mathematics

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Estimation and Control of Positive Complex Networks Using Linear Programming

Yan Zhang, Yuanyuan Wu *, Yishuang Sun and Pei Zhang

School of Information and Communication Engineering, Hainan University, Haikou 570228, China
* Correspondence: 995042@hainanu.edu.cn

Abstract: This paper focuses on event-triggered state estimation and control of positive complex networks. An event-triggered condition is provided for discrete-time complex networks by which an event-based state estimator and an estimator-based controller are designed through matrix decomposition technology. Thus, the system is converted to an interval uncertain system. The positivity and the L_1-gain stability of complex networks are ensured by resorting to a co-positive Lyapunov function. All conditions are solvable in terms of linear programming. Finally, the effectiveness of the proposed state estimator and controller are verified by a numerical example. The main contributions of this paper are as follows: (i) A positive complex network framework is constructed based on an event-triggered strategy, (ii) a new state estimator and an estimator-based controller are proposed, and (iii) a simple analysis and design approach consisting of a co-positive Lyapunov function and linear programming is presented for positive complex networks.

Keywords: positive complex networks; state estimation; co-positive Lyapunov function; linear programming

MSC: 90B15

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Citation: Zhang, Y.; Wu, Y.; Sun, Y.; Zhang, P. Estimation and Control of Positive Complex Networks Using Linear Programming. *Mathematics* **2024**, *12*, 2971. https://doi.org/10.3390/math12192971

Academic Editor: Sergio Gómez

Received: 27 July 2024
Revised: 11 September 2024
Accepted: 15 September 2024
Published: 25 September 2024

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1. Introduction

A complex network is an interconnected structure consisting of numerous nodes and connections, characterized by intricate relationships and interactions. With the continuous advancement of modern technology, the research on complex networks has received considerable attention due to their pervasive applications in biology [1], neural networks [2], physiological processes [3], and social sciences [4].

As expected, the work [5] was concerned with the synchronization problem of coupled complex-valued neural networks, thereby ensuring the stability of the state estimation system. The work [6] achieved fixed synchronization of complex networks by extensively borrowing network interconnections and interactions between nodes. On the basis of the pulse control theory, the work [7] provided sufficient conditions for the exponential stability of impulse perturbations. With the deepening of research, it has been gradually recognized that in some specific contexts, some networks present a special property-positivity, where all variables in the network remain in a non-negative state. This positive condition not only simplifies the network analysis, but also provides a new perspective for the study of positive systems. Positive systems are a class of complex dynamic systems whose state values and outputs are non-negative when the initial conditions of the system are non-negative [8,9]. In recent years, positive systems have gained significant traction in diverse fields such as multi-agent systems, biology, network communication, environmental science, and many others. For more examples of positive systems and complex networks, the reader can refer to [10,11]. Currently, few results are reported for positive complex networks, for the following reasons: (i) Determining how to deal with the positivity of complex network systems is a difficult task, (ii) determining how to design state observers

of positive complex networks is a considerable challenge, and (iii) determining how to design the synchronization control gain needs to introduce a novel approach.

Unsurprisingly, notable advancements have been achieved in the analysis of dynamic processes in complex networks [12]. Alongside the extensive research on synchronization control in complex networks, the equally important problem of state estimation should be emphasized due to its potential applications in understanding the internal dynamics of complex networks [13,14]. The work [15] introduced a new observer based on unknown disturbance models to reduce network complexity. In work [16], a set of event-based state estimators was constructed to reduce unnecessary data transmission in communication. While numerous studies have focused on state estimators and controllers in complex networks, a significant portion of them rely on quadratic forms to optimize expected values, leaving a gap in utilizing linear programming methods. Therefore, exploring research avenues that employ linear programming techniques can provide valuable insights and contribute to the advancement of this field, where a broad overview of the topic can be found in [17,18] and the references therein. With the rapid advancement of network communication technologies, numerous transmission strategies have emerged to effectively utilize limited network resources and prevent unnecessary waste. Another possible factor to consider is the limitation of network resources, which results in failures or interruptions in continuous communication between the sender and receiver in networks. As research progressed, it was discovered that time-triggering methods [19,20] may not respond immediately to events, potentially missing crucial state changes within fixed time intervals. Consequently, some new event-triggered mechanisms [21,22] were proposed and widely embraced in contemporary research. For general systems, the event-triggered conditions are usually presented in linear matrix inequalities and the quadratic Lyapunov function is commonly used. Since the states of positive systems are non-negative, it is unnecessary to apply the quadratic approach directly to positive systems. It had been shown that a linear approach is more powerful for handling the issues of positive systems [23–25]. Therefore, it is of significance to employ linear approach to describe event-triggered conditions of positive complex network systems.

With the motivations as described above, in this paper, the event-triggered problem of discrete-time complex networks using linear programming methods is investigated. Firstly, the research delves into the problem of event-triggered state estimation for discrete-time complex networks. Subsequently, by utilizing matrix decomposition technology, an event-triggered state estimator and controller are designed to address the problem at hand. Finally, the Lyapunov stability theory is employed to guarantee the positivity and stability of the system. The rest of the paper is organized as follows: Section 2 introduces a discrete-time complex network model, Section 3 provides the main results, simulation examples are included in Section 4, and Section 5 concludes the paper.

2. Preliminaries

We will model the observation and control of discrete-time complex networks. To improve readability, the notations used in this paper are standardized, as shown in Table 1.

Consider the dynamical networks with N coupled nodes in the following form:

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + \sum_{j=1}^{N} \varpi_{ij} \mathcal{T} x_j(k) + E_i \omega_i(k) + B_i u_i(k), \\ y_i(k) &= C_i x_i(k), \end{aligned} \tag{1}$$

where for the ith node, $x_i(k) \in \mathbb{R}^n$, $u_i(k) \in \mathbb{R}^n$ and $y_i(k) \in \mathbb{R}^m$ are the state vector, control input, and measurement output, respectively. $\omega_i(k) \in \mathbb{R}^n$ is the external disturbance, and the ℓ_1-norm of the vector $\omega(k)$ is $\|\omega(k)\|_{\ell_1} = \sum_{k=1}^{\infty} \|\omega(k)\|_1$. The notation $\mathcal{T} = diag\{t_1, t_2, \cdots, t_n\}$ is the inner coupling matrix between two connected nodes for all $1 \leq i$ and $j \leq N$. The notation $W = [\varpi_{ij}]_{N\times N}$ is the coupled configuration matrix representing the coupling structure of the dynamical network. If there is a connection between node i and j for $i \neq j$, then $\varpi_{ij} \succ 0$, otherwise $\varpi_{ij} = 0$. The notations $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times n}$, $C_i \in \mathbb{R}^{m\times n}$

and $E_i \in \mathbb{R}^{n\times n}$ are constant matrices with appropriate dimensions. The notation M_i represents the gain of the controller to be designed.

Table 1. Notations.

