mathematics

MDPI

Article

Fuzzy Evaluation Model for Lifetime Performance Using Type-I Censoring Data

Kuo-Ching Chiou [1], Tsun-Hung Huang [2], Kuen-Suan Chen [2,3,4,*] and Chun-Min Yu [2,*]

[1] Department of Finance, Chaoyang University of Technology, Taichung 413310, Taiwan; kcchiou@cyut.edu.tw
[2] Department of Industrial Engineering and Management, National Chin-Yi University of Technology, Taichung 411030, Taiwan; toby@ncut.edu.tw
[3] Department of Business Administration, Chaoyang University of Technology, Taichung 413310, Taiwan
[4] Department of Business Administration, Asia University, Taichung 413305, Taiwan
* Correspondence: kschen@ncut.edu.tw (K.-S.C.); march@ncut.edu.tw (C.-M.Y.)

Abstract: As global warming becomes increasingly serious, humans start to consider how to coexist with the natural environment. People become more and more aware of environmental protection and sustainable development. Therefore, in the pursuit of economic growth, it has become a consensus that enterprises should be responsible for the social and ecological environment. Regarding the manufacturing of electronic devices, as long as both component production quality and assembly quality are ensured, consumers can be provided with high-quality, safe, and efficient products. In light of this trend, enhancing product availability and reliability can reduce costs and carbon emissions resulting from repairing or replacing components, thus becoming a vital factor for corporate and environmental sustainability. Accordingly, enterprises enhance their economic benefits as well as have the effects of energy conservation and waste reduction by extending products' service lifetime and increasing their added value. According to several studies, it takes a long time to retrieve electronic products' lifetime data. Moreover, acquiring complete samples is often challenging. Consequently, when analyzing real cases, samples are usually collected using censoring techniques. The type-I right censoring data is suitable for industrial processes. Thus, this study utilized type-I right censoring sample data to estimate the lifetime performance index. It usually takes a large amount of time to access lifetime data for electronic products and it is often impossible to obtain complete samples since the size of the sample is usually small. Hence, to avoid misjudgment caused by sampling errors, this study followed suggestions from existing research and applied fuzzy tests built on confidence intervals to establish a fuzzy evaluation model for the lifetime performance index. This model helps relevant electronic industries not only evaluate the lifetime of their electronic components but also instantly seize opportunities for improvement.

Keywords: electronic component lifetime; relative lifetime performance index; type-I censoring; confidence-interval based fuzzy testing method

MSC: 62C05; 62C86

check for updates

Citation: Chiou, K.-C.; Huang, T.-H.; Chen, K.-S.; Yu, C.-M. Fuzzy Evaluation Model for Lifetime Performance Using Type-I Censoring Data. *Mathematics* **2024**, *12*, 1935. https://doi.org/10.3390/math12131935

Academic Editors: Yi-Kuei Lin, Cheng-Ta Yeh and Ping-Chen Chang

Received: 16 April 2024
Revised: 13 June 2024
Accepted: 19 June 2024
Published: 21 June 2024

Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https://creativecommons.org/licenses/by/4.0/).

1. Introduction

According to a number of studies, Taiwan's electronics industry owns a complete ecological chain in the world supply chain system involving information and communications technology, driven by a high clustering effect. As a result, it has played a pivotal role in the world electronics industry [1–4]. In the production of electronic products, it is essential to ensure both production quality and assembly quality of all components. This assures consumers of receiving high-quality, safe, and efficient products, thereby satisfying their demands [5,6]. Additionally, plenty of studies have highlighted the increasing seriousness of global warming, urging humans to contemplate how to coexist with the natural environment. Meanwhile, awareness of environmental protection and sustainable development

is gradually growing as well [7,8]. Consequently, corporate social responsibility (CSR) has emerged as the business philosophy that global enterprises promote. In the pursuit of economic growth, enterprises have reached a consensus that they must take greater responsibility for the social and ecological environment [9,10]. Under such a trend, boosting product availability and reliability can cut down on costs and carbon emissions caused by repairs or replacement of components, which is one of the key factors in the sustainable operation of enterprises and the environment. In this era of artificial intelligence, constant changes in technology have made manufacturing techniques sophisticated and complicated. Therefore, enterprises strive to extend the service lifetime of products as well as to increase the added value of products to increase their overall competitiveness and economic benefits.

As noted by some studies, the lifetime T of an electronic component has an exponential distribution with mean θ^{-1} [11–13]. Thus, the probability density function (*p.d.f.*) is $f_T(t) = \theta\, e^{-\theta t}$ for $t > 0$ and the cumulative function (*c.d.f.*) of T is $F_T(t) = 1 - e^{-\theta t}$ for $t > 0$. Building upon this assumption, some studies have proposed a relative lifetime performance index to offer the industry a more intuitive and user-friendly tool in practice [14–16]. According to Chiou [17] and Chen and Yu [16], the relative electronic component lifetime is denoted with $X = T/L$, where L represents the period of warranty. Thus, the relative electronic component lifetime X possesses a Gamma distribution with two parameters, 1 and λ_L, expressed as $X \sim G(1, \lambda_L)$. The second parameter $\lambda_L = L\theta$ denotes the relative lifetime performance index for an electronic component where parameter θ refers to the electronic component's mean time between failures (*MTBF*). Therefore, the relative electronic component lifetime X has an exponential distribution with the mean λ_L. Its probability density function (*p.d.f.*) is $f_X(x) = \lambda_L^{-1} \times \exp(-x/\lambda_L)$ for x > 0 and the cumulative distribution function (*c.d.f.*) is $F_X(x) = 1 - \exp(-x/\lambda_L)$, for x > 0. According to Chen et al. [18], the relative failure rate is denoted by λ_L^{-1}, and the reliability function of the relative lifetime X is $R_X(x) = \exp(-x/\lambda_L)$ for x > 0.

Based on Chen et al. [18], product reliability (p_r) represents the probability showing that the relative lifetime X is greater than 1, such that $p_r = e^{-\lambda_L^{-1}}$, indicating its direct mathematical relationship with the relative lifetime performance index λ_L. Obviously, when the value of θ^{-1} ($=\mu_T$) is larger than that of L, then the relative lifetime performance index is larger. Additionally, p_r increases as the value of the indicator λ_L rises. Therefore, the relative lifetime performance index λ_L can reveal the lifetime performance of the electronic component in a reasonable manner.

If the relative lifetime performance index λ_L is employed as an evaluation tool to establish an evaluation model for the electronic component lifetime, it will help the electronics industry improve the lifetime performance of electronic components through evaluation. Since this index has unknown parameters, it must be evaluated using sample data. However, Chiou [17] stated that it is time-consuming to obtain electronic products' lifetime data. In addition, often complete samples cannot be obtained. For example, not all observation values can be acquired due to constraints such as time, cost, or human negligence. Accordingly, when analyzing an actual case, it is common to encounter difficulty in obtaining a complete sample. Thus, samples are usually collected using censoring techniques. Censoring falls into three categories: type-I censoring, type-II censoring, and random censoring [19]. In many cases, data of type-II and type-I censoring are frequently applied to engineering [19]. This paper explores the study by Miller [19], emphasizing the right censoring type. This involves dividing the obtained data into two parts: the first part represents the individual's failure time X (focusing on the relative failure time), while the second part represents the censoring time C. If the failure time X of the experiment itself is less than or equivalent to the censoring time C, then the data for X is the data for uncensored observation. If the failure time X of the experiment itself is greater than the censoring time C, then X refers to the data for censored observation, symbolized by X^+. Concerning the censored observation data (X^+), the corresponding censoring time C is used as the imputation value of the individual's failure time X, and this imputation value is called censored

data. When the censoring time C is the order statistic X(m), where $m \leq$ n, the censoring type is called type-II right censoring. As the censoring time C is the fixed censoring time x_c, the censoring type is called type-I right censoring. When the censoring time C is a random variable (independent failure time X), the censoring type is called random right censoring. According to several studies, of the above three censoring types, random right censoring arises in medical applications with clinical trials or animal studies. Conversely, type-I right censoring data is suitable for industrial processes [19–22]. Therefore, in this paper, we utilize type-I right censoring sample data to estimate the lifetime performance index. As mentioned above, it is laborious to retrieve electronic products' lifetime data.

