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An Improved Laplacian Gravity Centrality-Based Consensus Method for Social Network Group Decision-Making with Incomplete ELICIT Information

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Abstract: With the advancement of information technology, social media has become increasingly prevalent. The complex networks of social relationships among decision-makers (DMs) have given rise to the problem of social network group decision-making (SNGDM), which has garnered considerable attention in recent years. However, most existing consensus-reaching methods in SNGDM only consider local network information when determining the influence of DMs within the social network. This approach fails to adequately reflect the crucial role of key DMs in regulating information propagation during the consensus-reaching process. Additionally, the partial absence of linguistic evaluations in the decision-making problems also poses obstacles to identifying the optimal alternative. Therefore, this paper proposes an improved Laplacian gravity centrality-based consensus method that can effectively handle incomplete decision information in social network environments. First, the extended comparative linguistic expressions with symbolic translation (ELICIT) are utilized to describe DMs' linguistic evaluations and construct the incomplete decision matrix. Second, the improved Laplacian gravity centrality (ILGC) is proposed to quantify the influence of DMs in the social network by considering local and global topological structures. Based on the ILGC measure, we develop a trust-driven consensus-reaching model to enhance group consensus, which can better simulate opinion interactions in real-world situations. Lastly, we apply the proposed method to a smart city evaluation problem. The results show that our method can more reasonably handle incomplete linguistic evaluations, more comprehensively capture the influence of DMs, and more effectively improve group consensus.

Keywords: consensus-reaching model; social network group decision-making; centrality measures; ELICIT information

MSC: 90-10



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1. Introduction

In this digital age, social media has permeated people's daily lives, fostering diverse social relationships and leading to the emergence of social network group decision-making (SNGDM) [1]. For instance, consider evaluating smart city initiatives, where input from various stakeholders is crucial for making informed decisions. The complex networks of social relationships among decision-makers (DMs) in such contexts present unique challenges and opportunities for group decision-making. Apart from social governance [2], scholars have also proposed various SNGDM methods and applied them in other domains, such as risk management [3–5], shelter site selection [6], failure analysis [7,8]. In practical decision-making scenarios, individuals tend to characterize qualitative attributes using linguistic information rather than precise numerical values due to the vagueness and uncertainty of human cognition. As a result, Zadeh's concepts of fuzzy sets and linguistic variables were initially employed to express DM judgments on the

performance of alternatives [9]. However, a single linguistic variable struggles to capture the hesitancy that DMs may exhibit during evaluation. Therefore, Rodriguez et al. [10] introduced the hesitant linguistic term set (HFLTS) to handle situations where individuals use multiple linguistic variables for assessment. Suppose there is a linguistic term set $S = \{s_0 = \text{bad}, s_1 = \text{slightlybad}, s_2 = \text{medium}, s_3 = \text{slightlygood}, s_4 = \text{good}\}$. A possible HFLTS expression can be $\{s_1, s_2\}$, which indicates that the DM hesitates between s_1 and s_2 when evaluating this alternative. Krishankumar et al. [11] extended the HFLTS and proposed the double hierarchy hesitant fuzzy linguistic term set (DHHFLTS) to describe uncertain information to solve group decision-making problems. Furthermore, Labella et al. [12] extended HFLTS by generalizing comparative linguistic expressions to a continuous domain and introduced the extended comparative linguistic expressions with symbolic translation (ELICIT) information. Assuming the use of the aforementioned linguistic term set, a possible ELICIT expression provided by the decision-maker can be “at least s_2 ”, which means that the DM believes this alternative is at least medium. Compared with other fuzzy linguistic expressions, the ELICIT information can better model the human reasoning process and enhance the aggregated result’s accuracy and interpretability in computing with words. Therefore, the ELICIT information is employed in this paper to express the uncertain evaluations collected from DMs in SNGDM.

Since individuals with diverse backgrounds and professional skills are engaged in SNGDM, substantial variations in evaluations may arise, necessitating the consensus-reaching process (CRP), especially when many DMs are involved [13]. The CRP can effectively reduce the divergence of individuals’ opinions, enhance the group consensus level (GCL), and facilitate the implementation of final decision outcomes [14]. Moreover, since people tend to interact more with those they trust, leveraging the trust relationships among DMs in SNGDM can facilitate information exchange and CRP [15]. Scholars have conducted extensive research in this area, and existing consensus-reaching strategies in SNGDM can be categorized into linear combination methods and consensus-reaching strategies based on optimization models [16].

Linear combination methods are also referred to as feedback mechanisms based on the identification and direction principles. In such approaches, the set of evaluations that are below the consensus threshold is first determined, and a reference for guiding the modifications to these identified evaluations is then constructed [17]. The updated evaluations are obtained according to the linear combination of the original assessment and its corresponding reference. For instance, Liang et al. [18] identified the opinions that should be adjusted based on their degree of contribution to the group consensus degree. Then, the authors employed the social network DeGroot model to implement the process of opinion updating. Cheng et al. [19] proposed a double recognition mechanism to identify the evaluations needing modification. Subsequently, the influence network was constructed, and the judgments of the DM that is situated at the greatest distance from the origin of the coordinate system were adopted as the modification reference. Gai et al. [20] developed a trust-chain-based evaluation transmission model to facilitate information transmission in the CRP. Then, the bidirectional feedback mechanism was designed to generate the modified evaluation. Li et al. [21] proposed a stochastic multi-criteria acceptability analysis-based consensus-reaching mechanism with weight stability interval values. Based on this mechanism, a two-stage feedback process was implemented to improve the group consensus without the need to present a consensus threshold. Wan et al. [22] designed a two-stage consensus-reaching strategy based on the linear combination approach. In the first stage, the DMs make self-adjustments based on how much the trust relationship aligns with the difference in evaluations. During the second stage, the authors identified four situations in CRP and proposed the corresponding modification strategies considering the binding force of clusters.

Different optimization models were constructed to update the opinion regarding the optimization-based consensus-reaching methods. For example, Yang et al. [23] developed a dual-path adjustment method to enhance the group consensus considering DMs’ trust

relationships and different types of behaviors. Next, the authors built the minimum cost model that considers the directions of modifications and the interactive paths between DMs. Wang et al. [24] proposed a minimum cost–maximum consensus model based on the harmonious power structure to enhance the group consensus. The cluster’s importance is obtained via social network analysis, and the group conflict is minimized to construct a harmonious power structure. Sun et al. [25] developed the minimum adjustment cost model to obtain the preferences of DMs that exhibit no herding behavior. For the low-consensus DMs with herding behavior, the authors proposed a punishment model that meets the consensus threshold requirement to determine the optimized weight of DMs. Zhao et al. [26] proposed the trust-polymerization degree-based adjustable minimum-cost consensus model for large-scale group decision-making problems. The proposed model considers the cluster diversity based on the improved affinity propagation algorithm and fluctuations in expert weights. Qin et al. [27] investigated the influential role of social relationships during the CRP from the perspective of structural holes. The authors developed the trust-driven bi-level minimum cost model to enhance group consensus, which can illustrate the Stackelberg game dynamics between DMs and the moderator. A particle swarm optimization algorithm was designed to solve the proposed model.

In most current consensus-reaching models in social network environments, local centrality measures, such as degree centrality, are employed to quantify the importance of DMs within the network [28]. However, such measures only reflect the local topology of the network, neglecting global information. When solving complex decision-making problems considering trust relationships, identifying influential nodes that have a prominent impact on the CRP in updating decision information is crucial since influencers play significant roles in structural and functional perspectives [29]. Hence, it is imperative to devise a more rational approach for assessing the influence of DMs within the social network considering both local and global network information. Then, the obtained importance degrees can be leveraged to facilitate the CRP. To address this issue, we propose an improved Laplacian gravity centrality-based consensus-reaching method for SNGDM considering incomplete ELICIT information. Subsequently, we applied the proposed method to a smart city evaluation problem. The primary contributions can be outlined as follows:

- (1) We propose an improved Laplacian gravity centrality to determine the influence of DMs in the social network. The improved Laplacian gravity centrality addresses the limitations of most centrality measures, which often focus solely on either local or global network information. By employing the gravity model, the proposed centrality captures nodes’ intrinsic characteristics and incorporates global network information through their connections with neighboring nodes. Additionally, the improved Laplacian gravity centrality utilizes effective distance to measure the distance between nodes, thus reflecting the hidden pattern geometry of the social network. Consequently, the proposed centrality measure offers a more comprehensive and rational depiction of the influence of DMs on information propagation within social networks, laying the groundwork for guiding subsequent CRP.
- (2) We propose the improved Laplacian gravity centrality-based consensus method, which utilizes trust relationships among DMs to handle incomplete ELICIT information. Since DMs are more inclined to consider opinions from individuals they trust during evaluation modification in the CRP, and that DMs with greater influence in the social network have more power to regulate information propagation, the proposed centrality measure is employed to guide the CRP. Specifically, the higher the improved Laplacian gravity centrality value of a DM, the greater the degree of reference in opinion modification. Simultaneously, we introduced a trust relationship-based approach to complete the missing ELICIT information and integrate it into the consensus-reaching model.
- (3) We apply the proposed consensus-reaching model to a smart city evaluation problem. An evaluation criteria system covering three aspects: infrastructure for smart cities, the digital economy, and smart living, comprising six indicators, has been established.