Notations	Expression
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{n\times m}$	$n \times m$ real matrices
a_{ij}	The element in the ith row and jth column of matrix A
$A \succeq 0 (\succ 0)$	All elements in A are non-negative (or positive)
$A^\top$	The transposition of matrix A
$diag_N\{A_i\}$	The block-diagonal matrix consisting of $A_1, A_2, \ldots A_N$
$\mathbf{1}_{m\times m} \in \mathbb{R}^{m\times m}$	An $m \times m$-dimensional matrix with all elements being 1
$\mathbf{1}_n$	An n-dimensional matrix with all elements being 1
I_N	The $N \times N$ identity matrix
$\mathbf{1}_n^{(\iota)}$	$\mathbf{1}_n^{(\iota)} = (\underbrace{0,\ldots,0}_{\iota-1}), 1, (\underbrace{0,\ldots,0}_{n-\iota})^\top$
$\otimes$	The Kronecker product

Lemma 1 ([8,26]). *System (1) is considered positive if and only if the matrices $A_i \succeq 0$, $B_i \succeq 0$, $C_i \succeq 0$, and $E_i \succeq 0$. Based on the assumptions made for systems (1), it can be concluded that the system is positive.*

Lemma 2 ([8,26]). *The equivalence between the following conditions holds for a matrix $A \succeq 0$: (i) The matrix A is a Schur matrix. (ii) There exists some vector $v \succ 0$ such that $(A - I)v \prec 0$.*

Definition 1 ([8,26]). *The system (1) is said to be ℓ_1-gain stable if the following statements hold: (i) When $\omega(k) = 0$, the system (1) is asymptotically stable. (ii) Under zero-initial conditions, the following inequality holds for $\omega(k) \neq 0$,*

$$\sum_{k=1}^{\infty} \|e(k)\|_1 \leq \sum_{k=1}^{\infty} \gamma \|\omega(k)\|_1,$$

where $\gamma > 0$ is the ℓ_1-gain value.

3. Main Results

This section will design the event-triggered observer and observer-based controller for positive complex networks.

3.1. Event-Triggered Observer

Construct the event-based state estimator for the ith node without input as

$$\widehat{x}_i(k+1) = A_i\widehat{x}_i(k) + \sum_{j=1}^{N} \varpi_{ij}\mathcal{T}\widehat{x}_j(k) - L_iC_i\widehat{x}_i(k) + K_i[y_i(k_t^i) - C_i\widehat{x}_i(k)], \tag{2}$$

where K_i and L_i denote the gains of the state estimator to be designed.

Remark 1. *It is clear that the state estimator gain $K_i(k)$ given in (2) is designed based on the event-triggered mechanism. However, in general complex network systems the state estimators are designed in the non-event-triggered form [27,28]. In addition, an additional state estimator gain $L_i(k)$ is introduced in (2) to ensure the positivity of the system. It implies that existing estimation design cannot be applied for positive complex networks. This is also one of novelties of (2).*

Let $\varrho_i(k) = y_i(k_t^i) - y_i(k)$ be defined as the difference between the ith sensor's measurements at the latest triggering instant and the current sampling instant. Then the estimator (2) can be rewritten as

$$\widehat{x}_i(k+1) = A_i\widehat{x}_i(k) + \sum_{j=1}^{N} \varpi_{ij}\mathcal{T}\widehat{x}_j(k) + K_i[C_ix_i(k) - C_i\widehat{x}_i(k) + \varrho_i(k)] - L_iC_i\widehat{x}_i(k). \tag{3}$$

By denoting $e_i(k) = x_i(k) - \widehat{x}_i(k)$, the estimator (3) and the estimation error dynamics without input can be rewritten as

$$\begin{array}{ll} \widehat{x}_i(k+1) & = (A_i - L_iC_i)\widehat{x}_i(k) + \sum_{j=1}^{N} \varpi_{ij}\mathcal{T}\widehat{x}_j(k) + K_iC_ie_i(k) + K_i\varrho_i(k), \\ e_i(k+1) & = (A_i - K_iC_i)e_i(k) + \sum_{j=1}^{N} \varpi_{ij}\mathcal{T}e_j(k) + E_i\omega_i(k) - K_i\varrho_i(k) + L_iC_i\widehat{x}_i(k). \end{array}$$

For convenience, denote

$$\begin{array}{c} A = diag_N\{A_i\}, C = diag_N\{C_i\}, E = diag_N\{E_i\}, K_N = diag_N\{K_i\}, L = diag_N\{L_i\}, \\ e(k) = \{e_1(k), e_2(k), \ldots, e_N(k)\}^\top, \omega(k) = \{\omega_1(k), \omega_2(k), \ldots, \omega_N(k)\}^\top, \\ \widehat{x}(k) = \{\widehat{x}_1(k), \widehat{x}_2(k), \ldots, \widehat{x}_N(k)\}^\top, \varrho(k) = \{\varrho_1(k), \varrho_2(k), \ldots, \varrho_N(k)\}^\top. \end{array}$$

By selecting $\widetilde{x}(k+1) = (\widehat{x}(k+1)^\top, e(k+1)^\top)^\top$ and applying the Kronecker product, the system without input can be expressed in the following form:

$$\widetilde{x}(k+1) = \begin{pmatrix} \hbar - LC & KC \\ LC & \hbar - KC \end{pmatrix} \begin{pmatrix} \widehat{x}(k) \\ e(k) \end{pmatrix} + \begin{pmatrix} K \\ -K \end{pmatrix} \varrho(k) + \begin{pmatrix} 0 \\ E \end{pmatrix} \omega(k), \tag{4}$$

where $\hbar = A + W \bigotimes \mathcal{T}$.

Theorem 1. *If there exist constants* $0 < \lambda_1 < \lambda_2 < 1$, $0 < \underline{\mu} < \mu < 1$, $\beta > 0$, $\varpi_{ii} < 0$, $\mathcal{R}^n$ *vectors* $v_{2i} \succ v_{1i} \succ 0$, $\delta_i^{(\iota)} \succ 0$, $\zeta_i^{(\iota)} \succ 0$, $\underline{\delta_i} \succ 0$, $\overline{\delta_i} \succ 0$, $\zeta_i \succ 0$ *such that*

$$\mathbf{1}_n^\top v_{2i}A_i - \sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\zeta_i^{(\iota)}}^\top C_i + \mathbf{1}_n^\top v_{2i}\varpi_{ii}\mathcal{T} - \sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top \beta \mathbf{1}_{m\times m}C_i \succeq 0, \tag{5}$$

$$\sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top C_i - \sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top \beta \mathbf{1}_{m\times m}C_i \succeq 0, \tag{6}$$

$$\sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\zeta_i^{(\iota)}}^\top C_i - \sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top \beta \mathbf{1}_{m\times m}C_i \succeq 0, \tag{7}$$

$$\mathbf{1}_n^\top v_{2i}A_i - \sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top C_i + \mathbf{1}_n^\top v_{2i}\varpi_{ii}\mathcal{T} - \sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top \beta \mathbf{1}_{m\times m}C_i \succeq 0, \tag{8}$$

