrence of landslide over the other indicator
2, 4, 6, 8	Intermediate Values	If a compromise between two indicators is required, intermediate values can be used

Step 3. Solving the judgment matrix.

The AHP method used in this work is the eigenvalue approach, which is mathematically based as a method for finding eigenvalues and eigenvectors, where $\widetilde{W} = [\widetilde{w}_1, \widetilde{w}_2, \cdots, \widetilde{w}_n]$. The eigenvector A is calculated using the square root method.

$$\widetilde{w}_i = \sqrt[n]{\prod_{j=1}^{n} a_{ij}} \tag{2}$$

$$W_i = \widetilde{W}_i / \sum_{i=1}^{n} \widetilde{W}_i \tag{3}$$

$$\lambda_{\max} = \sum_{i=1}^{n} \frac{(AW)_i}{nW_i} \tag{4}$$

where $\widetilde{W}_{ij}$ represents the value computed for the judgment matrix using column vector normalization, while W_i represents the value computed for the judgment matrix using row sum normalization, and $\lambda_{\max}$ represents the largest eigenvalue of the judgment matrix.

Step 4. Checking and correcting the consistency.

The consistency index (CI) formula for the consistency check is as follows:

$$CI = (\lambda_{\max} - n)/(n-1) \tag{5}$$

where n is the order of the judgment matrix.

The consistency ratio (CR) represents the random consistency ratio of the judgment matrix. When CR is less than 0.1, it indicates that the subjective weight ($\boldsymbol{W}1$) assigned to the indicators through AHP are valid. If the CR exceeds 0.1, it is necessary to adjust the evaluation factors of the judgment matrix, recompute the matrix, and iterate the process until the consistency criterion is met [47]. The formula for calculating the CR is as follows:

$$CR = CI/RI \tag{6}$$

In this formula, RI refers to the random consistency index of A, which can be found in Table 2, and CI denotes the consistency index of A.

Table 2. Randomness Index (*RI*) Table [25].

Number of Criteria	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

2.1.2. Objective Weights Method

An objective weighting method is adopted to determine the indicator weights for rockburst evaluation, since the measured values of the indicators vary in units and scales, and the subjective weighting method has high uncertainty [48]. The Criteria Importance through Intercriteria Correlation (CRITIC) method, proposed by D. Diakoulaki [49], is an objective weighting method that assigns weights based on the information content and correlation of the indicators [50]. The information content reflects the variance of the indicators, while the correlation reflects the conflict between them. The specific calculation steps are as follows:

Step 1. Dimensionless processing.

Dimensionless processing is required to eliminate the influence of different units and positive and negative indicators in the evaluation process. The formulas are shown below.

For positive indicators where a higher value is better:

$$x_{ik} = \frac{X_{ik} - \min(X_k)}{\max(X_k) - \min(X_k)} \tag{7}$$

For negative indicators where a lower value is better:

$$x_{ik} = \frac{\max(X_k) - X_{ik}}{\max(X_k) - \min(X_k)} \tag{8}$$

In these formulas, $k = 1, 2, 3, \cdots, n$; x_{ik} represents the k-th indicator of the i-th set of the data of the matrix X; X_k represents the k-th column of matrix X, while x_{ik} represents the data of X_{ik} after dimensionless processing, i = [1, 2, 3, . . ., m].

Step 2. Calculation of standard deviation for each indicator.

$$\sigma_k = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(x_{ik} - \overline{x}_k)^2} \tag{9}$$

Here, $\overline{x}_k$ represents the average value of the measured data for indicator X_k; n represents the number of measured data for indicator X_k; and σ_k represents the standard deviation of the measured data for indicator X_k.

Step 3. Construction of correlation coefficient matrix.

The correlation coefficient q_{kj} for n indicators is calculated as the linear correlation coefficient between indicator X_k and indicator X_j, where x_j represents the average value of the measured data for indicator X_j, as shown in Equation (10).

$$q_{kj} = \frac{\sum\limits_{k=1}^{m}(x_k - \overline{x}_k)(x_j - \overline{x}_j)}{\sqrt{\sum\limits_{k=1}^{m}(x_k - \overline{x}_k)^2 \sum\limits_{k=1}^{m}(x_j - \overline{x}_j)^2}} \tag{10}$$

Step 4. Calculation of information content for each indicator.

The information content C_k of each indicator is calculated by combining the standard deviation and the correlation coefficient, as shown in Equation (11).

$$C_k = \alpha_k \sum_{j=1}^{n}\left(1 - q_{kj}\right) \tag{11}$$

Step 5. Calculation of objective weights.

The objective weights of each indicator are obtained by normalizing the information content, $\boldsymbol{W}2 = [u_1, u_2, \cdots, u_n]$, where the calculation of u_k can be found in Equation (12).

$$u_k = \frac{C_k}{\sum\limits_{j=1}^{n} C_j} \tag{12}$$

2.1.3. Game Theory to Determine Combination Weights

The game theory method (GM) is a combination weighting method that integrates subjective and objective information to determine the relative importance of different indicators. This method views weight determination as a non-cooperative game process, where each indicator is an independent game player. The Nash equilibrium solution, i.e., the weight of each indicator, is obtained through game solving.