These include investment in information communication and technology (ICT), the extent of 5G base station coverage, the level of development in the ICT industry, the degree of digital financial inclusion, satisfaction with e-governance, and the extent of coverage by digital pilot hospitals. Sensitivity analysis and comparative analysis illustrate the effectiveness and superiority of our method in solving SNGDM problems.

The remainder of this paper is structured as follows: Section 2 outlines the preliminaries of this study. Section 3 details the improved Laplacian gravity centrality-based consensus model for SNGDM with incomplete ELICIT information. Section 4 presents the case study on the smart city evaluation problem. Section 5 provides discussions to showcase the practicability and benefits of the proposed method. Finally, Section 6 presents the conclusion and suggests directions for future research.

2. Preliminaries

This section provides a concise overview of the fundamental concepts of ELICIT information and social network analysis.

2.1. The ELICIT Information

The lack of adequate representation and computational models for comparative linguistic expressions underscores the need for a new approach. This approach must preserve both the interpretability and accuracy of outcomes in the process of computing with words. Therefore, to enhance the clarity and accuracy of linguistic computation, Labela et al. [12] introduced ELICIT, a flexible linguistic representation framework. The ELICIT linguistic model represents linguistic information through the generation of ELICIT information, which is an extension of comparative linguistic expressions generated by context-free grammar into a continuous domain using symbolic translation. ELICIT information leverages the main feature of comparative linguistic expressions, their interpretability, and when necessary, replaces the linguistic terms of the expressions with 2-tuple linguistic terms. In this way, the process of computing with words is performed without any approximations, providing accurate and easy-to-understand results.

The ELICIT representation model offers a novel and flexible method for modeling linguistic information. Due to its capacity to preserve more information throughout linguistic computation, ELICIT information is employed in this research to express expert evaluations.

Definition 1 ([12]). Let $S = s_0, s_1, \dots, s_g$ denote a linguistic term set; therefore, the granularity of S is $g + 1$. There are three potential ELICIT expressions: “at most(s_i, α) $^\gamma$ ”, “at least(s_i, α) $^\gamma$ ”, or “between(s_i, α_1) $^{\gamma_1}$ and (s_j, α_2) $^{\gamma_2}$ ”, where $\alpha \in [-0.5, 0.5)$ represents the symbolic translation variable, and $\gamma \in (-\frac{1}{2g}, \frac{1}{2g})$ denotes the adjustment coefficient, $j = 1, 2, \dots, g$.

For ELICIT information, the process of computing with words comprises three steps: translation, manipulation, and re-transformation, as presented below.

Definition 2 ([12]). Let x_{EL} denote any type of the three kinds of ELICIT expressions described above. Let $TFN(a, b, c, d)$ represent the trapezoidal fuzzy number equivalent to x_{EL} . The mapping from the ELICIT expression to its equivalent trapezoidal fuzzy number is shown as follows:

$$h^{-1} : x_{EL} \rightarrow TFN(a, b, c, d) \quad (1)$$

This transformation can be formulated differently depending on the particular ELICIT expression used. Please refer to [12] for detailed information.

Definition 3 ([12]). The manipulation stage involves performing fuzzy arithmetic operations with trapezoidal fuzzy numbers obtained in the transformation process defined in Definition 2. Let $TFN_A(a_1, b_1, c_1, d_1)$ and $TFN_B(a_2, b_2, c_2, d_2)$ denote two fuzzy envelopes obtained from two

corresponding trapezoidal fuzzy numbers. A shape function ς_{A+B} is defined to perform the addition operation of fuzzy envelopes, which is given as follows:

$$\varsigma_{A+B} = \begin{cases} \frac{(x-(a_1+a_2))^n}{(b_1+b_2)-(a_1+a_2)} & a_1 + a_2 \leq x \leq b_1 + b_2 \\ 1 & b_1 + b_2 \leq x \leq c_1 + c_2 \\ \frac{((d_1+d_2)-x)^n}{(d_1+d_2)-(c_1+c_2)} & c_1 + c_2 \leq x \leq d_1 + d_2 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Definition 4 ([12]). Another shape function ς_{A-B} is defined to perform the subtraction of two fuzzy envelopes $TFN_A(a_1, b_1, c_1, d_1)$ and $TFN_B(a_2, b_2, c_2, d_2)$, which is given as follows:

$$\varsigma_{A-B} = \begin{cases} \frac{(x-(a_1-d_2))^n}{(b_1-a_1)+(d_2-c_2)} & a_1 - d_2 \leq x \leq b_1 - c_2 \\ 1 & b_1 - c_2 \leq x \leq c_1 - b_2 \\ \frac{((d_1-a_2)-x)^n}{(d_1-c_1)-(b_2-a_2)} & c_1 - b_2 \leq x \leq d_1 - a_2 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

Definition 5 ([12]). The trapezoidal fuzzy number $\tilde{\delta}$ derived in the manipulation step needs to be transformed into its equivalent ELICIT expression in the re-translation step with the inverse function $\hbar : \tilde{\delta} \rightarrow x_{EL}$, which is presented as follows:

- (1) If $\tilde{\delta} = TFN(a, b, 1, 1)$, then $\hbar(\tilde{\delta}) = atleast(s_i, \alpha)^\gamma$
- (2) If $\tilde{\delta} = TFN(0, 0, c, d)$, then $\hbar(\tilde{\delta}) = atmost(s_i, \alpha)^\gamma$
- (3) If $\tilde{\delta} = TFN(a, b, c, d)$, then $\hbar(\tilde{\delta}) = between(s_i, \alpha_1)^{\gamma_1} and (s_j, \alpha_2)^{\gamma_2}$

Three examples are shown in Figures 1–3 to illustrate the relationship between the ELICIT expression and their shape functions.

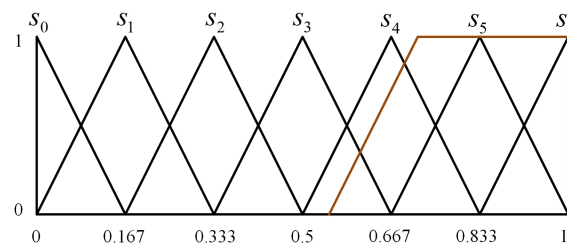


Figure 1. The shape function of ELICIT expression $atleast(s_4, 0.3)^{0.05}$.

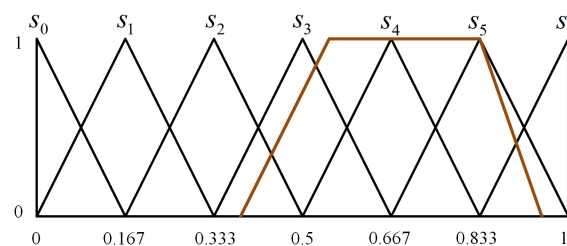


Figure 2. The shape function of ELICIT expression $between(s_3, 0.12)^{0.025} and (s_5, 0)^{-0.02}$.

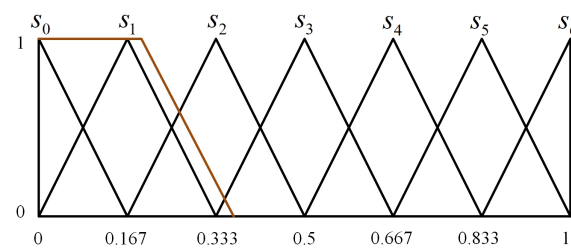


Figure 3. The shape function of ELICIT expression $atmost(s_1, 0.15)^{0.02}$.

Definition 6 ([12]). Let x_{EL1} and x_{EL2} denote two ELICIT expressions. With the transformation function defined in Definition 2, their equivalent trapezoidal fuzzy numbers of x_{EL1} and x_{EL2} can be expressed as $(\tilde{\delta}_1) = TFN_1(a_1, b_1, c_1, d_1)$ and $(\tilde{\delta}_2) = TFN_2(a_2, b_2, c_2, d_2)$, respectively. Therefore, the deviation of the two ELICIT expressions can be calculated as follows:

$$dev(x_{EL1}, x_{EL2}) = dev(\tilde{\delta}_1, \tilde{\delta}_2) = \sqrt{\frac{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2}{4}}. \quad (4)$$

Definition 7 ([12]). By determining the magnitude of the equivalent trapezoidal fuzzy number of the ELICIT expression x_{EL} , its expectation value can be calculated as follows:

$$E(x_{EL}) = Mag(\tilde{\delta}) = \frac{1}{12}(a + 5b + 5c + d). \quad (5)$$

2.2. Social Network Analysis

By treating DMs or stakeholders as nodes and their social relationships as edges, social network analysis provides valuable insights into the connections and interactions among individuals within a group [30]. By examining the structure and dynamics of these networks, people can better understand how information flows, influence spreads, and consensual decisions are made collectively [31]. Identifying influential nodes is particularly important, as these key nodes can regulate the flow of information during CRP in SNGDM [32]. Therefore, this subsection first introduces several representative centrality measures and then discusses the effective distance, which can reflect the hidden geometry of complex networks.

2.2.1. Centrality Measures

A social network can be represented by a directed graph $G = (E, L)$, where E and L indicate the set of nodes and the set of edges, respectively. N represents the number of nodes in the social network. Therefore, the adjacency matrix can be expressed as $T = [t_{ij}]_{N \times N}$, where $t_{ij} = 1$ indicates that there exists a social relationship between corresponding nodes; otherwise, $t_{ij} = 0$. An example of a social network among 6 DMs and its corresponding adjacency matrix is illustrated in Figure 4.