$$A^\top v_1 + (1-\lambda_1)C^\top \zeta + (W^\top \bigotimes \mathcal{T}^\top)v_1 + (1+\lambda_2)C^\top (I_N \bigotimes \beta \mathbf{1}_{m\times m})\overline{\delta} - \mu v_1 \prec 0, \tag{9}$$

$$\begin{array}{c} A^\top v_2 - (1-\lambda_2)C^\top \underline{\delta} + (W^\top \bigotimes \mathcal{T}^\top)v_2 \\ +(1+\lambda_2)C^\top (I_N \bigotimes \beta \mathbf{1}_{m\times m})\overline{\delta} + I_N \bigotimes \mathbf{1}_n - \mu v_2 \prec 0, \end{array} \tag{10}$$

$$E^\top v_2 - I_N \bigotimes \gamma \mathbf{1}_m \prec 0, \tag{11}$$

$$\lambda_1 v_2 \preceq v_1 \preceq \lambda_2 v_2, \tag{12}$$

$$\underline{\delta_i} \preceq \delta_i^{(\iota)} \preceq \overline{\delta_i}, \zeta_i^{(\iota)} \preceq \zeta_i, \tag{13}$$

hold for $\iota = 1, 2, \cdots, N$, *then under the event-based state estimator (2) with gain matrices satisfying*

$$K_i = \frac{\sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}, L_i = \frac{\sum_{\iota=1}^{n} \mathbf{1}_n^{(\iota)} {\zeta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}, \tag{14}$$

the resulting closed-loop system (4) is positive and stable.

Remark 2. *From the studies [15,16,27,28], it can be seen that the state estimator gain is designed based on linear matrix inequalities and the corresponding variables are matrices. However, in this paper, a new gain matrix design is introduced in (14) using a matrix decomposition approach and its gain variables are vectors. This matrix decomposition approach has been applied to the design of observers and controllers of positive systems [10,11,23,24]. Compared with the linear matrix inequality method, the linear programming method is more convenient and more suitable for solving positively complex networks. It should be noticed that the presented design approach can also be*

extended to other designs of positive complex networks such as compensators, synchronization controllers, and filters.

Proof. Using the event-triggering condition, it follows that

$$\|\varrho(k)\|_1 \leq \beta\|y(k)\|_1, \tag{15}$$

where $x_i(k) \succeq 0$ implies $y_i(k) \succeq 0$ through the equation $y_i(k) = C_i x_i(k)$. Then, it holds that

$$-\beta\mathbf{1}_{m\times m}y_i(k) \preceq \varrho_i(k) \preceq \beta\mathbf{1}_{m\times m}y_i(k). \tag{16}$$

From (4), the matrix form can be readily derived that

$$\widetilde{x}(k+1) \succeq \begin{pmatrix} \hbar - LC - \Lambda & KC - \Lambda \\ LC - \Lambda & \hbar - KC - \Lambda \end{pmatrix}\begin{pmatrix} \widehat{x}(k) \\ e(k) \end{pmatrix} + \begin{pmatrix} 0 \\ E \end{pmatrix}\omega(k), \tag{17}$$

where

$$\hbar - LC - \Lambda = \begin{pmatrix} \Theta_1 & \varpi_{12}\mathcal{T} & \cdots & \varpi_{1N}\mathcal{T} \\ \varpi_{21}\mathcal{T} & \Theta_2 & \cdots & \varpi_{2N}\mathcal{T} \\ \vdots & \vdots & \ddots & \vdots \\ \varpi_{N1}\mathcal{T} & \varpi_{N2}\mathcal{T} & \cdots & \Theta_N \end{pmatrix}, KC - \Lambda = diag_N\{\aleph_i\},$$
$$\hbar - KC - \Lambda = \begin{pmatrix} P_1 & \varpi_{12}\mathcal{T} & \cdots & \varpi_{1N}\mathcal{T} \\ \varpi_{21}\mathcal{T} & P_2 & \cdots & \varpi_{2N}\mathcal{T} \\ \vdots & \vdots & \ddots & \vdots \\ \varpi_{N1}\mathcal{T} & \varpi_{N2}\mathcal{T} & \cdots & P_N \end{pmatrix}, LC - \Lambda = diag_N\{\Upsilon_i\},$$

and $\Lambda = K(I_N \otimes \beta\mathbf{1}_{m\times m})C$, $\Theta_i = A_i - L_iC_i + \varpi_{ii}\mathcal{T} - K_i\beta\mathbf{1}_{m\times m}C_i$, $\aleph_i = K_iC_i - K_i\beta\mathbf{1}_{m\times m}C_i$, $\Upsilon_i = L_iC_i - K_i\beta\mathbf{1}_{m\times m}C_i$, $P_i = A_i - K_iC_i + \varpi_{ii}\mathcal{T} - K_i\beta\mathbf{1}_{m\times m}C_i$.

From (14), it follows that

$$\begin{aligned}
\Theta_i &= A_i - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\zeta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}C_i + \varpi_{ii}\mathcal{T} - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}\beta\mathbf{1}_{m\times m}C_i, \\
\aleph_i &= \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}C_i - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}\beta\mathbf{1}_{m\times m}C_i, \\
\Upsilon_i &= \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\zeta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}C_i - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}\beta\mathbf{1}_{m\times m}C_i, \\
P_i &= A_i - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}C_i + \varpi_{ii}\mathcal{T} - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}\beta\mathbf{1}_{m\times m}C_i.
\end{aligned}$$

With the fact that $\varpi_{1i}\mathcal{T} \geq 0$ and utilize Equations (5)–(8), it follows that $\hbar - LC - \Lambda \succeq 0$, $KC - \Lambda \succeq 0$, $LC - \Lambda \succeq 0$ and $\hbar - KC - \Lambda \succeq 0$. Thus, it can be inferred that system (4) is positive.

Next, the stability of system (4) is addressed. From (4) and (16), the system can be rewritten as

$$\widetilde{x}(k+1) \preceq \begin{pmatrix} Z + \Lambda & KC + \Lambda \\ LC + \Lambda & \Xi + \Lambda \end{pmatrix}\begin{pmatrix} \widehat{x}(k) \\ e(k) \end{pmatrix} + \begin{pmatrix} 0 \\ E \end{pmatrix}\omega(k), \tag{18}$$

where $Z = A - LC + W \otimes \mathcal{T}$, $\Xi = A - KC + W \otimes \mathcal{T}$.