If L weighting methods are selected, then weight set $\boldsymbol{w}_k = \{w_{k1}, w_{k2}, \cdots, w_{kn}\}$ $(k = 1, 2, \cdots, L)$ can be constructed, where n is the number of rockburst evaluation indicators.

Any linear combination of different vectors can be expressed as follows:

$$\boldsymbol{W} = \sum_{k=1}^{L} a_k \cdot \boldsymbol{w}_k^{\mathrm{T}}, a_k > 0 \tag{13}$$

In this formula, $\boldsymbol{W}$ represents the comprehensive weight vector and a_k represents the linear combination coefficient.

Taking the combination weight $\boldsymbol{W}$ of the indicator and the sum of the deviations as the objective of minimizing the sum of the deviations, the countermeasure modeling of the optimal solution of $\boldsymbol{W}$ as follows:

$$\min \left\| \sum_{k=1}^{L} a_k \boldsymbol{w}_k^T - \boldsymbol{w}_t \right\|_2 \tag{14}$$

where $t = 1, 2, 3, \cdots, L$.

According to Equation (14), Equation (15) can be calculated as follows:

$$\begin{bmatrix} \boldsymbol{w}_1 \cdot \boldsymbol{w}_1^T & \boldsymbol{w}_1 \cdot \boldsymbol{w}_2^T & \cdots & \boldsymbol{w}_1 \cdot \boldsymbol{w}_L^T \\ \boldsymbol{w}_2 \cdot \boldsymbol{w}_1^T & \boldsymbol{w}_2 \cdot \boldsymbol{w}_2^T & \cdots & \boldsymbol{w}_2 \cdot \boldsymbol{w}_L^T \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{w}_L \cdot \boldsymbol{w}_1^T & \boldsymbol{w}_L \cdot \boldsymbol{w}_2^T & \cdots & \boldsymbol{w}_L \cdot \boldsymbol{w}_L^T \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_L \end{bmatrix} = \begin{bmatrix} \boldsymbol{w}_1 \cdot \boldsymbol{w}_1^T \\ \boldsymbol{w}_2 \cdot \boldsymbol{w}_2^T \\ \vdots \\ \boldsymbol{w}_L \cdot \boldsymbol{w}_L^T \end{bmatrix} \tag{15}$$

After normalization, the subjective and objective weight coefficients (${a_k}^*$) are obtained:

$${a_k}^* = \frac{a_k}{\sum\limits_{k=1}^{L} a_k} \tag{16}$$

The combination weight vector ($\boldsymbol{W}$) can be obtained by substituting the ${a_k}^*$ calculated in Equation (16) into Equation (13).

2.2. Improved VIKOR Method

The VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje) method, originally introduced by Opricovic in 1998, is a multi-attribute decision-making approach aimed at discovering ideal alternatives for complex systems [51]. It begins by defining positive and negative ideal alternatives, representing the best and worst values for each evaluation criterion among all available alternatives [52]. Subsequently, it ranks these alternatives based on their proximity to these ideal alternatives. The VIKOR method generates a compromise solution that closely approaches the optimal solution while remaining feasible within the given constraints. The VIKOR method is traditionally based on the Minkowski distance formula [53], which has limitations in not taking into account data correlation, while its outcomes are affected by the dimensions of indicators. In contrast, the Mahalanobis distance effectively eliminates the influence of both dimensions and correlation, distinguishing it from the mentioned distance measures. Moreover, the Mahalanobis distance remains unaltered even when the original data undergo linear transformation. However, a challenge arises during the calculation of the Mahalanobis distance: if the covariance matrix becomes singular (i.e., the determinant of the covariance matrix equals 0), then $\boldsymbol{\Sigma}^{-1}$ becomes undefined, resulting in the inability to compute the Mahalanobis distance. This paper introduces the generalized Mahalanobis distance proposed by Chen [54]. The generalized Mahalanobis distance aims to prevent the non-existence of the Mahalanobis distance by substituting the inverse matrix with a pseudo-inverse matrix of the covariance array via singular value decomposition. This method inherits the relevance of the Mahalanobis distance and also solves the problem of the irreversibility of the covariance array. Additionally, it satisfies the three fundamental properties of distance, of which symmetry, non-negativity, and trigonometric inequality. Through singular value decomposition, the Moore–Penrose pseudo-inverse matrix $\boldsymbol{\Sigma}^{+}$ of $\boldsymbol{\Sigma}$ replaces the inverse matrix, which perfectly solves the problem of non-invertibility of the covariance matrix.

Therefore, this paper proposes a VIKOR evaluation method based on the generalized weighted Mahalanobis distance, and the specific calculation steps are as follows:

Step 1. Creation the decision matrix.

Let the decision matrix be P, c_{ij} be the j-th evaluation indicator of the i-th scheme, where m, n are the number of programs and the number of evaluation indicators, respectively. Then the original decision matrix can be expressed as:

$$P = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} \tag{17}$$

Step 2. Normalization of the decision matrix.

There are m evaluated objects, n evaluation indicators, the inverse indicator to take its inverse for the homothetic treatment, then the normalization method is shown in Equations (7) and (8).

Step 3. Determination of the ideal alternative an negative-ideal alternative.