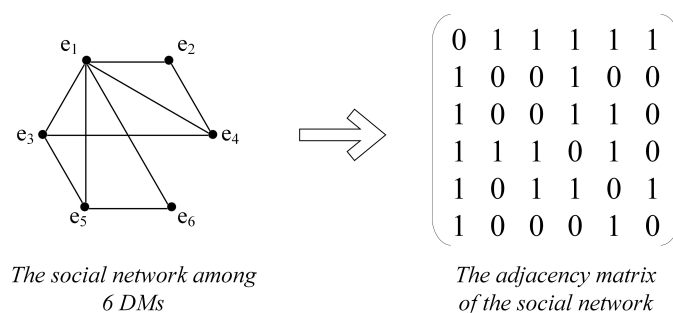


Figure 4. The social network among 6 DMs and its corresponding adjacency matrix.

Centrality measures are effective tools for determining the importance of nodes in social networks. The most representative centrality measures include degree centrality, closeness centrality, and betweenness centrality, which are introduced as follows:

Definition 8 ([33] Degree centrality). Degree centrality is the predominant measure of centrality, widely employed to assess the influence of nodes based on their degrees. The formula for degree centrality is shown as follows:

$$DC(i) = \sum_{j=1, j \neq i}^N t_{ij} \quad (6)$$

where N represents the number of nodes in this social network. The degree centrality of node i can also be expressed with $k(i)$.

Definition 9 ([33] Closeness centrality). Closeness centrality assesses the centrality of a node within a network by calculating the sum of the shortest distances from that node to all others. The calculation is shown as follows:

$$CC(i) = \frac{1}{\sum_{j \neq i} d_{ij}} \quad (7)$$

where d_{ij} denotes the shortest distance between node i and j .

Definition 10 ([33] Betweenness centrality). Betweenness centrality focuses on the level of concentration along paths, which can be determined as below:

$$BC(i) = \frac{\sum_{j \neq k \neq i} \text{Num}_{jk}(i)}{\sum_{j \neq k} \text{Num}_{jk}} \quad (8)$$

where Num_{jk} represents the count of shortest paths from node j to node k , while $\text{Num}_{jk}(i)$ represents the count of shortest paths from node j to node k passing through node i .

2.2.2. Effective Distance

Dirk Brockmann and Dirk Helbing proposed the effective distance, which is a distance measure motivated by probability [34]. It unveils the concealed geometric patterns within intricate networks. The core concept of effective distance lies in identifying the most probable path between two nodes based on the network information. The original definition of effective distance is outlined below.

Definition 11 ([34]). Let $0 \leq P_{mn} \leq 1$ represent the portion of travelers departing from node n and reaching node m . Thus, the effective distance from node n to node m can be calculated as follows:

$$Effd_{mn} = 1 - \log_2 P_{mn}, \quad (9)$$

where $P_{mn} = \frac{F_{mn}}{\sum_m F_{mn}}$, and F_{mn} denotes the traveler flux from n to m .

The adjacency matrix, also called the connection matrix, is a square matrix utilized to characterize the social network or a finite graph. Let $T = [t_{ij}]_{N \times N}$ represent the adjacency matrix of a social network that contains N nodes. In terms of the unweighted network, if a relationship exists from node i to node j , then $t_{ij} = 1$; otherwise, $t_{ij} = 0$. Thus, the original definition of effective distance can be extended based on the adjacency matrix, which is given as follows:

Definition 12. The effective distance from node i to node j , where they are directly connected, can be determined as follows:

$$EffD_{ij} = 1 - \log_2 \left(\frac{t_{ij}}{k(i)} \right), \quad (10)$$

where t_{ij} represents the entry in the adjacency matrix T of the social network, indicating the flux of flow from node i to node j . $k(i)$ represents the degree centrality of node i , denoting the cumulative

flux from node i to the other nodes. Thus, $\frac{t_{ij}}{k(i)}$ measures the proportion of flux from node i to node j , serving as a logical extension of P_{mn} in the original definition of effective distance.

The principle of transitivity is applied to calculate the effective distance for indirectly connected nodes. For example, the effective distance from node p to node q can be derived as $ED_{pq} = ED_{pi} + ED_{iq}$, assuming direct connections between nodes p and i and between nodes i and q . The effective distance is different from the conventional Euclidean distance in its directional and asymmetric nature. When multiple paths exist from node i to node j , the effective distance is determined by selecting the shortest path as the definitive measure.

3. The Improved Laplacian Gravity Centrality-Based Consensus Model for SNGDM with Incomplete ELICIT Information

This section elaborates on the improved Laplacian gravity centrality-based consensus-reaching model with incomplete ELICIT information. Section 3.1 briefly describes a typical GDM problem in social network environments. In Section 3.2, we introduce the improved Laplacian gravity centrality, which can leverage local and global network information to quantify the influence of DMs within the group. Subsequently, Section 3.3 proposes the improved Laplacian gravity centrality-based consensus-reaching strategy to enhance the group consensus for a consensual solution. Finally, in Section 3.4, the framework and steps of the proposed method are summarized.

3.1. Problem Description

In a typical SNGDM problem, assume that there is a set of m potential alternatives, $A = \{a_i | i = 1, 2, \dots, m\}$, and a set of n criteria represented as $C = \{c_j | j = 1, 2, \dots, n\}$. A group of r DMs is denoted as $E = \{e_k | k = 1, 2, \dots, r\}$. $w_j (j = 1, 2, \dots, n)$ is utilized to denote the weight of criteria that satisfies $w_j \in (0, 1)$ and $\sum_{j=1}^n w_j = 1$. Because of the vagueness and uncertainty in human cognition, we employ the ELICIT information to describe the performance of each alternative in terms of different criteria using a 7-scale linguistic term set $S = \{s_0 = \text{verybad}, s_1 = \text{bad}, s_2 = \text{slightlybad}, s_3 = \text{medium}, s_4 = \text{slightlygood}, s_5 = \text{good}, s_6 = \text{verygood}\}$. The matrix of decision information can be expressed as $X = [x_{ij}^k]_{m \times n}$, where x_{ij}^k is an ELICIT expression that represents e_k 's assessment on a_i in terms of c_j .

$$X^k = [x_{ij}^k]_{m \times n} = \begin{matrix} & \begin{matrix} c_1 & c_2 & \cdots & c_n \end{matrix} \\ \begin{matrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{matrix} & \begin{pmatrix} x_{11}^k & x_{12}^k & \cdots & x_{1n}^k \\ x_{21}^k & x_{22}^k & \cdots & x_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^k & x_{m2}^k & \cdots & x_{mn}^k \end{pmatrix} \end{matrix} \quad (k = 1, 2, \dots, r) \quad (11)$$

In this study, an undirected graph $G = (E, L)$ is constructed to describe the social network within this group of DMs. E and L represent the set of DMs and the trust relationships between them, respectively. The adjacency matrix of the social network is denoted as $T = [t_{kl}]_{r \times r}$, where t_{kl} is a binary variable denoting the presence of a trust connection between e_k and e_l .

3.2. The Improved Laplacian Gravity Centrality Measure

Most SNGDM studies either utilize local or global network information to determine the importance of DMs in the social network. Therefore, this section proposes an improved Laplacian gravity centrality measure to better capture interactions between nodes, considering both local and global network information.

The gravity model is also known as the inverse square law. In previous gravity centrality studies, the degree of a node is considered the mass of objects in the gravity model. Nonetheless, it is not always precise when pinpointing influential nodes with the degree centrality alone, particularly if they are connected with numerous nodes with low degrees. The capability of influencing other nodes on information propagation is one of the key aspects of influential nodes. If a node is connected to many nodes with a degree of 1,

it may be positioned at the center of a community, but it might not be able to effectively propagate information or influence nodes beyond the community. As information spreads through the network, each node gradually influences its nearest neighbors, and the degrees of these neighbors are an essential factor that needs to be considered.

To overcome the aforementioned limitations of the original gravity centrality method, we first introduce Laplacian centrality into the gravity centrality model to characterize the intrinsic characteristics of nodes. This approach considers not only the degree of nodes themselves but also the degrees of their neighboring nodes. Additionally, we incorporate the effective distance into the gravity centrality model. In the dynamic information propagation process of networks, distance often exhibits directionality; it may be easier for information to spread from node n to node m than from node m to node n . However, traditional gravity centrality models utilize Euclidean distance, which fails to capture the directional aspect of distance. Therefore, we introduce the effective distance to measure the distance between nodes, enabling a more effective consideration of dynamic information within the hidden topology of the network.

Let $T = [t_{kl}]_{r \times r}$ represent the adjacency matrix of the social network $G = (E, L)$. $\tilde{I} = \text{diag}(\lambda_i)$ is a diagonal matrix where the diagonal elements correspond to the nodes' degrees. Therefore, the Laplacian matrix of the social network is determined as $L = \tilde{I} - T$. The Laplacian centrality can be computed based on the alteration in Laplacian energy following the removal of a node [34]. The Laplacian energy of the social network is denoted as $\aleph(G) = \sum_{i=1}^n \gamma_i$, where $\gamma_i (i = 1, 2, \dots, n)$ represents the eigenvalues of matrix L . After the removal of node i , the loss of Laplacian energy is equivalent to the Laplacian centrality of node i , which is given as follows:

$$\text{LapC}(i) = \aleph(G) - \aleph(G_{-1}), \quad (12)$$

where G_{-1} is the social network (G) with the node (i) and edges connected to it removed.

