mathematics

Article

Efficient Decision Making for Sustainable Energy Using Single-Valued Neutrosophic Prioritized Interactive Aggregation Operators

Muhammad Riaz [1], Hafiz Muhammad Athar Farid [1], Jurgita Antucheviciene [2,*] and Gülay Demir [3]

[1] Department of Mathematics, University of the Punjab, Lahore 54590, Pakistan; mriaz.math@pu.edu.pk (M.R.); hmatharfarid@gmail.com (H.M.A.F.)
[2] Department of Construction Management and Real Estate, Vilnius Gediminas Technical University, Sauletekio al. 11, LT-10223 Vilnius, Lithuania
[3] Department of Medical Services and Techniques, Vocational School of Health Services, Sivas Cumhuriyet University, Sivas 58140, Turkey; gulaydemir@cumhuriyet.edu.tr
* Correspondence: jurgita.antucheviciene@vilniustech.lt

Abstract: To reduce greenhouse gas emissions, conserve the environment, and reduce dependency on fossil fuels, the transition from fossil energy to renewable energy is deemed essential. Several companies around the globe, especially big conglomerates, were pioneers in the use of renewable energy. For sustainable growth, Pakistani businesses are growing increasingly interested in the use of green sources in manufacturing and economic activities. In recent years, there has been a growth in the number of companies that are eager to use renewable energies to produce products that correspond to green standards, therefore boosting their competitiveness. Yet, the selection of an appropriate energy source for any industrially complex project is not a simple task, as numerous qualitative and quantitative characteristics must be considered. To arrive at a feasible conclusion, this research provides a multi-criteria paradigm for sustainable energy selection in a single-valued neutrosophic environment. This work developed an innovative aggregation operators approach that interprets the input evaluation using single-valued neutrosophic numbers. For this, a "single-valued neutrosophic prioritized interactive weighted averaging operator and single-valued neutrosophic prioritized interactive weighted geometric operator" has been introduced. Several additional appealing features of these aggregation operators are also discussed. The application of the recommended operators for sustainable energy related to the industrial complex is discussed. A comparison analysis proves the empirical existence of the suggested methodology's consistency and superiority.

Keywords: sustainable energy; interactive relation; aggregation operators; prioritized relation

MSC: 03E72; 94D05; 90B50

Citation: Riaz, M.; Farid, H.M.A.; Antuchevičienė, J.; Demir, G. Efficient Decision Making for Sustainable Energy Using Single-Valued Neutrosophic Prioritized Interactive Aggregation Operators. *Mathematics* **2023**, *11*, 2186. https://doi.org/10.3390/math11092186

Academic Editor: Anatoliy Swishchuk

Received: 9 March 2023
Revised: 25 April 2023
Accepted: 3 May 2023
Published: 5 May 2023

1. Introduction

At the moment, developing countries have enormous challenges in adopting sustainable energy policies. It raises a variety of issues, such as the creation and execution of energy policy, the selection of energy sources, and the evaluation of energy delivery technologies. Due to the critical, intricate, and subjective nature of the issues themselves, several scientific discoveries are targeted at establishing objective models of decision assistance. Furthermore, the concerns themselves have a shaky organisational structure. The source of this phenomenon should be sought, given that modelling this class of challenges necessitates the correct mapping of more than just the scenarios that have been analysed. In such a case, experts must evaluate the implications of evaluating the decision dilemma from a range of angles and points of view while taking into account a number of aspects that are diametrically opposed to one another. A process for planning sustainable energy, for example, must entail taking into account a variety of factors, including those linked to the environment, society, and the economy, while making decisions. The presence of interest groups has a substantial influence on the shape of the final ranking; hence, the created model must incorporate both objectification and correct mapping for this component. The fact that the model's measurement data are easily accessible, notwithstanding their intrinsic imprecision, is not without significance. At the same time, it is a research issue that is regularly raised and is still relevant.

Existing literature analyses verify, from a scientific standpoint, that an extension of the selection support technique beyond the traditional simulation study of a single goal function is generally accepted, and is characterized by a set of possible solutions. This can be found in the following sentence. Because of this development, it is now feasible to tackle multi-criteria issues, with the goal being to find a solution that meets a number of different, and sometimes competing, objectives. The model of multi-criteria group decision making (MCGDM) is characterized by the presence of several criteria, the occurrence of conflicts between those criteria, and the nature of the decision issue itself, which is complicated, subjective, and inadequately organized. Many different approaches to MCGDM have been developed on the basis of these principles. The current state of the art in this field makes it abundantly evident that MCGDM approaches are both practically and effectively capable of handling the challenge that was discussed before. An examination of the relevant published material reveals that MCGDM approaches are used in energy-related decision making difficulties on a rather regular basis. Methods of MCGDM are utilised as effective and practical instruments to tackle challenges linked to decision making on energy policy.

MCGDM is a cognitive approach used to select the best option among a limited set of alternatives by utilizing the expert opinions of decision makers (DMs). The "Fuzzy set" (FS) theory was pioneered by Zadeh [1] to address such problems by employing mathematical models to describe ambiguity. Atanassov [2] extended the concept of FS theory to "intuitionistic fuzzy set" (IFS) theory, which incorporates both membership and non-membership qualities. While FS can be characterized in terms of membership features, IFS is more comprehensive. Uncertainty is a critical factor when accurately evaluating an object. Consider the following scenario: an authority voiced their viewpoint on a certain problem; the chance of the claim being correct is 0.87, the likelihood of it being incorrect is 0.75, and the possibility of it being either true or false is 0.29. Smarandache [3] proposed neutrosophic sets (NSs) to address such issues. NSs contain variable degrees of "truth membership degree (TMD), indeterminacy membership degree (IMD), and falsity membership degree (FMD)" for each element, with values ranging from $]-0, 1+[$.

When it comes to the representation of discordant information, NS is seen as a more beneficial tool than IFS, according to philosophical perspectives. Yet, from the perspective of scientific investigation, NS and the predefined operators that go along with it need to be regarded as normative in order to put into practise practical applications. In order to solve this problem, Wang et al. [4] developed the idea of a "single-valued neutrosophic set" (SVNS), which possesses a variety of novel qualities and theorems. Ye [5] investigated SVNS operations in great detail and described them as a streamlined neutrosophic set (SNS). "Interval-valued neutrosophic sets" (IVNSs) were also established by Wang et al. [6] in order to simplify the implementation of NSs.

Scientists are currently focusing on SVNSs to tackle complex and unpredictable issues in real life. Several scholars have utilized these sets to develop selection processes based on distance/similarity measures [7–10], entropy [11], correlation coefficients [12], and score functions [13]. Ye [14] studied SVN similarity measurements based on the cotangent function. Additionally, Ye proposed SVN clustering techniques based on similarity metrics in [15]. Peng et al. [16] described an outranking approach for MCGDM issues that use SNSs. Biswas et al. [17] developed the entropy-based "grey relational analysis" (GRA) technique for MCGDM with SVNSs. Peng and Dai [18] conducted an exhaustive review of NS research published in various domains. In addition, Karaaslan [19] developed a number of similarity metrics that may function in an SVN refined as well as an interval neutrosophic refined environment. Kamacı [20] researched the soft extensions of linguistic concepts and offered an application in game theory.

In recent decades, there has been a rising interest in creating ways to construct unique aggregation operators (AOs) to integrate information in an effective and efficient manner. This interest has been spurred on by a number of different factors. Because of the many advantages they offer, AOs have developed into an essential component of the decision-making process. These AOs rely, in many instances, on operational rules that were developed with the intention of combining a restricted number of fuzzy numbers into a single fuzzy number, as described in [21–25].

In order to carry out an analysis on the SVN-based data, numerous AOs that are based on the operating rules of SVNS have been built. Ye [5] presented the basics of AOs, Peng et al. [26] suggested some improved AOs, Nancy and Garg [27] utilized Frank AOs, Han and Wei [28] introduced Choquet integral AOs, Li et al. [29] introduced Heronian mean AOs, Liu and Wang [30] investigated the Bonferroni mean (BM) AOs, Wang et al. [31] proposed a dual Bonferroni AO, Wei and Zhang [32] introduced Bonferroni power AOs, Wu et al. [11] initiated the prioritized AOs, and Ji et al. [33] proved the Frank prioritized BM in an SVN context.

Researchers have investigated various AOs for SVNSs, such as Dombi AOs, Einstein AOs, Harmonic AOs, and Muirhead AOs, in several studies [34–37]. Liu initially proposed the concept of AOs for SVNSs based on Archimedean t-norm and t-conorm, and this concept was further developed by kamacı [38] and Liu et al. [39], who established various AOs for SVNNs based on Hamacher operations and discussed their applications in selection. The use of generalized hybrid weighted averaging AOs of SVNNs was proposed by Zheng et al. [40] to develop MCGDM models, and Garg and Nancy [41] presented new logarithmic operating rules on SVNSs and their applicability to MCGDM. Other researchers focused on interdependent inputs of various kinds of SVNSs and developed an MCGDM strategy, such as in the studies by authors in [42,43]. Liu and Luo proposed correlated AOs for SNSs and a weighted distance measure-based MCGDM technique for the neutrosophic framework [44,45], while other studies proposed SVN linguistic mean AOs and their applications [46,47]. Lu and Ye developed exponential operations and their corresponding AOs for SVNNs [48]. Some extensive work related to AOs can be found in [49]. Entezari et al. [50] provided a bibliographic view of machine learning and artificial intelligence in energy systems. Izanloo et al. [51] presented a machine learning evaluation technique for sustainable energy investment choices, whereas Zahedi et al. [52] suggested the development of novel simulations for reservoir hydropower generation. The motivation and contribution of the current study are given as follows:

1. In line with the findings of previous research, one can conclude that the challenges that arise in the process of decision making in today's world are growing increasingly complicated. It is essential to communicate the unknown particulars more constructively to pick the best alternative for MCGDM problems.
2. In addition to this, it is absolutely necessary to have a solid understanding of how to properly handle the hierarchical connection that exists between the numerous criteria. According to Yager [53], while selecting an electric bike for a child based on both affordability and safety considerations, one should not allow the advantage to outweigh the loss of safety.
3. This is because of what Yager calls the "loss of protection paradox." After that, there is a relationship between these two criteria, but it is prioritized, with protection being given priority. This is what is known as an aggregation issue, and it arises from the fact that the characteristics have a priority connection.
4. Within the context of this situation, Yager presented prioritised AOs by providing attribute prioritising from the perspective of criterion weights, which varied depending on the degree to which higher value characteristics were met [53]. It seems that He et al. [54] are in the process of implementing interaction operational guide-

lines that take into consideration the interactions that occur between the various membership grades.
5. In light of the previous, hybridised AOs were conceived, which are a combination of SVN interactive AOs and SVN prioritised AOs.

The remaining parts of the paper are organized in the following fashion: In Section 2, you will find definitions not just for the SVNS but also for other key concepts. A number of SVN prioritised interactive AOs are included in Section 3. In Section 4, we constructed an MCGDM strategy by making use of the AOs that were given. In Section 5, the case study is described in greater depth, including with numerical examples and a comparison to the AOs that are currently in place. The most important findings of the research are outlined and discussed in Section 6.

2. Certain Basic Concepts

In this section, some basic ideas about SVNSs over the universal set Θ have been discussed.

Definition 1 ([4]). *A single-valued neutrosophic set (SVNS) in Θ is defined as*

$$\chi = \{\langle x, \mu_\chi(x), \nu_\chi(x), \tau_\chi(x) | x \in \Theta \rangle\} \tag{1}$$

where $\mu_\chi(x), \nu_\chi(x), \tau_\chi(x) \in [0,1]$, and $0 \le \mu_\chi(x) + \nu_\chi(x) + \tau_\chi(x) \le 3$ for all $x \in \Theta$. $\mu_\chi(x), \nu_\chi(x), \tau_\chi(x)$ denote TMD, IMD, and FMD, respectively, for some $x \in \Theta$.

The pair $\aleph^\chi = (\mu_{\aleph\chi}, \nu_{\aleph\chi}, \tau_{\aleph\chi})$ throughout this article is called an SVNN with the conditions $\mu_{\aleph\chi}, \nu_{\aleph\chi}, \tau_{\aleph\chi} \in [0,1]$ and $\mu_{\aleph\chi} + \nu_{\aleph\chi} + \tau_{\aleph\chi} \le 3$.

Definition 2 ([4]). *Let $\aleph^\chi{}_1 = \langle \mu^\chi{}_1, \nu^\chi{}_1, \tau^\chi{}_1 \rangle$ and $\aleph^\chi{}_2 = \langle \mu^\chi{}_2, \nu^\chi{}_2, \tau^\chi{}_2 \rangle$ be SVNNs, then*

$$\aleph^{\chi^c}_1 = \langle \tau^\chi{}_1, \nu^\chi{}_1, \mu^\chi{}_1 \rangle \tag{2}$$

$$\aleph^\chi{}_1 \vee \aleph^\chi{}_2 = \langle max\{\mu^\chi{}_1, \mu^\chi{}_2\}, min\{\nu^\chi{}_1, \nu^\chi{}_2\}, min\{\tau^\chi{}_1, \tau^\chi{}_2\} \rangle \tag{3}$$

$$\aleph^\chi{}_1 \wedge \aleph^\chi{}_2 = \langle min\{\mu^\chi{}_1, \mu^\chi{}_2\}, max\{\nu^\chi{}_1, \nu^\chi{}_2\}, max\{\tau^\chi{}_1, \tau^\chi{}_2\} \rangle \tag{4}$$

$$\aleph^\chi{}_1 \oplus \aleph^\chi{}_2 = \langle \mu^\chi{}_1 + \mu^\chi{}_2 - \mu^\chi{}_1\mu^\chi{}_2, \nu^\chi{}_1\nu^\chi{}_2, \tau^\chi{}_1\tau^\chi{}_2 \rangle \tag{5}$$

$$\aleph^\chi{}_1 \otimes \aleph^\chi{}_2 = \langle \mu^\chi{}_1\mu^\chi{}_2, \nu^\chi{}_1 + \nu^\chi{}_2 - \nu^\chi{}_1\nu^\chi{}_2, \tau^\chi{}_1 + \tau^\chi{}_2 - \tau^\chi{}_1\tau^\chi{}_2 \rangle \tag{6}$$

$$\sigma\aleph^\chi{}_1 = \langle 1 - (1 - \mu^\chi{}_1)^\sigma, \nu^{\chi^\sigma}_1, \tau^{\chi^\sigma}_1 \rangle \tag{7}$$

$$\aleph^{\chi^\sigma}_1 = \langle \mu^{\chi^\sigma}_1, 1 - (1 - \nu^\chi{}_1)^\sigma, 1 - (1 - \tau^\chi{}_1)^\sigma \rangle \tag{8}$$

Definition 3 ([11]). *Let $\aleph^\chi = \langle \mu^\chi{}_{\aleph\chi}, \nu^\chi{}_{\aleph\chi}, \tau^\chi{}_{\aleph\chi} \rangle$ be the SVNN, the score function (SF) can be characterized as given below.*

$$\lambda^ℸ(\aleph^\chi) = \frac{\mu^\chi{}_{\aleph\chi} + 1 - \nu^\chi{}_{\aleph\chi} + 1 - \tau^\chi{}_{\aleph\chi}}{3}$$

Consider two SVNNs, $\aleph^\chi$ and β, if $\lambda^ℸ(\aleph^\chi) > \lambda^ℸ(\beta)$, then $\aleph^\chi > \beta$.