If the sample size is large enough, the confidence interval of index λ_L is used to statistically test the life performance of electronic products. Since the length of the confidence interval will shorten as the sample size increases, the risk of misjudgment can be reduced. If the sample size is not large enough, the length of the confidence interval will be too long, which will make the statistical test error too large and increase the risk of misjudgment. Moreover, obtaining a complete sample is often challenging, particularly in the case of a small sample size [23]. In order to solve the risk of misjudgment caused by too-long confidence intervals caused by small samples, this study follows some suggestions from previous studies and adopts fuzzy tests built on confidence intervals to establish a fuzzy evaluation model for the lifetime performance index [16,24]. Fuzzy testing based on confidence intervals is not traditional fuzzy testing. Its advantage is that it can incorporate past accumulated data and expert experience. Therefore, the risk of misjudgment caused by too-long confidence intervals can be reduced. Obviously, since obtaining electronic product lifetime data is very time-consuming, in the case of a small sample, this model can assist relevant electronics industries with evaluating the lifetime of their electronic components as well as seizing opportunities for improvement in real time.

The remaining sections are organized in the following order. In Section 2, this study estimates the lifetime performance index and derives its 100 $(1-\alpha)$% confidence intervals using type-I right censoring sample data. Subsequently, in Section 3, these 100 $(1-\alpha)$% confidence intervals are employed to establish a fuzzy evaluation model for the lifetime performance index, and then testing rules are established. In Section 4, this study proposes an empirical case to clarify the fuzzy evaluation model so as to benefit its application and promotion in the industry. Finally, Section 5 gives the conclusions.

2. Confidence Intervals of the Relative Lifetime Performance Index

As mentioned above, the relative lifetime X of electronic components is distributed as an exponential distribution with mean λ_L, where λ_L denotes the relative lifetime performance index for an electronic component as follows:

$$\lambda_L = \frac{\mu_T}{L} = \frac{\theta^{-1}}{L} = \frac{1}{L\theta}, \tag{1}$$

When reliability is adopted to analyze an empirical case, data collection is incomplete for external factors or human factors. Consequently, the results obtained after analysis are often questionable. Considering labor costs and time, applying censoring types to data collection is one of the methods that can improve this shortcoming. The type-I progressive censoring scheme has become relatively widespread for analyzing lifetime data of highly reliable products [25–27]. In this paper, we took type-I right censoring (i.e., given censoring time) into consideration. According to Equation (2), as the type of censoring data for fixed $C = x_c$ is called type-I right censoring, then Y_i can be rewritten as follows:

$$Y_i = \begin{cases} X_i\,, X_i \leq x_c \ (\textit{uncensored data}) \\ x_c\,, \ X_i > x_c \ (\textit{censored data}) \end{cases}, \tag{2}$$

where $i = 1, 2, \ldots, n$. Let $\delta_i = I\ (X_i \leq\ x_c)$ be an indicator function, expressed as follows:

$$\delta_i = I\ (X_i \leq\ x_c)\ =\ \begin{cases} 1\ , & X_i \leq\ x_c \\ 0\ , & X_i >\ x_c \end{cases}. \tag{3}$$

Next, let m_u be the number of uncensored data, defined as follows:

$$m_u = \sum_{i=1}^{n} \delta_i\ =\ \sum_{i=1}^{n} I\ (X_i \leq\ x_c)\quad . \tag{4}$$

In fact, m_u follows a Binomial distribution, denoted by m_u~$B(n, p)$, where $p = P(X \leq x_c)$. According to the probability density function and the cumulative distribution function of X, we can find the distribution of estimate $\hat{\lambda}_L$ using type-I right censoring. Let the likelihood function of pair (y_i, δ_i) be as follows:

$$L(y_i, \delta_i) = (f_X(y_i))^{\delta_i}(R_X(y_i))^{1-\delta_i},\ \delta_i\ =\ 0.1, \tag{5}$$

where $y_1, y_2, \ldots, y_n$ are the type-I censored observation data for $Y_1, Y_2, \ldots, Y_n$. Let $\lambda_L^{-1} = \theta^*$. Then, the likelihood function of type-I right censoring is expressed as follows:

$$\begin{aligned} L(y_1,\ y_2, \ldots, y_n; \delta_1,\ \delta_2, \ldots, \delta_n) \quad &= \prod_{i=\ 1}^{n} L(y_i; \delta_i) = \prod_{x_i \in\ U} f_X(y_i) \prod_{x_i \in\ C^*} R_X(y_i) \\ &= \left((\theta^*)^{m_u} e^{-\sum\limits_{U} \theta^* x_i} \right) \left(e^{-\sum\limits_{C^*} \theta^*\ x_c} \right) = \theta^{m_u} e^{-\ (\sum\limits_{U} \theta^*\ x_i + \sum\limits_{C^*} \theta^*\ x_c)}\ , \\ &= (\theta^*)^{m_u} e^{-\ (\sum\limits_{i=1}^{n} \theta^*\ t_i)} \end{aligned} \tag{6}$$

where U represents the uncensored data and C^* represents the censored data. m_u represents the number of uncensored data as well as a random variable. Using Equation (6), the first- and second-order derivatives can be found below.

$$\frac{d}{d\theta^*} \ln L(\theta^* \mid y_1,\ y_2, \ldots, y_n) = \frac{d}{d\theta^*} \ln \left((\theta^*)^{m_u}\ e^{-(\sum\limits_{i=1}^{n} \theta^*\ y_i)} \right)\ =\ \frac{m_u}{\theta^*} - \sum_{i=1}^{n} y_i; \tag{7}$$

$$\frac{d^2}{d\theta^{*2}} \ln L(\theta^* \mid y_1,\ y_2, \ldots, y_n) =\ -\ \frac{m_u}{\theta^{*2}}. \tag{8}$$

Setting Equation (7) to 0, we can find the estimator of θ^* as follows:

$$\hat{\theta}^* =\ \frac{m_u}{\sum\limits_{i=1}^{n} Y_i}. \tag{9}$$

Using Equation (8), the Fisher information of $\hat{\theta}^*$ is defined as follows:

$$I(\theta^*)\ = E\left(-\frac{d^2}{d\theta^{*2}} \ln L(\theta^* \mid Y_1,\ Y_2, \ldots, Y_n) \right) =\ \frac{m_u}{\theta^{*2}}. \tag{10}$$

Based on the large-sample theorem, the asymptotic variance of the estimate $\hat{\theta}^*$ can be expressed as:

$$\mathrm{Var}(\hat{\theta}^*)\ =\ (I(\theta^*))^{-1}\ =\ \frac{\theta^{*2}}{m_u}. \tag{11}$$

Then, the asymptotic distribution of estimate $\hat{\theta}^*$ is presented as follows:

$$\sqrt{m_u}(\hat{\theta}^* - \theta^*) \rightarrow\ N\left(0, \theta^{*2}\right). \tag{12}$$

When $x_c \to \infty$ and $m_u \approx n$, the estimate $\hat{\theta}^*$ is defined as

$$\hat{\theta}^* \approx \frac{n}{\sum_{i=1}^{n} X_i} = \frac{1}{\overline{X}}. \tag{13}$$

Given that the censoring time is x_c and (Y_1, Y_2, …, Y_n) is a random sample, the number of uncensored data $m_u = \sum_{i=1}^{n} \delta_i$ in Equation (4) is unknown, and it follows a Binomial distribution (n, p) with $p = P(X \le x_c) = 1 - e^{-\theta^* x_c}$. Next, we can get the expected value $E(m_u) = np = n\left(1 - e^{-\theta^* x_c}\right)$.