Based on the definition of Laplacian centrality, upon removing node i , the degree of all neighboring nodes decreases by 1. Therefore, we can infer:

$$\text{LapC}(i) = \aleph(G) - \aleph(G_{-1}) = \lambda_i^2 + 2\lambda_i + \sum_{j \in \Omega_i} (\lambda_j^2 - (\lambda_j - 1)^2) = \lambda_i^2 + \lambda_i + 2 \sum_{j \in \Omega_i} \lambda_j \quad (13)$$

where λ_i and Ω_i denote the degree and the set of neighboring nodes i , respectively. From Equation (13), we can observe that the Laplacian centrality is associated with both the degree of the node itself and the degrees of its neighbors.

In the original inverse-square law, the numerator represents the mass of the object, which can be considered as a physical quantity reflecting the intrinsic characteristic of the object, while the denominator represents the distance between the objects. Based on the gravity model, this paper replaces the node degree in the original gravity centrality model with the Laplacian centrality defined in Equation (13) to characterize the intrinsic properties of the nodes (i.e., their importance). Additionally, by considering the dynamic information embedded in the hidden network topology, we introduce the effective distance to quantify the distance between nodes. Accordingly, we propose the improved Laplacian gravity model, defined as follows:

Definition 13. Let $\text{LapC}(i)$ denote the Laplacian centrality of node i and EffD_{ij} denote the effective distance from node i to node j . The improved Laplacian gravity centrality can be defined as follows:

$$\text{ILGC}(i) = \sum_{j=1, j \neq i}^n \frac{\text{LapC}(i)\text{LapC}(j)}{\text{EffD}_{ij}^2} \quad (14)$$

3.3. The Consensus-Reaching Model Based on the Improved Laplacian Gravity Centrality

In this subsection, the improved Laplacian gravity centrality-based consensus-reaching model is constructed to improve the group consensus degree. First, the estimation method

for the missing ELICIT decision information is presented. Then, the consensus measurement on three levels is given. Finally, the trust-driven consensus-reaching strategy is illustrated based on the proposed improved Laplacian gravity centrality.

3.3.1. Estimation of Missing ELICIT Decision Information

In GDM scenarios, especially when many participants are involved, it is common to encounter missing decision information during data collection. Therefore, the estimation and completion of missing decision information has become an important issue. This study uses the trust relationships between DMs to estimate the missing ELICIT information collected.

Definition 14. Let x_{ij}^l denote the ELICIT evaluation on a_i with respect to c_j given by e_l . Let $x_{ij}^k(i, j, k \in \Psi)$ denote the missing ELICIT information in the decision matrix. The estimated value of $x_{ij}^k(i, j, k \in \Psi)$ can be determined as follows:

$$\hbar^{-1}(x_{ij}^k) = \sum_{l=1, l \neq k}^r t_{kl} \frac{ILGC(e_l)}{\sum_{l=1}^2 t_{kl} ILGC(e_l)} \hbar^{-1}(x_{ij}^l) \quad (15)$$

where $\hbar^{-1}(\cdot)$ denotes the mapping from ELICIT expression to its equivalent trapezoidal fuzzy number. t_{kl} denotes the trust relationship between e_k and e_l . $ILGC(e_l)$ represents the improved Laplacian gravity centrality of e_l .

3.3.2. Consensus Measures

In SNGDM, it is crucial to guarantee that the group agreement achieves a certain degree of contentment before finalizing the decisions. Based on the complete decision information, the consensus levels can be calculated to measure the degree of agreement in the group. Palomares and colleagues [35] introduced a classification where consensus can be measured in two distinct manners: one is by the difference between individual perspectives and collective judgment, and the other is by the variation among personal assessments. This study adopts the latter method to establish consensus measurements on individual assessment, DM, and the group level.

Definition 15 (Consensus at the individual assessment level). Let x_{ij}^k and x_{ij}^l denote the individual assessments on the performance of $a_i (i = 1, 2, \dots, m)$ given by $e_k (k = 1, 2, \dots, r)$ and $e_l (l = 1, 2, \dots, r)$ in terms of $c_j (j = 1, 2, \dots, n)$, respectively. Therefore, the consensus at the individual assessment level can be computed as follows:

$$CL_{ij}^k = \frac{1}{r-1} \sum_{l=1, l \neq k}^r (1 - dev(x_{ij}^k, x_{ij}^l)), \quad (16)$$

where $dev(x_{ij}^k, x_{ij}^l)$ represents the deviation between individual assessments x_{ij}^k and x_{ij}^l . Therefore, $(1 - dev(x_{ij}^k, x_{ij}^l))$ indicates the similarity degree between individual assessments x_{ij}^k and x_{ij}^l . By taking the average, CL_{ij}^k can reflect the degree of consensus at the individual assessment level.

Definition 16 (Consensus at the DM level). The consensus at the DM level is considered as the consensus level of $DMe_k (k = 1, 2, \dots, r)$ on the set of alternatives with respect to all criteria, which can be obtained as follows:

$$CL^k = \frac{1}{mn} \sum_{i=1}^m \sum_{j=1}^n CL_{ij}^k, \quad (17)$$

where CL_{ij}^k denotes the degree of consensus at the individual assessment level obtained by Equation (16). By calculating the average of the degree of consensus for each alternative under each attribute, the consensus index at the DM level can be obtained. $CL_{ij}^k \in (0, 1)$ denotes the consensus at the individual level.

Definition 17 (Consensus at the group level). Based on the consensus at the DM level, the consensus at the group level can be aggregated as follows:

$$GCL = \frac{1}{r} \sum_{k=1}^r CL^k, \quad (18)$$

where $GCL \in (0, 1)$. Based on the consensus level at the DM level, the consensus at the group level can be derived. A higher GCL value signifies greater agreement among the group members.

Subsequently, the GCL needs to be compared with a predefined parameter η to ascertain whether the consensus among the existing assessments is acceptable. If $GCL \geq \eta$, the alternative ranking procedure can be conducted directly; otherwise, the trust-driven consensus-reaching strategy will be carried out to enhance the agreement level among the group of DMs.

3.3.3. Trust-Driven Consensus-Reaching Process

A 2-stage trust-driven feedback mechanism is designed to reach the preset satisfactory group consensus level. Initially, evaluations that contribute less to an adequate GCL are pinpointed using consensus measurements at two levels:

- (1) The DMs with insufficient consensus levels can be recognized as follows:

$$DIC = \{k | CL^k < \eta\} \quad (19)$$

- (2) In terms of the DMs, including in the DIC set, the evaluations that fall short of the consensus threshold are recognized as follows:

$$EIC = \{(i, j, k) | k \in DIC \wedge CL_{ij}^k < \eta\} \quad (20)$$

Let $x_{ij}^k(i, j, k \in EIC)$ denote the individual assessments that need to be adjusted and updated; then the direction rules are proposed to guide the opinion modification in the social network environments.

In SNGDM, the process of achieving consensus within the network structure is dependent on information propagation. The DMs possessing more influence hold a higher capacity to control the dissemination of information. The improved Laplacian gravity centrality defined in Section 3.2 can reflect the importance of DMs in the social network from local and global network topology and is employed to establish the consensus-reaching strategy. Therefore, the adjusted evaluations of the identified ones can be determined as follows:

$$\tilde{h}^{-1}(\tilde{x}_{ij}^k) = \frac{ILGC(e_k)}{\sum_{l=1}^r t_{kl} ILGC(e_l)} \tilde{h}^{-1}(x_{ij}^k) + \sum_{l=1, l \neq k}^r t_{kl} \left(\frac{ILGC(e_l)}{\sum_{l=1}^r t_{kl} ILGC(e_l)} \right) \tilde{h}^{-1}(x_{ij}^l), (i, j, k \in EIS) \quad (21)$$

where x_{ij}^k and $\tilde{x}_{ij}^k$ represent the initial individual assessments that are identified and the corresponding adjusted assessments, respectively. $\tilde{h}^{-1}(\cdot)$ denotes the mapping from ELICIT expression to its equivalent trapezoidal fuzzy number. t_{kl} denotes the trust relationship between e_k and e_l . $ILGC(e_l)$ is the improved Laplacian gravity centrality of e_l that represents the importance or influence of e_l in the social network.

The aforementioned procedures will be executed iteratively until the group consensus attains the preset satisfactory level η . Subsequently, the group assessments can be obtained by aggregating the individual opinions with the improved Laplacian gravity centrality and the criteria weight.

Definition 18. Let $w_j (j = 1, 2, \dots, n)$ denote the weight of criteria c_j , and the final group evaluation on the alternatives that satisfy the preset consensus threshold can be derived as follows:

$$\tilde{h}^{-1}(x_i^g) = \sum_{j=1}^n \sum_{k=1}^r \frac{ILGC(e_k)}{\sum_{k=1}^r ILGC(e_k)} w_j \tilde{h}^{-1}(x_{ij}^{k(fin)}) \quad (22)$$

where $x_{ij}^{k(fin)}$ represents the final evaluation of a_i on c_j from e_k . $\tilde{h}^{-1}(\cdot)$ denotes the mapping from ELICIT expression to its equivalent trapezoidal fuzzy number. $ILGC(e_k)$ is the improved Laplacian gravity centrality of e_k .

