

Article

Simplification of Electrode Profiles for Piezoelectric Modal Sensors by Controlling Gap-Phase Length

Marta Martín-Nieto ^{1,2}, Damián Castaño ³, Sergio Horta Muñoz ^{4,*} and David Ruiz ⁵

¹ Escuela de Ingeniería Industrial y Aeroespacial de Toledo, Universidad de Castilla-La Mancha, Av. Carlos III, Campus Fábrica de Armas, 45004 Toledo, Spain; marta.martinnieto@uclm.es

² CES Don Bosco, Universidad Complutense de Madrid, María Auxiliadora 9, 28040 Madrid, Spain

³ Geonum Group, Escuela de Ingeniería Industrial y Aeroespacial de Toledo, Universidad de Castilla-La Mancha, Av. Carlos III, Campus Fábrica de Armas, 45004 Toledo, Spain; damian.castano@uclm.es

⁴ Instituto de Investigación Aplicada a la Industria Aeronáutica, Escuela de Ingeniería Industrial y Aeroespacial de Toledo, Universidad de Castilla-La Mancha, Av. Carlos III, Campus Fábrica de Armas, 45004 Toledo, Spain

⁵ OMEVA Research Group, Escuela de Ingeniería Industrial y Aeroespacial de Toledo, Universidad de Castilla-La Mancha, Av. Carlos III, Campus Fábrica de Armas, 45004 Toledo, Spain; david.ruiz@uclm.es

* Correspondence: sergio.horta@uclm.es

Abstract: This paper presents a method to optimize and simplify the electrode profile of a piezoelectric modal sensor. At the same time that the electrode profile is optimized to maximize the charge, a null-polarity phase is introduced. This gap-phase is modeled using the normalized norm of the spatial gradient of the polarity profile along with a two-step filtering and projection technique. High-order vibration modes generate a complex polarization profile that makes the manufacturing process difficult. The novelty of the proposed method is the addition of a constraint on the length of the interface in the topology optimization problem. This constraint simplifies the optimal designs and facilitates the fabrication process. Several examples show the simplified electrode profile that maximizes the electrical charge produced by a vibration mode, while reducing the number of different polarization regions by means of the gap-length constraint.

Keywords: topology optimization; piezoelectric sensor; finite element method; material interface

MSC: 35Q74



Citation: Martín-Nieto, M.; Castaño, D.; Horta Muñoz, S.; Ruiz, D. Simplification of Electrode Profiles for Piezoelectric Modal Sensors by Controlling Gap-Phase Length. *Mathematics* **2024**, *12*, 1525. <https://doi.org/10.3390/math12101525>

Academic Editor: Fausto Sargeni

Received: 17 April 2024

Revised: 6 May 2024

Accepted: 10 May 2024

Published: 14 May 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Recent challenges in the energy sector require exploration of novel and sustainable approaches, with a particular emphasis on advancing smart materials and structures. Smart materials are those that adapt to environmental changes and function optimally under the prevailing conditions [1]. Structural health monitoring (SHM) relies on a network of sensors and actuators integrated into the structure, with smart materials offering an eco-friendly, cost-efficient, self-powered solution for in-service diagnostic systems within structures during their service life. Applications are found in numerous sectors, such as the aerospace [2,3], civil sector [4,5], or the energy and process industry [6].

Frequently, piezoelectric devices play a crucial role in providing both monitoring and actuation capabilities, such as damage sensing and self-healing [7,8]. In particular, piezoelectric materials serve as versatile components in the field of vibration monitoring. These materials exhibit rapid response, high coupling coefficients, and non-intrusive integration into structures [9]. Additionally, when they are conveniently placed, they can act as energy harvesters to generate autonomous sensation in terms of energy [10,11]. However, the application of piezoelectric materials is limited to large structures due to challenges in scaling up and integrating these materials effectively.

Therefore, a search for different algorithms that provide an optimal solution to the placement of sensors has been developed in scientific literature [12]. One commonly utilized approach is topology optimization (TO), a mathematical technique used to optimize material distribution within a specified design area. This approach is considered one of the most widely employed methods for creating smart structures [13] and energy harvesters [14]. TO, in combination with numerical techniques such as the Finite Element Method (FEM), allows for the modeling of different multi-physical problems [15–20]. It aims to obtain a design that minimizes an objective function subject to different constraints. By judiciously redistributing material within a given volume, novel configurations that enhance sensor and actuator performance can be discovered. In the context of modal sensors, TO aims to maximize sensitivity, bandwidth, and accuracy by redistributing piezoelectric material within the sensor structure. Optimized sensor layouts yield improved detection capabilities for structural vibrations [21–23].

While TO offers remarkable benefits, challenges remain. The intricate shapes generated may pose manufacturing difficulties, necessitating post-optimization adjustments. Researchers actively explore ways to enhance manufacturability while preserving some of the performance improvements [24,25]. This has applications not only in the design of piezoelectric sensors [26–28], but also in manufacturing parts in molding and additive manufacturing processes [29–33].

There are several TO problems for which it is critical to identify and control the interface on the boundary of the design.