Determining ideal alternative (y_j^+) and negative-ideal alternative (y_j^-) for each criterion, as shown in Equations (18) and (19).

$$\begin{cases} y_j^+ = \max\limits_{1 \le i \le m} y_{ij} \\ y_j^- = \min\limits_{1 \le i \le m} y_{ij} \end{cases} \tag{18}$$

$$\begin{cases} y^+ = ({y_1}^+, {y_2}^+, \cdots, {y_n}^+) \\ y^- = ({y_1}^-, {y_2}^-, \cdots, {y_n}^-) \end{cases} \tag{19}$$

where $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n$.

Step 4: Calculation of the values S_i and R_i by means of the following relations, as shown in Equations (20) and (21), respectively:

$$S_i = \sum_{j=1}^{n} \frac{\sqrt{(y^+ - y_i)W^{\mathrm{T}}\boldsymbol{\Sigma}^+ W(y^+ - y_i)^{\mathrm{T}}}}{\sqrt{(y^+ - y^-)\boldsymbol{\Sigma}^+(y^+ - y^-)^{\mathrm{T}}}} \tag{20}$$

$$R_i = \max_{1 \le j \le n} \left[\frac{\sqrt{(y^+ - y_i)W^{\mathrm{T}}\boldsymbol{\Sigma}^+ W(y^+ - y_i)^{\mathrm{T}}}}{\sqrt{(y^+ - y^-)\boldsymbol{\Sigma}^+(y^+ - y^-)^{T}}} \right] \tag{21}$$

where W is the comprehensive weight calculated by game theory, and the calculation of $\boldsymbol{\Sigma}^+$ is as follows:

The singular value decomposition form of $\boldsymbol{\Sigma}$ is $\boldsymbol{\Sigma} = UDV^T$, then $\boldsymbol{\Sigma}^+ = VTU^T$, where $D = diag(a_1, a_2, \cdots, a_r), a_i > 0$, r is the rank of matrix $\boldsymbol{\Sigma}$, U and V are orthogonal matrices. If $D(i,j) \neq 0$, then $T(i,j) = 1/D(i,j)$; if $D(i,j) = 0$, then $T(i,j) = 0$.

Step 5: Calculation of Q-value.

$$Q_i = \frac{\varepsilon(S_i - S^-)}{S^+ - S^-} + \frac{(1-\varepsilon)(R_i - R^-)}{R^+ - R^-}, i = 1, 2, \cdots, m \tag{22}$$

where S^+ is max$S$, S is min$S$, R^+ is min$R$, R^- is max$R$, and $\varepsilon(0 \le \varepsilon \le 1)$ is the compromise coefficient that reflects the decision maker's preference. The value of ε typically takes ε = 0.5, signifying equal significance of the largest group effect and the smallest individual regret [55,56].

Step 6: Determination of Rockburst Classification:

The rockburst intensity classification is determined by comparing the Q-value of the sample to the threshold Q-value.

3. Application

Rockburst is a complex phenomenon caused by the interaction of multiple factors in underground engineering. The selection of evaluation parameters is a crucial step for rockburst evaluation, as it determines the input variables and the output results of the evaluation model. Meanwhile, the selection of indicators for rockburst evaluation should balance the difficulty of data acquisition, the cost, and the comprehensiveness. Based on existing research [20,33,57–60], six indicators are selected to evaluate the rockburst intensity in this paper, considering the influencing factors, characteristics, and internal and external conditions of rockbursts. They are the maximum tangential stress of rock mass (σ_θ), the uniaxial compressive strength of rock mass (σ_c), the tensile strength of rock mass (σ_t), the stress coefficient (σ_θ/σ_c), the rock brittleness coefficient (σ_c/σ_t), and the elastic energy index (W_{et}). These parameters are:

(1) The maximum tangential stress of rock mass (σ_θ): This parameter reflects the stress condition of the rock mass around the excavation boundary;
(2) The uniaxial compressive strength of rock mass (σ_c): This parameter reflects the geological condition of the rock mass. It is a main rock characteristic that affects its resistance to failure under compression;
(3) The tensile strength of rock mass (σ_t): This parameter reflects another mechanical property of the rock mass. It is defined as the maximum tensile stress that a rock can withstand before failure. It is related to the occurrence of tensile fracture instabilities, which are also a cause of rockbursts;
(4) The stress coefficient (σ_θ/σ_c): This parameter reflects the ratio of the maximum tangential stress to the uniaxial compressive strength. It indicates how close or far a rock is from failure under shear stress;
(5) The rock brittleness coefficient (σ_c/σ_t): The uniaxial tensile strength is the other main rock characteristic, and the rock brittleness coefficient has often been applied to such engineering problems. It indicates how easily a rock can break under tension or shear stress;
(6) The elastic energy index (W_{et}): This parameter reflects the energy condition of the rock mass. W_{et} is defined as the proportion of retained strain energy to that dissipated, which can reflect the rock's ability to store elastic energy [12].

These parameters are comprehensive and representative of rockburst evaluation and can reflect the characteristics and differences of different rockburst intensity levels [36,57,58].

Based on the existing research [28,61], rockburst intensity can be classified into four levels: no rockburst (level I), weak rockburst (level II), mediate rockburst (level III), and strong rockburst (level IV). The relationship between these levels and the six indicators has been established, and the specific criteria for each single-factor indicator for rockburst are shown in Table 3.

Table 3. Specific criteria for each single-factor indicator.

