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Random "Decision and Experienced Utility", Adaptive "Consumer Memory and Choice": The Impact of Mind Fluctuations and Cognitive Biases on Consumption and Classification

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Abstract: In the study of consumer behavior, we believe that a distinction should be made between the subjective mental activity of consumption and the objective process of consumption experience, and that the deviation and fluctuation of "decision utility" and the randomness of "experience utility" have essential effects on consumer behavior. For one thing, purchase often precedes experience, and consumers cannot precisely predict utility and its distribution but can only make decisions based on adaptive expectations. Secondly, there is often uncertainty in the experience of goods, making predictions more difficult. Thirdly, the revision of consumer decision utility is carried out through memory in an adaptive process. We introduce two stochastic forms of decision and experience utility functions and build a two-period model. In the model, consumers make decisions based on a pre-determined decision utility function in the first period and a randomly realized experience utility function in the second period. The analysis shows that the volatility of "decision utility" and "experience utility" affects consumers in opposite directions; the former may trigger expansionary consumption, while the latter makes consumers more cautious. Finally, the consumption behaviors in the model can be divided into 24 categories based on the dimensions of chance, systematicity, luck, and deviance, corresponding to various scenarios. The total number of consumer behavior categories is a full ranking of the size relationship of the four factors mentioned above, thus 24 categories. For example, when good luck is accompanied by chance underestimation versus systematic underestimation, it leads to a better process experience for the consumer.

Keywords: decision utility; experiential utility; relative consumption; blind consumption; adaptive expectations

MSC: 91-10

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1. Introduction

In the utility theory of classical microeconomics, agents have a rational setting and will search for the combination of goods that maximizes their utility. In practice, however, classical microeconomics does not perfectly explain consumer behavior. To solve the inconsistency, economists have tried to revise the assumptions of rational people, complete information, and preference consistency in mainstream economics based on factors such as consumers' psychological value.

1.1. Literature Review and Raising of Problems

With the involvement of "consumer's psychological value", a utility function cannot be independent of time, and the utility of consumption no longer depends only on the current level of consumption but is also related past consumption and to the comparison of

certain social criteria. This is not a novel idea and can be traced in part to the "conspicuous consumption" proposed by Veblen [1], and after Dueseberry [2] put forward the relative income consumption theory, a certain micro-theoretical basis was formed. Since the Easterlin paradox was proposed, academic research in this field has continued to deepen. Van de Stadt et al. [3] suggested that utility is more dependent on relative consumption. Fuhrer [4] strongly rejects the hypothesis of time-separable preferences. Arrow and Dasgupta [5] pointed out that the effects of relative consumption may be offset as consuming more today will increase a person's relative consumption now, but it will worsen his relative consumption in the future. Thus, rational consumers will not make such decisions. So, we infer that deviations from the classical model may be influenced by other factors in consumer behavior.

Although relative consumption enjoys the memory effect (presents in a habit-formation way), the memory effect is not solely reflected in relative consumption; we argue that the "memory effect" and "temporal inseparability of preferences" are not only reflected in the comparison of consumption variables with the prior benchmark, but also in the correlation between the later decision utility [6] function, which is correlated with the form of the pre-experience utility function. Because both "psychological expectations" and "actual effects of the consumption experience" can be random variables and their realized values influence subsequent decisions, utility cannot be independent of time. In particular, psychological expectations of utility are likely to be highly imprecise.

Therefore, after examining the special nature of certain goods and the psychological tendencies and decision patterns of many actors, we propose a new idea to explain consumption: decision utility and experienced utility have random fluctuations, and rational people seek to maximize expected utility. The traditional "subjective expected utility model" is not applicable because consumers cannot obtain the true probability of utility per experience with a large sample, and the utility in each case in this model is still well-defined; only the utility distribution can be improved by Bayesian methods. However, we do not know the probability distribution nor the parameters of the utility function with certainty now. In addition, empirical studies have shown that this model does not match well with consumer behavior patterns, but consumers do not know the true expected value of decision utility because of their limited rationality and the limited number of experiences, and consumers' self-perceived "expectation" (i.e., the expected decision utility to participate in decision making) is essentially a random variable, while the actual experienced utility of the commodity is intrinsically random. The randomness will be reflected in two types of utility functions. This idea aims to address further issues that need to be discussed, such as the randomness of commodity experience and the deviation and fluctuation of consumers' psychological expectations. We conclude that the issues that should be focused on in commodity consumption are as follows:

- There are two kinds of utility: There is no time lag between the purchase and the actual use of goods or services in the traditional theoretical model of economics, where individuals are supposed to generate utility in their purchase and use, and thus make decisions based on the classical utility model. With further academic research on utility [6], it has been divided into two forms: experienced utility and decision utility. Experienced utility is commonly measured in terms of subjective well-being; decision utility is described in terms of consumer behavior. Empirical research has demonstrated that for most people, the two utilities are similar, but there are also cases where the two utility levels differ in the sub-group [7].
- Volatility on the consumer-side: past consumption experience, hedonic adaption, and "false memory" peak-end rule. In the case of repeated purchases, the "memory effect" should be taken into account in the utility function of the consumer. Warlop et al. [8] suggested that the memory of the consumption of a product and the difference between its competitors affect the consumer's welfare or utility. Hai et al. [9] and Gilboa et al. [10] study memorable goods as a type of good that distinguishes from durable goods or nondurable goods. The study of Robson and Samuelson [11] also

shows that after consumers enjoy the utility of goods in the first period, they will make decisions before the start of the second period by referring to the experienced utility of the first period, and their experienced utility in the second period is also influenced by the first period. For experiential utility, the studies by Van Boven and Gilovich [12] and Nicolao et al. [13] have shown that different types of consumption can bring people different levels of happiness (experienced utility). Meanwhile, hedonic adaptation [14] could also be the cause of volatility on the consumer side; it refers to the fact that consumer will generate a continuously decreasing hedonic response to the same stimuli. Furthermore, the study of Kahneman et al., [15] showed that consumers are not able to objectively describe the utility of a good or service but rather focus on the peak and end moments of the experience, which leads to a distorted evaluation of the consumer's true feelings. Some consumer psychology studies [16] have even found that consumers are often exposed to information that may contradict their consumption experience, leading to "false memories" of consumption aspects. Therefore, it is difficult for consumers to accurately perceive their own utility function and its related parameters.

- Randomness at the commodity end: intrinsic randomness vs. external randomness. The randomness of commodities includes the intrinsic randomness of commodities and the external randomness of commodities. For the intrinsic randomness of the commodity, it is mainly reflected in the random fluctuation of the quality of the commodity, which is usually described by "Quality Variations" [17]. In general, the higher quality of goods can provide consumers with higher utility. The effect of external randomness, i.e., the randomness of the external environment on the use of a good or service, can be positive or negative for consumers. For example, in the case of the commodity "travel", "weather" is a manifestation of the external randomness of the commodity, and differences in weather may lead to changes in consumer experience [18]. It was demonstrated [19] in a study of Scotland that bad weather was the main cause of traveler dissatisfaction.
- Uncertainty and randomness of the expectation of decision utility: Uncertainty and randomness might relate to the psychological state of consumers at that time [20] or mindset [21] or even the consumers' emotional evaluation of the retail environment [22]. As a result, the utility of real consumption is not only not easy to predict but is also entangled with the act of prediction itself [23]. With the development of behavioral economics, economists have gradually abandoned the old assumption that consumers are perfectly rational and replaced it with "bounded rationality" [24]; for instance, it is difficult for consumers to exercise self-control, which usually leads to unplanned consumption or so-called impulse consumption, which in turn produces various negative consequences [25]. It is documented [26] that consumers incur cognitive dissonance when making purchase decisions; it is also random, which can be classified into two types: first, the systematic bias of individual predetermined utility (mean of decision utility) relative to the mean of truly experienced utility (mean of experience utility) due to the level of cognition, described by the parametric mean. Consumer behavior is characterized by the systematic overestimation or underestimation of certain goods, with cautious consumers tending to underestimate utility and impulsive consumers tending to overestimate utility. Second, there is volatility in the individual predicted utility as a random variable due to the influence of mind or emotion, which can be described by the variance of the parameter. Consumers with a calm and stable mood may have a small variance of "decision utility" over multiple purchases, while consumers with a sensitive and variable mood may enjoy a large variance of "decision utility". The variance of "decision utility" is larger for sensitive and emotionally volatile consumers. We would like to know how the effects of "systematic bias" and "volatility" affect consumer behavior, respectively.

McFadden [26] used the constructed Random Utility Model to introduce stochasticity to consumers' bounded rationality; this model focuses on the display of the probability

distribution of preferences but is not easily applied to the functional analysis of consumer behavior. We introduce randomness into the utility function as an alternative modeling approach, which implies an important assumption: the randomness of consumer utility can be reflected by the randomness of the utility function parameters; although definite utility function forms do not necessarily exist, utility functions in the form of similar functions containing random parameters can describe consumer behavior.

1.2. Article Structure and Main Innovations

In Section 2 below, we first develop a simple two-stage deterministic model framework that includes general and relative consumption goods and considers the effect of relative consumption. Section 3 introduces the imprecise "decision utility" expectation and the subsequent stochastic "experience utility" correction to the later "decision utility", comparing the model results with the deterministic case. The differences between the model results and the deterministic scenario are examined to illustrate the impact of each factor on consumption. Section 4 examines the systematic effects of mindset fluctuations on consumption, i.e., the statistical analysis of the model results in Section 3 and the variation of the mean value with the variance of the parameters. Section 5 examines the systematic effects of bias on consumption when the decision utility is a biased estimate and classifies consumption behavior into 24 categories based on the relationship between the magnitudes of the parameter variables, each of which contain three basic consumption characteristics. Section 6 provides an additional discussion of the model and concluding remarks, among others. The innovations of this paper are as follows.

Three types of randomness are introduced and distinguished, dividing consumer factors from commodity factors: a. randomness from the objective level of commodity quality, b. randomness from the subjective level of consumers' "decision utility", and c. randomness from the subjective–objective combination of "experiential utility" randomness.

Adaptive expectations theory is used in the role of decision utility in relation to experienced utility. The imprecise expectations in this paper are similar to the adaptive expectations theory of macroeconomics, but they are also different. Adaptive expectations are the process of using the past record of an economic variable to predict the future, repeatedly testing and revising it, and taking the approach of trying again if it is wrong so that expectations gradually conform to the objective. Adaptive expectations theory is often applied to explain the relationship between inflation and unemployment. Macroeconomic variables are generally supported by large samples and are relatively easy to predict. In contrast, expectations of micro-commodity consumption are relatively difficult due to consumers' in-person experience. In the model of this paper, before the first consumption of the commodity, there is no a priori basis for determining utility expectations, and they can only be defined as imprecise, subjective expectations. When the good is consumed for the second time, the consumer has acquired the basis for the random experience of the first consumption, and adaptive expectations are applied.

The variance between "experienced utility" and "decision utility" is found to have opposite effects on consumption behavior. The volatility of the "experienced utility" tends to make consumers purchase less, while the volatility of the "decision utility" tends to trigger excessive consumption.