Finally, the performance of different alternatives can be ranked according to the expectation value of $x_i^g (i = 1, 2, \dots, m)$ by Equation (5).

3.4. The Framework of the Proposed Method

The flowchart of the proposed improved Laplacian gravity centrality-based consensus method for SNGDM with incomplete ELICIT information is shown in Figure 5. The steps for implementing this method are summarized as follows:

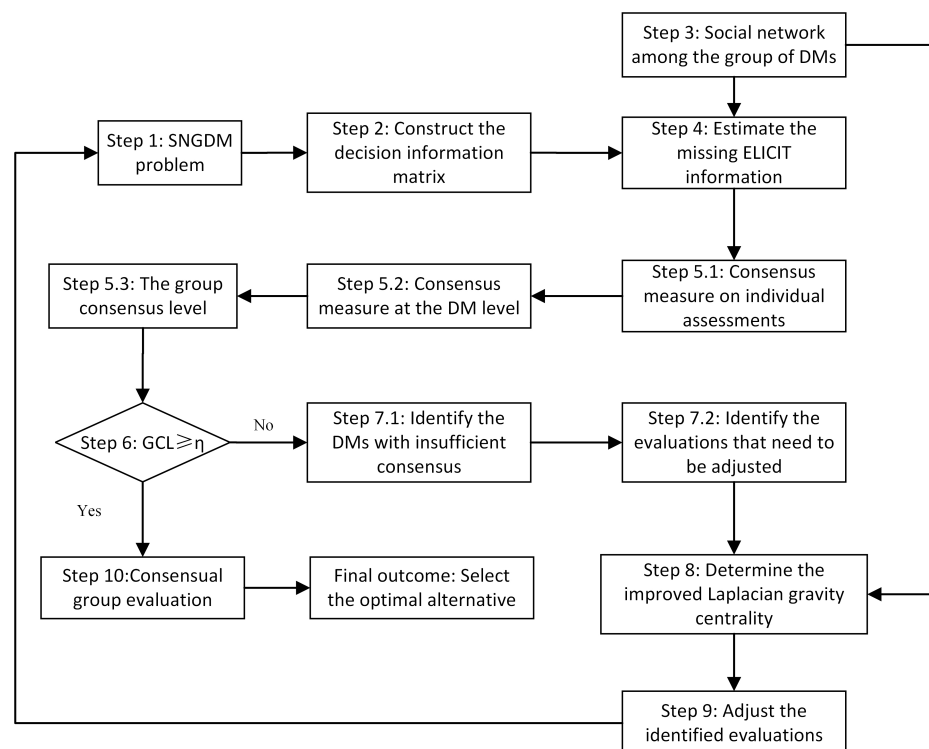


Figure 5. The flowchart of the improved Laplacian gravity centrality-based consensus method for SNGDM with incomplete ELICIT information.

Step 1. Determine the set of m potential alternatives $A = \{a_i | i = 1, 2, \dots, m\}$ and a set of n criteria represented as $C = \{c_j | j = 1, 2, \dots, n\}$.

Step 2. Invite a group of r DMs, denoted as $E = \{e_k | k = 1, 2, \dots, r\}$, to evaluate the performance of alternatives using ELICIT information and construct the decision information matrix.

Step 3. Analyze the social network among DMs and establish the adjacency matrix T based on the trust relationships between DM.

Step 4. Estimate the missing ELICIT information based on the social network analysis of the group of DMs.

Step 5. Determine three levels of consensus, i.e., consensus at the individual assessment level CL_{ij}^k , consensus at the DM level CL^k , and consensus at the group level GCL .

Step 6. If the present level of group consensus GCL meets or exceeds the specified threshold, that is $GCL \geq \eta$, proceed to Step 10. If not, the feedback mechanism is initiated to enhance the level of group consensus.

Step 7. Pinpoint the evaluations that need to be adjusted and updated based on the 2-stage trust-driven feedback mechanism.

Step 8. Determine the influence of DMs in the social network during CRP with the proposed improved Laplacian gravity centrality measure.

Step 9. Obtain the modified evaluations based on the trust-driven consensus-reaching strategy. Then, go back to Step 5 to check whether the preset consensus requirement can be met.

Step 10. Determine the alternative ranking based on the consensual group evaluation of the performance of alternatives.

4. Case Study on the Smart City Evaluation Problem

This section elaborates on the case study of a smart city evaluation problem to demonstrate the effectiveness and superiority of the proposed improved Laplacian gravity centrality-based consensus method for SNGDM with incomplete ELICIT information.

4.1. Problem Background

Many cities are facing imbalances between population growth, resource supply, urban infrastructure, and public services, negatively impacting their sustainable development [36]. Meanwhile, rapid technological advancements offer new opportunities to tackle urban development challenges. For instance, globally advanced cities like New York, Shanghai, and Seoul are actively utilizing digital technology to promote the development of smart transportation, smart grids, smart healthcare, and other areas, thereby facilitating the construction of smart cities. To realize the vision of constructing smart cities, it is particularly important to establish an indicator system to evaluate the 'smart' level of cities and to rank smart cities based on the evaluation information.

Therefore, this study will establish an indicator system that contains 8 criteria $c_j (j = 1, 2, \dots, 8)$ from three evaluation perspectives (i.e., smart living, smart infrastructure, and digital economy), which is presented in Table 1. Five cities, denoted as $a_i (i = 1, 2, \dots, 5)$, are selected for evaluation after preliminary screening. A group of DMs $e_k (k = 1, 2, \dots, 20)$ are invited from local institutions to assess the level of smartness of the four alternatives in terms of each evaluation indicator.

Table 1. The indicator system of the smart city evaluation problem.

Perspective	Symbol	Criterion	Meaning
Smart living	c_1	Satisfaction level with e-governance	Satisfaction level with e-governance measures how content citizens are with the quality, accessibility, and efficiency of government services provided through digital platforms.
	c_2	Coverage of digital hospitals	Coverage of digital hospitals measures the extent to which healthcare services are available and accessible through digital platforms and technologies.
	c_3	Coverage of smart bus services	Coverage of smart bus services measures the extent and efficiency of public bus systems utilizing digital technologies for enhanced operation and user experience.
Smart infrastructure	c_4	The level of ICT investment	The level of ICT investment measures the amount of resources allocated to information and communication technologies to improve infrastructure and services.
	c_5	Coverage of 5G base stations	Coverage of 5G base stations measures the extent and availability of 5G network infrastructure within a city.
	c_6	Development level of data centers	Development level of data centers measures the advancement and capacity of data storage and processing facilities within a city.
Digital economy	c_7	Development level of the ICT industry	Development level of the ICT industry measures the growth, innovation, and economic impact of the information and communication technology sector.
	c_8	Digital financial inclusion	Digital financial inclusion measures the extent to which digital financial services are accessible and used by all segments of the population.

4.2. Method Implementation

Given the subjective nature of human interpretation and the qualitative aspects of standards, the ELICIT information in the 7-scale linguistic term set is utilized to articulate expert assessments. The linguistic term set is represented as $S = \{s_0 = \text{verybad}, s_1 = \text{bad}, s_2 = \text{slightlybad}, s_3 = \text{medium}, s_4 = \text{slightlygood}, s_5 = \text{good}, s_6 = \text{verygood}\}$. For brevity, the evaluations given by e_1 on the alternatives regarding the set of criteria are presented in Table 2.

Table 2. The evaluation of alternatives regarding each criterion given by e_1 .

	a_1	a_2	a_3	a_4	a_5
c_1	$bets_2 \& s_3$	s_4	s_4	$atleast s_2$	$bets_2 \& s_3$
c_2	s_3	$atmost s_3$	$bets_1 \& s_2$	s_2	s_5
c_3		$bets_3 \& s_4$	s_2	$atmost s_4$	$bets_4 \& s_5$
c_4	s_3	$atleast s_2$		$bets_1 \& s_2$	s_6
c_5	$atmost s_4$	s_4	$bets_4 \& s_5$	s_3	
c_6	$bets_3 \& s_4$	s_3	s_4	s_5	$atleast s_3$
c_7	s_5	$bets_1 \& s_2$	$atmost s_3$	$bets_3 \& s_4$	s_4
c_8	s_3	s_5	s_1		s_1

The social network of the group of DMs, which is determined based on their trust relationships, is illustrated in Figure 6. Nodes represent the group of DMs, and edges represent the trust relationships between them. For example, a bidirectional trust relationship exists between e_8 and e_9 . Nodes that are more centrally located within the social network of Figure 6 have trust relationships with a larger number of DMs.

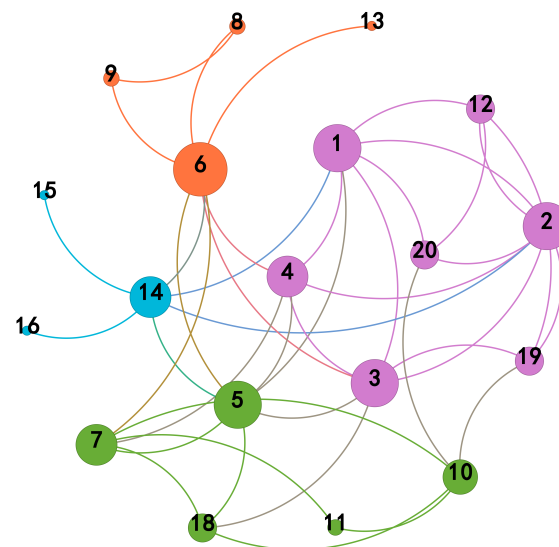


Figure 6. The social network among the group of DMs.

