

Article

SimSST: An R Statistical Software Package to Simulate Stop Signal Task Data †

Mohsen Soltanifar ^{1,2,*}  and Chel Hee Lee ^{3,4} 

- ¹ Biostatistics Division, Dalla Lana School of Public Health, University of Toronto, 620, 155 College Street, Toronto, ON M5T 3M7, Canada
- ² Analytics Division, College of Professional Studies, Northeastern University, 1400-410 West Georgia Street, Vancouver, BC V6B 1Z3, Canada
- ³ Department of Mathematics and Statistics, University of Calgary, 2500 University Drive NW, Calgary, AB T2N 1N4, Canada
- ⁴ Department of Critical Care Medicine, Alberta Heath Services, University of Calgary, 3260 Hospital Drive NW, Calgary, AB T2N 4Z6, Canada
- * Correspondence: mohsen.soltanifar@alum.utoronto.ca
- † This paper is an extended version of our paper published in "Soltanifar, M; Lee, C. SimSST: Simulated Stop Signal Task Data. R package version 0.0.5.2, Published on 9 January 2023, Access URL on Comprehensive R Archive Network: <https://CRAN.R-project.org/package=SimSST> (accessed on 13 January 2023)".

Abstract: The stop signal task (SST) paradigm with its original roots in 1948 has been proposed to study humans' response inhibition. Several statistical software codes have been designed by researchers to simulate SST data in order to study various theories of modeling response inhibition and their assumptions. Yet, there has been a missing standalone *statistical software package* to enable researchers to simulate SST data under generalized scenarios. This paper presents the R statistical software package "SimSST", available in Comprehensive R Archive Network (CRAN), to simulate stop signal task (SST) data. The package is based on the general non-independent horse race model, the copulas in probability theory, and underlying ExGaussian (ExG) or Shifted Wald (SW) distributional assumption for the involving go and stop processes enabling the researchers to simulate sixteen scenarios of the SST data. A working example for one of the scenarios is presented to evaluate the simulations' precision on parameter estimations. Package limitations and future work directions for its subsequent extensions are discussed.



Citation: Soltanifar, M.; Lee, C.H. SimSST: An R Statistical Software Package to Simulate Stop Signal Task Data. *Mathematics* **2023**, *11*, 500. <https://doi.org/10.3390/math11030500>

Keywords: R; simulation; stop signal task

MSC: 62-04; 68N01; 92-10

Academic Editor: José Antonio Roldán-Nofuentes

Received: 20 December 2022
Revised: 13 January 2023
Accepted: 14 January 2023
Published: 17 January 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

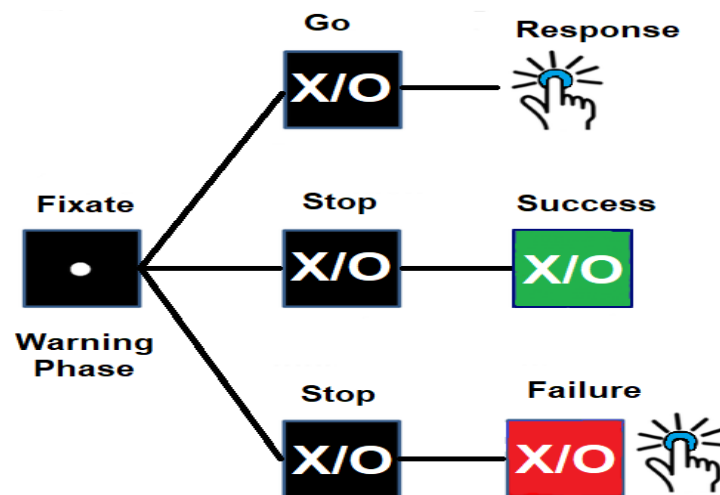
1. Introduction

This introduction is divided into five subsections as follows: Section 1.1 provides a brief review of the stop signal task. Section 1.2 presents the horse race model (independent/generally non-independent) as one of the main methods in which the stop signal task is studied. Section 1.3 provides a summary of two key parametric reaction times distributions in which the underlying processes in the horse race model in this work are modeled. Section 1.4 discusses the motivations behind the "SimSST" statistical software package. Finally, Section 1.5 presents the study outline.

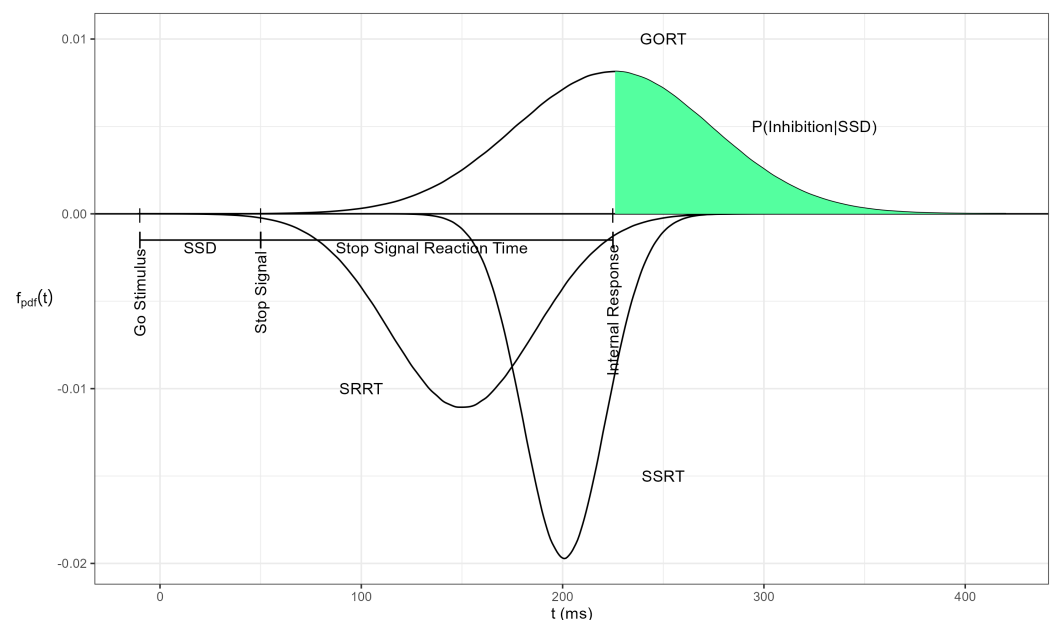
1.1. Stop Signal Task

The concept of "Response Inhibition" refers to the situations in which the current ongoing course of actions or thoughts must be changed, controlled, or stopped [1]. The inhibition can be either proactive or reactive [2,3], and the paradigms commonly used for its study are the go/no-go task (GNGT) [4] and the stop signal task (SST) [5–7]. The earliest model of the SST was proposed by Vince in 1948, and more complete versions were proposed in the following

decades for two-choice tasks, go and stop trials [7,8]. The experiment starts with a fixation period on the computer screen. In the go tasks, a random series of X or O symbols are shown to the participant on the computer screen. The participant is then instructed to press the correct gamepad button associated with either X or O. Approximately 25% (i.e., probability of stop signal delay $p_{ss} = 0.25$) of these trials have a stop signal after taking a short time of delay, which is called a stop signal delay time (SSD). The participant is instructed with an auditory beep sign to withhold pressing the game device buttons. The outcome from stop trials is categorized into one of successful inhibition (i.e., withholding pressing the device button) or failed inhibition (i.e., already have been pressing the device). Figure 1a presents the stop signal task.



