

Article

A Soft Computing Scaled Conjugate Gradient Procedure for the Fractional Order Majnun and Layla Romantic Story

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Abstract: The current study shows the numerical performances of the fractional order mathematical model based on the Majnun and Layla (FO-MML) romantic story. The stochastic computing numerical scheme based on the scaled conjugate gradient neural networks (SCGNNs) is presented to solve the FO-MML. The purpose of providing the solutions of the fractional derivatives is to achieve more accurate and realistic performances of the FO-MML romantic story model. The mathematical model is divided into four dynamics, while the exactness is authenticated through the comparison of obtained and reference Adam results. Moreover, the negligible absolute error enhances the accuracy of the stochastic scheme. Fourteen numbers of neurons have been taken and the information statics are divided into authorization, training, and testing, which are divided into 12%, 77% and 11%, respectively. The reliability, capability, and accuracy of the stochastic SCGNNs is performed through the stochastic procedures using the regression, error histograms, correlation, and state transitions for solving the mathematical model.

Keywords: fractional order; Majnun and Layla; mathematical system; scaled conjugate gradient; neural networks; reference results

MSC: 32W50

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1. Introduction

Psychology encompasses the scientific learning processes of the mind according to some American reports [1–4]. Few scientists and psychologists have presented the mathematical relationship in the form of a love story [5–7]. The psychological applications have been noticed in social nature, human development, health, clinical actions, and cognition. One of the psychological questions comes to mind: what are the meanings of love? Everyone has their own meaning. In general, love is divided into two categories, spiritual and physical love. The spiritual love is realistic and does not change according to the situation, while the physical love shows an organic attraction. Love is often maddening and sometimes becomes terrifying for the person involved. Those who are in true love feel that the earth is spinning. A person falls in love without any judgement of color, religion, and creed. These emotions are not specific to humans, as other creatures also have the feelings of love. There are various historical real love stories, some of them are Heer Ranjha, Sassi Punnuh, Sohni Mahiwal, Romeo Juliet, and Layla Majnun.

The present work shows the numerical solutions of the historical fractional order mathematical model based on the Majnun and Layla (FO-MML) romantic story. This story was recognized in Persia in the 9th century. The theme of this story has also been presented in different languages including Arabic, Persian, Pakistani, Indian, and Turkish. This love

story is not only reported in the literature, but these characters have been presented in various dramas and films. One of the super hit Hindi films “Aaja Nachle” is presented based on the love story of Layla–Majnun, which is directed by Anil Mehta and produced by Aditya Chopra. In this film, when Majnun has been beaten by his master, Layla faces the same wounds. Some believe that this is magic because Layla’s blood was shed when Majnun’s hand harmed her. According to people, it was a gift from God or something to do with a ghost or unholy magic. Her family forbid her from being close to him. They employed various techniques to expel the evil, such as by attaching a black cord to his arm. Childhood converted into youth with their growing love, and they promised to set such a true example of love, one that would be memorized in the minds of people for centuries. Layla’s brother Tabraiz was unhappy; he forcibly stopped Layla from meeting him. He tried to kill him, but Majnun was powerful. He snapped the weapon and killed Tabraiz. The court decided to kill Majnun with stones until his last breath. Layla tried to stop this, but it was all in vain, and then Majnun became unconscious. She requested, “do not hit Majnun, I will be married where my family wants, but with that condition to forgive Majnun and put him out of the city. I left my heartbeat for you (Majnun). I swear no one can touch me, if someone did, he will find the clay pot instead of body. The people will weep and repent when God will come to our graves with flowers”. She wedded and sacrificed herself. When her husband tried to touch her, she resisted with these words: “I am only for Majnun”. Her husband ordered his army chief to prepare against Majnun, “I want to see her lover and her friendship”. She caught the sword of her husband, injured herself and cried “see the blood of my hand, you cannot imagine how much we love each other”. Her husband started a fight with Majnun and shouted “Majnun you are high or my castle”. Majnun replied, “you are the husband of Layla, why are you shouting? Why are you expected to be a lover if you do not understand what love is? Layla is my love, life, and heartbeat. You married Layla, but you don’t understand the significance of love”. Majnun was asked a few questions by her husband, including: “is she your cremation?” and Majnun replied, “she is my faith”. “Do you respect her?” and Majnun retorted, “she is everything”. “I will cut your head; I will kill you today”, he said. Majnun replied, “Layla will pass away with me. I am going to keep her inside and put her in the deep well”. Majnun claimed, “no matter how much you dig the well, you cannot reach her. It would be better if you killed me because if I died, she would also die”. He had two choices: to let it go from his heart or accept her passing. Majnun accepted the second option. Her husband used a sword to kill him, but he was unaware that Layla would also receive the same wound. Both were scarified to God at the same time. Overall, nobody can protect themselves from love, even Zuleka, Ghalib, Iqbal, Bulay Shah, and Khusro.

We learned the value of love from all the major religions. Whether it be the Quran, the Geeta, the Bible, or the Torat, they have all taught us the importance of love. The holy prophet Muhammad (S.A.W) said: “Loving humanity is second only to having faith in Allah as one of the best deeds in Islam”.