As noted above, this study needs to estimate the lifetime performance index λ_L using the sample data since its real value is unknown. Let the electronic component lifetime T have an exponential distribution with mean θ^{-1}. Since the relative lifetime $X = T/L$, X is distributed as an exponential distribution with mean $(\theta^*)^{-1}$. Assume that (X_1, X_2, …, X_n) is a random sample set of X, while the type-I right censoring data (Y_1, Y_2, …, Y_n) is a random sample set of Y (given censoring time x_c).

In Equation (9), using the property of the invariance for the maximum likelihood estimator (*MLE*), we can find the *MLE* of index λ_L as follows:

$$\hat{\lambda}_L = \frac{1}{\hat{\theta}^*} = \sum_{i=1}^{n} Y_i / m_u \,. \tag{14}$$

Let the lifetime performance index λ_L be the function $h(\theta^*)$, $h(\theta^*) = \theta^{*-1}$. Using Equation (12) and the delta method (Casella and Berger [28]), we can get the asymptotic normal distribution for the estimator of $\hat{\lambda}_L = h(\hat{\theta}^*)$ as follows:

$$\sqrt{m_u}(h(\hat{\theta}^*) - h(\theta^*)) = \sqrt{m_u}(\hat{\lambda}_L - \lambda_L) \to N\left(0, \frac{1}{\theta^{*2}}\right), \tag{15}$$

where the asymptotic variance of $\hat{\lambda}_L$ (=$h(\hat{\theta}^*)$) is $\frac{1}{m_u\, \theta^{*2}} = \lambda_L^2 / m_u$. Thus, we have the following equation:

$$Z = \frac{h(\hat{\theta}^*) - h(\theta^*)}{\frac{1}{\sqrt{m_u}\, \theta^*}} = \frac{\hat{\lambda}_L - \lambda_L}{\frac{\lambda_L}{\sqrt{m_u}}} \to N(0, 1). \tag{16}$$

Given

$$\lim_{n\to\infty} E\left(\hat{\lambda}_L - \lambda_L\right)^2 = \lim_{n\to\infty} Var\left(\hat{\lambda}_L\right) = \lim_{m_u\to\infty} \frac{\lambda_L^2}{m_u} = 0, \tag{17}$$

the estimator of $\hat{\lambda}_L$ is unbiased and consistent with λ_L, displayed as follows:

$$E\left(\hat{\lambda}_L\right) = \lambda_L. \tag{18}$$

According to the Lehmann–Scheffé theorem, $\hat{\lambda}_L$ represents the uniformly minimum variance unbiased estimator (*UMVUE*) of λ_L. Using Equation (16), we can get the asymptotic standard normal distribution. The random variable Z, as noted above, is displayed in the following equation:

$$p\{-z_{\alpha/2} \le Z \le z_{\alpha/2}\} = p\left\{-z_{\alpha/2} \le \frac{\hat{\lambda}_L - \lambda_L}{\lambda_L/\sqrt{m_u}} \le z_{\alpha/2}\right\} = p\left\{-z_{\alpha/2} \le \sqrt{m_u}\left(\frac{\hat{\lambda}_L}{\lambda_L} - 1\right) \le z_{\alpha/2}\right\} = 1 - \alpha. \tag{19}$$

Thus, the $(1-\alpha)\times 100\%$ confidence interval of λ_L is $\left[L(\hat{\lambda}_L), U(\hat{\lambda}_L)\right]$, where the functions $L(\hat{\lambda}_L)$ and $U(\hat{\lambda}_L)$ are written as follows:

$$L(\hat{\lambda}_L) = \frac{\hat{\lambda}_L}{1+(z_{\alpha/2}/\sqrt{m_u})} \tag{20}$$

and

$$U(\hat{\lambda}_L) = \frac{\hat{\lambda}_L}{1-(z_{\alpha/2}/\sqrt{m_u})}, \tag{21}$$

where $z_{\alpha/2}$ represents the upper $\alpha/2$ quantile of the standard normal distribution. As mentioned above, the fuzzy test proposed in this article is not a traditional fuzzy test. It is based on the confidence interval. Under the same sample size, the shorter the length of the confidence interval, the smaller the error of the interval estimate, and the uniformly minimum variance unbiased estimator (*UMVUE*) is the estimator with the shortest confidence interval among the unbiased estimators, which can improve the accuracy of the test. Let the confidence interval length of λ_L be

$$l(\hat{\lambda}_L) = \hat{\lambda}_L\left(\frac{1}{1-z_{\alpha/2}/\sqrt{m_u}} - \frac{1}{1+z_{\alpha/2}/\sqrt{m_u}}\right). \tag{22}$$

Then the expected value of $l(\hat{\lambda}_L)$ is written as follows:

$$E(l(\hat{\lambda}_L)) = \lambda_L\left(\frac{1}{1-z_{\alpha/2}/\sqrt{m_u}} - \frac{1}{1+z_{\alpha/2}/\sqrt{m_u}}\right). \tag{23}$$

The electronic component lifetime *(T)* has an exponential distribution with mean θ^{-1}, while the relative lifetime X (=T/L) has an exponential distribution with mean $(\theta^*)^{-1}$. Let P_k be the k*th* percentile of the relative lifetime X, k = 1, 2, . . .,99, such that $P(X \le P_k) = k\%$. Then, the *kth* percentile (P_k) is expressed as follows:

$$P_k = \frac{\ln((100-k)\%)}{-\theta^*}. \tag{24}$$

Given the confidence interval level $(1-\alpha)\times 100\% = 95\%$, sample size n = 100, relative lifetime performance index λ_L = 1(1)5 and censoring time $x_c = P_k$ with k = 10(10)100, the number of uncensored data m_u is unknown, and then m_u is replaced with $E(m_u)$. When the censoring time is denoted as $x_c = P_k$, then the value of $E(m_u)$ is received as follows:

$$E(m_u) = np = n\times\ (X \le\ x_c) = n\times\ P(Y \le\ P_k)\ =\ n\ \times\ (k)\%. \tag{25}$$

When the number of samples, n = 100, is fixed, based on Equation (22), we can get the distribution for $E(l(\hat{\lambda}_L))$, as shown in Figure 1. As λ_L is fixed, the larger the censoring time x_c (=P_k) is, the smaller the average confidence interval length of $E(l(\hat{\lambda}_L))$ is, which means the estimate of the index λ_L is better. That is, when the number of uncensored data m_u is higher, the estimated indicator λ_L is better.

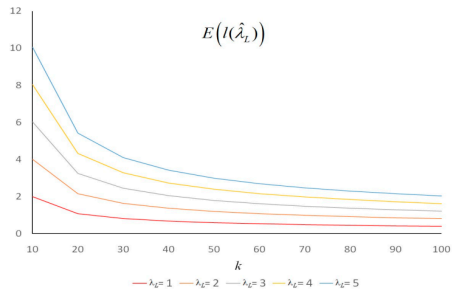


Figure 1. Curves for P_k, k = 10(10)100, λ_L = 1(1)5, n = 100, α = 0.05.