For more basic definitions, $\sum_{h=1}^{e} = ℶ_h$ for the sake of convenience.

Definition 4 ([11]). *Let $\aleph^\chi{}_h = \langle \mu^\chi{}_h, \nu^\chi{}_h, \tau^\chi{}_h \rangle$ be the accumulation of SVNNs, and SVNPWA $\mathfrak{P}^n \to \mathfrak{P}$ be the mapping, if*

$$SVNPWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_e) = \left(\frac{\breve{\amalg}_1}{\beth_h \breve{\amalg}_h} \aleph^{\chi}{}_1 \oplus \frac{\breve{\amalg}_2}{\beth_h \breve{\amalg}_h} \aleph^{\chi}{}_2 \oplus \ldots, \oplus \frac{\breve{\amalg}_e}{\beth_h \breve{\amalg}_h} \aleph^{\chi}{}_e \right) \tag{9}$$

then the mapping SVNPWA is called a "single-valued neutrosophic prioritized weighted averaging (SVNPWA) operator", where $\breve{\amalg}_h = \prod_{h=1}^{g-1} \lambda^{\daleth}(\aleph^{\chi}{}_h)$ $(\urcorner = 2 \ldots, e)$, $\breve{\amalg}_1 = 1$, *and* $\lambda^{\daleth}(\aleph^{\chi}{}_h)$ *is the score of the hth SVNN.*

Theorem 1 ([11])**.** *Let* $\aleph^{\chi}{}_h = \langle \mu^{\chi}{}_h, \nu^{\chi}{}_h, \tau^{\chi}{}_h \rangle$ *be the accumulation of SVNNs, then*

$$SVNPWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots, \aleph^{\chi}{}_e) = \left(1 - \prod_{h=1}^{e} (1 - \mu^{\chi}{}_h)^{\frac{\breve{\amalg}_h}{\beth_h \breve{\amalg}_h}}, \prod_{h=1}^{e} (\nu^{\chi}{}_h)^{\frac{\breve{\amalg}_h}{\beth_h \breve{\amalg}_h}}, \prod_{h=1}^{e} (\tau^{\chi}{}_h)^{\frac{\breve{\amalg}_h}{\beth_h \breve{\amalg}_h}} \right) \tag{10}$$

Definition 5 ([11])**.** *Let* $\aleph^{\chi}{}_h = \langle \mu^{\chi}{}_h, \nu^{\chi}{}_h, \tau^{\chi}{}_h \rangle$ *be the accumulation of SVNNs, and SVNPWG* $\mathfrak{P}^n \to \mathfrak{P}$ *be the mapping. If*

$$SVNPWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_e) = \left(\aleph^{\chi}{}_1^{\frac{\breve{\amalg}_1}{\beth_h \breve{\amalg}_h}} \otimes \aleph^{\chi}{}_2^{\frac{\breve{\amalg}_2}{\beth_h \breve{\amalg}_h}} \otimes \ldots, \otimes \aleph^{\chi}{}_e^{\frac{\breve{\amalg}_e}{\beth_h \breve{\amalg}_h}} \right) \tag{11}$$

then the mapping SVNPWG is called a "single-valued neutrosophic prioritized weighted geometric (SVNPWG) operator", where $\breve{\amalg}_h = \prod_{h=1}^{g-1} \lambda^{\daleth}(\aleph^{\chi}{}_h)$ $(\urcorner = 2 \ldots, e)$, $\breve{\amalg}_1 = 1$ *and* $\lambda^{\daleth}(\aleph^{\chi}{}_h)$ *is the score of the hth SVNN.*

Theorem 2 ([11])**.** *Let* $\aleph^{\chi}{}_h = \langle \mu^{\chi}{}_h, \nu^{\chi}{}_h, \tau^{\chi}{}_h \rangle$ *be the accumulation of SVNNs, then*

$$SVNPWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots, \aleph^{\chi}{}_e) = \left(\prod_{h=1}^{e} (\mu^{\chi}{}_h)^{\frac{\breve{\amalg}_h}{\beth_h \breve{\amalg}_h}}, 1 - \prod_{h=1}^{e} (1 - \nu^{\chi}{}_h)^{\frac{\breve{\amalg}_h}{\beth_h \breve{\amalg}_h}}, 1 - \prod_{h=1}^{e} (1 - \tau^{\chi}{}_h)^{\frac{\breve{\amalg}_h}{\beth_h \breve{\amalg}_h}} \right) \tag{12}$$

SVN Interactive Operations

Definition 6. *Let* $\aleph^{\chi}$, $\aleph^{\chi}{}_1$, *and* $\aleph^{\chi}{}_2$ *be the two SVNNs; the interactive operations for SVNNs are given below:*

1. $\aleph^{\chi}{}_1 \oplus \aleph^{\chi}{}_2 = (\mu^{\chi}{}_1 + \mu^{\chi}{}_2 - \mu^{\chi}{}_1 \mu^{\chi}{}_2, \nu^{\chi}{}_1 + \nu^{\chi}{}_2 - \nu^{\chi}{}_1 \nu^{\chi}{}_2 - \nu^{\chi}{}_1 \mu^{\chi}{}_2 - \mu^{\chi}{}_1 \nu^{\chi}{}_2,$ $\tau^{\chi}{}_1 + \tau^{\chi}{}_2 - \tau^{\chi}{}_1 \tau^{\chi}{}_2 - \tau^{\chi}{}_1 \mu^{\chi}{}_2 - \mu^{\chi}{}_1 \tau^{\chi}{}_2)$
2. $\aleph^{\chi}{}_1 \otimes \aleph^{\chi}{}_2 = (\mu^{\chi}{}_1 + \mu^{\chi}{}_2 - \mu^{\chi}{}_1 \mu^{\chi}{}_2 - \mu^{\chi}{}_1 \nu^{\chi}{}_2 - \nu^{\chi}{}_1 \mu^{\chi}{}_2, \nu^{\chi}{}_1 + \nu^{\chi}{}_2 - \nu^{\chi}{}_1 \nu^{\chi}{}_2,$ $\tau^{\chi}{}_1 + \tau^{\chi}{}_2 - \tau^{\chi}{}_1 \tau^{\chi}{}_2)$
3. $\lambda \aleph^{\chi} = (1 - (1 - \mu^{\chi})^{\lambda}, (1 - \mu^{\chi})^{\lambda} - (1 - (\mu^{\chi} + \nu^{\chi}))^{\lambda}, (1 - \mu^{\chi})^{\lambda} - (1 - (\mu^{\chi} + \tau^{\chi}))^{\lambda}),$ $\lambda > 0$
4. $\aleph^{\chi \lambda} = \left((1 - \nu^{\chi})^{\lambda} - (1 - (\nu^{\chi} + \mu^{\chi}))^{\lambda}, 1 - (1 - \nu^{\chi})^{\lambda}, 1 - (1 - \tau^{\chi})^{\lambda} \right), \quad \lambda > 0$

Definition 7. *Let* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *be the accumulation of SVNNs, and SVNIWA :* $\breve{\$}^n \to \breve{\$}$ *is the mapping,*

$$SVNIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \bigoplus_{\urcorner=1}^{r} \omega_{\urcorner} \aleph^{\chi}{}_{\urcorner} \tag{13}$$

then the mapping SVNIWA is called a "single-valued neutrosophic interactive weighted averaging operator", where $(\omega_1, \omega_2, \ldots \omega_r)$ *is the WV, with the constraints* $\omega_j > 0$, $\omega_{\urcorner} \in [0, 1]$ *and* $\sum_{\urcorner=1}^{r} \omega_{\urcorner} = 1$.

Theorem 3. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *as the accumulation of SVNNs, then*

$$SVNIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \bigoplus_{\daleth=1}^{r} \omega_{\daleth} \aleph^{\chi}{}_{\daleth}$$

$$= \Bigg(1 - \prod_{\daleth=1}^{r}\Big(1-(\mu^{\chi}{}_{\daleth})\Big)^{\omega_{\daleth}}, \prod_{\daleth=1}^{r}\Big(1-(\mu^{\chi}{}_{\daleth})\Big)^{\omega_{\daleth}} - \prod_{\daleth=1}^{r}\Big(1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth}))\Big)^{\omega_{\daleth}}, \prod_{\daleth=1}^{r}\Big(1-(\mu^{\chi}{}_{\daleth})\Big)^{\omega_{\daleth}} - \prod_{\daleth=1}^{r}\Big(1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth}))\Big)^{\omega_{\daleth}}\Bigg) \tag{14}$$

Definition 8. *Assume $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ is the accumulation of SVNNs, and $SVNIWG : \check{\$}^n \to \check{\$}$ is a mapping,*

$$SVNIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \bigotimes_{\daleth=1}^{r} (\aleph^{\chi}{}_{\daleth})^{\omega_{\daleth}} \tag{15}$$

then the mapping SVNIWG is called a "single-valued neutrosophic fuzzy interactive weighted geometric operator", where $(\omega_1, \omega_2, \ldots \omega_r)$ is the WV of the considered SVNNs with the condition that $\omega_j > 0$, $\omega_{\daleth} \in [0,1]$, and $\sum_{\daleth=1}^{r} \omega_{\daleth} = 1$

Theorem 4. *Consider $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ as the accumulation of SVNNs, then*

$$\begin{aligned} SVNIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) &= \bigotimes_{\daleth=1}^{r} (\aleph^{\chi}{}_{\daleth})^{\omega_{\daleth}} \\ &= \Bigg(\prod_{\daleth=1}^{r}\Big(1-(\nu^{\chi}{}_{\daleth})\Big)^{\omega_{\daleth}} - \prod_{\daleth=1}^{r}\Big(1-((\nu^{\chi}{}_{\daleth})+(\mu^{\chi}{}_{\daleth}))\Big)^{\omega_{\daleth}}, \\ &\quad 1-\prod_{\daleth=1}^{r}\Big(1-(\nu^{\chi}{}_{\daleth})\Big)^{\omega_{\daleth}}, 1-\prod_{\daleth=1}^{r}\Big(1-(\tau^{\chi}{}_{\daleth})\Big)^{\omega_{\daleth}}\Bigg) \end{aligned} \tag{16}$$

3. SVN Prioritised Interactive AOs

In this part, we will introduce a type of hybrid AOs called SVN prioritised interactive AOs.

3.1. SVNPIWA Operator

Definition 9. *Consider $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ as the accumulation of SVNNs, and $SVNPIWA : \check{\$}^n \to \check{\$}$ is a mapping,*

$$SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_1 \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_2 \oplus \ldots, \oplus \frac{\zeta^{\eta}{}_r}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_r \tag{17}$$

then the mapping SVNPIWA is called a "single-valued neutrosophic prioritized interactive weighted averaging (SVNPIWA) operator", where $\zeta^{\eta}{}_j = \prod_{k=1}^{j-1} \widehat{F}(\aleph^{\chi}{}_k)$ $(j = 2\ldots, n)$, $\zeta^{\eta}{}_1 = 1$, and $\widehat{F}(\aleph^{\chi}{}_k)$ is the score of the kth SVNN.

Based on SVN interactive operations we have the following theorem. By this theorem, we can easily find the value of the SVNPIWA operator.

Theorem 5. *Consider $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ as the accumulation of SVNNs, then*

$$
\begin{aligned}
&SVNIPWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \\
&= \Bigg(1 - \prod_{\daleth=1}^{r}\left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \\
&\prod_{\daleth=1}^{r}\left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r}\left(1 - ((\mu^{\chi}{}_{\daleth}) + (\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \\
&\prod_{\daleth=1}^{r}\left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r}\left(1 - ((\mu^{\chi}{}_{\daleth}) + (\tau^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\Bigg)
\end{aligned}
\tag{18}
$$

Proof. This is given in Appendix A.1. □

Below are several highly appealing traits of the SVNPIWA operator that have been identified.

Theorem 6. *Assume* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *is the accumulation of SVNNs, where* $\zeta^{\eta}{}_q = \prod_{\daleth=1}^{q-1}\widehat{F}(\aleph^{\chi}{}_{\daleth})$ $(q = 2\ldots, n)$*,* $\zeta^{\eta}{}_1 = 1$*, and* $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of the* g^{th} *SVNN. If all* $\aleph^{\chi}{}_r$ *are equal, i.e.,* $\aleph^{\chi}{}_r = \aleph^{\chi}$ *for all q, then*

$$SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \aleph^{\chi}$$

Proof. This is given in Appendix A.2. □

By Theorem 6, if all SVNNs are identical, then the output of the SVNPIWA operator will also be identical.

Corollary 1. *If* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *is the accumulation of the largest SVNNs, i.e.,* $\aleph^{\chi}{}_r = (1, 0, 0)$ *for all j, then*

$$SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = (1, 0, 0)$$

Proof. Proof of this corollary can be readily obtained by utilizing Theorem 6. □

Corollary 2. *If* $\aleph^{\chi}{}_1 = \langle \mu^{\chi}{}_1, \nu^{\chi}{}_1, \tau^{\chi}{}_1 \rangle$ *is the smallest SVNN, i.e.,* $\aleph^{\chi}{}_1 = (0, 0, 1)$*, then*

$$SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = (0, 0, 1)$$

Proof. This is given in Appendix A.3. □

Corollary 2 states that when the highest priority attribute is the smallest SVNN, even if other criteria are satisfied, no rewards will be obtained.

Theorem 7. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$*, and* $\aleph^{\chi *}_r = \langle \mu^{\chi *}_r, \nu^{\chi *}_r, \tau^{\chi *}_r \rangle$ *as the accumulations of SVNNs, where* $\zeta^{\eta}{}_q = \prod_{\daleth=1}^{q-1}\widehat{F}(\aleph^{\chi}{}_{\daleth})$*,* $\zeta^{\eta *}_q = \prod_{\daleth=1}^{q-1}\widehat{F}(\aleph^{\chi *}_{\daleth})$ $(\daleth = 2\ldots, n)$*,* $\zeta^{\eta}{}_1 = 1$*,* $\zeta^{\eta *}_1 = 1$*,* $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of* $\aleph^{\chi}{}_{\daleth}$ *SVNN, and* $\widehat{F}(\aleph^{\chi *}_{\daleth})$ *is the score of* $\aleph^{\chi *}_{\daleth}$ *SVNN. If* $\mu^{\chi *}_r \geq \mu^{\chi}{}_r$*,* $\nu^{\chi *}_r \leq \nu^{\chi}{}_r$*, and* $\tau^{\chi *}_r \leq \tau^{\chi}{}_r$ *for all r, then*

$$SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \leq SVNPIWA(\aleph^{\chi *}_1, \aleph^{\chi *}_2, \ldots \aleph^{\chi *}_r)$$

Proof. This is given in Appendix A.4. □

By Theorem 7, if we have two collections of SVNNs and one collection is entirely larger than the other, we can conclude that the aggregate output is also greater than the other collection.