Several models based on the complex variables have been used in various applications, e.g., optical systems [8], plasma physics [9], high-energy accelerators [10], rotor dynamics [11], and some other areas [12–15]. The motive of this research is to present the numerical solutions of FO-MML by applying the stochastic computing performances of the scaled conjugate gradient neural networks (SCGNNs). The time-fractional based derivatives are used in various real-world applications [16,17]. The stochastic procedures based on the artificial neural networks have been applied to solve various stiff, complicated, singular, nonlinear, linear, and grim differential systems [18–20]. Some noteworthy applications of the stochastic solvers are the COVID-19 model [21–23], food chain model [24], smoking models [25], HIV systems [26], delayed differential model [27], and forth order differential model [28]. However, the stochastic SCGNNs paradigms have never been implemented to solve the FO-MML. Few novel characteristics of the stochastic method to solve the FO-MML are shown as the following:

- The integer model based on the Majnun and Layla model is converted into the FO-MML to find more realistic results;
- The stochastic solvers have not been applied before to solve the FO-MML love story model;
- The artificial stochastic procedures together with the SCGNNs are accessible to stimulate the FO-MML love story system;
- The study of the mathematical system is presented for three variations based on the fractional kind of the mathematical Majun Layla model;
- The correctness of the scheme is observed by using the comparison of the obtained and reference results;
- The absolute error (AE) authenticates the accuracy of solutions based on the fractional kind of the mathematical Majun Layla model;
- The regression measures, error histogram (EHs) performances, correlation presentations, and state transitions (STs) standards approve the dependability of proposed SCGNNs for an FO-MML love story model.

The other sections of the paper are presented as follows: Section 2 shows the fractional order mathematical Majnun Layla model; Section 3 presents the SCGNNs' performances; Section 4 signifies the simulations of the FO-MML love story model; and the conclusions are given in the Section 5.

2. Fractional Order Majnun Layla Model

This section provides the FO-MML romantic story along with the real and complex relationships. The simple form of the model along with its two types of the complex variables is provided as [29,30]

$$\begin{cases} \frac{dM(u)}{du} = \lambda_a + \lambda_c M(u) + L^2(u), & M_0 = c_1, \\ \frac{dL(u)}{du} = \lambda_b + \lambda_d L(u) + M^2(u), & L_0 = c_2, \end{cases} \tag{1}$$

where $\lambda_c < 0$, $\lambda_a > 0$, $\lambda_b < 0$ and $\lambda_d < 0$, while c_1, c_2, c_3 and c_4 are the initial conditions. The feelings of Majnun and Layla have been expressed in the form of $M(u)$ and $L(u)$. λ_a and λ_b are the parameter constants that have been used to indicate the environmental properties based on their spirits. λ_a is taken as positive and fixed, which represents the sympathy and kindness for Majnun. $\lambda_b < 0$ means the unkind behavior of the people to Layla. M^2 and L^2 indicate their love at extreme level. $\lambda_b < 0$ and $\lambda_d < 0$ indicate the true love along with their feelings. The updated form of Equation (1) is achieved by taking the complex values $M(u) = M_r(u) + iM_i(u)$ and $L(u) = L_r(u) + iL_i(u)$ as [30]

$$\begin{cases} \frac{d}{du} M_r(u) = \lambda_a + \lambda_c M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = c_1, \\ \frac{d}{du} M_i(u) = \lambda_c M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = c_2, \\ \frac{d}{du} L_r(u) = \lambda_b + \lambda_d L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = c_3, \\ \frac{d}{du} L_i(u) = \lambda_d L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = c_4, \end{cases} \tag{2}$$

where, $M_i(u)$ and $L_i(u)$ are the emotions of Majnun and Layla using the imaginary parts, while $M_r(u)$ and $L_r(u)$ are their real feelings. This work presents the fractional order mathematical Majnun and Layla (FO-MML) romantic story by applying the procedures of SCGNNs. The FO-MML based on the love relationship is written as

$$\begin{cases} \frac{d^v}{du^v} M_r(u) = \lambda_a + \lambda_c M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = c_1, \\ \frac{d^v}{du^v} M_i(u) = \lambda_c M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = c_2, \\ \frac{d^v}{du^v} L_r(u) = \lambda_b + \lambda_d L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = c_3, \\ \frac{d^v}{du^v} L_i(u) = \lambda_d L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = c_4, \end{cases} \tag{3}$$

where v signifies the FO Caputo derivative. The values of FO have been chosen between 0 and 1. The idea to apply the fractional derivatives is to obtain precise and accurate solutions. In fractional types of systems, the minute particulars (super-fast and low evolution) are examined. This shows more details of the system’s dynamics based on the fractional calculus, which is difficult to handle by applying the integer order. The derivatives based on the fractional order are performed with more competency as compared to the integer order with the obtainability of the condition. The fractional derivatives are used to validate the system’s performance using the real applications [31,32]. Furthermore, the fractional derivatives are widely examined to solve various applications that arise in engineering, control networks, and mathematical models. The fractional calculus has been widely applied over the past three decades by using the substantial operators, such as in the works of Caputo [33], Weyl-Riesz [34], Riemann-Liouville [35], Grnwald-Letnikov [36] and Erdlyi-Kober [37]. All above mentioned operators have their own implications. However, the Caputo derivatives are used to solve the non-homogeneous/homogeneous initial conditions and are considered easy for implementations. By keeping the importance of these fractional calculus applications, authors are motivated to perform the numerical solutions of the FO-MML by using the designed stochastic SCGNNs, see [37–39].

3. Proposed Scheme: SCGNNs

The current section presents the designed SCGNNs to solve the FO-MML story model. The necessary procedures are presented to describe the stochastic SCGNNs along with their implementations. Figure 1 shows the optimization measures using the multi-layer structure of SCGNNs, which are divided into two parts, the mathematical model and simulation performances. The designed operator performances are provided in two steps:

- (i) The significant measures using the proposed SCGNNs.
- (ii) The execution through the stochastic SCGNNs framework is presented to solve the FO-MML story model.

The numerical measures are executed with the default parameter setting to produce the dataset. Thirteen neurons have been selected with the data selection to solve the FO-MML story model. The artificial intelligence aptitudes using the supervised learning based on the SCGNNs have been provided with the best collaboration in the directories, including intricacy, overfitting, premature convergence, and underfitting performances. In addition, the system’s parameters are adjusted after comprehensive simulations, knowledge, experience, and small disparities in the networks.