When the k value of quantile P_k is fixed, both the sample number n and the value of $E(m_u)$ become larger. When other conditions remain unchanged, the sample number n gets larger, whereas the value of $E\left(l\left(\hat{\lambda}_L\right)\right)$ gets smaller, indicating the estimate of the index λ_L is better, as shown in Figure 2.

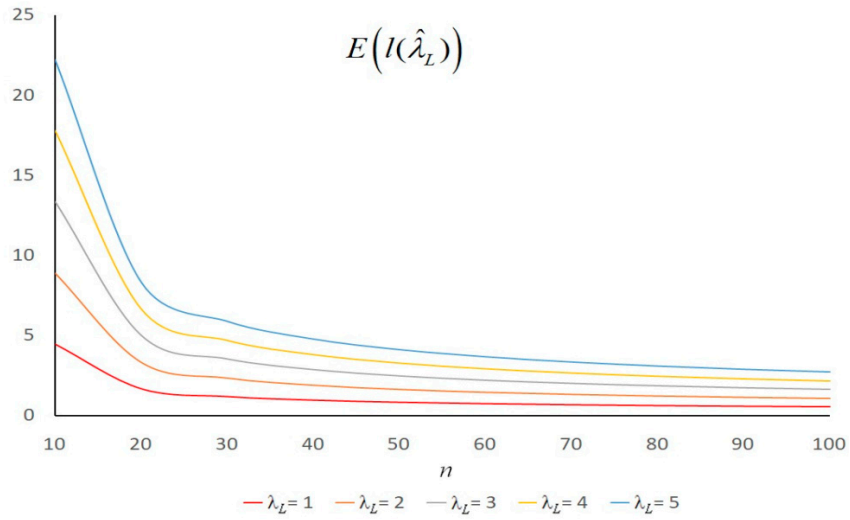


Figure 2. Curves for n = 10(10)100, λ_L = 1(1)5, P_k (k = 60), α = 0.05.

Based on Equations (23) and (25), the expected value $E\left(l\left(\hat{\lambda}_L\right)\right)$ of the confidence interval length for the relative lifetime performance index λ_L is related to the number of uncensored observations m_u. When m_u is larger, then $E\left(l\left(\hat{\lambda}_L\right)\right)$ is smaller, that is, the estimate of index λ_L is better. This result can be verified and illustrated in Figures 1 and 2. These findings, demonstrated by interval estimates in statistics, carry significant implications. Consequently, these factors can be dominated by research to boost interval estimates' accuracy.

3. Fuzzy Testing Method Built on Confidence Intervals

According to [16], when the sample size is large enough, due to the short length of the confidence interval, the statistical test for the lifetime performance of electronic products can be directly performed using the confidence interval of index λ_L. Since it is very time-consuming to obtain the lifetime data of electronic products, the sample size is usually not large enough and the confidence interval is wide. In order to reduce the risk of misjudgment caused by small samples, this article will use the confidence interval of index λ_L derived from Section 3 to construct the confidence interval-based fuzzy test. The observed value of this UMVUE is 50% of the quantile of this confidence interval. As mentioned above, the length of its confidence interval is the shortest estimator among unbiased estimators, which can not only improve the accuracy of statistical tests but also improve fuzzy tests based on confidence intervals.

Then, this section presents a fuzzy testing method—null hypothesis H_0: $\lambda_L \geq e$ versus alternative hypothesis H_1: $\lambda_L < e$—to evaluate if the lifetime performance index λ_L has reached the desired level. Before performing this method, we examine the testing rules of statistical hypotheses as follows:

(1) Given that $\hat{\lambda}_L \geq d_c$, do not reject H_0 and conclude that $\lambda_L \geq e$.
(2) Given that $\hat{\lambda}_L < d_c$, reject H_0 and conclude that $\lambda_L e$.

In the rules, the critical value d_0 is obtained using Equation (16). Then, the significant level α is received and expressed as

$$\alpha = min\, P\left\{\hat{\lambda}_L < d_c \middle| \lambda_L \in H_0\right\} = P\left\{Z < \frac{d_c - \lambda_L}{\lambda_L/\sqrt{m_u}} \middle| \lambda_L = e\right\}. \quad (26)$$

Next, we obtain the critical value d_c as follows:

$$d_c = e - z_\alpha \frac{e}{\sqrt{m_u}}. \tag{27}$$

Using Equations (2), (9), and (14), the observed value of estimate $\hat{\lambda}_L$ is calculated using the following equation:

$$\hat{\lambda}_{L0} = \frac{1}{\hat{\theta}^*} = \sum_{i=1}^{n} y_i / m_u. \tag{28}$$

Based on Buckley's approach [29], Equations (20) and (21), the $\alpha-$cuts of the triangular fuzzy number $\hat{\lambda}_{L0}$ is denoted by the following equation [16,17]:

$$\widetilde{\lambda}_{L0}[\alpha] = \begin{cases} [\lambda_{L01}(\alpha), \lambda_{L02}(\alpha)], \text{ for } 0.01 \le \alpha \le 1 \\ [\lambda_{L01}(0.01), \lambda_{L02}(0.01)], \text{ for } 0 \le \alpha \le 0.01 \end{cases}, \tag{29}$$

where

$$\lambda_{L01}(\alpha) = \frac{\hat{\lambda}_{L0}}{1+z_{\alpha/2}/\sqrt{m_u}} \tag{30}$$

and

$$\lambda_{L02}(\alpha) = \frac{\hat{\lambda}_{L0}}{1-z_{\alpha/2}/\sqrt{m_u}}. \tag{31}$$

Clearly, the value of $\lambda_{L01}(\alpha)$ is not equivalent to that of $\lambda_{L02}(\alpha)$ given that $\alpha < 1$. As $\alpha = 1$ and $-z_{\alpha/2} = z_{1-\alpha/2}$, we obtain the result $\lambda_{L01}(1)\lambda_{L02}(1) = \hat{\lambda}_{L0}$, where $\hat{\lambda}_{L0} = \hat{\lambda}_{L0}/(1 + z_{0.5}/\sqrt{m_u})$. Therefore, let $\lambda^*_{L0} = \hat{\lambda}_{L0}$, such that the $\alpha-$cuts of the triangular fuzzy number $\widetilde{\lambda}^*_{L0}$ is defined as

$$\widetilde{\lambda}^*_{L0}[\alpha] = \begin{cases} [\lambda^*_{L01}(\alpha), \lambda^*_{L02}(\alpha)], \text{ for } 0.01 \le \alpha \le 1 \\ [\lambda^*_{L01}(0.01), \lambda^*_{L02}(0.01)], \text{ for } 0 \le \alpha \le 0.01 \end{cases}, \tag{32}$$

where

$$\lambda^*_{L01}(\alpha) = \frac{1}{1+z_{\alpha/2}/\sqrt{m_u}}\,\hat{\lambda}_{L0} \tag{33}$$

and

$$\lambda^*_{L02}(\alpha) = \frac{1}{1-z_{\alpha/2}/\sqrt{m_u}}\hat{\lambda}_{L0}. \tag{34}$$