Theorem 8. *(Boundary) Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *as the family of SVNNs, and*

$$\aleph^{\chi -} = (min_q\,(\mu^{\chi}{}_r), max_q\,(\nu^{\chi}{}_r)) \quad and \quad \aleph^{\chi +} = (max_q\,(\mu^{\chi}{}_r), min_q\,(\nu^{\chi}{}_r))$$

Then,

$$\aleph^{\chi -} \leq SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \leq \aleph^{\chi +}$$

where $\zeta^{\eta}{}_{\daleth} = \prod_{\daleth=1}^{q-1} \widehat{F}(\aleph^{\chi}{}_{\daleth})$ $(q = 2\ldots, r)$, $\zeta^{\eta}{}_1 = 1$ *and* $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of the* g^{th} *SVNN.*

By Theorem 8, we have the upper and lower bounds of the proposed SVNPIWA operator.

Theorem 9. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *and* $\breve{\$}^{\beth}{}_r = \langle \sigma_r, \xi_r, \delta_r \rangle$ *as two accumulations of SVNNs, where* $\zeta^{\eta}{}_{\daleth} = \prod_{\daleth=1}^{r-1} \widehat{F}(\aleph^{\chi}{}_{\daleth})$ $(\daleth = 2\ldots, r)$, $\zeta^{\eta}{}_1 = 1$, *and* $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of the* j^{th} *SVNN. If* $R > 0$ *and* $\breve{\$}^{\beth} = \langle \mu^{\chi}{}_{\breve{\$}^{\beth}}, \nu^{\chi}{}_{\breve{\$}^{\beth}}, \tau^{\chi}{}_{\breve{\$}^{\beth}} \rangle$ *is an SVNN, then*

1. $SVNPIWA(\aleph^{\chi}{}_1 \oplus \breve{\$}^{\beth}, \aleph^{\chi}{}_2 \oplus \breve{\$}^{\beth}, \ldots \aleph^{\chi}{}_r \oplus \breve{\$}^{\beth}) = SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \oplus \breve{\$}^{\beth}$
2. $SVNPIWA(R\aleph^{\chi}{}_1, R\aleph^{\chi}{}_2, \ldots R\aleph^{\chi}{}_r) = R\ SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r)$
3. $SVNPIWA(\aleph^{\chi}{}_1 \oplus \breve{\$}^{\beth}{}_1, \aleph^{\chi}{}_2 \oplus \breve{\$}^{\beth}{}_2, \ldots \aleph^{\chi}{}_r \oplus \breve{\$}^{\beth}{}_n) = SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \oplus$ $SVNPIWA(\breve{\$}^{\beth}{}_1, \breve{\$}^{\beth}{}_2, \ldots \breve{\$}^{\beth}{}_r)$
4. $SVNPIWA\ R\aleph^{\chi}{}_1 \oplus \breve{\$}^{\beth}, R\aleph^{\chi}{}_2 \oplus \breve{\$}^{\beth}, \ldots \oplus R\aleph^{\chi}{}_r \oplus \breve{\$}^{\beth}) = R\ SVNPIWA(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \oplus \breve{\$}^{\beth}$

Proof. This is given in Appendix A.5. □

Theorem 9 indicated some algebraic properties of the SVNPIWA operator.

3.2. SVN Prioritized Interactive Geometric AOs

Definition 10. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *as the accumulation of SVNNs, and SVNPIWG :* $\breve{\$}^n \to \breve{\$}$ *as a mapping. If*

$$SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \aleph^{\chi}{}_1^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \otimes \aleph^{\chi}{}_2^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \otimes \ldots, \otimes \aleph^{\chi}{}_r^{\frac{\zeta^{\eta}{}_r}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \tag{19}$$

then the mapping SVNPIWG is called a "single-valued neutrosophic prioritized interactive weighted geometric (SVNPIWG) operator", where $\zeta^{\eta}{}_j = \prod_{k=1}^{j-1} \widehat{F}(\aleph^{\chi}{}_k)$ $(j = 2\ldots, n)$, $\zeta^{\eta}{}_1 = 1$, *and* $\widehat{F}(\aleph^{\chi}{}_k)$ *is the score of the kth SVNN.*

The following theorem is derived from SVN interactive operations.

Theorem 10. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *as the accumulation of SVNNs, then*

$$\begin{aligned} &SVNIPWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \\ &= \left(\prod_{\daleth=1}^{r} \left(1 - (\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r} \left(1 - ((\nu^{\chi}{}_{\daleth}) + (\mu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \right. \\ &\left. 1 - \prod_{\daleth=1}^{r} \left(1 - (\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, 1 - \prod_{\daleth=1}^{r} \left(1 - (\tau^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right) \end{aligned} \tag{20}$$

Proof. This is given in Appendix B.1. □

Below are several highly appealing traits of the SVNPIWG operator that have been identified.

Theorem 11. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *as the accumulation of SVNNs, where* $\zeta^{\eta}{}_q = \prod_{\daleth=1}^{q-1} \widehat{F}(\aleph^{\chi}{}_{\daleth})$ $(q = 2\ldots, n)$, $\zeta^{\eta}{}_1 = 1$, *and* $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of the* g^{th} *SVNN. If all* $\aleph^{\chi}{}_r$ *are equal, i.e.,* $\aleph^{\chi}{}_r = \aleph^{\chi}$ *for all q, then*

$$SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = \aleph^{\chi}$$

Proof. This is given in Appendix B.2. □

Corollary 3. *If* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ $q = (1, 2, \ldots n)$ *is the accumulation of the largest SVNNs, i.e.,* $\aleph^{\chi}{}_r = (1, 0, 0)$ *for all j, then*

$$SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = (1, 0, 0)$$

Proof. A corollary similar to Theorem 6 can be readily derived. □

Corollary 4. *(Non-compensatory) If* $\aleph^{\chi}{}_1 = \langle \mu^{\chi}{}_1, \nu^{\chi}{}_1 \rangle$ *is the smallest SVNN, i.e.,* $\aleph^{\chi}{}_1 = (0, 0, 1)$, *then*

$$SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) = (0, 0, 1)$$

Proof. This is given in Appendix B.3. □

The implication of Corollary 4 is that when the minimum SVNN satisfies the more important criteria, the other criteria will not receive any rewards even if they are also met.

Theorem 12. *(Monotonicity) Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *and* $\aleph^{\chi*}_r = \langle \mu^{\chi*}_r, \nu^{\chi*}_r, \tau^{\chi*}_r \rangle$ *as the accumulations of SVNNs, where* $\zeta^{\eta}{}_q = \prod_{\daleth=1}^{q-1} \widehat{F}(\aleph^{\chi}{}_{\daleth})$, $\zeta^{\eta*}_q = \prod_{\daleth=1}^{q-1} \widehat{F}(\aleph^{\chi*}_{\daleth})$ $(\daleth = 2\ldots, n)$, $\zeta^{\eta}{}_1 = 1$, $\zeta^{\eta*}_1 = 1$, $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of* $\aleph^{\chi}{}_{\daleth}$ *SVNN, and* $\widehat{F}(\aleph^{\chi*}_{\daleth})$ *is the score of* $\aleph^{\chi*}_{\daleth}$ *SVNN. If* $\mu^{\chi*}_r \geq \mu^{\chi}{}_r$, $\nu^{\chi*}_r \leq \nu^{\chi}{}_r$ *and* $\tau^{\chi*}_r \leq \tau^{\chi}{}_r$ *for all q, then*

$$SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \leq SVNPIWG(\aleph^{\chi*}_1, \aleph^{\chi*}_2, \ldots \aleph^{\chi*}_r)$$

Theorem 13. *Consider* $\aleph^{\chi}{}_r = \langle \mu^{\chi}{}_r, \nu^{\chi}{}_r, \tau^{\chi}{}_r \rangle$ *and* $\check{\$}^{\beth}{}_r = \langle \sigma_r, \xi_r, \delta_r \rangle$ *as two accumulations of SVNNs, where* $\zeta^{\eta}{}_{\daleth} = \prod_{\daleth=1}^{r-1} \widehat{F}(\aleph^{\chi}{}_{\daleth})$ $(\daleth = 2\ldots, r)$, $\zeta^{\eta}{}_1 = 1$, *and* $\widehat{F}(\aleph^{\chi}{}_{\daleth})$ *is the score of the* j^{th} *SVNN. If* $R > 0$ *and* $\check{\$}^{\beth} = \langle \mu^{\chi}{}_{\check{\$}^{\beth}}, \nu^{\chi}{}_{\check{\$}^{\beth}}, \tau^{\chi}{}_{\check{\$}^{\beth}} \rangle$ *is an SVNN, then*

1. $SVNPIWG(\aleph^{\chi}{}_1 \otimes \check{\$}^{\beth}, \aleph^{\chi}{}_2 \otimes \check{\$}^{\beth}, \ldots \aleph^{\chi}{}_r \otimes \check{\$}^{\beth}) = SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \otimes \check{\$}^{\beth}$
2. $SVNPIWG(R\aleph^{\chi}{}_1, R\aleph^{\chi}{}_2, \ldots R\aleph^{\chi}{}_r) = R\ SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r)$
3. $SVNPIWG(\aleph^{\chi}{}_1 \otimes \check{\$}^{\beth}{}_1, \aleph^{\chi}{}_2 \otimes \check{\$}^{\beth}{}_2, \ldots \aleph^{\chi}{}_r \otimes \check{\$}^{\beth}{}_n) = SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \otimes$ $SVNPIWG(\check{\$}^{\beth}{}_1, \check{\$}^{\beth}{}_2, \ldots \check{\$}^{\beth}{}_r)$
4. $SVNPIWG\ (R\aleph^{\chi}{}_1 \otimes \check{\$}^{\beth}, R\aleph^{\chi}{}_2 \otimes \check{\$}^{\beth}, \ldots \otimes R\aleph^{\chi}{}_r \otimes \check{\$}^{\beth}) = R\ SVNPIWG(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \otimes \check{\$}^{\beth}$

4. Methodology for MCGDM Using Proposed AOs

Consider a set of alternatives denoted by $\mathscr{L}^{\beth} = \{\mathscr{L}^{\beth}_1, \mathscr{L}^{\beth}_2, \ldots, \mathscr{L}^{\beth}_m\}$ and a collection of criteria denoted by $\kappa^{\gamma} = \{\kappa^{\gamma}{}_1, \kappa^{\gamma}{}_2, \ldots, \kappa^{\gamma}{}_n\}$, where the priority between the criteria is determined using a linear orientation. Specifically, $\kappa^{\gamma}{}_1 \succ \kappa^{\gamma}{}_2 \succ \kappa^{\gamma}{}_3 \ldots \kappa^{\gamma}{}_n$ indicates that criterion $\kappa^{\gamma}{}_j$ is assigned a higher priority than criterion $\kappa^{\gamma}{}_i$ if $j > i$. Let $\tau^{\zeta} = \{\tau^{\zeta}{}_1, \tau^{\zeta}{}_2, \ldots, \tau^{\zeta}{}_p\}$ denote the decision makers, where the prioritization between the decision makers is given by $\tau^{\zeta}{}_1 \succ \tau^{\zeta}{}_2 \succ \tau^{\zeta}{}_3 \ldots \tau^{\zeta}{}_p$. In this setting, each decision maker provides a matrix $D^{(p)} = (\mathscr{B}^{(p)}_{ij})m \times n$ representing their own perspective on the alternatives with respect to the attributes.

If all performance parameters are of the same type, no normalization is necessary. However, in the case of MCGDM where there are two types of assessment criteria (benefit criteria τ^{χ}_{b} and cost criteria τ^{χ}_{c}), the matrix $D(p)$ is normalized using the formula $Y^{(p)} = (\mathscr{P}^{(p)}ij)m \times n$ to produce a normalized matrix.

$$(\mathscr{P}^{(p)}_{ij})_{m\times n} = \begin{cases} (\mathscr{B}^{(p)}_{ij})^{c}; & j \in \tau^{\chi}{}_{c} \\ \mathscr{B}^{(p)}_{ij}; & j \in \tau^{\chi}{}_{b}. \end{cases} \tag{21}$$

where $(\mathscr{B}^{(p)}_{ij})^{c}$ shows the compliment of $\mathscr{B}^{(p)}_{ij}$.

The suggested operators will be implemented to the MCGDM, which will require the preceding steps (Algorithm 1).

Algorithm 1 Decision-making algorithm

Step 1: The decision matrix $D^{(p)} = (\mathscr{B}^{(p)}ij)m \times n$ can be obtained into SVNN format from the DMs.

Step 2: Find the normalization matrix $Y^{(p)} = (\mathscr{P}^{(p)}_{ij})_{m\times n}$ using Equation (21).

Step 3: Evaluate the values of $\breve{\varpi}^{(p)}_{ij}$ by given formula.

$$\breve{\varpi}^{(p)}_{ij} = \overline{\prod}_{k=1}^{p-1}\mho(\mathscr{P}^{(k)}_{ij}) \quad (p = 2\ldots,n), \tag{22}$$

$$\breve{\varpi}^{(1)}_{ij} = 1$$

Step 4: Use one of the provided AOs to combine all of the independent SVN decision matrices $Y^{(p)} = (\mathscr{P}^{(p)}_{ij})_{m\times n}$ into one combined evaluation matrix of the alternatives $W^{(p)} = (\mathcal{E}_{ij})_{m\times n}$.

$$\begin{aligned} \mathcal{E}_{ij} &= \mathrm{SVNPIWA}(\mathscr{P}^{(1)}_{ij}, \mathscr{P}^{(2)}_{ij}, \ldots \mathscr{P}^{(p)}_{ij}) \\ &= \left(1 - \prod_{z=1}^{p}\left(1 - ((\mu^{\chi^{z}_{ij}})\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}}, \prod_{z=1}^{p}\left(1 - (\mu^{\chi^{z}_{ij}})\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}} - \prod_{z=1}^{p}\left(1 - ((\mu^{\chi^{z}_{ij}}) + (\nu^{\chi^{z}_{ij}}))\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}},\right. \\ &\quad \left.\prod_{z=1}^{p}\left(1 - (\mu^{\chi^{z}_{ij}})\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}} - \prod_{z=1}^{p}\left(1 - ((\mu^{\chi^{z}_{ij}}) + (\tau^{\chi^{z}_{ij}}))\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}}\right) \end{aligned} \tag{23}$$

or

$$\begin{aligned} \mathcal{E}_{ij} &= \mathrm{SVNPIWG}(\mathscr{P}^{(1)}_{ij}, \mathscr{P}^{(2)}_{ij}, \ldots \mathscr{P}^{(p)}_{ij}) \\ &= \left(\prod_{z=1}^{p}\left(1 - (\nu^{\chi^{z}_{ij}})\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}} - \prod_{z=1}^{p}\left(1 - ((\nu^{\chi^{z}_{ij}}) + (\mu^{\chi^{z}_{ij}}))\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}}, 1 - \prod_{z=1}^{p}\left(1 - ((\nu^{\chi^{z}_{ij}})\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}},\right. \\ &\quad \left.1 - \prod_{z=1}^{p}\left(1 - ((\tau^{\chi^{z}_{ij}})\right)^{\frac{\breve{\varpi}^{z}_{j}}{\sum_{j=1}^{n}\breve{\varpi}^{z}_{j}}}\right) \end{aligned} \tag{24}$$

Step 5: Calculate the values of $\breve{\varpi}_{ij}$ by the following formula:

$$\breve{\varpi}_{ij} = \overline{\prod}_{k=1}^{j-1}\mho(\mathcal{E}_{ik}) \quad (j = 2\ldots,n), \tag{25}$$

$$\breve{\varpi}_{i1} = 1$$

Algorithm 1 *Cont.*

Step 6: Aggregate the SVN values $\mathcal{E}_{ij}$ for each alternative $\mathscr{L}_i^{\daleth}$ by the SVNPIWA (or SVNPIWG) operator.

$$\begin{aligned}\mathcal{E}_{ij} &= \text{SVNPIWA}(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \ldots \mathscr{P}_{in}) \\ &= \left(1 - \prod_{j=1}^{n}\left(1 - \mu^{\chi}{}_{ij}\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}}, \prod_{j=1}^{n}\left(1 - \mu^{\chi}{}_{ij}\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}} - \prod_{j=1}^{n}\left(1 - (\mu^{\chi}{}_{ij} + \nu^{\chi}{}_{ij})\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}},\right. \\ &\left.\prod_{j=1}^{n}\left(1 - \mu^{\chi}{}_{ij}\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}} - \prod_{j=1}^{n}\left(1 - (\mu^{\chi}{}_{ij} + \tau^{\chi}{}_{ij})\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}}\right) \end{aligned} \tag{26}$$

or

$$\begin{aligned}\mathcal{E}_{ij} &= \text{SVNPIWG}(\mathscr{P}_{i1}, \mathscr{P}_{i2}, \ldots \mathscr{P}_{in}) \\ &= \left(\prod_{j=1}^{n}\left(1 - \nu^{\chi}{}_{ij}\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}} - \prod_{j=1}^{n}\left(1 - (\nu^{\chi}{}_{ij} + \mu^{\chi}{}_{ij})\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}}, 1 - \prod_{j=1}^{n}\left(1 - \nu^{\chi}{}_{ij}\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}},\right. \\ &\left. 1 - \prod_{j=1}^{n}\left(1 - \tau^{\chi}{}_{ij}\right)^{\frac{\breve{\omega}_j}{\sum_{j=1}^{n}\breve{\omega}_j}}\right)\end{aligned} \tag{27}$$

Step 7: Analyze the score for all cumulative alternative assessments.

