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A New Fractional-Order Adaptive Sliding-Mode Approach for Fast Finite-Time Control of Human Knee Joint Orthosis with Unknown Dynamic

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Abstract: This study delves into the implementation of Fast Finite Time Fractional-Order Adaptive Sliding Mode Control (FFOASMC) for knee joint orthosis (KJO) in the presence of undisclosed dynamics. To achieve this, a novel approach introduces a Fractional-Order Sliding Surface (FOSS). In the context of limited knowledge regarding the dynamics of knee joint arthrosis, Fractional-Order Fast Adaptive Sliding Mode Control (FOFASMC) is devised. Its purpose is to ensure both finite-time stability and prompt convergence of the KJO's state to the desired trajectory. This controller employs adaptive rules to estimate the enigmatic dynamic parameters of KJO. Through the application of the Lyapunov theorem, the attained finite-time stability of the closed loop is demonstrated. Simulation results effectively showcase the viability of these approaches and offer a comparative analysis against conventional integer-order sliding mode controllers.



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MSC: 93C10

1. Introduction

Today, as a result of the increasing demand for assistive technologies, interest in expanding portable rehabilitation systems has increased [1,2]. The knee joint arthrosis, referred to by various terms in the literature such as lower limb exoskeleton, shank arthrosis system, or knee rehabilitation robot, serves predominantly to facilitate the rehabilitation of individuals afflicted by impaired limb functionality at the knee joint level. Its primary objective is to enable these patients to regain control over their limbs [3–6]. In the past few years, a variety of methods have been used for the stabilizing of KJOs, such as PD controller [7], cascaded PID controller [3], active impedance control [8,9], model reference adaptive control [10], robust adaptive neural control scheme [11], neural network sliding mode control (SMC) [12], Lyapunov-based controllers [13,14], non-singular terminal SMC (TSMC) [15], SMC based on disturbance observer [16], fast TSMC [17], adaptive-backstepping controller [18] and fuzzy backstepping SMC [19].

In the work of Huo et al. [9], an active impedance control approach was devised for knee joint arthrosis systems (KJOs). This approach aimed to calculate human joint torque and mitigate the impedance of the orthosis system to achieve a satisfactory level. Rifai et al. [20] had used a model reference adaptive control methodology for online parameter regulation in a human-driven KJO to obtain the best tracking performance.

Mushage et al. [21] introduced a hybrid state observer alongside a fuzzy neural network controller for limb exoskeleton robots. This dual approach was designed to accurately estimate system states and capture the intricate non-linear dynamics involved in trajectory-tracking challenges. Mefoued [11] devised an adaptive neural control strategy that accounts for situations where the dynamic model of a system is not known. This approach combines neural network control with repetitive learning control, which was specifically tailored to stabilize lower limb rehabilitation systems [22]. Finally, Wehbi et al. [8] used active impedance control for stabilizing a KJO system during the swing phase, which uses two observers based on SMC for estimating both human joint torque and non-linear disturbance.

In reality, KJO systems are subjected to external disturbances, modeling uncertainties, and unmolded dynamics. KJO parameters vary between subjects, and measuring human effort, an external disturbance, is challenging [23]. So, a robust method is needed to maintain the tracking performance in the system. The robot should also meet the demand of the individual difference of the patients' force, height, and weight. Therefore, the robustness of the control algorithm for a KJO system has become a research hotspot.

SMC is one of the most reliable and well-known methods of variable structure control systems, which is robust against these perturbations and has been used a lot for the control of robotic rehabilitation systems, including KJOs [24–29]. Ding et al. [6] introduced an adaptive proxy-based sliding mode control (SMC) to attain resilient trajectory tracking for shank-orthosis systems. The concept involves creating a virtual entity referred to as a “proxy”, which is governed by a conventional PID control system. Mohammed et al. [16] proposed a TSMC with a non-linear observer to achieve robust path tracking of a shank-orthosis system. Lazaro et al. [24] put forth an output feedback Adaptive Sliding Mode Control (ASMC) for lower limb rehabilitation, aiming to achieve comprehensive trajectory tracking across all degrees of freedom within the system. Adaptive or Robust-Adaptive SMC is a controller that uses adaptive laws for estimating some controller parameters [24,25]. These parameters are mostly upper bounds of system perturbations including disturbances and uncertainties [25]. Additionally, Long et al. [26] employed a combined position control strategy that incorporates robust SMC and a cerebellar model articulation neural network for lower limb exoskeletons.

Cao et al. [27] proposed multi-output SMC for the simultaneous control of angular trajectory and knee joint adaptation in a pneumatic muscle-driven lower limb exoskeleton. SMC is also utilized in a cable-axis rehabilitation exoskeleton for low-level torque control [28]. Sliding mode observers (SMOs) are employed for joint arthrosis to estimate model uncertainties and human muscular torques [29]. In the above-mentioned SMC methods, asymptotic stability is achieved, which cannot guarantee finite-time convergence. The fast convergence in a KJO system can improve tracking accuracy and enhance transient response. TSMC has recently been proposed to extend the traditional SMC and achieve finite-time stability, fast response, and more robustness and accuracy. The authors of [30] introduced a model-free adaptive non-singular fast TSMC, involving an Intelligent PI controller, delay estimation, and adaptive sliding compensator. This guarantees finite-time convergence of the path tracking error to zero. Moreover, [31] suggested swift TSMC integration into the proposed controller for finite-time stability in velocity and position. Additionally, a robust non-singular TSMC was formulated for stabilizing active rehabilitation exoskeletons amidst singularities.

The main problem of SMC or TSMC is undesirable oscillations called chattering. This phenomenon results in high-frequency oscillations because of the discontinuity in the control law. In real implementations, SMC produces these oscillations in KOJ system actuators. Chattering can cause a KOJ system to vibrate, leading to joint wear. On the other hand, due to the lack of a chattering mathematical model, it is not possible to predict or estimate its effects in real applications [32]. Therefore, reducing chattering is very important in the implementation of SMC methods in real systems.

To minimize chattering, options include using smooth functions like sigmoidal, saturation, or hyperbolic tangent functions rather than the abrupt sign function. Alternatively,

a continuous auxiliary control can stabilize the main system, which is complemented by a discontinuous controller to manage disturbances, preventing chattering in the closed-loop system [33]. Despite the reduction in chattering, using these methods causes a loss of accuracy and robustness and, therefore, poor performance of the system. Implementing the SMC in a KJO system based on the aforementioned methods for approximating the discontinuous sign function is used in [27,34]. Bkekri et al. [35] suggested an adaptive super-twisting SMC that diminishes chattering and establishes a constant upper bound for disturbances using adaptive laws.

A recent method for addressing chattering while maintaining system performance involves employing fractional-order sliding mode controllers (FOSMCs) [36–38]. The application of fractional-order SMCs not only mitigates chattering but also enhances closed-loop system performance. Saif et al. recently used a fractional-order sliding mode controller for achieving tracking in quadcopters and had also compared the performance with integer-order SMC. Yu et al. proposed a model-free fractional-order SMC along with a non-linear disturbance observer for electric drive systems in [37], and Zahraoui et al. [38] proposed a fractional-order SMC based on machine learning techniques for speed control in permanent magnet motors. There are few papers on the fractional-order TSMC of KJO systems. Delavari et al. [39] introduced an amalgamation of non-linear disturbance observers and fuzzy fractional-order terminal sliding mode control (TSMC) to enhance tracking accuracy and diminish the impact of muscle torque uncertainties. Additionally, a robust adaptive TSMC employing fractional-order surfaces was proposed for trajectory tracking in lower limb exoskeleton systems under perturbations [40]. However, a recent commentary [41] has highlighted significant errors in the stability proof of finite-time stability when perturbations are present. One notable mistake pertains to the inability to prove finite-time stability for the error in the sliding mode of the system (when $s=0$) given the inherently non-linear KJO system model, uncertainties encompassing human and orthosis parameters, coupled with the challenge of identifying these varying parameters and establishing an accurate dynamic model for the system [41].