(a) Stop Signal Task Trials



(b) The Horse Race Model

Figure 1. (a) The standard stop signal task; (b) Graphical representation of the complete horse race model: GORT: go reaction times, SRRT: signal respond reaction times, SSRT: stop signal reaction times, SSD: stop signal delay.

The stop signal task has been proposed through four methods: the deadline model [9], the independent horse race model [10], the interactive horse race model [11], and the Hanes–

Carpenter model [12]. We focus on the independent horse race model in details in the next section and its generalized non-independent version.

1.2. The Horse Race Model

The race model is founded on the notion of the race between two go process (GORT) and the stop process (SSRT) that is initiated upon triggering stop signal delay (SSD). Here, if $GORT > SSD + SSRT$, then a successful inhibition of the primary go task is executed. Otherwise, a failed inhibition of the primary go task is recorded. The GORT in this type of trial is called the signal respond reaction times (SRRT). Figure 1b illustrates the complete independent horse race model. There are three presumptions:

First, there is stochastic independency (Stoch. Ind.) where GORT in go trials and SSRT in stop trials are independent (i.e., $GORT \perp SSRT$).

Second, there is contextual independency where GORT in go trials and GORT in stop trials have the same distribution (i.e., $GORT \stackrel{d}{=} (GORT|SSD)$).

Third, there is a non-constant status for SSRT as a random variable.

We refer to the general non-independent horse race model as a complete horse race model in which the assumption of the stochastic independency of GORT and SSRT processes is relaxed. Since a non-zero Spearman correlation implies statistical dependence, [13] (Proposition 7.1.3(i)), we will quantify the dependence of GORT and SSRT in terms of their non-zero Spearman correlation.

The entire distribution of SSRT can be modeled with non-parametric Frequentist methods [14] or Bayesian Parametric Approaches (BPA) with various features such as Individual BPA (IBPA) and Hierarchical BPA (HBPA) [15] and mixture methods [16]. In the next section, we discuss two useful distributions to model GORT and SSRT: Exponentially modified Gaussian (ExG) and Shifted Wald (SW).

1.3. The Key Reaction Times Distributions

Distributions proposed to model reaction time can be grouped into two families [17–25]. One is the family of convolutionary distributions such as Exponentially modified Gaussian (ExG) and Exponentially modified Wald (ExW) [19,20]. The other is the family of non-convolutionary distributions such as Gumbel, Weibull, Lognormal, Gamma and Shifted Wald (SW) distributions [21–25]. Our work in this paper focuses on the ExG distribution from the convolutionary family with a monotonically increasing hazard function and the SW distribution from the non-convolutionary family with a peaked hazard function. Table 1 presents key information regarding ExG and SW distributions.

Table 1. A summary of Exponentially modified Gaussian (ExG) and Shifted Wald (SW) distributions' key features.

Distribution	ExG(μ, σ, τ)	SW(μ, σ, τ)
Representation	$N(\mu, \sigma^2) \oplus \text{Exp}(\tau)$	$\text{Wald}(\mu, \sigma^2) \oplus \delta_\tau$
Domain	$t \in \mathbb{R}$	$t \in \mathbb{R}_\tau^+$
Parameters	$\mu \in \mathbb{R}, \sigma \in \mathbb{R}^+, \tau \in \mathbb{R}^+$	$\mu \in \mathbb{R}^+, \sigma \in \mathbb{R}^+, \tau \in \mathbb{R}_0^+$
PDF	$\frac{1}{\tau} \exp(-\frac{t-\mu}{\tau} + \frac{\sigma^2}{2\tau^2}) \Phi(\frac{t-\mu}{\sigma} - \frac{\sigma}{\tau})$	$\frac{1}{\sqrt{2\pi}\sigma(t-\tau)^{\frac{3}{2}}} \exp(-\frac{(t-\tau-\mu)^2}{2\sigma^2\mu^2(t-\tau)})$
CDF	$\Phi(\frac{t-\mu}{\sigma}) - \exp(-\frac{t-\mu}{\tau} + \frac{\sigma^2}{2\tau^2}) \Phi(\frac{t-\mu}{\sigma} - \frac{\sigma}{\tau})$	$\Phi(\frac{t-\tau-\mu}{\sigma\mu\sqrt{t-\tau}}) + \exp(\frac{2}{\mu\sigma^2}) \Phi(\frac{-(t-\tau)-\mu}{\sigma\mu\sqrt{t-\tau}})$
Mean	$\mu + \tau$	$\mu + \tau$
Variance	$\sigma^2 + \tau^2$	$\mu^3\sigma^2$
Skewness	$2\sigma^{-3}\tau^3(1 + \sigma^{-2}\tau^2)^{-\frac{3}{2}}$	$3(\mu\sigma^2)^{\frac{1}{2}}$
Kurtosis	$3(1 + 2\sigma^{-2}\tau^2 + 3\sigma^{-4}\tau^4)(1 + \sigma^{-2}\tau^2)^{-2}$	$15\mu\sigma^2 + 3$
Hazard Function	\nearrow	\curvearrowright

Note: Φ is the CDF of the standard normal distribution.

Note that both ExG and SW are three parametric, right-skewed, leptokurtic, and, generally non-members of the exponential family. However, they differ in terms of domains

and functional behaviors. For example, ExG is defined on all real values, while SW is defined on only non-negative real values (or real values not smaller than $\tau \geq 0$). In addition, the ExG hazard function is monotonically increasing, while the SW hazard function is peaked at some values in the domain. Figure 2 presents these distributions hazard function behaviors under a variety of scenarios of their associated parameter changes.