Rockburst Level	Evaluation Index of Rockburst					
	σ_θ	σ_c	σ_t	σ_θ/σ_c	σ_c/σ_t	W_{et}
No rockburst (I)	0–24	0–80	0–5	0–0.3	>40	0–2.0
Weak rockburst (II)	24–60	80–120	5–7	0.3–0.5	26.7–40	2.0–3.5
Mediate rockburst (III)	60–126	120–180	7–9	0.5–0.7	14.5–26.7	3.5–5.0
Strong rockburst (IV)	≥126	≥180	≥9	0.7–1.0	0–14.5	≥5.0

Based on the selected rockburst evaluation indicators and rockburst intensity levels, 60 groups of rockburst engineering examples were selected from the existing literature as sample data for rockburst intensity levels, as shown in Appendix A.

In this study, three experts were selected for evaluation and scoring, one of them is a professor in geotechnical engineering and two experts are engineers with senior titles in the field of mining. Based on Equations (1)–(6), the maximum eigenvalues obtained are CI = 0.0509 and CR = 0.0454. Since *CR* < 0.1, the matrix passes the consistency test, so the AHP subjective weights $\boldsymbol{W}1$ = (0.1010, 0.0929, 0.1744, 0.2355, 0.2533, 0.1429). According to Equations (7)–(12), the CRITIC objective weight $\boldsymbol{W}2$ = (0.1145, 0.1527, 0.1699, 0.1561, 0.0693, 0.3375) can be calculated. The competitive characteristic of game theory is reflected in the calculation of subjective weights $\boldsymbol{W}1$ and objective weights $\boldsymbol{W}2$. The optimal solution is the one that balances the AHP subjective weighting method and the CRITIC objective weighting method. Substituting $\boldsymbol{W}1$ and $\boldsymbol{W}2$ into Equations (15) and (23) can be obtained.

$$\begin{bmatrix} \boldsymbol{W}1 \cdot \boldsymbol{W}1 & \boldsymbol{W}1 \cdot \boldsymbol{W}2 \\ \boldsymbol{W}2 \cdot \boldsymbol{W}1 & \boldsymbol{W}2 \cdot \boldsymbol{W}2 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \boldsymbol{W}1 \cdot \boldsymbol{W}1^{\mathrm{T}} \\ \boldsymbol{W}2 \cdot \boldsymbol{W}2^{\mathrm{T}} \end{bmatrix} \tag{23}$$

By solving Equations (23) and (24) can be computed.

$$\begin{cases} a_1 = 0.4507 \\ a_2 = 0.6584 \end{cases} \tag{24}$$

Combining Equations (16), (24) and (25) can be calculated as follows:

$$\begin{cases} {a_1}^* = 0.4064 \\ {a_2}^* = 0.5936 \end{cases} \tag{25}$$

Thus, based on Equations (13) and (25), the comprehensive weight, $\boldsymbol{W}$, can be calculated as shown in Equation (26), and the specific weighting results are shown in Table 4 and Figure 3.

$$\boldsymbol{W} = (0.1090, 0.1284, 0.1717, 0.1884, 0.1441, 0.2584) \tag{26}$$

Table 4. The weights of each indicator of rockburst intensity.

Methods	σ_θ	σ_c	σ_t	σ_θ/σ_c	σ_c/σ_t	W_{et}
AHP	0.1010	0.0929	0.1744	0.2355	0.2533	0.1429
CRITIC	0.1145	0.1527	0.1699	0.1561	0.0693	0.3375
GM	0.1090	0.1284	0.1717	0.1884	0.1441	0.2584

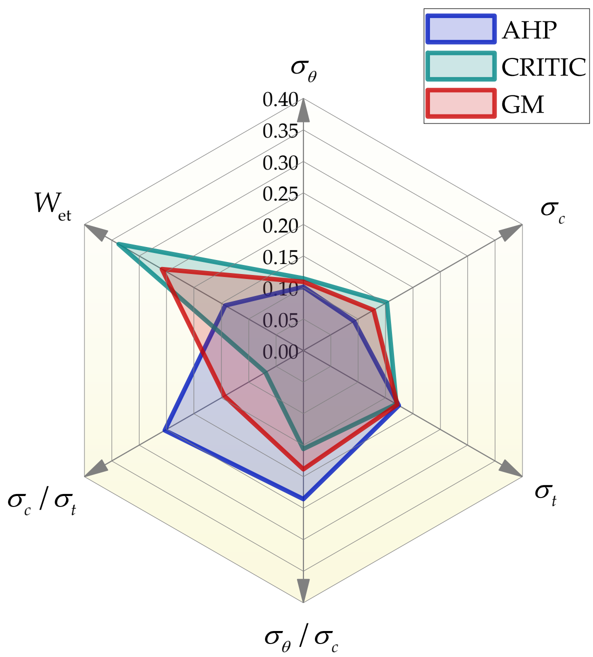

Figure 3. Visual comparison indicator weights.

The positive and negative ideal alternatives of each sample can be obtained by Equations (18)–(21). Q can be calculated by Equation (22), where the compromise coefficient ε is generally equal to 0.5 [62,63]. The critical Q values are 0.0583, 0.2519, and 0.4808, respectively. Therefore, it can be concluded that there is no rockburst (level I) when $0 \leq Q_i < 0.227$; weak rockburst (level II) when $0.227 \leq Q_i < 0.483$; moderate rockburst (level III) when $0.483 \leq Q_i < 0.773$; and strong rockburst (level IV) when $0.773 \leq Q_i \leq 1$. Figure 4 shows the grade results of the rockburst evaluations for each sample.