On the one hand, there are challenges associated with the mechanical and geometrical design of bi-material shell-type structures. The coatings serve to protect or enhance certain functionalities by covering the surface of the substrate. They also act as an interface to improve certain functionalities. Wang and Kang [34] and Fu et al. [35,36] have proposed approaches using the level-set method to address these issues. This method enables the acquisition of clear and smooth boundary representations; however, these works are limited to achieving a uniform thickness in the skin/coating. Moreover, Harvey and Hubert [37] apply coating control to the design of sandwich-type composite materials, allowing for optimizing the stiffness of a bi-material structure with a lightweight core and rigid skin made of reinforced composite material. The main difference in Harvey and Hubert's approach compared to previous works [34–36] is the application of the well-known Solid Isotropic Material with Penalization (SIMP) method [38] combined with a two-step filtering approach. Similarly, Yoon and Yi [39] develop an approach to the topology optimization problem of porous structure coating using density filter and p-norm approach. This methodology minimizes compliance subject to mass constraint, where optimal layouts are influenced by considering either the coating or layered structure. Later, Yi et al. [40] improved on their previous work by enhancing the ability to smoothly transition from both void and base materials to the coating, thus facilitating the easy merging of isolated coating structures when necessary.

On the other hand, another issue that requires interface control, which is more relevant to the current study, is the design of smart structures using piezoelectric coatings. This allows for the creation of piezoelectric modal sensors and actuators that optimize the measured/generated electric charge. Therefore, complex geometries can result in improved modal response characteristics such as enhanced sensitivity, wider bandwidth, and reduced cross-coupling effects. These improvements offer potential enhancements in sensor accuracy and actuator efficiency. However, manufacturing such intricate structures may be expensive and time-consuming while also being prone to defects. As a result, the practical application of optimized designs is hindered by production challenges. Thus, achieving optimal performance while addressing practical manufacturability continues to pose an ongoing challenge. The electrode profile of piezoelectric modal sensors is a critical aspect that needs to be simplified for practicality without significantly sacrificing performance.

Donoso et al. [26–28] address the issue of fabricating piezo-modal transducers. The study [26] emphasizes the need for electrodes to include null-polarity phases (also known

as gap-phases) of a certain width between areas of opposite polarity during manufacturing to prevent short-circuiting and ensure proper functioning. By controlling electrode profile interpolation using density-based and virtual temperature methods, the researchers incorporate the effect of gap-phase width into the design problem formulation. In their subsequent work [27], designs exhibit isolated features with like-polarity that become more pronounced as additional modes are considered, posing challenges from a manufacturing and operation perspective due to complex wiring schemes. To simplify electrode phase connectivity and required wiring schemes, this work introduces a connectivity constraint on electrode profiles. Similarly, in [28], the authors present a new systematic method for designing two-phase electrodes while maintaining optimal behavior by imposing connectivity constraints on each phase so that only one wire per phase is necessary.

In the present work, the aim is to rigorously explore the simplification of electrodes in modal sensors through topology optimization (TO) with a maximum gap-phase constraint. A methodology is being developed to reduce the complexity of a piezoelectric sensor when it experiences vibration modes that result in multiple zones with different polarities in the optimal design. By limiting the maximum size of the existing gap-phase, it becomes possible to decrease the number of electrodes with different polarities. This leads to a design that may not be optimal for generating electrical charge, but it significantly reduces manufacturing challenges while maintaining an acceptable level of performance. Additionally, this approach enhances the robustness of the device by eliminating small details in the electrode profile. The interface between the electrodes is modeled by incorporating the normalized norm of the spatial gradient of the polarity profile, along with a two-step filtering and projection technique. Through the discussion of relevant case studies and the consideration of electromechanical and manufacturability aspects, researchers and engineers are provided with practical strategies for achieving a balance between performance-driven design and practical manufacturability.

2. Interface Problems

The first motivation for introducing an interface between two phases is presented in [41]. In this work, the authors incorporate coated structures in the context of minimizing the compliance in a topology optimization problem. This procedure allows for the identification of the material interface through the spatial gradient of the design field, enabling the separation and control of the length scales for both the base structure and the coating.

This approach is extended to the optimal design of electrodes in [26]. In this work, the interface represents a null-polarity phase, which separates areas of opposite polarity with the objective of avoiding short-circuiting.

With independence of the application, the base of the method is a two-step filtering/projection approach. Considering ρ as the initial design field, the smoothed field $\tilde{\rho}$ is obtained as the solution of the Helmholtz partial differential equation (PDE) with Neumann boundary conditions [42]:

$$\begin{cases} -r_1^2 \nabla^2 \tilde{\rho} + \tilde{\rho} = \rho \\ \frac{\partial \tilde{\rho}}{\partial \mathbf{n}} = 0, \end{cases}$$

where the boundary is defined by the normal vector $\mathbf{n}$. From now on, the usual filter radius, defined as R_1 , is used to compute the scalar r_1 :

$$r_1 = \frac{R_1}{2\sqrt{3}}.$$

Note that, as opposed to the classical conic density filter [43,44], the PDE-filter is solved for nodal density variables instead of element density variables.

The filtering approach introduces intermediate densities whose area is difficult to manufacture. This issue is overcome with a projection method [45], where the projected variable $\bar{\rho}$ is computed with the following equation:

$$\bar{\rho} = \frac{\tanh(\beta_1 \eta_1) + \tanh(\beta_1 (\bar{\rho} - \eta_1))}{\tanh(\beta_1 \eta_1) + \tanh(\beta_1 (1 - \eta_1))},$$

where β_1 determines the sharpness of the projection, and $\eta_1 \in [0, 1]$ is the threshold parameter. At this point, with a high value of β_1 , the desired black and white design (in any context) is perfectly defined; however, this step represents a problem for the computation of the spatial gradient due to the non-smooth projected field. A second PDE filter is applied to obtain the smoothed variable φ :

$$-r_2^2 \nabla^2 \varphi + \varphi = \bar{\rho},$$

with

$$r_2 = \frac{R_2}{2\sqrt{3}}.$$

The spatial gradient of the variable φ must be normalized in order to model a sharp material interface:

$$\|\nabla \varphi\|_\gamma = \gamma \|\nabla \varphi\|$$

where $\gamma = R_2/\sqrt{3}$ [41]. Finally, this field is projected again with β_2 and η_2 :

$$\overline{\|\nabla \varphi\|_\gamma} = \frac{\tanh(\beta_2 \eta_2) + \tanh(\beta_2 (\|\nabla \varphi\|_\gamma - \eta_2))}{\tanh(\beta_2 \eta_2) + \tanh(\beta_2 (1 - \eta_2))}.$$

In Figure 1, a circular crown is shown to illustrate the effect of the two-step filtering. All the design fields take values between 0 and 1.

