

## Article

# Data Envelopment Analysis Approaches for Multiperiod Two-Level Production and Distribution Planning Problems

Tomohiro Hayashida <sup>\*,†</sup>, Ichiro Nishizaki <sup>†</sup>, Shinya Sekizaki  and Junya Okabe

Graduate School of Advanced Science and Engineering, Hiroshima University, Higashihiroshima 739-8527, Japan; nisizaki@hiroshima-u.ac.jp (I.N.); sekizaki@hiroshima-u.ac.jp (S.S.); j-okabe@ml.hiroshima-u.ac.jp (J.O.)

\* Correspondence: hayashida@hiroshima-u.ac.jp

<sup>†</sup> These authors contributed equally to this work.

**Abstract:** This paper deals with two-level production and distribution planning problems in supply chain management where the leader is a distributor and the follower is a manufacturer. Assuming that the distributor can observe the input–output data in the production process, we formulated the data envelopment analysis (DEA) production problem corresponding to the production planning problem of the manufacturer. This paper proposes a novel data envelopment analysis (DEA) approach to solve a challenging multiperiod two-level production and distribution planning problem in supply chain management. The innovative idea behind the proposed approach is to allow the distributor to observe the input–output data regarding the production activities of the manufacturer, even if the distributor cannot fully comprehend all parameters of the manufacturer’s production cost minimization problem. This approach addresses the challenge of uncertain demands by employing a two-stage model with simple recourse and considering the usage of the input–output data. The paper demonstrates the validity of the proposed DEA approaches through computational experiments and discusses the accuracy, reliability, and importance of the input–output data. The proposed approach provides a practical and effective solution to the multiperiod two-level production and distribution planning problem in supply chain management, and can help decision-makers improve the efficiency and effectiveness of their operations. The innovative idea of allowing the distributor to observe the input–output data about the production activities of the manufacturer is a significant contribution to the field of supply chain management and has the potential to advance the state of the art in this area.

**Keywords:** DEA approaches; multiperiod two-level production; distribution planning problem; two-stage model; simple recourse; uncertain demands

**MSC:** 49N30



**Citation:** Hayashida, T.; Nishizaki, I.; Sekizaki, S.; Okabe, J. Data Envelopment Analysis Approaches for Multiperiod Two-Level Production and Distribution Planning Problems. *Mathematics* **2023**, *11*, 4492. <https://doi.org/10.3390/math11214492>

Academic Editors: Ming-Min Yu, Bo Hsiao and Li-Hsueh Chen

Received: 9 September 2023

Revised: 13 October 2023

Accepted: 23 October 2023

Published: 30 October 2023



**Copyright:** © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

## 1. Introduction

In supply chain management, a two-level production and distribution planning problem arises when the distributor assumes the role of a leader and the manufacturer becomes the follower. This hierarchical structure can present intricate decision-making challenges for the distributor. When the manufacturer rationally reacts to the distributor’s choices, the distributor is tasked with formulating the entire two-level production and distribution planning problem [1]. Essential to this formulation is a deep understanding of the manufacturer’s technological processes. However, the distributor often finds themselves in a predicament, as they may not always possess comprehensive knowledge of the manufacturer’s technological intricacies and operations.

For such a situation where the opponent player’s complete information is not available, the Bayesian game approach has often been adopted [2–11]. In contrast to such an approach, Nishizaki et al. [12] employed the data environment analysis (DEA) approach for the two-level production and distribution planning problem to identify incomplete

information on production technologies of the manufacturer at the lower level. In their study, assuming that although it is not possible for the distributor to fully understand technological coefficients expressing the relationship between resources and products, the distributor can observe the input–output data such as the amount of resources purchased and the amount of products produced. They also considered a two-level production and distribution planning problem in which the manufacturer’s production cost minimization problem was formulated according to the concept of DEA [13,14]

However, Nishizaki et al. [12] targeted a single-period planning problem, which as the following literature shows, is important to extend to a multiperiod planning problem. In this paper, we investigate a multiperiod production and distribution planning problem in a more realistic setting.

Saberi et al. [15] point out that there are multiple opportunities for supply chain entities to acquire returns and to make sustainable operational and strategic decisions, and, therefore, the study in multiperiod planning horizon settings is of great importance in theory and practice related to dynamic green production and pricing policies. We can find many other instances of multiperiod models for supply chain management [16–21].

In the research field of the DEA models, for example, the overall efficiency is evaluated by aggregating the data at all periods or using the average [22,23]. Furthermore, a framework called the network DEA has been introduced to develop a general multistage model [24,25]. For further information about two-stage DEA models, Halkos et al. [26] present a related review paper. These studies focus on the issues of how to express the efficiency of multiperiod activities. Thus, it should be noted that these previous studies are indirectly related to this study, which attempts to express the production possibility set of multiperiod activities from the perspective of DEA.