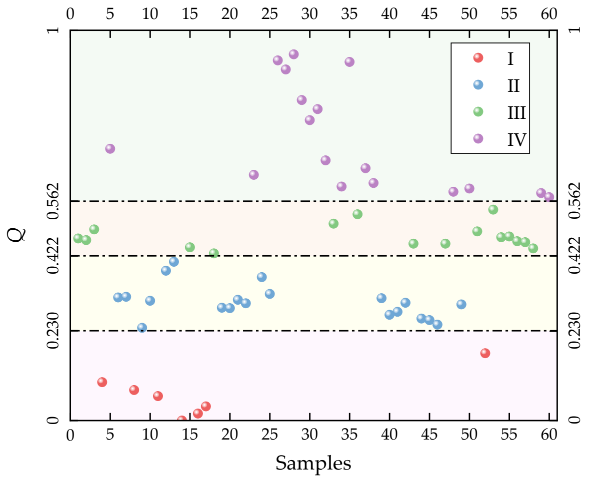

Figure 4. Distribution of Q-values.

To verify the rationality of the model, the unimproved VIKOR, the VIKOR algorithm improved by Euclidean distance, and the VIKOR algorithm improved by Euclidean distance were selected as comparison models. The detailed evaluation results are shown in Table 5, Figures 5 and 6.

Table 5. Comparative statistics of the case samples.

Samples	VIKOR-GWMD	VIKOR-ED	VIKOR-CD	VIKOR	Actual Grade
1	III	III	III	III	III
2	III	III	III	IV●	III
3	III	III	III	III	III
4	I	I	I	II●	I
5	IV	IV	IV	III●	IV
6	II	II	II	II	II
7	II	II	II	III●	II
8	I	I	I	II●	I
9	II	II	II	II	II
10	II	II	II	III●	II
11	I	I	I	II●	I
12	II	III●	III●	III●	II
13	II	II	II	III●	II
14	I	I	I	II●	I
15	III	III	III	III	III
16	I	I	I	I	I
17	I	I	I	II●	I
18	III	III	III	III	III
19	II	II	II	II	II
20	II	II	II	III●	II
21	II	II	II	II	II
22	II●	II●	II●	II●	I
23	IV	IV	IV	IV	IV
24	II	II	II	III●	II

Table 5. *Cont.*

Samples	VIKOR-GWMD	VIKOR-ED	VIKOR-CD	VIKOR	Actual Grade
25	II	II	II	III•	II
26	IV	IV	IV	IV	IV
27	IV	IV	IV	IV	IV
28	IV	IV	IV	IV	IV
29	IV	IV	IV	IV	IV
30	IV	IV	IV	IV	IV
31	IV	IV	IV	IV	IV
32	IV	IV	IV	IV	IV
33	III	III	III	III	III
34	IV	IV	IV	IV	IV
35	IV	IV	IV	IV	IV
36	III	III	III	III	III
37	IV	IV	IV	IV	IV
38	IV	IV	IV	IV	IV
39	II	II	II	III•	II
40	II	II	II	III•	II
41	II	II	II	III•	II
42	II	II	II	III•	II
43	III	III	III	III	III
44	II	II	II	III•	II
45	II	II	II	III•	II
46	II	II	II	II	II
47	III	III	III	III	III
48	IV	IV	IV	III•	IV
49	II	II	II	III•	II
50	IV•	III	III	III	III
51	III	III	III	III	III
52	I	II•	III•	II•	I
53	III•	III•	III•	III•	II
54	III	III	III	III	III
55	III	IV•	IV•	III	III
56	III	III	III	III	III
57	III•	III•	III•	II	II
58	III	III	III	III	III
59	IV•	IV•	IV•	III	III
60	IV	IV	IV	IV	IV

VIKOR-GWMD represents the VIKOR method improved by generalized weighted Mahalanobis distance; VIKOR-ED represents the VIKOR method improved by Euclidean distance; VIKOR-CD represents the VIKOR method improved by Canberra distance; and VIKOR represents the unimproved VIKOR method. The symbol "•" indicates an incorrect judgment.

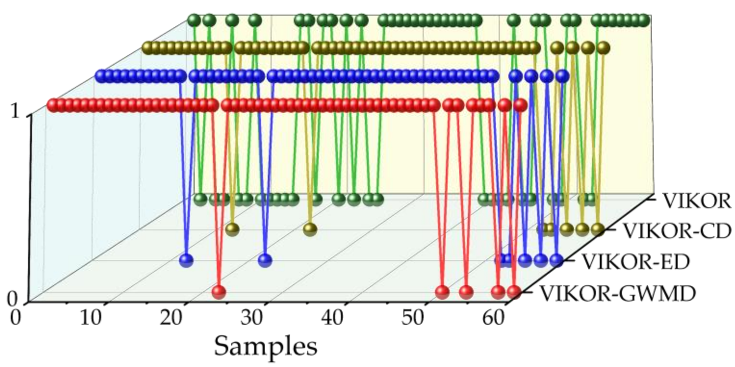

Figure 5. Comparison of visualized rockburst evaluation results.