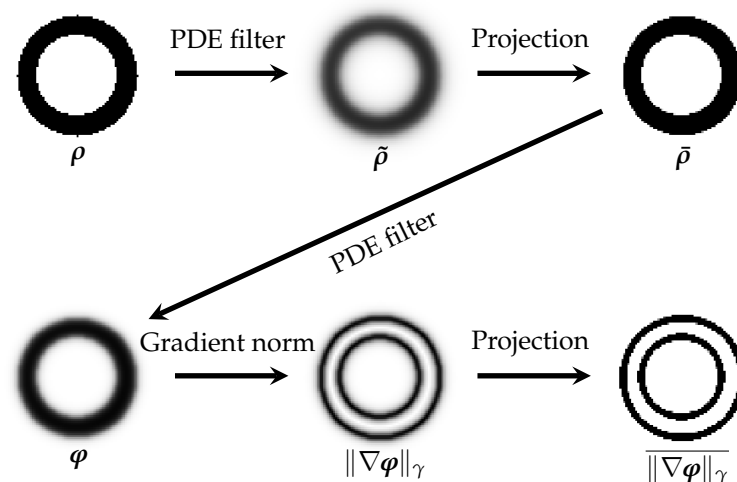


Figure 1. Two-step filter process.

3. Modal Sensors

Having in mind the pioneer work [46] where the piezoelectric polarization is optimized for one-dimensional beams, this idea has been extrapolated to two-dimensional plates. The objective is to maximize the electric charge generated by a vibration mode. The piezoelectric stress/charge constants in x and y directions are represented as e_{31} and e_{32} . The first subscript indicates the direction of the polarization field, in this case the z -axis. The second subscript is related to the direction where the strain is applied, 1 for the x -axis and 2 for the y -axis. From now on, the piezoelectric material used is considered isotropic, $e_{31} = e_{32}$, and the polarization axes are coincident with the geometric ones of the plate [47]. Based on these assumptions, it is important to notice that the piezoelectric properties are not needed in the problem formulation and just appear as a scaling factor that will be omitted in the discrete problem for the sake of simplicity.

$$q = \int_{\Omega} P(x, y) [e_{31}\varepsilon_{11} + e_{32}\varepsilon_{22}] d\Omega = e_{31} \int_{\Omega} P(x, y) \left[\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + z \left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} \right) \right] d\Omega$$

where $P(x, y)$ is the polarization profile, (u_1, u_2, u_3) represents the displacement field (vibration mode) and Ω is the design domain.

From the numerical point of view, it is convenient to define a new variable: $P(x, y) = 2\chi(x, y) - 1$, with $\chi(x, y) \in \{0, 1\}$. The domain Ω is discretized in finite elements, where the integer variable χ is relaxed and discretized into a density variable $\rho \in [0, 1]$. This variable represents the polarity of the electrode placed over the piezoelectric layer, with negative or positive values, respectively, for each element.

The Kirchhoff–Love plate theory is used to model the plate, using plate-type finite elements with four nodes and three degrees of freedom per node: two rotations over the x - and y -axis and a displacement over the z -axis. The thickness of the electrode layer is negligible compared to the global thickness of the plate. Additionally, this thickness is small enough compared to the x - and y - lengths of the plate to consider a plane stress state.

For the i -th vibration mode, the optimization problem is formulated as follows:

$$\begin{aligned} \max_{\rho} : q_i &= \mathbf{P}^T(\rho) \mathbf{B}^T \mathbf{U}_i = (2\rho^T - 1) \mathbf{B}^T \mathbf{U}_i \\ \text{s.t.} : \\ &\begin{cases} (\mathbf{K} - \omega_i^2 \mathbf{M}) \mathbf{U}_i = \mathbf{0} \\ \rho_e \in [0, 1], \quad e = 1, \dots, n_e, \end{cases} \end{aligned} \quad (1)$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness and mass matrices, respectively, $(\omega_i, \mathbf{U}_i)$ are the i -th eigenvalue and the associated eigenmode, $\mathbf{B}$ is the displacement-strain matrix [48] and n_e is the number of elements. The eigenproblem is solved once, as the vibration mode $\mathbf{U}_i$ does not depend on the design variable ρ .

The maximization of the electrical charge is written in terms of a topology optimization problem. It is important to emphasize the linear behavior of the objective function in Equation (1) with respect to the design variable ρ . This leads us to solve a linear programming problem, where the variable always take the extreme values and, unlike the classical TO problems, the design variable does not need any kind of regularization [49].

These devices are particularly useful in adaptive structures, where real-time monitoring and control of vibrations are essential for performance optimization and structural integrity.

4. Problem Formulation

Considering the objective of maximizing the electric charge generated by a vibration mode, while simplifying the polarization profile, a constraint over the maximum length of the interface is included in the optimization problem.

One vibration mode of high-order is selected, with the motivation of obtaining a big alternation in the electrode profile.