The authors note that prior research has not tackled the issue of finite-time stability for the KJO system assuming unknown dynamic parameters. Given the uncertainties and uncharted dynamics within the non-linear coupled KJO model, coupled with the necessity of achieving finite-time convergence inherent to this category of robots, the primary impetus behind devising a finite-time control law for KJO systems is to circumvent the challenge of unknown dynamic parameters due to their uncertain nature. Therefore, the novelty and the primary contributions of this paper include the following:

- (1) It is assumed that KJO dynamic parameters are completely unknown. Then, a new FFOASMC is designed in order to achieve finite-time stability in which the unknown dynamic parameters of the KJO system are estimated via adaptive laws.
- (2) Introducing a novel approach for mitigating chattering by utilizing the developed fractional-order sliding mode control (SMC).
- (3) Notably, the design methodology presented in this paper can be readily extended to encompass the broader spectrum of high-order non-linear affine systems, encompassing entities like robot manipulators and rigid bodies.

In summary, the main motivation for this paper, which has been applied to the KJO system, is finite-time stability under the condition that all of the dynamic parameters of the KJO system are unknown (using adaptive sliding mode laws).. Its stability is also demonstrated, though it can be extended to apply to nonlinear affine systems in general.

The paper's structure is outlined as follows: Preliminaries are detailed in Section 2. System modeling and problem formulation are elaborated upon in Section 3. The design procedure of the controller and its stability proofs are expounded in Section 4. Section 5 presents numerical simulation results to validate the methodologies. The paper concludes in Section 6 with the final remarks.

2. Preliminaries

Fractional-order calculus, an established mathematical field, provides a broader framework encompassing derivatives and integrals spanning from integer to non-integer orders. A general operator ${}_a D_t^\alpha$ is used to express the fractional-order differentiator as shown below [42]:

$${}_a D_t^\alpha = \begin{cases} \frac{d^\alpha}{dt^\alpha} & , R(\alpha) > 0 \\ 1 & , R(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha} & , R(\alpha) < 0 \end{cases} \quad (1)$$

where α represents the fractional order and 'a' is a constant reflecting the initial condition. Various definitions for fractional differentiation and integration exist in the literature, such as Grünwald–Letnikov (G-L), Riemann Liouville (R-L), and Caputo. However, the most common case for describing dynamic systems is the Caputo definition. This is because the initial conditions of fractional-order differential equations using Caputo derivatives align with those of integer-order counterparts [42]. Hence, Caputo's definition proves more suitable for describing fractional-order dynamic systems in practical applications. The Caputo derivative of a function, such as $f(t)$, is defined as follows [43]:

$$D_t^\alpha f(t) = \frac{1}{\Gamma''([\alpha] - \alpha)} \int_0^t \frac{f^{([\alpha])}(\tau)}{(t - \tau)^{\alpha - [\alpha] + 1}} d\tau, \quad 0 < \alpha \notin N \quad (2)$$

Here, $\Gamma''(\cdot)$ signifies the Gamma function, and $[\alpha]$ represents the smallest integer greater than or equal to α [43]. Concluding this subsection, let us examine the following two lemmas, which will be instrumental in establishing the stability of the controllers in the subsequent sections.

Lemma 1 ([44]). Consider $V(x)$ as a Lyapunov function candidate where V_0 is its initial condition. Then, if the following inequality holds:

$$\dot{V}(x) + \alpha V(x) + \beta V^\gamma(x) \leq 0 \quad (3)$$

in which $\alpha, \beta > 0$ and $0 < \gamma < 1$. Next, the function $V(x)$ will converge to the origin within a finite time, with the indicated settling time:

$$T \leq \alpha^{-1}(1 - \gamma)^{-1} \ln(1 + \alpha\beta^{-1}V_0^{1-\gamma}) \quad (4)$$

Lemma 2 ([44]). Consider the following fractional-integer differential equation:

$$\dot{x} + kD^{\lambda-1}[\text{sig}(x)^a] = 0 \quad (5)$$

where $x \in R^n$, $0 < a, \lambda < 1$ and $k = \text{diag}(k_i)$ is a constant positive matrix in which $k_i \in R^+$, $i = 1, 2, \dots, n$ and $\text{sig}(x)^a$ is defined as:

$$\text{sig}(x)^a = [|x_1|^a \text{sign}(x_1), \dots, |x_n|^a \text{sign}(x_n)]^T \quad (6)$$

Subsequently, the state x will converge to the origin within a finite time denoted as T_x .

The following section details the Problem Formulation, encompassing the description of the KJO system and the article's objective.

3. Description of Knee-Orthosis System

As depicted in Figure 1, the system involves an individual wearing an orthosis while seated with the shank having free movement around the knee joint. The orthosis consists

of upper and lower jointed sections, and the upper segment is powered by a brushless DC (BLDC) motor. This orthosis generates torque, confining knee joint movement to a range of 0° to 120° . In this range, 0° denotes full knee extension, 120° signifies maximum knee flexion, and 90° represents the neutral position (Figure 2).



Figure 1. A person that is wearing a knee orthosis in a sitting position [45].

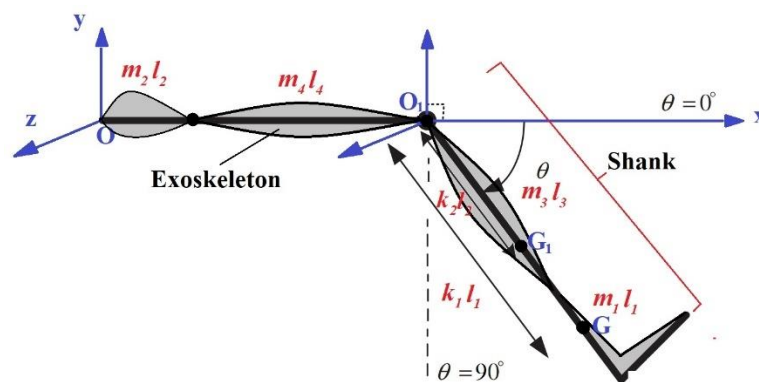


Figure 2. The KJO system in the plane.

Center of mass positions of the orthosis and the lower limb with respect to coordinate center O_1 are given by [46]:

$$\begin{aligned} \vec{O_1 G} &= [K_1 l_1 \cos(\theta), -K_1 l_1 \sin(\theta)] \\ \vec{O_1 G_1} &= [K_2 l_3 \cos(\theta), -K_2 l_3 \sin(\theta)] \end{aligned} \quad (7)$$

where K_1 and K_2 denote the mass repartition coefficients of the lower part, G and G_1 indicate the lower part center of gravity, and l_1 and l_3 , respectively, denote the length of the shank and the lower part of the orthosis (Figure 2).