Consider the choice of a positive Lyapunov function $V(x(k)) = \tilde{x}^\top(k)v$, where $v = (v_1^\top, v_2^\top)^\top$, $\lambda_1 v_2 \leq v_1 \leq \lambda_2 v_2$, $v_1 = (v_{11}^\top, v_{12}^\top, \cdots v_{1N}^\top)^\top$, $v_2 = (v_{21}^\top, v_{22}^\top, \cdots v_{2N}^\top)^\top$ and $v_{1i} \in \mathbb{R}^n$, $v_{2i} \in \mathbb{R}^n$. Considering the case $\omega(k) = 0$, it is not hard to derive that

$$\begin{aligned} \triangle V(k) &= V(x(k+1)) - V(x(k)) \\ &\leq \begin{pmatrix} \widehat{x}^\top(k) & e^\top(k) \end{pmatrix} \begin{pmatrix} \begin{pmatrix} A^\top v_1 + C^\top L^\top (v_2 - v_1) + (W^\top \otimes \mathcal{T}^\top) v_1 \\ +C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) K^\top (v_1 + v_2) - v_1 \end{pmatrix} \\ \begin{pmatrix} A^\top v_2 + C^\top K^\top (v_1 - v_2) + (W^\top \otimes \mathcal{T}^\top) v_2 \\ +C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) K^\top (v_1 + v_2) - v_2 \end{pmatrix} \end{pmatrix}. \end{aligned} \tag{19}$$

From (13) and (14), it holds that $\frac{\mathbf{1}_n \underline{\delta}_i^\top}{\mathbf{1}_n^\top v_{2i}} \preceq K_i \preceq \frac{\mathbf{1}_n \overline{\delta}_i^\top}{\mathbf{1}_n^\top v_{2i}}$, $L_i \preceq \frac{\mathbf{1}_n \zeta_i^\top}{\mathbf{1}_n^\top v_{2i}}$. From (19), it follows that

$$\triangle V(k) + (1-\mu)V(k) \leq \begin{pmatrix} \widehat{x}^\top(k) & e^\top(k) \end{pmatrix} \begin{pmatrix} \begin{pmatrix} A^\top v_1 + (1-\lambda_1) C^\top \zeta + (W^\top \otimes \mathcal{T}^\top) v_1 \\ +(1+\lambda_2) C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) \overline{\delta} - \mu v_1 \end{pmatrix} \\ \begin{pmatrix} A^\top v_2 - (1-\lambda_2) C^\top \underline{\delta} + (W^\top \otimes \mathcal{T}^\top) v_2 \\ +(1+\lambda_2) C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) \overline{\delta} - \mu v_2 \end{pmatrix} \end{pmatrix}. \tag{20}$$

With the facts that $\widehat{x}(k) \succeq 0$, $e(k) \succeq 0$ in mind, using (9) and (10) leads to $\triangle V(K) + (1-\mu)V(K) \leq 0$. Consequently, it can be obtained that $V(k+1) \leq \mu V(k)$. As a result, the system (4) is exponentially stable.

Considering the case $\omega(k) \neq 0$, the equation (19) can be rewritten as

$$\triangle V(k) \leq \begin{pmatrix} \widehat{x}^\top(k) & e^\top(k) \end{pmatrix} \begin{pmatrix} \begin{pmatrix} A^\top v_1 + C^\top L^\top (v_2 - v_1) + (W^\top \otimes \mathcal{T}^\top) v_1 \\ +C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) K^\top (v_1 + v_2) - v_1 \end{pmatrix} \\ \begin{pmatrix} A^\top v_2 + C^\top K^\top (v_1 - v_2) + (W^\top \otimes \mathcal{T}^\top) v_2 \\ +C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) K^\top (v_1 + v_2) - v_2 \end{pmatrix} \end{pmatrix} + \omega^\top(k) E^\top v_2. \tag{21}$$

Define $\psi(k) = \gamma \|\omega(k)\|_1 - \|e(k)\|_1$. Then the inequality holds that

$$\begin{aligned} \triangle V(k) + (1-\mu)V(k) \leq \begin{pmatrix} \widehat{x}^\top(k) & e^\top(k) \end{pmatrix} & \begin{pmatrix} \begin{pmatrix} A^\top v_1 + (1-\lambda_1) C^\top \zeta + (W^\top \otimes \mathcal{T}^\top) v_1 \\ +(1+\lambda_2) C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) \overline{\delta} - \mu v_1 \end{pmatrix} \\ \begin{pmatrix} A^\top v_2 - (1-\lambda_2) C^\top \underline{\delta} + (W^\top \otimes \mathcal{T}^\top) v_2 \\ +(1+\lambda_2) C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) \overline{\delta} - \mu v_2 \end{pmatrix} \end{pmatrix} \\ & + \omega^\top(k)(E^\top v_2 - I_N \otimes \gamma \mathbf{1}_m). \end{aligned} \tag{22}$$

Applying (9)–(11), it yields that $\triangle V(k) + (1-\mu)V(k) + \psi(k) \leq 0$. Therefore, it can be guaranteed that $V(K+1) \leq \mu V(K) + \psi(K)$. Then, it yields that

$$\begin{aligned} V(k+1) - V(0) \leq (\mu - 1)(V(k) + V(k-1) + \ldots + V(0)) \\ + \textstyle\sum_{k=1}^{\infty} \gamma \|\omega(k)\|_1 - \sum_{k=1}^{\infty} \|e(k)\|_1. \end{aligned} \tag{23}$$

With the fact that $V(k+1) \geq 0$, $V(0) = 0$, $\mu - 1 \leq 0$, the inequality (23) can be rewritten as $\sum_{k=1}^{\infty} \|e(k)\|_1 \leq \sum_{k=1}^{\infty} \gamma \|\omega(k)\|_1$. This implies that the system exhibits ℓ_1-gain stability with a performance level of γ. □

Remark 3. *Different from the state estimation of general complex networks [5], the positivity of state observer and the stability of the error dynamics are required for the state estimation of positive complex networks. In Theorem 1, a novel observer is proposed for positive complex networks by virtue of matrix decomposition approach. Due to the property that all states of positive systems are non-negative, quadratic Lyapunov functions and linear matrix inequalities may no longer be the optimal choice [13–16]. A co-positive Lyapunov function $V(x(k)) = \tilde{x}^\top(k)v$ used in Theorem 1 is presented to analyze the stability of the corresponding systems. In contrast to the linear matrix inequalities, the conditions (5)–(11) and (26)–(28) are given in the form of linear programming. It is less difficult to calculate and more convenient to deal with the problems related to large-scale systems. It should be pointed out that the presented state estimation framework in Theorem 1 can be developed for other issues of positive complex networks.*

3.2. Observer-Based Control

Design an observer-based controller: $u_i(k) = M_i\widehat{x}_i(k)$, where M_i is the control gain. Conduct an event-based state estimator for the ith node:

$$\begin{aligned}\widehat{x}_i(k+1) = A_i\widehat{x}_i(k) + \textstyle\sum_{j=1}^{N}\varpi_{ij}\mathcal{T}\widehat{x}_j(k) - L_iC_i\widehat{x}_i(k) + B_iM_i\widehat{x}(k)\\ + K_i[y_i(k_t^i) - C_i\widehat{x}_i(k)].\end{aligned} \tag{24}$$

