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Finite-Time Bounded Tracking Control for a Class of Neutral Systems

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Abstract: In this paper, we investigate finite-time bounded (FTB) tracking control for a class of neutral systems. Firstly, the dynamic equation of the tracking error signal is given based on the original neutral system. Then, we combine it with the equations of the state vector to construct an error system, where the reference signal and the disturbance signal are fused in a new vector. Next, about the error system, we study the input–output finite-time stability problem of the closed-loop system by utilizing the Lyapunov–Krasovskii functional. We also give a finite-time stability condition in the form of linear matrix inequalities (LMIs). Furthermore, the delay-dependent and delay-independent finite-time bounded tracking controllers are designed separately for the original system. Finally, a realistic example is given to show the effectiveness of the controller design method in the paper.

Keywords: finite-time bounded tracking; linear matrix inequalities (LMIs); Lyapunov–Krasovskii functional; neutral systems

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1. Introduction

As time tends to infinity, asymptotic stability can guarantee the asymptotic convergence of tracking errors and state trajectories. So it has always been one of the focuses in the control research [1,2]. However, in practice, it is often required that the stability is reached in a desired time interval, not in infinite time. Thus, the research of finite-time stability emerged as the actuality requires. As an important role in the study of the transient behavior of control systems, finite-time stability can help to improve the control precision, achieve better anti-interference and robustness over a time interval [3–5]. Therefore, finite-time stability has drawn considerable attention, and many results have been developed during the past decades [6–9].

Early in 1961, Dorato proposed the short-time stability in the stability analysis of linear time-varying systems [10]. Finite-time stability (FTS) was firstly put forward by Weiss and Infante during the research on the stability of nonlinear systems in 1967 [11]. Moulay et al. [12] inspired FTS by the theory of differential equations for systems with input delay and state delay. Furthermore, they obtained the responding finite-time controllers, respectively, for scalar linear systems and for the chain of integrators with input delay. Artstein's model reduction was extended to nonlinear feedback for handling input delays [12]. Some new methods and tools are used to analyze FTS. For example, the polynomial Lyapunov functional approach and the sum-of-squares (SOS) theory were used to investigate FTS of continuous-time polynomial fuzzy systems, and all the proposed conditions are given in the form of SOS [13].

With the development of FTS, the finite-time control problem for various systems has also been widely studied [4,14–18]. Fu and Xu proposed a terminal sliding mode disturbance observer to estimate the augmented disturbance. Additionally, they came up with an algorithm to guarantee fast convergence of the tracking error for a class of MIMO nonlinear systems [19]. However, in [17,20], the authors regarded the output as a component of states, which is not common in control systems. Finite-time bounded (FTB) tracking, belonging to finite-time control, was valued in recent years [6,21–23]. It was proposed on the basis of input–output FTS by Amato et al. [24]. FTB tracking is such that, given a bound on the initial condition, the output does not exceed a certain threshold during a specified time interval. Amato et al. extended input–output FTS to impulse systems and linear time-varying systems, and they gave sufficient conditions to ensure input–output FTS timewith different types of input [25–28].

However, there are few research results related to FTB tracking control for neutral-type systems. On one hand, the achievements on FTB tracking control focus on ordinary time-delay systems and fractional-order systems [21,23]. On the other hand, studies of neutral systems concentrate on H_∞ control, guaranteed cost control and observer-based control [29–33]. Additionally, although Ref. [34] studied FTB for a class of neutral systems, it only synthesized an observer but not a tracking controller.

In the paper, we investigate FTB tracking control for a class of neutral systems with disturbance. To figure out this problem, the input–output FTS of the error system is studied. Using a Lyapunov–Krasovskii functional, we obtain two FTS criteria and a FTB corollary in terms of LMIs. On the basis of these FTS or FTB conclusions, we design a controller based on time-delay state feedback for the error system. Thus, the delay-dependent and -independent FTB tracking controllers for the original neutral system are given.

The paper is organized as follows. Section 2 is devoted to problem statements and necessary descriptions of definitions, assumptions, and lemmas. The main results for stability analysis and control design are shown in Section 3. In Section 4, we give a simulation example to show that the designed controllers are effective. Finally, the conclusion is obtained in Section 5.

2. Preliminaries

In the sequel, the following notations are used. $\mathbf{R}^n$ denotes n-dimensional Euclidean space. $\mathbf{R}^{n\times m}$ is the set of all $n \times m$ real matrices. $*$ refers to the symmetric part of a matrix A. A^T is the transpose matrix of A. $P > 0$ represents that P is a positive definite matrix. I denotes an identity matrix with appropriate dimensions.

2.1. Problem Statements

Consider the following neutral system:

$$\begin{cases} \dot{x}(t) - G\dot{x}(t-\tau) = Ax(t) + A_1x(t-\tau) + Bu(t) + Ew(t), \\ y(t) = Cx(t), t \geq 0, \\ x(t) = \phi(t), t \in [-\tau, 0]. \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$, $y(t) \in \mathbf{R}^p$ and $w(t) \in \mathbf{R}^q$ are the state, the input, the output, and the disturbance vector, respectively. $\tau > 0$ denotes the constant time delay, which appears in both the state and the derivative terms of (1). $A \in \mathbf{R}^{n\times n}$, $A_1 \in \mathbf{R}^{n\times n}$, $C \in \mathbf{R}^{p\times n}$, $G \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times m}$, and $E \in \mathbf{R}^{n\times q}$ are known matrices. The initial condition $\phi(t)$ is a continuous vector-valued initial function of $t \in [-\tau, 0]$. Define the operator $\mathcal{D}x(t) = x(t) - Gx(t-\tau)$.

Let $y_d(t) \in \mathbf{R}^p$ be the reference signal. Then the tracking error is defined as

$$e(t) = y(t) - y_d(t). \quad (2)$$

For $\forall t \in [-\tau, 0]$, we define $y_d(t) = \eta(t)$, where $\eta(t)$ is a continuous vector-valued initial function. Then the initial condition of tracking error is $e(t) = C\phi(t) - \eta(t)$.

The purpose of this paper is to investigate the finite-time bounded tracking of system (1). Finite-time bounded tracking means that the output $y(t)$ is always in some neighborhood of $y_d(t)$. It is the promotion of input–output finite-time stability, which is described as follows.