Using the Lagrange equation, the dynamic model of the KJO system is obtained as follows [16]:

$$\begin{aligned} (J_k + J_{eqex})\ddot{\theta} + (f_{vk} + f_{vex})\dot{\theta} - (m_1 k_1 l_1 + m_3 k_2 l_3)g \cos(\theta) \\ = \tau_{ex} + \tau_k - (f_{sex} + f_{sk})\text{sign}(\dot{\theta}) \end{aligned} \quad (8)$$

where $\theta, \dot{\theta}, \ddot{\theta} \in \mathbb{R}$ are posture variables of the KJO system, $\theta(0) \in \mathbb{R}$ is the vector of the initial condition, m_1 and J_k stand for the mass and the inertia of the shank foot, J_{eqex} signifies the inertia of the lower part of the orthosis, and f_{vk} and f_{vex} are, respectively, the coefficients of viscous damping for the knee joint and the actuated orthosis. f_{sk} and f_{sex} indicate the respective solid friction coefficients. The term $(m_1 k_1 l_1 + m_3 k_2 l_3)g \cos(\theta)$ is the gravitational

torque, while g indicates the gravitational constant. τ_{ex} and τ_k , respectively, show the torque of the actuated orthosis and the torque of the human knee joint. An orthosis is a knee joint robot that moves through a combination of actuators and human thigh muscles.

The orthosis joint is driven by a BLDC motor with mechanical transmission employed to amplify the orthosis' torque. Since the time constant of the current control system is negligible in comparison to the mechanical time constant and given the characteristics of the BLDC motor, the following equation can be employed [31]:

$$\tau_{ex} = k_m i(t) \quad (9)$$

where k_m is a positive constant value, $i(t)$ is the input current of the BLDC motor, and τ_{ex} is the exerted torque in the orthosis.

Using both electrical and mechanical equations as in Equations (8) and (9) and defining $J = J_k + J_{eqex}$, $B = f_{vk} + f_{vex}$, $A = f_{sk} + f_{sex}$ and $\tau_g = (m_1 k_1 l_1 + m_3 k_2 l_3) g$, the model of the orthosis can be rewritten as:

$$M\ddot{\theta} + N(\theta, \dot{\theta}) = f \quad (10)$$

where

$$\begin{aligned} N(\theta, \dot{\theta}) &= \frac{B}{k_m} \dot{\theta} - \frac{\tau_g}{k_m} \cos(\theta) - \frac{\tau_k}{k_m} + \frac{A}{k_m} \text{sign}(\dot{\theta}) \\ M &= J/k_m \\ f &= i(t) \end{aligned} \quad (11)$$

In general, the dynamic parameters of a KJO robot, including the mass and inertia of both the shank foot and the orthosis, viscous damping coefficients, solid friction coefficients, and so on, are always exposed to model uncertainties and unmodeled dynamics. In addition, the KJO in Equation (10) is exposed to unmodeled dynamics, external disturbances, and unknown human knee joint torques. Therefore, due to external disturbances and model uncertainties, most conventional controllers are not viable for tracking control in these orthosis systems. As a starting point, we make the following assumption in this paper:

Assumption 1. Assuming the dynamic parameters of the KJO model in Equation (10) are elusive due to substantial uncertainties, the dynamic model of the KJO, can then be considered as follows:

$$\hat{M}\ddot{\theta} + \hat{N}(\theta, \dot{\theta}) = f \quad (12)$$

where $\hat{M}$ and $\hat{N}(\theta, \dot{\theta})$ are an estimation of dynamic system parts.

In this paper, two new FFOASMC approaches for finite-time tracking control of KJO are presented, which can be easily adapted to the n-DOF general dynamic model of robot systems (meaning famous rigid body dynamic equation) and even (considering some conditions such as controllability, the stability of zero dynamics and so on) the input affine high-order non-linear models such as $\dot{x} = f(x) + g(x)u(t)$, $x \in R^n$. This approach is summarized as follows.

In Section 4, given the absence of information regarding dynamic parameters of the KJO, as stipulated in Assumption 1, the objective is to formulate a novel FFOASMC that achieves finite-time tracking of the knee joint angle $\theta(t)$ to follow a predefined trajectory $\theta_d(t)$. In this context, an adaptive law is utilized to estimate the dynamic parameter vector of the system.

4. FFOASMC Design Considering Unknown Dynamics

4.1. Rewriting the Dynamic Model of KJO System

The dynamic equation presented in Equation (12) can be reformulated as follows:

$$\bar{M}\ddot{\theta} + L(\theta, \dot{\theta}) = f \quad (13)$$

where $\bar{M}$ is an arbitrary constant positive value, f is the input signal and $L(\theta, \dot{\theta})$ is as follows:

$$L(\theta, \dot{\theta}) = [\hat{M} - \bar{M}]\ddot{\theta} + \hat{N}(\theta, \dot{\theta}) = Y(\theta, \dot{\theta})\hat{\psi} \quad (14)$$

where $Y(\theta, \dot{\theta})\hat{\psi}$ indicates the regressor form of the dynamic part $[\hat{M} - \bar{M}]\ddot{\theta} + \hat{N}(\theta, \dot{\theta})$, which is a pseudo-linear description with respect to unknown dynamic parameters $\hat{\psi} \in R^m$ and a known regressor matrix $Y(\theta, \dot{\theta}) \in R^{1 \times m}$.

4.2. Fractional-Order Sliding Surface and Reaching Law Formulation

Taking into account the knee joint angle tracking error as $\tilde{\theta} = \theta_d - \theta$, to attain finite-time convergence in tracking error, while employing Lemma 2, the fractional-order sliding mode surface is formulated as follows:

$$s = \dot{\tilde{\theta}} + kD^{\beta-1}[\text{sig}(\tilde{\theta})^\alpha] \quad (15)$$

where $\text{sig}(\tilde{\theta})^\alpha = |\tilde{\theta}|^\alpha \text{sign}(\tilde{\theta})$ with $0 < \alpha, \beta < 1$ and $k \in R^+$ is a desired constant positive number.

When the system is on the sliding surface ($s = 0, \dot{s} = 0$), its behavior is described by $\dot{\tilde{\theta}} + kD^{\beta-1}[\text{sig}(\tilde{\theta})^\alpha] = 0$. Based on the findings of Lemma 2, the tracking error $\tilde{\theta}$ converges to zero within a finite time. Consequently, the knee joint angle adheres to its desired trajectory within a finite duration.

Now, the primary control objective is to induce the dynamic of the system described in Equation (13) to converge to the sliding surface within a finite time. In order to achieve rapid finite-time convergence of s to zero, a terminal reaching law is introduced in the following manner [44]:

$$\dot{s} = -\rho s - \mu \text{sig}(s)^\lambda \quad (16)$$

where ρ and μ are positive constant values and $0 < \lambda < 1$. Leveraging the reaching law outlined in Equation (16) and accounting for Assumption 1, the subsequent subsection introduces a novel FFOASMC strategy. This approach is devised to drive the sliding surface s to the origin within a predetermined finite time.