The electric charge associated to the i -th vibration mode is defined as an interpolation of $\bar{\rho}$ and $\|\nabla \varphi\|_{\gamma}$:

$$q_i = \mathbf{P}^T(\bar{\rho}, \|\nabla \varphi\|_{\gamma}) \mathbf{B}^T \mathbf{U}_i = \sum_{e=1}^{n_e} (1 - \|\nabla \varphi_e\|_{\gamma}) (2\bar{\rho}_e - 1) \mathbf{B}_e^T \mathbf{U}_i^{(e)}$$

where n_e is the number of finite elements, $\mathbf{B}_e$ is the column of the global matrix $\mathbf{B}$ corresponding to the degrees of freedom of the e -th finite element and $\mathbf{U}_i^{(e)}$ is the vector that stores the degrees of freedom of the e -th element. Those elements where the gap-phase takes the value 0 generate the electric charge presented in the previous section, while the rest of the structure (where the interface takes the value 1) does not collect any electrical charge. The problem is formulated in the framework of classical topology optimization, but the interpolation scheme does not need any power law. In addition, the use of a penalization exponent leads to the problem of facilitating the occurrence of one polarity over the other. This fact is useful in the context of minimizing the compliance, the weight, or any mechanical property, but not in the optimal design of the electrode profile.

The optimization problem is defined in the following way:

$$\begin{aligned} \max_{\rho} : \quad & q_i(\rho) \\ \text{s.t.:} \quad & \begin{cases} (\mathbf{K} - \omega_i^2 \mathbf{M}) \mathbf{U}_i = \mathbf{0} \\ \mathbf{1}^T \|\nabla \boldsymbol{\varphi}\|_{\gamma} \leq V_{\max} \\ \rho_e \in [0, 1], \quad e = 1, \dots, n_e, \end{cases} \end{aligned}$$

with the scalar value $V_{\max}$ representing the maximum amount of null-phase allowed in the problem. For a simpler understanding of the constraint, it will be defined in percentage form as $V = V_{\max}/n_e$. The optimization problem is numerically solved by MMA [50], a gradient method that requires the computation of the derivatives of the objective function and the constraints.

5. Sensitivity Analysis

The electric charge q_i is defined in terms of the projected field $\bar{\rho}$ and the projected normalized gradient $\|\nabla \boldsymbol{\varphi}\|_{\gamma}$. For the sake of simplicity, this scalar parameter can be defined as follows:

$$q_i = \mathbf{P}^T(\bar{\rho}, \|\nabla \boldsymbol{\varphi}\|_{\gamma}) \mathbf{B}^T \mathbf{U}_i, \quad (2)$$

where $\mathbf{P}$ represents the polarization profile of the piezoelectric layer. The rest of the terms in Equation (2) do not depend on the design variable. The derivative of the electrode profile $\mathbf{P}$ with respect to the design variable ρ_e is computed as follows:

$$\frac{\partial \mathbf{P}}{\partial \rho_e} = \sum_{i=1}^{n_e} \left(\frac{\partial \mathbf{P}}{\partial \bar{\rho}_i} \frac{\partial \bar{\rho}_i}{\partial \rho_e} + \frac{\partial \mathbf{P}}{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}} \frac{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}}{\partial \rho_e} \right).$$

The last term is expressed as follows:

$$\frac{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}}{\partial \rho_e} = \frac{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}}{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}} \frac{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}}{\partial \rho_e} = \frac{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}}{\partial \rho_e}$$

with

$$\frac{\partial \|\nabla \boldsymbol{\varphi}_i\|_{\gamma}}{\partial \rho_e} = \frac{1}{\|\nabla \boldsymbol{\varphi}_i\|_{\gamma}} \left(\frac{\partial \varphi_i}{\partial x} \frac{\partial}{\partial \rho_e} \left(\frac{\partial \varphi_i}{\partial x} \right) + \frac{\partial \varphi_i}{\partial y} \frac{\partial}{\partial \rho_e} \left(\frac{\partial \varphi_i}{\partial y} \right) \right)$$

and

$$\begin{cases} \frac{\partial}{\partial \rho_e} \left(\frac{\partial \varphi_i}{\partial x} \right) = \frac{\partial}{\partial \bar{\rho}_k} \left(\frac{\partial \varphi_i}{\partial x} \right) \frac{\partial \bar{\rho}_k}{\partial \rho_e} \\ \frac{\partial}{\partial \rho_e} \left(\frac{\partial \varphi_i}{\partial y} \right) = \frac{\partial}{\partial \bar{\rho}_k} \left(\frac{\partial \varphi_i}{\partial y} \right) \frac{\partial \bar{\rho}_k}{\partial \rho_e} \end{cases}$$

where the derivative $\partial \bar{\rho}_k / \partial \rho_e$ represents the standard modification of sensitivities due to filtering and projection.

6. Results

This section presents the numerical examples. Some parameters are identical for all examples: the mesh consists of 100×100 rectangular elements, and the plate dimensions are $1 \text{ cm} \times 1 \text{ cm} \times 0.01 \text{ cm}$. The sensor has three layers: a silicon substrate, an AlN piezoelectric layer, and an aluminum electrode. Only the properties of the silicon layer are considered in the mechanical model due to the negligible effect of the other layers on the plate stiffness. The Young's modulus of silicon is $E = 130 \text{ GPa}$, the Poisson's ratio is $\nu = 0.28$, and the density is $\rho_m = 2650 \text{ kg/m}^3$. These parameters are representative of actual modal sensors but can be adapted for different meshes or materials.

For the two-step filtering, the filter radii are $R_1 = 0.1 \text{ cm}$ and $R_2 = 0.05 \text{ cm}$. The thresholds are $\eta_1 = \eta_2 = 0.5$, and the sharpness of both projections starts with $\beta_1 = 4$ and

$\beta_2 = 0.5$. These values are doubled every 50 iterations, up to a maximum of 300 iterations. The boundary conditions of the rectangular plate are clamped at its left and right sides.