The model parameters categorize underestimation and overestimation of consumption in terms of various dimensions: chance, systematic, and luck; they then classify consumption into 24 possibilities. For example, a distinction is made between the "chance overestimation/underestimation" of the utility of a good, which occurs in a specific purchase behavior, and "systematic overestimation/underestimation", which is statistically significant in nature. That is, it is not excluded that a larger realized value may have a smaller expected value, and a smaller realized value may have a larger expected value but with a smaller probability of occurrence. In other words, a prudent person may also overestimate the utility of a good by chance. This helps to enhance the strength of the model's explanation of reality.

2. General and Relatively Consumptive Goods—A Model Framework for Deterministic Phase II Commodity Selection

To simplify the illustration, we start our discussion with a deterministic model of commodity purchases, where our consumers are inward-looking, i.e., they compare only with their own past consumption levels and not with others. This paper is a micro study of our base, dealing only with inward-looking consumers and not with macro variables. Studies of outward-looking relative consumption such as "keeping up with the Joneses" need to take into account average social consumption, economic growth, social welfare, economics. We leave this to the macro perspective papers that follow. Without loss of generality, imagine a two-period consumption scenario for a two-period good: consumption is divided into two periods, and consumers consume in both periods, receiving a certain amount of income in each period m. The goods are divided into two categories, one for ordinary goods without climbing effects and the other for goods that induce relative consumption effects. We call them relative goods or memory goods, and their utility is influenced by factors such as past standards, consumption habits, and psychological expectations.

To simplify and highlight the main issue, the price of each commodity is set to remain constant in both periods. In the second period, in addition to the direct utility generated by the second category of goods, there is also an additional utility effect due to the comparison of consumption with the first period. This additional utility will be relatively low when the consumption in the second period is less than that in the first period, and relatively high if the consumption is more than that in the first period.

To keep the model parsimonious, the utility function of the second period is set to be related to the ratio of the first period to the second period consumption of the second category of goods. Variable a is the first category of goods, b is the second category of goods, and P_a, P_b are their prices, respectively. The utility of the first period is $u_1(a_1, b_1)$, and the utility of the second period is $u_2(a_2; b_2, b_2/b_1)$. The utility function of the second period is consistent with the classical utility function assumption for all three variables, i.e., for each variable there is an increasing function, and the utility obeys the law of marginal decrease. Let the income in each period be m. We consider the consumer's borrowing behavior by introducing savings s. The problem faced by the consumer when there is saving or borrowing is shown below.

$$\max u_1(a_1, b_1) + (1+\rho)^{-1} u_2(a_2; b_2, b_2/b_1) \tag{1}$$

s.t.

$$a_1 p_a + b_1 p_b + s = m \tag{2}$$

$$a_2 p_a + b_2 p_b = m + (1+r)s \tag{3}$$

where s is the savings in the first period, and r is the interest rate corresponding to the first period; a_i is the consumption of the first category of goods in period I, and b_i is the consumption of the second category of goods in period i; ρ is the subjective discounting parameter.

The equations of the general form are not easy to analyze and require the determination of a form of utility function to be calculated. In research, it sometimes does not matter what form of the utility function is chosen. This is because where a monotonic composite function is transformed, its commodity optimization results are consistent. For example, when the commonly used Cobb Douglas function and logarithmic utility function (the Cobb Douglas utility function to take the logarithm of the logarithmic utility function) are used in the corresponding parameters and budget constraints, the optimization results are the same. In addition, considering the "logarithmic linearization" as a common method in economic theory, many complex functions can be approximated as logarithmic forms. Therefore, the choice of a logarithmic utility function has some general significance. Let us consider an example of a logarithmic utility function.

$$u_1(a_1, b_1) = \alpha \ln a_1 + \beta \ln b_1 \tag{4}$$

$$u_2(a_2; b_2, b_2/b_1) = \alpha \ln a_2 + \beta \ln b_2 + \gamma \ln(b_2/b_1) \tag{5}$$

We scaled the parameter α, β, γ to the interval $[0, 1)$. Note that the parameter γ measures the relative consumption effect, reflecting the utility satisfaction of the second period relative to the ratio of the number of consumed goods b_2 to b_1, i.e., positive utility if the second period consumes more than the first period, and negative utility if the second period does not consume as much as the first period. Thus, the parameter γ can be interpreted as the degree of consumer's obsession with the first period of consumption of the second category of goods. The larger γ is, the more the person is obsessed with the incremental consumption of that good. In this logarithmic utility setting, there must be $\beta(1+\rho) > \gamma$. Otherwise, a rational consumer would not choose to consume b_1 because of the negative utility it entails. This relationship is also something that should be noted later in the numerical simulation.

Solving the problem consisting of (1)–(5) yields

$$\begin{cases} a_1^* = \frac{\alpha(1+\rho)(2+r)}{(\alpha+\beta)(2+\rho)(1+r)} \frac{m}{p_a} & \text{(6a)} \\ b_1^* = \frac{(\beta(1+\rho)-\gamma)(2+r)}{(\alpha+\beta)(2+\rho)(1+r)} \frac{m}{p_b} & \text{(6b)} \\ a_2^* = \frac{\alpha(2+r)}{(\alpha+\beta)(2+\rho)} \frac{m}{p_a} & \text{(6c)} \\ b_2^* = \frac{(\beta+\gamma)(2+r)}{(\alpha+\beta)(2+\rho)} \frac{m}{p_b} & \text{(6d)} \end{cases}$$

We derive the result (4) with respect to the parameter γ.

$$\begin{cases} \frac{\partial a_1^*}{\partial \gamma} = \frac{\partial a_2^*}{\partial \gamma} = 0 & \text{(7a)} \\ \frac{\partial b_1^*}{\partial \gamma} = \frac{-(2+r)}{(\alpha+\beta)(2+\rho)(1+r)} \frac{m}{p_b} < 0 & \text{(7b)} \\ \frac{\partial b_2^*}{\partial \gamma} = \frac{2+r}{(\alpha+\beta)(2+\rho)} \frac{m}{p_b} > 0 & \text{(7c)} \end{cases}$$

In the log utility function model with savings, the consumption of the common good in both periods is not related to the relative consumption effect and does not vary with parameter γ. The consumption of good b in the first period is negatively related to parameter γ, and the consumption of good b in the second period is positively related to parameter γ. It is easy to understand that when consumers value the incremental consumption of good b more, they will take the decision of consuming relatively lower consumption in the first period and consuming relatively more consumption in the second period, and the incremental consumption will be significantly enhanced under this decision, which will result in higher additional utility.

In the above model, when $\gamma = 0$, the model degenerates to the traditional deterministic two-commodity choice problem with no relative consumption effects, which we label with the "classic" superscript.

$$\begin{cases} a_1^{classic} = a_2^{classic} = \alpha(\alpha+\beta)^{-1} m {p_a}^{-1} & \text{(8a)} \\ b_1^{classic} = b_2^{classic} = \beta(\alpha+\beta)^{-1} m p_b^{-1} & \text{(8b)} \end{cases}$$

Compared to the traditional model without relative consumption effects, we find that when relative consumption effects are taken into account, the consumption of common goods in the first period is more so than in the traditional model, and the consumption of common goods in the second period is less so than in the traditional model; the consumption of memory goods in the first period is less so than in the traditional model, and the consumption of memory goods in the second period is more so than in the traditional. The memory effect reflects a consumer behavior that delays gratification.

3. Consumption of Goods under Imprecise Decision Utility Expectations and Their Adaptive Corrections

3.1. Model Framework in General Functional Form

Section 2 is our basic model framework, and here, we start to cut to the chase by introducing stochasticity and studying imprecise utility expectations and their effects. In addition to relative consumption effects, imprecise utility expectations are responsible for consumption deviations from the classical model. As mentioned in the introduction, in reality, many consumer goods acquire experiential utility gradually after purchase. For example, for goods such as pharmaceuticals, health care products, and durable goods, people can only make decisions based on the expectations of their decision utility until which time they use them to obtain the true experiential utility. In the model, consumers first decide to purchase two goods based on their psychological expectations in the first period and then make the actual consumption in the first period. When consumers get the true experiential utility at the end of the first period, their expectations of the decision utility in the second period will also be revised, and then they will change their utility functions and improve their purchase mix. This reasoning is consistent with the idea of adaptive expectations.

We still assume that a is a conventional good, while b has the expectation of determining utility and the uncertainty of experiencing utility, in addition to the relative consumption nature. When there is saving or borrowing, in the first period, the distribution of utility is not known because of the difficulty in obtaining information and the limitation of rational ability, and consumers do not know the specific expectation of the utility function but can only make a subjective rational decision based on a general expectation in their mind. We use E_1[-] to denote the consumer's "subjective expectation" of utility in the early first period for both periods", and at the beginning of the first period, i.e., before the consumer makes a purchase decision, the consumer faces the problem of

$$\max E_1[u_1(a_1, b_1)] + (1+\rho)^{-1} E_1[u_2(a_2; b_2, b_2/b_1)] \tag{9}$$

s.t.

$$a_1 p_a + b_1 p_b + s = m \tag{10}$$

$$a_2 p_a + b_2 p_b = m + (1+r)s \tag{11}$$

The result $\{a_1^*, b_1^*, a_2^*, b_2^*, s_1^*\}$ was obtained, where a_1^*, b_1^*, s_1^* was purchased and realized in the first period, while a_2^*, b_2^* was not really implemented, but was only a plan for the second period at the beginning of the first period. When the first period is consumed, it is known exactly what $u_1(a_1^*, b_1^*)$ is. After that, the consumer corrects the expectation of the utility function based on the consumption experience of the first period to get the expectation of the utility function of the second period and make the decision for the second period. That is, the second period sought is the optimization under the conditional expectation of the variables already realized in the first period and its planning problem.

$$\max E_2[u_2(a_2; b_2, b_2/b_1^*) | u(a_1^*, b_1^*), s_1^*] \tag{12}$$

s.t.

$$a_2 p_a + b_2 p_b = m + (1+r)s_1^* \tag{13}$$

For example, the utility function used to determine the utility expectation in the second period should be replaced by the "randomly realized" experience utility function in the first period.

3.2. Analysis of the Logarithmic Form Utility Function

Still using the logarithmic utility function as an example to illustrate the problem,

$$u_1(a_1, b_1) = \alpha \ln a_1 + (\beta + \varepsilon) \ln b_1 \tag{14}$$

$$u_2(a_2; b_2, b_2/b_1) = \alpha \ln a_2 + (\beta + \varepsilon) \ln b_2 + \gamma \ln(b_2/b_1) \tag{15}$$

Variable ε is a random variable with zero mean and unknown distribution. Again, consumers are not informed of β. After all, the individual consumption experience of some goods is different from statistics of large samples of data, and predicting utility is difficult. At the beginning of the first period, the consumer's initial estimate of β is β_0, i.e., the consumer first makes a decision based on the subjective utility expectation estimated internally in the first period. This expectation differs from the objective expectation and is expressed as the subjective parameter β_0. As explained in the introduction, why not use the "subjective expected utility model" to set the framework—assuming the distribution of various experiential utilities with subjective probabilities and then weighting—and why do we not use the "subjective expected utility model" framework here, which assumes the distribution of various experience utilities under subjective probabilities, and then weigh and optimize them mathematically to derive the subjective expectation of utility? The reason is that, considering the fact that the distribution of consumers' experience utility is not known and the subjective probabilities are also not known, it is better to express the subjective expected utility in terms of the parameter estimates of the utility function directly on the model, and it is easier to analyze the phenomena of overestimation, underestimation, and psychological fluctuations).