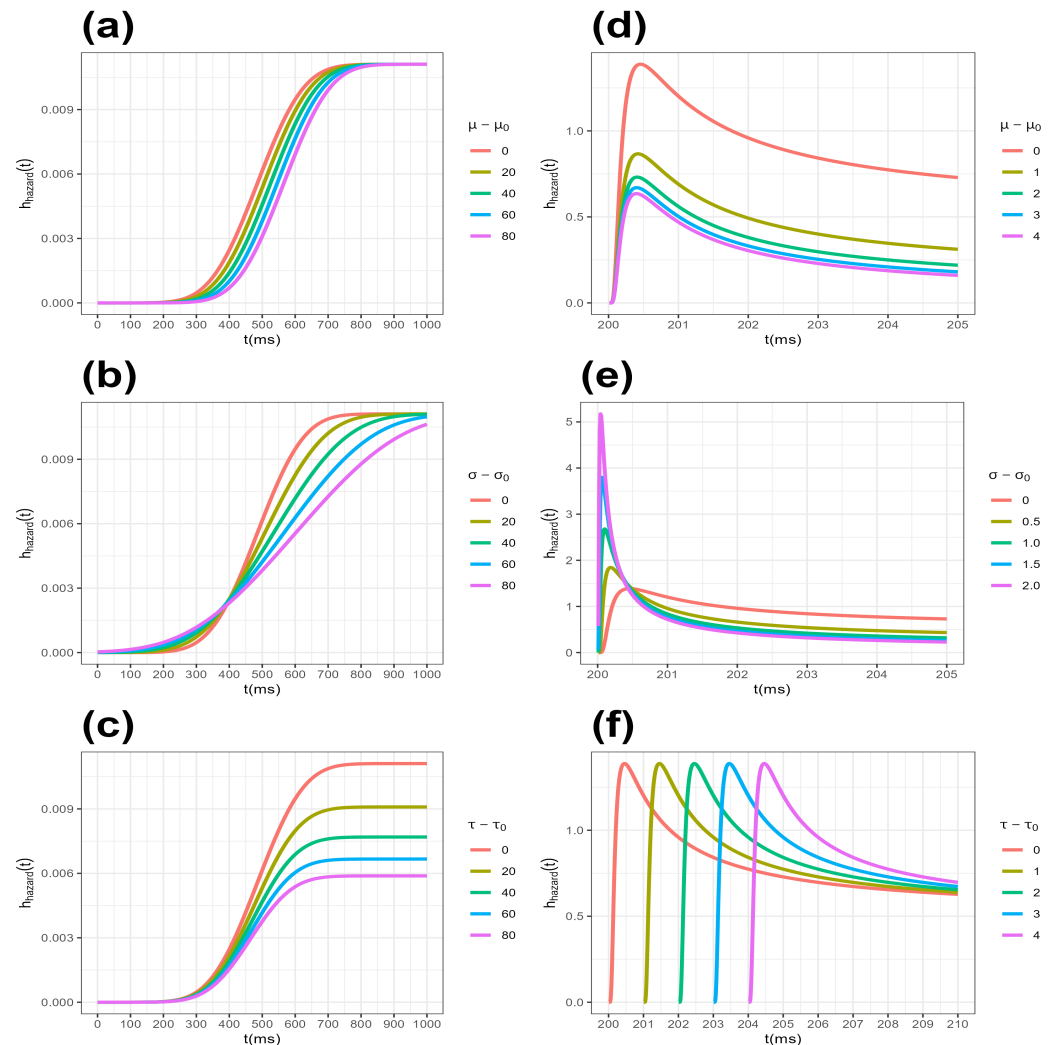


Figure 2. (a–c) ExG distribution hazard function change by parameters given $\mu_0 = 440, \sigma_0 = 90, \tau_0 = 90$; (d–f) SW distribution hazard function change by parameters given $\mu_0 = 1, \sigma_0 = 1, \tau_0 = 200$.

1.4. Motivation

Until the early 2020s, several studies have tested various hypotheses in the stop signal paradigm concerning inhibition functions, the validity of assumptions of the horse race model, and a new index estimation of SSRT using simulations [26–32]. One may find the codes used for the simulations of the stop signal task in C [26], Matlab [27], R [28–30], and Python [31,32]. However, the authors believe further improvement is needed for the following reasons.

First, based on our literature review, we could not find a standalone *statistical software package* that allows researchers to simulate the stop signal task data effectively, easily, and timely.

Second, the scripts noted above are based on the assumptions that conceptualize the go-and-stop processes using the ExG distribution, which violates peak hazard behavior observed in many empirical reaction times. This violation ultimately restricts the use of their implementations.

Third, either fixed SSD or the tracking methods are written in the scripts noted above—there are no tools allowing researchers to work simultaneously with both methods yet.

Fourth, the above methods all use an independent horse race model, yet no statistical software code has been proposed until now to simulate the generally non-independent horse race model.

Finally, R provides a flexible and rich environment to work with other data simulation packages successfully established in various medical fields, such as psychology and neuroscience [33–38]. In addition, the researchers can take advantage of open-source and free software across platforms with active community development [39,40].

Hence, the authors' main motivation to present the R package, SimSST [41], available on CRAN is to enable the researchers to simulate the stop signal task data easier, freely, faster, more efficient, effectively, and with continuous community support.

1.5. Study Outline

This paper is organized as follows. We start by providing minimal but required theories and instructions for installing the SimSST package. The main functions simulating SST data with fixed SSD and tracking methods are then illustrated. Next, we present a simulation study for estimating the parameters used in distributions of modeling GORT and SSRT using the package. Finally, we discuss the contribution of the SimSST package to the stop signal task study, current limitations, and future development plans.

2. The SimSST Package

2.1. The Background and Installation

The “SimSST” package is based on the general non-independent horse-race model. Each round of simulations is carried out with the following steps:

1. Set the simulation method (fixed SSD or tracking).
2. Set the stochastic independency status of GORT and SSRT processes (Yes or No).
3. Set distribution type for GORT and SSRT and also set starting SSD (one of seven distributions: ExW, ExG, SW, Gumbel, Weibull, Lognormal and Gamma).
4. Simulate each process (either independently or using Copulas).
5. Test if the response from the first stop trial meets $SSRT + SSD < GORT$. If so, the trial is marked as successful inhibition. Otherwise, failed inhibition.
6. Test and record the status of all successive stop trials.
 - (a) Fixed SSD Method—Repeat checking if the following stop trial status is similar to the first one without a change in the value of SSD in each stop trial.
 - (b) Tracking Method—Check the status of the first stop trial. If a stop is successful, add a redesigned constant d (e.g., 50 ms) to the SSD to check the status of the second stop trial. Otherwise, subtract d (e.g., 50 ms) from the SSD to check that trial status. Repeat this algorithm for the successive stop trials until the end.

The simulation process presented above may involve $2 \times 2 \times 7^2 = 196$ potential scenarios. Table 2 presents sixteen ($2 \times 2 \times 2^2 = 16$) scenarios offered by the package SimSST (version 0.0.5.2).

The package “SimSST” depends on three R packages: “gamlss.dist” [42], “MASS” [43], and “dplyr” [44], and it was developed under R-4.1.3 [45]. The package (version 0.0.5.2) is available from the Comprehensive R Archive Network (CRAN) at <https://CRAN.R-project.org/package=SimSST> (accessed on 9 January 2023). The following code chunk shows how to install and load the package:

```
line #1:> install.packages("SimSST", dependencies=TRUE)
line #2:> library(SimSST)
```

The “SimSST” package involves three major multi-variate functions `simssfixed()`, `simsstrack()` and `simssgen()`. The first two are based on an independent horse-race model, and the third one is based on a general non-independent horse-race model. Details are

described in the coming sections. The reader may try to type `?simssfired`, `?simsstrack` or `?simssgen` in R for the details.

Table 2. A summary of potential simulation scenarios by R package “SimSST” (version 0.0.5.2)).