Then the system with input can be expressed in the following form:

$$\widetilde{x}(k+1) = \begin{pmatrix}\hbar - LC + BM & KC\\ LC & \hbar - KC\end{pmatrix}\begin{pmatrix}\widehat{x}(k)\\ e(k)\end{pmatrix} + \begin{pmatrix}K\\ -K\end{pmatrix}\varrho(k) + \begin{pmatrix}0\\ E\end{pmatrix}\omega(k). \tag{25}$$

Theorem 2. *If there exist constants* $0 < \lambda_1 < \lambda_2 < 1$, $0 < \underline{\mu} < \mu < 1$, $\beta > 0$, $\varpi_{ii} < 0$, $\varphi > 0$, $\mathcal{R}^n$ *vectors* $v_{2i} \succ v_{1i} \succ 0$, $\delta_i^{(\iota)} \succ 0$, $\zeta_i^{(\iota)} \succ 0$, $\underline{\delta_i} \succ 0$, $\overline{\delta_i} \succ 0$, $\underline{\zeta_i} \succ 0$, $\varepsilon_i^{(\iota)} \succ 0$, $\varepsilon_i \succ 0$ *such that*

$$\begin{aligned}&\mathbf{1}_n^\top B_i^\top v_{2i}A_i - \varphi\textstyle\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\zeta_i^{(\iota)}}^\top C_i + B_i\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\varepsilon_i^{(\iota)}}^\top\\ &+\mathbf{1}_n^\top B_i^\top v_{2i}\varpi_{ii}\mathcal{T} - \varphi\textstyle\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top\beta\mathbf{1}_{m\times m}C_i \succeq 0,\end{aligned} \tag{26}$$

$$\begin{aligned}&A^\top v_1 + (1-\lambda_1)C^\top\zeta + (W^\top\otimes\mathcal{T}^\top)v_1\\ &+(1+\lambda_2)C^\top(I_N\otimes\beta\mathbf{1}_{m\times m})\overline{\delta} + \lambda_1\varepsilon - \mu v_1 \prec 0,\end{aligned} \tag{27}$$

$$\begin{aligned}&A^\top v_2 - (1-\lambda_2)C^\top\underline{\delta} + (W^\top\otimes\mathcal{T}^\top)v_2\\ &+(1+\lambda_2)C^\top(I_N\otimes\beta\mathbf{1}_{m\times m})\overline{\delta} + I_N\otimes\mathbf{1}_n - \mu v_2 \prec 0,\end{aligned} \tag{28}$$

$$\mathbf{1}_n^\top B_i^\top v_{2i} \preceq \varphi\mathbf{1}_n^\top v_{2i}, \tag{29}$$

$$\varepsilon_i^{(\iota)} \preceq \varepsilon_i, \tag{30}$$

hold for $\iota = 1, 2, \cdots, N$, *then the event-based state estimator with gain matrix the same as (14) and the event-triggered control law with gain matrix satisfy*

$$M_i = \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\varepsilon_i^{(\iota)}}^\top}{\mathbf{1}_n^\top B^\top v_{2i}}, \tag{31}$$

the resulting closed-loop system (25) is positive and stable.

Proof. From (25), the matrix is straightforward to obtain

$$\widetilde{x}(k+1) \succeq \begin{pmatrix}\Xi - \Lambda & KC - \Lambda\\ LC - \Lambda & \hbar - KC - \Lambda\end{pmatrix}\begin{pmatrix}\widehat{x}(k)\\ e(k)\end{pmatrix} + \begin{pmatrix}0\\ E\end{pmatrix}\omega(k), \tag{32}$$

where $\Xi = A - LC + BM + W\otimes\mathcal{T}$, and Λ, $\hbar$, $KC - \Lambda$, $LC - \Lambda$, $\hbar - KC - \Lambda$ are the same as (17).

$$\Xi - \Lambda = \begin{pmatrix}T_1 & \varpi_{12}\mathcal{T} & \cdots & \varpi_{1N}\mathcal{T}\\ \varpi_{21}\mathcal{T} & T_2 & \cdots & \varpi_{2N}\mathcal{T}\\ \vdots & \vdots & \ddots & \vdots\\ \varpi_{N1}\mathcal{T} & \varpi_{N2}\mathcal{T} & \cdots & T_N\end{pmatrix},$$

with $T_i = A_i - L_iC_i + \varpi_{ii}\mathcal{T} + B_iM_i - K_i\beta\mathbf{1}_{m\times m}C_i$.

From (14) and (31), it follows that $T_i = A_i - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\zeta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}C_i + \varpi_{ii}\mathcal{T} + B_i\frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\varepsilon_i^{(\iota)}}^\top}{\mathbf{1}_n^\top B^\top v_{2i}} - \frac{\sum_{\iota=1}^{n}\mathbf{1}_n^{(\iota)}{\delta_i^{(\iota)}}^\top}{\mathbf{1}_n^\top v_{2i}}\beta\mathbf{1}_{m\times m}C_i$.

With the fact that $\varpi_{1i}\mathcal{T} \geq 0$, by utilizing Equation (26), it follows that $\Xi - \Lambda \succeq 0$ and from (6)–(8), it obtains that $KC - \Lambda \succeq 0$, $LC - \Lambda \succeq 0$ and $\hbar - KC - \Lambda \succeq 0$. Thus, it is not hard to conclude that system (25) is positive.

Next, it comes time to prove the stability of system (25). By using (25) and (16), the system can be rewritten as

$$\widetilde{x}(k+1) \preceq \begin{pmatrix} \Xi+\Lambda & KC+\Lambda \\ LC+\Lambda & \hbar-KC+\Lambda \end{pmatrix} \begin{pmatrix} \widehat{x}(k) \\ e(k) \end{pmatrix} + \begin{pmatrix} 0 \\ E \end{pmatrix} \omega(k), \tag{33}$$

Considering the case $\omega(k) = 0$, it derives that

$$\triangle V(k) \leq \begin{pmatrix} \widehat{x}^\top(k) & e^\top(k) \end{pmatrix} \begin{pmatrix} \Pi_1 \\ \Pi_2 \end{pmatrix}, \tag{34}$$

where $\Pi_1 = A^\top v_1 + C^\top L^\top (v_2 - v_1) + (W^\top \otimes \mathcal{T}^\top) v_1 + C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) K^\top (v_1 + v_2) + M^\top B^\top v_1 - v_1$, and $\Pi_2 = A^\top v_2 + C^\top K^\top (v_1 - v_2) + (W^\top \otimes \mathcal{T}^\top) v_2 + C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) K^\top (v_1 + v_2) - v_2$. In addition, from (30) it drives that $M_i \preceq \frac{\mathbf{1}_n \varepsilon_i^\top}{\mathbf{1}_n^\top B_i^\top v_{2_i}}$. Thus $\Pi_1 \preceq A^\top v_1 + (1 - \lambda_1) C^\top \zeta + (W^\top \otimes \mathcal{T}^\top) v_1 + (1 + \lambda_2) C^\top (I_N \otimes \beta \mathbf{1}_{m\times m}) \overline{\delta} + \lambda_1 \varepsilon - \mu v_1$.