Step 8: The alternatives were classified by the SF and, eventually, the most suitable alternative was selected.

5. Case Study

It is widely accepted that a transition away from fossil hydrocarbons and toward renewable sources of energy will inevitably take place in order to curtail gas emissions, mitigate environmental impacts, and lessen reliance on fossil fuels. As a result of this trend, numerous businesses all over the world, particularly multinational enterprises with a significant amount of power, have been pioneers in the use of renewable energy. The fact that an increasing number of businesses are declaring their support for renewable energy and taking concrete steps to expedite the transition to clean energy sources is regarded as a positive indicator in the context of the global community's collective efforts to create a greener future. Gas consumption by industry in Pakistan can be seen in Figure 1, oil/petroleum consumption by industry in Pakistan can be seen in Figure 2, and electricity consumption by industry in Pakistan can be seen in Figure 3 [55]. The sustainable industry is very important to the growth and improvement of a nation's economy. In past few decades, there has been a phenomenal rise in the amount of energy that is required. The rise in economic activity, population expansion, and the fast technological revolution that is occurring across the globe are the main primary drivers of the development related to the need for energy. Corporations in Pakistan are paying considerable attention to the usage of green sources in manufacturing as well as commercial operations for the purpose of achieving environmental sustainability. Recently, a growing number of companies have taken a keen interest in adopting the use of alternative energy options. These companies are looking for eco-friendly alternatives to develop goods that are compliant with green requirements in order to increase their market competition. However, choosing an appropriate energy supply for any industrial complex project is not an easy process since it involves a number of qualitative and quantitative standards [56,57].

The amount of oil that Pakistan is manufacturing is only a very small fraction of what the country needs to fulfil its total demand. The development of indigenous oil is hampered by scientific, managerial, and economical limitations. Because of this, a considerable portion of the overall demand must be satisfied by importing substantial volumes of petroleum and other forms of crude oil. The most recent information suggests that the cost of importing oil climbed by 95.9 percent during the first four months of the fiscal year 2022, reaching USD 17.03 billion compared to USD 8.69 billion during the same time period in the previous year. Oil is becoming more costly as a result of higher oil prices on the worldwide market as well as the enormous devaluation of the Pakistani rupee. This is leading to pressure on Pakistan's external sector and a widening trade imbalance for the country. The rise in the cost of importing oil may be ascribed to two factors: first, an increase in the value of imported oil; second, an increase in the amount of imported oil. The value of imported petroleum products increased by 121.15 percent while the quantity increased by 24.18 percent. During the time period under consideration, the value of the country's crude oil imports increased by 75.34 percent, while the quantity increased by 1.4 percent. During the same time period, the value of imports of liquefied natural gas increased by 82.90 percent, while imports of liquefied petroleum gas (LPG) increased by 39.86 percent. Both of these increases occurred during the first four months of the 2022 fiscal year [55].

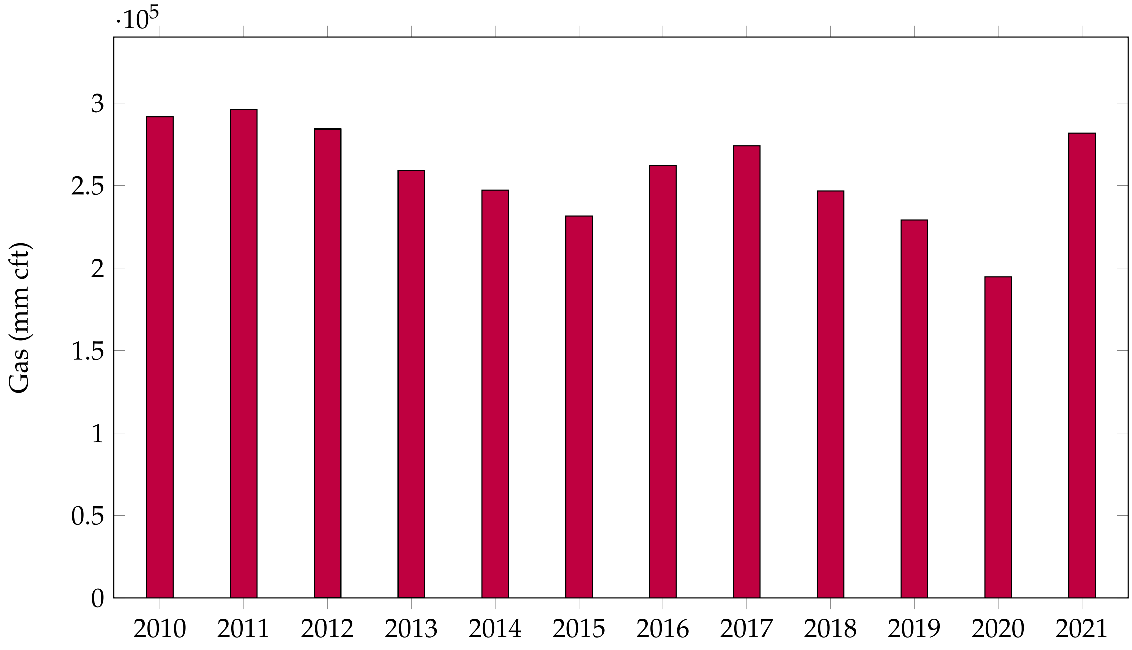

Figure 1. Gas consumption by industry in Pakistan.

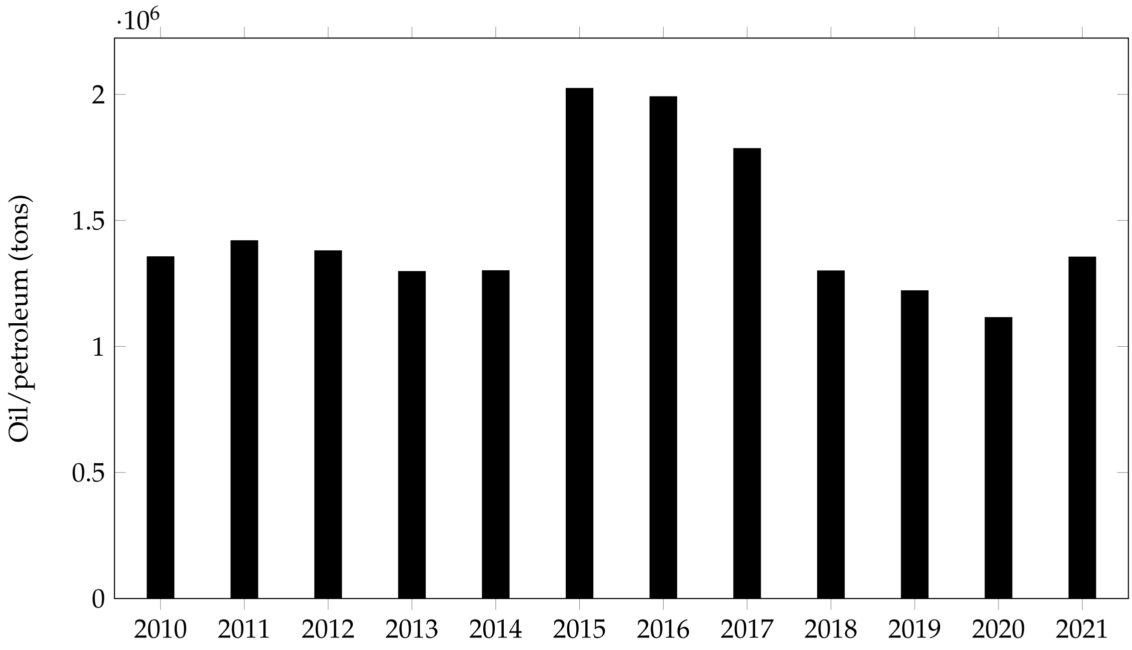

Figure 2. Oil/petroleum consumption by industry in Pakistan.

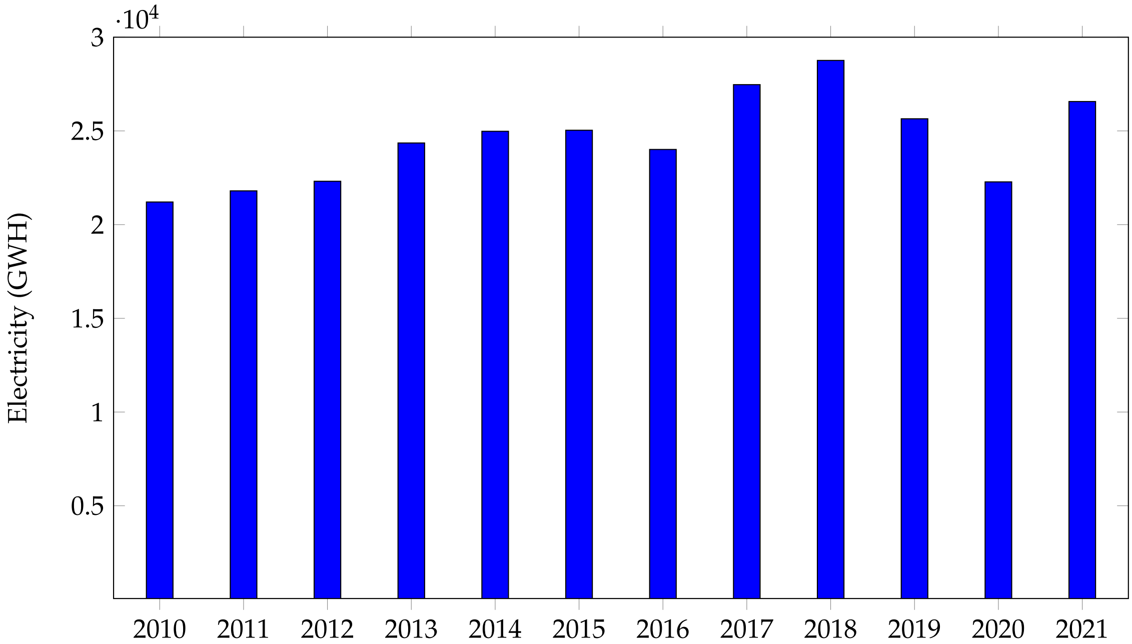

Figure 3. Electricity consumption by industry in Pakistan.

There has been a significant huge potential market for gas, which is placing enormous burden on the meager gas reserves that the country has available. As a result, the country's finite natural gas reserves are swiftly running out. The government is seeking reasonable options that may be implemented either in the near term or over a lengthy period in order to efficiently meet the enormous energy demand. The administration is putting a lot of effort into developing new boreholes in order to enhance the amount of national gas that is available to meet the growing demand for electricity. In addition to it, natural gas in the form of LNG and piped gas is being brought in. During the 2021 fiscal year, around 373 million MMBTU of LNG gas with a value of approximately USD 34 billion was brought in. This accounts for perhaps around 30 percent of the total natural gas use across the nation. Domestic production of gas accounts for 75.64 percent of the total during the first two months of the 2022 fiscal year, while imports account for 24.36 percent of the total [55].

Wind corridors can be found throughout Pakistan, and the country has an enormous capacity to produce energy from windmill power. It is anticipated that Pakistan can produce 50,000 MW through wind. Wind energy accounts for 4.8 percent of the total generation capacity and now stands at 1985 MW as its contribution. There is a significant immense capacity for solar electricity in Pakistan. Nearly the whole nation is bathed in copious natural light all day long. According to the Independent and Renewable Energy Policy 2019, the capacity share of these renewable resources is rather low at the moment, but the policy anticipates that it will significantly expand over the next several years. In total, 600 megawatts (MW) of solar panels have been installed, which is approximately 1.4 percent of the total installed capacity. The potential for Pakistan to generate energy from water is great, and the country is extremely wealthy in hydroelectric resources. It is predicted that Pakistan has a total hydropower potential of around 60,000 MW. The country is not utilising its full potential and is utilising almost 16 percent of the overall hydropower capacity. The significant initial investment required for the construction of hydropower facilities, the establishment of an energy transmission network, and the relocation of the people who would be displaced are only a few of the reasons why hydropower is not being utilised to its full potential. Hydroelectricity accounts for about 25 percent of the total installed capacity, with its current installed capacity of 10,251 MW [58].

This demonstrates that Pakistan has an immense capability for the expansion of renewable energy sources, as well as an abundance of space for future growth. However, the execution of sustainable electricity projects is today confronted with additional obstacles, such as grid infrastructure and power system dispatching procedures, as a direct result of the requirement to maximise the integration of new electricity sources into the sector.

To illustrate the suggested MCGDM technique, we will choose the optimal renewable energy sources for industrial complex utilizing SVNNs based on a variety of attributes.