Thus $E_1[u_1(a_1, b_1)] + (1+\rho)^{-1} E_1[u_2(a_2; b_2, b_2/b_1)]$ takes the following form:

$$(\alpha \ln a_1 + \beta_0 \ln b_1) + (1+\rho)^{-1}(\alpha \ln a_2 + \beta_0 \ln b_2 + \gamma \ln(b_2/b_1)) \tag{16}$$

The consumer uses (16) for global decision making and obtains the following result:

$$\begin{cases} a_1^* = \frac{\alpha(1+\rho)(2+r)}{(\alpha+\beta_0)(2+\rho)(1+r)} \frac{m}{p_a},\ b_1^* = \frac{(\beta_0(1+\rho)-\gamma)(2+r)}{(\alpha+\beta_0)(2+\rho)(1+r)} \frac{m}{p_b} & \text{(17a)} \\ a_2^* = \frac{\alpha(2+r)}{(\alpha+\beta_0)(2+\rho)} \frac{m}{p_a},\ b_2^* = \frac{(\beta_0+\gamma)(2+r)}{(\alpha+\beta_0)(2+\rho)} \frac{m}{p_b} & \text{(17b)} \end{cases}$$

Individual micro-consumption behavior lacks statistics of large sample data, and consumers' own experience is only limited, so it is difficult to know exactly the distribution and expectations of the parameters. At this point the consumer has no way of knowing β, and we call the result of decision making with the true parameter mean β the ideal consumption, representing the consumer's (thoroughly rational expectations and without randomness) consumption strategy when accurately expected, i.e.,

$$\begin{cases} a_1^{\text{ideal}} = \frac{\alpha(1+\rho)(2+r)}{(\alpha+\beta)(2+\rho)(1+r)} \frac{m}{p_a},\ b_1^{\text{ideal}} = \frac{(\beta(1+\rho)-\gamma)(2+r)}{(\alpha+\beta)(2+\rho)(1+r)} \frac{m}{p_b} & \text{(18a)} \\ a_2^{\text{ideal}} = \frac{\alpha(2+r)}{(\alpha+\beta)(2+\rho)} \frac{m}{p_a},\ b_2^{\text{ideal}} = \frac{(\beta+\gamma)(2+r)}{(\alpha+\beta)(2+\rho)} \frac{m}{p_b} & \text{(18b)} \end{cases}$$

Equations (8) and (18) are identical, indicating that the initial decision outcome with accurate consumer expectations is consistent with the deterministic model. We know that the above a_2^*, b_2^* is not implemented in the first period, and the savings in the first period can be calculated from a_1^*, b_1^* as

$$s_1^* = \frac{(\alpha+\beta_0)(r-\rho) + \gamma(2+r)}{(\alpha+\beta_0)(2+\rho)(1+r)} m \tag{19}$$

Consumers make the first period decision according to the psychological expectation parameter β_0, and the actual utility experienced is a utility function with the parameter β_1. That is, the utility function realized in the first period is $\alpha \ln a_1^* + \beta_1 \ln b_1^*$, and although the parameter β_1 is also a randomly realized outcome, it leaves a memory of the experience for the consumers and corrects their initial expectations. A natural consequence is that consumers start the second period with the utility they experienced in the first period, i.e., they use the utility function parameterized by β_1 to make the second-period decision. Of course, the consumer still does not know the true expectation of β. The model uses the

parameter β_1 as the expectation parameter of the utility function for the second period decision and substitutes the savings s_1^*, and the second period problem is

$$\max(\alpha \ln a_2 + \beta_1 \ln b_2 + \gamma \ln(b_2/b_1)) \tag{20}$$

s.t.

$$a_2 p_a + b_2 p_b = m + (1+r)s_1^* \tag{21}$$

$$a_2 p_a + b_2 p_b = (\alpha + \beta_0 + \gamma)(2+r)(\alpha+\beta_0)^{-1}(2+\rho)^{-1} m \tag{22}$$

Solving this problem yields the second period consumption combination as

$$\begin{cases} a_2^{**} = \frac{\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)(2+\rho)} \frac{m}{p_a} = \frac{(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)} a_2^{\text{ideal}} & \text{(23a)} \\ b_2^{**} = \frac{(\beta_1+\gamma)(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)(2+\rho)} \frac{m}{p_b} = \frac{(\beta_1+\gamma)(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\beta+\gamma)(\alpha+\beta_1+\gamma)(\alpha+\beta_0)} b_2^{\text{ideal}} & \text{(23b)} \end{cases}$$

Comparative static analysis of the parameters of β_1 is

$$\frac{\partial a_2^{**}}{\partial \beta_1} = \frac{-\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_1+\gamma)^2(\alpha+\beta_0)(2+\rho)} < 0 \tag{24}$$

$$\frac{\partial b_2^{**}}{\partial \beta_1} = \frac{\alpha(\alpha+\beta_0+\gamma)(2+r)m}{(\alpha+\beta_1+\gamma)^2(\alpha+\beta_0)(2+\rho)p_b} > 0 \tag{25}$$

Since the size of β_1 indicates the size of the actual experienced utility of the first period consumers for the second category of goods (the size of the randomly realized value), it is clear that the purchase of the second period goods b increases with the increase in the experienced utility of the first period, which is also in line with logical common sense. It is not difficult to calculate the difference in the quantity of the second period goods in the two decisions as follows.

$$a_2^{**} - a_2^* = \frac{(\beta_0 - \beta_1)(\alpha+\beta)}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)} a_2^{\text{ideal}} \tag{26}$$

$$b_2^{**} - b_2^* = -\frac{(\beta_0 - \beta_1)(\alpha+\beta)\alpha}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)(\beta+\gamma)} b_2^{\text{ideal}} \tag{27}$$

If $\beta_0 - \beta_1 > 0$ and the consumer randomly achieves an experience utility in the first period that is less than the initial predicted decision utility, then $b_2^{**} - b_2^* < 0$, and the consumer will spend less in the second period than originally planned. If $\beta_0 - \beta_1 < 0$ and the consumer randomly achieves an experience utility in the first period that is greater than the initial estimated decision utility, then $b_2^{**} - b_2^* > 0$, and the consumer will spend more in the second period than originally planned. This result is consistent with common sense.

Considering that results (17) and (23) are random variables that can be used to explain chance purchases, they cannot be used to directly describe consumption behavior in a statistical sense. Therefore, we need to analyze their statistical characteristics, such as mean values.

4. Effect of Psychological Fluctuations on Consumption Mean under Unbiased Estimation

In Section 3 of the model, parameter β_0 is the consumer's estimate of β. β_0's expectation and variance enjoy different meanings, the former describing the level of the consumers' decision utility in a statistical sense and the latter describing the volatility of the consumers' decision utility, i.e., consumers' psychological fluctuations. We are more interested in knowing the impact of psychological fluctuations on consumption than in estimating the impact of the mean on consumption behavior, which is a matter of explaining

the economics of the "state of mind" of consumption. This part assumes that the decision utility is an unbiased estimate of the true experience utility expectation, i.e.,

$$E[\beta_0] = \beta \tag{28}$$

4.1. Expected Consumption under Unbiased Estimation

We calculate the expectation for the first period of commodity consumption (13).

$$\begin{aligned} E\left[a_1^*\right] &= \tfrac{\alpha(1+\rho)(2+r)}{(2+\rho)(1+r)} \tfrac{m}{p_a} E\left[\tfrac{1}{(\alpha+\beta_0)}\right] \\ &= \tfrac{\alpha(1+\rho)(2+r)}{(2+\rho)(1+r)} \tfrac{m}{p_a} \tfrac{1}{(\alpha+\beta)} E\left[(1+\tfrac{\beta_0-\beta}{\alpha+\beta})^{-1}\right] \end{aligned}$$

Since β_0 generally does not deviate too much from β, and this parameter is a decimal between the interval (0, 1), the absolute value is also generally much smaller than around $\alpha+\beta$, so we can use the Taylor expansion and ignore the higher-order decimals in the later calculations.

$$E[a_1^*] = \frac{\alpha(1+\rho)(2+r)}{(2+\rho)(1+r)} \frac{m}{p_a} \frac{1}{(\alpha+\beta)} E\left[1 - \frac{\beta_0-\beta}{\alpha+\beta} + \frac{(\beta_0-\beta)^2}{(\alpha+\beta)^2} - \cdots\right]$$

$$E[a_1^*] = \left(1 + \textstyle\sum_{n=1}^{\infty} E\left[\dfrac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right) a_1^{\text{ideal}} \tag{29}$$

Similarly, the calculation process is shown in Appendix A,

$$E[b_1^*] = \left(1 - \frac{\alpha(1+\rho)+\gamma}{\beta(1+\rho)-\gamma} \textstyle\sum_{n=1}^{\infty} E\left[\dfrac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right) b_1^{\text{ideal}} \tag{30}$$

Expanding the results of the second-period yields, the calculation process is shown in Appendix A:

$$E[a_2^{**}] = \left[1 + \sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right] \left(1 + \frac{\gamma}{\alpha+\beta+\gamma} \sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right) a_2^{\text{ideal}} \tag{31}$$

$$E[b_2^{**}] = \left(1 - \frac{\alpha}{\beta+\gamma} \sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right) \left(1 + \frac{\gamma}{\alpha+\beta+\gamma} \sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right) b_2^{\text{ideal}} \tag{32}$$

In the expectation of the four variables above, if the distribution of the random variables is unbiased, the higher-order central distance of the odd order is zero in all cases. Although the even central moments are not zero, the higher-order moments can be disregarded in the approximation calculation if we consider the deviation of the prediction from the true value to be a small quantity. In this way, ignoring the higher-order small quantities, only the second-order central moments—the variances—remain, summarizing the above results.

$$\begin{cases} E\left[a_1^*\right] \approx a_1^{\text{ideal}} + \frac{D[\beta_0]}{(\alpha+\beta)^2} a_1^{\text{ideal}} & \text{(33a)} \\ E\left[b_1^*\right] \approx b_1^{\text{ideal}} - \frac{D[\beta_0]}{(\alpha+\beta)^2} \frac{\alpha(1+\rho)+\gamma}{\beta(1+\rho)-\gamma} b_1^{\text{ideal}} & \text{(33b)} \\ E[a_2^{**}] \approx \left(1 + \frac{D[\beta_1]}{(\alpha+\beta+\gamma)^2}\right)\left(1 + \frac{\gamma}{\alpha+\beta+\gamma} \frac{D[\beta_0]}{(\alpha+\beta)^2}\right) a_2^{\text{ideal}} & \text{(33c)} \\ E[b_2^{**}] \approx \left(1 - \frac{\alpha D[\beta_1]}{(\alpha+\beta+\gamma)^2(\beta+\gamma)}\right)\left(1 + \frac{\gamma}{\alpha+\beta+\gamma} \frac{D[\beta_0]}{(\alpha+\beta)^2}\right) b_2^{\text{ideal}} & \text{(33d)} \end{cases}$$

In (33), since $E[b_2^{**}]$ cannot be less than zero and the expectation of a nonnegative random variable cannot be negative, taking the approximation also requires that the parameters satisfy $D[\beta_1] < \alpha^{-1}(\beta+\gamma)(\alpha+\beta+\gamma)^2$. In fact, as we will show later, this holds in general.