In order to extend the study of Nishizaki et al. [12] to a multiperiod model, we introduce the following elements. It is natural to consider that uncertainty about demands increases as the period progresses in the multiperiod model. Therefore, in this paper, the demands at retailers are expressed as discrete random variables, and the difference between supply and demand is manipulated by a two-stage model with simple recourse. It is assumed in the proposed multiperiod model, the residual demand is added to the demand in the next period. Since we deal with a multiperiod model, it is assumed that products at a warehouse can be held the next period and the corresponding storage is costly. The distributor makes a decision on whether or not to place an order for the products at the beginning of each period, and placing orders is also costly. For the input–output data of the manufacturer in the multiperiod model, we assume that the input–output data are observed in multiple periods before an optimal plan for the planning periods is obtained by solving the production and distribution planning problem. The observation periods may be successive, or they may be composed of separated periods.

The novelty of this paper can be summarized below. Assuming that although it is not possible for the distributor to fully comprehend all parameters of the manufacturer’s production cost minimization problem, the distributor can observe the input–output data about the production activities of the manufacturer, we examine the multiperiod production and distribution planning problem. Considering uncertain demands due to the multiperiod planning, we employ the two-stage model with simple recourse. Furthermore, we discuss the accuracy, reliability, and importance of the input–output data.

The structure of this paper is delineated as follows: Section 2 delineates the connection between the lower-level production planning problem and the DEA production problem. It introduces the formulation of the two-level multiperiod production and distribution problem with technological coefficients and emphasizes the significance of input–output data in this context. Furthermore, we will detail the methodologies employed to address the formulated challenges. Section 3 provides a comparative analysis, juxtaposing the formulation using technological coefficients with the methodologies presented in the DEA approaches. This section underpins the validity and advantages of our proposed methods through demonstrative numerical examples. Section 4 encapsulates the conclusions drawn

from our research, highlighting the contributions and potential implications for future endeavors in this domain.

## 2. Formulations

Nishizaki et al. [12] investigated a two-level production and distribution problem where the leader was a distributor and the follower was a manufacturer. This problem dealt with single period planning without any uncertainty, and the manufacturer's production problem at the lower level was formulated as the DEA production problem. In this paper, we extend this model to a multiperiod problem under uncertainty. The distributor asks the manufacturer to produce the products so that the demands at the retailers are met, and this situation is formulated as a two-stage model with simple recourse. The products produced in factories of the manufacturer are transported to warehouses of the distributor. The distributor delivers the products to the retailers with uncertain demands represented by discrete random variables. We assume that the manufacturer determines which factories to produce the products ordered by the distributor to minimize the production cost including the purchase cost of resources, specifically based on the manufacturer's rational response. Due to the nature of multiperiod problems, the distributor does not always place an order with the manufacturer at every period, and it is costly to place an order. It is possible but costly to hold the products in a warehouse over a few periods. We also assume that the warehouses already exist, and no decision is made as to whether or not a warehouse should be open.

### 2.1. Multiperiod Problem with Technological Coefficients

As shown in Figure 1, we consider a two-level multiperiod production and distribution problem in which the distributor places an order to the manufacturer, and then it transports multiple products supplied from the manufacturer possessing multiple factories to the retailers with stochastic demands. The manufacturer with multiple factories produces the products ordered by the distributor, and it decides in which factory each product should be produced. The manufactured products are delivered to the distributor's warehouses, and the cost for delivering the products is paid by the distributor. The distributor delivers the products from the multiple warehouses to the multiple retailers with stochastic demands.

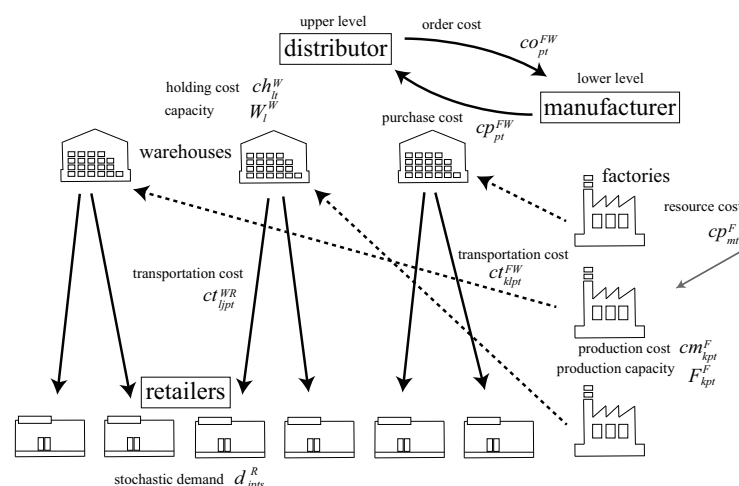


**Figure 1.** Two-level production and distribution network.

We summarize the indices, decision variables, and parameters as follows.

#### indices

- $l \in \{1, \dots, L\}$ : warehouse:  $W$
- $k \in \{1, \dots, K\}$ : factory:  $F$
- $j \in \{1, \dots, J\}$ : retailer:  $R$

$p \in \{1, \dots, P\}$ : product  
 $m \in \{1, \dots, M\}$ : resource  
 $t \in \{1, \dots, T\}$ : period  
 $s \in \{1, \dots, S\}$ : scenario

The variables and parameters shown below are superscripted by  $W$ ,  $F$ , and  $R$ , indicating that they are related to warehouses, factories, and retailers, respectively. A two-letter word indicates that it is related to two entities. For example, a variable with  $WR$  is related to both a warehouse and a retailer.