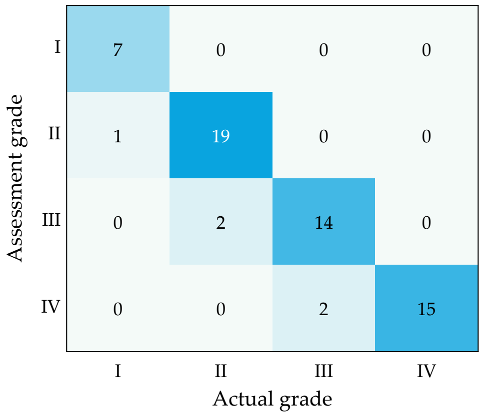

Figure 6. The confusion matrix of VIKOR-GWMD.

The accuracy of the VIKOR method with distance formulas is more than 85%, as shown in Tables 5 and 6 and Figures 5 and 7, which is much higher than that of the VIKOR method without improvement. The accuracy of the VIKOR method with generalized weighted Mahalanobis distance improvement is 91.67%, which is higher than that of the VIKOR methods improved by Euclidean distance and Canberra distance. Moreover, when misjudgments occurred in samples 22, 50, 53, 57, and 59, the results tend to be safe, which is acceptable from an engineering safety perspective. Although the VIKOR methods enhanced by the Euclidean distance and the Canberra distance have similar accuracies of 88.33%, discrepancies arise in Sample 52. In this sample, VIKOR-ED evaluates a rockburst intensity level of II, which is only one level below the actual level I, while VIKOR-CD evaluates a level of III with a two-level error.

Table 6. Evaluation results of different models.

Methods	VIKOR-GWMD	VIKOR-ED	VIKOR-CD	VIKOR
Accurate	55	53	53	35
Misjudge	5	7	7	25
Accuracy (%)	91.67	88.33	88.33	58.33

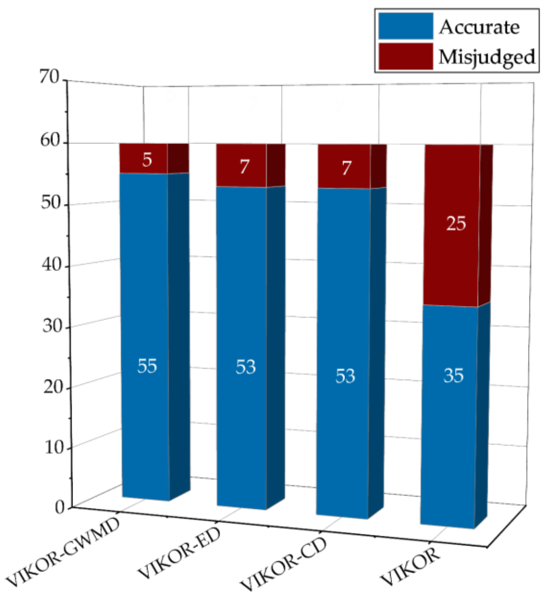

Figure 7. Comparison of the accuracy of different models.

4. Discussion

In this paper, we proposed a novel method for rockburst intensity classification evaluation based on the VIKOR method improved by the generalized weighted Mahalanobis distance. The method considers six evaluation parameters: the maximum tangential stress of rock mass (σ_θ), the uniaxial compressive strength of rock mass (σ_c), the tensile strength of rock mass (σ_t), the stress coefficient (σ_θ/σ_c), the rock brittleness coefficient (σ_c/σ_t), and the elastic energy index (W_{et}). The method is applied to 60 rockburst cases from all over the world and compared with other VIKOR methods improved by Euclidean distance and Canberra distance. The results show that our proposed method has higher evaluation accuracy than other methods in both case studies. The purpose of this discussion section is to explain the rationale behind our parameter selection and distance formula choice, and to justify the superiority of our proposed method over other methods.

4.1. Model Selection

Another aspect of our proposed method is the choice of the VIKOR method over other multi-criteria decision-making (MCDM) methods for rockburst intensity classification evaluation. The VIKOR method is a compromise ranking method that can provide a solution that is closest to the ideal alternative from the perspective of the majority of decision makers [56]. The VIKOR method has some advantages over other methods, as on the one hand, it can handle both quantitative and qualitative criteria, as well as conflicting and non-commensurable criteria, which are common in rockburst evaluation problems. On the other hand, it can generate a compromise solution that can satisfy most of the decision makers, as well as a set of acceptable solutions that can provide more flexibility and alternatives for decision making.

The main reason for this consistency and agreement is that the VIKOR method uses two coefficients: the closeness coefficient and the regret coefficient, which can reflect both the positive and negative aspects of each alternative. The closeness coefficient measures how close an alternative is to the ideal alternative, while the regret coefficient measures how far an alternative is from the anti-ideal alternative. Therefore, using both closeness coefficient and regret coefficient can provide a more comprehensive and balanced evaluation of each alternative. In contrast, other methods may have some limitations or drawbacks in their evaluation processes. Such as the TOPSIS method uses only the relative closeness coefficient, which measures how close an alternative is to the ideal alternative relative to the anti-ideal alternative. However, this coefficient may not be able to capture the absolute distance or difference between alternatives and reference values, which may affect its accuracy and discrimination power.