With the facts that $\widehat{x}(k) \succeq 0$, $e(k) \succeq 0$ in mind, it can be inferred that $V(k+1) \leq \mu V(k)$. As a result, the system (25) is exponentially stable.

Considering the case $\omega \neq 0$, and define $\psi(k) = \gamma \|\omega(k)\|_1 - \|e(k)\|_1$. Then the following inequality holds as

$$\begin{aligned} \triangle V(k) + (1-\mu) V(k) - \psi(k) \leq \begin{pmatrix} \widehat{x}^\top(k) & e^\top(k) \end{pmatrix} \begin{pmatrix} \Pi_1 \\ \Pi_2 + I_N \otimes \mathbf{1}_n \end{pmatrix} \\ + \omega^\top(k) (E^\top v_2 - I_N \otimes \gamma \mathbf{1}_m). \end{aligned} \tag{35}$$

With the fact that $V(k+1) \geq 0$, $V(0) = 0$, $\mu - 1 \leq 0$, the inequality (35) can be rewritten as $\sum_{k=1}^{\infty} \|e(k)\|_1 \leq \sum_{k=1}^{\infty} \gamma \|\omega(k)\|_1$. As a result, the system is ℓ_1-gain stable with performance γ. □

Remark 4. *Theorem 1 provides a design scheme for the gain matrices K_i and L_i of the state estimator, while Theorem 2 outlines the design method for the gain matrix M_i of the controller, building upon the foundation established in Theorem 1. It is worth noting that, in many cases, complex networks are commonly addressed using quadratic forms, as evident in references [5–7]. However, such an approach is not practical for positive complex networks. Therefore, it becomes crucial to carefully select appropriate methods for designing the state estimator and controller in order to ensure their effectiveness in positive complex networks.*

4. Numerical Example

In this section, a numerical simulation example is given to demonstrate the validity of the proposed design. Consider system (1), where

$$A_1 = \begin{pmatrix} 0.34 & 0.36 & 0.33 \\ 0.34 & 0.32 & 0.34 \\ 0.32 & 0.34 & 0.33 \end{pmatrix}, A_2 = \begin{pmatrix} 0.33 & 0.36 & 0.32 \\ 0.34 & 0.32 & 0.36 \\ 0.32 & 0.35 & 0.35 \end{pmatrix}, A_3 = \begin{pmatrix} 0.32 & 0.36 & 0.36 \\ 0.34 & 0.33 & 0.34 \\ 0.33 & 0.35 & 0.33 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} 0.02 & 0.01 & 0.03 \\ 0.01 & 0.03 & 0.02 \\ 0.03 & 0.01 & 0.01 \end{pmatrix}, B_2 = \begin{pmatrix} 0.01 & 0.01 & 0.01 \\ 0.03 & 0.03 & 0.02 \\ 0.02 & 0.02 & 0.01 \end{pmatrix}, B_3 = \begin{pmatrix} 0.03 & 0.02 & 0.01 \\ 0.02 & 0.03 & 0.02 \\ 0.01 & 0.02 & 0.03 \end{pmatrix},$$
$$C_1 = \begin{pmatrix} 0.01 & 0.03 & 0.02 \\ 0.02 & 0.03 & 0.01 \end{pmatrix}, C_2 = \begin{pmatrix} 0.02 & 0.02 & 0.01 \\ 0.01 & 0.03 & 0.01 \end{pmatrix}, C_3 = \begin{pmatrix} 0.03 & 0.01 & 0.02 \\ 0.02 & 0.02 & 0.03 \end{pmatrix},$$
$$E_1 = \begin{pmatrix} 0.4 & 0.2 \\ 0.2 & 0.1 \\ 0.3 & 0.3 \end{pmatrix}, E_2 = \begin{pmatrix} 0.3 & 0.3 \\ 0.2 & 0.1 \\ 0.1 & 0.2 \end{pmatrix}, E_3 = \begin{pmatrix} 0.2 & 0.1 \\ 0.2 & 0.2 \\ 0.3 & 0.3 \end{pmatrix},$$
$$W = \begin{pmatrix} -0.02 & 0.01 & 0.01 \\ 0.01 & -0.02 & 0.01 \\ 0.01 & 0.01 & -0.02 \end{pmatrix}, \mathcal{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Given the values $\beta = 0,13$, $\varphi = 0.08$, $\lambda_1 = 0.4$, $\lambda_2 = 0.5$, $\mu = 0.98$, $\gamma = 354.9149$ and the disturbance signal $\omega(k) = (20 \times \frac{1}{(1+k)^{6/7}})^\top$, the event-based state estimator with gain matrices satisfy:

$$K_1 = \begin{pmatrix} 0.8744 & 4.2970 \\ 0.8740 & 4.2967 \\ 0.8743 & 4.2970 \end{pmatrix}, K_2 = \begin{pmatrix} 5.5099 & 0.0669 \\ 5.5098 & 0.0668 \\ 5.5099 & 0.0699 \end{pmatrix}, K_3 = \begin{pmatrix} 0.0767 & 6.1071 \\ 0.0764 & 6.1068 \\ 0.0769 & 6.1073 \end{pmatrix},$$
$$L_1 = \begin{pmatrix} 0.6965 & 0.6970 \\ 0.6882 & 0.6890 \\ 0.6950 & 0.6953 \end{pmatrix}, L_2 = \begin{pmatrix} 0.9133 & 0.6093 \\ 0.9105 & 0.6104 \\ 0.9134 & 0.6093 \end{pmatrix}, L_3 = \begin{pmatrix} 0.7842 & 0.8658 \\ 0.7915 & 0.8502 \\ 0.7947 & 0.8635 \end{pmatrix},$$

and the event-triggered control law with gain matrices:

$$M_1 = \begin{pmatrix} -3.7188 & -3.1974 & -3.5826 \\ -3.8179 & -3.1867 & -3.6875 \\ -3.6253 & -3.1845 & -3.5325 \end{pmatrix}, M_2 = \begin{pmatrix} -3.2985 & -2.5194 & -3.0847 \\ -3.2985 & -2.5194 & -3.0847 \\ -1.5763 & -1.6570 & -1.5947 \end{pmatrix}, M_3 = \begin{pmatrix} -3.2569 & -3.5010 & -3.3832 \\ -6.9703 & -5.3239 & -6.5186 \\ -3.3310 & -3.5015 & -3.3699 \end{pmatrix},$$

Figure 1 illustrates the event-triggering signal. Unlike the time-triggering strategy, the number of the event-triggering strategy is smaller than the number of the time-triggering strategy at the same instant, which reduces unnecessary communication and avoids a waste of resources. Additionally, Figures 2–4 display the initial states of x and the corresponding state estimates. It can be found that the state of the estimator converges with the state of the system node. This shows that the state estimator designed in this paper has a good effect on the system. Figure 5 presents the states of the error system and all three subsystems eventually converge to 0. It reaches the positivity and stability of the systems.