Scenario	Method	Stoch. Ind.	GORT	SSRT	Function
1	Fixed SSD	Yes	ExG	ExG	simssfired()/simssgen()
2				SW	simssfired()/simssgen()
3			SW	ExG	simssfired()/simssgen()
4				SW	simssfired()/simssgen()
5		No	ExG	ExG	simssgen()
6				SW	simssgen()
7			SW	ExG	simssgen()
8				SW	simssgen()
9	Tracking	Yes	ExG	ExG	simsstrack()/simssgen()
10				SW	simsstrack()/simssgen()
11			SW	ExG	simsstrack()/simssgen()
12				SW	simsstrack()/simssgen()
13		No	ExG	ExG	simssgen()
14				SW	simssgen()
15			SW	ExG	simssgen()
16				SW	simssgen()

2.2. The Fixed SSD Based Simulated SST Data

The function `simssfired()` simulates the SST data based on the independent horse race model and the fixed SSDs and needs the following nine input parameters to simulate SST data. For given $b \geq 1$, *id* is a b -sized character vector for participants' IDs. The parameters `block`, `n`, and `m` are b -sized numeric vectors of block numbers, total trials per block, and stop trials per block, respectively. `SSD.b` is a b -sized vector of initial stop signal delays on each block. The `dist.go` and `theta.go` parameters are b -sized character vectors, specifying the type of GORT distributions, and a $b \times 3$ numeric matrix having the specification of their distributions, respectively. Similarly, `dist.stop` and `theta.stop` are a b -sized numeric vector specifying the type of SSRT distributions and a $b \times 3$ numeric matrix having the specification of their distributions. The function general code in R is:

```
simssfired(pid,block,n,m,SSD.b,dist.go,theta.go,dist.stop,theta.stop)
```

2.3. The Tracking Method Based Simulated SST Data

The function `simsstrack()` similarly simulates the SST data based on the independent horse race model and the tracking method in which at each stage a constant of 50 ms is added or subtracted to the following SSDs. Its parameters are the same as those of the former function `simssfired()` and its code in R is the following:

```
simsstrack(pid,block,n,m,SSD.b,dist.go,theta.go,dist.stop,theta.stop)
```

2.4. The General Tracking Method Based Simulated Correlated SST Data

The functions `simssfired()` and `simsstrack()` proposed in the package “SimSST” have two limitations. First, both are based on the stochastic independence of GORT and SSRT processes. Recent studies have shown a severe violation of the assumptions needed for the independent horse race paradigm [46]. Solutions to overcome this issue are considering SSDs from 200 ms and incorporation of the Spearman's rank correlation [47] for the case of the relaxation of the assumption of stochastic independence between GORT and SSRT. (Note that for simulation purposes, the Pearson correlation is an inappropriate parameter as it may not be preserved under monotonic transformations of the original simulated data [47].) Second, there is no mechanism to connect the functions `simssfired()` and `simsstrack()`. In particular, we know that replacing the constant added (subtracted) value of 50 ms to the

SSD in the subsequent stop trials with 0 converts `simsstrack()` function to the `simssfired()`. To solve these limitations, we may consider the generalized simulation function `simssgen()` based on the theory of copulas with eleven parameters as follows:

`simssgen(pid, block, n, m, SSD.b, dist.go, theta.go, dist.stop, theta.stop, rho, d)`

where in which the first nine parameters are exactly as those in the functions `simssfired()` and `simsstrack()` and the additional parameters ρ and d refer to b -sized numeric vectors of the Spearman correlation of GORT and SSRT processes and the b -sized numeric vectors of measure of added (subtracted) constant to the initial SSD values, respectively. In particular, denoting the first nine parameters with “–”, we will have:

$$\text{simssgen}(-, \rho = (0)_{b \times 1}, d = (0)_{b \times 1}) = \text{simssfired}(-) \quad (1)$$

$$\text{simssgen}(-, \rho = (0)_{b \times 1}, d = (50)_{b \times 1}) = \text{simsstrack}(-). \quad (2)$$

As the first working example for the fixed SSD-based simulated SST data, let us consider a participant John Smith for two rounds (i.e., blocks) of stop signal task trials, each of size ten trials, including four random stop trials. We assume that for these rounds of SST trials, the initiating stop signal delays are 220 ms and 240 ms, respectively. A further assumption is that the GORT and SSRT are independently distributed as $ExG(\mu = 440, \sigma = 90, \tau = 90)$ and $ExG(\mu = 120, \sigma = 80, \tau = 70)$, respectively. The R code given bellow produces the simulated dataset presented in Figure 3a (note that the probability of stop signal is given by $p_{ss} = m/n = 0.40$):

```
line #1:> mySSTdata1<- simssgen(
  pid = c("John.Smith", "John.Smith"),
  block = c(1,2),
  n = c(10,10),
  m = c(4,4),
  SSD.b = c(220,240),
  dist.go = c("ExG", "ExG"),
  theta.go = as.matrix(rbind(c(440,90,90),c(440,90,90))),
  dist.stop = c("ExG", "ExG"),
  theta.stop = as.matrix(rbind(c(120,80,70),c(120,80,70))),
  rho=c(0,0),
  d=c(0,0))
line #2:> mySSTdata1
```

As the second working example for the tracking method-based simulated SST data, we now consider another participant called Jane McDonald with the same parameter specification used for the former participant, John Smith. Then, with given tracking method in which $d = +50$ ms, the R code given below produces Figure 3b:

```
line #1:> mySSTdata2<- simssgen(
  pid = c("Jane.McDonald", "Jane.McDonald"),
  block = c(1,2),
  n = c(10,10),
  m = c(4,4),
  SSD.b = c(220,240),
  dist.go = c("ExG", "ExG"),
  theta.go = as.matrix(rbind(c(440,90,90),c(440,90,90))),
  dist.stop = c("ExG", "ExG"),
  theta.stop = as.matrix(rbind(c(120,80,70),c(120,80,70))),
  rho=c(0,0),
  d=c(50,50))
line #2:> mySSTdata2
```

	Participant.id	Block	Trial	Inhibition	GORT	SSRT	SRRT	SSD
1	John.Smith	1	Stop	0	-999	117.4	250.6	220
2	John.Smith	1	Go	-999	450.8	-999	-999	-999
3	John.Smith	1	Go	-999	257.4	-999	-999	-999
4	John.Smith	1	Go	-999	453	-999	-999	-999
5	John.Smith	1	Go	-999	689.3	-999	-999	-999
6	John.Smith	1	Stop	1	-999	115.3	-999	220
7	John.Smith	1	Stop	1	-999	170.4	-999	220
8	John.Smith	1	Go	-999	534.9	-999	-999	-999
9	John.Smith	1	Go	-999	514.8	-999	-999	-999
10	John.Smith	1	Stop	1	-999	115.7	-999	220
11	John.Smith	2	Go	-999	490.9	-999	-999	-999
12	John.Smith	2	Go	-999	480.2	-999	-999	-999
13	John.Smith	2	Go	-999	650.3	-999	-999	-999
14	John.Smith	2	Stop	1	-999	217.1	-999	240
15	John.Smith	2	Stop	1	-999	253.3	-999	240
16	John.Smith	2	Go	-999	601.2	-999	-999	-999
17	John.Smith	2	Stop	1	-999	202.5	-999	240
18	John.Smith	2	Go	-999	505.8	-999	-999	-999
19	John.Smith	2	Go	-999	511.6	-999	-999	-999
20	John.Smith	2	Stop	1	-999	57.1	-999	240

Showing 1 to 20 of 20 entries, 8 total columns

(a) Fixed SSD method

	Participant.id	Block	Trial	Inhibition	GORT	SSRT	SRRT	SSD
1	Jane.McDonald	1	Stop	1	-999	263.8	-999	220
2	Jane.McDonald	1	Go	-999	609.7	-999	-999	-999
3	Jane.McDonald	1	Stop	1	-999	78.6	-999	270
4	Jane.McDonald	1	Go	-999	350	-999	-999	-999
5	Jane.McDonald	1	Stop	0	-999	138.8	392.1	320
6	Jane.McDonald	1	Go	-999	593.4	-999	-999	-999
7	Jane.McDonald	1	Stop	1	-999	81.1	-999	270
8	Jane.McDonald	1	Go	-999	524.2	-999	-999	-999
9	Jane.McDonald	1	Go	-999	427.1	-999	-999	-999
10	Jane.McDonald	1	Go	-999	430.1	-999	-999	-999
11	Jane.McDonald	2	Stop	1	-999	175.5	-999	240
12	Jane.McDonald	2	Go	-999	457.6	-999	-999	-999
13	Jane.McDonald	2	Stop	0	-999	175.6	443.4	290
14	Jane.McDonald	2	Go	-999	351.2	-999	-999	-999
15	Jane.McDonald	2	Stop	1	-999	74	-999	240
16	Jane.McDonald	2	Go	-999	479.3	-999	-999	-999
17	Jane.McDonald	2	Stop	0	-999	277.6	436.1	290
18	Jane.McDonald	2	Go	-999	709.8	-999	-999	-999
19	Jane.McDonald	2	Go	-999	521.9	-999	-999	-999
20	Jane.McDonald	2	Go	-999	555.1	-999	-999	-999

Showing 1 to 20 of 20 entries, 8 total columns

(b) Tracking method

Figure 3. Sample SimSST R software package(R studio version 4.1.3) simulated SST data output with input variables as in the text.