The stochastic SCGNNs performances are implemented in the ‘Matlab’ software by using the ‘nftool’ command to obtain the selection of appropriate hidden neurons, learning methods, testing measures, and verification. The execution of SCGNNs for solving the FO-MML model together with the parameter setting is given in Table 1. The training of the networks is performed through the SCGNNs, and the backpropagation is provided to progress the Jacobian ‘ JoB ’, i.e., mean square error (MSE) to regulate the weights together with bias variables (B). The amendment of the decision variables using the SCG is written as

$$\begin{aligned} JoJ &= JoB \times JoB, \\ Je &= JoB \times e, \\ dB &= \frac{-(J+I \times mu)}{Je}. \end{aligned}$$

In the above system, I presents the identity vector and e is the error. The parameter performances of the SCGNNs are provided in Table 1, together with minor variation/disparsity/adjustment. Hence, these parameter settings will be incorporated with widespread consideration.

The numerical simulations through the stochastic SCGNNs are provided to solve the FO-MML love story model. Thirteen numbers of neurons have been provided in Figure 2 using the statics of authorization, training, and testing, which are taken as 12%, 77%, and 11% to solve the dynamics of the FO-MML. It is not always necessary to select the

same statics for authorization, training, and testing. The training values have been chosen as > 77% to achieve improved and better presentations due to bias input performances. If the statics of training performance is <77%, then the precision of stochastic SCGNNs is degraded suggestively. Therefore, the sample statics for the interval based on the unbiased and biased values should be selected with full attention to avoid both premature convergence and divergence.

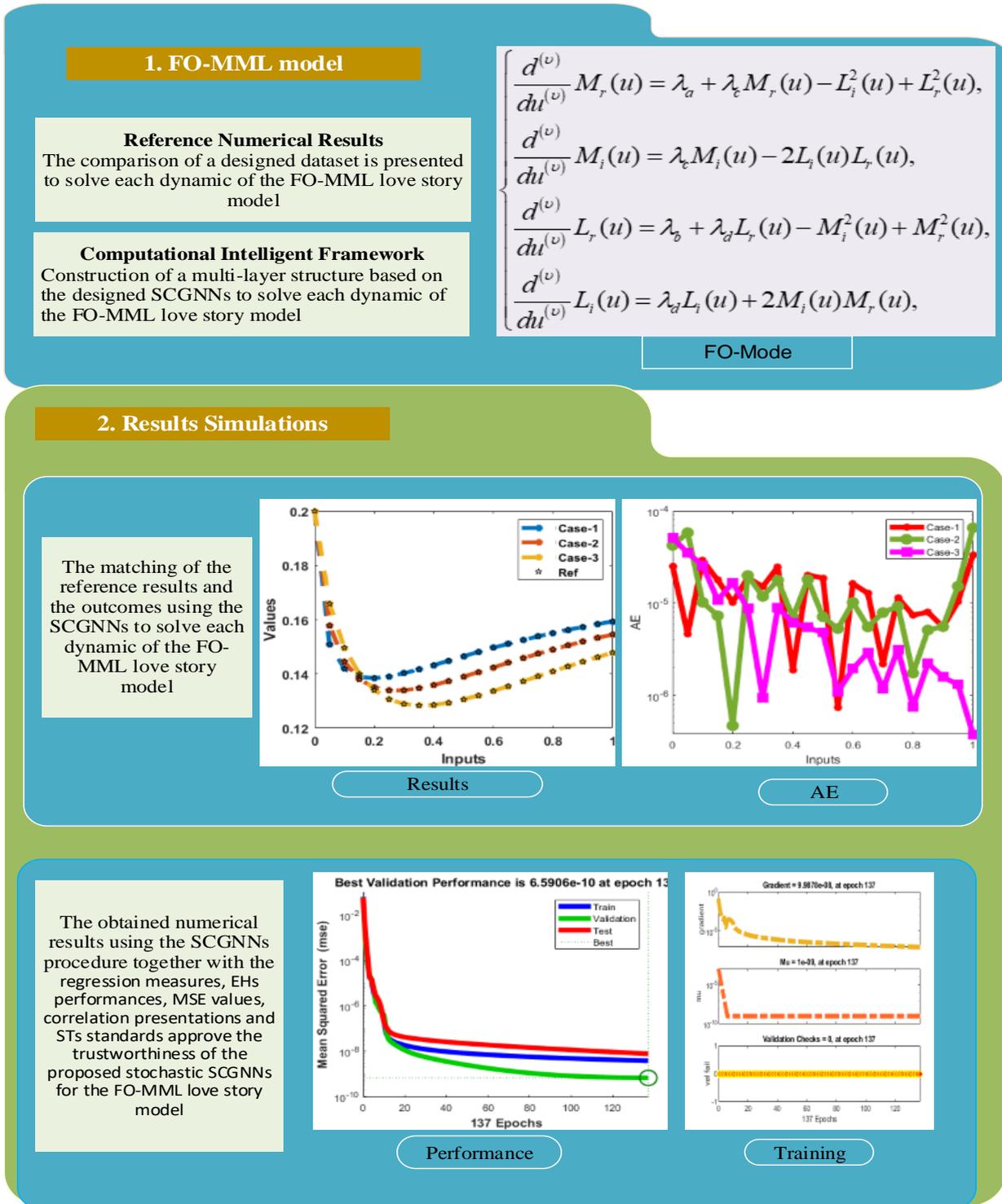


Figure 1. Workflow illustrations of scheme for FO-MML system.

Table 1. Parameter values to perform the SCGNNs.