### decision variable

#### upper-level

$x_{l,j,p,t}^{WR}$ : transportation amount of product  $p$  from warehouse  $l$  to retailer  $j$  at period  $t$ .  
 $o_{p,t}^W$ : order of product  $p$  at period  $t$ . (If ordered,  $o_{p,t}^W = 1$ , otherwise,  $o_{p,t}^W = 0$ .)  
 $y_{p,l,t}^W$ : inventory amount of product  $p$  in warehouse  $l$  at period  $t$ .  
 $\delta_{j,p,t,s}^+$ : supply shortage of product  $p$  for retailer  $j$  at period  $t$  and scenario  $s$ .  
 $\delta_{j,p,t,s}^-$ : oversupply amount of product  $p$  for retailer  $j$  at period  $t$  and scenario  $s$ .

#### lower level

$y_{k,l,p,t}^{FW}$ : transportation amount of product  $p$  from factory  $k$  to warehouse  $l$  at period  $t$ .  
 $z_{k,p,t}^F$ : production amount of product  $p$  in factory  $k$  at period  $t$ .  
 $b_{m,t}^F$ : purchase amount of resource  $m$  at period  $t$ .

### parameter

$ct_{l,j,p,t}^{WR}$ : transportation cost of one unit of product  $p$  from warehouse  $l$  to retailer  $j$  at period  $t$ .  
 $ct_{k,l,p,t}^{FW}$ : transportation cost of one unit of product  $p$  from factory  $k$  to warehouse  $l$  at period  $t$ .  
 $ch_{l,t}^W$ : holding cost of one unit volume in warehouse  $l$  at period  $t$ .  
 $s_p^W$ : storage volume of one unit of product  $p$ .  
 $cp_{p,t}^{FW}$ : purchase price of one unit of product  $p$  from the manufacturer at period  $t$ .  
 $co_{p,t}^{FW}$ : ordering cost of product  $p$  for the manufacturer at period  $t$ .  
 $P_{j,p,t}^{+R}$ : penalty for the supply shortage to retailer  $j$  for one unit of product  $p$  at period  $t$ .  
 $P_{j,p,t}^{-R}$ : penalty for the oversupply to retailer  $j$  for one unit of product  $p$  at period  $t$ .  
 $d_{j,p,t,s}^R$ : demand of product  $p$  for retailer  $j$  at period  $t$  and scenario  $s$ .  
 $cp_{m,t}^F$ : purchase price of one unit of resource  $m$  at period  $t$ .  
 $cm_{k,p,t}^F$ : production cost for product  $p$  at factory  $k$  at period  $t$ .  
 $a_{m,p}^F$ : technological coefficient: an amount of resource  $m$  required to produce one unit of product  $p$ .  
 $W_l^W$ : capacity of warehouse  $l$ .  
 $F_{k,p,t}^F$ : production capacity for product  $p$  in factory  $k$  at period  $t$ .

The two-level multiperiod production and distribution planning problem of the distributor assuming the rational response of the manufacturer can be formulated as follows.

$$\begin{aligned} \text{minimize } & \sum_{t=1}^T \left\{ \sum_{l=1}^L \sum_{j=1}^J \sum_{p=1}^P ct_{l,j,p,t}^{WR} x_{l,j,p,t}^{WR} + \sum_{k=1}^K \sum_{l=1}^L \sum_{p=1}^P ct_{k,l,p,t}^{FW} y_{k,l,p,t}^{FW} + \sum_{l=1}^L ch_{l,t}^W \sum_{p=1}^P s_p^W y_{p,l,t}^W \right. \\ & \left. + \sum_{p=1}^P cp_{p,t}^{FW} \sum_{k=1}^K \sum_{l=1}^L y_{k,l,p,t}^{FW} + \sum_{p=1}^P co_{p,t}^O o_{p,t}^W + \sum_{j=1}^J \sum_{p=1}^P P_{j,p,t}^{+R} \delta_{j,p,t,s}^+ + \sum_{j=1}^J \sum_{p=1}^P P_{j,p,t}^{-R} \delta_{j,p,t,s}^- \right\} \end{aligned} \quad (1a)$$

$$\begin{aligned} \text{subject to } & \sum_{l=1}^L x_{l,j,p,t}^{WR} + \delta_{j,p,t,s}^+ - \delta_{j,p,t,s}^- = d_{j,p,t,s}^R, \quad j = 1, \dots, J, \quad p = 1, \dots, P, \\ & t = 1, \dots, T, \quad s = 1, \dots, S \end{aligned} \quad (1b)$$

$$\delta_{j,p,t,s}^+ \delta_{j,p,t,s}^- = 0, \quad j = 1, \dots, J, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (1c)$$

$$o_{p,t}^W \in \{0, 1\}, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (1d)$$

$$\begin{aligned} x_{l,j,p,t}^{WR} y_{p,l,t}^W \delta_{j,p,t,s}^+ \delta_{j,p,t,s}^- & \geq 0, \quad j = 1, \dots, J, \quad k = 1, \dots, K, \quad l = 1, \dots, L, \\ & p = 1, \dots, P, \quad t = 1, \dots, T \end{aligned} \quad (1e)$$

$$\text{minimize } \sum_{t=1}^T \left\{ \sum_{m=1}^M cp_{m,t}^F b_{m,t}^F + \sum_{k=1}^K \sum_{p=1}^P cm_{k,p,t}^F z_{k,p,t}^F \right\} \quad (1f)$$