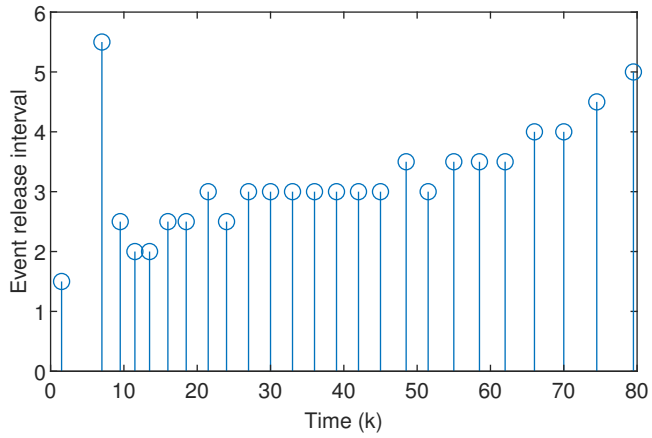


Figure 1. Event-triggering signal.

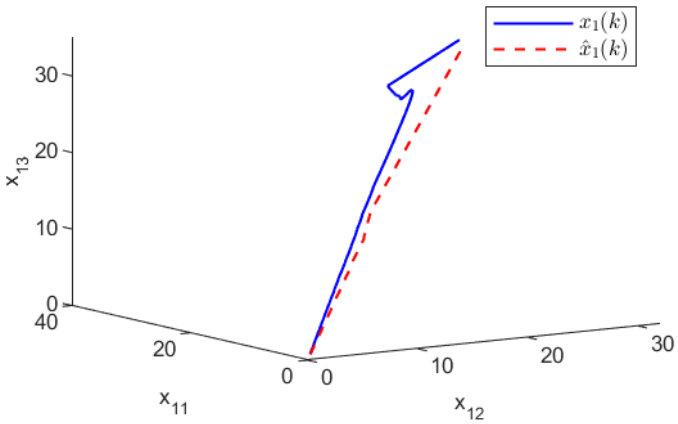


Figure 2. Observation signal and status signal of x_1.

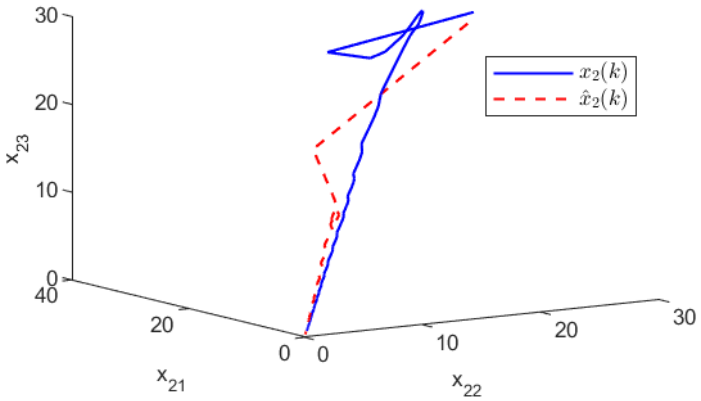


Figure 3. Observation signal and status signal of x_2.

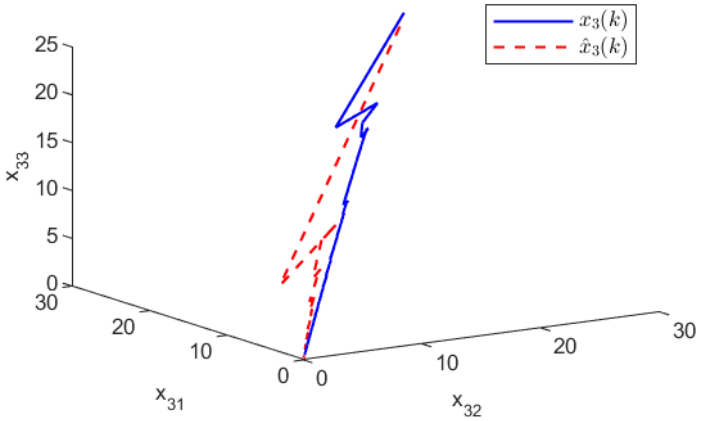


Figure 4. Observation signal and status signal of x_3.

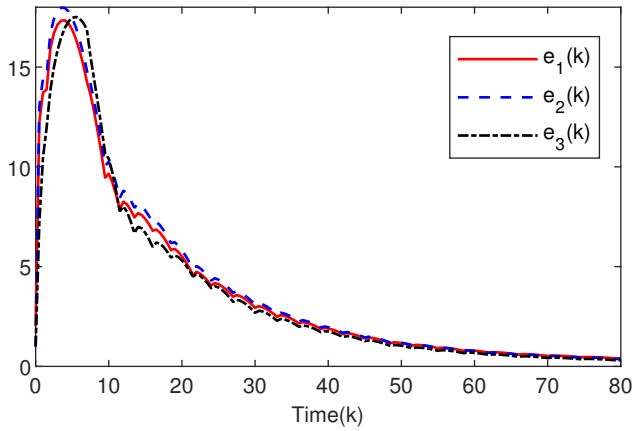


Figure 5. The status of the error system.

5. Conclusions

This paper designs an event-triggered state estimator and an estimator-based controller for discrete-time complex network systems with disturbances. The event-triggered conditions for discrete complex networks are formulated using the 1-norm. Matrix factorization techniques are utilized to design the gain matrices for the state estimator and the estimator-based controller. The positivity and stability of the lower bound and upper bound of the system are rigorously proven through the use of linear positive Lyapunov functions and linear programming, thereby ensuring the positivity and stability of the L_1-gain of the system. Future research could further explore the identification of key nodes

and critical paths in the networks to improve the precision of control. It could also include how nonlinear interactions in the network affect system stability and how these interactions can be exploited to achieve precise guidance of network behavior. Additionally, some possible research can be devoted to understanding how small interventions guide the network from one state to another.

Author Contributions: Conceptualization, Y.W.; methodology, Y.W., Y.Z. and P.Z.; software, Y.S. and P.Z.; validation, Y.Z., P.Z. and Y.W.; formal analysis, Y.W.; investigation, Y.Z., Y.W., P.Z. and Y.S.; resources, Y.W.; data curation, Y.S. and P.Z.; writing—original draft preparation, Y.Z., Y.W., P.Z. and Y.S.; writing—review and editing, Y.W.; visualization, Y.Z., P.Z. and Y.S.; supervision, Y.W.; project administration, Y.W. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by Hainan Provincial Natural Science Foundation of China (622RC618), Science Research Funding of Hainan University (KYQD(ZR)21007) and Postgraduate Innovative Research Funding of Hainan Province (Qhys2023-279, Qhys2023-278 and Qhys2023-280).

Data Availability Statement: No new data were created or analyzed in this study.