Definition 1 ([24])**.** *System* (1) *with $u(t) = 0$ is said to be input–output finite-time stable with respect to (c_1, c_2, Γ, T), where the scalars $c_1 > 0$, $c_2 > 0$, $T > 0$, and $\Gamma > 0$, if under the zero initial condition $\phi(t) = 0, \forall t \in [-\tau, 0]$, system*(1) *satisfies*

$$\int_0^T w^T(t)w(t)\mathrm{d}t \le c_1^2 \Rightarrow y^T(t)\Gamma y(t) \le c_2^2, \forall t \in [0, T].$$

Based on Definition 1, the definition of finite-time bounded tracking is shown as follows.

Definition 2 ([23])**.** *Given scalars$c_1 > 0$, $c_2 > 0$, $T > 0$, and the weight matrix $\Gamma > 0$, system* (1) *is said to achieve the finite-time bounded tracking as for $y_d(t)$ with respect to (c_1, c_2, Γ, T) , if under the zero-initial condition*

$$\phi(t) = 0, e(t) = C\phi(t) - \eta(t) = 0, \forall t \in [-\tau, 0],$$

system (1) *satisfies*

$$\int_0^T w^T(t)w(t)\mathrm{d}t \le c_1^2 \Rightarrow e^T(t)\Gamma e(t) \le c_2^2, \forall t \in [0, T].$$

2.2. Some Related Assumptions and Lemmas

To acquire the desired controller, the following assumptions and lemmas are employed in the subsequent developments.

Assumption 1. $|\lambda_i(G)| < 1 (i = 1, 2, \ldots, n)$, *where $\lambda_i(G)$ is the ith eigenvalue of matrix G.*

Remark 1. *When $||G|| < 1$, the nominal system of system* (1) *is stable, i.e., system (1) with $u(t) = w(t) = 0$ converges to zero. We also call the operator $\mathcal{D}x(t) = x(t) - Gx(t - \tau)$ stable. If $|\lambda_i(G)| < 1 (i = 1, 2, \ldots, n)$, we can easily obtain that $||G|| < 1$ because $|\lambda_i(G)| < ||G||$. So $|\lambda_i(G)| < 1$ can ensure $\mathcal{D}x(t) = x(t) - Gx(t - \tau)$ is stable [35,36].*

Assumption 2. *The reference signal $y_d(t)$ satisfies*

$$\int_0^T \dot{y}_d^T(t)\dot{y}_d(t)\mathrm{d}t \le c^2,$$

where c is a given positive constant.

Assumption 3. *The disturbance $w(t)$ satisfies*

$$\int_0^T \dot{w}^T(t)\dot{w}(t)\mathrm{d}t \le d^2,$$

where d is a given positive constant.

Lemma 1 ([37])**.** *Given constant symmetric matrices Ω_1, Ω_2 and Ω_3, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$, the matrix inequality*

$$\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$$

holds, if and only if

$$\begin{bmatrix} \Omega_1 & {\Omega_3}^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0, \text{ or } \begin{bmatrix} -\Omega_2 & {\Omega_3}^T \\ \Omega_3 & \Omega_1 \end{bmatrix} < 0.$$

Lemma 2 ([38])**.** *For any scalar $\tau > 0$, a positive definite matrix $M \in \mathbf{R}^{n\times n}$ and $x(t) \in \mathbf{R}^n$, the following inequality holds:*

$$-\int_{t-\tau}^{t} \dot{x}^T(s) M \dot{x}(s) \mathrm{d}s \le \frac{1}{\tau} \begin{bmatrix} x^T(t) & x^T(t-\tau) \end{bmatrix} \begin{bmatrix} -M & M \\ M & -M \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}.$$

3. Main Results

In this section, we derive sufficient conditions on finite-time bounded tracking of system (1). Firstly, an error system is constructed. Then, the finite-time stability criterion of the error system is proposed.

3.1. Construction of an Error System

According to Equation (2), deriving $e(t) - ae(t-\tau)$ yields

$$\dot{e}(t) - a\dot{e}(t-\tau) = C\dot{x}(t) - aC\dot{x}(t-\tau) - \dot{y}_d(t) + a\dot{y}_d(t-\tau), \tag{3}$$

where a is a constant, and $|a| < 1$.

Making the derivative on both sides of system (1), we can obtain

$$\ddot{x}(t) - G\ddot{x}(t-\tau) = A\dot{x}(t) + A_1\dot{x}(t-\tau) + B\dot{u}(t) + E\dot{w}(t). \tag{4}$$

Denote $X(t) = \begin{bmatrix} e(t) \\ \dot{x}(t) \end{bmatrix}$. By Equations (1), (3) and (4), it follows that

$$\begin{cases} \dot{X}(t) - G_0\dot{X}(t-\tau) = A_0X(t) + A_{01}X(t-\tau) + B_0\dot{u}(t) + E_0\dot{w}(t) - G_d[\dot{y}_d(t) - a\dot{y}_d(t-\tau)], \\ e(t) = C_0X(t), \\ X(t) = \phi_0(t), t \in [-\tau, 0]. \end{cases} \tag{5}$$

where

$$G_0 = \begin{bmatrix} aI & 0 \\ 0 & G \end{bmatrix}, A_0 = \begin{bmatrix} 0 & C \\ 0 & A \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ B \end{bmatrix},$$

$$A_{01} = \begin{bmatrix} 0 & -aC \\ 0 & A_1 \end{bmatrix}, E_0 = \begin{bmatrix} 0 \\ E \end{bmatrix}, G_d = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

$$C_0 = \begin{bmatrix} I & 0 \end{bmatrix}, \phi_0(t) = \begin{bmatrix} C\phi(t) - \eta(t) \\ \dot{\phi}(t) \end{bmatrix}.$$

Based on the construction of G_0 in system (5), the eigenvalues of G_0 are respectively the eigenvalues of G and a. For $|a| < 1$ and $|\lambda_i(G)| < 1 (i = 1, 2, \ldots, n)$ in Assumption 1, it is easy to obtain that $|\lambda_j(G_0)| < 1 (j = 1, 2, \ldots, n + p)$. According to Remark 1, the operator $\dot{X}(t) - G_0\dot{X}(t-\tau)$ is stable.