According to the trust relationships and the corresponding adjacency matrix of DMs, the unimproved Laplacian gravity centrality of DMs can be computed. Based on the improved Laplacian gravity centrality, the missing values of the original decision information can be estimated with Equation (15). With the complete decision information and the consensus metrics, the consensus at the individual assessment, DM, and group levels can be determined with Equations (16)–(18). The consensus at the DM level is presented in Table 3.

Table 3. The consensus at the DM level.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
CL^k	0.852	0.855	0.861	0.673	0.850	0.857	0.532	0.854	0.563	0.853
	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}
CL^k	0.869	0.627	0.856	0.554	0.856	0.861	0.675	0.860	0.626	0.436

The initial group consensus level is calculated as $GCL = 0.746$ according to Equation (18). In order to make the final decision agreeable to most DMs, the consensus threshold is established at $\eta = 0.85$. Given that the group consensus presently falls short of this threshold, a trust-driven strategy for reaching consensus needs to be employed to enhance the group consensus.

In accordance with the identification rule at the individual assessment and DM levels, the particular evaluations that require adjustments can be identified as follows:

$$EIC = \left\{ \begin{array}{l} ((1, 6, 4), (2, 4, 4), (2, 7, 4), (2, 8, 4), (4, 6, 4), (2, 5, 7), (3, 1, 7), (2, 8, 9), \\ (3, 7, 12), (5, 4, 12), (5, 6, 12), (1, 1, 14), (4, 3, 14), (2, 6, 17), (4, 8, 17), \\ (5, 3, 17), (2, 3, 19), (4, 1, 19), (2, 7, 20), (3, 6, 20)) \end{array} \right\} \quad (23)$$

Based on the proposed trust-driven consensus-reaching strategy, the identified DMs who are required to adjust their respective viewpoints will refer to the individuals they trust within the social network. The more prominent an individual's influence within the network, the more they will be referred to when updating their assessments. In this study, the improved Laplacian gravity centrality is utilized to reflect the influence of DMs in the social network and guide the CRP. Take $(1, 6, 4)$ in EIC as an example, the DMs that e_4 trusts are $e_k (k = 1, 2, 3, 5, 6, 7)$. According to the direction rule elaborated in Section 3.3.3, the updated individual assessment can be determined as follows:

$$\tilde{h}^{-1}(x_{16}^4) = \frac{ILGC(e_4)}{\sum_{k=1}^7 ILGC(e_k)} \tilde{h}^{-1}(x_{16}^4) + \sum_{k=1, k \neq 4}^7 \frac{ILGC(e_k)}{\sum_{k=1}^7 ILGC(e_k)} \tilde{h}^{-1}(x_{16}^k) \quad (24)$$

where $\tilde{h}^{-1}(x_{16}^4)$ and $\tilde{h}^{-1}(x_{16}^k)$ denote the original and updated evaluation of x_{16}^4 , respectively. $ILGC(e_k)$ represents the improved Laplacian gravity centrality of e_k . By referring to the evaluations of the DMs trusted by e_4 , the revised evaluation of x_{16}^4 can be obtained. In this process, the greater the influence (i.e., the larger the $ILGC(e_k)$) of a DM in the social network, the more their opinion is considered during the update.

Similarly, the rest of the identified individual assessments can be updated, and the modified ones are given in Table 4.

According to the updated evaluations, the consensus levels on the individual assessments, DMs, and group levels are computed via Equations (16)–(18). The consensus on the DM level after the first iteration is shown in Table 5.

Subsequently, the updated group consensus level after the first iteration can be obtained as $GCL^{(1)} = 0.752$. We can observe that the level of agreement within the group has been increased, yet it has not met the predetermined consensus requirement. Consequently, we continue to iterate the aforementioned procedures until the group consensus satisfies the threshold criteria. First, the DMs with insufficient consensus levels are identified with Equation (19). Second, the specific evaluations that fall short of the consensus threshold are recognized by Equation (20). Then, the identified evaluations are updated based on their improved Laplacian gravity centrality with Equation (21). Subsequently, the consensus on the three levels is checked again. Since the methodology utilized for the iteration process is the same, the detailed procedure is omitted here. After another five iterations, the GCL increased to 0.854, which satisfies the preset consensus requirement. The total execution time of the proposed algorithm for this group of DMs to reach the satisfied consensus

is 2.81 min. The enhancement in the group consensus is illustrated in Figure 7. Table 6 presents the satisfactory consensual group evaluations.

Table 4. The updated individual assessments after the first iteration.

$\overline{x}_{16}^4$	$\overline{x}_{24}^4$	$\overline{x}_{27}^4$
$bet(s_2, 0.354)^{-0.001} \& (s_3, 0.144)^{0.253}$	$bet(s_3, 0.255)^{0.125} \& (s_4, -0.243)^{0.013}$	$bet(s_3, 0.123)^{0.255} \& (s_4, 0.554)^{-0.013}$
$\overline{x}_{28}^4$	$\overline{x}_{46}^4$	$\overline{x}_{25}^7$
$bet(s_1, 0.424)^{0.167} \& (s_2, 0.283)^{0.343}$	$bet(s_2, 0.092)^{-0.005} \& (s_3, 0.043)^{0.021}$	$bet(s_4, 0.394)^{0.445} \& (s_5, 0.213)^{0.340}$
$\overline{x}_{31}^7$	$\overline{x}_{28}^9$	$\overline{x}_{37}^{12}$
$bet(s_3, -0.412)^{0.065} \& (s_4, -0.386)^{0.092}$	$bet(s_2, 0.306)^{0.254} \& (s_3, 0.471)^{0.163}$	$bet(s_1, 0.027)^{-0.015} \& (s_2, 0.044)^{0.003}$
$\overline{x}_{54}^{12}$	$\overline{x}_{56}^{12}$	$\overline{x}_{11}^{14}$
$bet(s_2, 0.074)^{0.233} \& (s_3, 0.102)^{0.341}$	$bet(s_4, 0.265)^{0.405} \& (s_5, 0.368)^{0.209}$	$bet(s_2, 0.317)^{-0.036} \& (s_3, 0.296)^{-0.014}$
$\overline{x}_{43}^{14}$	$\overline{x}_{26}^{17}$	$\overline{x}_{48}^{17}$
$bet(s_3, 0.375)^{0.003} \& (s_4, 0.208)^{-0.011}$	$bet(s_1, 0.018)^{0.027} \& (s_2, 0.077)^{0.124}$	$bet(s_4, 0.331)^{-0.154} \& (s_5, 0.276)^{0.201}$
$\overline{x}_{53}^{17}$	$\overline{x}_{23}^{19}$	$\overline{x}_{41}^{19}$
$bet(s_2, 0.309)^{0.116} \& (s_3, 0.425)^{0.147}$	$bet(s_3, 0.207)^{-0.023} \& (s_4, 0.356)^{0.159}$	$bet(s_3, -0.132)^{0.491} \& (s_4, -0.268)^{0.284}$
$\overline{x}_{27}^{20}$	$\overline{x}_{36}^{20}$	
$bet(s_4, 0.228)^{0.015} \& (s_5, 0.351)^{0.043}$	$bet(s_3, 0.492)^{-0.132} \& (s_4, 0.380)^{-0.087}$	

Table 5. The consensus at the DM level after the first iteration.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}
CL^k	0.854	0.856	0.863	0.679	0.851	0.857	0.540	0.855	0.571	0.855
	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}
CL^k	0.871	0.633	0.858	0.567	0.857	0.863	0.679	0.862	0.631	0.443

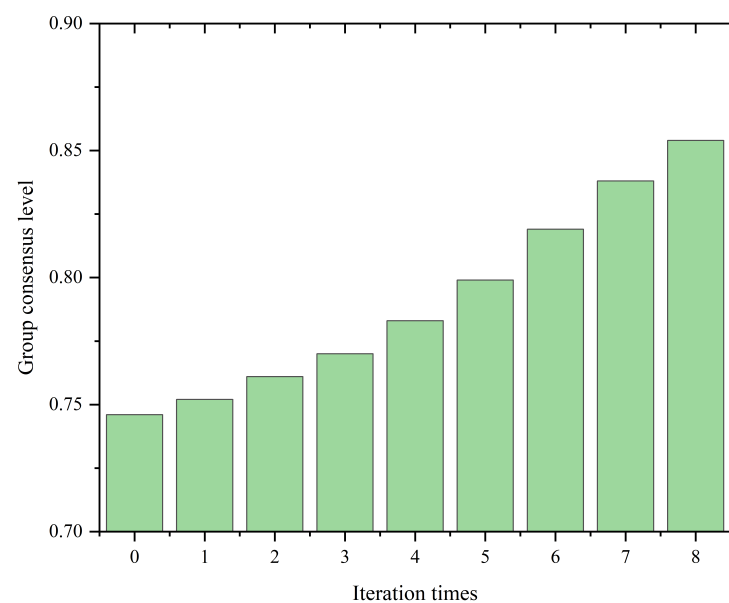


Figure 7. The enhancement in the group consensus.