5.1. Numerical Example

There are five alternatives $\mathscr{L}^{\daleth}{}_i$ $(i = 1,2,3,4,5)$, where $\mathscr{L}^{\daleth}{}_1 =$ solar energy, $\mathscr{L}^{\daleth}{}_2 =$ solid waste energy, $\mathscr{L}^{\daleth}{}_3 =$ biomass energy, $\mathscr{L}^{\daleth}{}_4 =$ wind energy, and $\mathscr{L}^{\daleth}{}_5 =$ nuclear energy. Here, $\kappa^{\curlyvee}{}_1 =$ social factor (social benefits, social acceptance), $\kappa^{\curlyvee}{}_2 =$ technical factor (national energy security, national economic benefits), $\kappa^{\curlyvee}{}_3 =$ economic factor (technical maturity, grid availability, efficiency), $\kappa^{\curlyvee}{}_4 =$ environmental factor (carbon emissions), and $\kappa^{\curlyvee}{}_5 =$ political factor (investment cost, management cost) are attributes. Priorities are assigned betwixt the criteria provided by the linear orientation in this case. $\kappa^{\curlyvee}{}_1 \succ \kappa^{\curlyvee}{}_2 \succ \kappa^{\curlyvee}{}_3 \ldots \kappa^{\curlyvee}{}_5$ indicates criteria $\kappa^{\curlyvee}{}_J$ has a high priority than $\kappa^{\curlyvee}{}_i$ if $j > i$. In this example, we use SVNNs as input data for ranking the given alternatives under the given attributes. Here, three DMs are involved, i.e., $\tau^{\zeta}{}_1, \tau^{\zeta}{}_2$, and $\tau^{\zeta}{}_3$. DMs are not given the same priority. Prioritization is provided by a linear pattern betwixt the DMs, given as $\tau^{\zeta}{}_1 \succ \tau^{\zeta}{}_2 \succ \tau^{\zeta}{}_3$, which shows that DM $\tau^{\zeta}{}_{\zeta}$ has a higher importance than $\tau^{\zeta}{}_{\varrho}$ if $\zeta > \varrho$.

Using SVNPIWA Operator

Step 1: Obtain the decision matrix $D^{(p)} = (\mathscr{B}_{ij}^{(p)})_{m \times n}$ in the format of SVNNs by the DMs, given in Tables 1–3.

Table 1. SVN decision matrix from $\tau^{\zeta}{}_1$.

	$\kappa^{\curlyvee}{}_1$	$\kappa^{\curlyvee}{}_2$	$\kappa^{\curlyvee}{}_3$	$\kappa^{\curlyvee}{}_4$	$\kappa^{\curlyvee}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.961, 0.754, 0.431)	(0.675, 0.253, 0.435)	(0.285, 0.213, 0.237)	(0.723, 0.311, 0.322)	(0.237, 0.342, 0.543)
$\mathscr{L}^{\daleth}{}_2$	(0.543, 0.135, 0.533)	(0.545, 0.355, 0.532)	(0.154, 0.745, 0.432)	(0.567, 0.435, 0.422)	(0.345, 0.345, 0.421)
$\mathscr{L}^{\daleth}{}_3$	(0.745, 0.435, 0.432)	(0.335, 0.321, 0.542)	(0.422, 0.243, 0.532)	(0.321, 0.342, 0.533)	(0.432, 0.423, 0.543)
$\mathscr{L}^{\daleth}{}_4$	(0.454, 0.332, 0.432)	(0.723, 0.456, 0.643)	(0.426, 0.163, 0.543)	(0.432, 0.223, 0.543)	(0.345, 0.353, 0.432)
$\mathscr{L}^{\daleth}{}_5$	(0.732, 0.323, 0.543)	(0.543, 0.765, 0.654)	(0.234, 0.764, 0.543)	(0.543, 0.554, 0.435)	(0.542, 0.265, 0.542)

Table 2. SVN decision matrix from $\tau^{\zeta}{}_2$.

	$\kappa^{\curlyvee}{}_1$	$\kappa^{\curlyvee}{}_2$	$\kappa^{\curlyvee}{}_3$	$\kappa^{\curlyvee}{}_4$	$\kappa^{\curlyvee}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.342, 0.342, 0.675)	(0.342, 0.346, 0.435)	(0.342, 0.434, 0.564)	(0.564, 0.134, 0.543)	(0.453, 0.234, 0.534)
$\mathscr{L}^{\daleth}{}_2$	(0.432, 0.654, 0.342)	(0.542, 0.342, 0.762)	(0.453, 0.654, 0.453)	(0.432, 0.654, 0.453)	(0.773, 0.342, 0.654)
$\mathscr{L}^{\daleth}{}_3$	(0.854, 0.632, 0.321)	(0.513, 0.243, 0.432)	(0.321, 0.524, 0.432)	(0.621, 0.455, 0.542)	(0.232, 0.234, 0.543)
$\mathscr{L}^{\daleth}{}_4$	(0.432, 0.323, 0.432)	(0.532, 0.432, 0.451)	(0.322, 0.134, 0.543)	(0.321, 0.234, 0.532)	(0.321, 0.671, 0.572)
$\mathscr{L}^{\daleth}{}_5$	(0.369, 0.542, 0.441)	(0.624, 0.342, 0.431)	(0.521, 0.127, 0.579)	(0.343, 0.334, 0.431)	(0.342, 0.331, 0.348)

Table 3. SVN decision matrix from $\tau^{\zeta}{}_3$.

	$\kappa^{\curlyvee}{}_1$	$\kappa^{\curlyvee}{}_2$	$\kappa^{\curlyvee}{}_3$	$\kappa^{\curlyvee}{}_4$	$\kappa^{\curlyvee}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.532, 0.112, 0.332)	(0.752, 0.223, 0.436)	(0.843, 0.432, 0.649)	(0.462, 0.345, 0.431)	(0.745, 0.253, 0.524)
$\mathscr{L}^{\daleth}{}_2$	(0.454, 0.245, 0.422)	(0.354, 0.135, 0.422)	(0.531, 0.335, 0.421)	(0.432, 0.343, 0.521)	(0.532, 0.323, 0.321)
$\mathscr{L}^{\daleth}{}_3$	(0.431, 0.135, 0.457)	(0.575, 0.243, 0.632)	(0.555, 0.532, 0.522)	(0.434, 0.356, 0.632)	(0.342, 0.324, 0.527)
$\mathscr{L}^{\daleth}{}_4$	(0.642, 0.354, 0.521)	(0.441, 0.521, 0.539)	(0.531, 0.349, 0.531)	(0.353, 0.531, 0.451)	(0.183, 0.223, 0.451)
$\mathscr{L}^{\daleth}{}_5$	(0.541, 0.231, 0.432)	(0.531, 0.153, 0.522)	(0.434, 0.254, 0.522)	(0.335, 0.254, 0.529)	(0.535, 0.585, 0.234)

Step 2: Using Equation (21), normalize the decision matrices gained by DMs. $\kappa^{\Upsilon}{}_2$ is a cost-type criterion and others are benefit-type criterions, given in Tables 4–6.

Table 4. Normalized SVN decision matrix from $\tau^{\zeta}{}_1$.

	$\kappa^{\Upsilon}{}_1$	$\kappa^{\Upsilon}{}_2$	$\kappa^{\Upsilon}{}_3$	$\kappa^{\Upsilon}{}_4$	$\kappa^{\Upsilon}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.461, 0.254, 0.431)	(0.435, 0.253, 0.275)	(0.285, 0.213, 0.237)	(0.123, 0.311, 0.322)	(0.237, 0.342, 0.543)
$\mathscr{L}^{\daleth}{}_2$	(0.343, 0.135, 0.533)	(0.532, 0.355, 0.145)	(0.154, 0.745, 0.432)	(0.167, 0.435, 0.422)	(0.345, 0.345, 0.421)
$\mathscr{L}^{\daleth}{}_3$	(0.345, 0.435, 0.432)	(0.542, 0.321, 0.335)	(0.422, 0.243, 0.532)	(0.121, 0.342, 0.533)	(0.232, 0.423, 0.543)
$\mathscr{L}^{\daleth}{}_4$	(0.354, 0.332, 0.132)	(0.143, 0.456, 0.123)	(0.226, 0.163, 0.143)	(0.432, 0.223, 0.543)	(0.345, 0.353, 0.432)
$\mathscr{L}^{\daleth}{}_5$	(0.132, 0.323, 0.143)	(0.654, 0.165, 0.243)	(0.234, 0.164, 0.243)	(0.143, 0.554, 0.435)	(0.142, 0.265, 0.542)

Table 5. Normalized SVN decision matrix from $\tau^{\zeta}{}_2$.

	$\kappa^{\Upsilon}{}_1$	$\kappa^{\Upsilon}{}_2$	$\kappa^{\Upsilon}{}_3$	$\kappa^{\Upsilon}{}_4$	$\kappa^{\Upsilon}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.342, 0.342, 0.475)	(0.435, 0.346, 0.342)	(0.342, 0.134, 0.264)	(0.164, 0.134, 0.543)	(0.453, 0.234, 0.534)
$\mathscr{L}^{\daleth}{}_2$	(0.432, 0.354, 0.342)	(0.262, 0.342, 0.142)	(0.153, 0.654, 0.453)	(0.132, 0.154, 0.453)	(0.173, 0.342, 0.654)
$\mathscr{L}^{\daleth}{}_3$	(0.854, 0.332, 0.321)	(0.432, 0.243, 0.113)	(0.321, 0.324, 0.432)	(0.121, 0.455, 0.542)	(0.232, 0.234, 0.543)
$\mathscr{L}^{\daleth}{}_4$	(0.332, 0.323, 0.432)	(0.451, 0.432, 0.132)	(0.322, 0.134, 0.125)	(0.321, 0.234, 0.532)	(0.321, 0.671, 0.572)
$\mathscr{L}^{\daleth}{}_5$	(0.369, 0.542, 0.441)	(0.431, 0.342, 0.224)	(0.121, 0.127, 0.279)	(0.343, 0.334, 0.431)	(0.342, 0.331, 0.348)

Table 6. Normalized SVN decision matrix from $\tau^{\zeta}{}_3$.

	$\kappa^{\Upsilon}{}_1$	$\kappa^{\Upsilon}{}_2$	$\kappa^{\Upsilon}{}_3$	$\kappa^{\Upsilon}{}_4$	$\kappa^{\Upsilon}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.432, 0.112, 0.232)	(0.436, 0.223, 0.152)	(0.143, 0.432, 0.369)	(0.162, 0.345, 0.431)	(0.745, 0.253, 0.524)
$\mathscr{L}^{\daleth}{}_2$	(0.454, 0.245, 0.422)	(0.422, 0.135, 0.354)	(0.131, 0.335, 0.421)	(0.132, 0.343, 0.521)	(0.132, 0.323, 0.321)
$\mathscr{L}^{\daleth}{}_3$	(0.431, 0.135, 0.457)	(0.632, 0.243, 0.175)	(0.155, 0.132, 0.522)	(0.134, 0.356, 0.632)	(0.342, 0.324, 0.527)
$\mathscr{L}^{\daleth}{}_4$	(0.242, 0.354, 0.521)	(0.239, 0.521, 0.441)	(0.131, 0.149, 0.531)	(0.353, 0.531, 0.451)	(0.183, 0.223, 0.451)
$\mathscr{L}^{\daleth}{}_5$	(0.181, 0.231, 0.432)	(0.522, 0.153, 0.131)	(0.434, 0.254, 0.222)	(0.335, 0.254, 0.529)	(0.135, 0.585, 0.234)

Step 3: Determine the $\breve{\varpi}_{ij}^{(p)}$ values using Equation (22).

$$\breve{\varpi}_{ij}^{(1)} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\breve{\varpi}_{ij}^{(2)} = \begin{pmatrix} 0.592 & 0.502 & 0.612 & 0.697 & 0.451 \\ 0.625 & 0.544 & 0.326 & 0.570 & 0.526 \\ 0.626 & 0.629 & 0.549 & 0.482 & 0.489 \\ 0.563 & 0.488 & 0.573 & 0.555 & 0.520 \\ 0.622 & 0.449 & 0.309 & 0.518 & 0.578 \end{pmatrix}$$

$$\breve{\varpi}_{ij}^{(3)} = \begin{pmatrix} 0.261 & 0.292 & 0.274 & 0.438 & 0.296 \\ 0.299 & 0.341 & 0.146 & 0.252 & 0.311 \\ 0.397 & 0.352 & 0.249 & 0.261 & 0.237 \\ 0.315 & 0.242 & 0.314 & 0.287 & 0.187 \\ 0.287 & 0.219 & 0.187 & 0.272 & 0.320 \end{pmatrix}$$

Step 4: Use SVNPIWA to aggregate all individual SVN decision matrices $Y^{(p)} = (\mathscr{P}_{ij}^{(p)})_{m\times n}$ into one cumulative assessments matrix of the alternatives $W^{(p)} = (\mathcal{E}_{ij})_{m\times n}$ using Equation (23) given in Table 7.

Table 7. Collective SVN decision matrix.

	$\kappa^{\gamma}{}_1$	$\kappa^{\gamma}{}_2$	$\kappa^{\gamma}{}_3$	$\kappa^{\gamma}{}_4$	$\kappa^{\gamma}{}_5$
$\mathscr{L}^{\daleth}{}_1$	(0.421324, 0.263951, 0.428712)	(0.418807, 0.27189, 0.273659)	(0.285225, 0.217551, 0.264106)	(0.144484, 0.264965, 0.432365)	(0.200718, 0.306102, 0.540719)
$\mathscr{L}^{\daleth}{}_2$	(0.391077, 0.250206, 0.458212)	(0.445786, 0.346762, 0.193779)	(0.151387, 0.71087, 0.435542)	(0.151393, 0.508091, 0.501523)	(0.265399, 0.346564, 0.486485)
$\mathscr{L}^{\daleth}{}_3$	(0.365273, 0.35376, 0.415507)	(0.528653, 0.294306, 0.269752)	(0.359884, 0.261262, 0.539298)	(0.135601, 0.398085, 0.584573)	(0.247895, 0.36302, 0.542863)
$\mathscr{L}^{\daleth}{}_4$	(0.361542, 0.334378, 0.335592)	(0.256165, 0.47959, 0.279706)	(0.242114, 0.15179, 0.211368)	(0.388347, 0.302402, 0.552899)	(0.321625, 0.570959, 0.490232)
$\mathscr{L}^{\daleth}{}_5$	(0.224492, 0.470171, 0.349079)	(0.586917, 0.205028, 0.245532)	(0.235804, 0.177498, 0.250159)	(0.224411, 0.438496, 0.462201)	(0.20775, 0.356832, 0.43955)

Step 5: Determine the values of $\breve{\varpi}_{ij}$ by using Equation (25).

$$\breve{\varpi}_{ij} = \begin{pmatrix} 1 & 0.576 & 0.359 & 0.216 & 0.104 \\ 1 & 0.561 & 0.356 & 0.119 & 0.045 \\ 1 & 0.532 & 0.348 & 0.181 & 0.069 \\ 1 & 0.564 & 0.276 & 0.173 & 0.088 \\ 1 & 0.468 & 0.333 & 0.201 & 0.089 \end{pmatrix}$$

Step 6: Aggregate the SVN values $\mathcal{E}_{ij}$ for each alternative $\mathscr{L}^{\daleth}{}_i$ by the SVNPIWA operator using Equation (26) given in Table 8.

Table 8. SVN aggregated values $\mathcal{E}_i$.