Index	Settings
Hidden neurons	13
Decreasing Mu performances	0.1
Adaptive parameter, i.e., mu	5×10^{-03}
Fitness values (MSE)	0
Increasing Mu values	10
Maximum Mu performances	10^{11}
Maximum Epochs	450
Validation tests	12%
Layers (hidden, output, input)	Single
Minimum values of the gradient	10^{-07}
Testing measures	11%
Sample selection	Random
Training performance	77%
Generated dataset	Adam method
Stoppage standards	Default

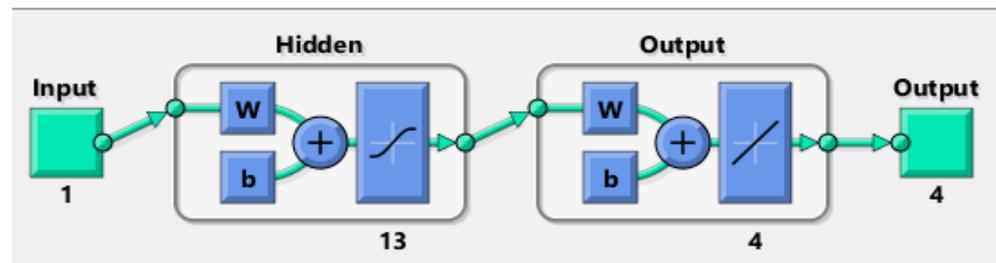


Figure 2. Input/hidden/output layers performances to solve the FO-MML model.

4. Numerical Measures

This section presents three variations of the FO-MML model based on the values of the FO to obtain the numerical performances.

Case 1: Suppose the FO-MML with $v = 0.6, \lambda_a = 1 \lambda_b = -1, \lambda_c = -1$ and $\lambda_d = -1$ is provided as

$$\begin{cases} \frac{d^{0.6}}{du^{0.6}} M_r(u) = 1 - M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = 0.1, \\ \frac{d^{0.6}}{du^{0.6}} M_i(u) = -M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = 0.2, \\ \frac{d^{0.6}}{du^{0.6}} L_r(u) = -1 - L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = 0.3, \\ \frac{d^{0.6}}{du^{0.6}} L_i(u) = -L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = 0.4. \end{cases} \tag{4}$$

Case 2: Consider the FO-MML with $v = 0.7, \lambda_a = 1 \lambda_b = -1, \lambda_c = -1$ and $\lambda_d = -1$ is provided as

$$\begin{cases} \frac{d^{0.7}}{du^{0.7}} M_r(u) = 1 - M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = 0.1, \\ \frac{d^{0.7}}{du^{0.7}} M_i(u) = -M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = 0.2, \\ \frac{d^{0.7}}{du^{0.7}} L_r(u) = -1 - L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = 0.3, \\ \frac{d^{0.7}}{du^{0.7}} L_i(u) = -L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = 0.4, \end{cases} \tag{5}$$

Case 3: Let FO-MML with $v = 0.8$, $\lambda_a = 1$, $\lambda_b = -1$, $\lambda_c = -1$ and $\lambda_d = -1$ is provided as

$$\begin{cases} \frac{d^{0.8}}{du^{0.8}} M_r(u) = 1 - M_r(u) - L_i^2(u) + L_r^2(u), & (M_r)_0 = 0.1, \\ \frac{d^{0.8}}{du^{0.8}} M_i(u) = -M_i(u) - 2L_i(u)L_r(u), & (M_i)_0 = 0.2, \\ \frac{d^{0.8}}{du^{0.8}} L_r(u) = -1 - L_r(u) - M_i^2(u) + M_r^2(u), & (L_r)_0 = 0.3, \\ \frac{d^{0.8}}{du^{0.8}} L_i(u) = -L_i(u) + 2M_i(u)M_r(u), & (L_i)_0 = 0.4. \end{cases} \tag{6}$$

The numerical values are illustrated in Figures 3–7 for solving the FO-MML story model by using the stochastic framework based on SCGNNs. The obtained numerical simulations of the FO-MML love story model have been provided in Figures 3–5 through the computational stochastic framework. Figure 3 indicates the performances of MSE using the FO-MML story model. The achieved best measures of the FO-MML story model have been performed, giving 8.07596×10^{-10} , 1.51683×10^{-09} , and 6.59063×10^{-10} with iterations 174, 145, and 137 for case 1 to 3. The gradient measures have been reported as 9.47×10^{-08} , 9.98×10^{-08} , and 9.99×10^{-08} for case 1 to 3. The achieved performances and the values of the EHs to solve the FO-MML love story model are presented in Figure 4. The values of the EHs for each case of the FO-MML model have been calculated as 2.85×10^{-05} , 4.18×10^{-05} , and 5.39×10^{-05} . The optimal performances of testing, justification, and training have been plotted in Figure 4. The correlation representations are illustrated in Figure 5 through the SCGNNs procedures to solve the FO-MML love story model. It is observed that each variation of the FO-MML love story model shows the performances of the correlation as being 1, which represents the case of a perfect system. The accuracy of stochastic SCGNNs for the FO-MML love story model is presented by using the testing, justification, and training procedures. Table 2 designates the MSE to solve the FO-MML love story model using the stochastic SCGNNs.

Table 2. SCGNNs performances for the FO-MML romantic model.

Case	MSE			Mu	Performance	Iterations	Gradient	Time
	Training	Testing	Justification					
1	6.80×10^{-09}	8.07×10^{-10}	3.28×10^{-09}	1×10^{-09}	6.80×10^{-09}	175	9.99×10^{-08}	04
2	6.29×10^{-09}	1.51×10^{-09}	7.93×10^{-10}	1×10^{-09}	6.29×10^{-09}	145	7.60×10^{-07}	04
3	3.75×10^{-09}	6.59×10^{-10}	7.67×10^{-09}	1×10^{-09}	3.75×10^{-09}	137	2.69×10^{-06}	03

The comparison of the results and AE to solve the FO-MML love story model using the stochastic computing SCGNNs scheme has been presented in Figures 6 and 7. The solutions of each dynamic of the FO-MML model are shown in Figure 6. The accuracy of SCGNNs is approved via matching of the proposed and reference solutions. The values of the AE for the dynamics $M_r(u)$, $M_i(u)$, $L_r(u)$ and $L_i(u)$ of the FO-MML love story model are presented in Figure 7. The AE values for $M_r(u)$ are shown in Figure 7a, which were found to be around 10^{-04} to 10^{-06} , 10^{-04} to 10^{-07} , and 10^{-05} to 10^{-06} for the variations 1 to 3. The AE measures for the dynamics $M_i(u)$ are presented in Figure 7b, which lie around 10^{-04} to 10^{-06} , 10^{-05} to 10^{-06} , and 10^{-05} to 10^{-07} for case 1 to 3. The AE performances for the dynamics $L_r(u)$ and $L_i(u)$ are presented in Figure 7c, which are found to be around 10^{-04} to 10^{-05} , 10^{-04} to 10^{-06} , and 10^{-05} to 10^{-06} for case 1 to 3. The matching of the results and small AE values indicates the correctness and precision of the stochastic SCGNNs approach to solve the FO-MML love story model.