Thus, the monotonicity of consumption expectations for both commodities for both periods is evident in result (33) for both $D[\beta_1]$ and $D[\beta_0]$. The instability experienced by commodity b $D[\beta_1]$, which decreases its consumption in the second period, increases the consumption of commodity a in general and has no effect on the first period. The expected consumption of commodity a in both periods is an increasing function of $D[\beta_0]$; the expected consumption of commodity b in the first period is a decreasing function of $D[\beta_0]$, and the expected consumption of commodity b in the second period is an increasing function of $D[\beta_0]$.

$$\frac{\partial E[b_2^{**}]}{\partial D[\beta_0]} = \left(1 - \frac{\alpha D[\beta_1]}{(\alpha+\beta+\gamma)^2(\beta+\gamma)}\right) \frac{\gamma b_2^{\text{ideal}}}{(\alpha+\beta)^2(\alpha+\beta+\gamma)} \tag{34}$$

When $D[\beta_1] < \alpha^{-1}(\beta+\gamma)(\alpha+\beta+\gamma)^2$, $E[b_2^{**}]$ is an increasing function of $D[\beta_0]$.
When $D[\beta_1] > \alpha^{-1}(\beta+\gamma)(\alpha+\beta+\gamma)^2$, $E[b_2^{**}]$ is a decreasing function of $D[\beta_0]$.
When $D[\beta_1] = \alpha^{-1}(\beta+\gamma)(\alpha+\beta+\gamma)^2$, $E[b_2^{**}]$ is independent of $D[\beta_0]$.

The variance of parameter β_0 describes the volatility of the actor's initial subjective prediction of the commodity's utility (determines utility volatility), and a larger variance of $D[\beta_0]$ indicates a more unstable initial consumer psychological utility expectation; while the variance of parameter β_1 describes the volatility of the commodity's experience effect, a larger variance of $D[\beta_1]$ indicates a more unstable commodity experience. As mentioned earlier, these two are different. From the comparative static analysis, it is known that the volatility of the actual experience reduces the expected consumption of the good, i.e., the more volatile the actual experience is in the first period, the less they tend to purchase the good in the second period. For example, a consumer goes to book a long-distance trip and pays for different itineraries and services before departure. His expected satisfaction with the trip is measured by β_0. If his psychological desire to consume is high and low, the expected fluctuation is likely to be large, which is expressed when parameter $D[\beta_0]$ is larger. When the actual satisfaction of the travel process becomes very uncertain under the influence of random factors—such as changeable weather, the guide's attitude, traffic congestion, sleeping out, scenic spot crowd, food quality, etc.—after the person has joined the trip, it means that the fluctuation of the consumption experience itself is very large, which is expressed in the model when $D[\beta_1]$ is very large. In addition, this consumer is relatively new to this travel route, having been there only once or twice or never, and really has no way of knowing the true average experience utility. As reflected in the model, parameter expectation β cannot be known. After all, the average person rarely experiences the same trip many times over. Each person's experience is subjective and unlikely to be consistent, making it difficult to obtain true parameter expectations from a microscopic perspective.

For better numerical simulation, we first investigate the range of values of the parameter variance. Considering that both β_0 and β_1 are random variables within the interval $[0, 1)$, their variances have a range of values.

Lemma 1. *The variance of a random variable x with values in the interval (0, 1) cannot be greater than 0.25.*

In numerical simulation, $\alpha+\beta+\gamma$ is often about 1, α does not exceed $\beta+\gamma$ (consumer psychological satisfaction is mainly brought by the second category of goods), and $D[\beta_1]$ again does not exceed 0.25, so in general, $(\alpha+\beta+\gamma)^{-2}(\beta+\gamma)^{-1}\alpha D[\beta_1]$ is difficult to exceed 1. At this time, $E[b_2^{**}]$ is the incremental function of $D[\beta_0]$; that is, the expected consumption of commodity b in the second period will increase due to the increased volatility of psychological expectations. That is, if one's perception of the consumption of a commodity is not clear, one is likely to consume more of this commodity in the later period. This is an interesting conclusion. In other words, the more unstable the psychological expectation is, the more likely it is that people with fluctuating emotions will expand their

consumption in the future, but this is different from "impulsive" consumption. What people used to understand as "impulsive" may be derived from high psychological expectations of goods, which is different from the psychological fluctuations described in our model. Because high expectations are not the same as high volatility, the variance and mean are two concepts. High psychological fluctuations may not necessarily only come from high expectations, but also from sudden under-expectations. Our model shows that the higher and lower a person's consumption desire is, the more likely it is to cause excessive consumption in the future. This consumer behavior is better described as "ignorant blind consumption" rather than "impulsive consumption".

Many examples of this consumer behavior can actually be found in reality. For example, after a lottery draw, many lottery ticket buyers, whether he/she wins or not, will claim that they will not buy any more tickets and then immediately buy the next lottery ticket while doing so. In fact, many gamblers behave very similarly, with some people tending to buy further chips the worse the prediction of the gambling outcome and the greater the fluctuation. This is a reflection of blind consumption, which comes from the instability of psychological expectations or cognitive bias.

When $\beta + \gamma$ is very, very small, $E[b_2^{**}]$ can be a decreasing function of $D[\beta_0]$, and the consumption expectation of the second-period memory goods decreases as the misjudgment fluctuation of the first period increases. In fact, because $\beta + \gamma$ is very small and α is very large, at this point, the consumer simply does not care much about the second category of consumption with random experiences, and the person is not too fond of comparison and does not care much about relative consumption. His psychological satisfaction comes mainly from the first category, perhaps the daily necessities. At this time, the fluctuation of the psychological expectation of commodity b will reduce the future consumption of commodity b. At this point, $E[b_2^{**}] < 0$, and the optimal solution for commodity b in the second period may be negative, meaning that rationally expected consumers will no longer consume commodity b in the second period.

In any case, rational people are averse to the volatility of parameter β_1, which describes the actual experiential utility of the first period of the good. This is the difference between "psychological prediction volatility" and "commodity experience volatility".

4.2. Numerical Simulation

Let us look at two examples of values.

In the two examples above, all parameters are the same, except for the variances of β_0 and β_1, which yield different results. Case A is characterized by psychological expectations that underestimate the fluctuations in the experience of the consumption of the good; Case B is characterized by the psychological expectations that overestimate the fluctuations in the experience of the consumption of the good, with the result that good b is consumed more in the system of Case B in the second period. In other words, if the variance of the consumers' psychological utility expectations is estimated to be large, but the actual experience is relatively stable, consumers will consume more of commodity b in the second period. Overall, the data results are relatively different from the classical model's ideal consumption.

To show the trend of the results with the two variances, we treated the two variances as independent variables and set the values of the parameters to $\alpha = 0.1, \beta = 0.6, \gamma = 0.3$, $\rho = 0$ and plotted the results (27) using the mathematical software "Maple", as shown below (Figures 1–4).

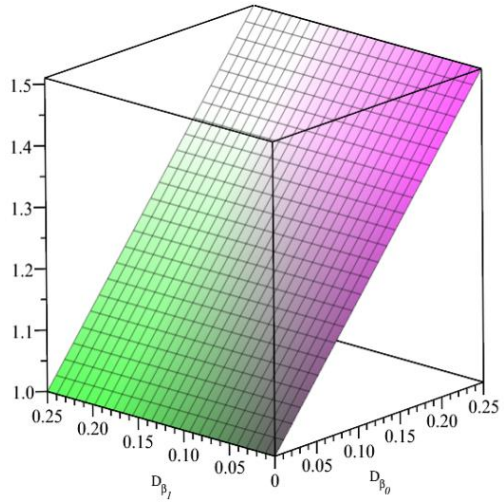

Figure 1. The plot of $E\left[a_1^*\right]/a_1^{\text{ideal}}$ with the variances of β_1 and β_0.

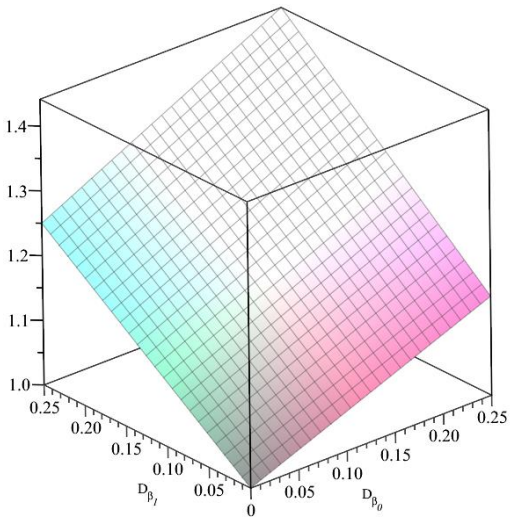

Figure 2. The plot of $E\left[a_2^{**}\right]/a_2^{\text{ideal}}$ with the variances of β_1 and β_0.

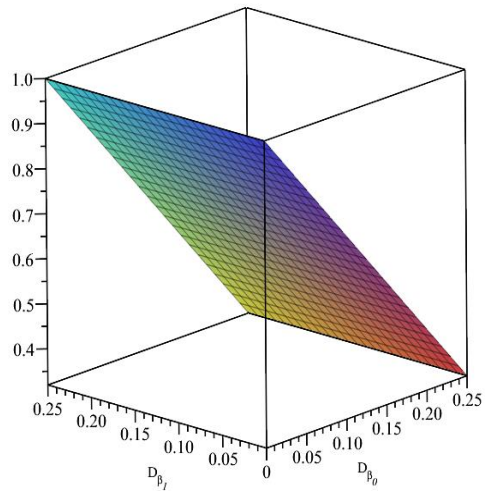

Figure 3. The plot of $E\left[b_1^*\right]/b_1^{\text{ideal}}$ with the variances of β_1 and β_0.

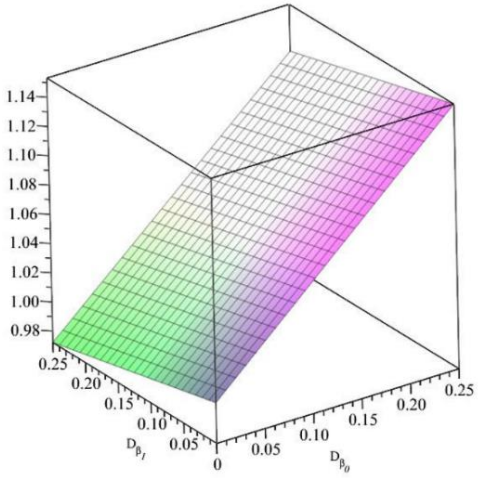

Figure 4. The plot of $E\left[b_2^{**}\right]/b_2^{\text{ideal}}$ with the variances of β_1 and β_0.

To see the effect of the relative consumption effect and the psychological misspecification variance on the results, we set the parameter $\alpha = 0.1, \rho = 0, D[\beta_1] = 0.1$ and ensured that $\beta + \gamma = 1 - \alpha = 0.9$ holds; let γ vary from 0 to 0.45, and let $D[\beta_0]$ vary from 0 to 0.25, and plot (25) as shown below (Figures 5–8).

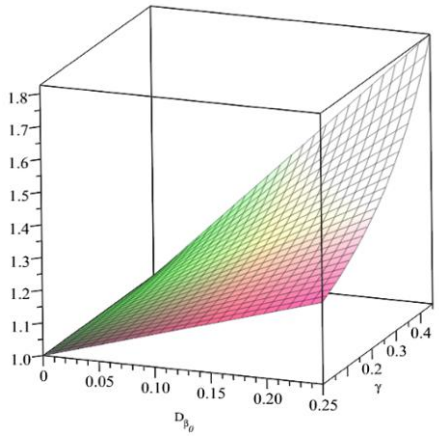

Figure 5. The plot of $E\left[a_1^{*}\right]/a_1^{\text{ideal}}$ with γ and $D[\beta_0]$.