4.3. Fractional-Order Adaptive SMC

Given the estimation of dynamic parameter ψ , which is denoted as $\hat{\psi}$ from Equation (14), the FFOASMC designed for achieving finite-time tracking of the KJO, based on the dynamic equation in Equation (12), is as follows. This control strategy incorporates the sliding manifold from Equation (15) and the fractional-order reaching law in Equation (16):

$$f = \bar{M}u + Y(\theta, \dot{\theta})\hat{\psi} \quad (17)$$

where u is as follows:

$$u = \ddot{\theta}_d + kD^\beta[\text{sig}(\tilde{\theta})^\alpha] + \rho s + \mu \text{sig}(s)^\lambda + \eta \text{sign}(s) \quad (18)$$

The value of $\hat{\psi}$ in Equation (17) is derived through the subsequent adaptive law:

$$\dot{\hat{\psi}} = \frac{1}{M}(\Gamma Y^T s) \quad (19)$$

where $\Gamma \in R^{m \times m}$ is an arbitrary positive definite matrix.

The main result of this section, which is the finite-time stabilizing of the system, is provided in Theorem 1 as follows.

Theorem 1. Consider the KJO model in Equation (10) when its dynamic parameters are unknown according to Assumption 1. By applying the FFOASMC in Equations (17)–(19), where s is defined in Equation (15) and $k, \mu, \rho \in \mathbb{R}^+, 0 < \alpha, \beta < 1$ and $\bar{M}$ are arbitrary constant values, the knee joint angle $\theta(t)$ converges to the desired trajectory $\theta_d(t)$ in finite time.

Proof. Applying the control signal in Equation (17) into the rewritten KJO dynamic equation in Equation (13), we have:

$$\bar{M}\ddot{\theta} + L(\theta, \dot{\theta}) = \bar{M}u + Y(\theta, \dot{\theta})\hat{\psi} \quad (20)$$

Now, inserting $L(\theta, \dot{\theta})$ and u , respectively, from (14) and (18) into (20) results in:

$$\begin{aligned} \bar{M}\ddot{\theta} = & \bar{M}(\ddot{\theta}_d + kD^\beta[\text{sig}(\tilde{\theta})^\alpha] + \rho s \\ & + \mu \text{sig}(s)^\lambda + \eta \text{sign}(s)) + Y(\theta, \dot{\theta})\tilde{\psi} \end{aligned} \quad (21)$$

where $\tilde{\psi} = \hat{\psi} - \psi$ are dynamic parameter errors. Multiplying both sides of the equation in Equation (21) with $\bar{M}^{-1}$ yields:

$$\begin{aligned} \ddot{\theta} + kD^\beta[\text{sig}(\tilde{\theta})^\alpha] + \rho s + \mu \text{sig}(s)^\lambda \\ + \eta \text{sign}(s) + \bar{M}^{-1}Y(\theta, \dot{\theta})\tilde{\psi} = 0 \end{aligned} \quad (22)$$

Now, consider the following candidate for the Lyapunov function:

$$V = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\psi}^T\Gamma^{-1}\tilde{\psi} \quad (23)$$

Taking the time derivative of V results in the following:

$$\dot{V} = s\dot{s} + \tilde{\psi}^T\Gamma^{-1}\dot{\tilde{\psi}} \quad (24)$$

Using (15) and given that $\dot{\tilde{\psi}} = \dot{\hat{\psi}}$ (due to the fact that the dynamic parameters are constant over time), we have:

$$\dot{V} = s(\ddot{\theta} + kD^\beta[\text{sig}(\tilde{\theta})^\alpha]) + \tilde{\psi}^T\Gamma^{-1}\dot{\hat{\psi}} \quad (25)$$

By substituting the dynamic error from (22) into (25), we obtain:

$$\begin{aligned} \dot{V} = & s(-\rho s - \mu \text{sig}(s)^\lambda - \eta \text{sign}(s) - \bar{M}^{-1}Y(\theta, \dot{\theta})\tilde{\psi}) \\ & + \tilde{\psi}^T\Gamma^{-1}\dot{\hat{\psi}} \end{aligned} \quad (26)$$

Now, applying the adaptive rule from (19) to (26), we obtain:

$$\dot{V} = -\rho|s|^2 - \mu|s|^\lambda - \eta|s| \quad (27)$$

Utilizing the definition of $\text{sig}(s)^\lambda$ as $|s|^\lambda \text{sign}(s)$, we can determine the upper bound of $\dot{V}$ in Equation (27) as follows:

$$\dot{V} = -\rho|s|^2 - \mu|s|^{\lambda+1} - \eta|s| \leq 0 \quad (28)$$

Consequently, due to the positivity of μ, ρ , and η , the Lyapunov function derivative presented in Equation (28) exhibits negative semi-definiteness. Now, define $\tilde{w}(t) = \rho|s|^2 + \mu|s|^{\lambda+1} + \eta|s|$ and integrate both sides of (28); this leads to the conclusion that $V(0) \geq V(t) + \int_0^t \tilde{w}(v)dv$. This inequality shows that $\infty > V(0) \geq \lim_{t \rightarrow \infty} \int_0^t \tilde{w}(v)dv$.

Hence, using Barbalat lemma ([47]), $\tilde{w}(t)$ and therefore $s(t)$ will converge to the origin where $t \rightarrow \infty$. So far, we have only shown the asymptotic stability of $s(t)$, not finite-time stability. So, we have to continue the proof.

By defining the sliding surface as outlined in Equation (15) and in accordance with Lemma 2, when $s(t)$ reaches zero, the tracking error $\tilde{\theta}$ converges to zero within a finite time (denoted as $T_{\tilde{\theta}}$). Next, we need to establish that by implementing the FFOASMC from Equation (17), the system attains the sliding surface ($s(t) = 0$) within a finite time (referred to as T_s). T_s denotes the finite duration in which $s(t)$ converges to zero from its initial state. As a result, it follows that the knee joint angle-tracking error will also reach zero within a defined finite time $T_{final} = T_{\tilde{\theta}} + T_s$.

To determine T_s , we introduce another Lyapunov function as follows:

$$V_s = \frac{1}{2}s^2 \quad (29)$$

Differentiating vs. and applying (15) results in the following:

$$\dot{V}_s = s(\ddot{\theta} + kD^\beta[\text{sig}(\tilde{\theta})^\alpha]) \quad (30)$$

Substituting (22) into (30) yields:

$$\dot{V}_s = -s(\rho s + \mu \text{sig}(s)^\lambda + \eta \text{sign}(s) - \varepsilon) \quad (31)$$

Here, $\varepsilon = -\bar{M}^{-1}Y(\theta, \dot{\theta})\tilde{\psi}$ stands for the residual error vector subsequent to implementing the controller. Leveraging the outcomes of Theorem 1 and considering (19), both $\tilde{\psi}$ and $Y(\theta, \dot{\theta})$ are confined, thereby bounding ε as $|\varepsilon| \leq \zeta$, where $\zeta > 0$. Consequently, the equation in Equation (31) can be expressed as follows:

$$\dot{V}_s \leq -\rho|s|^2 - \mu|s|^{\lambda+1} - \eta|s| + |s||\varepsilon| \quad (32)$$

And thus:

$$\dot{V}_s \leq -\rho|s|^2 - \mu|s|^{\lambda+1} - \eta|s| + \zeta|s| \quad (33)$$

Assuming $\eta > \zeta$ and defining $\bar{\rho} = \eta - \zeta > 0$, then:

$$\dot{V}_s \leq -\rho|s|^2 - \mu|s|^{\lambda+1} - \bar{\rho}|s| \quad (34)$$

Given that $-\bar{\rho}|s| < 0$, we have:

$$\dot{V}_s \leq -\rho|s|^2 - \mu|s|^{1+\lambda} \quad (35)$$

Now, utilizing (29) and substituting $|s| = \sqrt{2V_s}$ into (35) leads to:

$$\dot{V}_s \leq -2\rho V_s - 2^{\frac{1+\lambda}{2}}\mu V_s^{\frac{1+\lambda}{2}} \quad (36)$$

Hence, with the application of Lemma 1 and through a straightforward comparison between Equations (3) and (36), which are, respectively, $\dot{V}(x) + \alpha V(x) + \beta V^\gamma(x) \leq 0$ and $\dot{V}_s + 2\rho V_s + 2^{\frac{1+\lambda}{2}}\mu V_s^{\frac{1+\lambda}{2}} \leq 0$, it can be easily concluded that the Lyapunov function V_s and as a result the sliding surface $s(t)$ (see Equation (29)) will converge to zero with the following settling time:

$$T_s \leq \frac{1}{\rho(1-\lambda)} \ln \left(1 + \frac{2\rho V_0^{\frac{(1-\lambda)}{2}}}{2^{\frac{(1+\lambda)}{2}}\mu} \right) \quad (37)$$

This assures finite-time convergence of the system to the sliding surface $s(t) = 0$.