6.1. First Example

The vibration mode selected for this example is the fifth and it is depicted in Figure 2. The displacements (eigenvector) are scaled in these figures to show the deformation of the plate.

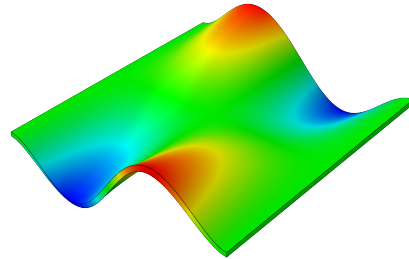


Figure 2. 5th vibration mode.

The electric charge depends linearly on the vibration mode. It is convenient to normalize this parameter with the maximum value q_i^* , which is obtained by solving the optimization problem, taking into account neither the gap-phase nor the volume constraint. For the following examples, the parameter used to compare them is $\alpha = q_i / q_i^*$.

The electrode profile of the 5th mode, without volume constraint, is shown in Figure 3. The purple and yellow areas represent areas of opposite polarization, while the orange color is associated with the gap-phase (that it is introduced after the optimization process). The values obtained are $\alpha = 1$ for the first case, and $\alpha = 0.9771$ for the second. As expected, imposing the null-polarity phase over the optimal design reduces the electric charge generated for two reasons: the gap-phase does not generate a charge and these areas are not taken into account in the optimization process.

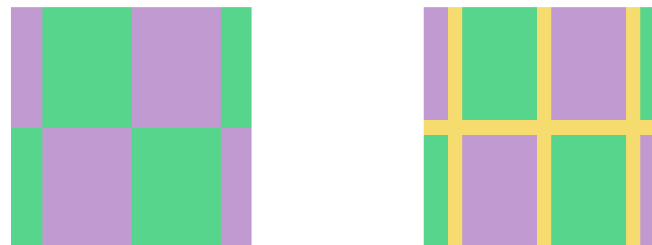


Figure 3. Electrode profile without (left) and with (right) gap-phase.

The results of the new approach are shown in Figure 4. The constraint over the maximum length of the projected gradient (gap-phase) simplifies the electrode pattern. For the first case, with the constraint fixed at $V = 0.20$, the value of the objective function is $\alpha = 0.9766$. This parameter decreases when the limitation over the length is stronger, with a value of $\alpha = 0.8396$ for $V = 0.15$. The convergence of α is represented in Figure 5.

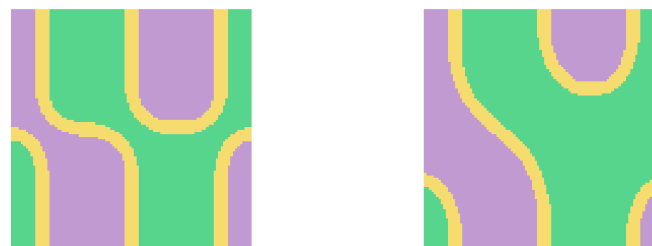


Figure 4. Electrode profile for the 5th mode. $V = 0.20$ (left) and $V = 0.15$ (right).

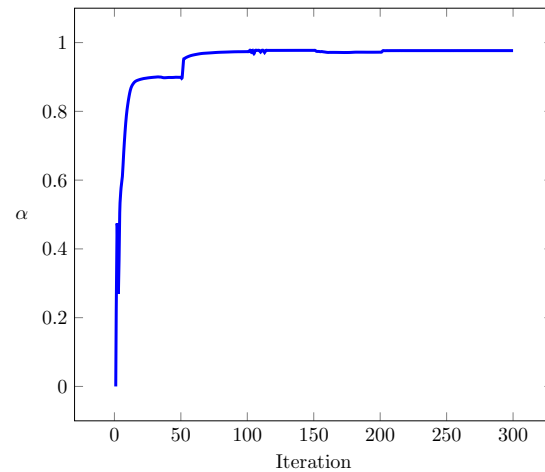


Figure 5. Convergence history of α for the case study of Figure 4.

6.2. Second Example

For the second example, the vibration mode to be optimized is the 6th one; the rest of the parameters remain constant. The eigenvector is displayed in Figure 6.

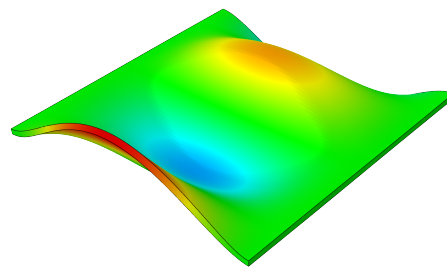


Figure 6. 6th vibration mode.

The optimized electrode profile is depicted in Figure 7, without constraint ($\alpha = 0.9484$), with $V = 0.20$ ($\alpha = 0.9779$) and with $V = 0.15$ ($\alpha = 0.9503$).

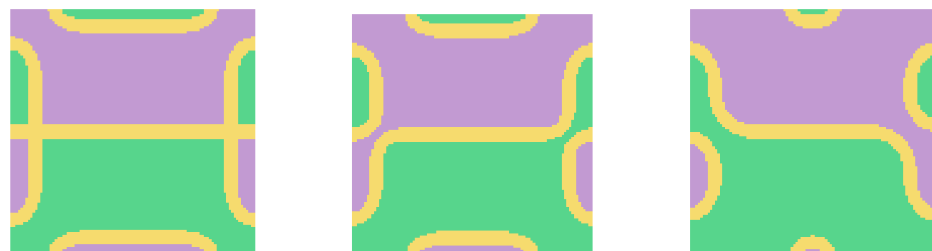


Figure 7. Electrode profile for the 6th mode. Without volume constraints (**left**), $V = 0.20$ (**center**) and $V = 0.15$ (**right**).