Conflicts of Interest: The authors declare no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; or in the decision to publish the results.

References

1. Gupta, O.P.; Deshmukh, R.; Kumar, A.; Singh, S.K.; Sharma, P.; Ram, S.; Singh, G.P. From gene to biomolecular networks: A review of evidences for understanding complex biological function in plants. *Curr. Opin. Biotechnol.* **2022**, *74*, 66–74. [CrossRef] [PubMed]
2. Long, H.; Ci, J.; Guo, Z.; Wen, S.; Huang, T. Synchronization of coupled switched neural networks subject to hybrid stochastic disturbances. *Neural Netw.* **2023**, *166*, 459–470. [CrossRef] [PubMed]
3. Sethi, A.; Melamud, E. Joint inference of physiological network and survival analysis identifies factors associated with aging rate. *Cell Rep. Methods* **2022**, *2*, 100356. [CrossRef] [PubMed]
4. Jia, C.; Zhang, R.; Wang, D. Evolutionary game of cooperative behavior among social capitals in PPP projects: A complex network perspective. *Ain Shams Eng. J.* **2023**, *14*, 102006. [CrossRef]
5. Liu, Y.; Shen, B.; Zhang, P. Synchronization and state estimation for discrete-time coupled delayed complex-valued neural networks with random system parameters. *Neural Netw.* **2022**, *150*, 181–193. [CrossRef]
6. Lu, R.; Yu, W.; Lü, J.; Xue, A. Synchronization on complex networks of networks. *IEEE Trans. Neural Netw. Learn. Syst.* **2014**, *25*, 2110–2118. [CrossRef]
7. Liu, Y.; Wang, Z.; Liang, J.; Liu, X. Synchronization and state estimation for discrete-time complex networks with distributed delays. *IEEE Trans. Syst. Man Cybern. Part B* **2008**, *38*, 1314–1325.
8. Farina, L.; Rinaldi, S. *Positive Linear Systems: Theory and Applications*; John Wiley & Sons: Hoboken, NJ, USA, 2000.
9. Zhao, P.; Kang, Y.; Niu, B.; Zhao, Y. Input-to-state stability and stabilization for switched nonlinear positive systems. *Nonlinear Anal. Hybrid Syst.* **2023**, *47*, 101298. [CrossRef]
10. Zhang, J.; Zheng, G.; Feng, Y.; Chen, Y. Event-triggered state-feedback and dynamic output-feedback control of PMJSs with intermittent faults. *IEEE Trans. Autom. Control* **2022**, *68*, 1039–1046. [CrossRef]
11. Su, H.; Wu, H.; Chen, X.; Chen, M.Z. Positive edge consensus of complex networks. *IEEE Trans. Syst. Man Cybern. Syst.* **2017**, *48*, 2242–2250. [CrossRef]
12. Li, Q.; Wang, Z.; Sheng, W.; Alsaadi, F.E.; Alsaadi, F.E. Dynamic event-triggered mechanism for H_∞ non-fragile state estimation of complex networks under randomly occurring sensor saturations. *Inf. Sci.* **2020**, *509*, 304–316. [CrossRef]
13. Wang, L.; Wang, Z.; Huang, T.; Wei, G. An event-triggered approach to state estimation for a class of complex networks with mixed time delays and nonlinearities. *IEEE Trans. Cybern.* **2015**, *46*, 2497–2508. [CrossRef] [PubMed]
14. Liu, Y.; Wang, Z.; Ma, L.; Alsaadi, F.E. A partial-nodes-based information fusion approach to state estimation for discrete-time delayed stochastic complex networks. *Inf. Fusion* **2019**, *49*, 240–248. [CrossRef]
15. Wu, Z.; Chen, J.; Zhang, X.; Xiao, Z.; Tao, J.; Wang, X. Dynamic event-triggered synchronization of complex networks with switching topologies: Asynchronous observer-based case. *Appl. Math. Comput.* **2022**, *435*, 127413. [CrossRef]
16. Zhao, Y.; Li, X.; Rao, R. Synchronization of nonidentical complex dynamical networks with unknown disturbances via observer-based sliding mode control. *Neurocomputing* **2021**, *454*, 441–447. [CrossRef]
17. Zhou, X.; Zhang, J.; Lin, P.; Shi, X. Event-triggered PI control of positive switched systems. In Proceedings of the 2022 41st Chinese Control Conference (CCC), Hefei, China, 25–27 July 2022; IEEE: Piscataway, NJ, USA, 2022; pp. 1593–1598.
18. Liu, L.J.; Karimi, H.R.; Zhao, X. New approaches to positive observer design for discrete-time positive linear systems. *J. Frankl. Inst.* **2018**, *355*, 4336–4350. [CrossRef]

19. Kirner, R.; Puschner, P. A qualitative cybersecurity analysis of time-triggered communication networks in automotive systems. *J. Syst. Archit.* **2023**, *136*, 102835. [CrossRef]
20. Zhang, X.; Wang, Q.; Fu, B. Further stabilization criteria of continuous systems with aperiodic time-triggered intermittent control. *Commun. Nonlinear Sci. Numer. Simul.* **2023**, *125*, 107387. [CrossRef]
21. Liu, Y.; Shen, B.; Shu, H. Finite-time resilient H_∞ state estimation for discrete-time delayed neural networks under dynamic event-triggered mechanism. *Neural Netw.* **2020**, *121*, 356–365. [CrossRef]
22. Liu, C.; Li, H.; Shi, Y.; Xu, D. Codesign of event trigger and feedback policy in robust model predictive control. *IEEE Trans. Autom. Control* **2019**, *65*, 302–309. [CrossRef]
23. Zhang, J.F.; Liu, L.Y.; Fu, S.Z.; Li, S. Event-triggered control of positive switched systems with actuator saturation and time-delay. *Int. J. Autom. Comput.* **2021**, *18*, 141–154. [CrossRef]
24. Zhang, S.; Zhang, J.; Li, S.; Yu, S. Event-triggered control of positive semi-markovian jump systems without/with input saturation. *J. Shanghai Jiaotong Univ.* **2022**, *27*, 723–736. [CrossRef]
25. Xiao, S.; Zhang, Y.; Zhang, B. Event-triggered networkbased state observer design of positive systems. *Inf. Sci.* **2018**, *469*, 30–43. [CrossRef]
26. Kaczorek, T. *Positive 1D and 2D Systems*; Springer: London, UK, 2001.
27. Shen, B.; Wang, Z.; Liu, X. Bounded H_∞ synchronization and state estimation for discrete time-varying stochastic complex networks over a finite horizon. *IEEE Trans. Neural Netw.* **2010**, *22*, 145–157. [CrossRef]
28. Shen, B.; Wang, Z.; Ding, D.; Shu, H. H_∞ state estimation for complex networks with uncertain inner coupling and incomplete measurements. *IEEE Trans. Neural Netw. Learn. Syst.* **2013**, *24*, 2027–2037. [CrossRef]

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