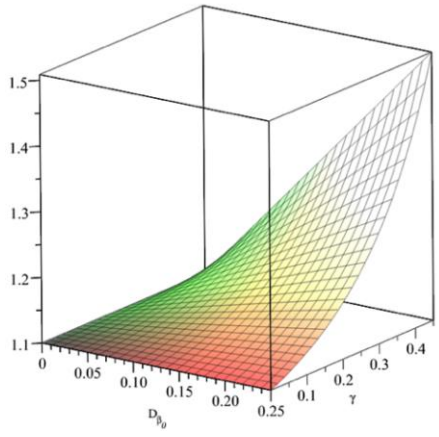

Figure 6. The plot of $E\left[a_2^{**}\right]/a_2^{\text{ideal}}$ with γ and $D[\beta_0]$.

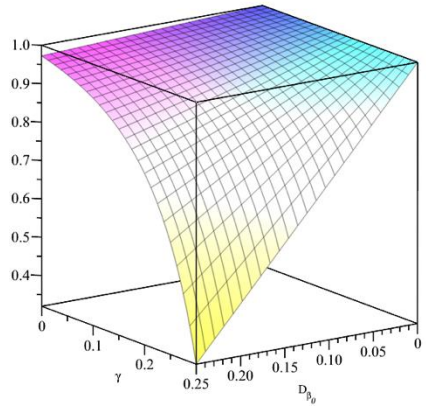

Figure 7. The plot of $E\left[b_1^*\right]/b_1^{\text{ideal}}$ with γ and $D[\beta_0]$.

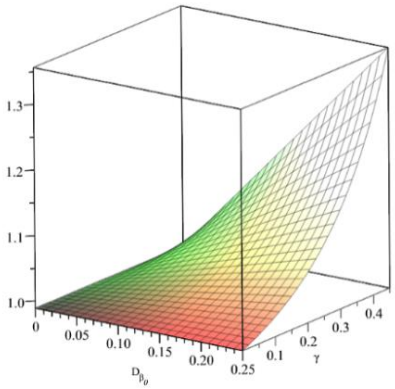

Figure 8. The plot of $E\left[b_2^{**}\right]/b_2^{\text{ideal}}$ with γ and $D[\beta_0]$.

In all the numerical simulations above, the consumption of commodity b in the second period tends to be large and is positively associated with both the relative consumption effect and the psychological misspecification variance.

Under log utility, if $\gamma = 0$, there is no relative consumption effect for the good, and the model degenerates to its conventional form, but the effects of psychological expectation fluctuations and experience fluctuations remain, and Equation (33) simplifies to the following form.

$$\begin{cases} E\left[a_1^*\right] = a_1^{\text{ideal}} + a_1^{\text{ideal}} D[\beta_0]/(\alpha+\beta)^2,\ E\left[b_1^*\right] = b_1^{\text{ideal}} - \alpha D[\beta_0]/\left(\beta(\alpha+\beta)^2\right) b_1^{\text{ideal}} & \text{(35a)} \\ E[a_2^{**}] = \left[1 + D[\beta_1]/(\alpha+\beta)^2\right] a_2^{\text{ideal}},\ E[b_2^{**}] = \left(1 - \alpha D[\beta_1]/\left(\beta(\alpha+\beta)^2\right)\right) b_2^{\text{ideal}} & \text{(35b)} \end{cases}$$

In the results of the model without relative consumption effects (35), the expected consumption of goods in the first period deviates from the ideal value due to fluctuations in psychological expectations, and the expected consumption of goods in the second period deviates from the ideal value due to fluctuations in actual experience, but the consumption of goods in the second period is not related to the psychological prediction in the early period. This is due to the loss of the memory nature of relative consumption. In conclusion, the tendency to consume less of item b in both periods is similar to the "risk aversion" effect in consumers.

5. Systematic Biased Estimation of Consumption Impact and Classification of Consumption Behavior

The difference between the one-time realized and expected values of a random variable is that the former represents chance, and the latter represents systematicity. Here, we

distinguish between one-time bias and systematic bias. The systematic bias variables that determine the utility parameters are defined as follows.

$$bais = E[\beta_0] - \beta \tag{36}$$

Taylor expansions of the corresponding consumptions of results (13) and (19) (see Appendix C for the procedure) and neglecting higher-order minima yield

$$\begin{cases} E\left[a_1^*\right] \approx \frac{\alpha+\beta}{\alpha+\beta+bais}\left(1+\frac{D[\beta_0]}{(\alpha+\beta+bais)^2}\right)a_1^{\text{ideal}} & \text{(37a)} \\ E\left[b_1^*\right] \approx \left(\frac{\alpha+\beta}{(1+\rho)\beta-\gamma}\frac{(1+\rho)(\beta+bias)-\gamma}{\alpha+\beta+bias} - \frac{(\alpha+\beta)(\alpha(1+\rho)+\gamma)}{\beta(1+\rho)-\gamma}\frac{D[\beta_0]}{(\alpha+\beta+bais)^3}\right)b_1^{\text{ideal}} & \text{(37b)} \\ E[a_2^{**}] \approx \left[1+\frac{D[\beta_1]}{(\alpha+\beta+\gamma)^2}\right]\left(\begin{array}{l}\frac{(\alpha+\beta)(\alpha+\beta+\gamma+bais)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)} \\ +\frac{\gamma(\alpha+\beta)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)}\end{array}\quad\frac{D[\beta_0]}{(\alpha+\beta+bais)^2}\right)a_2^{\text{ideal}} & \text{(37c)} \\ E[b_2^{**}] \approx \frac{(\alpha+\beta)(\alpha+\beta+bais+\gamma)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)}\left(1-\frac{(\beta+\gamma)^{-1}\alpha D[\beta_1]}{(\alpha+\beta+\gamma)^2}\right)\left(1+\frac{(\alpha+\beta+bais+\gamma)^{-1}D[\beta_0]}{(\alpha+\beta+bais)^2\gamma^{-1}}\right)b_2^{\text{ideal}} & \text{(37d)} \end{cases}$$

In the absence of relative consumption effects, (37) reduces to

$$\begin{cases} E\left[a_1^*\right] \approx \frac{\alpha+\beta}{\alpha+\beta+bais}\left(1+\frac{D[\beta_0]}{(\alpha+\beta+bais)^2}\right)a_1^{\text{ideal}} & \text{(38a)} \\ E\left[b_1^*\right] \approx \frac{\alpha+\beta}{\beta}\left(\frac{\beta+bias}{\alpha+\beta+bias} - \frac{\alpha D[\beta_0]}{(\alpha+\beta+bais)^3}\right)b_1^{\text{ideal}} & \text{(38b)} \\ E[a_2^{**}] \approx E[a_2^{**}] \approx \left[1+\frac{D[\beta_1]}{(\alpha+\beta)^2}\right]a_2^{\text{ideal}} & \text{(38c)} \\ E[b_2^{**}] \approx \left(1-\frac{\alpha D[\beta_1]}{(\alpha+\beta)^2\beta}\right)b_2^{\text{ideal}} & \text{(38d)} \end{cases}$$

Consistent with the discussion in Section 4, this is when the variance of the decision utility parameter no longer affects the consumption of the second-period b goods. In conclusion, the study of the effect of consumption mind fluctuations cannot leave the relative consumption effect (memory effect).

The above results are derived for bias, and it is easy to see that $\frac{\partial E\left[a_1^*\right]}{\partial bais} < 0$, $\frac{\partial E\left[b_1^*\right]}{\partial bais} > 0$, $\frac{\partial E\left[a_2^*\right]}{\partial bais} < 0$, $\frac{\partial E\left[b_2^*\right]}{\partial bais} < 0$, and an increase in bias implies a progressive overestimation of decision utility over experience utility. Distinct from (26) and (27), because the derivatives of the second-period consumption expectations for both goods are negative with respect to the bias, the systematic overestimation reduces the overall second-period consumption expectations. However, in (26) and (27), the chance overestimation reduces the second-period consumption of category b goods, but it increases the consumption of category a good in the second period. However, with systematic overestimation, the consumption levels of both in the second period are expected to decrease. This shows that the potential impact of systematic overestimation is more severe than that of chance overestimation.

The probability density functions of β_0 and β_1 are drawn as in the figure, and the horizontal distance between the two density function waves (which coincide with the mean when the plural is unbiased) is the absolute value of bias. According to the size relationship between the four, β_0, $E[\beta_0]$, β, β_1, consumption behaviors and outcomes can be classified into 24 categories, i.e., one full-ranking number of them. The basic six groups of twelve two-parameter relationships and the corresponding consumption behavior states are as follows:

The term "chance overestimation" refers to the fact that the decision utility of random selection is larger than the experience utility, corresponding to $\beta_0 > \beta_1$; the term "chance underestimation" refers to the fact that the decision utility of random selection is smaller than the experience utility, corresponding to $\beta_0 < \beta_1$.

A "subjective overestimate of chance" means that the decision utility of a random choice is larger than the expectation of the decision utility and a chance decision that is more optimistic than one's consumption habits, corresponding to $\beta_0 > E[\beta_0]$; conversely, a "subjective underestimate of chance" corresponds to $\beta_0 < E[\beta_0]$.

"Objective overestimation by chance", meaning that the decision utility of random selection exceeds exactly the mean value of the experienced utility, corresponds to $\beta_0 > \beta$, and, conversely, "objective underestimation by chance" corresponds to $\beta_0 < \beta$.

"Systematic overestimation" refers to the expectation that the subjectively determined utility is greater than the experienced utility and is a comparison in the statistical mean sense, corresponding to $E[\beta_0] > \beta$ and, conversely, "systematic underestimation" corresponding to $E[\beta_0] < \beta$.

The term "accidental super-habitual realization" refers to the fact that the randomly realized experiential utility is greater than the expected decision utility, which corresponds to the randomly realized consumption utility exceeding the consumer's habitual decision mean, corresponding to $\beta_1 > E[\beta_0]$ and, conversely, "accidental low habitual realization" corresponding to $\beta_1 < E[\beta_0]$.

The "good luck consumption" is worthy of a randomly realized experience utility greater than the mean of the experience utility, indicating that the consumer experienced an extraordinary consumption process corresponding to $\beta_1 > \beta$ and, conversely, the "bad luck consumption" corresponding to $\beta_1 < \beta$.

That is, "chance high/underestimate; chance subjective high/underestimate; chance objective high/underestimate; systematic high/underestimate; chance super/low habitual realization; good or bad luck" These six sets of relationships can be used as factors to classify consumption behavior.

The other parameter relationships can be considered as a composite of the two relationships above. The "super underestimation" corresponds to $\beta_0 < E[\beta_0] < \beta$, which is the simultaneous occurrence of "contingent subjective underestimation" and "systematic underestimation". For "super bad luck" $\beta_1 < \beta < E[\beta_0] < \beta_0$ is a combination of "bad luck consumption", "contingent subjective overestimation" and "systematic overestimation". The combination of "bad luck consumption", "contingent subjective overestimation" and "systematic overestimation", i.e., a state of poor-luck consumption under super high consumer expectations, with a large utility gap. The consumption states corresponding to the four-parameter structures are schematically labeled in the figure. For example, in the first figure, $\beta_0 < E[\beta_0] < \beta < \beta_1$, and the decision utility random variable is lower than the level of systematic underestimation, i.e., this is a super low anticipation, but the actual experienced utility is above its mean value, indicating the existence of systematic underestimation, contingent subjective underestimation, and good luck for consumers (Figures 9–12).