6.3. Third Example

In this last example, the vibration mode selected is the 7th. Figure 8 shows the displacement of the structure, and the optimized designs are presented in Figure 9, without volume constraint ($\alpha = 0.9743$), with $\alpha = 0.9228$ for $V = 0.20$ and $\alpha = 0.8270$ for $V = 0.15$.

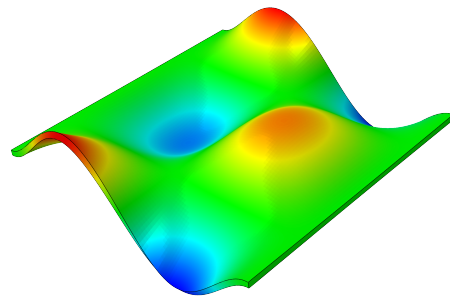


Figure 8. 7th vibration mode.



Figure 9. Electrode profile for the 7th mode. Without volume constraints (left), $V = 0.20$ (center) and $V = 0.15$ (right).

7. Discussion

This work focuses on maximizing the electric charge for a single vibration mode. This approach can be extrapolated to the design of modal filters, where multiple eigenvectors are involved in the optimization process. The width and steepness of the interface are determined by the parameters of the two-step filtering and projection, but they can be modeled as described in [41].

The boundary conditions and vibration mode have been chosen to create a significant alternation in the electrode profile. However, this approach can be applied to any combination of free, clamped, or simply-supported conditions. The dimensions of the plate are $1 \text{ cm} \times 1 \text{ cm} \times 0.01 \text{ cm}$ to assume plane stress, but they can be changed to adapt the structure to a macro-scale. Similarly, the material properties are fixed but can be adapted to any.

It is important to note that null-polarity phases of the electrode profile variable ($\bar{\rho} = 0.5$) cannot occur. This is because of the projection method and the assumption in the two-step approach that there is a change in polarity, introducing the gap-phase due to the nonzero gradient.

8. Conclusions

The objective of this work is to simplify the electrode profile obtained by maximizing the electric charge that generates a vibration mode. This aim is achieved by introducing a constraint over the maximum length of the gap-phase that separate both electrodes.

The novel methodology demonstrates good performance, allowing for the simplification of the pattern for different vibration modes. In every example, the simplification also involves a reduction in the amount of charge that is generated, as the set of admissible solutions becomes smaller. The three examples show a decrease of the generated charge ranging from 5 to 18%, alongside a reduction of more than 30% of the gap-phase length. This reduction is due to the normalization of the cost. It is important to notice that the parameter α is computed as the ratio between the charge obtained from the optimization problem and the maximum charge of the sensor. This last value is computed by solving a linear optimization problem without any constraint.

It is worthwhile to emphasize that this approach is focused on the design of the polarization profile, but it can be also extended to a mechanical context, where this phase can represent a coating of a different material.

Author Contributions: Conceptualization, D.R.; methodology, M.M.-N., D.C., S.H.M. and D.R.; software, D.C. and S.H.M.; validation, M.M.-N., D.C. and S.H.M.; formal analysis, D.C. and D.R.; investigation, M.M.-N., D.C., S.H.M. and D.R.; resources, M.M.-N. and D.C.; data curation, D.R.; writing—original draft preparation, M.M.-N., D.C., S.H.M. and D.R.; writing—review and editing, M.M.-N., D.C., S.H.M. and D.R.; visualization, S.H.M. and D.R.; supervision, M.M.-N., D.C., S.H.M. and D.R.; project administration, M.M.-N., D.C., S.H.M. and D.R.; funding acquisition, D.C., S.H.M. and D.R. All authors have read and agreed to the published version of the manuscript.

Funding: This publication is part of the grant PID2020-116207GB-I00, the grant PID2019-109652GB-I00 and the grant PID2022-137387OB-I00 funded by MICIU/AEI/10.13039/501100011033 and “ERDF UE A way of making Europe”, and 2023-GRIN-34320 provided by the University of Castilla-La Mancha.

Data Availability Statement: Data will be made available on request to the corresponding author.

Acknowledgments: The authors would like to thank Alberto Donoso for his kind support.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