$$\text{subject to } \sum_{k=1}^K \sum_{p=1}^P a_{m,p,t}^F z_{k,p,t}^F \leq b_{m,t}^F, \quad m = 1, \dots, M, \quad t = 1, \dots, T \quad (1g)$$

$$\sum_{l=1}^L y_{k,l,p,t}^{FW} \leq z_{k,p,t}^F, \quad k = 1, \dots, K, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (1h)$$

$$\mathcal{M}o_{p,t}^W \geq \sum_{k=1}^K z_{k,p,t}^F, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (1i)$$

$$\sum_{p=1}^P s_p^W \left( \sum_{k=1}^K y_{k,l,p,t}^{FW} + y_{p,l,t}^W \right) \leq W_l^W, \quad l = 1, \dots, L, \quad t = 1, \dots, T \quad (1j)$$

$$\begin{aligned} y_{p,l,t-1}^W - \sum_{j=1}^J x_{l,j,p,t}^{WR} + \sum_{k=1}^K y_{k,l,p,t}^{FW} & = y_{p,l,t}^W, \quad p = 1, \dots, P, \\ & l = 1, \dots, L, \quad t = 1, \dots, T \end{aligned} \quad (1k)$$

$$z_{k,p,t}^F \leq F_{k,p,t}^F, \quad k = 1, \dots, K, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (1l)$$

$$\begin{aligned} b_{m,t}^F, z_{k,p,t}^F, y_{k,l,p,t}^{FW} & \geq 0, \quad m = 1, \dots, M, \quad k = 1, \dots, K, \quad p = 1, \dots, P, \\ & t = 1, \dots, T, \quad l = 1, \dots, L \end{aligned} \quad (1m)$$

The distributor's objective function (1a) to be minimized is the sum of the transportation cost of the products from warehouses to retailers, the transportation cost from factories to warehouses, the holding cost of the products in warehouses, the purchase cost of the products from the manufacturer, and the penalty costs for supply shortage and oversupply. Equation (1b) is a demand constraint under uncertainty. Suppose that demand  $d_{j,p,t,s}^R$  is realized when scenario  $s$  occurs. Then, if  $\sum_{l=1}^L x_{l,j,p,t}^{WR} \geq d_{j,p,t,s}^R$ , product  $p$  is excessively supplied to retailer  $j$ , or if  $\sum_{l=1}^L x_{l,j,p,t}^{WR} \leq d_{j,p,t,s}^R$ , it is insufficiently supplied. The relation between the supply amount to retailer  $j$  and the demand of retailer  $j$  is represented by (1b), introducing non-negative variables  $\delta_{j,p,t,s}^+$  and  $\delta_{j,p,t,s}^-$  where one of the two variables is zero for retailer  $j = 1, \dots, J$ , product  $p = 1, \dots, P$ , period  $t = 1, \dots, T$ , and scenario  $s = 1, \dots, S$ . The variables  $\delta_{j,p,t,s}^+$  and  $\delta_{j,p,t,s}^-$  imply the supply shortage and oversupply, respectively. Constraint (1c) shows that either  $\delta_{j,p,t,s}^+$  or  $\delta_{j,p,t,s}^-$  is zero, but it can be deleted because the supply shortage and oversupply are penalized in the objective function (1a). In constraint (1d), if the distributor places an order for product  $p$  at period  $t$ ,  $o_{p,t}^W = 1$ , otherwise  $o_{p,t}^W = 0$ . The

objective function (1f) of the manufacturer at the lower level to be minimized is the sum of the purchase cost of resources and the production costs at all factories. It is assumed that the resources are procured in bulk rather than purchased individually for each factory. The left-hand side of constraint (1g) means the resource usage represented by using the technological coefficient  $a_{m,p,t}^F$ , and the right-hand side is the purchase amount of the resource. Then, constraint (1g) implies the resource constraint that the usage does not exceed the purchase amount. Constraint (1h) means that the transportation amount from a factory to warehouses does not exceed the production amount in the factory. Constraint (1i) means that the product will not be manufactured if there is no order, and coefficient  $M$  in the left-hand side is a sufficiently large positive number. Constraint (1j) represents the capacity limitation of a warehouse. Constraint (1k) shows that the number of products obtained by subtracting the shipment from the inventory at period  $t - 1$  and adding the arrival amount is the inventory at period  $t$ . Constraint (1l) represents the production capacity of a factory.

As pointed out in the previous section, since it is difficult to estimate the technological coefficients appearing in constraint (1g) of the manufacturer's problem, assuming that the amount of resources purchased and the amount of products produced can be observed, we will reformulate the follower's production planning problem as the DEA production problem in Section 2.3.

## 2.2. Production Planning Problem with Technological Coefficients and DEA Production Problem

Before dealing with the two-level multiperiod production and distribution planning problem including the DEA production problem minimizing the costs of production and purchase for the manufacturer at the lower level, we describe the relation between the production planning problem with technological coefficients and the DEA production problem.