Obviously, the value of $\lambda^*_{L01}(\alpha)$ is equal to that of $\lambda^*_{L02}(\alpha)$ with $\alpha = 1$, and the newly transformed triangular fuzzy number is represented as $\widetilde{\lambda}^{**}_{L0}=\Delta(\lambda^{**}_{L0}, \lambda^{**}_{M0}, \lambda^{**}_{R0})$, where $\lambda^{**}_{M0}=\hat{\lambda}_{L0}$,

$$\lambda^{**}_{L0} = \frac{1}{1+z_{0.005}/\sqrt{m_u}}\,\hat{\lambda}_{L0} \tag{35}$$

and

$$\lambda^{**}_{R0} = \frac{1}{1-z_{0.005}/\sqrt{m_u}}\,\hat{\lambda}_{L0}. \tag{36}$$

Furthermore, the membership function of the fuzzy number $\widetilde{\lambda}^{**}_{L0}$ is denoted by

$$g(x) = \begin{cases} 0 & , if\ x < \lambda^{**}_{L0} \\ 2\left(1 - F_Z\left(\sqrt{m_u}\left(\frac{\hat{\lambda}_{L0}}{x} - 1\right)\right)\right) & , if\ \lambda^{**}_{L0} \le x < \hat{\lambda}_{L0} \\ 1 & , if\ x = \hat{\lambda}_{L0} \\ 2\left(1 - F_Z\left(\sqrt{m_u}\left(1 - \frac{\hat{\lambda}_{L0}}{x}\right)\right)\right) & , if\ \hat{\lambda}_{L0} < x \le \lambda^{**}_{R0} \\ 0 & , if\ \lambda^{**}_{R0} < x \end{cases}, \tag{37}$$

where F_Z is a cumulative distribution function of the standard normal distribution. Similar to the fuzzy number $\widetilde{\lambda}^*_{L0}$, the $\alpha-$cuts of triangular-shaped fuzzy critical value number $\widetilde{d_c}$ is expressed as

$$\widetilde{d_c}[\alpha] = \begin{cases} [d_{c1}(\alpha), d_{c2}(\alpha)], \ for\ 0.01 \le \alpha \le 1 \\ [d_{c1}(0.01), d_{c2}(0.01)], \ for\ 0 \le \alpha \le 0.01 \end{cases}, \tag{38}$$

where

$$d_{c1}(\alpha) = \frac{1}{1 + z_{\alpha/2}/\sqrt{m_u}}\, d_c \tag{39}$$

and

$$d_{c2}(\alpha) = \frac{1}{1 - z_{\alpha/2}/\sqrt{m_u}}\, d_c\,. \tag{40}$$

Clearly, the value of $d_{c1}(\alpha)$ is equivalent to that of $d_{c2}(\alpha)$ given that $\alpha = 1$ Then, a newly transformed triangular fuzzy number is denoted by $\widetilde{d_c} = \Delta\,(d_{cL}, d_{cM}, d_{cR})$, where $d_{cM} = d_c$,

$$d_{cL} = \frac{1}{1 + z_{0.005}/\sqrt{m_u}}\, d_c \tag{41}$$

and

$$d_{cR} = \frac{1}{1 - z_{0.005}/\sqrt{m_u}}\, d_c\,. \tag{42}$$

Furthermore, the membership function of the fuzzy number $\widetilde{d_c}$ is represented as

$$g_c(x) = \begin{cases} 0 & , \ if\ x < d_{cL} \\ 2\left(1 - F_Z\left(\sqrt{m_u}\left(\frac{d_c}{x} - 1\right)\right)\right) & , if\ d_{cL} \le x < d_c \\ 1 & , if\ x = d_c \\ 2\left(1 - F_Z\left(\sqrt{m_u}\left(1 - \frac{d_c}{x}\right)\right)\right) & , \ if\ d_c < x \le d_{cR} \\ 0 & , \ if\ d_{cR} < x \end{cases}, \tag{43}$$

where F_Z is a cumulative distribution function of the standard normal distribution. Subsequently, the curves of $g(x)$ and $g_c(x)$ are depicted below:

Based on Buckley [29] and Chiou [17], we let A_R be the area in the curved graph of $g_c(x)$ and be defined as

$$A_R = \{\,(x, \alpha) | d_{c1}(\alpha) \le x \le d_{c2}(\alpha), 0 \le \alpha \le 1\}. \tag{44}$$

Buckley [29], Chen and Yu [16], and Chiou [17] have stated that it is complicated to utilize integration to find the area of set A_R. A Riemann sum is performed in the study to find the area of the block. The procedures are described as follows:

(1) We divide A_R with n = 100 into 100 horizontal blocks. (2) Each block is calculated using a quasi-trapezoid area. (3) We find the sum of the areas for these 100 horizontal blocks. Suppose that $j = [100 \times \alpha]$, and $j = 0, 1, 2, \ldots, 100$ for $0 \le \alpha \le 1$. $[100 \times \alpha]$ represents the largest integer, less than or equivalent to $100 \times \alpha$. Similarly, $\alpha = j \times 0.01$, $j = 0, 1, 2, \ldots,$ 100. Set A_R is cut into 100 trapezoidal blocks by these 101 horizontal lines. Therefore, the area of the jth block is denoted by

$$A_{R\,j} = \{\,(x, \alpha) | d_{c1}(0.01 \times j) \le x \le d_{c2}(\alpha)(0.01 \times j), 0.01 \times (j-1) \le \alpha \le 0.01 \times j\},\ j = 0,\ 1,\ 2, \ldots,\ 100. \tag{45}$$

Next, the length of the jth horizontal line is depicted as

$$lr_j = \left(\frac{1}{1 - \frac{z_{0.005 \times j}}{\sqrt{m_u}}} - \frac{1}{1 + \frac{z_{0.005 \times j}}{\sqrt{m_u}}}\right)\ d_c\ ,\ j = 0,\ 1,\ 2, \ldots,\ 100. \tag{46}$$

Obviously, when $lr_0 = lr_1$ and $lr_{100} = 0$, then the area is defined as

$$A_R = \sum_{j=1}^{100} (0.01) \times \left(\frac{lr_{j-1} + lr_j}{2} \right) = 0.5 \times lr_1 + 0.01 \times \sum_{j=1}^{99} lr_j. \tag{47}$$

Moreover, let A_S be the area in the curved graph of $g_c(x)$ but to the right of the vertical line $x = \hat{\lambda}_{L0}$, expressed as

$$A_S = \left\{ (x, \alpha) | \hat{\lambda}_{L0} \le x \le d_{c2}(\alpha), 0 \le \alpha \le b \right\}, \tag{48}$$

where $\alpha = b$. Then $d_{c2}(b) = \hat{\lambda}_{L0}$. Similar to A_R, let $h = [100 \times b]$, and then $j = 0, 1, 2, \ldots, h$ for $0 \le \alpha \le b$. $[100 \times b]$ represents the largest integer, less than or equivalent to $100 \times b$. Clearly, $b = 0.01 \times h$ and $h + 1$ horizontal lines of $\alpha = j \times 0.01$, ($j = 0, 1, 2, \ldots, h$) cut A_S into h quasi-trapezoid blocks. Therefore, the area of the jth block is written as

$$A_{Sj} = \left\{ (x, \alpha) | \hat{\lambda}_{L0} \le x \le d_{c2}(0.01 \times j), 0.01 \times (j-1) \le \alpha \le 0.01 \times j \right\}, \; j = 0, 1, 2, \ldots, h. \tag{49}$$

Then, the length ls_j of the jth horizontal line is denoted by

$$ls_j = \frac{1}{1 - \frac{z_{0.005}}{\sqrt{m_u}}} d_c - \hat{\lambda}_{L0}, \; j = 0, 1, 2, \ldots, h. \tag{50}$$