In conclusion, the knee joint angle tracking error $\tilde{\theta}$ converges to the origin within the finite time $T_{final} = T_{\tilde{\theta}} + T_s$, where T_s is determined from (37), and $T_{\tilde{\theta}}$ signifies a finite time during the sliding mode condition ($s(t) = 0$), leading to the tracking error $\tilde{\theta}$ reaching zero as Lemma 2. $\square$

In Equation (14), the dynamics of the system are regressed as $L(\theta, \dot{\theta}) = [\hat{M} - \bar{M}]\ddot{\theta} + \hat{N}(\theta, \dot{\theta}) = Y(\theta, \dot{\theta})\hat{\psi}$, where $\hat{\psi} \in R^m$ is the vector of the unknown dynamic parameters of the knee joint orthosis such as mass, inertia, and motor constant. In Equation (19), the estimation of this vector is obtained in the form of an adaptive law and is used in the control law in (17). It should be noted that if we did not design the adaptive law in (19), then instead of $\hat{\psi}$, we should have used the real values of the vector of dynamic parameters of the system in (17) meaning ψ . This is while we had already claimed that we had solved the problem with unknown dynamic of orthosis.

It should also be noted that $\bar{M}$ is supposed to be an arbitrary constant positive value. However, if we select a very large value for $\bar{M}$, it will make the amplitude of the control input in (17) a very large value. Then, firstly, it goes to saturation in real applications and does not accept any value, and secondly, this action greatly increases the control effort and the energy spent for achieving trajectory tracking in finite time.

Remark 1. The presence of the discontinuous sign function in the control inputs (18) leads to chattering. To address this, a pseudo-sliding function is proposed, approximating the sign function as $\text{sign}(s) \approx \frac{s}{(|s| + \chi(t))}$. Here, $\chi(t) > 0$ is a bounded positive function satisfying $\int_0^\infty \chi(t) dt < \infty$, with a suitable choice being $\chi(t) = \frac{1}{(1+t^n)}$, where $n \geq 2$.

Remark 2. In addition to mitigating chattering, using fractional-order sliding surfaces offers the advantage of more controller parameters (e.g., parameter β in Equation (15)) compared to integer-order approaches. These extra parameters empower designers to tailor system output specifications more effectively.

Remark 3. Referring to (15), the error equation within the sliding mode of FFOASMC ($s(t) = 0$) is $\ddot{\tilde{\theta}} + kD^{\beta-1}[\text{sign}(\tilde{\theta})^\alpha] = 0$, which is similar to the integer-order counterpart, $\ddot{\tilde{\theta}} + k \text{sign}(\tilde{\theta})^\alpha = 0$. Considering $k = 0.7$, $\alpha = 0.01$ and $\beta = 0.99$, Figure 3 illustrates the error equation on the sliding surface for integer and fractional-order cases. Notably, the chattering layer in the fractional-order switching manifold is narrower compared to the integer-order counterpart. This underscores a key advantage of fractional-order SMC over integer-order systems.

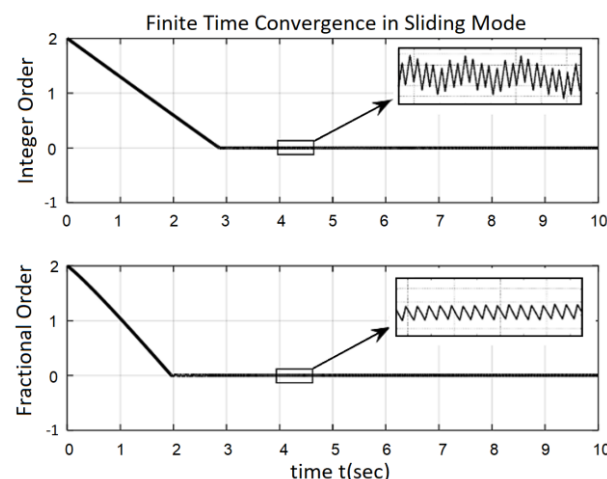


Figure 3. Finite-time stability of system error within the sliding mode, considering both fractional and integer-order surfaces.

Remark 4. The references [46–48] demonstrate that this distinction arises due to the fact that in a stable fractional-order system, system states converge to zero as $t^{-\alpha}$, whereas in a classical integer-order counterpart, this convergence is asymptotic and follows e^{-kt} . The rate of convergence of these functions to zero favors the reduction in chattering when employing fractional-order sliding surfaces. To explain further, the general form of the error equation in Fractional-Order Sliding Mode Control (FOSMC) is denoted as $D_t^\alpha x(t) = f(x(t))$, while for Integer-Order Sliding Mode Control (SMC), it is represented as $\dot{x}(t) = f(x(t))$. As per the solutions derived from linear fractional-order systems, as expounded in [48], the error of the fractional-order system demonstrates a decay to zero characterized by $t^{-\alpha}$, whereas the error in the integer-order scenario converges toward zero with a decay pattern akin to e^{-kt} . This discrepancy implies that when initiated from an initial state error, the chattering layer of the fractional-order non-linear system encircling the switching manifold $s(t) = 0$ is perceptibly smaller in comparison to its integer-order counterpart. This serves as one of the discernible advantages of fractional-order sliding mode control over classical integer-order systems.

In Figure 4, this contrast is vividly depicted for both an integer-order system (i.e., $\dot{e}(t) = -ke(t)$) and a fractional-order system (i.e., ${}_0D_t^\alpha e(t) = -ke(t)$) when $e(t) \in \mathbb{R}^2$. As the system approaches the sliding surface from the initial states (e_{10}, e_{20}) , the states in the fractional-order system attenuate toward $t^{-\alpha}$ ($1 \rightarrow 2 \rightarrow 3 \dots$), while the states in the integer-order case regress toward the origin following a pattern akin to e^{-kt} ($1' \rightarrow 2' \rightarrow 3' \dots$). Figure 4 provides a quantitative illustration signifying that within the framework of FOSMC for Non-linear Fractional-Order (NFO) systems, chattering is notably diminished in comparison to conventional integer-order SMC.