Suppose that in the linear production process, a manufacturer produces  $P$  types of final products by using  $M$  kinds of resources. The available amounts of resources are represented by a vector  $\mathbf{b} \in \mathbb{R}^M$ , and the production amounts are represented by a vector  $\mathbf{y} \in \mathbb{R}^P$ . Let  $A \in \mathbb{R}^{M \times P}$  be a technological matrix of the manufacturer, and then an element  $a_{mp}$  of  $A$  represents the amount of resource  $m$  required to manufacture one unit of product  $p$ . Let  $\mathbf{c} \in \mathbb{R}^P$  be a price vector of the products, and the manufacturer maximizes the total revenue  $\mathbf{c}^T \mathbf{y}$  from selling the  $M$  types of products, where the superscript  $T$  represents the transposition of a vector or matrix. The linear production process described by the triplet  $(A, \mathbf{c}, \mathbf{b})$  can be optimized by solving the following linear programming problem.

$$\text{maximize } \mathbf{c}^T \mathbf{y} \quad (2a)$$

$$\text{subject to } A\mathbf{y} \leq \mathbf{b} \quad (2b)$$

$$\mathbf{y} \geq \mathbf{0}. \quad (2c)$$

Based on the concept of DEA [13,14], Lozano [27] considered the DEA production game extended from the linear production game defined by [28], and formulated the DEA production problem with data regarding the resource inputs and product outputs observed in the linear production process. In the DEA production problem, it is interpreted that the production technology can be implicitly found in the input–output data observed in the production process.

Suppose that the following input–output data are observed in the linear production process.

$$\mathcal{D} = \{(X^d, Y^d) \in \mathbb{R}^M \times \mathbb{R}^P, d = 1, \dots, D\}. \quad (3)$$

The number of the observed data is  $D$ ,  $X^d$  is the  $d$ th input data such as resource inputs, and  $Y^d$  is the  $d$ th output data such as product outputs. Let  $X \in \mathbb{R}^{M \times D}$  be a matrix such that the  $d$  column vector is  $X^d$ , and let  $Y \in \mathbb{R}^{P \times D}$  be a matrix such that the  $d$  column vector is  $Y^d$ .

The following optimization problem called the DEA production problem for finding the input level  $\mathbf{x}$  and the output level  $\mathbf{y}$  yielding the maximal profit can be formulated by using the observed input–output data (3) instead of the technological matrix  $A$ .

$$\text{maximize } \mathbf{c}^T \mathbf{y} \quad (4a)$$

$$\text{subject to } X\lambda \leq \mathbf{x} \quad (4b)$$

$$Y\lambda \geq \mathbf{y} \quad (4c)$$

$$\mathbf{x} \leq \mathbf{b} \quad (4d)$$

$$\lambda \in \Lambda, \quad (4e)$$

where  $\lambda \in \mathbb{R}^D$  is a variable characterizing the production possibility set, and the set  $\Lambda$  is defined according to an assumption of returns to scale. For the assumption of constant returns to scale (CRS), the set  $\Lambda$  is defined as  $\Lambda = \Lambda_C \equiv \{\lambda \in \mathbb{R}^D \mid \lambda \geq \mathbf{0}\}$ . For the assumption of variable returns to scale (VRS), the set  $\Lambda$  is defined as  $\Lambda = \Lambda_V \equiv \{\lambda \in \mathbb{R}^D \mid \lambda \geq \mathbf{0}, \sum_{d=1}^D \lambda_d = 1\}$ . In this paper, we employ the VRS model, i.e.,  $\Lambda = \Lambda_V$ , and this means that any input–output level  $(\mathbf{x}, \mathbf{y})$  is in the convex hull including all the observed input–output data. Constraint (4b) means that any convex combination of the observed input data does not exceed the input level  $\mathbf{x}$ , and constraint (4c) means that any convex combination of the observed output data does not fall below the output level  $\mathbf{y}$ . Constraint (4d) means that the input level  $\mathbf{x}$  does not exceed the limit of available resources  $\mathbf{b}$ . It should be noted that the input level  $\mathbf{x}$  in problem (2a)–(2c) is represented by  $A\mathbf{y}$  using the technological matrix  $A$ , and the relation between the input level  $\mathbf{x}$  and the output level  $\mathbf{y}$  is uniquely determined.

### 2.3. Multiperiod DEA Production and Distribution Planning Problem

We summarize indices, data (parameters), and decision variables introduced in the multiperiod DEA production and distribution planning problem.

#### indices

$\tau \in \{1, \dots, T\}$ : period that input–output data are observed.

$d \in \{1, \dots, D_\tau\}$ : index of input–output data at period  $\tau$ .

#### aggregated data

$\beta_p^{\tau,d}$ :  $d$ th output data (production amount) of product  $p$  observed at period  $\tau$ .

$\alpha_m^{\tau,d}$ :  $d$ th input data (purchase amount) of resource  $m$  observed at period  $\tau$ .

#### disaggregated data

$\beta_{p,k}^{\tau,d}$ :  $d$ th output data (production amount) of product  $p$  from factory  $k$  observed at period  $\tau$ .

$\alpha_{m,k}^{\tau,d}$ :  $d$ th input data (purchase amount) of resource  $m$  for factory  $k$  observed at period  $\tau$ .

#### decision variables

$\lambda_{\tau,d}^{k,t}$ : weighting variable of planning period  $t$  for the  $d$ th input–output data in factory  $k$  observed at period  $\tau$ .