Letting $W(t) = \begin{bmatrix} \dot{y}_d(t) \\ \dot{y}_d(t-\tau) \\ \dot{w}(t) \end{bmatrix}$, we can receive the error systems as follows:

$$\begin{cases} \dot{X}(t) - G_0\dot{X}(t-\tau) = A_0X(t) + A_{01}X(t-\tau) + B_0\dot{u}(t) + G_W W(t), \\ e(t) = C_0X(t), \\ X(t) = \phi_0(t), t \in [-\tau, 0], \end{cases} \tag{6}$$

where $G_W = \begin{bmatrix} -G_d & aG_d & E_0 \end{bmatrix}$. Obviously, system (6) has the same form as system (1), and $W(t)$ can be regarded as the disturbance. Furthermore, if the closed-loop of system (6) realizes finite-time tracking control with respect to $e(t)$, then the output vector $y(t)$ can track $y_d(t)$ in finite time.

Remark 2. *It can be found that $W(t)$ in system (6) contains the derivatives of the disturbance signal $\dot{w}(t)$ of system (1), instead of $w(t)$. Since the input–output finite-time stability of system (6) is the object of our discussion, we put constraints on $\dot{y}_d(t)$ and $\dot{w}(t)$ in Assumptions 2 and 3, rather than on $y_d(t)$ or $w(t)$ as described in Definition 1.*

3.2. Finite-Time Bounded Tracking Control of the Error System

Design the delay-dependent feedback controller for system (6) with the form of

$$\dot{u}(t) = K_1X(t) + K_2X(t-\tau),$$

where K_1 and K_2 are the feedback gain matrices with appropriate dimensions. Then the closed-loop of system (6) is

$$\begin{cases} \dot{X}(t) - G_0\dot{X}(t-\tau) = \hat{A}_0X(t) + \hat{A}_{01}X(t-\tau) + G_W W(t), \\ e(t) = C_0X(t), \\ X(t) = \phi_0(t), t \in [-\tau, 0]. \end{cases} \tag{7}$$

where $\hat{A}_0 = A_0 + B_0K_1, \hat{A}_{01} = A_{01} + B_0K_2$.

For system (7), we have the following theorem.

Theorem 1. *Under Assumptions 2 and 3, for a given scalar $\gamma \geq 0$, if there exist matrices $R > 0, P_i > 0 (i = 1, 2, 3, 4)$ satisfying*

$$\begin{bmatrix} \hat{A}_0^{\mathrm{T}}P_1 + P_1\hat{A}_0 - \gamma P_1 + P_3 - \frac{1}{\tau}P_4 & P_1\hat{A}_{01} + \frac{1}{\tau}P_4 & P_1G_W & P_1G_0 & \tau\hat{A}_0^{\mathrm{T}}P_4 \\ \hat{A}_{01}^{\mathrm{T}}P_1 + \frac{1}{\tau}P_4 & -\frac{1}{\tau}P_4 & 0 & 0 & \tau\hat{A}_{01}^{\mathrm{T}}P_4 \\ G_W^{\mathrm{T}}P_1 & 0 & -R & 0 & \tau G_W^{\mathrm{T}}P_4 \\ G_0^{\mathrm{T}}P_1 & 0 & 0 & -P_2 & \tau G_0^{\mathrm{T}}P_4 \\ \tau P_4\hat{A}_0 & \tau P_4\hat{A}_{01} & \tau P_4G_W & \tau P_4G_0 & -\tau P_4 \end{bmatrix} < 0 \tag{8}$$

$$C_0{}^T\Gamma C_0 \leq P_1, \tag{9}$$

$$R \leq \frac{c_2^2}{c_1^2\exp(\gamma T)}I, \tag{10}$$

then $e^T(t)\Gamma e(t) \leq c_2^2$ $(\forall t \in [0, T])$ holds, i.e., system (7) is input–output finite time with respect to (c_1, c_2, Γ, T), where $c_1^2 \geq 2c^2 + d^2$.

Proof. For system (7), a Lyapunov–Krasovskii functional candidate is given by

$$V = V_1 + V_2 + V_3 + V_4, \tag{11}$$

where

$$V_1 = X^T(t)P_1X(t), \tag{12}$$

$$V_2 = \int_{t-\tau}^{t} \dot{X}^T(s)P_2\dot{X}(s)\mathrm{d}s, \tag{13}$$

$$V_3 = \int_{t-\tau}^{t} e^{\gamma(t-\theta)}X^T(\theta)P_3X(\theta)\mathrm{d}\theta, \tag{14}$$

$$V_4 = \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{X}^T(s)P_4\dot{X}(s)\mathrm{d}s\mathrm{d}\theta \tag{15}$$

with $P_i > 0$ $(i = 1,2,3,4)$. Equations (12)–(14) are the common forms of the Lyapynov–Krasovskii functional for time-delay continuous systems, and Equation (15) is designed for neutral systems. Taking the time derivatives of Equations (12)–(15), respectively, along the trajectory of system (7) yields

$$\begin{aligned}\dot{V}_1 =& \dot{X}^{\mathrm{T}}(t)P_1X(t) + X^{\mathrm{T}}(t)P_1\dot{X}(t)\\ =& X^{\mathrm{T}}(t)\left[\hat{A}_0^{\mathrm{T}}P_1 + P_1\hat{A}_0\right]X(t) + X^{\mathrm{T}}(t-\tau)\hat{A}_{01}^{\mathrm{T}}P_1X(t) + W^{\mathrm{T}}(t)G_W^{\mathrm{T}}P_1X(t)\\ &+ \dot{X}^{\mathrm{T}}(t-\tau)G_0^{\mathrm{T}}P_1X(t) + X^{\mathrm{T}}(t)P_1\hat{A}_{01}X(t-\tau) + X^{\mathrm{T}}(t)P_1G_WW(t) + X^{\mathrm{T}}(t)P_1G_0\dot{X}(t-\tau)\end{aligned} \tag{16}$$

$$\dot{V}_2 = \dot{X}^T(t)P_2\dot{X}(t) - \dot{X}^T(t-\tau)P_2\dot{X}(t-\tau), \tag{17}$$

$$\begin{aligned}\dot{V}_3 &= \gamma\int_{t-\tau}^{t} e^{\gamma(t-s)}X^T(\theta)P_3X(\theta)\mathrm{d}\theta + X^T(t)P_3X(t) - e^{\gamma\tau}X^T(t-\tau)P_3X(t-\tau)\\ &\le \gamma\int_{t-\tau}^{t} e^{\gamma(t-s)}X^T(\theta)P_3X(\theta)\mathrm{d}\theta + X^T(t)P_3X(t),\end{aligned} \tag{18}$$