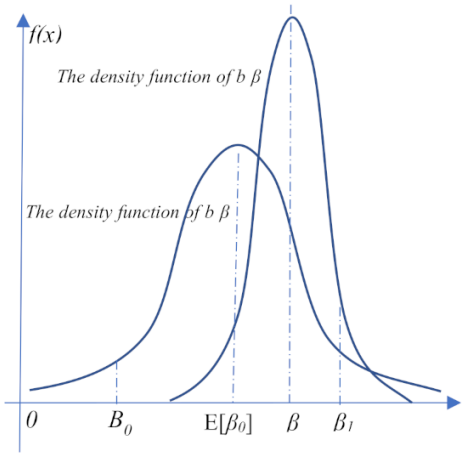

Figure 9. Accidental ultra-low estimates under systematic undervaluation accompanied by good luck.

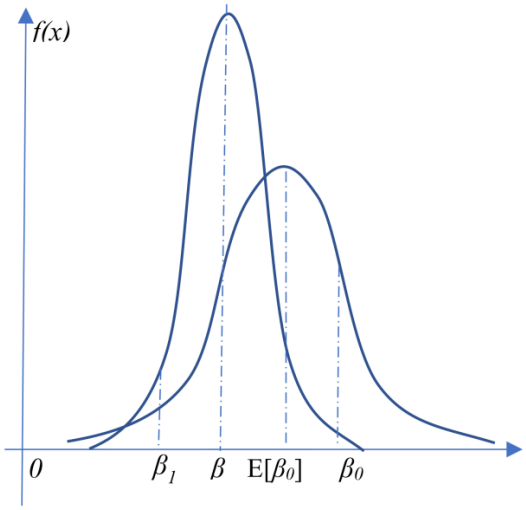

Figure 10. Occasional high estimates under systematic overvaluation accompanied by poor luck.

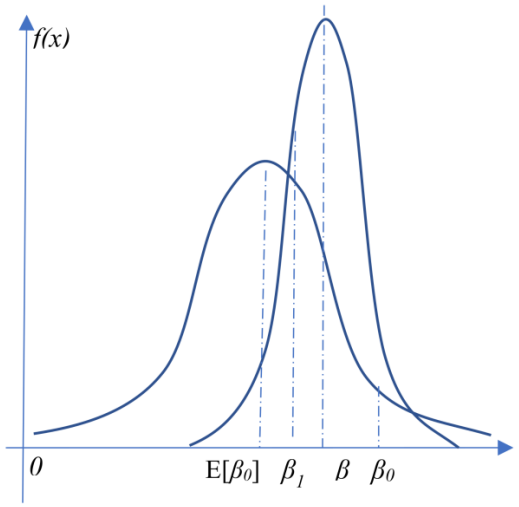

Figure 11. Systematic underestimation with occasional overestimation accompanied by bad luck.

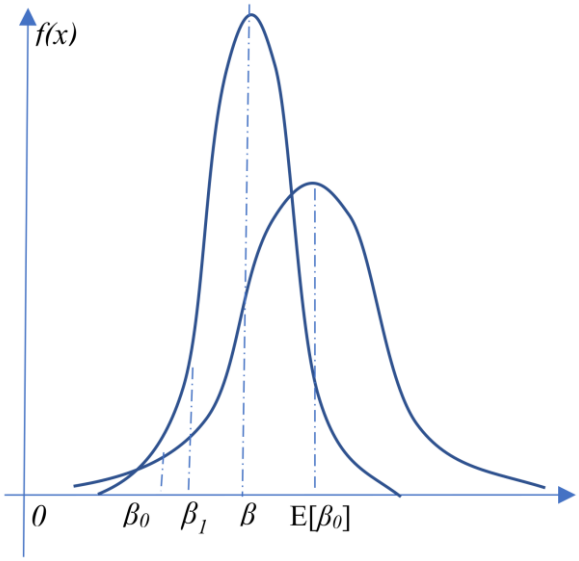

Figure 12. Accidental overestimation under systematic overestimation comes with not-so-good luck.

Table A1 in Appendix D gives the consumption states and their underlying factors for all twenty-four-size relationships of the four parameters. Any random consumption can be classified into these 24 states, although their probabilities vary in size and in some cases are smaller probability events, such as a "cautious" person suddenly having a very high expectation and encountering very bad luck. In Table 1, the second column shows the significance of the relationships between the magnitudes of the adjacent parameter variables, each consisting of three sets of independent and unrelated factors; the other three states are labeled in the third column. They can also correspond to realistic scenarios, as illustrated by the first three relationships: $\beta_0 < E[\beta_0] < \beta < \beta_1$ describes a scenario in which a cautious person encounters good luck with ultra-low expectations; $\beta_0 < E[\beta_0] < \beta_1 < \beta$ describes a scenario in which a cautious person encounters bad luck with ultra-low expectations; and $\beta_0 < \beta < E[\beta_0] < \beta_1$ describes a scenario in which an optimistic person encounters extraordinary luck with occasional ultra-low expectations. We will not list them all—the rest are similar.

Table 1. Results of two numerical simulation cases.

$\alpha = 0.3, \beta = 0.5$ $\gamma = 0.2, \rho = 0$	First Period		Second Period	
	$E[a_1^*]/a_1^{\text{ideal}}$	$E[b_1^*]/b_1^{\text{ideal}}$	$E[a_2^{**}]/a_2^{\text{ideal}}$	$E[b_2^{**}]/b_2^{\text{ideal}}$
A. $D[\beta_0] = 0.1, D[\beta_1] = 0.2$	115.6%	74.0%	123.7%	94.3%
B. $D[\beta_0] = 0.2, D[\beta_1] = 0.1$	131.5%	44.4%	116.9%	101.7%

The above categorical findings obtained through numerical simulations can also be found in the empirical evidence [27], wherein the service quality corresponds to β_1 in this paper, the experience quality is the psychological outcome of consumers corresponding to β_0 in this paper, and the satisfaction is reflected in the difference between the predestinated utility and the real experience utility in the first period, which also affects the consumption in the second period.

Previous studies do recognize the consumer problems caused by quality of experience, but they do not combine quality of experience with decision utility, i.e., they do not study the pre-consumption estimation of quality of experience and its distribution. They also lack the random consideration of consumer psychology, quality of experience and other parameters. We believe that it is the distribution and random realization of the prediction, not the real quality of experience, that plays a decisive role in the decision and quantity of purchase.

6. Discussion and Conclusions

6.1. Discussion

6.1.1. Parameter-Adaptive Expectation Model for Linear Combinatorial

In the third part of the model, the parameters used for the decision utility function in the second-period are those of the first-period experience utility function. In the adaptive expectations framework, if consumers are more confident in their initial predictions, they may take some linear combination of the parameters of the first-period decision utility function β_0 and the parameters of the experience utility function β_1, e.g., the decision utility function for good b in the second period is chosen as

$$(\sigma\beta_0 + (1-\sigma)\beta_1)\ln b_2$$

when $\sigma = 0$ corresponds to the model in the third part of the text. With the above settings, the results of the second-period consumption decisions are calculated as shown below.

$$\begin{cases} a_2^{**} = \frac{(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\alpha+\sigma\beta_0+(1-\sigma)\beta_1+\gamma)(\alpha+\beta_0)} a_2^{\text{ideal}} & \text{(39a)} \\ b_2^{**} = \frac{(\sigma\beta_0+(1-\sigma)\beta_1+\gamma)(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\beta+\gamma)(\alpha+\sigma\beta_0+(1-\sigma)\beta_1+\gamma)(\alpha+\beta_0)} b_2^{\text{ideal}} & \text{(39b)} \end{cases}$$

The calculation is more complicated, but the rest of the analysis is similar and will not be repeated.

It is important to note that the reason why $\alpha + \beta + \gamma = 1$ or $\alpha + \beta = 1$ cannot be defined in the model setting is that the consumer knows the parameters of his utility function, i.e., the person can estimate β once he/she knows α, which brings him back to the traditional model. In our model, the consumer does not know β because the consumer has no previous experience in the corresponding consumption, and the consumption is full of randomness, so the consumer can only estimate his utility function in various ways.

6.1.2. Discussion of the Classification of Consumption Scenarios

In Section 5 of the study, we divided the consumption scenarios into six groups of basic relationships with twenty-four types of consumption states due to the different parameter structures. However, their probability distributions were not discussed further. The probabilities of these categories are not equally distributed, and the probabilities of some classifications may be small, which can be estimated from images or data in an empirical study. Support for this paper's study of "the effect of decision utility bias and fluctuations in experienced utility on consumption outcomes" can be found in a related empirical study [28], as well as in a study of "fluctuations in decision utility" that they do not mention. In a subsequent study, we model the probabilities of the distribution of various states for specific distributions, which in turn can be used to analyze consumer behavior with a random utility model with subjective probabilities.

6.1.3. Third-Order Central Moment Problem with Skewed Distribution

If the distribution of decision utility and commodity experience is skewed, then at this point the third-order central moments can be retained for the further analysis of the mindset characteristics in consumption behavior and their effects during the approximation treatment in Section 4.

Results (25) and (26) of the expansion of the consumption decision volume for the second period are approximated to give

$$E[a_2^{**}] \approx \left(1 + \frac{D[\beta_1]}{(\alpha+\beta+\gamma)^2} - \frac{K[\beta_1]}{(\alpha+\beta+\gamma)^3}\right)\left(1 + \frac{\gamma}{\alpha+\beta+\gamma}\left(\frac{D[\beta_0]}{(\alpha+\beta)^2} - \frac{K[\beta_0]}{(\alpha+\beta)^3}\right)\right)a_2^{\text{ideal}} \tag{40a}$$

$$E[b_2^{**}] \approx \left(1 - \frac{\alpha}{\beta+\gamma}\left(\frac{D[\beta_1]}{(\alpha+\beta+\gamma)^2} - \frac{K[\beta_1]}{(\alpha+\beta+\gamma)^3}\right)\right)\left(1 + \frac{\gamma}{\alpha+\beta+\gamma}\left(\frac{D[\beta_0]}{(\alpha+\beta)^2} - \frac{K[\beta_0]}{(\alpha+\beta)^3}\right)\right)b_2^{\text{ideal}} \tag{40b}$$

where $K[\beta_0]$ and $K[\beta_1]$ denote the third-order central moments of β_0 and β_1, i.e., bias. It is also common to have biased psychological states, which manifest as consumers' asymmetric perceptions of certain consumer experience expectations, and this is a direction that can be expanded subsequently.

6.1.4. Discussion of Findings

Distinguishing the process of decision utility from experience utility occurrence in consumption behavior helps to enhance the theoretical explanatory power of behavioral economics, and introducing the randomness of both utilities and distinguishing randomness as originating from subjective psychological fluctuations and objective commodity experiences in order to analyze their different consumption effects is our first initiative. With this as a basis, we classify consumption behavior into 24 categories of universal significance, based on people's subjective perceptions and objective experiences, i.e., overestimation, underestimation, and psychological estimation of the stability of the mean and randomly realized values of utility (described by the variance of the variables of interest) and the random realization in the objective commodity experience (good or bad luck).