PDE	Partial Differential Equation
TO	Topology Optimization

References

1. Syriac, R.M.; Bhasi, A.B. Structural Health Monitoring of Smart Composites: A Review. In *Proceedings of the Green Buildings and Sustainable Engineering*; Drück, H., Pillai, R.G., Tharian, M.G., Majeed, A.Z., Eds.; Springer: Singapore, 2019; pp. 49–59. [\[CrossRef\]](#)
2. Elahi, H.; Munir, K.; Eugeni, M.; Abrar, M.; Khan, A.; Arshad, A.; Gaudenzi, P. A Review on Applications of Piezoelectric Materials in Aerospace Industry. *Integr. Ferroelectr.* **2020**, *211*, 25–44. [\[CrossRef\]](#)
3. Sheeraz, M.A.; Malik, M.S.; Rahman, K.; Elahi, H.; Khurram, M.; Eugeni, M.; Gaudenzi, P. Multimodal piezoelectric wind energy harvester for aerospace applications. *Int. J. Energy Res.* **2022**, *46*, 13698–13710. [\[CrossRef\]](#)
4. Chen, Y.; Xue, X. Advances in the Structural Health Monitoring of Bridges Using Piezoelectric Transducers. *Sensors* **2018**, *18*, 4312. [\[CrossRef\]](#) [\[PubMed\]](#)
5. Zhang, H.; Zhou, Y.; Quan, L. Identification of a moving mass on a beam bridge using piezoelectric sensor arrays. *J. Sound Vib.* **2021**, *491*, 115754. [\[CrossRef\]](#)
6. Benavides-Serrano, A.; Mannan, M.; Laird, C. A quantitative assessment on the placement practices of gas detectors in the process industries. *J. Loss Prev. Process Ind.* **2015**, *35*, 339–351. [\[CrossRef\]](#)
7. Liseli, J.B.; Agnus, J.; Lutz, P.; Rakotondrabe, M. An Overview of Piezoelectric Self-Sensing Actuation for Nanopositioning Applications: Electrical Circuits, Displacement, and Force Estimation. *IEEE Trans. Instrum. Meas.* **2020**, *69*, 2–14. [\[CrossRef\]](#)
8. Gonçalves, J.F.; Silva, E.C.N.; De Leon, D.M.; Perondi, E.A. A Controllability-Based TO Approach for the Piezoelectric Actuator Design Considering Multimodal Vibration Control. *Int. J. Struct. Stab. Dyn.* **2020**, *20*, 2043009. [\[CrossRef\]](#)
9. Zhou, X.; Wu, S.; Wang, X.; Wang, Z.; Zhu, Q.; Sun, J.; Huang, P.; Wang, X.; Huang, W.; Lu, Q. Review on piezoelectric actuators: materials, classifications, applications, and recent trends. *Front. Mech. Eng.* **2024**, *19*, 6. [\[CrossRef\]](#)
10. Yao, S.; Peralta-Braz, P.; Alamdari, M.; Ruiz, R.; Atroshchenko, E. Optimal design of piezoelectric energy harvesters for bridge infrastructure: Effects of location and traffic intensity on energy production. *Appl. Energy* **2024**, *355*, 122285. [\[CrossRef\]](#)
11. Peralta-Braz, P.; Alamdari, M.; Ruiz, R.; Atroshchenko, E.; Hassan, M. Design optimisation of piezoelectric energy harvesters for bridge infrastructure. *Mech. Syst. Signal Process.* **2023**, *205*, 110823. [\[CrossRef\]](#)
12. Hassani, S.; Dackermann, U. A Systematic Review of Optimization Algorithms for Structural Health Monitoring and Optimal Sensor Placement. *Sensors* **2023**, *23*, 3293. [\[CrossRef\]](#) [\[PubMed\]](#)
13. Horta Muñoz, S.; Ruiz, D. Optimization of the Polarization Profile of Conical-Shaped Shells Piezoelectric Sensors. *Sensors* **2023**, *23*, 442. [\[CrossRef\]](#) [\[PubMed\]](#)
14. Homayouni-Amlashi, A.; Mohand-Ousaid, A.; Rakotondrabe, M. Topology optimization of 2DOF piezoelectric plate energy harvester under external in-plane force. *J. Micro Bio Robot.* **2020**, *16*, 65–77. [\[CrossRef\]](#)
15. Ruiz, D.; Bellido, J.C.; Donoso, A.; Sánchez-Rojas, J.L. Design of in-plane piezoelectric sensors for static response by simultaneously optimizing the host structure and the electrode profile. *Struct. Multidiscip. Optim.* **2013**, *48*, 1023–1026. [\[CrossRef\]](#)