Obviously, as $ls_0 = ls_1$ and $ls_h = 0$, then the area of A_R is written as

$$A_S = \sum_{j=1}^{h} (0.01) \times \left(\frac{ls_{j-1} + ls_j}{2} \right) = 0.5 \times ls_1 + 0.01 \times \sum_{j=1}^{h-1} ls_j. \tag{51}$$

According to some studies, the ratio of A_S to A_R (A_S / A_R) can be applied to a fuzzy decision, as shown below:

$$\frac{A_S}{A_R} = \frac{0.5 \times ls_1 + 0.01 \times \sum_{j=1}^{h-1} ls_j}{0.5 \times lr_1 + 0.01 \times \sum_{j=1}^{99} lr_j}. \tag{52}$$

It is extremely complicated to calculate the block areas of A_S and A_R using Equations (47) and (51), respectively. As a result, the calculation of the value of A_S / A_R is relatively complex (in Equation (52)). In order to facilitate industrial applications, the membership functions of $g(x)$ and $g_c(x)$ in Figure 3 both fall within the asymmetric graphical distributions. Based on the method by Chen and Yu [16], this paper employs the bottom length $l_S = d_{cR} - \hat{\lambda}_{L0}$ of block A_S to correspond to the area for A_s, since the function $g_c(x)$ is of the asymmetric graphical distribution. Additionally, in Figure 3, this paper uses the bottom length $l_R = 2(d_{cR} - d_c)$ of block A_R to correspond to the area for A_R, where $l_R \ge (d_{cR} - d_{cL})$. Next, $\frac{l_S}{l_R}$ corresponding to $\frac{A_S}{A_R}$ is utilized as a fuzzy evaluation tool, where l_S and l_R are defined as follows:

$$l_S = d_{cR} - \hat{\lambda}_{L0} = \frac{1}{1 - z_{0.005} / \sqrt{m_u}} d_c - \hat{\lambda}_{L0}, \tag{53}$$

and

$$l_R = 2(d_{cR} - d_c) = 2d_c \left(\frac{1}{1 - \frac{z_{0.005}}{\sqrt{m_u}}} - 1 \right). \tag{54}$$

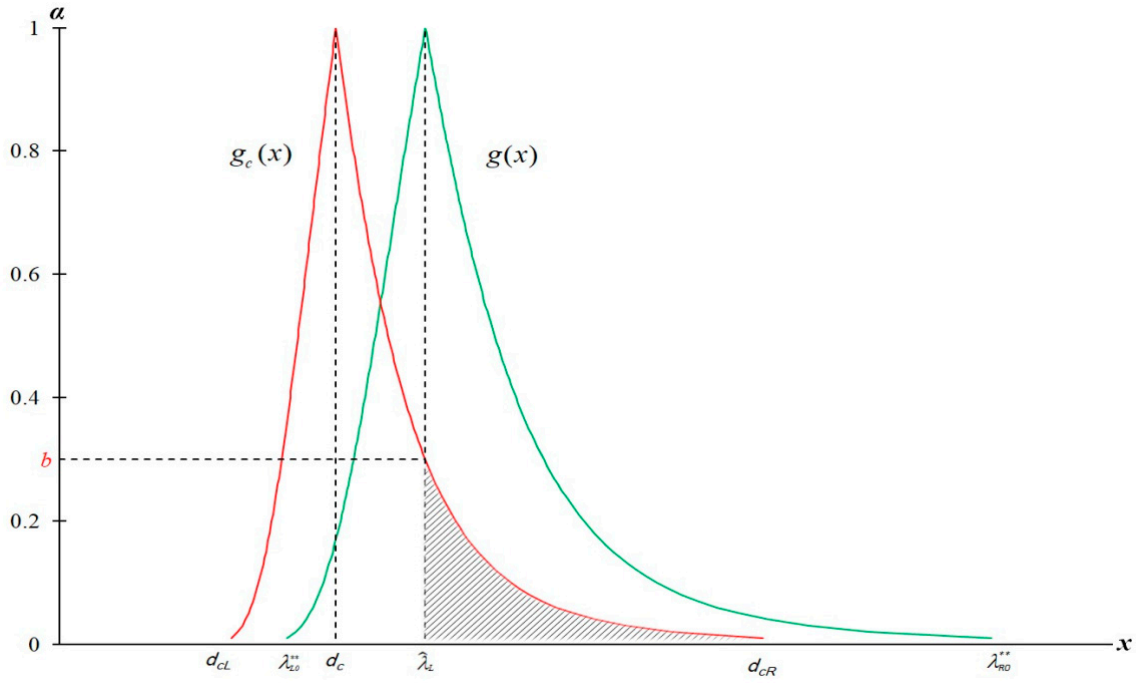


Figure 3. The membership functions of $g(x)$ and $g_c(x)$.

According to Chen and Yu [16], if we let $\eta = \frac{l_S}{l_R}$, then process engineers can determine two values, η_1 and η_2, as the basis for fuzzy decision-making using their past accumulated experiences or long-term production data analysis cases. As noted by Buckley [29], we let $0 < \eta_1 < \eta_2 < 0.5$. Applying these two numbers should be taken into consideration for fuzzy testing. Therefore, the rules for fuzzy testing are established below [17]:

(1) If $\eta < \eta_1$ then H_0 is rejected and $\lambda_L \geq e$ is concluded.
(2) If $\eta_1 \leq \eta \leq \eta_2$, then a decision on "reject/not reject" is not made.
(3) If $\eta_2 < \eta < 0.5$, then H_0 is rejected and $\lambda_L < e$ is concluded.

4. Practical Example

This section illustrates the fuzzy testing method demonstrated in Section 3 using a numerical example. According to Chen and Yu [16], the value of the lifetime performance index must be greater than 3. Therefore, the null hypothesis is H_0: $\lambda_L \geq 3$, whereas the alternative hypothesis is H_1: $\lambda_L < 3$.

Here are 30 sample data points $(t_1, t_2, \ldots, t_{30})$ =

(2.31869876,	0.450464354,	5.959543721,	0.19815643,	2.36292942,
1.711931047,	1.463690428,	2.177143634,	1.336923286,	0.160569092,
0.117929117,	2.045192427,	5.126371449,	0.718544566,	0.279190037,
5.722989698,	0.527396462,	0.760546501,	2.090882944,	2.353623477,
1.492238468,	0.967454182,	1.050979265,	1.878164671,	2.85121394,
1.06125565,	8.876851026,	1.889987275,	0.185337992,	1.2043438),

representing the lifetimes T of electronic components generated from an exponential distribution with a mean of 3.

We work with the relative lifetimes X (=T/L), $(x_1, x_2, \ldots, x_{30}) = (t_1, t_2, \ldots, t_{30})$ with $L = 1$ of electronic components. Additionally, let the given censoring time be $x_c = P_{60}$. Based on Equation (28), we obtain the number of uncensored data, $m_u = n \times k = 18$, and then the censoring time $x_c = P_{60}$. $= x_{(18)} = 1.878164671$. Using Equation (2), the type-I observation censored data, $(y_1, y_2, \ldots, y_{30})$ =

(0.117929117,	0.160569092,	0.185337992	0.19815643,	0.279190037,
0.450464354,	0.527396462,	0.718544566,	0.760546501,	0.967454182,
1.06125565,	1.2043438,	1.336923286,	1.463690428,	1.492238468,
1.711931047,	1.878164671,	1.878164671,	1.878164671,	1.878164671,
1.878164671	1.878164671	1.878164671	1.878164671	1.878164671
1.878164671	1.878164671	1.878164671	1.878164671	1.878164671).