4.2. Distance Formula Comparison

The choice of the distance formula is a critical component of multicriteria decision-making methods like VIKOR and TOPSIS. It quantifies an alternative's proximity or distance from a reference value (e.g., an ideal alternative or an anti-ideal alternative) based on multiple evaluation parameters. Different distance formulas possess distinct properties and assumptions that may impact their suitability and accuracy for various decision-making problems. In this study, we compared three commonly used distance formulas in VIKOR methods: Euclidean distance (ED), Canberra distance (CD), and generalized weighted Mahalanobis distance (GWMD). Below are the summarized advantages and disadvantages of each formula.

Euclidean distance (ED) has limitations, including:

(1) It is sensitive to outliers or extreme values in parameters. If some parameters have much larger or smaller values than others, they may dominate the distance calculation and overshadow the effects of other parameters. Therefore, using ED may require normalization or standardization of parameters to avoid scale effects;

(2) It is linear and symmetric, which means that it does not consider any nonlinearities or asymmetries in the relationship between parameters and rockburst levels. For

example, a small increase in a parameter may have a larger impact on rockburst intensity than a large decrease, or vice versa. Therefore, using ED may not capture the complexity and diversity of rockburst phenomena;

(3) It assumes that all parameters have equal importance and are independent of each other. However, this assumption may not be valid in reality, as some parameters may have more influence or correlation than others on rockbursts. Therefore, using equal weights or ignoring correlations may lead to inaccurate or biased results.

Canberra distance (CD) has limitations, including:

(1) It still assumes that all parameters have equal importance and are independent of each other. Therefore, using CD may still lead to inaccurate or biased results if some parameters have more influence or correlation than others on rockbursts;

(2) It is linear and symmetric, which means that it does not consider any nonlinearities or asymmetries in the relationship between parameters and rockburst levels. Therefore, using CD may not capture the complexity and diversity of rockburst phenomena.

We chose the generalized weighted Mahalanobis distance over other distance formulas for improving the VIKOR method for several reasons. Firstly, it considers both the parameter importance and correlation in the distance calculation. The weights reflect the relative importance of each parameter, while the covariance matrix reflects the variance and covariance of each parameter. Therefore, using GWMD can better reflect the similarity or difference between alternatives and reference values. Secondly, it can deal with some nonlinearities, fuzziness, uncertainties, inconsistencies, outliers, or extreme values in parameters, as it uses a quadratic form that can capture the variance and covariance of parameters. Thirdly, it has some desirable mathematical properties that make it suitable for multi-attribute decision-making problems, including:

(1) It is invariant to linear transformations of parameters, such as scaling or shifting. This means that it does not depend on the units or ranges of parameters and does not require normalization or standardization of parameters to avoid scale effects;

(2) It is a metric that satisfies the four axioms of distance: non-negativity, identity, symmetry, and triangle inequality. This means that it has a clear geometric interpretation and can be used to measure the actual distance between alternatives and reference values.

In this paper, as can be seen from Tables 5 and 6 and Figures 5 and 7, the accuracy of using VIKOR-GWMD is higher than that of VIKOR-ED and VIKOR-CD, which may be due to the fact that the martensitic distance overcomes the correlation between the indicators and makes them independent of each other, and the results are more reasonable. The precision of the results using the VIKOR-CD method is the same as that using the VIKOR-ED method, but the discrepancy between the evaluation results using the VIKOR-CD method and the actual outcomes is more considerable than that using the VIKOR-ED in Sample 52. Consequently, the VIKOR improved by Euclidean distance proves to be somewhat more accurate than that with Canberra distance improvement to some extent.

Our study has some implications and limitations for rockburst evaluation and prevention. On one hand, it provides a simple, reliable, and effective tool and reference basis for rockburst evaluation, which can help engineers and decision makers to assess the rockburst risk and take appropriate measures to reduce the damage and loss caused by rockburst risk. On the other hand, it also has some limitations, such as the parameter selection, the data source, the model validation, etc., which need to be further improved and refined in future research.

5. Conclusions

In this work, a novel method for rockburst intensity classification evaluation based on the VIKOR method improved by the game theory and the generalized weighted Mahalanobis distance is proposed. The main work and conclusions of this paper are as follows:

(1) The combined weights of the evaluation indicators were obtained by calculating the combined weights of the hierarchical analysis method subjective assignment method and CRITIC objective assignment method through the game theory combined assignment method;
(2) The generalized weighted Mahalanobis distance was introduced to improve the VIKOR algorithm, which overcame some limitations of other distance formulas or methods, such as inconsistent indicator dimensions, variations in indicator importance, correlations between indicators;
(3) The method was applied to 60 examples of rock blasting projects worldwide, and the accuracy of the VIKOR method improved by the distance formula was 88.3–91.77%, which was higher than the accuracy of the unimproved VIKOR method of 51.3%. Some samples are not consistent with expectations, but the security levels are all improved. This indicates that it is feasible to improve the distance formula for the traditional VIKOR method;
(4) The comparison of the method proposed in this paper with the commonly used Euclidean and Canberra distances shows that the accuracy of the improved VIKOR method using the Mahalanobis distances is 91.7%, which is higher than that of the 88.33% using the Euclidean and Canberra distances distance improvement. This indicates that the accuracy of VIKOR method improved using Ma distances is higher than that of VIKOR method improved using Euclidean and Canberra distances;
(5) The method proposed in this work is an effective tool for rockburst evaluation. It assists operating system practitioners in detecting and preventing potential rockburst hazards. Moreover, it ensures the safety of the practitioners and reduces the losses by taking appropriate precautions in advance.