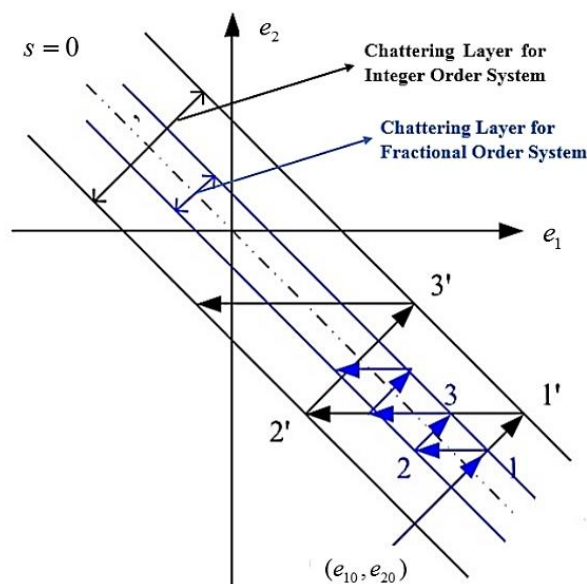


Figure 4. The sliding mode surface in Fractional-Order Sliding Mode Control (FOSMC) in comparison to that in Integer-Order Sliding Mode Control (SMC).

5. Simulation Results

This section presents numerical examples to demonstrate the efficacy of the proposed controllers. The dynamic parameters of the KJO model are set according to [16]. The simulations are conducted using MATLAB/Simulink software. As the example, the FFOASMC in Equations (17) and (18) is simulated. To do this, the desired trajectory is considered as [46]:

$$\theta_d = \frac{\pi}{6} \sin(2\pi f) + \frac{\pi}{4} \quad (38)$$

where $f = 0.16$. The initial conditions for the simulation are set as follows:

$$\theta(0) = 0, \dot{\theta}(0) = 0 \quad (39)$$

The parameters of the controller and adaptive law in Equation (17)–(19) are selected as follows according to Theorem 1:

$$\begin{aligned} \rho &= 3, k = 1.5, a = 0.8, \bar{M} = 5 \\ \lambda &= 0.9, \beta = 0.99, \eta = 0.1 \end{aligned} \quad (40)$$

In order to use adaptive law in Equation (19), the regressor vector is obtained as $Y(\theta, \dot{\theta}) = [\dot{\theta}, -\cos(\theta), \text{sign}(\dot{\theta}), 1]$ and also $\Gamma = 3I_4$ where I_4 denotes identity matrix.

The numerical simulation results presented in Figures 5–7 clearly show that using Theorem 1, finite-time convergence is achieved on the KOJ angle by applying the control law shown in Equations (17) and (18), and the estimated adaptive parameters will remain bounded and converge to some limited values.

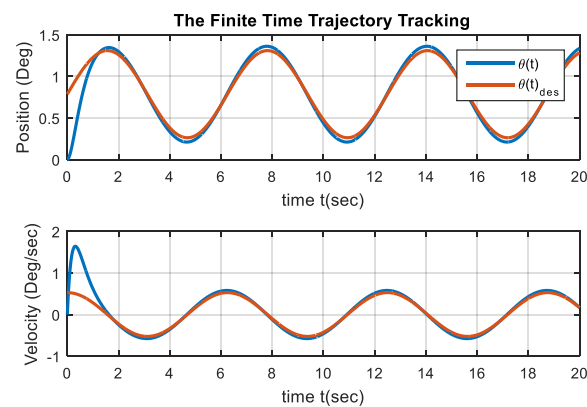


Figure 5. Position trajectory tracking and finite-time convergence of KOJ angle using FFOASMC in Equations (17)–(19).

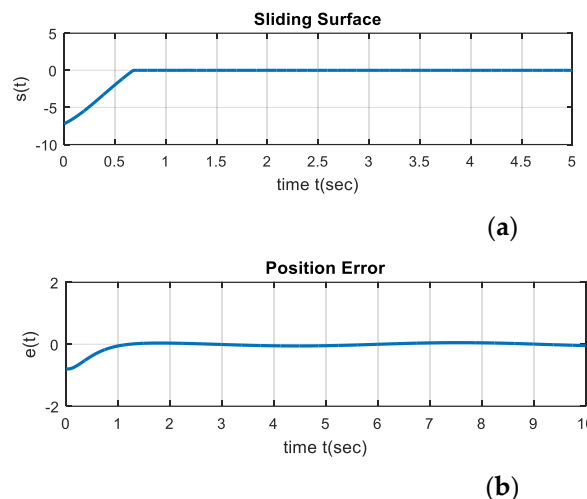


Figure 6. Finite-time convergence in KJO system applying FFOASMC in Equations (17)–(19). (a) Sliding mode surface. (b) Position tracking error.

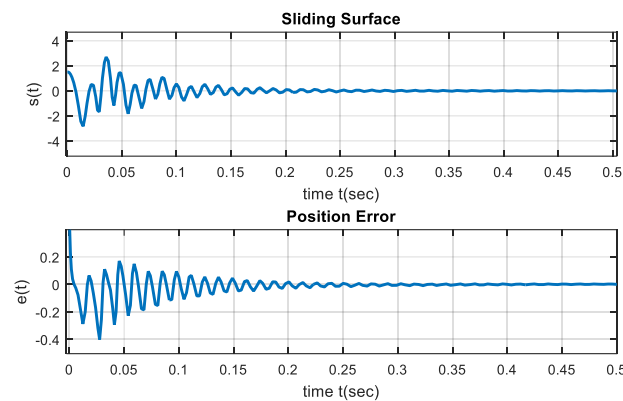


Figure 7. Convergence in KJO system applying FFOASMC in Equations (17)–(19) without using the chattering avoidance idea. First figure: sliding mode surface. Second figure: position tracking error.

After reaching the sliding surface, the system enters the sliding phase, where error values converge to zero within a limited time, as shown in Figures 5 and 6. Without applying the chattering avoidance approach from Remark 1, simulation results, like those in Figure 7, would lack chattering reduction. A direct comparison between Figures 6 and 7 demonstrates the substantial chattering reduction achieved through the proposed technique in this paper.

Figure 5 depicts the position and velocity of the KJO angle, validating finite-time convergence to the desired trajectory. In Figure 6a, the sliding surface converges to zero within finite time, and Figure 6b demonstrates error convergence, affirming Theorem 1's outcomes. As anticipated from the stability proof in Theorem 1, Figure 8 showcases the dynamic parameters—outputs of the adaptive law in Equation (19)—converging to bounded values.

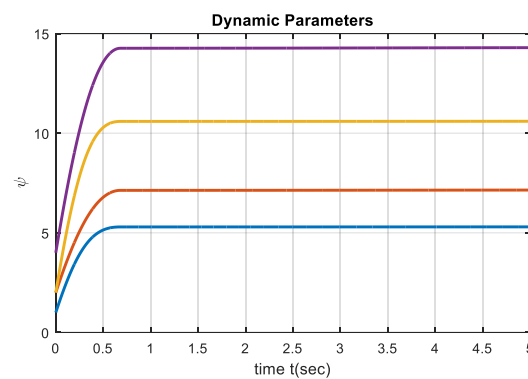


Figure 8. Dynamic parameters of adaptive law in Equation (19) by using FFOASMC in Equations (17)–(18).

In Figure 9, the bounded input signal (current of the BLDC joint motor) is depicted during convergence.

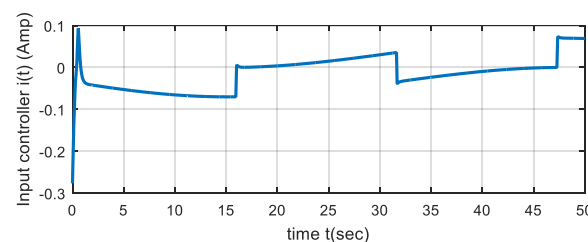


Figure 9. Input controller using FFOASMC in Equations (17) and (18).