Let $\dot{F}(s) = \dot{X}^{\mathrm{T}}(s)P_4\dot{X}(s)$, then

$$\begin{aligned}V_4 = \textstyle\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{F}(s)\mathrm{d}s\mathrm{d}\theta &= \textstyle\int_{-\tau}^{0}(F(t) - F(t+\theta))\mathrm{d}\theta\\ &= \tau F(t) - \textstyle\int_{-\tau}^{0}F(t+\theta)\mathrm{d}\theta\\ &= \tau F(t) - \textstyle\int_{t-\tau}^{t}F(l)\mathrm{d}l.\end{aligned}$$

So

$$\dot{V}_4 = \tau\dot{F}(t) - (F(t) - F(t-\tau)) = \tau\dot{F}(t) - \int_{t-\tau}^{t}\dot{F}(s)\mathrm{d}s = \tau\dot{F}(t) - \int_{t-\tau}^{t}\dot{X}^{\mathrm{T}}(s)P_4\dot{X}(s)\mathrm{d}s. \tag{19}$$

Denote $\xi = \begin{bmatrix} X(t)\\ X(t-\tau)\\ W(t)\\ \dot{X}(t-\tau)\end{bmatrix}.$

Based on Equations (16), (17) and (19), and inequality (18), we can obtain

$$\begin{aligned}\dot{V} \le& X^{\mathrm{T}}(t)\left(\hat{A}_0^{\mathrm{T}}P_1 + P_1\hat{A}_0 + P_3\right)X(t) + X^{\mathrm{T}}(t)P_1\hat{A}_{01}X(t-\tau) + X^{\mathrm{T}}(t)P_1G_WW(t)\\ &+ X^{\mathrm{T}}(t)P_1G_0\dot{X}(t-\tau) + X^{\mathrm{T}}(t-\tau)\hat{A}_{01}^{\mathrm{T}}PX(t) + W^T(t)G_W^{\mathrm{T}}P_1X(t) + \dot{X}^{\mathrm{T}}(t-\tau)G_0^{\mathrm{T}}P_1X(t)\\ &- \dot{X}^{\mathrm{T}}(t-\tau)P_2\dot{X}(t-\tau) + \gamma\int_{t-\tau}^{t} e^{\gamma(t-s)}X^{\mathrm{T}}(\theta)P_3X(\theta)\mathrm{d}\theta\\ &+ \tau\xi^{\mathrm{T}}(t)\begin{bmatrix}\hat{A}_0 & \hat{A}_{01} & G_W & G_0\end{bmatrix}^{\mathrm{T}}P_4\begin{bmatrix}\hat{A}_0 & \hat{A}_{01} & G_W & G_0\end{bmatrix}\xi(t) - \int_{t-\tau}^{t}\dot{X}^{\mathrm{T}}(s)P_4\dot{X}(s)\mathrm{d}s\end{aligned} \tag{20}$$

According to Lemma 2, we have

$$
\begin{aligned}
-\int_{t-\tau}^{t} \dot{X}^T(s) P_4 \dot{X}(s) \mathrm{d}s &\leq \frac{1}{\tau} \begin{bmatrix} X^T(t) \\ X^T(t-\tau) \end{bmatrix}^T \begin{bmatrix} -P_4 & P_4 \\ P_4 & -P_4 \end{bmatrix} \begin{bmatrix} X(t) \\ X(t-\tau) \end{bmatrix} \\
&= \frac{1}{\tau} \xi^T(t) \begin{bmatrix} -P_4 & P_4 & 0 & 0 \\ P_4 & -P_4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xi(t).
\end{aligned} \tag{21}
$$

Combining inequality (21) with inequality (20), we have

$$
\dot{V} \leq \xi^T(t) \Omega_1 \xi(t) + \tau \xi^T(t) \begin{bmatrix} \hat{A}_0^T \\ \hat{A}_{01}^T \\ G_W^T \\ G_0^T \end{bmatrix} P_4 \begin{bmatrix} \hat{A}_0 & \hat{A}_{01} & G_W & G_0 \end{bmatrix} \xi(t) + \gamma \int_{t-\tau}^{t} e^{\gamma(t-s)} X^T(\theta) P_3 X(\theta) \mathrm{d}\theta, \tag{22}
$$

where

$$
\Omega_1 = \begin{bmatrix} \hat{A}_0^{\mathrm{T}} P_1 + P_1 \hat{A}_0 + P_3 - \frac{1}{\tau} P_4 & P_1 \hat{A}_{01} + \frac{1}{\tau} P_4 & P_1 G_W & P_1 G_0 \\ * & -\frac{1}{\tau} P_4 & 0 & 0 \\ * & * & 0 & 0 \\ * & * & * & -P_2 \end{bmatrix}. \tag{23}
$$

Next, we utilize inequalities (8), (9), (10) and (22) to study the FTS for system (7). In view of $-\tau P_4 < 0$, according to Lemma 1, inequality (8) equals to

$$
\Omega_2 - \tau \begin{bmatrix} \hat{A}_0^T P_4 \\ \hat{A}_{01}^T P_4 \\ G_W^T P_4 \\ G_0^T P_4 \end{bmatrix} (-\tau P_4)^{-1} \begin{bmatrix} \tau P_4 \hat{A}_0 & \tau P_4 \hat{A}_{01} & \tau P_4 G_W \tau P_4 G_0 \end{bmatrix} < 0, \tag{24}
$$

where

$$
\Omega_2 = \begin{bmatrix} \hat{A}_0^{\mathrm{T}} P_1 + P_1 \hat{A}_0 - \gamma P_1 + P_3 - \frac{1}{\tau} P_4 & P_1 \hat{A}_{01} + \frac{1}{\tau} P_4 & P_1 G_W & P_1 \, G_0 \\ * & -\frac{1}{\tau} P_4 & 0 & 0 \\ * & * & -R & 0 \\ * & * & * & -P_2 \end{bmatrix}.
$$

Based on the expression of Ω_1 in Equation (23), inequality (24) can be further transformed into the following forms:

$$
\Omega_1 + \tau \begin{bmatrix} \hat{A}_0^T \\ \hat{A}_{01}^T \\ G_W^T \\ G_0^T \end{bmatrix} P_4 \begin{bmatrix} \hat{A}_0 & \hat{A}_{01} & G_W & G_0 \end{bmatrix} + \begin{bmatrix} -\gamma P_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -R & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} < 0. \tag{25}
$$