6.2. Conclusions

At the same time, these are the shortcomings of the current research on this topic, and this paper aims to fill this gap from a theoretical perspective; we will also explore

this theory empirically in future research. It is planned that the distribution of β_0, for example, can be obtained from a satisfaction survey with multiple repeated consumptions by conducting two questionnaires for a specific group of respondents.

In behavioral economics research, it is necessary to distinguish between intrinsic psychological factors (subjective biases and fluctuations) and extrinsic fluctuations in commodity experience. These fluctuations are not market fluctuations, as considered in traditional economics, but arise from the dual uncertainty of imperfectly rational psychological judgments and consumption processes. The real world is very complex, and the distribution probabilities, utility levels, and mean values of random factors such as the utility of psychological expectation decisions and consumption experience processes are difficult to predict, so consumers' imprecise expectations and random commodity consumption experiences will inevitably affect the consumers' purchase mix and make them deviate from the classical model explanation. Our model finds that consumers' "psychological expectation volatility" has a different or even opposite impact on consumers' "real experience volatility", as the former may trigger expansionary consumption, while the latter makes consumers more cautious. Therefore, in future studies on the utility of goods, it is important to consider not only the quantity of goods and relative consumption, but also the random volatility of consumer psychology and the market.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A

$$\begin{aligned}
E\left[b_1^*\right] &= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}E\left(\tfrac{(\beta_0+\alpha-\alpha)(1+\rho)}{(\alpha+\beta_0)} - \tfrac{\gamma}{\alpha+\beta}\tfrac{1}{1+(\beta_0-\beta)/(\alpha+\beta)}\right)\\
&= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}E\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta}\tfrac{1}{1+(\beta_0-\beta)/(\alpha+\beta)}\right)\\
&= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}E\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta}\left[1-\tfrac{\beta_0-\beta}{\alpha+\beta}+\tfrac{(\beta_0-\beta)^2}{(\alpha+\beta)^2}-\cdots\right]\right)\\
&= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta}\left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right]\right)\\
&= \tfrac{(\beta(1+\rho)-\gamma)(2+r)}{(\alpha+\beta)(2+\rho)(1+r)}\tfrac{m}{p_b} - \tfrac{(\alpha(1+\rho)+\gamma)(2+r)}{(\alpha+\beta)(2+\rho)(1+r)}\tfrac{m}{p_b}\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]
\end{aligned}$$

$$E[b_1^*] = \left(1-\frac{\alpha(1+\rho)+\gamma}{\beta(1+\rho)-\gamma}\sum_{n=1}^{\infty}E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right)b_1^{\text{ideal}} \tag{A1}$$

Noting that $E[\beta_1|\beta_0] = E[\beta_1] = \beta$ and first treating the initial expectation β_0 as given, we calculate the conditional expectation for the second period of commodity consumption.

$$\begin{aligned}
a_2^{**} &= \tfrac{1}{1+\frac{\beta_1-\beta}{\alpha+\beta+\gamma}}\tfrac{\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta+\gamma)(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_a}\\
&= \left[1-\tfrac{\beta_1-\beta}{\alpha+\beta+\gamma}+\tfrac{(\beta_1-\beta)^2}{(\alpha+\beta+\gamma)^2}-\cdots\right]\tfrac{\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta+\gamma)(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_a}
\end{aligned}$$

Then,

$$E[a_2^{**}|\beta_0] = \left[1+\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\frac{(\alpha+\beta_0+\gamma)\alpha(2+r)}{(\alpha+\beta+\gamma)(\alpha+\beta_0)(2+\rho)}\frac{m}{p_a} \tag{A2}$$

$$E[a_2^{**}|\beta_0] = \left[1+\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\left(\frac{(\alpha+\beta)}{(\alpha+\beta+\gamma)}+\frac{\gamma(\alpha+\beta)}{(\alpha+\beta_0)(\alpha+\beta+\gamma)}\right)a_2^{\text{ideal}} \tag{A3}$$

$$\begin{aligned} b_2^{**} &= \tfrac{(\alpha+\beta_1+\gamma-\alpha)(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} \\ &= \tfrac{(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} - \tfrac{\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_1+\gamma)(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} \\ &= \tfrac{(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} \\ &\quad -\left[1-\tfrac{\beta_1-\beta}{\alpha+\beta+\gamma}+\tfrac{(\beta_1-\beta)^2}{(\alpha+\beta+\gamma)^2}-\cdots\right]\tfrac{\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta+\gamma)(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} \end{aligned} \tag{A4}$$

$$\begin{aligned} E[b_2^{**}|\beta_0] &= \tfrac{(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} \\ &-\left[1+\textstyle\sum_{n=1}^{\infty} E\left[\tfrac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\tfrac{\alpha(\alpha+\beta_0+\gamma)(2+r)}{(\alpha+\beta+\gamma)(\alpha+\beta_0)(2+\rho)}\tfrac{m}{p_b} \\ &= \tfrac{(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\alpha+\beta_0)(\beta+\gamma)}b_2^{\text{ideal}} \\ &-\left[1+\textstyle\sum_{n=1}^{\infty} E\left[\tfrac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\tfrac{\alpha(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\alpha+\beta+\gamma)(\alpha+\beta_0)(\beta+\gamma)}b_2^{\text{ideal}} \end{aligned} \tag{A5}$$

Continue to expand and ask for expectations.

$$E[a_2^{**}] = \left[1+\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\left(1+\frac{\gamma}{\alpha+\beta+\gamma}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right)a_2^{\text{ideal}} \tag{A6}$$

$$\begin{aligned} E[b_2^{**}] &= E\begin{pmatrix} \frac{(\alpha+\beta)}{(\alpha+\beta+\gamma)}+\left(1-\frac{\beta_0-\beta}{\alpha+\beta}+\frac{(\beta_0-\beta)^2}{(\alpha+\beta)^2}-\ldots\right)\frac{\gamma}{(\alpha+\beta+\gamma)} \\ -\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]E\begin{pmatrix} \frac{\alpha(\alpha+\beta)}{(\alpha+\beta+\gamma)(\beta+\gamma)} \\ +\left(1-\frac{\beta_0-\beta}{\alpha+\beta}+\frac{(\beta_0-\beta)^2}{(\alpha+\beta)^2}\right)\frac{\alpha\gamma}{(\alpha+\beta+\gamma)(\beta+\gamma)} \end{pmatrix} \end{pmatrix} b_2^{\text{ideal}} \\ &= \begin{pmatrix} 1+\frac{\gamma}{(\alpha+\beta+\gamma)}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right] \\ -\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\left(\frac{\alpha}{\beta+\gamma}+\frac{\alpha\gamma}{(\alpha+\beta+\gamma)(\beta+\gamma)}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right) \end{pmatrix} b_2^{\text{ideal}} \\ &= \left(1-\frac{\alpha}{\beta+\gamma}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right)\left(1+\frac{\gamma}{(\alpha+\beta+\gamma)}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-\beta)^n}{(\alpha+\beta)^n}\right]\right)b_2^{\text{ideal}} \end{aligned} \tag{A7}$$

Appendix B

Proof. Taking n samples of x x_1, x_2, … x_n, the standard deviation of the samples of x squared is

$$\begin{aligned} \hat{\sigma}^2 &= \tfrac{\sum_{i=1}^{n}(x_i-\overline{x_i})^2}{n-1} = \tfrac{\sum_{i=1}^{n}(x_i-\frac{1}{n}\sum_{k=1}^{n}x_k)^2}{n-1} \\ &= \tfrac{\sum_{i=1}^{n}(x_i-\frac{1}{n}(x_i+\sum_{k\neq i}x_k))^2}{n-1} \\ &= \tfrac{\sum_{i=1}^{n}((n-1)x_i-\sum_{k\neq i}x_k)^2}{n^2(n-1)} \end{aligned}$$

For any i, viewing $x_k, k \neq i$ as an argument, the above equation is a quadratic function of x_i with the opening upward, so that the function takes its maximum value on the interval [0,1] at both ends of the interval, 0 or 1. For each i that this is the case, then the above

equation obtains its overall maximum value when x_i is either 0 or 1. Without loss of generality, suppose that there are m values taken as 1 and *n-m* values taken as 0, then

$$\begin{aligned}
&\overline{x_i} = \tfrac{m}{n} \\
&\hat{\sigma}^2 = \tfrac{\sum_{i=1}^{n}(x_i-\frac{m}{n})^2}{n-1} = \tfrac{\sum_{i=1}^{n}(n^2x_i^2-2nmx_i+m^2)}{n^2(n-1)} = \tfrac{n^2\sum_{i=1}^{n}x_i^2-2nm\sum_{i=1}^{n}x_i+nm^2}{n^2(n-1)} \\
&\quad = \tfrac{n^2m-2nm^2+nm^2}{n^2(n-1)} = \tfrac{m(n-m)}{n(n-1)} \le \left(\tfrac{m+n-m}{2}\right)^2\tfrac{1}{n(n-1)} = \tfrac{n}{4(n-1)} \\
&\sigma^2 = \lim_{n\to\infty}\hat{\sigma}^2 \le \lim_{n\to\infty}\tfrac{n}{4(n-1)} = \tfrac{1}{4}
\end{aligned}$$

□

Appendix C

Estimation is biased by assuming $bais = E[\beta_0] - \beta$, Taylor expansions, and ignoring higher-order minima.

$$\begin{aligned}
E\left[\tfrac{1}{\alpha+\beta_0}\right] &= E\left[\tfrac{1}{\alpha+\beta+bais+\beta_0-E[\beta_0]}\right] = \tfrac{1}{\alpha+\beta+bais}E\left[\left(1+\tfrac{\beta_0-E[\beta_0]}{\alpha+\beta+bais}\right)^{-1}\right] \\
&= \tfrac{1}{\alpha+\beta+bais}\left(1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bais)^n}\right]\right) \\
&\approx \tfrac{1}{\alpha+\beta+bais}\left(1+\tfrac{D[\beta_0]}{(\alpha+\beta+bais)^2}\right)
\end{aligned}$$

$$\begin{aligned}
E[b_1^*] &= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}E\left(\tfrac{(\beta_0(1+\rho)-\gamma)}{(\alpha+\beta_0)}\right) \\
&= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}E\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta+bias}\tfrac{1}{1+\frac{\beta_0-\beta-bias}{\alpha+\beta+bias}}\right) \\
&= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}E\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta+bias}\left[1-\tfrac{\beta_0-E[\beta_0]}{\alpha+\beta+bias}+\tfrac{(\beta_0-E[\beta_0])^2}{(\alpha+bias)^2}-\cdots\right]\right) \\
&= \tfrac{(2+r)}{(2+\rho)(1+r)}\tfrac{m}{p_b}\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta+bias}\left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bias)^n}\right]\right]\right) \\
&= \tfrac{(\alpha+\beta)}{(\beta(1+\rho)-\gamma)}\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta+bias}\left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bias)^n}\right]\right]\right)b_1^{\text{ideal}} \\
&= \tfrac{(\alpha+\beta)}{(\beta(1+\rho)-\gamma)}\left(1+\rho-\tfrac{\alpha(1+\rho)+\gamma}{\alpha+\beta+bias}\left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bias)^n}\right]\right]\right)b_1^{\text{ideal}} \\
&= \begin{pmatrix} \tfrac{\alpha+\beta}{(1+\rho)\beta-\gamma}\tfrac{(1+\rho)(\beta+bias)-\gamma}{\alpha+\beta+bias} \\ -\tfrac{(\alpha+\beta)(\alpha(1+\rho)+\gamma)}{(\beta(1+\rho)-\gamma)(\alpha+\beta+bias)}\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bias)^n}\right] \end{pmatrix}b_1^{\text{ideal}}
\end{aligned}$$