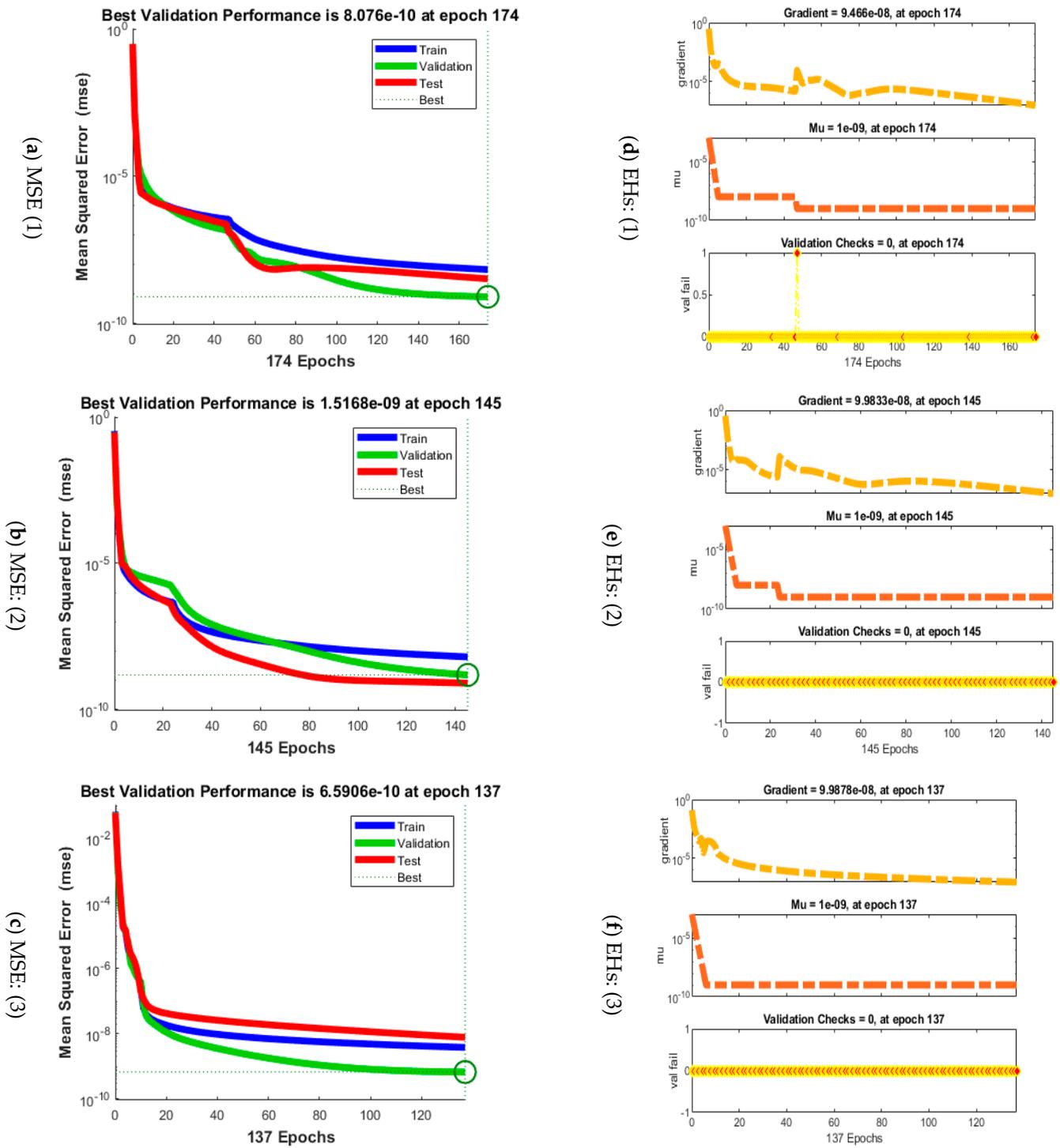


Figure 3. MSE and STs for the FO-MML love story model.