$\mathcal{E}_1$	(0.372423, 0.266908, 0.388898)
$\mathcal{E}_2$	(0.356465, 0.380501, 0.402695)
$\mathcal{E}_3$	(0.390243, 0.331658, 0.416333)
$\mathcal{E}_4$	(0.320105, 0.381764, 0.365156)
$\mathcal{E}_5$	(0.249393, 0.313219, 0.289075)

Step 7: Compute the score for all SVN aggregated values $\mathcal{E}_i$.

$$\widehat{F}(\mathcal{E}_1) = 0.572206$$

$$\widehat{F}(\mathcal{E}_2) = 0.524423$$

$$\widehat{F}(\mathcal{E}_3) = 0.547417$$

$$\widehat{F}(\mathcal{E}_4) = 0.524395$$

$$\widehat{F}(\mathcal{E}_5) = 0.549033$$

Step 8: Rank according to score values.

$$\mathcal{E}_1 \succ \mathcal{E}_5 \succ \mathcal{E}_3 \succ \mathcal{E}_2 \succ \mathcal{E}_4$$

So,

$$\mathscr{L}^{\daleth}{}_1 \succ \mathscr{L}^{\daleth}{}_5 \succ \mathscr{L}^{\daleth}{}_3 \succ \mathscr{L}^{\daleth}{}_2 \succ \mathscr{L}^{\daleth}{}_4$$

$\mathscr{L}^{\daleth}{}_1$ is the best alternative among all other alternatives.

5.2. Comparison Analysis

In this section, we compare our proposed AOs with several existing AOs. By applying our proposed AOs to the data and obtaining comparable optimal solutions, we demonstrate their validity and durability. Our technique is advantageous over some existing AOs because it operates in a fair and neutral manner for SVNNs, making it more practical. After applying our proposed AOs, we obtain the ranking $\mathscr{L}^{\daleth}_4 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5$. To validate our optimal solution, we also run the problem through other existing operators. The fact that we reach the same optimum conclusion using both our proposed AOs and other existing AOs demonstrates the validity of our proposed AOs. Table 9 provides a comparison of our suggested AOs with some current operators.

Table 9. Comparison of proposed AOs with some existing operators.

Authors	AOs	Ranking of Alternatives	The Optimal Alternative
Wu at el. [11]	SNNPWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Garg and Nancy [37]	SVNPMM	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Wei snd Wei [36]	SVNDPWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Liu [59]	SVNNWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Li at el. [34]	SNNEWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Nancy and Garg [27]	SVNFWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Li at el. [29]	IGWHM	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Wei and Zhang [32]	SVNWBPM	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Wei and Wei [36]	SVNDPWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Garg and Nancy [41]	L-SVNWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Peng at el. [26]	SNNWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$
Proposed Algorithm	SVNPIWA	$\mathscr{L}^{\daleth}_1 \succ \mathscr{L}^{\daleth}_5 \succ \mathscr{L}^{\daleth}_3 \succ \mathscr{L}^{\daleth}_2 \succ \mathscr{L}^{\daleth}_4$	$\mathscr{L}^{\daleth}_1$

6. Conclusions

To address the challenges of reducing greenhouse gas emissions, conserving the environment, and decreasing reliance on fossil fuels, it is essential to transition from fossil energy to renewable energy sources. Numerous multinational corporations, especially large conglomerates, have taken the initiative to incorporate renewable energy technologies. Businesses in Pakistan are becoming increasingly aware of the significance of incorporating renewable energy sources into their manufacturing and economic activities in order to achieve sustainable growth. In recent years, there has been a significant increase in the number of companies actively pursuing the use of renewable energies to satisfy ecological standards and increase their market competitiveness. However, choosing a suitable energy source for industrially complex projects is not a straightforward endeavour. It requires a thorough consideration of various qualitative and quantitative characteristics to ensure that the renewable energy source selected meets the project's specific needs and requirements. Failure to perceive attribute relationships in an uncertain environment might affect the conclusions of various MCGDM issues. To overcome these challenges, we proposed a unique method for selecting the ideal sustainable energy alternative using SVN data, in which the SVNNs accounted for the viewpoint of the decision maker. The judgements of decision makers were communicated through SVNNs, and the ambiguity and inadequacy of information were effectively managed. Due to the importance of AOs in decision making, this paper offers two hybrid AOs: the "single-valued neutrosophic prioritised interactive weighted averaging (SVNPIWA) operator and the single-valued neutrosophic prioritised interactive weighted geometric (SVNPIWG) operator". Some essential characteristics of the developed operators were highlighted. Lastly, a comprehensive illustration of the technique's possible uses was presented.

In the future, the theoretical framework will be improved by merging several rankings resulting from different multi-criteria techniques with metamethods. In addition, the established processes for ranking sustainable energy alternatives will be modified to allow for estimating uncertainty by employing a range of fuzzy set types. In addition, we want to develop new hybrid methods for assessing sustainable energy systems by combining algorithms for calculating weights with MCGDM methodology.

Author Contributions: M.R.: methodology, formal analysis; H.M.A.F.: methodology, formal analysis, writing—review and editing; J.A.: investigation, methodology, supervision; G.D.: methodology, investigation; All authors made a significant scientific contribution to the research in the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data used to support the findings of the study are included within the article.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

Appendix A.1

The initial statement enables a straightforward derivation of Definition 9 and Theorem 5, which we demonstrate in the subsequent section.

$$
\begin{aligned}
&\mathrm{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \\
=& \frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_1 \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_2 \oplus \ldots, \oplus \frac{\zeta^{\eta}{}_r}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_r \\
=& \left(1 - \prod_{\daleth=1}^{r} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \prod_{\daleth=1}^{r} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \prod_{\daleth=1}^{r} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\tau^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right)
\end{aligned}
$$

The proof of this theorem utilizes mathematical induction to establish its validity. When $\daleth = 2$

$$
\mathrm{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2) = \frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_1 \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_2
$$

By interactive laws of SVNNs, we have

$$
\begin{aligned}
\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_1 = & \left(1 - \left(1 - \mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \left(1 - \mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \left(1 - (\mu^{\chi}{}_1 + \nu^{\chi}{}_1)\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \left(1 - \mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \left(1 - (\mu^{\chi}{}_1 + \tau^{\chi}{}_1)\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right)
\end{aligned}
$$

$$\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}\aleph^{\chi}{}_2 = \left(1-\left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_2+\nu^{\chi}{}_2)\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_2+\tau^{\chi}{}_2)\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)$$

Then,

$$\begin{aligned}
&\mathrm{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2) \\
=\ & \frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}\aleph^{\chi}{}_1 \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}\aleph^{\chi}{}_2 \\
=\ & \left(1-\left(1-\mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \left(1-\mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_1+\nu^{\chi}{}_1)\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \left(1-\mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_1+\tau^{\chi}{}_1)\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right) \\
& \oplus \left(1-\left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_2+\nu^{\chi}{}_2)\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_2+\tau^{\chi}{}_2)\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right) \\
=\ & \left(1-\left(1-\mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left(1-\mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_1+\nu^{\chi}{}_1)\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\left(1-(\mu^{\chi}{}_2+\nu^{\chi}{}_2)\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \\
& \left. \left(1-\mu^{\chi}{}_1\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\left(1-\mu^{\chi}{}_2\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-(\mu^{\chi}{}_1+\tau^{\chi}{}_1)\right)^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\left(1-(\mu^{\chi}{}_2+\tau^{\chi}{}_2)\right)^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right) \\
=\ & \left(1-\prod_{\daleth=1}^{2}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \prod_{\daleth=1}^{2}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{2}\left(1-(\mu^{\chi}{}_{\daleth}+\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \prod_{\daleth=1}^{2}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{2}\left(1-(\mu^{\chi}{}_{\daleth}+\tau^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)
\end{aligned}$$

Suppose the result holds for $\daleth = d$

$$
\begin{aligned}
&\mathrm{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_d) \\
=\;& \bigoplus_{\daleth=1}^{d} \frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_{\daleth} \\
=\;& \left(1 - \prod_{\daleth=1}^{d} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \prod_{\daleth=1}^{d} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{d} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \prod_{\daleth=1}^{d} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{d} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\tau^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right)
\end{aligned}
$$

Now, we shall prove it for $\daleth = d + 1$

$$
\begin{aligned}
&\mathrm{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_d, \aleph^{\chi}{}_{d+1}) \\
=\;& \bigoplus_{\daleth=1}^{d} \frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_{\daleth} \oplus \frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_{d+1} \\
=\;& \left(1 - \prod_{\daleth=1}^{d} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \prod_{\daleth=1}^{d} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{d} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \prod_{\daleth=1}^{d} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{d} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\tau^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right) \\
& \oplus \left(1 - \left(1 - \mu^{\chi}{}_{d+1}\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \left(1 - \mu^{\chi}{}_{d+1}\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \left(1 - (\mu^{\chi}{}_{d+1} + \nu^{\chi}{}_{d+1})\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \right. \\
& \left. \left(1 - \mu^{\chi}{}_{d+1}\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \left(1 - (\mu^{\chi}{}_{d+1} + \tau^{\chi}{}_{d+1})\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right) \\
=\;& \left(1 - \prod_{\daleth=1}^{d+1} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}}, \prod_{\daleth=1}^{d+1} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{d+1} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right. \\
& \left. \prod_{\daleth=1}^{d+1} \left(1 - (\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{d+1} \left(1 - ((\mu^{\chi}{}_{\daleth}) + (\tau^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \right).
\end{aligned}
$$

Appendix A.2

From Definition 9, we have

$$
\begin{aligned}
\mathrm{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) &= \frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_1 \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_2 \oplus \ldots, \oplus \frac{\zeta^{\eta}{}_r}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi}{}_r \\
&= \frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi} \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi} \oplus \ldots, \oplus \frac{\zeta^{\eta}{}_r}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi} \\
&= \frac{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}} \aleph^{\chi} \\
&= \aleph^{\chi}
\end{aligned}
$$

Appendix A.3

Here, $\aleph^{\chi}{}_1 = (0,0,1)$, then by the SF we have

$$\widehat{F}(\aleph^{\chi}{}_1) = 0$$

Since,

$$\zeta^{\eta}{}_q = \prod_{\urcorner=1}^{q-1} \widehat{F}(\aleph^{\chi}{}_{\urcorner}) \qquad (q = 2\ldots,n), \quad \text{and} \quad \zeta^{\eta}{}_1 = 1$$

$\widehat{F}(\aleph^{\chi}{}_{\urcorner})$ is the score of gth SVNN.

Then,

$$\zeta^{\eta}{}_q = \prod_{\urcorner=1}^{q-1} \widehat{F}(\aleph^{\chi}{}_{\urcorner}) = \widehat{F}(\aleph^{\chi}{}_1) \times \widehat{F}(\aleph^{\chi}{}_2) \times \ldots \times \widehat{F}(\aleph^{\chi}{}_{q-1}) = 0 \times \widehat{F}(\aleph^{\chi}{}_2) \times \ldots \times \widehat{F}(\aleph^{\chi}{}_{q-1}), (q = 2\ldots,n) \prod_{\urcorner=1} \zeta^{\eta}{}_{\urcorner} = 1$$

From Definition 9, we have

$$\begin{aligned} \text{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) &= \frac{\zeta^{\eta}{}_1}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}} \aleph^{\chi}{}_1 \oplus \frac{\zeta^{\eta}{}_2}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}} \aleph^{\chi}{}_2 \oplus \ldots, \oplus \frac{\zeta^{\eta}{}_r}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}} \aleph^{\chi}{}_r \\ &= \frac{1}{1} \aleph^{\chi}{}_1 \oplus \frac{0}{1} \aleph^{\chi}{}_2 \oplus \ldots \frac{0}{1} \aleph^{\chi}{}_r \\ &= \aleph^{\chi}{}_1 = (0,0,1) \end{aligned}$$

Appendix A.4

Here, $\mu^{\chi *}{}_{\urcorner} \geq \mu^{\chi}{}_{\urcorner}, \forall g$

If $\mu^{\chi *}{}_{\urcorner} \geq \mu^{\chi}{}_{\urcorner}$.

$\Leftrightarrow (\mu^{\chi *}{}_{\urcorner}) \geq (\mu^{\chi}{}_{\urcorner}) \Leftrightarrow (\mu^{\chi *}{}_{\urcorner}) \geq (\mu^{\chi}{}_{\urcorner}) \Leftrightarrow 1 - (\mu^{\chi *}{}_{\urcorner}) \leq 1 - (\mu^{\chi}{}_{\urcorner})$

$\Leftrightarrow (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq (1 - (\mu^{\chi}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}}$

$\Leftrightarrow \prod_{\urcorner=1}^{r} (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq \prod_{\urcorner=1}^{r} (1 - (\mu^{\chi}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}}$

$\Leftrightarrow 1 - \prod_{\urcorner=1}^{r} (1 - (\mu^{\chi}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq 1 - \prod_{\urcorner=1}^{r} (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}}$

Now, we take

$\mu^{\chi *}{}_{\urcorner} \geq \mu^{\chi}{}_{\urcorner}$ and $\nu^{\chi *}{}_{\urcorner} \leq \nu^{\chi}{}_{\urcorner}, \forall g$

If $\mu^{\chi *}{}_{\urcorner} \geq \mu^{\chi}{}_{\urcorner}$.

$\Leftrightarrow (\mu^{\chi *}{}_{\urcorner}) \geq (\mu^{\chi}{}_{\urcorner}) \Leftrightarrow (\mu^{\chi *}{}_{\urcorner}) \geq (\mu^{\chi}{}_{\urcorner}) \Leftrightarrow 1 - (\mu^{\chi *}{}_{\urcorner}) \leq 1 - (\mu^{\chi}{}_{\urcorner})$

$\Leftrightarrow (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq (1 - (\mu^{\chi}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}}$

$\Leftrightarrow \prod_{\urcorner=1}^{r} (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r} \left(1 - ((\mu^{\chi *}{}_{\urcorner}) + (\nu^{\chi *}{}_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq$

$\prod_{\urcorner=1}^{r} (1 - (\mu^{\chi}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r} \left(1 - ((\mu^{\chi}{}_{\urcorner}) + (\nu^{\chi}{}_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}}$

Now, again we take

$\mu^{\chi *}{}_{\urcorner} \geq \mu^{\chi}{}_{\urcorner}$ and $\tau^{\chi *}{}_{\urcorner} \leq \tau^{\chi}{}_{\urcorner}, \forall g$

If $\mu^{\chi *}{}_{\urcorner} \geq \mu^{\chi}{}_{\urcorner}$.