In this case study, the weight of the criteria is assumed to be equal. Therefore, the group assessment of smart cities can be determined via Equation (22). The ranking of

the smart cities can be obtained by their expectation values with Equation (5). Table 7 demonstrates the consensual group assessments, expectation value, and alternative ranking. City a_4 is identified as the city with the highest ‘smart’ degree under the evaluation criteria.

Table 6. The consensual group evaluation of alternatives in terms of each criterion.

	a_1	a_2	a_3	a_4	a_5
c1	$Bet(s2, -0.285)^{0.186}$ & $(s3, -0.319)^{0.152}$	$Bet(s1, 0.394)^{0.226}$ & $(s2, 0.180)^{0.346}$	$Bet(s1, 0.221)^{0.010}$ & $(s2, 0.364)^{0.126}$	$Bet(s4, 0.415)^{0.031}$ & $(s5, 0.376)^{0.269}$	$Bet(s1, 0.487)^{0.202}$ & $(s2, 0.263)^{0.017}$
c2	$Bet(s1, 0.244)^{0.164}$ & $(s2, 0.412)^{0.335}$	$Bet(s1, 0.044)^{0.312}$ & $(s2, 0.352)^{0.054}$	$Bet(s2, 0.324)^{0.038}$ & $(s3, 0.107)^{0.031}$	$Bet(s3, 0.281)^{0.144}$ & $(s4, 0.503)^{0.107}$	$Bet(s1, 0.057)^{0.263}$ & $(s2, 0.434)^{0.198}$
c3	$Bet(s3, -0.051)^{0.116}$ & $(s4, -0.137)^{0.034}$	$Bet(s1, 0.286)^{0.374}$ & $(s2, 0.495)^{0.231}$	$Bet(s2, -0.012)^{0.351}$ & $(s3, 0.036)^{0.402}$	$Bet(s3, 0.546)^{0.053}$ & $(s4, 0.176)^{0.145}$	$Bet(s2, 0.411)^{0.329}$ & $(s3, 0.355)^{0.016}$
c4	$Bet(s2, 0.493)^{0.304}$ & $(s3, 0.372)^{0.065}$	$Bet(s2, 0.309)^{0.237}$ & $(s3, 0.316)^{0.062}$	$Bet(s2, 0.393)^{0.114}$ & $(s3, 0.177)^{0.025}$	$Bet(s2, 0.037)^{-0.014}$ & $(s3, 0.055)^{-0.009}$	$Bet(s2, 0.246)^{0.355}$ & $(s3, 0.130)^{0.447}$
c5	$Bet(s4, 0.258)^{0.170}$ & $(s5, 0.106)^{0.274}$	$Bet(s2, 0.308)^{0.441}$ & $(s3, 0.188)^{0.341}$	$Bet(s4, 0.231)^{-0.301}$ & $(s5, 0.304)^{-0.445}$	$Bet(s3, 0.185)^{0.160}$ & $(s4, 0.310)^{0.105}$	$Bet(s3, -0.461)^{0.438}$ & $(s4, -0.141)^{0.233}$
c6	$Bet(s2, 0.437)^{-0.110}$ & $(s3, 0.409)^{-0.278}$	$Bet(s1, 0.265)^{0.167}$ & $(s2, 0.103)^{0.064}$	$Bet(s1, 0.076)^{0.492}$ & $(s2, 0.174)^{0.386}$	$Bet(s2, 0.253)^{0.004}$ & $(s3, 0.393)^{0.015}$	$Bet(s1, 0.207)^{0.151}$ & $(s2, 0.295)^{0.336}$
c7	$Bet(s3, 0.381)^{0.461}$ & $(s4, 0.227)^{0.205}$	$Bet(s1, 0.396)^{0.284}$ & $(s2, 0.463)^{0.056}$	$Bet(s2, 0.367)^{0.443}$ & $(s3, 0.474)^{0.409}$	$Bet(s4, 0.489)^{-0.263}$ & $(s5, 0.506)^{-0.177}$	$Bet(s2, 0.368)^{0.454}$ & $(s3, 0.140)^{0.271}$
c8	$Bet(s3, 0.024)^{0.257}$ & $(s4, 0.243)^{0.416}$	$Bet(s2, 0.348)^{-0.115}$ & $(s3, 0.197)^{-0.216}$	$Bet(s3, 0.316)^{0.187}$ & $(s4, 0.393)^{0.405}$	$Bet(s3, 0.196)^{0.144}$ & $(s4, 0.375)^{0.153}$	$Bet(s2, 0.358)^{0.294}$ & $(s3, 0.236)^{0.180}$

Table 7. The consensual group assessments, expectation value, and alternative ranking.

	The Consensual Group Assessments	Expectation Value	Alternative Ranking
a_1	$Bet(s3, 0.226)^{-0.157}$ & $Bet(s4, 0.153)^{-0.049}$	0.603	2
a_2	$Bet(s1, 0.489)^{0.193}$ & $Bet(s2, 0.147)^{0.346}$	0.281	5
a_3	$Bet(s3, 0.081)^{0.042}$ & $Bet(s4, 0.039)^{0.037}$	0.554	3
a_4	$Bet(s4, 0.625)^{0.331}$ & $Bet(s5, 0.539)^{0.143}$	0.748	1
a_5	$Bet(s2, 0.379)^{-0.083}$ & $Bet(s3, 0.196)^{-0.122}$	0.366	4

5. Discussions

In this section, the validity analysis and comparative analysis are conducted to illustrate the validity and superiority of the improved Laplacian gravity centrality-based consensus method for SNGDM with incomplete ELICIT information.

5.1. Validity Analysis

First, we will assess the validity of the proposed SNGDM method from three perspectives. (1) The optimal alternative identified by our proposed approach should remain unchanged when a non-optimal alternative is replaced with a worse one; (2) a valid SNGDM approach needs to adhere to the transitivity property; (3) if the SNGDM problem is divided into multiple sub-problems and the same approach is applied to solve these sub-problems, the obtained ranking needs to be consistent with that of the original SNGDM problem.

To verify the validity of the proposed method from the first perspective, the original group assessment on city a_5 is modified, and the other assessments remain unchanged. Table 8 shows the original and the modified group evaluation on a_5 . According to our proposed approach and the modified group evaluation, the new ranking of the alternatives can be determined as $a_4 > a_1 > a_3 > a_5 > a_2$. Compared to the original ranking established in Section 4, the optimal alternative is still a_4 , demonstrating the validity of the proposed SNGDM method.

The original set of alternatives is divided into four subsets for the other two perspectives, as detailed in Table 9. Using the proposed SNGDM approach, we determined the

ranking of alternatives within each subset. The rankings within these subsets align with the ranking of the original set of alternatives, further validating our method.

Table 8. The original and modified group assessments on alternative a_5 .

	The Original Group Assessment on a_5	The Modified Group Assessment on a_5
c_1	$Bet(s1, 0.487)^{0.202} \& (s2, 0.263)^{0.017}$	$Bet\ s1 \& s2$
c_2	$Bet(s1, 0.057)^{0.263} \& (s2, 0.434)^{0.198}$	$Bet\ s1 \& s2$
c_3	$Bet(s2, 0.411)^{0.329} \& (s3, 0.355)^{0.016}$	$Bet\ s1 \& s2$
c_4	$Bet(s2, 0.246)^{0.355} \& (s3, 0.130)^{0.447}$	$Bet\ s1 \& s2$
c_5	$Bet(s3, -0.461)^{0.438} \& (s4, -0.141)^{0.233}$	$Bet\ s1 \& s2$
c_6	$Bet(s1, 0.207)^{0.151} \& (s2, 0.295)^{0.336}$	$Bet\ s1 \& s2$
c_7	$Bet(s2, 0.368)^{0.454} \& (s3, 0.140)^{0.271}$	$Bet\ s1 \& s2$
c_8	$Bet(s2, 0.358)^{0.294} \& (s3, 0.236)^{0.180}$	$Bet\ s1 \& s2$

Table 9. The subsets, ranking indices, and rankings of alternatives.

Subsets of Alternatives	Ranking Indices of Alternatives	Rankings
a_1, a_2, a_3, a_4	$RI(a_1) = 0.653, RI(a_2) = 0.320, RI(a_3) = 0.401, RI(a_4) = 0.748$	$a_4 > a_1 > a_3 > a_2$
a_1, a_3, a_4, a_5	$RI(a_1) = 0.691, RI(a_3) = 0.489, RI(a_4) = 0.755, RI(a_5) = 0.343$	$a_4 > a_1 > a_3 > a_5$
a_2, a_3, a_4, a_5	$RI(a_2) = 0.351, RI(a_3) = 0.680, RI(a_4) = 0.769, RI(a_5) = 0.594$	$a_4 > a_3 > a_5 > a_2$
a_1, a_2, a_4, a_5	$RI(a_1) = 0.774, RI(a_2) = 0.445, RI(a_4) = 0.803, RI(a_5) = 0.682$	$a_4 > a_1 > a_5 > a_2$

5.2. Comparative Analysis

In this subsection, qualitative and quantitative comparisons are conducted with several typical GDM methods and graph convolutional network techniques (GCN) to demonstrate the advantages and features of our approach. First, the qualitative comparison is carried out from the following aspects: the expression structure of the decision information, the estimation method to complete the missing decision information, the determination method of the DMs' importance or influence, and the consensus-reaching strategy. The detailed comparison is summarized in Table 10.