By inequalities (22) and (25), it follows that

$$\begin{aligned}\dot{V} &\le \xi^T(t)\begin{bmatrix} \gamma P_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\xi(t) + \gamma\int_{t-\tau}^{t} e^{\gamma(t-s)}X^T(\theta)P_3X(\theta)\mathrm{d}\theta \\ &= \gamma X^T(t)P_1X(t) + W^T(t)RW(t) + \gamma\int_{t-\tau}^{t} e^{\gamma(t-s)}X^T(\theta)P_3X(\theta)\mathrm{d}\theta \\ &= \gamma V_1 + \gamma V_3 + W^T(t)RW(t) \\ &\le \gamma V + W^T\,(t)RW(t), \end{aligned} \tag{26}$$

so we have

$$\dot{V}(t) - \gamma V(t) \le W^T(t)RW(t).$$

The following inequality is established:

$$\exp(-\gamma t)\dot{V}(t) - \gamma\exp(-\gamma t)V(t) \le \exp(-\gamma t)W^{\mathrm{T}}(t)RW(t).$$

Noticing that $\frac{\mathrm{d}}{\mathrm{d}t}(\exp(-\gamma t)V(t)) = \exp(-\gamma t)\dot{V}(t) - \gamma\exp(-\gamma t)V(t)$, we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}(\exp(-\gamma t)V(t)) \le \exp(-\gamma t)W^{\mathrm{T}}(t)RW(t), \tag{27}$$

Based on the zero-initial condition in Definition 2, from inequality (27), it can be obtained that

$$\exp(-\gamma t)\ V(t) \le \int_0^t \exp(-\gamma s)\ W^T(s)RW(s)\mathrm{d}s.$$

Thus,

$$\begin{aligned} V(t) &\le \exp(\gamma t)\int_0^t \exp(-\gamma s)W^T(s)RW(s)\mathrm{d}s \\ &\le \exp(\gamma t)\lambda_{\max}(R)\int_0^t W^T(s)W(s)\mathrm{d}s \\ &\le \exp(\gamma T)\lambda_{\max}(R)\int_0^T W^T(s)W(s)\mathrm{d}s. \end{aligned} \tag{28}$$

Since $e(t) = C_0X(t)$, based on inequality (9), Equations (11) and (12), we obtain

$$e^T(t)\Gamma e(t) = X^T(t){C_0}^T\Gamma C_0X(t) \le X^T(t)P_1X(t) \le V(t). \tag{29}$$

In combination with inequalities (28) and (29), it follows that

$$e^T(t)\Gamma e(t) \le \exp(\gamma T)\ \lambda_{\max}(R)\int_0^T W^T(t)W(t)\mathrm{d}t. \tag{30}$$

In view of $W(t) = \begin{bmatrix} \dot{y}_d(t) \\ \dot{y}_d(t-\tau) \\ \dot{w}(t) \end{bmatrix}$, we have

$$\begin{aligned} \int_0^T W^T(t)W(t)dt &= \int_0^T \begin{bmatrix} \dot{y}_d^T(t) & \dot{y}_d^T(t-\tau) & \dot{w}^T(t) \end{bmatrix}\begin{bmatrix} \dot{y}_d(t) \\ \dot{y}_d(t-\tau) \\ \dot{w}(t) \end{bmatrix}\mathrm{d}t \\ &= \int_0^T \dot{y}_d^T(t)\dot{y}_d(t)\mathrm{d}t + \int_0^T \dot{y}_d^T(t-\tau)\dot{y}_d(t-\tau)\mathrm{d}t + \int_0^T \dot{w}^T(t)\dot{w}(t)\mathrm{d}t. \end{aligned} \tag{31}$$

Then substituting Equation (31) into Equation (30), according to inequality (10), Assumptions 2 and 3, we can obtain

$$e^T(t)\Gamma e(t) \leq e^{\gamma T}\lambda_{max}(R)(2c^2+d^2) \leq e^{\gamma T}\lambda_{max}(R)c_1^2 \leq e^{\gamma T}\frac{c_2^2}{e^{\gamma T}c_1^2}c_1^2 \leq c_2^2.$$

This proves Theorem 1. □

Furthermore, we give the feedback gain matrices K_1 and K_2 in system (7).

Theorem 2. *Under Assumptions 2 and 3, for a given scalar $\gamma \geq 0$, if there exist M_1, M_2, $Z_i (i = 1,2,3,4)$ with appropriate dimensions satisfying*

$$\begin{bmatrix} \Phi_1 & A_{01}Z_1 + B_0M_2 + \frac{1}{\tau}Z_4 & G_W & G_0Z_1 & \tau(A_0Z_1+B_0M_1)^{\mathrm{T}} \\ * & -\frac{1}{\tau}Z_4 & 0 & 0 & \tau(A_{01}Z_1+B_0M_2)^{\mathrm{T}} \\ * & * & -R & 0 & \tau G_W^{\mathrm{T}} \\ * & * & * & -Z_2 & \tau Z_1 G_0^{\mathrm{T}} \\ * & * & * & * & \tau(Z_4 - 2Z_1) \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} -Z_1 & Z_1{C_0}^T \\ * & -\Gamma^{-1} \end{bmatrix} < 0, \tag{33}$$

$$R \leq \frac{c_2^2}{e^{\gamma T}c_1^2}I, \tag{34}$$

where $\Phi_1 = (A_0Z_1 + B_0M_1)^T + (A_0Z_1 + B_0M_1) - \gamma Z_1 + Z_3 - \frac{1}{\tau}Z_4$, $c_1^2 \geq 2c^2 + d^2$. Then system (7) is input–output finite-time stable with respect to (c_1, c_2, Γ, T), when $K_1 = M_1Z_1^{-1}$, $K_2 = M_2Z_1^{-1}$.