$\Leftrightarrow (\mu^{\chi *}{}_{\urcorner}) \geq (\mu^{\chi}{}_{\urcorner}) \Leftrightarrow (\mu^{\chi *}{}_{\urcorner}) \geq (\mu^{\chi}{}_{\urcorner}) \Leftrightarrow 1 - (\mu^{\chi *}{}_{\urcorner}) \leq 1 - (\mu^{\chi}{}_{\urcorner})$

$\Leftrightarrow (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq (1 - (\mu^{\chi}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}}$

$\Leftrightarrow \prod_{\urcorner=1}^{r} (1 - (\mu^{\chi *}{}_{\urcorner}))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r} \left(1 - ((\mu^{\chi *}{}_{\urcorner}) + (\tau^{\chi *}{}_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r} \zeta^{\eta}{}_{\urcorner}}} \leq$

$$\prod_{\daleth=1}^{r}(1-(\mu^{\chi}{}_{\daleth}))^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}-\prod_{\daleth=1}^{r}\left(1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}$$

Let

$$\overline{\aleph\chi}=\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1},\aleph^{\chi}{}_{2},\ldots\aleph^{\chi}{}_{r})$$

and

$$\overline{\aleph\chi^{*}}=\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1}^{*},\aleph^{\chi}{}_{2}^{*},\ldots\aleph^{\chi}{}_{r}^{*})$$

One can obtain that $\overline{\aleph\chi^{*}}\geq\overline{\aleph\chi}$. So,

$$\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1},\aleph^{\chi}{}_{2},\ldots\aleph^{\chi}{}_{r})\leq\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1}^{*},\aleph^{\chi}{}_{2}^{*},\ldots\aleph^{\chi}{}_{r}^{*})$$

Appendix A.5

Here, we provide proof of part 1 and part 3,

1. By Theorem 5,

$$\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1}\oplus\breve{\ddot{\$}}\beth,\aleph^{\chi}{}_{2}\oplus\breve{\ddot{\$}}\beth,\ldots\aleph^{\chi}{}_{r}\oplus\breve{\ddot{\$}}\beth)$$

$$=\left((1-\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})(1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}},\right.$$

$$\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})(1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}-\prod_{\daleth=1}^{r}\left((1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth}))(1-((\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})+(\nu^{\chi}{}_{\breve{\ddot{\$}}\beth}))))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}},$$

$$\left.\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})(1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}-\prod_{\daleth=1}^{r}\left((1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth}))(1-((\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})+(\tau^{\chi}{}_{\breve{\ddot{\$}}\beth}))))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\right)$$

$$=\left((1-\left(1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\prod_{\daleth=1}^{r}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}},\left((1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\right.$$

$$\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}-\left(1-((\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})+(\nu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\prod_{\daleth=1}^{r}(1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth})))^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}},$$

$$\left((1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}-\left(1-((\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})+(\tau^{\chi}{}_{\breve{\ddot{\$}}\beth})\right.$$

$$\left.))^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\prod_{\daleth=1}^{r}(1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth})))^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}$$

$$=\left((1-\left(1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})\right)\prod_{\daleth=1}^{r}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}},\left((1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\right.$$

$$-\left(1-((\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})+(\nu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)\prod_{\daleth=1}^{r}(1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth})))^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}},$$

$$\left((1-(\mu^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}$$

$$\left.-\left(1-((\mu^{\chi}{}_{\breve{\ddot{\$}}\beth})+(\tau^{\chi}{}_{\breve{\ddot{\$}}\beth}))\right)\prod_{\daleth=1}^{r}(1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth})))^{\frac{\zeta^{\eta}\daleth}{\Sigma_{\daleth=1}^{r}\zeta^{\eta}\daleth}}\right)$$

Now, by operational laws of SVNNs,

$$
\begin{aligned}
&\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1}, \aleph^{\chi}{}_{2}, \ldots \aleph^{\chi}{}_{r}) \oplus \breve{\$}^{\gimel} \\
&= \left(1 - \prod_{\daleth=1}^{r}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \prod_{\daleth=1}^{r}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r}\left(1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right. \\
&\left.\prod_{\daleth=1}^{r}\left(1-(\tau^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r}\left(1-((\tau^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right) \oplus \langle \mu^{\chi}{}_{\breve{\$}^{\gimel}}, \nu^{\chi}{}_{\breve{\$}^{\gimel}}, \tau^{\chi}{}_{\breve{\$}^{\gimel}} \rangle
\end{aligned}
$$

$$
\begin{aligned}
&= \left((1 - \left(1-(\mu^{\chi}{}_{\breve{\$}^{\gimel}})\right)\prod_{\daleth=1}^{r}\left(1-(\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right. \\
&\left((1-(\mu^{\chi}{}_{\breve{\$}^{\gimel}}))\right)\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-((\mu^{\chi}{}_{\breve{\$}^{\gimel}})+(\nu^{\chi}{}_{\breve{\$}^{\gimel}}))\right)\prod_{\daleth=1}^{r}(1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth})))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \\
&\left.\left((1-(\mu^{\chi}{}_{\breve{\$}^{\gimel}}))\right)\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \left(1-((\mu^{\chi}{}_{\breve{\$}^{\gimel}})+(\tau^{\chi}{}_{\breve{\$}^{\gimel}}))\right)\prod_{\daleth=1}^{r}(1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth})))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)
\end{aligned}
$$

Thus,
$\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1} \oplus \breve{\$}^{\gimel}, \aleph^{\chi}{}_{2} \oplus \breve{\$}^{\gimel}, \ldots \aleph^{\chi}{}_{r} \oplus \breve{\$}^{\gimel}) = \mathrm{SVNPIWA}(\aleph^{\chi}{}_{1}, \aleph^{\chi}{}_{2}, \ldots \aleph^{\chi}{}_{r}) \oplus \breve{\$}^{\gimel}$
3. According to Theorem 5,

$$
\begin{aligned}
&\mathrm{SVNPIWA}(\aleph^{\chi}{}_{1} \oplus \breve{\$}^{\gimel}{}_{1}, \aleph^{\chi}{}_{2} \oplus \breve{\$}^{\gimel}{}_{2}, \ldots \aleph^{\chi}{}_{r} \oplus \breve{\$}^{\gimel}{}_{n}) \\
&= \left((1 - \prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})(1-(\sigma_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right. \\
&\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})(1-(\sigma_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} - \prod_{\daleth=1}^{r}((1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth}))(1-((\sigma_{\daleth})+(\xi_{\daleth})))))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \\
&\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})(1-(\sigma_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} \\
&\left.- \prod_{\daleth=1}^{r}((1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth}))(1-((\sigma_{\daleth})+(\delta_{\daleth})))))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \left((1 - \prod_{\daleth=1}^{r}\left(1-(\sigma_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\prod_{\daleth=1}^{r}\left(1-\mu^{\chi}{}_{\daleth}\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right. \\
&\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\prod_{\daleth=1}^{r}\left((1-(\sigma_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} \\
&- \prod_{\daleth=1}^{r}((1-((\mu^{\chi}{}_{\daleth})+(\nu^{\chi}{}_{\daleth}))))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\prod_{\daleth=1}^{r}((1-((\sigma_{\daleth})+(\xi_{\daleth})))))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}, \\
&\prod_{\daleth=1}^{r}\left((1-\mu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\prod_{\daleth=1}^{r}\left((1-(\sigma_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}} \\
&\left.- \prod_{\daleth=1}^{r}((1-((\mu^{\chi}{}_{\daleth})+(\tau^{\chi}{}_{\daleth}))))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\prod_{\daleth=1}^{r}((1-((\sigma_{\daleth})+(\delta_{\daleth})))))^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)
\end{aligned}
$$

Now,

$$\text{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \oplus \text{SVNPIWA}(\breve{\$}^{\beth}{}_1, \breve{\$}^{\beth}{}_2, \ldots \breve{\$}^{\beth}{}_r)$$
$$= \left(1 - \prod_{\urcorner=1}^{r}\left(1 - (\mu^{\chi}{}_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}, \prod_{\urcorner=1}^{r}\left(1 - (\mu^{\chi}{}_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r}\left(1 - ((\mu^{\chi}{}_{\urcorner}) + (\nu^{\chi}{}_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}},\right.$$
$$\left.\prod_{\urcorner=1}^{r}\left(1 - (\mu^{\chi}{}_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r}\left(1 - ((\mu^{\chi}{}_{\urcorner}) + (\tau^{\chi}{}_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}\right)$$
$$\bigoplus\left(1 - \prod_{\urcorner=1}^{r}\left(1 - (\sigma_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}, \prod_{\urcorner=1}^{r}\left(1 - (\sigma_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r}\left(1 - ((\sigma_{\urcorner}) + (\xi_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}},\right.$$
$$\left.\prod_{\urcorner=1}^{r}\left(1 - (\sigma_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r}\left(1 - ((\sigma_{\urcorner}) + (\delta_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}\right)$$
$$= \left((1 - \prod_{\urcorner=1}^{r}\left(1 - (\sigma_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \prod_{\urcorner=1}^{r}\left(1 - \mu^{\chi}{}_{\urcorner}\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}},\right.$$
$$\prod_{\urcorner=1}^{r}\left((1 - \mu^{\chi}{}_{\urcorner}\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \prod_{\urcorner=1}^{r}\left((1 - (\sigma_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r}((1 - ((\mu^{\chi}{}_{\urcorner}) + (\nu^{\chi}{}_{\urcorner})))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}$$
$$\prod_{\urcorner=1}^{r}((1 - ((\sigma_{\urcorner}) + (\xi_{\urcorner})))))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}, \prod_{\urcorner=1}^{r}\left((1 - \mu^{\chi}{}_{\urcorner}\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \prod_{\urcorner=1}^{r}\left((1 - (\sigma_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}$$
$$\left.- \prod_{\urcorner=1}^{r}((1 - ((\mu^{\chi}{}_{\urcorner}) + (\tau^{\chi}{}_{\urcorner})))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \prod_{\urcorner=1}^{r}((1 - ((\sigma_{\urcorner}) + (\delta_{\urcorner})))))^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}\right)$$

Thus, $\text{SVNPIWA}(\aleph^{\chi}{}_1 \oplus \breve{\$}^{\beth}{}_2, \aleph^{\chi}{}_2 \oplus \breve{\$}^{\beth}{}_2, \ldots \aleph^{\chi}{}_r \oplus \breve{\$}^{\beth}{}_r) = \text{SVNPIWA}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) \oplus \text{SVNPIWA}(\breve{\$}^{\beth}{}_1, \breve{\$}^{\beth}{}_2, \ldots \breve{\$}^{\beth}{}_r)$

Appendix B

Appendix B.1

Definition 10 and Theorem 10 are easily preceded by the first statement. This is shown in the following parts.

$$\text{SVNPIWG}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r)$$
$$= \aleph^{\chi}{}_1^{\frac{\zeta^{\eta}{}_1}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \otimes \aleph^{\chi}{}_2^{\frac{\zeta^{\eta}{}_2}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \otimes \ldots, \otimes \aleph^{\chi}{}_r^{\frac{\zeta^{\eta}{}_r}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}$$
$$= \left(\prod_{\urcorner=1}^{r}\left(1 - (\nu^{\chi}{}_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} - \prod_{\urcorner=1}^{r}\left(1 - ((\nu^{\chi}{}_{\urcorner}) + (\mu^{\chi}{}_{\urcorner}))\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}},\right.$$
$$\left.1 - \prod_{\urcorner=1}^{r}\left(1 - (\nu^{\chi}{}_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}, 1 - \prod_{\urcorner=1}^{r}\left(1 - (\tau^{\chi}{}_{\urcorner})\right)^{\frac{\zeta^{\eta}{}_{\urcorner}}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}\right).$$

The validity of this theorem is established through the use of mathematical induction. when $\urcorner = 2$

$$\text{SVNPIWG}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2) = \aleph^{\chi}{}_1^{\frac{\zeta^{\eta}{}_1}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}} \otimes \aleph^{\chi}{}_2^{\frac{\zeta^{\eta}{}_2}{\sum_{\urcorner=1}^{r}\zeta^{\eta}{}_{\urcorner}}}$$

By interactive laws of SVNS, we have

$$
\begin{aligned}
&\aleph\chi_1^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\\
=\;&\left(\left(1-\nu\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\left(1-(\nu\chi_1+\mu\chi_1)\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\nu\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\tau\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\right)\\
&\aleph\chi_2^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\\
=\;&\left(\left(1-\nu\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\left(1-(\nu\chi_2+\mu\chi_2)\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\nu\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\tau\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\right)
\end{aligned}
$$

Then,

$$
\begin{aligned}
&\mathrm{SVNPIWG}(\aleph\chi_1,\aleph\chi_2)\\
=\;&\aleph\chi_1^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\otimes\aleph\chi_2^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\\
=\;&\left(\left(1-\nu\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\left(1-(\nu\chi_1+\mu\chi_1)\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\nu\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\tau\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\right)\\
&\otimes\left(\left(1-\nu\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\left(1-(\nu\chi_2+\mu\chi_2)\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\nu\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\tau\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\right)\\
=\;&\left(\left(1-\nu\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\left(1-\nu\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\left(1-(\nu\chi_1+\mu\chi_1)\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\left(1-(\nu\chi_2+\mu\chi_2)\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},\right.\\
&\left.1-\left(1-\nu\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\left(1-\nu\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\left(1-\tau\chi_1\right)^{\frac{\zeta\eta_1}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\left(1-\tau\chi_2\right)^{\frac{\zeta\eta_2}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\right)\\
=\;&\left(\prod_{\daleth=1}^{2}\left(1-(\nu\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\prod_{\daleth=1}^{2}\left(1-(\nu\chi_\daleth+\mu\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\prod_{\daleth=1}^{2}\left(1-(\nu\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},\right.\\
&\left.1-\prod_{\daleth=1}^{2}\left(1-(\tau\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\right).
\end{aligned}
$$

Assume that the result holds for $\daleth = d$

$$
\begin{aligned}
&\mathrm{SVNPIWG}(\aleph\chi_1,\aleph\chi_2,\ldots\aleph\chi_d)\\
=\;&\bigotimes_{\daleth=1}^{d}\aleph\chi_\daleth^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}\\
=\;&(\prod_{\daleth=1}^{d}\left(1-(\nu\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}}-\prod_{\daleth=1}^{d}\left(1-((\nu\chi_\daleth)+(\mu\chi_\daleth))\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},\\
&1-\prod_{\daleth=1}^{d}\left(1-(\nu\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}},1-\prod_{\daleth=1}^{d}\left(1-(\tau\chi_\daleth)\right)^{\frac{\zeta\eta_\daleth}{\sum_{\daleth=1}^{r}\zeta\eta_\daleth}})
\end{aligned}
$$

Now, we shall prove it for $\daleth = d + 1$

$$
\begin{aligned}
&\mathrm{SVNPIWG}(\aleph^{\chi}{}_{1},\aleph^{\chi}{}_{2},\ldots\aleph^{\chi}{}_{d},\aleph^{\chi}{}_{d+1})\\
=\;& \bigotimes_{\daleth=1}^{d}\aleph^{\chi}{}_{\daleth}^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\otimes\aleph^{\chi}{}_{d+1}^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\\
=\;& \left(\prod_{\daleth=1}^{d}\left(1-(\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}-\prod_{\daleth=1}^{d}\left(1-((\nu^{\chi}{}_{\daleth})+(\mu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},1-\prod_{\daleth=1}^{d}\left(1-(\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right.\\
&\left.1-\prod_{\daleth=1}^{d}\left(1-(\tau^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)\\
&\otimes\left(\left(1-\nu^{\chi}{}_{d+1}\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}-\left(1-(\nu^{\chi}{}_{d+1}+\mu^{\chi}{}_{d+1})\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},1-\left(1-\nu^{\chi}{}_{d+1}\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right.\\
&\left.1-\left(1-\tau^{\chi}{}_{d+1}\right)^{\frac{\zeta^{\eta}{}_{d+1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right)\\
=\;& \left(\prod_{\daleth=1}^{d+1}\left(1-(\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}-\prod_{\daleth=1}^{d+1}\left(1-((\nu^{\chi}{}_{\daleth})+(\mu^{\chi}{}_{\daleth}))\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},1-\prod_{\daleth=1}^{d+1}\left(1-(\nu^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}},\right.\\
&\left.1-\prod_{\daleth=1}^{d+1}\left(1-(\tau^{\chi}{}_{\daleth})\right)^{\frac{\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\right).
\end{aligned}
$$

Appendix B.2

From Definition 10, we have

$$
\begin{aligned}
\mathrm{SVNPIWG}(\aleph^{\chi}{}_{1},\aleph^{\chi}{}_{2},\ldots\aleph^{\chi}{}_{r}) &= \aleph^{\chi}{}_{1}^{\frac{\zeta^{\eta}{}_{1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\otimes\aleph^{\chi}{}_{2}^{\frac{\zeta^{\eta}{}_{2}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\otimes\ldots,\otimes\aleph^{\chi}{}_{r}^{\frac{\zeta^{\eta}{}_{r}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\\
&= \aleph^{\chi\frac{\zeta^{\eta}{}_{1}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\otimes\aleph^{\chi\frac{\zeta^{\eta}{}_{2}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\otimes\ldots,\otimes\aleph^{\chi\frac{\zeta^{\eta}{}_{r}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\\
&= \aleph^{\chi\frac{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}{\sum_{\daleth=1}^{r}\zeta^{\eta}{}_{\daleth}}}\\
&= \aleph^{\chi}
\end{aligned}
$$

Appendix B.3

Here, $\aleph^{\chi}{}_{1}=(0,0,1)$, then by definition of the SF we have

$$\widehat{F}(\aleph^{\chi}{}_{1})=0$$

Since,

$$\zeta^{\eta}{}_{q}=\prod_{\daleth=1}^{q-1}\widehat{F}(\aleph^{\chi}{}_{\daleth})\qquad(q=2\ldots,n),\quad\text{and}\quad\zeta^{\eta}{}_{1}=1$$

$\widehat{F}(\aleph^{\chi}{}_{\daleth})$ is the score of g^{th} SVNN.