There are two possibilities for observing the input–output data of the manufacturer. The first possibility is that the input–output data can be observed in an aggregated form, namely the input–output data aggregated for all factories are observed collectively. The second possibility is that the input–output data can be observed in a disaggregated form, namely the input–output data of each factory can be observed separately.

In the aggregated form,  $\alpha_m^{\tau,d}$  is the  $d$ th purchase amount of resource  $m$  observed at period  $\tau$ . Although the distributor at the upper level does not know the usage of resource  $m$  in each factory for the  $d$ th observed data at period  $\tau$ , it only knows that the total usage of resource  $m$  for all factories is  $\alpha_m^{\tau,d}$ . In the disaggregated form,  $\alpha_{m,k}^{\tau,d}$  is the  $d$ th purchase amount of resource  $m$  for factory  $k$  observed at period  $\tau$ . The distributor knows the usage of resource  $m$  in factory  $k$  from the  $d$ th observed data at period  $\tau$ .

In a similar way, there are two possibilities for the output data. In the aggregated form,  $\beta_p^{\tau,d}$  is the  $d$ th output amount of product  $p$  observed at period  $\tau$ . Although the distributor at the upper level does not know the output of product  $p$  in each factory for the  $d$ th observed data at period  $\tau$ , it only knows that the total output of product  $p$  for all factories is  $\beta_p^{\tau,d}$ . In the disaggregated form,  $\beta_{p,k}^{\tau,d}$  is the  $d$ th production amount of product  $p$  in factory  $k$  observed at period  $\tau$ . The distributor knows the output of product  $p$  in factory  $k$  from the  $d$ th observed data at period  $\tau$ .

As seen in Figure 2, the input–output data have been observed in  $\mathcal{T}$  periods, and the number of input–output data at period  $\tau$  is  $D_\tau$ . Using all the input–output data of  $\mathcal{T}$  periods, we formulate the manufacturer’s production planning problem for all planning periods  $t = 1, \dots, T$  as the DEA production problem.

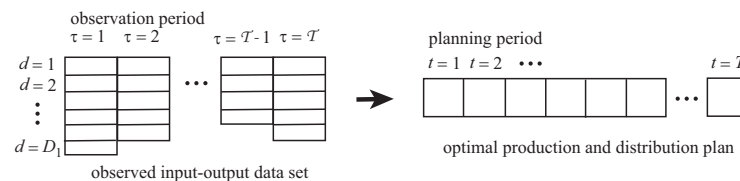


**Figure 2.** Observed input–output data and optimal production and distribution plan.

### 2.3.1. Aggregated Data

Assume that the manufacturer’s resource inputs and product outputs can be observed as follows.

$$\{(\alpha^{\tau,d}, \beta^{\tau,d}) \in \mathbb{R}^M \times \mathbb{R}^P, \tau = 1, \dots, \mathcal{T}, d = 1, \dots, D_\tau\}, \quad (5)$$

where  $\alpha^{\tau,d} = (\alpha_1^{\tau,d}, \dots, \alpha_M^{\tau,d})^\top$  is the  $d$ th  $M$ -dimensional resource input vector at observation period  $\tau$ , and  $\beta^{\tau,d} = (\beta_1^{\tau,d}, \dots, \beta_P^{\tau,d})^\top$  is the  $d$ th  $P$ -dimensional product output vector at observation period  $\tau$ . To define the production possibility set represented by convex combinations of the aggregated input–output data (5), we introduce a variable  $\lambda_{\tau,d}$  which is a weighting variable for the  $d$ th data at observation period  $\tau$ . Under the assumption of VRS, the production possibility set can be defined by

$$\sum_{\tau=1}^{\mathcal{T}} \sum_{d=1}^{D_\tau} \lambda_{\tau,d} = 1, \lambda_{\tau,d} \geq 0, \tau = 1, \dots, \mathcal{T}, d = 1, \dots, D_\tau. \quad (6)$$

However, since we deal with multiple factories  $k = 1, \dots, K$  and multiple planning periods  $t = 1, \dots, T$ , we need to express production in factory  $k$  at planning period  $t$ . To do so, we consider the usage rate of the input in factory  $k$  at period  $t$  or the production ratio of the output in factory  $k$  at period  $t$ , and then let these ratios be denoted by  $\gamma_{kt}$ . Setting  $\lambda_{\tau,d}^{k,t} \equiv \gamma_{kt} \lambda_{\tau,d}$ , we introduce the weighting variable  $\lambda_{\tau,d}^{k,t}$  for the  $d$ th data in the observation period  $\tau$  in factory  $k$  at planning period  $t$ .