Proof. Multiplying $\Lambda = diag(P_1^{-1}, P_1^{-1}, I, P_1^{-1}, P_4^{-1})$ by the right and left on both sides of inequality (8) yields

$$\begin{bmatrix} \hat{\Phi}_0 & \hat{A}_{01}P_1^{-1} + \frac{1}{\tau}P_1^{-1}P_4P_1^{-1} & G_W & G_0P_1^{-1} & \tau P_1^{-1}\hat{A}_0^{\mathrm{T}} \\ * & -\frac{1}{\tau}P_1^{-1}P_4P_1^{-1} & 0 & 0 & \tau P_1^{-1}\hat{A}_{01}^{\mathrm{T}} \\ * & * & -R & 0 & \tau G_W^{\mathrm{T}} \\ * & * & * & -P_1^{-1}P_2P_1^{-1} & \tau P_1^{-1}G_0^{\mathrm{T}} \\ * & * & * & * & -\tau P_4^{-1} \end{bmatrix} < 0 \tag{35}$$

where $\widehat{\Phi}_0 = P_1^{-1}\hat{A}_0^T + \hat{A}_0P_1^{-1} - \gamma P_1^{-1} + P_1^{-1}P_3P_1^{-1} - \frac{1}{\tau}P_1^{-1}P_4P_1^{-1}$. Substituting $\hat{A}_0 = A_0 + B_0K_1$ and $\hat{A}_{01} = A_{01} + B_0K_2$ into inequality (35), we obtain

$$\begin{bmatrix} \Phi_0 & A_{01}P_1^{-1} + B_0K_2P_1^{-1} + \frac{1}{\tau}P_1^{-1}P_4P_1^{-1} & G_W & G_0P_1^{-1} & \tau P_1^{-1}(A_0+B_0K_1)^{\mathrm{T}} \\ * & -\frac{1}{\tau}P_1^{-1}P_4P_1^{-1} & 0 & 0 & \tau P_1^{-1}(A_{01}+B_0K_2)^{\mathrm{T}} \\ * & * & -R & 0 & \tau G_W^T \\ * & * & * & -P_1^{-1}P_2P_1^{-1} & \tau P_1^{-1}G_0^T \\ * & * & * & * & -\tau P_4^{-1} \end{bmatrix} < 0 \tag{36}$$

where

$$\Phi_0 = (A_0P_1^{-1} + B_0K_1P_1^{-1})^T + (A_0P_1^{-1} + B_0K_1P_1^{-1}) - \gamma P_1^{-1} + P_1^{-1}P_3P_1^{-1} - \frac{1}{\tau}P_1^{-1}P_4P_1^{-1}.$$

Denote $Z_1 = P_1^{-1}$, $M_1 = K_1P_1^{-1}$, $M_2 = K_2P_1^{-1}$, $Z_2 = P_1^{-1}P_2P_1^{-1}$, $Z_3 = P_1^{-1}P_3P_1^{-1}$ and $Z_4 = P_1^{-1}P_4P_1^{-1}$, it can be obtained that inequality (36) equals to the following inequality:

$$\begin{bmatrix} \Phi_1 & A_{01}Z_1 + B_0M_2 + \frac{1}{\tau}Z_4 & G_W & G_0Z_1 & \tau(A_0Z_1 + B_0M_1)^{\mathrm{T}} \\ * & -\frac{1}{\tau}Z_4 & 0 & 0 & \tau(A_{01}Z_1 + B_0M_2)^{\mathrm{T}} \\ * & * & -R & 0 & \tau G_W^T \\ * & * & * & -Z_2 & \tau Z_1 G_0^T \\ * & * & * & * & -\tau P_4^{-1} \end{bmatrix} < 0 \tag{37}$$

where

$$\Phi_1 = (A_0Z_1 + B_0M_1)^T + (A_0Z_1 + B_0M_1) - \gamma Z_1 + Z_3 - \frac{1}{\tau}Z_4.$$

Obviously, inequality (37) is not an LMI because of the last matrix block $-\tau {P_4}^{-1}$. To solve inequality (37) by LMI toolbox of MATLAB, we notice that

$$\left({P_4}^{-1} - Z_1\right)^{\mathrm{T}} P_4 \left({P_4}^{-1} - Z_1\right) = {P_4}^{-1} - 2Z_1 + {Z_1}^{\mathrm{T}}{P_4}^{-1}Z_1 = {P_4}^{-1} - 2Z_1 + Z_4. \tag{38}$$

Because the congruent transformation does not change the positive definiteness of the matrix, the following inequality holds:

$$\left({P_4}^{-1} - Z_1\right)^{\mathrm{T}} P_4 \left({P_4}^{-1} - Z_1\right) \geq 0 \tag{39}$$

From Equation (38) and inequality (39), we obtain ${P_4}^{-1} - 2Z_1 + Z_4 \geq 0$, i.e.,

$$-P_4^{-1} \leq Z_4 - 2Z_1.$$

Therefore, if

$$\begin{bmatrix} \Phi_1 & A_{01}Z_1 + B_0M_2 + \frac{1}{\tau}Z_4 & G_W & G_0Z_1 & \tau(A_0Z_1 + B_0M_1)^{\mathrm{T}} \\ * & -\frac{1}{\tau}Z_4 & 0 & 0 & \tau(A_{01}Z_1 + B_0M_2)^{\mathrm{T}} \\ * & * & -R & 0 & \tau G_W^{\mathrm{T}} \\ * & * & * & -Z_2 & \tau Z_1 G_0^{\mathrm{T}} \\ * & * & * & * & \tau(Z_4 - 2Z_1) \end{bmatrix} < 0,$$

then inequality (37) holds.

Based on inequality (9), we can derive

$$Z_1{C_0}^T\Gamma C_0Z_1 - Z_1 \leq 0,$$

i.e.,

$$-Z_1 - Z_1{C_0}^T(-\Gamma^{-1})^{-1}C_0Z_1 \leq 0. \tag{40}$$

Then according to Lemma 1 and inequality (40), we can acquire

$$\begin{bmatrix} -Z_1 & Z_1{C_0}^T \\ * & -\Gamma^{-1} \end{bmatrix} < 0.$$

Consequently, if inequalities (32) to (34) hold, then inequalities (8) to (10) are satisfied, i.e., Theorem 1 is true. This proves Theorem 2. □

Denote $K_1 = \begin{bmatrix} K_{1e} & K_{1x} \end{bmatrix}$ and $K_2 = \begin{bmatrix} K_{2e} & K_{2x} \end{bmatrix}$. As $X(t) = \begin{bmatrix} e(t) \\ \dot{x}(t) \end{bmatrix}$, it follows that

$$\dot{u}(t) = K_{1e}e(t) + K_{1x}\dot{x}(t) + K_{2e}e(t-\tau) + K_{2x}\dot{x}(t-\tau). \tag{41}$$

Integrating Equation (41) on interval $[0, t]$, we have

$$u(t) = K_{1e}\int_0^t e(\theta)\mathrm{d}\theta + K_{1x}x(t) - K_{1x}x(0) + K_{2e}\int_{-\tau}^{t-\tau} e(\theta)\mathrm{d}\theta + K_{2x}x(t-\tau) - K_{2x}x(-\tau). \quad (42)$$

$u(t)$ is the delay-dependent finite-time bounded tracking controller of system (1). If we want to design a delay-independent controller, we can take $K_2 = 0$, namely, take $M_2 = 0$ in inequality (32). Thus, the following corollary is received.