Then,

$$\zeta^{\eta}{}_{q}=\prod_{\daleth=1}^{q-1}\widehat{F}(\aleph^{\chi}{}_{\daleth})=\widehat{F}(\aleph^{\chi}{}_{1})\times\widehat{F}(\aleph^{\chi}{}_{2})\times\ldots\times\widehat{F}(\aleph^{\chi}{}_{q-1})=0\times\widehat{F}(\aleph^{\chi}{}_{2})\times\ldots\times\widehat{F}(\aleph^{\chi}{}_{q-1})(q=2\ldots,n)\prod_{\daleth=1}\zeta^{\eta}{}_{\daleth}=1$$

From Definition 9, we have

$$\begin{aligned} \mathrm{SVNPIWG}(\aleph^{\chi}{}_1, \aleph^{\chi}{}_2, \ldots \aleph^{\chi}{}_r) &= \aleph^{\chi}{}_1^{\frac{\zeta^{\eta}{}_1}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \otimes \aleph^{\chi}{}_2^{\frac{\zeta^{\eta}{}_2}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \otimes \ldots, \otimes \aleph^{\chi}{}_r^{\frac{\zeta^{\eta}{}_r}{\sum_{\daleth=1}^{r} \zeta^{\eta}{}_{\daleth}}} \\ &= \aleph^{\chi}{}_1^{\frac{1}{1}} \otimes \aleph^{\chi}{}_2^{\frac{0}{1}} \otimes \ldots \aleph^{\chi}{}_r^{\frac{0}{1}} \\ &= \aleph^{\chi}{}_1 = (0, 0, 1) \end{aligned}$$

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
3. Smarandache, F. *Neutrosophy: Neutrosophic Probability, Set, and Logic: Analytic Synthesis & Synthetic Analysis*; American Research Press: Champaign, IL, USA, 1998.
4. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Smarandache, R. Single valued neutrosophic sets. *Multispace Multistruct.* **2010**, *4*, 410–413.
5. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst.* **2014**, *26*, 2459–2466. [CrossRef]
6. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Smarandache, R. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*; Hexis: Phoenix, AZ, USA, 2005.
7. Garg, H.N. Some new biparametric distance measures on single-valued neutrosophic sets with applications to pattern recognition and medical diagnosis. *Information* **2017**, *8*, 162. [CrossRef]
8. Hashim, R.M.; Gulistan, M.; Smarandache, F. Applications of neutrosophic bipolar fuzzy sets in hope foundation for planning to build a children hospital with different types of similarity measures. *Symmetry* **2018**, *10*, 331. [CrossRef]
9. Peng, X.; Smarandache, F. New multiparametric similarity measure for neutrosophic set with big data industry evaluation. *Artif. Intell. Rev.* **2020**, *53*, 3089–3125. [CrossRef]
10. Uluçay, V.; Deli, I.; Şahin, M. Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. *Neural Comput. Appl.* **2018**, *29*, 739–748. [CrossRef]
11. Wu, X.H.; Wang, J.Q.; Peng, J.J.; Chen, X.H. Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. *Int. J. Fuzzy Syst.* **2016**, *18*, 1104–1116. [CrossRef]
12. Kamacı, H. Simplified neutrosophic multiplicative refined sets and their correlation coefficients with application in medical pattern recognition. *Neutrosophic Sets Syst.* **2021**, *41*, 270–285.
13. Nancy, G.H. An improved score function for ranking Neutrosophic sets and its application to decision-making process. *Int. J. Uncertain Quantif.* **2016**, *6*, 377–385. [CrossRef]
14. Ye, J. Single valued neutrosophic similarity measures based on cotangent function and their application in the fault diagnosis of steam turbine. *Soft Comput.* **2017**, *21*, 817–825. [CrossRef]
15. Ye, J. Single-valued neutrosophic clustering algorithms based on similarity measures. *J. Classif.* **2017**, *34*, 148–162. [CrossRef]
16. Peng, J.-J.; Wang, J.-Q.; Wang, J.; Zhang, H.-Y.; Chen, X.-H. An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. *Appl. Soft Comput.* **2014**, *25*, 336–346. [CrossRef]
17. Biswas, P.; Pramanik, S.; Giri, B.C. Entropy based grey relational analysis method for multiattribute decision making under single valued neutrosophic assessments. *Neutrosophic Sets Syst.* **2014**, *2*, 102–110.
18. Peng, X.; Dai, J. A bibliometric analysis of neutrosophic set: Two decades review from 1998–2017. *Artif. Intell. Rev.* **2020**, *53*, 199–255. [CrossRef]
19. Karaaslan, F. Multicriteria decision-making method based on similarity measures under single-valued neutrosophic refined and interval neutrosophic refined environments. *Int. J. Intell. Syst.* **2018**, *33*, 928–952. [CrossRef]
20. Kamacı, H. Linguistic single-valued neutrosophic soft sets with applications in game theory. *Int. J. Intell. Syst.* **2021**, *36*, 3917–3960. [CrossRef]
21. Ashraf, S.; Abdullah, S. Emergency decision support modeling for COVID-19 based on spherical fuzzy information. *Int. J. Intell. Syst.* **2020**, *35*, 1–45. [CrossRef]
22. Ashraf, S.; Abdullah, S.; Mahmood, T.; Aslam, M. Cleaner production evaluation in gold mines using novel distance measure method with cubic picture fuzzy numbers. *Int. J. Fuzzy Syst.* **2019**, *21*, 2448–2461. [CrossRef]
23. Riaz, M.; Farid, H.M.A.; Aslam, M.; Pamucar, D.; Bozanic, D. Novel approach for third-party reverse logistic provider selection process under linear Diophantine fuzzy prioritized aggregation operators. *Symmetry* **2021**, *13*, 1152. [CrossRef]
24. Iampan, A.; Garcia, G.S.; Riaz, M.; Farid, H.M.A.; Chinram, R. Linear Diophantine fuzzy Einstein aggregation operators for multi-criteria decision-making problems. *J. Math.* **2021**, *2021*, 5548033. [CrossRef]
25. Farid, H.M.A.; Riaz, M. Some generalized q-rung orthopair fuzzy Einstein interactive geometric aggregation operators with improved operational laws. *Int. J. Intell. Syst.* **2021**, *36*, 7239–7273. [CrossRef]
26. Peng, J.-J., Wang, J.-Q., Wang, J., Zhang, H.-Y., Chen, X.-H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *Int. J. Syst. Sci.* **2016**, *47*, 2342–2358. [CrossRef]
27. Nancy, G.H. Novel single-valued neutrosophic decision making operators under Frank norm operations and its application. *Int. J. Uncertain. Quantif.* **2016**, *6*, 361–375. [CrossRef]

28. Han, L.; Wei, C. Group decision making method based on single valued neutrosophic Choquet integral operator. *Oper. Res.* **2017**, *21*, 110–118.
29. Li, Y.; Liu, P.; Chen, Y. Some single valued neutrosophic number Heronian mean operators and their application in multiple attribute group decision making. *Informatica* **2016**, *27*, 85–110. [CrossRef]
30. Liu, P.; Wang, Y. Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural. Comput. Appl.* **2014**, *25*, 2001–2010. [CrossRef]
31. Wang, J.; Tang, X.; Wei, G. Models for multiple attribute decision-making with dual generalized single-valued neutrosophic Bonferroni mean operators. *Algorithms* **2018**, *11*, 2. [CrossRef]
32. Wei, G.; Zhang, Z. Some single-valued neutrosophic bonferroni power aggregation operators in multiple attribute decision making. *J. Ambient Intell. Humaniz. Comput.* **2019**, *10*, 863–882. [CrossRef]
33. Ji, P.; Wang, J.Q.; Zhang, H.Y. Frank prioritized Bonferroni mean operator with single-valued neutrosophic sets and its application in selecting third-party logistics providers. *Neural. Comput. Appl.* **2018**, *30*, 799–823. [CrossRef]
34. Li, B.; Wang, J.; Yang, L.; Li, X. Novel generalized simplified neutrosophic number Einstein aggregation operator. *Int. J. Appl. Math.* **2016**, *48*, 1–6.
35. Mondal, K.; Pramanik, S.; Giri, B.C.; Smarandache, F. NN-Harmonic mean aggregation operators-based MCGDM strategy in a neutrosophic number environment. *Axioms* **2018**, *7*, 12. [CrossRef]
36. Wei, G.; Wei, Y. Some single-valued neutrosophic dombi prioritized weighted aggregation operators in multiple attribute decision making. *J. Intell. Fuzzy Syst.* **2018**, *35*, 2001–2013. [CrossRef]
37. Garg, H.N. Multi-criteria decision-making method based on prioritized Muirhead mean aggregation operator under neutrosophic set environment. *Symmetry* **2018**, *10*, 280. [CrossRef]
38. Kamacı, H. Neutrosophic cubic Hamacher aggregation operators and their applications in decision making. *Neutrosophic Sets Syst.* **2020**, *33*, 234–255.
39. Liu, P.; Chu, Y.; Li, Y.; Chen, Y. Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making. *Int. J. Fuzzy Syst.* **2014**, *16*, 242–255.
40. Zheng, E.Z.; Teng, F.; Liu, P.D. Multiple attribute group decision-making method based on neutrosophic number generalized hybrid weighted averaging operator. *Neural Comput. Appl.* **2017**, *28*, 2063–2074. [CrossRef]
41. Garg, H.N. New logarithmic operational laws and their applications to multiattribute decision making for single-valued neutrosophic numbers. *Cogn. Syst. Res.* **2018**, *52*, 931–946. [CrossRef]
42. Broumi, S., Smarandache, F.: Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making. *Bull. Pure Appl. Sci.* **2014**, *33*, 135–155. [CrossRef]
43. Liang, R.; Wang, J.Q.; Li, L. Multi-criteria group decision-making method based on interdependent inputs of single-valued trapezoidal neutrosophic information. *Neural Comput. Appl.* **2018**, *30*, 241–260. [CrossRef]
44. Liu, C.F.; Luo, Y.S. Correlated aggregation operators for simplified neutrosophic set and their application in multi-attribute group decision making. *Int. J. Fuzzy Syst.* **2016**, *30*, 1755–1761. [CrossRef]
45. Liu, C.F.; Luo, Y.S. The weighted distance measure based method to neutrosophic multiattribute group decision making. *Math. Probl. Eng.* **2016**, *2016*, 3145341. [CrossRef]
46. Tan, R.; Zhang, W.; Chen, S. Some generalized single valued neutrosophic linguistic operators and their application to multiple attribute group decision making. *J. Syst. Sci. Inf.* **2017**, *5*, 148–162. [CrossRef]
47. Liu, P.; Shi, L. Some neutrosophic uncertain linguistic number Heronian mean operators and their application to multi-attribute group decision making. *Neural Comput. Appl.* **2017**, *28*, 1079–1093. [CrossRef]
48. Lu, Z.K.; Ye, J. Exponential operations and an aggregation method for single-valued neutrosophic numbers in decision making. *Information* **2017**, *8*, 62. [CrossRef]
49. Mahmood, T.; Haleemzai, I.; Ali, Z.; Pamucar, D.; Marinkovic, D. Power Muirhead Mean Operators for Interval-Valued Linear Diophantine Fuzzy Sets and Their Application in Decision-Making Strategies. *Mathematics* **2022**, *10*, 70. [CrossRef]
50. Entezari, A.; Aslani, A.; Zahedi, R.; Noorollahi, Y. Artificial intelligence and machine learning in energy systems: A bibliographic perspective. *Energy Strategy Rev.* **2003**, *45*, 101017. [CrossRef]
51. Izanloo, M.; Aslani, A.; Zahedi, R. Development of a Machine learning assessment method for renewable energy investment decision making. *Appl. Energy* **2022**, *327*, 120096. [CrossRef]
52. Zahedi, R.; Eskandarpanah, R.; Akbari, M.; Rezaei, N.; Farahani, P.M.O.N. Development of a New Simulation Model for the Reservoir Hydropower Generation. *Water Resour. Manag.* **2022**, *36*, 2241–2256. [CrossRef]
53. Yager, R.R. Prioritized aggregation operators. *Int. J. Approx. Reason.* **2008**, *48*, 263–274. [CrossRef]
54. He, Y.D.; Chen, H.Y.; Zhou, L.G. Generalized intuitionistic fuzzy geometric interaction operators and their application to decision making. *Expert Syst. Appl.* **2014**, *41*, 2484–2495. [CrossRef]
55. Available online: https://www.finance.gov.pk/survey/chapter_22/PES14-ENERGY.pdf (accessed on 17 January 2023)
56. Thanh, N.V. Sustainable Energy Source Selection for Industrial Complex in Vietnam: A Fuzzy MCDM Approach. *IEEE Access* **2022**, *10*, 50692–50701. [CrossRef]
57. Mishra, A.R.; Rani, P.; Saha, A.; Senapati, T.; Hezam, I.M.; Yager, R.R. Fermatean fuzzy copula aggregation operators and similarity measures-based complex proportional assessment approach for renewable energy source selection. *Complex Intell. Syst.* **2022**, *8*, 5223–5248. [CrossRef] [PubMed]

58. Pakistan Energy Demand Forecast (2021–2030), Integrated Energy Planning for Sustainable Development (IEP) Ministry of Planning, Development & Special Initiatives Government of Pakistan. Available online: https://www.pc.gov.pk/uploads/report/IEP_Report_FINAL.pdf (accessed on 12 January 2023).
59. Liu, P. The aggregation operators based on Archimedean t-conorm and t-norm for single-valued neutrosophic numbers and their application to decision making. *Int. J. Fuzzy Syst.* **2016**, *18*, 849–863. [CrossRef]