Using the weighting variables  $\lambda_{\tau,d}^{k,t}$ ,  $\tau = 1, \dots, \mathcal{T}$ ,  $d = 1, \dots, D_\tau$ ,  $k = 1, \dots, K$ ,  $t = 1, \dots, T$  to form the production possibility set by convex combinations of the observed input–output data (5), we reformulate the production planning problem (1a)–(1m) of the manufacturer at the lower level in problem (1a)–(1m) as the following DEA production problem under the assumption of VRS. Henceforth, for the sake of simplicity, we will call problem (1a)–(1m) the technological coefficient problem, and also call the DEA production problem with aggregated input–output data the aggregated DEA production problem.

$$\text{minimize } \sum_{t=1}^T \left\{ \sum_{m=1}^M c p_{m,t}^F \sum_{k=1}^K b_{m,k,t}^F + \sum_{k=1}^K \sum_{p=1}^P c m_{k,p,t}^F \sum_{l=1}^L y_{k,l,p,t}^{FW} \right\} \quad (7a)$$

$$\text{subject to } \sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} \alpha_m^{\tau,d} \leq b_{m,k,t}^F, \quad m = 1, \dots, M, \quad k = 1, \dots, K, \quad t = 1, \dots, T \quad (7b)$$

$$\sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} \beta_p^{\tau,d} \geq \sum_{l=1}^L y_{k,l,p,t}^{FW}, \quad p = 1, \dots, P, \quad k = 1, \dots, K, \quad t = 1, \dots, T \quad (7c)$$

$$\mathcal{M}o_{p,t}^W \geq \sum_{k=1}^K \sum_{l=1}^L y_{k,l,p,t}^{FW}, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (7d)$$

$$\sum_{p=1}^P s_p^W \left( \sum_{j=1}^J y_{k,l,p,t}^{FW} + y_{p,l,t}^W \right) \leq W_l^W, \quad l = 1, \dots, L, \quad t = 1, \dots, T \quad (7e)$$

$$y_{p,l,t-1}^W - \sum_{j=1}^J x_{l,j,p,t}^{WR} + \sum_{k=1}^K y_{k,l,p,t}^{FW} = y_{p,l,t}^W, \quad p = 1, \dots, P, \quad l = 1, \dots, L, \quad t = 1, \dots, T \quad (7f)$$

$$\sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} \beta_p^{\tau,d} \leq F_{k,p,t}^F, \quad k = 1, \dots, K, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (7g)$$

$$\sum_{k=1}^K \sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} = 1, \quad t = 1, \dots, T \quad (7h)$$

$$b_{m,k,t}^F, y_{k,l,p,t}^{FW}, \lambda_{\tau,d}^{k,t} \geq 0, \quad m = 1, \dots, M, \quad k = 1, \dots, K, \quad p = 1, \dots, P, \\ t = 1, \dots, T, \quad l = 1, \dots, L \quad (7i)$$

Compared to the technological coefficient problem (1a)–(1m), since the production amount of each product in each factory at each period is not explicitly shown in the aggregated DEA production problem (7a)–(7i), not only the technological coefficients  $a_{m,p}^F$  but also the production amounts  $z_{k,p,t}^F$  are not used. Although the input–output data are aggregated, the weighting variable  $\lambda_{\tau,d}^{k,t}$  is assigned to each factory. Therefore, we introduce the decision variable  $b_{m,k,t}^F$  which represents the purchase amount of resource  $m$  for factory  $k$  at period  $t$ , instead of the decision variable  $b_{m,t}^F$  used in the technological coefficient problem (1a)–(1m). It is noted that  $\sum_{k=1}^K b_{m,k,t}^F = b_{m,t}^F$  holds.

The objective function (7a) is the sum of the cost of purchasing resources and cost of manufacturing products, and it is the same as the objective function of the technological coefficient problem (1a)–(1m). However, variable  $b_{m,k,t}^F$  for factory  $k$  is employed as the purchase amount of resource  $m$ , and the production amount of factory  $k$  is represented by the sum  $\sum_{l=1}^L y_{k,l,p,t}^{FW}$  of transportation amounts from factory  $k$  to all warehouses instead of the production amount  $z_{k,p,t}^F$  in factory  $k$ . Constraint (7b) means that the input of resource  $m$  in factory  $k$  at period  $t$  does not exceed the purchase amount of resource  $m$ , and the resource input is represented by a convex combination of the observed input data  $\alpha_m^{\tau,d}$ . Constraint (7b) corresponds to constraint (1g) in the technological coefficient problem. Therefore, constraint (7b) ensures that the observed input–output data are consistent with the production process. Specifically, it requires that the observed output data can be produced using the observed input data and the unknown production parameters. This constraint is formulated as a linear combination of the input data and the unknown parameters, where the coefficients are the observed output data. The resulting linear equation represents the production process, and the constraint ensures that the observed output data are consistent with this process. Constraint (7c) means that the output of product  $p$  in factory  $k$  at period  $t$  is larger than or equal to the sum  $\sum_{l=1}^L y_{k,l,p,t}^{FW}$  of transportation amounts of product  $p$  from factory  $k$  to all warehouses, and the product output is represented by

a convex combination of the observed output data  $\beta_p^{\tau,d}$ . Constraint (7c) corresponds to constraint (1h) in the technological coefficient problem. Therefore, constraint (7c) ensures that the observed input–output data are consistent with the production process. Specifically, it requires that the observed output data can be produced using the observed input data and the unknown production parameters. This constraint is formulated as a linear combination of the input data and the unknown parameters, where the coefficients are the observed output data. The resulting linear equation represents the production process, and the constraint ensures that the observed output data are consistent with this process. Together, constraints (7b) and (7c) estimate the production possibility set using input–output data observations. They ensure that the observed output data are consistent with the production process, and that the estimated production process is efficient. By estimating the production possibility set in this way, the DEA approach can identify the best performing units and the optimal production plan for the manufacturer, given the input–output data and the uncertain demands. Constraint (7d) is an ordering constraint, and corresponds to constraint (1i). Constraints (7e) and (7f) are the same as those of the technological coefficient problem (1a)–(1m), and they mean the capacity constraint of a warehouse and the inventory constraint, respectively. Constraint (7g) means that the production amount of product  $p$  in factory  $k$  expressed by a convex combination of the observed output data  $\beta_p^{\tau,d}$  does not exceed the production capacity of the factory. Constraint (7h) indicates that this aggregated DEA production problem is under the assumption of VRS.