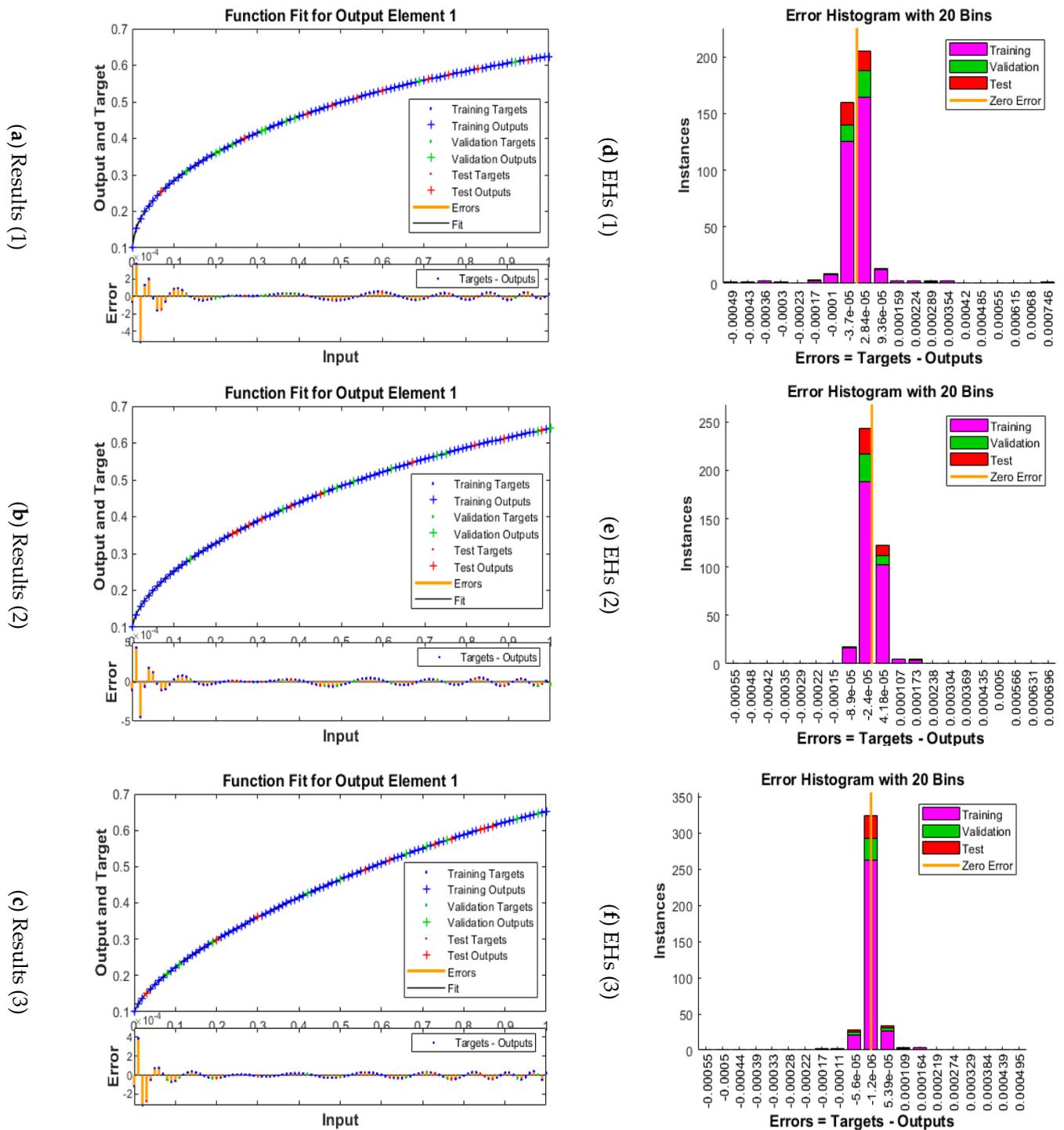


Figure 4. Assessments of the outcomes along with the EHs to solve the FO-MML love story model.