16. Osanov, M.; Guest, J.K. Topology Optimization for Architected Materials Design. *Annu. Rev. Mater. Res.* **2016**, *46*, 211–233. [\[CrossRef\]](#)
17. Wong, J.; Ryan, L.; Kim, I.Y. Design optimization of aircraft landing gear assembly under dynamic loading. *Struct. Multidiscip. Optim.* **2018**, *57*, 1357–1375. [\[CrossRef\]](#)
18. Wu, J.; Sigmund, O.; Groen, J.P. Topology optimization of multi-scale structures: A review. *Struct. Multidiscip. Optim.* **2021**, *63*, 1455–1480. [\[CrossRef\]](#)
19. Ruiz, D.; Horta Muñoz, S. Optimal design of electrode polarization in piezoelectric unimorph beams to induce traveling waves. *Appl. Math. Model.* **2021**, *99*, 1–13. [\[CrossRef\]](#)
20. Ruiz, D.; Horta Muñoz, S.; García-Contreras, R. Simultaneous Design of the Host Structure and the Polarisation Profile of Piezoelectric Sensors Applied to Cylindrical Shell Structures. *Mathematics* **2022**, *10*, 2753. [\[CrossRef\]](#)
21. Wang, S.Y.; Tai, K.; Quek, S.T. Topology optimization of piezoelectric sensors/actuators for torsional vibration control of composite plates. *Smart Mater. Struct.* **2006**, *15*, 253. [\[CrossRef\]](#)
22. Ruiz, D.; Bellido, J.C.; Donoso, A. Optimal Design of Piezoelectric Modal Transducers. *Arch. Comput. Methods Eng.* **2018**, *25*, 313–347. [\[CrossRef\]](#)
23. Guzmán, D.G.; Silva, E.C.N.; Rubio, W.M. Topology optimization of piezoelectric sensor and actuator layers for active vibration control. *Smart Mater. Struct.* **2020**, *29*, 085009. [\[CrossRef\]](#)
24. Lazarov, B.S.; Wang, F.; Sigmund, O. Length scale and manufacturability in density-based topology optimization. *Arch. Appl. Mech.* **2016**, *86*, 189–218. [\[CrossRef\]](#)
25. Vatanabe, S.L.; Lippi, T.N.; de Lima, C.R.; Paulino, G.H.; Silva, E.C. Topology optimization with manufacturing constraints: A unified projection-based approach. *Adv. Eng. Softw.* **2016**, *100*, 97–112. [\[CrossRef\]](#)
26. Donoso, A.; Sigmund, O. Topology optimization of piezo modal transducers with null-polarity phases. *Struct. Multidiscip. Optim.* **2016**, *53*, 193–203. [\[CrossRef\]](#)
27. Donoso, A.; Guest, J.K. Topology optimization of piezo modal transducers considering electrode connectivity constraints. *Comput. Methods Appl. Mech. Eng.* **2019**, *356*, 101–115. [\[CrossRef\]](#)
28. Donoso, A.; Aranda, E.; Ruiz, D. A new method for designing piezo transducers with connected two-phase electrode. *Comput. Struct.* **2023**, *275*, 106936. [\[CrossRef\]](#)
29. Ransing, R.S.; Lewis, R.W. Optimal design of die coating thickness using the Lewis—Ransing correlation. *Int. J. Cast Met. Res.* **1997**, *9*, 269–277. [\[CrossRef\]](#)
30. Guest, J.K.; Prévost, J.H.; Belytschko, T. Achieving minimum length scale in topology optimization using nodal design variables and projection functions. *Int. J. Numer. Methods Eng.* **2004**, *61*, 238–254. [\[CrossRef\]](#)
31. Li, Q.; Chen, W.; Liu, S.; Tong, L. Structural topology optimization considering connectivity constraint. *Struct. Multidiscip. Optim.* **2016**, *54*, 971–984. [\[CrossRef\]](#)
32. Gaynor, A.T.; Guest, J.K. Topology optimization considering overhang constraints: Eliminating sacrificial support material in additive manufacturing through design. *Struct. Multidiscip. Optim.* **2016**, *54*, 1157–1172. [\[CrossRef\]](#)
33. Tromme, E.; Kawamoto, A.; Guest, J.K. Topology optimization based on reduction methods with applications to multiscale design and additive manufacturing. *Front. Mech. Eng.* **2020**, *15*, 151–165. [\[CrossRef\]](#)
34. Wang, Y.; Kang, Z. A level set method for shape and topology optimization of coated structures. *Comput. Methods Appl. Mech. Eng.* **2018**, *329*, 553–574. [\[CrossRef\]](#)
35. Fu, J.; Li, H.; Xiao, M.; Gao, L.; Chu, S. Topology optimization of shell-infill structures using a distance regularized parametric level-set method. *Struct. Multidiscip. Optim.* **2019**, *59*, 249–262. [\[CrossRef\]](#)
36. Fu, J.; Li, H.; Gao, L.; Xiao, M. Design of shell-infill structures by a multiscale level set topology optimization method. *Comput. Struct.* **2019**, *212*, 162–172. [\[CrossRef\]](#)
37. Harvey, D.; Hubert, P. Extensions of the coating approach for topology optimization of composite sandwich structures. *Compos. Struct.* **2020**, *252*, 112682. [\[CrossRef\]](#)
38. Bendsoe, M.P.; Sigmund, O. Extensions and applications. In *Topology Optimization: Theory, Methods, and Applications*; Springer: Berlin/Heidelberg, Germany, 2004; pp. 71–158.
39. Yoon, G.H.; Yi, B. A new coating filter of coated structure for topology optimization. *Struct. Multidiscip. Optim.* **2019**, *60*, 1527–1544. [\[CrossRef\]](#)
40. Yi, B.; Yoon, G.H.; Peng, X. A simple density filter for the topology optimization of coated structures. *Eng. Optim.* **2021**, *53*, 2088–2107. [\[CrossRef\]](#)
41. Clausen, A.; Aage, N.; Sigmund, O. Topology optimization of coated structures and material interface problems. *Comput. Methods Appl. Mech. Eng.* **2015**, *290*, 524–541. [\[CrossRef\]](#)
42. Lazarov, B.S.; Sigmund, O. Filters in topology optimization based on Helmholtz-type differential equations. *Int. J. Numer. Methods Eng.* **2011**, *86*, 765–781. [\[CrossRef\]](#)
43. Bourdin, B. Filters in topology optimization. *Int. J. Numer. Methods Eng.* **2001**, *50*, 2143–2158. [\[CrossRef\]](#)
44. Sigmund, O. Morphology-based black and white filters for topology optimization. *Struct. Multidiscip. Optim.* **2007**, *33*, 401–424. [\[CrossRef\]](#)
45. Wang, F.; Lazarov, B.S.; Sigmund, O. On projection methods, convergence and robust formulations in topology optimization. *Struct. Multidiscip. Optim.* **2011**, *43*, 767–784. [\[CrossRef\]](#)

46. Lee, C.K.; Moon, F.C. Modal Sensors/Actuators. *J. Appl. Mech.* **1990**, *57*, 434–441. [[CrossRef](#)]
47. Moheimani, S.; Fleming, A. *Piezoelectric Transducers for Vibration Control and Damping*, 2nd ed.; Springer: Berlin/Heidelberg, Germany, 2006.
48. Cook, R.D.; Malkus, D.S.; Plesha, M.E.; Witt, R.J. *Concepts and Applications of Finite Element Analysis*, 4th ed.; John Wiley Sons: Hoboken, NJ, USA, 2001.
49. Donoso, A.; Bellido, J.C. Systematic design of distributed piezoelectric modal sensors/actuators for rectangular plates by optimizing the polarization profile. *Struct. Multidiscip. Optim.* **2009**, *38*, 347–356. [[CrossRef](#)]
50. Svanberg, K. The method of moving asymptotes—a new method for structural optimization. *Int. J. Numer. Methods Eng.* **1987**, *24*, 359–373. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.