$$E[a_2^{**}|\beta_0] = \left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\left(\tfrac{(\alpha+\beta)}{(\alpha+\beta+\gamma)}+\tfrac{\gamma(\alpha+\beta)}{(\alpha+\beta_0)(\alpha+\beta+\gamma)}\right)a_2^{\text{ideal}}$$

$$\begin{aligned}
E[a_2^{**}] &= \left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\begin{pmatrix} \tfrac{(\alpha+\beta)}{(\alpha+\beta+\gamma)} \\ \tfrac{\gamma(\alpha+\beta)E\left[\left(1+\frac{\beta_0-E[\beta_0]}{\alpha+\beta+bais}\right)^{-1}\right]}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)} \end{pmatrix}a_2^{\text{ideal}} \\
&= \left[1+\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\begin{pmatrix} \tfrac{(\alpha+\beta)(\alpha+\beta+\gamma+bais)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)} \\ +\tfrac{\gamma(\alpha+\beta)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)}\textstyle\sum_{n=1}^{\infty}E\left[\tfrac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bais)^n}\right] \end{pmatrix}a_2^{\text{ideal}}
\end{aligned}$$

$$
\begin{aligned}
&E[b_2^{**}|\beta_0] = \left(1 - \frac{\alpha}{\alpha+\beta+\gamma}\left[1 + \sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right]\right)\frac{(\alpha+\beta_0+\gamma)(\alpha+\beta)}{(\alpha+\beta_0)(\beta+\gamma)} b_2^{\text{ideal}} \\
&= \frac{(\alpha+\beta)}{(\beta+\gamma)}\left(\frac{\beta+\gamma}{\alpha+\beta+\gamma} - \frac{\alpha}{\alpha+\beta+\gamma}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right)\left(1 + \frac{\gamma}{\alpha+\beta_0}\right) b_2^{\text{ideal}} \\
&= \frac{(\alpha+\beta)}{(\beta+\gamma)(\alpha+\beta+\gamma)}\begin{pmatrix} \beta+\gamma \\ -\alpha\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right] \end{pmatrix}\left(1 + \frac{\frac{\gamma}{\alpha+\beta+bais}}{1+\frac{\beta_0-E[\beta_0]}{\alpha+\beta+bais}}\right) b_2^{\text{ideal}} \\
&= \frac{(\alpha+\beta)}{(\beta+\gamma)(\alpha+\beta+\gamma)}\left(\beta+\gamma-\alpha\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right) \\
&\cdot\left(1 + \frac{\gamma}{\alpha+\beta+bais}\left(1 + \sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bais)^n}\right]\right)\right) b_2^{\text{ideal}} \\
&= \frac{(\alpha+\beta)}{(\beta+\gamma)(\alpha+\beta+\gamma)}\left(\beta+\gamma-\alpha\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right) \\
&\cdot\begin{pmatrix} \frac{\alpha+\beta+bais+\gamma}{\alpha+\beta+bais} \\ +\frac{\gamma}{\alpha+\beta+bais}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bais)^n}\right] \end{pmatrix} b_2^{\text{ideal}} \\
&= \frac{(\alpha+\beta)(\alpha+\beta+bais+\gamma)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)}\left(1 - \frac{\alpha}{\beta+\gamma}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_1-\beta)^n}{(\alpha+\beta+\gamma)^n}\right]\right) \\
&\cdot\left(1 + \frac{\gamma(\alpha+\beta+bais)}{(\alpha+\beta+bais+\gamma)(\alpha+\beta+bais)}\sum_{n=1}^{\infty} E\left[\frac{(-1)^n(\beta_0-E[\beta_0])^n}{(\alpha+\beta+bais)^n}\right]\right) b_2^{\text{ideal}}
\end{aligned}
$$

The above result, neglecting higher order minima yields

$$
\begin{cases}
E\left[a_1^*\right] \approx \frac{\alpha+\beta}{\alpha+\beta+bais}\left(1 + \frac{D[\beta_0]}{(\alpha+\beta+bais)^2}\right) a_1^{\text{ideal}} \\
E\left[b_1^*\right] \approx \left(\frac{\alpha+\beta}{(1+\rho)\beta-\gamma}\frac{(1+\rho)(\beta+bias)-\gamma}{\alpha+\beta+bias} - \frac{(\alpha+\beta)(\alpha(1+\rho)+\gamma)}{\beta(1+\rho)-\gamma}\frac{D[\beta_0]}{(\alpha+\beta+bais)^3}\right) b_1^{\text{ideal}} \\
E[a_2^{**}] \approx E[a_2^{**}] \approx \left[1 + \frac{D[\beta_1]}{(\alpha+\beta+\gamma)^2}\right]\begin{pmatrix} \frac{(\alpha+\beta)(\alpha+\beta+\gamma+bais)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)} \\ +\frac{\gamma(\alpha+\beta)}{(\alpha+\beta+\gamma)(\alpha+\beta+bais)}\frac{D[\beta_0]}{(\alpha+\beta+bais)^2} \end{pmatrix} a_2^{\text{ideal}} \\
E[b_2^{**}] \approx \frac{(\alpha+\beta)(\alpha+\beta+bais+\gamma)}{(\alpha+\beta+bais)(\alpha+\beta+\gamma)}\left(1 - \frac{(\beta+\gamma)^{-1}\alpha D[\beta_1]}{(\alpha+\beta+\gamma)^2}\right)\left(1 + \frac{(\alpha+\beta+bais+\gamma)^{-1}\gamma D[\beta_0]}{(\alpha+\beta+bais)^2}\right) b_2^{\text{ideal}}
\end{cases}
$$

Appendix D

Table A1. Parameter relationships and random consumption states.

Parameter Structure	Basic State (Extremely Information-Independent Group)	Accompanying States
$\beta_0 < E[\beta_0] < \beta < \beta_1$	Good luck, systematic underestimation, contingent subjective underestimation	Serendipity objective underestimation, serendipity super-habitual realization, serendipity underestimation
$\beta_0 < E[\beta_0] < \beta_1 < \beta$	Serendipity subjective underestimation, serendipity super-habitual realization, bad luck	Serendipitous objective underestimation, contingent underestimation, systematic underestimation
$\beta_0 < \beta < E[\beta_0] < \beta_1$	Serendipitous objective underestimation, systematic overestimation, and serendipitous super-habitual realization	Serendipity underestimation, good luck, serendipity subjective underestimation
$\beta_0 < \beta < \beta_1 < E[\beta_0]$	Serendipity objective underestimation, good luck, serendipity low habit realization	Contingent underestimation, systematic overestimation, and contingent subjective underestimation
$\beta_0 < \beta_1 < \beta < E[\beta_0]$	Serendipitous undervaluation, bad luck, systematic overvaluation	Serendipity objective underestimation, serendipity low habit realization, serendipity subjective underestimation
$\beta_0 < \beta_1 < E[\beta_0] < \beta$	Serendipitous underestimation, serendipitous low habit realization, systematic underestimation	Objective underestimation of chance, subjective underestimation of chance, bad luck
$\beta_1 < E[\beta_0] < \beta < \beta_0$	Serendipitous low habit realization, systematic underestimation, and serendipitous objective overestimation	Serendipitous overestimation, bad luck, serendipitous subjective overestimation

Table A1. *Cont.*

Parameter Structure	Basic State (Extremely Information-Independent Group)	Accompanying States
$\beta_1 < E[\beta_0] < \beta_0 < \beta$	Serendipitous low habit realization, serendipitous subjective overestimation, serendipitous objective underestimation	Systematic undervaluation, contingent overvaluation, bad luck
$\beta_1 < \beta < E[\beta_0] < \beta_0$	Bad luck, systematic overvaluation, fortuitous subjective overvaluation	Serendipitous overestimation, serendipitous low habit realization, serendipitous objective overestimation
$\beta_1 < \beta < \beta_0 < E[\beta_0]$	Bad luck, chance objective overestimation, chance subjective underestimation	Serendipitous overestimation, serendipitous low habit realization, systematic overestimation
$\beta_1 < \beta_0 < \beta < E[\beta_0]$	Contingent overestimation, contingent objective underestimation, systematic overestimation	Serendipity low habit realization, bad luck, serendipity subjective underestimation
$\beta_1 < \beta_0 < E[\beta_0] < \beta$	Contingent overestimation, contingent subjective underestimation, systematic underestimation	Serendipity objective underestimation, serendipity low habit realization, bad luck
$E[\beta_0] < \beta_0 < \beta < \beta_1$	Serendipity subjective overestimation, good luck consumption, serendipity objective underestimation	Serendipitous underestimation, systematic underestimation, serendipitous super-habitual realization
$E[\beta_0] < \beta_0 < \beta_1 < \beta$	Serendipity subjective overestimation, serendipity underestimation, bad luck	Serendipitous objective underestimation, systematic underestimation, serendipitous super-habitual realization
$E[\beta_0] < \beta < \beta_0 < \beta_1$	Systematic Underestimation, Contingent Objective Overestimation, Contingent Underestimation	Good luck, serendipity subjective overestimation, serendipity super habitual realization
$E[\beta_0] < \beta < \beta_1 < \beta_0$	Systematic undervaluation, contingent overvaluation, good luck	Serendipitous subjective overestimation, serendipitous objective overestimation, serendipitous super-habitual realization
$E[\beta_0] < \beta_1 < \beta < \beta_0$	Serendipity super-habit realization, bad luck, objective overestimation of serendipity	Systematic underestimation, contingent overestimation, and contingent subjective overestimation
$E[\beta_0] < \beta_1 < \beta_0 < \beta$	Serendipity super-habit realization, serendipity overestimation, serendipity objective underestimation	Systematic underestimation, bad luck, serendipitous subjective overestimation
$\beta < E[\beta_0] < \beta_0 < \beta_1$	Systematic overestimation, contingent subjective overestimation, contingent underestimation	Good luck, chance objective overestimation, chance super habit realization
$\beta < E[\beta_0] < \beta_1 < \beta_0$	systematic overestimation, accidental super-habitual realization, serendipitous overestimation, and	Good luck, subjective overestimation by chance, objective overestimation by chance
$\beta < \beta_0 < E[\beta_0] < \beta_1$	Objective overestimation of chance, subjective underestimation of chance, and realization of chance over habit	Serendipitous undervaluation, good luck, systematic overvaluation
$\beta < \beta_0 < \beta_1 < E[\beta_0]$	Chance objective overestimation, chance underestimation, chance low habit realization	Good luck, systematic overestimation, contingent subjective underestimation
$\beta < \beta_1 < \beta_0 < E[\beta_0]$	Good luck, serendipitous overestimation, serendipitous subjective underestimation	Serendipitous low habit realization, systematic overestimation, serendipitous objective overestimation
$\beta < \beta_1 < E[\beta_0] < \beta_0$	Good luck, serendipitous low habit realization, serendipitous subjective overestimation	Serendipitous overestimation, systematic overestimation, and serendipitous objective overestimation

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