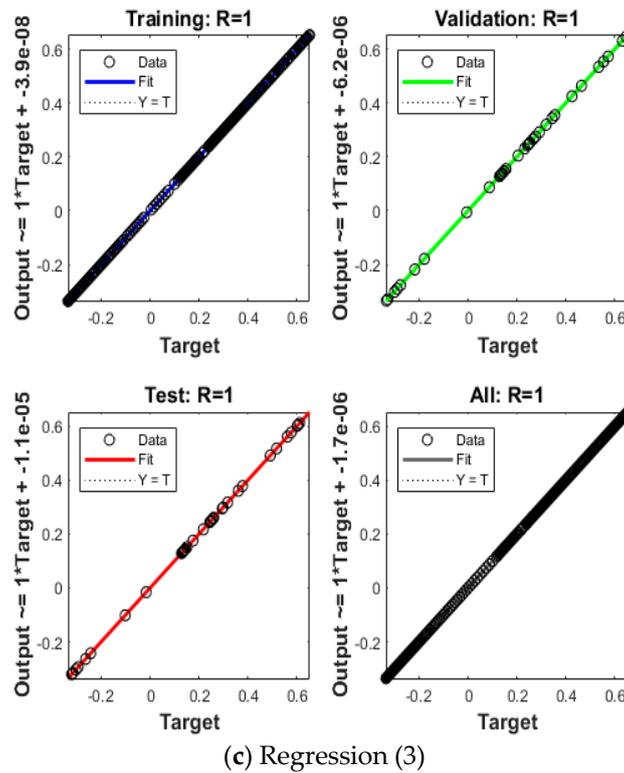
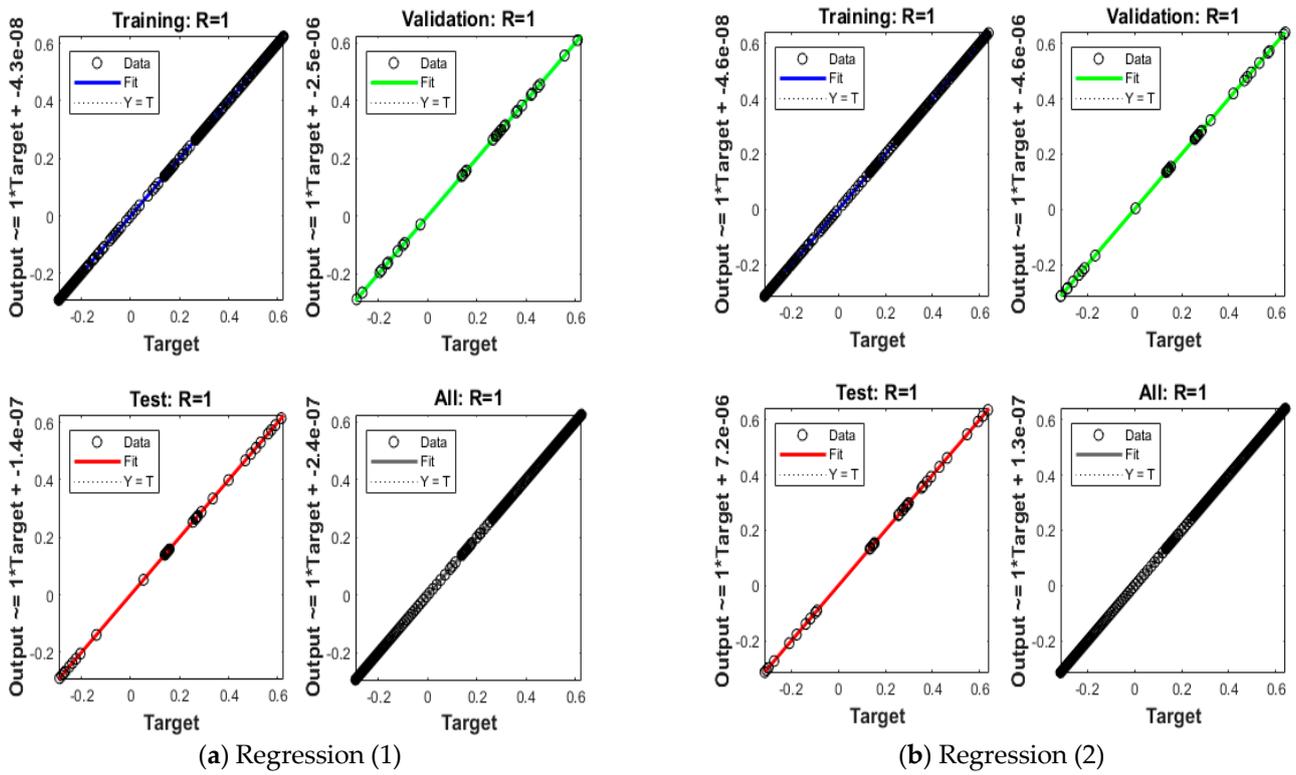
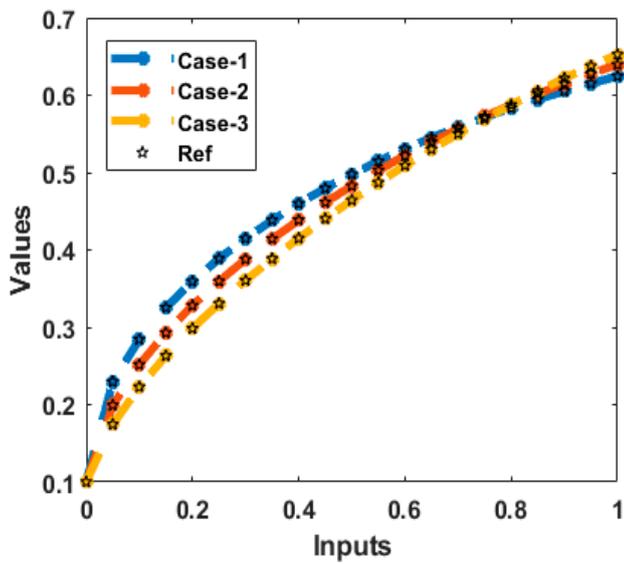
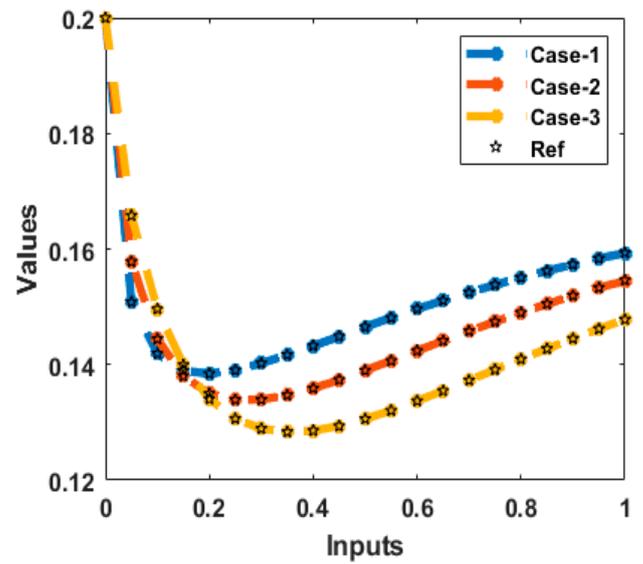


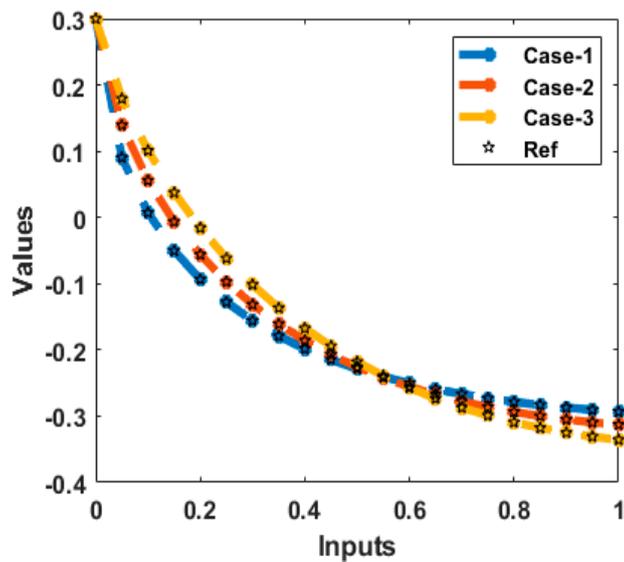
Figure 5. Regression values to solve the FO-MML love story model.