Table 10. Qualitative comparisons of typical SNGDM methods from four aspects.

Method	Decision Information Expression	Estimation of Missing Values	Determination of DMs' Importance	Consensus-Reaching Strategy
This paper	ELICIT information	Estimated with trust relationships	The improved Laplacian gravity centrality	Trust-driven consensus model based on the improved Laplacian gravity centrality
[16]	Crisp numbers	Not considered	Degree centrality	Maximum consensus model
[37]	Crisp numbers	Not considered	Degree centrality	Bi-level consensus model with social influence
[23]	Preference matrices	Not considered	Trust values and individual influence	Minimum adjustment model
[20]	Numerical preference relations	Not considered	Degree centrality	Trust chain driven bidirectional feedback mechanism
[38]	Numerical preference relations	Not considered	Calculated based on the consistency level of preferences and the limited compromise behavior of DMs	Maximum satisfaction consensus model
[25]	Linguistic variables	Not considered	Evaluation similarity and degree centrality	Feedback mechanism considering herding behavior

Regarding the expression of decision information, methods in [16,37] use crisp numbers to describe the evaluators' assessments of alternatives. However, precise numbers

or intervals fail to capture the inherent uncertainty and fuzziness in human cognition. Methods in [20,23,38] utilize preference relations to depict the performance of alternatives in pairwise comparisons as decision information. In SNGDM problems, comparing each pair of alternatives for every attribute significantly increases the method's execution time, especially when the number of alternatives is large. Additionally, using preference relations for pairwise comparisons may reduce the consistency of evaluations. The method in [25] introduces linguistic variables to describe experts' linguistic assessments, but standard linguistic variable expressions struggle to reflect the potential hesitation in evaluations. This paper uses ELICIT expressions to describe the DMs' evaluation of alternatives. Compared to other linguistic expressions, ELICIT is closer to the human reasoning process and enhances the accuracy and interpretability of linguistic computations. In addition, this study addresses the issue of missing decision information and proposes a trust-based estimation method for completing missing values, enhancing the practicality of the approach.

When determining the importance or influence of DMs within a group, most methods utilize degree centrality, such as methods in [16,20,25,37]. However, degree centrality is a local measure and overlooks the global information of the network. The method in [38] uses the level of evaluation consistency and the extent of DMs' limited compromise behavior to determine DM's importance, but it neglects the social relationships among them in a social network context, failing to fully leverage the trust degree to facilitate the SNGDM process. In contrast, this study proposes the improved Laplacian gravity centrality, which incorporates both local and global network topology information and reflects the hidden pattern geometry of the social network.

Regarding consensus-reaching strategies, typical SNGDM methods generally involve two approaches: the identification-guidance approach and the optimal model-based consensus model. In the identification-guidance approach, the method in [16] proposes a bi-level consensus model considering social influence to achieve consensus, method in [20] designs a trust chain-driven bidirectional feedback mechanism to identify and adjust decision units with low consensus levels, and the method in [25] accounts for potential herding behavior among decision-makers to construct a feedback mechanism.

In the optimal model-based consensus approach, the method in [16] employs a maximum consensus model to guide the consensus process, where the updated evaluations are directly output by the optimal model. The method in [23] uses a minimum adjustment model to derive adjusted evaluations, aiming to achieve consensual decisions by minimizing the extent of evaluation changes. The method in [38] applies a maximum satisfaction consensus model to facilitate consensus, ensuring that the adjusted evaluations meet the highest satisfaction degree. While the optimal model-based consensus approach is efficient, it does not fully leverage the trust relationships among DMs in the social network. For instance, in the maximum satisfaction consensus model, the updated evaluations are directly output by the model without considering the fact that decision-makers are more likely to refer to those they trust. This study uses the improved Laplacian gravity centrality to construct a trust-driven consensus model. Identified evaluations are adjusted concerning the evaluations of trusted DMs, and the degree of reference increases with the DM's influence in the social network. This approach better simulates real-world SNGDM processes, making it more reasonable and effective.

Then, the proposed method is compared with GCN techniques to demonstrate the features of our approach.

In terms of scope and application, our method addresses the specific problem of SNGDM by focusing on the consensus-reaching process among DMs within a social network. The emphasis is on handling incomplete linguistic evaluations and enhancing group consensus through an improved Laplacian gravity centrality-based consensus method. GCN techniques are primarily designed for learning node representations by aggregating features from neighboring nodes, which is highly effective for tasks such as node classification, link prediction, and graph classification. The underlying principle is information

spreading across the graph, which may not directly align with the specific requirements of SNGDM.

Regarding the methodology, we utilize ELICIT to handle incomplete decision information and construct an incomplete decision matrix. The improved Laplacian gravity centrality is introduced to quantify the influence of DMs, considering both local and global topological structures. This method is tailored to enhance group consensus by accurately capturing the influence of key DMs and simulating real-world opinion interactions. GCNs aggregate information from a node's neighbors to learn embeddings representing each node in its graph context. While this technique is powerful for various graph-based learning tasks, it does not inherently address the specific challenges of consensus-reaching in SNGDM, such as handling incomplete linguistic evaluations and incorporating the unique influence dynamics of DMs.

For the consensus-reaching process, our consensus-reaching model is trust-driven and leverages the ILGC measure to enhance group consensus, particularly in scenarios where decision information is incomplete. The focus is on simulating opinion interactions and improving the decision-making process within social networks. While GCNs facilitate information propagation across the graph, they are not specifically designed to model the consensus-reaching process among a group of DMs. Therefore, applying GCNs directly to SNGDM may not address the unique requirements of this problem domain.

Following the qualitative comparisons, we chose several typical methods for quantitative analysis. Given that different approaches use various evaluation expression structures, we standardize by using crisp numbers to represent DMs' opinions and apply the same example to compute the alternative rankings. The alternative ranking indices and ranking results obtained using the methods from references [16,23,36], as well as the method proposed in this paper, are shown in Table 11. It can be observed that the optimal alternative identified by all methods is a_4 , and the least favorable alternative is a_2 . This demonstrates the effectiveness of the group decision-making method proposed in this paper.

Table 11. The alternative ranking determined by different methods.

Alternative	[16]	[23]		[36]		This Paper		
	Ranking Indices	Ranking	Ranking Indices	Ranking	Ranking Indices	Ranking	Ranking Indices	Ranking
a_1	0.405	2	0.586	3	0.581	3	0.645	2
a_2	0.184	5	0.304	5	0.311	5	0.104	5
a_3	0.292	4	0.749	2	0.695	2	0.437	3
a_4	0.691	1	0.836	1	0.728	1	0.792	1
a_5	0.367	3	0.477	4	0.405	4	0.251	4

6. Conclusions

To ensure the decision-making outcome is widely accepted and efficiently executed, consensus-reaching strategies are employed to align the perspectives of DMs and enhance group consensus. With the proliferation of social media, social network relationships have become a powerful tool for information propagation. Therefore, it is crucial to explore the topological structure of social networks and utilize network information to simulate opinion exchanges among DMs and achieve group consensus. To address this issue, we propose an improved Laplacian gravity centrality-based consensus model that can effectively handle incomplete ELICIT information in social network environments. The conclusions are summarized as follows based on theoretical examination and case study.

- (1) The improved Laplacian gravity centrality measure is proposed. This centrality measure addresses the shortcomings of traditional centrality metrics that typically focus on either local or global network information. By leveraging the gravity model, our proposed centrality captures both the inherent characteristics of nodes and their broader network connections. Additionally, it utilizes effective distance to better reflect

the underlying geometric patterns of social networks, providing a more accurate representation of the influence of DMs on information dissemination.

- (2) A consensus-reaching method based on this improved centrality measure is developed, effectively managing incomplete ELICIT information through trust relationships among DMs. Recognizing that DMs are more likely to consider inputs from trusted peers and that influential DMs play a critical role in information regulation, our method uses the improved Laplacian gravity centrality to guide the consensus-reaching process. DMs with higher centrality values have a greater impact on opinion modifications, ensuring that trust-based adjustments are incorporated into the model to address missing information.
- (3) The proposed method is applied to a smart city evaluation problem for method validation. We developed an evaluation criteria system that includes indicators such as ICT investment, 5G coverage, ICT industry development, digital financial inclusion, e-governance satisfaction, and digital hospital coverage. Our method demonstrated its effectiveness and superiority in addressing SNGDM challenges through sensitivity and comparative analyses.

However, this paper still has some limitations. For instance, the proposed method only uses a single linguistic expression to describe the linguistic assessments of DMs, without considering the personalized individual semantics. Additionally, the paper does not take into account the dynamic changes in the strength of social network relationships as opinions are updated during the consensus-reaching procedure. In the future, we will utilize multi-granular and heterogeneous linguistic terms to provide a more nuanced representation of DMs' opinions, thereby improving the accuracy of the consensus outcomes. Second, we will investigate the integration of machine learning algorithms to dynamically adjust the influence of DMs based on real-time interactions and evolving network structures. Additionally, the application of the proposed method to different domains, such as healthcare, education, and disaster management, could be explored to validate its versatility and effectiveness across various decision-making scenarios.

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