2.5. Simulation Study: An Example

This section provides an example (scenario #9 in Table 2) showing the analysis of SST trial data generated by the SimSST package. Consider a single participant for one block of trials with the following input parameters (we assume that the proportion of stop stimuli is $p_{ss} = m/n = 0.25$).

Method:	Tracking
Stoch. Ind. Status:	Independent
Initial SSD:	220 ms
d parameter:	50 ms
total trials:	$n = 100$ k ($1 \leq k \leq 10$)
stop trials:	$m = 25$ k ($1 \leq k \leq 10$)
GORT distribution:	ExGaussian(ExG)
$\theta_{go} = (\mu_{go}, \sigma_{go}, \tau_{go})$	(440, 90, 90)
SSRT distribution:	ExGaussian(ExG)
$\theta_{stop} = (\mu_{stop}, \sigma_{stop}, \tau_{stop})$	(120, 80, 70).

We now generate the first of ten simulated data using the function `simsstrack` in the SimSST package.

```
line #1:> library(SimSST)
line #2:> SimSST_Data1 <- simssgen(
  pid = c("FNLN1"),
  block = 1,
  n = 100,
  m = 25,
  SSD.b = 220,
  dist.go = "ExG",
  theta.go = as.matrix(rbind(c(440,90,90))),
  dist.stop = "ExG",
  theta.stop = as.matrix(rbind(c(120,80,70))),
  rho=0,
  d=50)
line #3:> SimSST_Data1
```

After saving all ten datasets in .csv format, we convert them as input to the BEESTS software [48] with the following minor modifications: (i) Record positive SSRT values in the GORT column, (ii) Rename the columns of *Trial*, *Inhibition*, *SSD* and *GORT* to *ss-presented*, *inhibited*, *ssd* and *rt*, respectively, and (iii) Sort the dataset using the *ss-presented* column in ascending order. We provide the R code in Appendix A to avoid these tedious manual modifications. The reader can access the simulated datasets in the supplementary materials.

We now have the BEESTS formatted data for analysis. The following prior specification and MCMC setting are used to apply the standard BEESTS IBPA [15,48]:

Prior Setting Information:

$\mu_{go} \sim \text{Uniform}[1, 1000]$:	start value: 500
$\sigma_{go} \sim \text{Uniform}[1, 500]$:	start value: 150
$\tau_{go} \sim \text{Uniform}[1, 500]$:	start value: 150
$\mu_{stop} \sim \text{Uniform}[1, 1000]$:	start value: 300
$\sigma_{stop} \sim \text{Uniform}[1, 500]$:	start value: 150
$\tau_{stop} \sim \text{Uniform}[1, 500]$:	start value: 150

MCMC Setting Information:

Number of chains:	3
Samples:	50,000
Burn in:	5000
Thining:	5
Limits of Integration:	1–10,000
With Trigger Failure(WTF):	No.

Figure 4 shows each parameter estimate of GORT and SSRT with their 95% CrI fitted by the BEESTS IBPA. We confirmed that for all cases, the parameters used for generating the SST trial dataset are captured by the 95% CrI. Note that the estimation of the GORT parameters can generally be improved by increasing the total trial sample size, resulting in a narrower 95% CrI. On the other hand, estimating the parameters associated with SSRT generally produces a wider 95% CrI, and some oscillations for the τ_{stop} parameter are found.

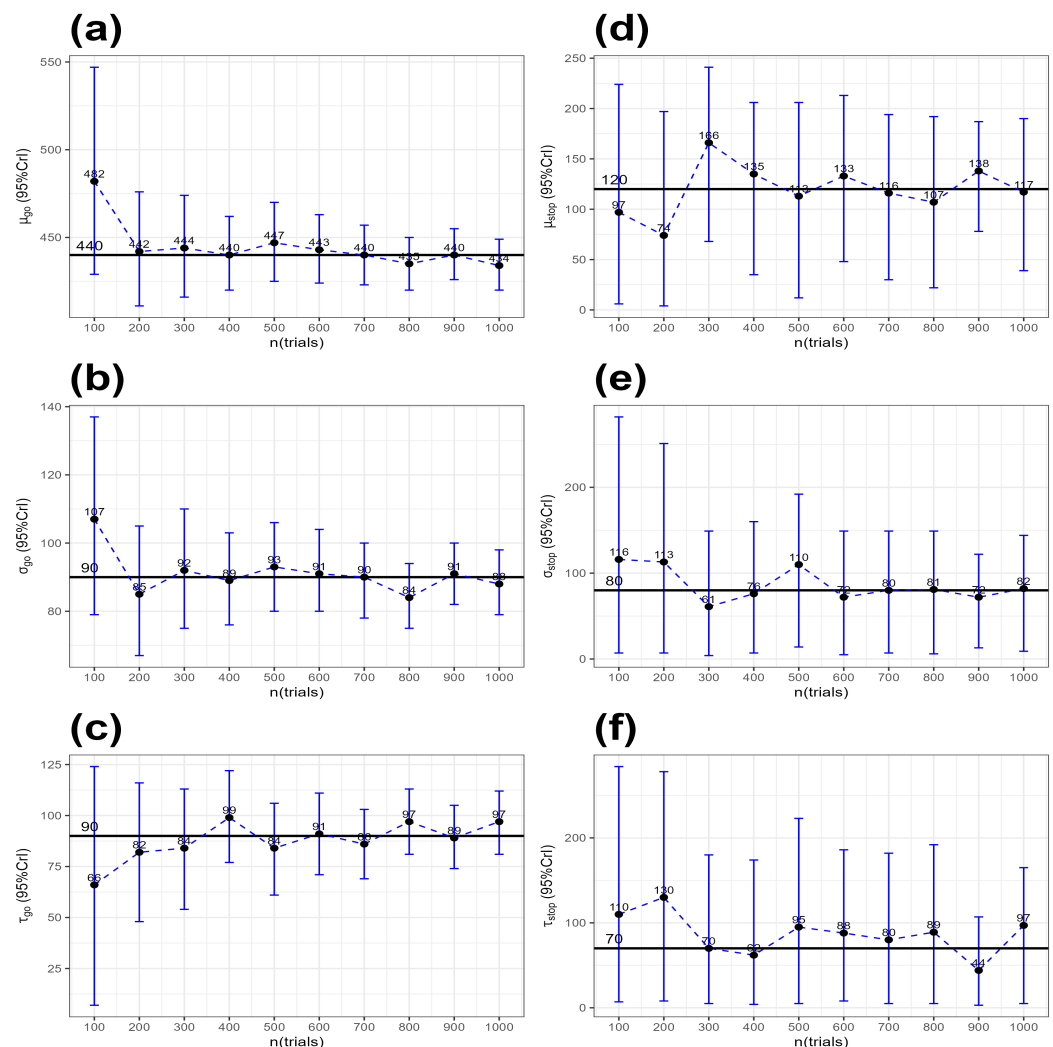


Figure 4. BEETS method parameters estimations for the SimSST-based simulated SST data with true parameter values on the black horizontal line: (a–c) GORT ExG parameters; (d–f) SSRT ExG parameters.