A scrutiny of the simulation outcomes in this section validates the theoretical results presented in this paper. The FFOASMC controllers from Theorems 1 exhibit effective performance in achieving finite-time stabilization of the KJO system, even when dynamic parameters are entirely unknown.

6. Conclusions

This paper introduces fast finite-time adaptive sliding mode controllers based on fractional-order surfaces for enhancing the stability of KJO rehabilitation systems. The proposed approach involves designing an FFOASMC that ensures finite-time stability even when system dynamic parameters are unknown. Stability verification of the controller's performance in the presence of uncertainties and disturbances is demonstrated through numerical examples. Analytically, it is established that incorporating fractional-order sliding surfaces reduces chattering, thereby enhancing sliding mode controller performance. Further improvement is attained by substituting the sign function with a new continuous function. The generalization of this method for models with more degrees of freedom such as mechanical arms, solving the problem by considering the input saturation constraint and also providing a method for optimal calculation of the obtained fractional-order SMC parameters, can be used as a continuation of the research conducted in this paper.

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References

1. Rupal, B.; Rafique, S.; Singla, A.; Singla, E.; Isaksson, M.; Virk, G. Lower-limb exoskeletons: Research trends and regulatory guidelines in medical and non-medical applications. *Int. J. Adv. Robot. Syst.* **2017**, *14*, 1–27. [\[CrossRef\]](#)
2. Li, Z.G.; Ren, Z.; Zhao, K.; Deng, C.; Feng, Y. Human-cooperative control design of a walking exoskeleton for body weight support. *IEEE Trans. Ind. Inform.* **2020**, *16*, 2985–2996. [\[CrossRef\]](#)
3. Yang, M.X.; Wang, X.S.; Zhu, Z.Y.; Xi, R.; Wu, Q.C. Development and control of a robotic lower-limb exoskeleton for paraplegic patients. *Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci.* **2019**, *233*, 1087–1098. [\[CrossRef\]](#)
4. Moreno, J.C.; Figueiredo, J.; Pons, J.L. Chapter 7—exoskeletons for lower-limb rehabilitation. In *Rehabilitation Robotics*; Colombo, R., Sanguineti, V., Eds.; Academic Press: Cambridge, MA, USA, 2018; pp. 89–99.
5. Chen, G.; Chan, C.; Guo, Z.; Yu, H. A review of lower extremity assistive robotic exoskeletons in rehabilitation therapy. *Crit. Rev. Biomed. Eng.* **2013**, *41*, 343–363. [\[CrossRef\]](#)
6. Ding, G.; Huo, W.; Huang, J.; Amirat, Y.; Mohammed, S. Robust and Safe Control of a Knee Joint Orthosis. In Proceedings of the IEEE International Conference on Intelligence and Safety for Robotics (ISR), Shenyang, China, 24–27 August 2018; pp. 343–348.
7. Tsukahara, A.; Kawanishi, R.; Hasegawa, Y.; Sankai, Y. Sit-to-Stand and Stand-to-Sit Transfer Support for Complete Paraplegic Patients with Robot Suit HAL. *Adv. Robot.* **2010**, *24*, 161–1638. [\[CrossRef\]](#)
8. Wehbi, F.Z.; Huo, W.; Amirat, Y.; Rafei, M.E.; Khalil, M.; Mohammed, S. Active impedance control of a knee-joint orthosis during swing phase. In Proceedings of the 2017 International Conference on Rehabilitation Robotics (ICORR), London, UK, 17–20 July 2017; pp. 435–440.
9. Huo, W.; Mohammed, S.; Amirat, Y. Observer-based active impedance control of a knee-joint assistive orthosis. In Proceedings of the IEEE International Conference on Rehabilitation Robotics (ICORR), Singapore, 11–14 August 2015; pp. 313–318.
10. Daachi, B.; Madani, T.; Daachi, M.E.; Djouani, K. MLPNN adaptive controller based on a reference model to drive an actuated lower limb orthosis. In Proceedings of the 5th IEEE RAS/EMBS International Conference on Biomedical Robotics and Biomechatronics, Sao Paulo, Brazil, 12–15 August 2014; pp. 638–643.
11. Mefoued, S. A robust adaptive neural control scheme to drive an actuated orthosis for assistance of knee movements. *Neurocomputing* **2014**, *140*, 27–40. [\[CrossRef\]](#)
12. Mefoued, S. A second-order sliding mode control and a neural network to drive a knee joint actuated orthosis. *Neurocomputing* **2015**, *155*, 71–79. [\[CrossRef\]](#)
13. Rifai, H.; Mohammed, S.; Hassani, W.; Amirat, Y. Nested saturation based control of an actuated knee joint orthosis. *Mechatronics* **2013**, *23*, 1141–1149. [\[CrossRef\]](#)