Using Equation (28), the observed value of the estimate is expressed as follows.

$$\hat{\lambda}_{L0} = \frac{\sum_{i=1}^{n} y_i}{m_u} = \frac{38.10309}{18} = 2.11684 \tag{55}$$

According to Equations (35) and (36), the values of λ_{L0}^{**} and λ_{R0}^{**} can be calculated as follows:

$$\lambda_{L0}^{**} = \frac{1}{1+\frac{z_{0.005}}{\sqrt{m_u}}}\,\hat{\lambda}_{L0} = \frac{1}{1+\frac{2.57583}{\sqrt{18}}} \times 2.11684 = 1.31716 \tag{56}$$

and

$$\lambda_{R0}^{**} = \frac{1}{1-\frac{z_{0.005}}{\sqrt{m_u}}}\,\hat{\lambda}_{L0} = \frac{1}{1-\frac{2.57583}{\sqrt{18}}} \times 2.11684 = 5.38813 \tag{57}$$

Applying Equations (56) and (57) to Equation (37), we define the membership function of the fuzzy number $\widetilde{\lambda}_{L0}^{**}$ as

$$d(x) = \begin{cases} 0 & , if\ x < 1.31716 \\ 2\left(1 - F_Z\left(\sqrt{18}\left(\frac{2.11684}{x} - 1\right)\right)\right) & , if\ 1.31716 \le x < 2.11684 \\ 1 & , if\ x = 2.11684 \\ 2\left(1 - F_Z\left(\sqrt{18}\left(1 - \frac{2.11684}{x}\right)\right)\right) & , if\ 2.11684 < x \le 5.38813 \\ 0 & , if\ 5.38813 < x \end{cases} \tag{58}$$

The two hypotheses are denoted by H_0: $\lambda_L \ge 3$ versus H_1: $\lambda_L < 3$, given the significant level $\alpha = 0.05$. Using Equation (27), we can obtain the critical value as follows:

$$d_c = e - z_\alpha \frac{e}{\sqrt{m_u}}\bigg|_{e=3} = 1.83681. \tag{59}$$

According to Equations (41) and (42), we find the values of d_{cL} and d_{cR} below:

$$d_{cL} = \frac{1}{1+\frac{2.57583}{\sqrt{18}}} \times 1.83681 = 1.14291 \tag{60}$$

and

$$d_{cR} = \frac{1}{1-\frac{2.57583}{\sqrt{18}}} \times 1.83681 = 4.67535. \tag{61}$$

Applying Equations (60) and (61) to Equation (43), we define the membership function of the fuzzy number $\widetilde{d_c}$ as

$$d_c(x) = \begin{cases} 0 & , if\ x < 1.14291 \\ 2\left(1 - F_Z\left(\sqrt{18}\left(\frac{1.83681}{x} - 1\right)\right)\right) & , if\ 1.14291 \le x < 1.83681 \\ 1 & , if\ x = 1.83681 \\ 2\left(1 - F_Z\left(\sqrt{18}\left(1 - \frac{1.83681}{x}\right)\right)\right) & , if\ 1.83681 < x \le 4.67535 \\ 0 & , if\ 4.67535 < x \end{cases} \tag{62}$$

Subsequently, the curves of $d(x)$ and $d_c(x)$ are depicted in Figure 4.

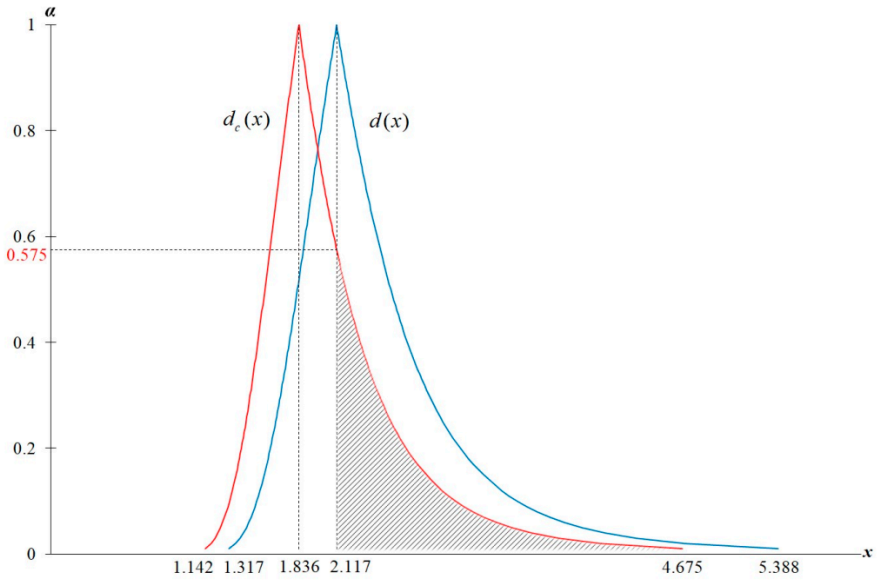


Figure 4. Membership functions $d(x)$ and $d_c(x)$ applied in the numerical example.

Using Equations (53) and (54), we obtain the values of l_S and l_R:

$$l_S = d_{cR} - \hat{\lambda}_{L0} = \frac{1}{1 - \frac{z_{0.005}}{\sqrt{m_u}}} d_c - \hat{\lambda}_{L0} = \frac{1}{1 - \frac{2.57583}{\sqrt{18}}} \times 1.83681 - 2.11684 = 2.55851 \quad (63)$$

and

$$l_R = 2(d_{cR} - d_c) = 2d_c \left(\frac{1}{1 - \frac{z_{0.005}}{\sqrt{m_u}}} - 1 \right) = 2 \times 1.83681 \times \left(\frac{1}{1 - \frac{2.57583}{\sqrt{18}}} - 1 \right) = 5.67708 \quad (64)$$

Therefore, $\eta = l_S / l_R = 2.55851 / 5.67708 = 0.45067$. Based on statistical inference, $\hat{\lambda}_{L0} = 2.11684$ is in the region of "do not reject H_0" ($\{\hat{\lambda}_{L0} < 1.83681\}$; using Equation (59)), the conclusion is "do not reject H_0, and we conclude that $\lambda_L \geq 3$". According to the rules for fuzzy testing, the value of η_1 is 0.2, and the value of η_2 is 0.4 [16,17]. Therefore, we get $\eta_2 < \eta < 0.5$, indicating that H_0 needs to be rejected and $\lambda_L < 3$ is concluded. Nevertheless, $\hat{\lambda}_{L0} = 2.11684$ is much lower than $\lambda_L = 3$. Accordingly, based on the fuzzy testing method proposed in this study, H_0 needs to be rejected and $\lambda_L < 3$ is concluded. According to the above fuzzy testing rules, this result seems reasonable in view of practicability.

5. Conclusions

This paper places great emphasis on developing a model for evaluating a product's lifetime performance, utilizing the assessment process to enhance its lifetime performance, thereby increasing its value. It can also help attain the goals of green production—saving energy and diminishing waste. In this paper, the electronic component, lifetime (T), was adopted from the exponential distribution that is often used in general studies. In addition, we also employed the electronic component lifetime and the relative electronic component lifetime X (=T/L) of the minimum required unit time L as well as constructed a relative lifetime performance index. The larger the index value, the better the electronic component performance. When collecting electronic component lifetime data, we often fail to collect complete data for several reasons, such as time, cost, and manpower. Consequently, we employed type-I censoring and derived the best estimator (*UMVUE*) for the lifetime performance index. In the meantime, a fuzzy testing model was proposed and built on the derived confidence intervals. Moreover, this fuzzy testing model prevented misjudgments

caused by sampling errors, making it highly suitable for the service and manufacturing industries, which place utmost value on timeliness. Additionally, this model is also applicable to experiments that require destructive testing, higher costs, or longer durations. Since passive components are a cornerstone of the electronics industry and play a vital role in driving peripheral equipment and industries, passive components were utilized as the basis to discover and explain the proposed model in this paper. At the same time, a numerical example was also put forward to illustrate the proposed model.