Simulation performance is discussed using ten mean posterior estimates of each the six parameters $\theta \in (\mu_{go}, \sigma_{go}, \tau_{go}, \mu_{stop}, \sigma_{stop}, \tau_{stop})$. We tested $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$, where θ_0 is the predefined true parameter value used for generating data, using one-sample t -test and one sample Wilcoxon signed-rank test. As reported in Table 3, all six parameters

are not significantly different from the one assumed given non-multiplicity (note that we have stronger evidence for non-significant results as well when controlling for multiplicity with Holm's method for $m = 6$, [49]). However, relatively high PE values associated with the parameter τ_{stop} are shown in both tests. This result might be attributed to the small stop trials sample size and wider prior distributions specified in the BEESTS IBPA. This finding cautions the researchers to choose more informative stop priors than those used in the standard BEESTS IBPA.

Table 3. A summary of one sample t -test and one sample Wilcoxon signed-ranked test for each of individual parameters ($n = 10$).

$H_0 : \theta = \theta_0$ $H_1 : \theta \neq \theta_0$						
$\theta = \theta_0$	$\mu_{go} = 440$	$\sigma_{go} = 90$	$\tau_{go} = 90$	$\mu_{stop} = 120$	$\sigma_{stop} = 80$	$\tau_{stop} = 70$
(a) t -test						
$\hat{\theta}(95\%CI)$	445(435,454)	91(86,96)	88(81,94)	120(102,138)	86(72,100)	86(69,104)
Sig.(2-sided)	0.305	0.629	0.436	0.961	0.332	0.061
PE(%)	1.1	1.1	2.2	0.0	7.5	22.9
(b) Wilcoxon test						
$\hat{\theta}(95\%CI)$	443(435,464)	90(87,98)	89(81,94)	120(102,138)	87(71,110)	89(66,110)
Sig.(2-sided)	0.447	0.953	0.721	1.000	0.635	0.076
PE(%)	0.7	0.0	1.1	0.0	8.8	27.1

Note: Percentage Error (PE) was calculated as $PE = |(\hat{\theta} - \theta_0) / \theta_0| \times 100$.

3. Discussion

3.1. Summary and Contributions

We provide an R package, SimSST, which is the first statistical software package implementation of simulating stop signal task data since its concept was introduced in 1948. The package can simulate data (1) using either a fixed SSD or tracking method and/or (2) either stochastically independent or dependent GORT and SSRT processes and/or (3) either monotone increasing hazard or peaked hazard reaction times distribution describing GORT and SSRT processes. In addition, flexibility on controlling the number of stop trials and total trials allows researchers to explore the asymptotic property through testing the hypothesis. We anticipate that the package assists the researchers to simulate SST under various conditions using a general non-independent horse race paradigm efficiently. As an application, one may simulate SST data under various scenarios and investigate its connection to fMRI-EEG data to investigate the missing angles of the associated results to the empirical data in the past studies [50].

3.2. Limitations and Future Work

Our work has several limitations, but they are taken as new perspectives for extensions as follows:

First, the current version of the SimSST package supports a limited number of distributions, such as ExG and SW for a three-parameter family of distributions. The key two parametric candidates such as Weibull, Gamma, and Lognormal [51] need inclusion. One may ask to include an Exponentially modified Wald (ExW) distribution [52] for the three-parameter family for the next version.

Second, the package “SimSST” does not yet address simulation trials with trigger failures (TF). This issue creates another dimension for its extension in the subsequent version.

Third, the scope of a simulation study using the SimSST package depends on the current support of BEESTS software. For example, BEESTS does not include SW distributional assumption. This limits parameter estimations scenarios.

Fourth, the illustrative example with this SimSST package is based on one participant with a single block trial with various sizes of trials. The extension of this example to the case with several participants (each with several blocks of trials) needs investigation. The implementation of this scenario requires the hierarchical Bayesian parametric approach (HBPA) supported by BEESTS.

Aside from the above four potential future work directions, there is another potential future work direction through the presented simulations in the SimSST package: investigation of behavior of inhibition function $P(\text{Inhibition}|\text{SSD})$ (Figure 1b) in terms of relative position of GORT and SSRT processes when they are simulated from the same parametric distributions [53]. Here, given scenarios 1, 4, 5, 8, 9, 12, 13 and 16 in Table 2, it is plausible to explore the behavior of the inhibition function when the GORT process is weakly faster or strongly faster (or none) versus the SSRT process. A final classification of models of response inhibition over these results [54] and across a spectrum of clinical groups will widen our understanding of the underlying brain inhibition processes.

3.3. Conclusions

The free SimSST R package enables one to simulate stop signal task data under a generally non-independent horse race paradigm, ExG and SW parametric assumptions for the GORT/SSRT processes and both fixed SSD and tracking methods. The simulated datasets help researchers test their plausible hypothesis regarding various stopping processes within the stop signal task literature. This saves researchers the required time and costs associated to the empirical SST data.

Supplementary Materials: The following supporting information can be downloaded at: <https://www.mdpi.com/article/10.3390/math11030500/s1>, (i) “SimSST” R package CRAN documentation; (ii) ten simulated SST datasets by the package “SimSST”; (iii) their reformatted versions for the BEESTS package; and (iv) the BEESTS software output for each reformatted simulated dataset.

Author Contributions: Conceptualization, M.S.; methodology, M.S.; software, M.S. and C.H.L.; validation, M.S. and C.H.L.; formal analysis, M.S.; investigation, M.S. and C.H.L.; resources, M.S.; data curation, M.S.; writing—original draft preparation, M.S. and C.H.L.; writing—review and editing, M.S. and C.H.L.; visualization, M.S. and C.H.L.; supervision, M.S. and C.H.L.; project administration, M.S. and C.H.L.; funding acquisition, M.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work had no external funding support.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not Applicable.

Data Availability Statement: Not applicable.