14. Rifai, H.; Mohammed, S.; Djouani, K.; Amirat, Y. Toward lower limbs functional rehabilitation through a knee-joint exoskeleton. *IEEE Trans. Control Syst. Technol.* **2017**, *25*, 712–719. [\[CrossRef\]](#)
15. Madani, T.; Daachi, B.; Djouani, K. Non-singular terminal sliding mode controller: Application to an actuated exoskeleton. *Mechatronics* **2016**, *33*, 136–145. [\[CrossRef\]](#)
16. Mohammed, S.; Huo, W.; Huang, J.; Rifai, H.; Amirat, Y. Non-linear disturbance observer-based sliding mode control of a human-driven knee joint orthosis. *Robot. Auton. Syst.* **2016**, *75*, 41–49. [\[CrossRef\]](#)
17. Madani, T.; Daachi, B.; Djouani, K. Modular-controller-design-based fast terminal sliding mode for articulated exoskeleton systems. *IEEE Trans. Control Syst. Technol.* **2017**, *25*, 1133–1140. [\[CrossRef\]](#)
18. Chen, S.; Yao, B.; Zhu, X.; Chen, Z.; Wang, Q.; Zhu, S.; Song, Y. Adaptive robust backstepping force control of 1-DOF joint exoskeleton for human performance augmentation. *IFAC-Pap. Online* **2015**, *48*, 142–147. [\[CrossRef\]](#)
19. Yang, P.; Zhang, G.; Wang, J.; Wang, X.; Zhang, L.; Chen, L. Command Filter Backstepping Sliding Model Control for Lower-Limb Exoskeleton. *Math. Probl. Eng.* **2017**, *2017*, 1064535. [\[CrossRef\]](#)
20. Rifai, H.; Mohammed, S.; Daachi, B.; Amirat, Y. Adaptive control of a human-driven knee joint orthosis. In Proceedings of the IEEE International Conference on Robotics and Automation, Saint Paul, MN, USA, 14–18 May 2012; pp. 2486–2491.
21. Mushage, B.O.; Chedjou, J.C.; Kyamakya, K. Fuzzy neural network and observer based fault-tolerant adaptive non-linear control of uncertain 5-DOF upper-limb exoskeleton robot for passive rehabilitation. *Nonlinear Dyn.* **2017**, *87*, 2021–2037. [\[CrossRef\]](#)
22. Yang, Y.; Huang, D.; Dong, X. Enhanced neural network control of lower limb rehabilitation exoskeleton by add-on repetitive learning. *Neurocomputing* **2020**, *323*, 256–264. [\[CrossRef\]](#)
23. Huang, J.; Huo, W.; Xu, W.; Mohammed, S.; Amirat, Y. Control of upper-limb power-assist exoskeleton using a human-robot interface based on motion intention recognition. *IEEE Trans. Autom. Sci. Eng.* **2015**, *12*, 1257–1270. [\[CrossRef\]](#)
24. Lazaro, R.; Salgado, I.; Chairez, I. Adaptive sliding-mode controller of a lower limb mobile exoskeleton for active rehabilitation. *ISA Trans.* **2020**, *109*, 210–228.
25. Arabtelgerd, Z.; Koochakzadeh, A.; Soorki, M.N.; Yasoubi, S.M. Path Tracking Control of Bioflexible Probes Exposed to Uncertainties and Internal Tissues Disturbances with Unknown Upper Bonds Using Robust-Adaptive Sliding Mode Control. In *Control Engineering in Mechatronics*; Springer Nature: Singapore, 2023; pp. 103–121.
26. Long, Y.; Du, Z.J.; Wang, W.D.; Dong, W. Robust sliding mode control based on GA optimization and CMAC compensation for lower limb exoskeleton. *Appl. Bionics Biomech.* **2016**, *2016*, 5017381. [\[CrossRef\]](#) [\[PubMed\]](#)
27. Cao, J.; Xie, S.Q.; Das, R. MIMO sliding mode controller for gait exoskeleton driven by pneumatic muscles. *IEEE Trans. Control Syst. Technol.* **2018**, *26*, 274–281. [\[CrossRef\]](#)
28. Jarrett, C.; McDaid, A.J. Robust Control of a Cable-Driven Soft Exoskeleton Joint for Intrinsic Human-Robot Interaction. *IEEE Trans. Neural Syst. Rehabil. Eng.* **2017**, *25*, 976–986. [\[CrossRef\]](#) [\[PubMed\]](#)
29. Achili, B.; Madani, T.; Daachi, B. Adaptive observer based on MLPNN and sliding mode for wearable robots: Application to an active joint orthosis. *Neurocomputing* **2016**, *197*, 69–77. [\[CrossRef\]](#)
30. Han, S.; Wang, H.; Tian, Y. Model-free based adaptive non-singular fast terminal sliding mode control with time-delay estimation for a 12 DOF multi-functional lower limb exoskeleton. *Adv. Eng. Softw.* **2018**, *119*, 38–47. [\[CrossRef\]](#)
31. Madani, T.; Daachi, B.; Djouani, K. Finite-time control of an actuated orthosis using the fast terminal sliding mode. *IFAC Proc. Vol.* **2015**, *47*, 4607–4612. [\[CrossRef\]](#)
32. Levant, A. Chattering Analysis. *IEEE Trans. Autom. Control* **2010**, *55*, 1380–1389. [\[CrossRef\]](#)
33. Boiko, I. Analysis of chattering in sliding mode control systems with continuous boundary layer approximation of discontinuous control. In Proceedings of the American Control Conference, San Francisco, CA, USA, 29 June–1 July 2011; pp. 757–762.
34. Liu, J.; Zhang, Y.; Wang, J.; Chen, W. Adaptive sliding mode control for a lower-limb exoskeleton rehabilitation robot. In Proceedings of the 13th IEEE Conference on Industrial Electronics and Applications, Wuhan, China, 31 May–2 June 2018; pp. 1481–1486.
35. Bkekri, R.; Benamor, A.; Messaoud, H.; Mohammed, S.; Amirat, Y. Adaptive sliding mode control with application super-twisting algorithm of a Human-Driven Knee Joint Orthosis. In Proceedings of the 2nd International Conference on Bio-Engineering for Smart Technologies, Paris, France, 30 August–1 September 2017; pp. 1–4.
36. Saif, A.-W.A.; Gaufan, K.B.; El Ferik, S.; Al-Dhaifallah, M. Fractional Order Sliding Mode Control of Quadrotor Based on Fractional Order Model. *IEEE Access* **2023**, *11*, 79823–79837. [\[CrossRef\]](#)
37. Yu, Y.; Liu, X. Model-Free Fractional-Order Sliding Mode Control of Electric Drive System Based on Nonlinear Disturbance Observer. *Fractal Fract.* **2022**, *6*, 603. [\[CrossRef\]](#)
38. Zahraoui, Y.; Zaihidee, F.M.; Kermadi, M.; Mekhilef, S.; Mubin, M.; Tang, J.R.; Zaihidee, E.M. Fractional Order Sliding Mode Controller Based on Supervised Machine Learning Techniques for Speed Control of PMSM. *Mathematics* **2023**, *11*, 1457. [\[CrossRef\]](#)
39. Delavari, H.; Jokar, R. Fractional order adaptive fuzzy terminal sliding mode controller design for a knee joint orthosis with non-linear disturbance observer. In Proceedings of the 5th International Conference on Control, Instrumentation, and Automation, Shiraz, Iran, 21–23 November 2017; pp. 49–54.
40. Ahmed, A.; Wang, H.; Tian, Y. Robust adaptive fractional-order terminal sliding mode control for the lower-limb exoskeleton. *Asian J. Control* **2019**, *21*, 1–10. [\[CrossRef\]](#)

41. Samaei, M.H.; Sheikh Ahmadi, S.; Naderi Soorki, M.; Amini, A.S. Comment on S. Ahmed, H. Wang, and Y. Tian, “Robust adaptive fractional-order terminal sliding mode control for lower-limb exoskeleton,” *Asian J. Control*, vol. 21, no. 1, pp. 1–10 (2019). *Asian J. Control* **2023**, *25*, 2433–2436. [[CrossRef](#)]
42. Podlubny, I. *Fractional Differential Equations, Mathematics in Science and Engineering*; Academic Press: Cambridge, MA, USA, 1999.
43. Soorki, M.N.; Tavazoei, M.S. Adaptive robust control of fractional-order systems in the presence of model uncertainties and external disturbances. *IET Control Theory Appl.* **2018**, *12*, 961–969. [[CrossRef](#)]
44. Wang, Y.; Gu, L.; Xu, Y.; Cao, X. Practical Tracking Control of Robot Manipulators with Continuous Fractional-Order Nonsingular Terminal Sliding Mode. *IEEE Trans. Ind. Electron.* **2016**, *63*, 6194–6204. [[CrossRef](#)]
45. Barrera Sánchez, A.; Blanco Ortega, A.; Martínez Rayón, E.; Gómez Becerra, F.A.; Abúndez Pliego, A.; Campos Amezcua, R.; Guzmán Valdivia, C.H. State of the Art Review of Active and Passive Knee Orthoses. *Machines* **2022**, *10*, 865. [[CrossRef](#)]
46. Mefoued, S.; Mohammed, S.; Amirat, Y. Knee joint movement assistance through robust control of an actuated orthosis. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, San Francisco, CA, USA, 25–30 September 2011; pp. 1749–1754.
47. Popov, V.M. *Hyperstability of Control System*; Springer-Verlag: Berlin, Germany, 1973.
48. Soorki, M.N.; Tavazoei, M.S. Fractional-order linear time-invariant swarm systems: Asymptotic swarm stability and time response analysis. *Cent. Eur. J. Phys.* **2013**, *11*, 845–854. [[CrossRef](#)]

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