Author Contributions: Conceptualization, K.-C.C. and K.-S.C.; methodology, K.-C.C. and K.-S.C.; software, T.-H.H.; validation, T.-H.H.; formal analysis, K.-C.C., K.-S.C. and C.-M.Y.; resources, T.-H.H.; data curation, C.-M.Y.; writing—original draft preparation, K.-C.C., T.-H.H., K.-S.C. and C.-M.Y.; writing—review and editing, K.-C.C., K.-S.C. and C.-M.Y.; visualization, T.-H.H.; supervision, K.-S.C.; project administration, K.-C.C. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Suppipat, S.; Hu, A.H. Achieving sustainable industrial ecosystems by design: A study of the ICT and electronics industry in taiwan. *J. Clean. Prod.* **2022**, *369*, 133393. [CrossRef]
2. Wang, C.T.; Chiu, C.S. Competitive strategies for Taiwan's semiconductor industry in a new world economy. *Technol. Soc.* **2014**, *36*, 60–73. [CrossRef]
3. Amindoust, A.; Ahmed, S.; Saghafinia, A.; Bahreininejad, A. Sustainable supplier selection: A ranking model based on fuzzy inference system. *Appl. Soft Comput.* **2012**, *12*, 1668–1677. [CrossRef]
4. Kao, F.C.; Huang, S.C.; Lo, H.W. A rough-fermatean DEMATEL approach for sustainable development evaluation for the manufacturing industry. *Int. J. Fuzzy Syst.* **2022**, *24*, 3244–3264. [CrossRef]
5. Pearn, W.L.; Cheng, Y.C. Measuring production yield for processes with multiple characteristics. *Int. J. Prod. Res.* **2010**, *48*, 4519–4536. [CrossRef]
6. Wu, M.F.; Chen, H.Y.; Chang, T.C.; Wu, C.F. Quality evaluation of internal cylindrical grinding process with multiple quality characteristics for gear products. *Int. J. Prod. Res.* **2019**, *57*, 6687–6701. [CrossRef]
7. Yang, C.M. An improved multiple quality characteristic analysis chart for simultaneous monitoring of process mean and variance of steering knuckle pin for green manufacturing. *Qual. Eng.* **2021**, *33*, 383–394. [CrossRef]
8. Menon, R.R.; Ravi, V. Using ANP and QFD methodologies to analyze eco-efficiency requirements in an electronic supply chain. *Clean. Eng. Technol.* **2021**, *5*, 100350. [CrossRef]
9. Simmou, W.; Govindan, K.; Sameer, I.; Hussainey, K.; Simmou, S. Doing good to be green and live clean!—Linking corporate social responsibility strategy, green innovation, and environmental performance: Evidence from maldivian and moroccan small and medium-sized enterprises. *J. Clean. Prod.* **2023**, *384*, 135265. [CrossRef]
10. Yang, Y.; Jiang, Y. Buyer-supplier CSR alignment and firm performance: A contingency theory perspective. *J. Bus. Res.* **2023**, *154*, 113340. [CrossRef]
11. Anderson, D.R.; Sweeney, D.J.; Williams, T.A. *Statistics for Business and Economics*; West Publ. Co.: Saint Paul, MN, USA, 1990.
12. Keller, G.; Warrack, B.; Bartel, H. *Statistics for Management and Economics*; Duxbury Press: Belmont, CA, USA, 1994.
13. Lee, H.M.; Wu, J.W.; Lei, C.L. Assessing the lifetime performance index of exponential products with step-stress accelerated life-testing data. *IEEE Trans. Reliab.* **2013**, *62*, 296–304. [CrossRef]
14. Lee, W.C.; Wu, J.W.; Hong, C.W.; Hong, S.F. Evaluating the lifetime performance index based on the bayesian estimation for the rayleigh lifetime products with the upper record values. *J. Appl. Math.* **2013**, *2013*, 547209. [CrossRef]
15. Hsu, B.M.; Shu, M.H.; Chen, B.S. Evaluating lifetime performance for the pareto model with censored and imprecise information. *J. Stat. Comput. Simul.* **2011**, *81*, 1817–1833. [CrossRef]
16. Chen, K.S.; Yu, C.M. Lifetime performance evaluation and analysis model of passive component capacitor products. *Ann. Oper. Res.* **2022**, *311*, 51–64. [CrossRef]
17. Chiou, K.C. Building up of Fuzzy Evaluation Model of Life Performance Based on Type-II Censored Data. *Mathematics* **2023**, *11*, 3686. [CrossRef]
18. Chen, K.S.; Chiou, K.C.; Yu, C.M. Lifetime performance index of electronic products. *Microelectron. Reliab.* **2020**, *113*, 113941. [CrossRef]
19. Miller, R.G. *Survival Analysis*; John Wiley and Sons: New York, NY, USA, 1981.

20. Khorram, E.; Farahani, Z.S.M. Statistical inference of weighted Exponential lifetimes under progressive type-II censoring scheme. *Qual. Technol. Quant. Manag.* **2014**, *11*, 433–451. [CrossRef]
21. Wu, S.F.; Hsieh, Y.T. The assessment on the lifetime performance index of products with Gompertz distribution based on the progressive type I interval censored sample. *J. Comput. Appl. Math.* **2019**, *351*, 66–76. [CrossRef]
22. Wu, S.F.; Wu, Y.C.; Wu, C.H.; Chang, W.T. Experimental design for the lifetime performance index of weibull products based on the progressive type i interval censored sample. *Symmetry* **2021**, *13*, 1691. [CrossRef]
23. Wang, T.C.; Wu, C.W.; Shu, M.H. A variables-type multiple-dependent-state sampling plan based on the lifetime performance index under a weibull distribution. *Ann. Oper. Res.* **2022**, *311*, 381–399. [CrossRef]
24. Chen, H.Y.; Lin, K.P. Fuzzy supplier selection model based on lifetime performance index. *Expert Syst. Appl.* **2022**, *208*, 118135. [CrossRef]
25. Ahmed, N.; Ali, S.; Shah, I. Control charts for monitoring mean of generalized exponential distribution with type-I censoring. *Qual. Reliab. Eng. Int.* **2022**, *38*, 592–614. [CrossRef]
26. Raza, S.M.M.; Riaz, M.; Ali, S. EWMA Control Chart for Poisson-Exponential Lifetime Distribution under Type i Censoring. *Qual. Reliab. Eng. Int.* **2016**, *32*, 995–10051. [CrossRef]
27. Wu, S.F.; Jheng, J.J.; Chang, W.T. Sampling design for the lifetime performance index of exponential lifetime distribution under progressive type I interval censoring. *Commun. Stat.-Theory Methods* **2023**, *52*, 2766–2782. [CrossRef]
28. Casella, G.; Berger, R.L. *Statistical Inference*, 2nd ed.; Duxbury Press: Pacific Grove, CA, USA, 2002.
29. Buckley, J.J. Fuzzy statistics: Hypothesis testing. *Soft Comput.* **2005**, *9*, 512–518. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.