Acknowledgments: This paper is dedicated to the memory of Professor Emeritus Paul N. Corey (1941–2022) for his half-century service to the academic family at the University of Toronto, Canada. The corresponding author is grateful to the following researchers for their direct and indirect influence on him on the prior years yielding to this paper: Mike Escobar; Annie Dupuis, Wendy Lou; Russell Schachar; Keith Knight (University of Toronto, Canada); and Gordon Logan (Vanderbilt University, USA). Both authors are grateful to journal reviewers for constructive comments on the first draft of the paper.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

BPA: Bayesian Parametric Approach; CDF: Cumulative Distribution Function; CRAN: Comprehensive R Archive Network; CrI: Credible Interval; EEG: Electroencephalogram; ExG: Ex-Gaussian Distribution; fMRI: Functional Magnetic Resonance Imaging; GNGT: Go No-Go Task; HBPA: Hier-

archical Bayesian Parametric Approach; IBPA: Individual Bayesian Parametric Approach; MCMC: Markov Chain Monte Carlo; FNLN: First Name Last Name; PDF: Probability Density Function; PE: Percentage Error; RT: Reaction Times; SRRT: Reaction Time in a failed stop trial; SSRT: Stop Signal Reaction Times in a stop trial; SSD: Stop Signal Delay; SST: Stop Signal Task; Stochastic Independency: Stoch. Ind.; SW: Shifted Wald Distribution; TF: Trigger Failure; WTF: With Trigger Failure.

Appendix A. The R Code to Convert SimSST Files to BEESTS Input Files

```
line #1:> mypackages <- c("dplyr", "SimSST")
line #2:> lapply(mypackages, require, character.only = TRUE)
line #3:> Data1 <- cbind.data.frame(
  ss_presented = recode(SimSST_Data1[,3], 'Stop' = "1", 'Go' = "0"),
  inhibited = SimSST_Data1[,4],
  ssd = SimSST_Data1[,8],
  rt = SimSST_Data1[,5],
  srrt = SimSST_Data1[,7]
)
line #4:> for(i in 1:100)
  if(Data1$inhibited[i]==0) Data1$rt[i] <- Data1$srrt[i]
line #5:> BEESTS_Data1 <- (Data1[, -5])[order(Data1$ss_presented),]
line #6:> BEESTS_Data1
```

References

- Schachar, R.J.; Logan, G.D.; Robaey, P.; Chen, S.; Ickowicz, A.; Barr, C. Restraint and Cancellation: Multiple Inhibition Deficits in Attention Deficit Hyperactivity Disorder. *J. Abnorm. Child Psychol.* **2007**, *35*, 229–238. [\[CrossRef\]](#) [\[PubMed\]](#)
- van den Wildenberg, W.P.M.; Ridderinkhof, K.R.; Wylie, S.A. Towards Conceptual Clarification of Proactive Inhibitory Control: A Review. *Brain Sci.* **2022**, *12*, 1638. [\[CrossRef\]](#) [\[PubMed\]](#)
- Soltanifar, M.; Escobar, M.; Dupuis, A.; Chevrier, A.; Schachar, R. The Asymmetric Laplace Gaussian (ALG) Distribution as the Descriptive Model for the Internal Proactive Inhibition in the Standard Stop Signal Task. *Brain Sci.* **2022**, *12*, 730. [\[CrossRef\]](#) [\[PubMed\]](#)
- Trommer, B.L.; Hoepfner, J.B.; Lorber, R.; Armstrong, K.J. The Go-No-Go Paradigm in Attention Deficient Disorder. *Ann. Neurol.* **1988**, *24*, 610–614. [\[CrossRef\]](#) [\[PubMed\]](#)
- Logan, G.D.; Cowan, W.B. On the Ability to Inhibit Thought and Action: A Theory of an Act of Control. *Psychol. Rev.* **1984**, *91*, 295–327. [\[CrossRef\]](#)
- Verbruggen, F.; Aron, A.R.; Band, G.H.; Beste, C.; Bissett, P.G.; Brockett, A.T.; Brown, J.W.; Chamberlain, S.R.; Chambers, C.D.; Colonius, H.; et al. A consensus guide to capturing the ability to inhibit actions and impulsive behaviors in the stop-signal task. *eLife* **2019**, *8*, e46323. [\[CrossRef\]](#)
- Logan, G.D.; van Zandt, T.; Verbruggen, F.; Wagenmakers, E.J. Ability to Inhibit Thought and Action: General and Special Theories of an Act of Control. *Psychol. Rev.* **2014**, *121*, 66–95. [\[CrossRef\]](#)
- Vince, M.A. The intermittency of control movements and the psychological refractory period. *Br. J. Psychology. Gen. Sect.* **1948**, *38*, 149–157.
- Ollman, R.T.; Billington, M.J. The Deadline Model for Simple Reaction Times. *Cogn. Psychol.* **1972**, *3*, 311–336. [\[CrossRef\]](#)
- Logan, G.D. Attention, Automaticity, and the Ability to Stop a Speeded Choice Response. In *Attention and Performance IX*; Long, J., Baddeley, A.D., Eds.; Hillsdale: Erlbaum, NJ, USA, 1981.
- Boucher, L.; Palmeri, T.J.; Logan, G.D.; Schall, J.D. Inhibitory Control in Mind and Brain: An Interactive Race Model of Countermanding Saccades. *Psychol. Rev.* **2007**, *114*, 376–397. [\[CrossRef\]](#)
- Hanes, D.P.; Carpenter, R.H.S. Countermanding Saccades in Humans. *Vis. Res.* **1999**, *39*, 2777–2791. [\[CrossRef\]](#) [\[PubMed\]](#)
- Athreya, K.B.; Lahiri, S.M. *Measure Theory and Probability Theory*; Springer: New York, NY, USA, 2006.
- Colonius, H. A Note on the Stop Signal Paradigm, or How to Observe the Unobservable. *Psychol. Rev.* **1990**, *97*, 309–312. [\[CrossRef\]](#)
- Matzke, D.; Dolan, C.V.; Logan, G.D.; Brown, S.D.; Wagenmakers, E.J. Bayesian Parametric Estimation of Stop signal Reaction Time Distributions. *J. Exp. Psychol. Gen.* **2013**, *142*, 1047–1073. [\[CrossRef\]](#) [\[PubMed\]](#)
- Soltanifar, M.; Escobar, M.; Dupuis, A.; Schachar, R. A Bayesian Mixture Modelling of Stop Signal Reaction Time Distributions: The Second Contextual Solution for the Problem of Aftereffects of Inhibition on SSRT Estimations. *Brain Sci.* **2021**, *11*, 1102. [\[CrossRef\]](#)
- Del Prado Martin, F.M. A Theory of Reaction Times Distributions. 2008. (Unpublished). Available online: <http://cogprints.org/6310> (accessed on 16 December 2022).