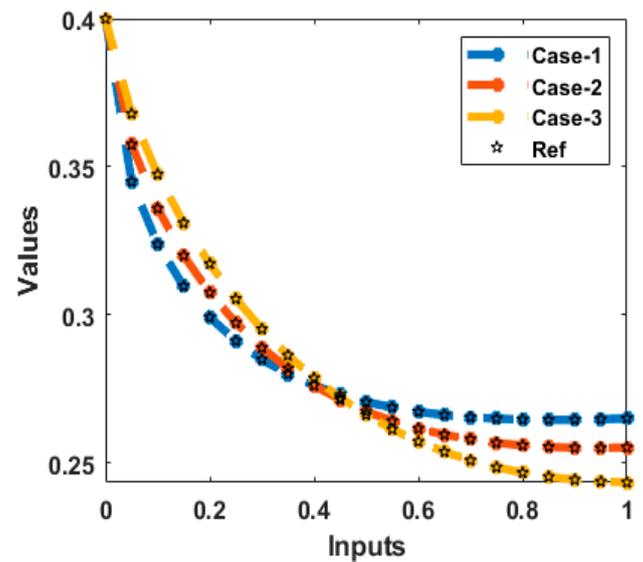
(a) Results of $M_r(u)$



(b) Results of $M_i(u)$



(c) Results of $L_r(u)$



(d) Results of $L_i(u)$

Figure 6. Comparison of the achieved performances along with the reference solutions to solve the FO-MML love story model.

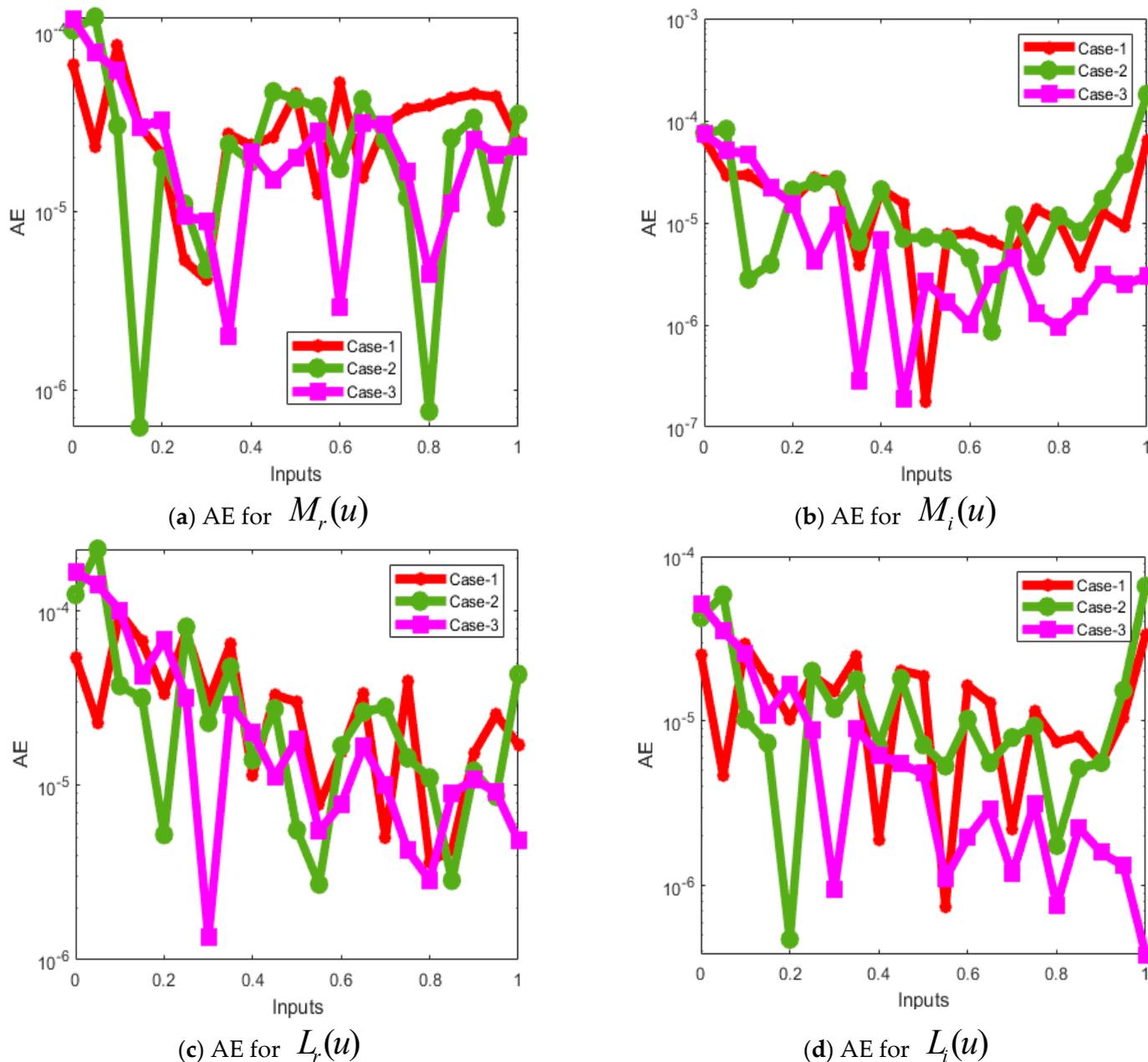


Figure 7. AE performances to solve the FO-MML love story model using the SCGNNs.

5. Conclusions

The current study presents the fractional order mathematical model based on the Majnun and Layla romantic story by applying the SCGNNs. The fractional types of derivatives have been implemented to achieve more accurate and realistic results with the Majnun–Layla romantic story model. The exactness of the model is authenticated by using the comparison of obtained and reference results. The parameter values have been adjusted to find the numerical solutions of the model. Thirteen numbers of neurons have been taken using the statics of the authorization, training, and testing, which have been used as 12%, 77%, and 11% for solving the dynamics of the FO-MML model. To prove the dependability, accuracy, and capability of the stochastic SCGNNs scheme, the regression measures, EHs performances, correlation presentations, and STs standards have been provided to ensure the reliability and consistency. Moreover, the small values of the absolute error enhance the exactness of the proposed scheme.

In future, these computational numerical scheme SCGNNs can be used for stiff nature systems, fluid dynamical systems, and biological models [40–50].

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