Corollary 1. *Under Assumptions 2 and 3, for a given scalar $\gamma \geq 0$, if there exist M, Z_i $(i = 1,2,3,4)$ satisfying*

$$\begin{bmatrix} \Phi_1 & A_{01}Z_1 + \frac{1}{\tau}Z_4 & G_W & G_0Z_1 & \tau(A_0Z_1 + B_0M)^{\mathrm{T}} \\ * & -\frac{1}{\tau}Z_4 & 0 & 0 & \tau Z_1^{\mathrm{T}}A_{01}^{\mathrm{T}} \\ * & * & -R & 0 & \tau G_W^{\mathrm{T}} \\ * & * & * & -Z_2 & \tau Z_1 G_0^{\mathrm{T}} \\ * & * & * & * & \tau(Z_4 - 2Z_1) \end{bmatrix} < 0 \quad (43)$$

$$\begin{bmatrix} -Z_1 & Z_1 {C_0}^T \\ * & -\Gamma^{-1} \end{bmatrix} < 0, \quad (44)$$

$$R \leq \frac{c_2^2}{e^{\gamma T}c_1^2} I, \quad (45)$$

where $\Phi_1 = (A_0Z_1 + B_0M)^T + (A_0Z_1 + B_0M) - \gamma Z_1 + Z_3 - \frac{1}{\tau}Z_4$, $c_1^2 \geq 2c^2 + d^2$, then the output of system (1) *realizes the finite-time bounded tracking to the reference signal $y_d(t)$ under the controller*

$$u(t) = K_e\int_0^t e(\theta)\mathrm{d}\theta + K_x x(t) - K_x x(0), \quad (46)$$

where $\begin{bmatrix} K_e & K_x \end{bmatrix} = MZ_1^{-1}$.

4. Numerical Simulation

In this section, we conduct a realistic delay differential equation of neutral type (NDDE) problem originated from [39] to validate the effectiveness of the results in Section 3.

Example. Figure 1a represents a small metal strip with two cells, and Figure 1b is the partial element equivalent circuits (PEEC) model of the metal strip, which includes the partial inductances $L_{p_{ij}}$ and the partial coefficients of potential p_{ij}. The state vector and the input represent the partial inductance branch currents and the unknown nodal voltages, respectively.

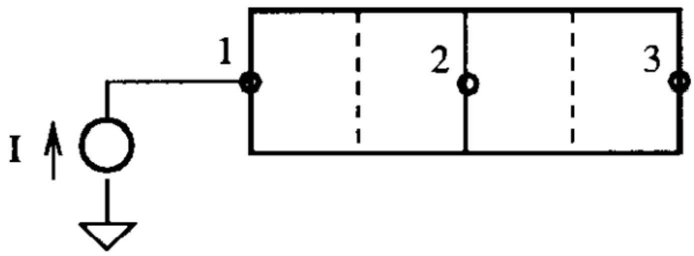
In the model, the coefficient matrices are

$$\frac{A}{100} = \begin{bmatrix} -7 & 1 & 2 \\ 3 & -9 & 0 \\ 1 & 2 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix},$$

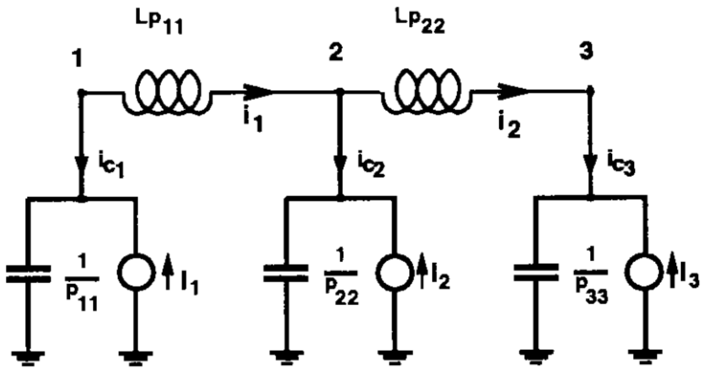
$$\frac{A_1}{100} = \begin{bmatrix} 1 & 0 & -3 \\ -0.5 & -0.5 & -1 \\ -0.5 & -1.5 & 0 \end{bmatrix}, E = 0.05\begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix},$$

$$G = \frac{1}{72}\begin{bmatrix} -1 & -5 & 2 \\ 4 & 0 & 3 \\ -2 & 4 & 1 \end{bmatrix}, \frac{C}{100} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix},$$

and $\phi(t) = 0$, for all $t \in [-\tau, 0]$.



(**a**)



(**b**)

Figure 1. (**a**) Metal strip with two L_p cells and (**b**) PEEC model for the metal strip.

Choose the parameters $\Gamma = I,\ c_1 = 1,\ c_2 = 2,\ T = 10$ and $\gamma = 0.2$. By using LMI toolbox in MATLAB, we conclude that there exists a feasible solution of inequalities (32) to (34) when the time-delay upper bound is 0.003. At this time, the feedback gain matrices are

$$K_1 = \begin{bmatrix} K_{1e} & K_{1x} \end{bmatrix} = \begin{bmatrix} 63.3591 & -0.6543 & 102.1523 & 60.3751 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} K_{2e} & K_{2x} \end{bmatrix} = \begin{bmatrix} -0.3725 & -0.4602 & 41.2559 & 70.8512 \end{bmatrix}.$$

Case 1 . When

$$y_d(t) = \begin{cases} 0.2\sin t, t \in [0, T], \\ 0, t \in [-\tau, 0], \end{cases}$$

and

$$w(t) = 0.15\sin(2t),$$

we have

$$\int_0^T \dot{y}_d^T(t)\dot{y}_d(t)\mathrm{d}t \leq 0.25 \overset{\text{def}}{=} c^2$$

and

$$\int_0^T \dot{w}^T(t)\dot{w}(t)\mathrm{d}t \leq 0.5 \overset{\text{def}}{=} d^2.$$

Thus, $2c^2 + d^2 \leq c_1^2$, and Theorem 2 is satisfied. If we take $\eta(t) = 0, \forall t \in [-\tau, 0]$, then $e(t) = C\phi(t) - \eta(t) = 0, \forall t \in [-\tau, 0]$, and

$$e^T(t)\Gamma e(t) \leq c_2^2\ (\forall t \in [-\tau, 0])$$

holds. The output response is shown in Figure 2.