18. Palmer, E.M.; Horowitz, T.S.; Torralba, A.; Wolfe, J.M. What are the Shapes of Response Times Distributions in Visual Search? *J. Exp. Psychol.* **2011**, *37*, 58–71. [CrossRef]
19. Heatcote, A. RTSYS: A DOS Application for the Analysis of Reaction Times Data. *Behav. Res. Methods Instrum. Comput.* **1996**, *28*, 427–445. [CrossRef]
20. Schwarz, W. The Ex-Wald Distribution as a Descriptive Model of Reaction Time Data. *Behav. Res. Methods Instruments Comput.* **2001**, *33*, 457–469. [CrossRef]
21. Gumbel, E.J. *The Statistics of Extremes*; Columbia University Press: New York, NY, USA, 1958.
22. Weibull, W. A Statistical Distribution Function of wide Applicability. *J. Appl. Mech. Transform. ASME* **1951**, *18*, 293–297. [CrossRef]
23. Johnson, N.L.; Kotz, S.; Balakrishnan, N. *Continuous Univariate Distributions*, 2nd ed.; “14: Lognormal Distributions”; John Wiley & Sons: New York, NY, USA, 1994; Volume 1.
24. Lancaster, H.O. Forerunners of the Pearson chi-square. *Aust. J. Stat.* **1966**, *8*, 117–126. [CrossRef]
25. Royce, A.; Alario, F.X.; Van Maanen, L. The Shifted Wald Distribution for Response Time Data Analysis. *Psychol. Methods* **2016**, *21*, 309–327.
26. Band, G.P.; van der Molen, M.W.; Logan, G.D. Horse-race model simulations of the stop-signal procedure. *Acta Psychol.* **2003**, *112*, 105–142. [CrossRef] [PubMed]
27. Hannah, R.; Muralidharan, V.; Aron, A.R. Failing to attend versus failing to stop: Single-trial decomposition of action-stopping in the stop signal task. *Behav. Res. Methods* **2022**, 1–19. [CrossRef]
28. Weise, L.; Boecker, M.; Gauggel, S.; Falkenburger, B.; Druke, B. A reaction-time adjusted PSI method for estimating performance in the stop-signal task. *PLoS ONE* **2018**, *13*, e0210065. [CrossRef] [PubMed]
29. Soltanifar, M.; Dupuis, A.; Schachar, R.; Escobar, M. A frequentist mixture modelling of stop signal reaction times. *Biostat. Epidemiol.* **2019**, *3*, 90–108. [CrossRef]
30. Soltanifar, M.; Knight, K.; Dupuis, A.; Schachar, R.; Escobar, M. A Time Series-Based Point Estimation of Stop Signal Reaction Times: More Evidence on the Role of Reactive Inhibition-Proactive Inhibition Interplay on the SSRT Estimations. *Brain Sci.* **2020**, *10*, 598. [CrossRef] [PubMed]
31. Ye, W. Dynamics of a revised neural mass model in the stop-signal task. *Chaos Solitons Fractals* **2020**, *139*, 110004. [CrossRef]
32. Bissett, P.G.; Hagen, M.P.; Jones, H.M.; Poldrack, R.A. Design issues and solutions for stop-signal data from the Adolescent Brain Cognitive Development (ABCD) study. *ELife* **2021**, *10*, e60185. [CrossRef]
33. Nieuwoudt, C.; Brooks-Wilson, A.; Graham, J. SimRVSequences. an R package to simulate genetic sequence data for pedigrees. *Bioinformatics* **2020**, *36*, 2295–2297. [CrossRef]
34. Tripathi, S.; Lloyd-Price, J.; Ribeiro, A.; Yli-Harja, O.; Dehmer, M.; Emmert-Streib, F. sgnR: An R package for simulating gene expression data from an underlying real gene network structure considering delay parameters. *BMC Bioinform.* **2017**, *18*, 325. [CrossRef]
35. Nilforooshan, M.A. pedSimulate – An R package for simulating pedigree, genetic merit, phenotype, and genotype data. *Rev. Bras. De Zootec.* **2022**, *51*, e20210131. [CrossRef]
36. Technow, F.R. Package hypred: Simulation of Genomic Data in Applied Genetics. Ph.D. Thesis, University of Hohenheim, Institute of Plant Breeding, Seed Science and Population Genetics, Stuttgart, Germany, 2011.
37. Brilleman, S.L.; Wolfe, R.; Moreno-Betancur, M.; Crowther, M.J. Simulating Survival Data Using the simsurv R Package. *J. Stat. Softw.* **2021**, *97*, 1–27. [CrossRef]
38. Welvaert, M.; Durnez, J.; Moerkerke, B.; Verdoolaege, G.; Rosseel, Y. neuRosim: AnRPackage for Generating fMRI Data. *J. Stat. Softw.* **2011**, *44*, 1–19. [CrossRef]
39. Hackenberger, B.K. R software: Unfriendly but probably the best. *Croat. Med. J.* **2020**, *61*, 66–68. [CrossRef] [PubMed]
40. Mizumoto, A.; Plonsky, L. R as a Lingua Franca: Advantages of Using R for Quantitative Research in Applied Linguistics. *Appl. Linguist.* **2015**, *37*, 284–291. [CrossRef]
41. Soltanifar, M.; Lee, C. SimSST: Simulated Stop Signal Task Data. R Package Version 0.0.5.2. 2023. Available online: <https://CRAN.R-project.org/package=SimSST> (accessed on 9 January 2023).
42. Stasinopoulos, M.; Rigby, R. gamlss.dist: Distributions for Generalized Additive Models for Location Scale and Shape. R Package Version 6.0-5. 2022. Available online: <https://CRAN.R-project.org/package=gamlss.dist> (accessed on 9 January 2023).
43. Venables, W.N.; Ripley, B.D. *Modern Applied Statistics with S*, 4th Ed.; Springer: New York, NY, USA, 2002; ISBN 0-387-95457-0.
44. Wickham, H.; François, R.; Henry, L.; Müller, K. dplyr: A Grammar of Data Manipulation. R Package Version 1.0.10. 2022. Available online: <https://CRAN.R-project.org/package=dplyr> (accessed on 9 January 2023).
45. R Core Team. *R: A Language and Environment for Statistical Computing*; R Foundation for Statistical Computing: Vienna, Austria, 2022. Available online: <https://www.R-project.org/> (accessed on 9 January 2023).
46. Bissett, P.G.; Jones, H.M.; Poldrack, R.A.; Logan, G.D. Severe violations of independence in response inhibition tasks. *Sci. Adv.* **2021**, *7*, eabf4355. [CrossRef]
47. Wicklin, R. *Simulating Data with SAS*, 1st ed.; SAS Institute Inc.: Cary, NC, USA, 2013; pp. 169–174.
48. Matzke, D.; Love, J.; Wiecki, T.V.; Brown, S.D.; Logan, G.D.; Wagenmakers, E.J. Release the BEESTS: Bayesian Estimation of Ex-Gaussian Stop Signal Reaction Time Distributions. *Front. Psychol.* **2013**, *4*, 918. [CrossRef]
49. Holm, S. A Simple Sequentially Rejective Multiple Test Procedure. *Scand. J. Stat.* **1979**, *6*, 65–70.

50. Ko, L.-W.; Shih, Y.-C.; Chikara, R.K.; Chuang, Y.-T.; Chang, E.C. Neural Mechanisms of Inhibitory Response in a Battlefield Scenario: A Simultaneous fMRI-EEG Study. *Front. Hum. Neurosci.* **2016**, *10*, 1–15. [\[CrossRef\]](#)
51. Rouder, J.F. Are Un-shifted Distributional Models Appropriate for Response Time? *Psychometrika* **2005**, *70*, 377–381. [\[CrossRef\]](#)
52. Heathcote, A. Fitting Wald and ex-Wald distributions to response time data: An example using functions for the S-PLUS package. *Behav. Res. Methods Instruments Comput.* **2004**, *36*, 678–694. [\[CrossRef\]](#)
53. Soltanifar, M. A Look at the Primary Order Preserving Properties of Stochastic Orders: Theorems, Counterexamples and Applications in Cognitive Psychology. *Mathematics* **2022**, *10*, 4362. [\[CrossRef\]](#)
54. Verbruggen, F.; Logan, G.D. Models of response inhibition in the stop-signal and stop-change paradigms. *Neurosci. Biobehav. Rev.* **2009**, *33*, 647–661. [\[CrossRef\]](#) [\[PubMed\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.