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List of Symbols

σ_θ	the maximum tangential stress of rock mass
σ_c	the uniaxial compressive strength of rock mass
σ_t	the tensile strength of rock mass
σ_θ/σ_c	the stress coefficient
σ_c/σ_t	the rock brittleness coefficient
W_{et}	the elastic energy index
AHP	analytic hierarchy process
CRITIC	criteria importance though intercriteria correlation
GM	Game Theory
VIKOR	VlseKriterijumska Optimizacijia IKompromisno Resenje
ED	Euclidean distance
CD	Canberra distance
GWMD	generalized weighted Mahalanobis distance

Appendix A

Table A1. Evaluation samples of rockburst intensity classification.

Samples	σ_θ	σ_c	σ_t	σ_θ/σ_c	σ_c/σ_t	W_{et}	Actual Grade
1	75.00	180.00	8.30	0.42	21.69	5.00	III
2	89.00	236.00	8.30	0.38	28.43	5.00	III
3	98.60	120.00	6.50	0.82	18.46	3.80	III
4	18.32	96.41	2.01	0.19	47.93	1.87	I
5	110.30	167.19	12.67	0.66	13.20	6.83	IV
6	32.40	140.88	11.96	0.23	11.78	2.86	II
7	34.89	151.70	10.66	0.23	14.23	3.17	II
8	9.74	88.51	2.98	0.11	29.70	1.77	I
9	46.22	140.07	2.01	0.33	69.69	3.29	II
10	30.95	123.79	12.67	0.25	9.77	2.57	II
11	7.28	52.00	3.70	0.14	14.05	1.30	I
12	60.00	86.03	7.14	0.70	12.05	2.85	II
13	60.00	136.79	10.42	0.44	13.13	2.12	II
14	2.60	20.00	3.00	0.13	6.67	1.39	I
15	70.40	110.00	4.50	0.64	24.40	6.31	III
16	3.80	20.00	3.00	0.19	6.67	1.39	I
17	4.60	20.00	3.00	0.23	6.67	1.39	I
18	73.20	120.00	5.00	0.61	24.00	5.10	III
19	46.40	100.00	4.90	0.46	20.40	2.00	II
20	46.20	105.00	5.30	0.44	19.70	2.30	II
21	35.00	133.40	9.30	0.26	14.34	2.90	II
22	29.80	132.20	7.80	0.23	16.95	4.60	I
23	109.90	128.50	9.63	0.86	13.34	8.10	IV
24	59.90	96.50	8.00	0.62	12.06	1.80	II
25	50.60	63.83	5.06	0.79	12.61	2.23	II
26	120.80	151.60	10.10	0.80	15.01	20.00	IV
27	119.32	138.60	7.74	0.86	17.91	30.00	IV
28	95.67	127.37	10.51	0.75	12.12	30.00	IV
29	114.44	174.71	14.42	0.66	12.12	10.00	IV
30	127.60	145.42	13.70	0.88	10.61	10.00	IV
31	126.41	158.03	14.32	0.80	11.04	10.00	IV
32	108.53	113.37	10.43	0.96	10.87	10.00	IV
33	47.50	86.30	15.60	0.55	5.53	6.30	III
34	77.00	86.30	15.60	0.89	5.53	6.30	IV
35	77.00	91.30	14.50	0.84	6.30	21.00	IV
36	67.18	132.20	16.40	0.51	8.06	3.97	III
37	80.04	171.30	22.60	0.47	7.58	7.27	IV
38	72.56	304.20	20.90	0.24	14.56	10.57	IV
39	52.00	117.00	4.80	0.44	24.38	3.20	II
40	42.00	117.00	4.80	0.36	24.38	3.20	II
41	57.97	96.16	3.77	0.46	16.20	2.53	II
42	57.97	70.68	4.19	0.60	25.51	2.87	II
43	98.02	148.52	6.66	0.66	22.30	3.23	III
44	43.21	116.78	3.93	0.37	29.73	3.52	II
45	45.92	109.33	3.34	0.42	32.77	2.97	II
46	38.12	100.32	3.49	0.38	28.77	3.02	II
47	102.38	142.20	5.17	0.72	27.52	4.30	III
48	110.62	160.32	9.69	0.69	16.55	5.72	IV
49	40.99	97.60	6.30	0.42	15.50	3.20	II
50	81.75	125.77	12.14	0.65	10.36	5.75	III
51	90.99	146.75	7.58	0.62	19.35	4.50	III
52	30.90	238.00	7.60	0.13	31.20	7.40	I
53	75.50	151.00	18.20	0.50	8.30	3.10	II
54	75.60	194.00	8.90	0.39	21.70	5.00	III
55	57.90	181.00	7.50	0.32	24.10	9.30	III
56	72.60	173.00	8.00	0.42	21.70	5.20	III

Table A1. *Cont.*

Samples	σ_θ	σ_c	σ_t	σ_θ/σ_c	σ_c/σ_t	W_{et}	Actual Grade
57	54.90	183.00	9.00	0.30	20.40	5.10	II
58	62.70	196.00	9.00	0.32	21.70	5.00	III
59	61.60	162.00	9.20	0.38	17.60	9.00	III
60	132.40	172.00	9.80	0.77	17.50	5.50	IV

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