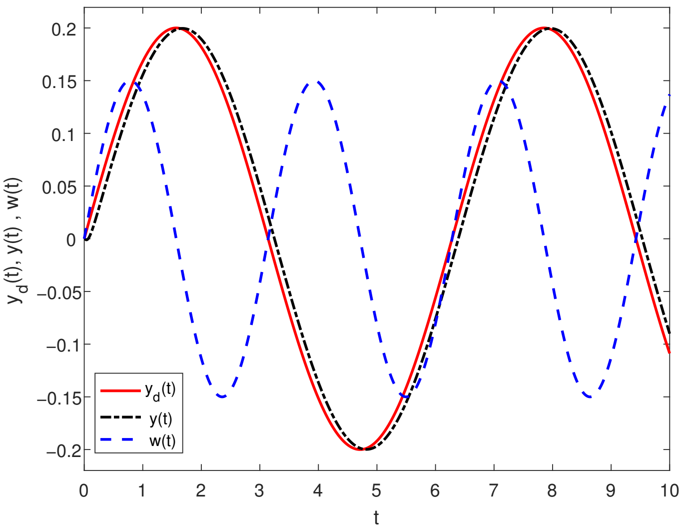


Figure 2. Output response by controller (42).

Case 2. When taking

$$y_d(t) = \begin{cases} \frac{1}{1+e^{-2t}} - 0.5, t \in [0, T], \\ 0, t \in [-\tau, 0], \end{cases}$$

and

$$w(t) = \frac{2}{2+e^{-t}} - 0.65,$$

one can obtain

$$\int_0^T \dot{y}_d^T(t)\dot{y}_d(t)\mathrm{d}t \le 0.2 \overset{\text{def}}{=} c^2$$

and

$$\int_0^T \dot{w}^T(t)\dot{w}(t)\mathrm{d}t \le 0.1 \overset{\text{def}}{=} d^2.$$

As a result, $2c^2 + d^2 \le c_1^2$, and Theorem 2 holds. The output response is shown in Figure 3.

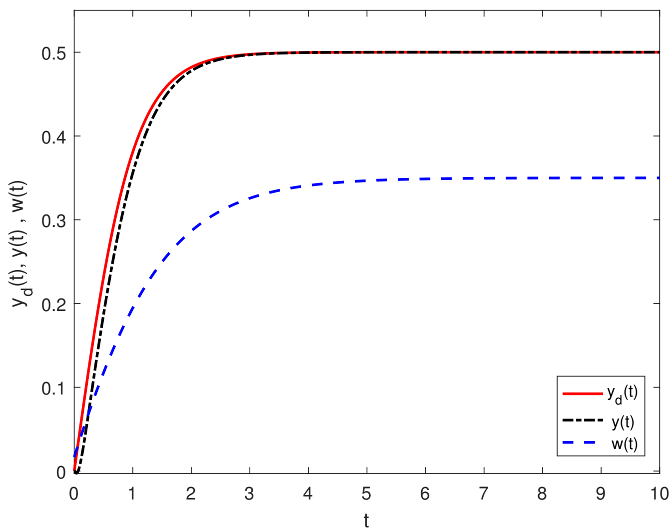


Figure 3. Output response by controller (42).

Figures 2 and 3 indicate that during time period $[0, T]$, by the controller (42), the output $y(t)$ stays in the c_2 neighborhood of $y_d(t)$, i.e., the output of (1) achieves bounded time tracking to the reference signal $y_d(t)$.

Case 3. For the disturbance signal and the reference signal in *case 1*, if we take Equation (46) as the controller, another output response is obtained. The feedback gain matrices are

$$K_1 = \begin{bmatrix} K_{1e} & K_{1x} \end{bmatrix} = \begin{bmatrix} 47.7525 & -36.6115 & 119.4409 & 107.9395 \end{bmatrix}.$$

Putting the output responses in *case 1* and the new one together, we can obtain Figure 4.

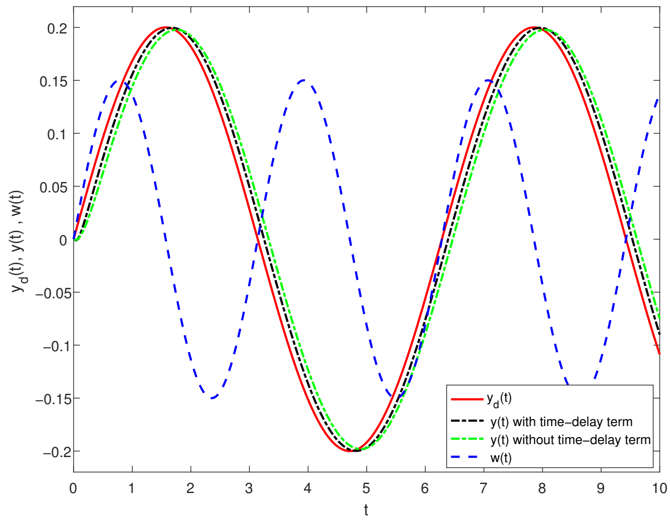


Figure 4. Output response by controller with and without time-delay term

Figure 4 shows that the tracking effect by using Equation (42) is better than Equation (46). That means the time-delay feedback term is helpful in the tracking problem of neutral systems.

5. Conclusions

In this work, we study FTB tracking control for a class of neutral systems. In order to realize these objectives, we construct an error system for the original neutral systems. Thus, FTB tracking of the original system is transferred to input–output FTS of the error system. For the closed-loop system of error system, we employ the Lyapunov–Krasovskii functional, which contains not only the time-delay term but also the exponential term. The sufficient conditions for the stability of the system are obtained in terms of LMIs. We further derive the finite-time bounded controllers that are delay dependent and delay independent. For further research, the finite-time bounded control problem will be extended to more types of control systems.

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