### 2.3.2. Disaggregated Data

We examine the DEA production problem with the disaggregated input–output data, and shortly call it the disaggregated DEA production problem. Assume that we can observe the input–output data of each factory separately. Specifically, the data of the manufacturer's resource inputs and product outputs can be represented as follows:

$$\{(A^{\tau,d}, B^{\tau,d}) \in \mathbb{R}^{M \times K} \times \mathbb{R}^{P \times K}, \tau = 1, \dots, \mathcal{T}, d = 1, \dots, D_\tau\}, \quad (8)$$

where  $A^{\tau,d}$  is an  $M \times K$  matrix whose elements are  $\alpha_{m,k}^{\tau,d}$ ,  $m = 1, \dots, M$ ,  $k = 1, \dots, K$ , and it is the input data on  $M$  resource inputs to  $K$  factories at observation period  $\tau$ ;  $B^{\tau,d}$  is a  $P \times K$  matrix whose elements are  $\beta_{p,k}^{\tau,d}$ ,  $p = 1, \dots, P$ ,  $k = 1, \dots, K$ , and it is the output data on  $P$  product outputs from  $K$  factories at observation period  $\tau$ .

Although the production constraint (7g) means that the product outputs do not exceed the capacity of production is required in the aggregated DEA production problem, the constraint corresponding to (7g) is not always necessary in the disaggregated DEA production problem because the product outputs do not exceed the maximal values in the observed output data. The disaggregated DEA production problem using the input–output data (8) can be formulated as follows:

$$\text{minimize } \sum_{t=1}^T \left\{ \sum_{m=1}^M c p_{m,t}^F \sum_{k=1}^K b_{m,k,t}^F + \sum_{k=1}^K \sum_{p=1}^P c \tilde{m}_{k,p,t}^F \sum_{l=1}^L y_{k,l,p,t}^{FW} \right\} \quad (9a)$$

$$\text{subject to } \sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} \alpha_{m,k}^{\tau,d} \leq b_{m,k,t}^F, \quad m = 1, \dots, M, \quad k = 1, \dots, K, \quad t = 1, \dots, T \quad (9b)$$

$$\sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} \beta_{p,k}^{\tau,d} \geq \sum_{l=1}^L y_{k,l,p,t}^{FW}, \quad p = 1, \dots, P, \quad k = 1, \dots, K, \quad t = 1, \dots, T \quad (9c)$$

$$\mathcal{M}o_{p,t}^W \geq \sum_{k=1}^K \sum_{l=1}^L y_{k,l,p,t}^{FW}, \quad p = 1, \dots, P, \quad t = 1, \dots, T \quad (9d)$$

$$\sum_{p=1}^P s_p^W \left( \sum_{j=1}^J y_{k,j,p,t}^{FW} + y_{p,l,t}^W \right) \leq W_l^W, \quad l = 1, \dots, L, \quad t = 1, \dots, T \quad (9e)$$

$$y_{p,l,t-1}^W - \sum_{j=1}^J x_{l,j,p,t}^{WR} + \sum_{k=1}^K y_{k,l,p,t}^{FW} = y_{p,l,t}^W, \quad p = 1, \dots, P, \quad l = 1, \dots, L, \quad t = 1, \dots, T \quad (9f)$$

$$\sum_{\tau=1}^T \sum_{d=1}^{D_\tau} \lambda_{\tau,d}^{k,t} = 1, \quad t = 1, \dots, T, \quad k = 1, \dots, K \quad (9g)$$

$$b_{m,k,t}^F, y_{k,l,p,t}^{FW}, \lambda_{\tau,d}^{k,t} \geq 0, \quad m = 1, \dots, M, \quad k = 1, \dots, K, \quad p = 1, \dots, P, \quad l = 1, \dots, L, \\ t = 1, \dots, T, \quad \tau = 1, \dots, T, \quad t = d, \dots, D_\tau. \quad (9h)$$

The objective function (9a) is the same as the objective function (7a) of the aggregated DEA production problem. Inequality (9b) is the resource constraint where the left-hand side is a convex combination of the input data  $\alpha_k^{\tau,d} = (\alpha_{1,k}^{\tau,d}, \dots, \alpha_{M,k}^{\tau,d})^T$  for factory  $k$  at observation period  $\tau$ . Showing similarity to constraint (9b), constraint (9c) is imposed to the output data from factory  $k$  at observation period  $\tau$ , and it expresses a relationship between the product outputs and the transportation amounts. Constraints (9d)–(9f) are the same as (7d)–(7f) of the aggregated DEA production problem. The weighting variable  $\lambda_{\tau,d}^{k,t}$  is used in a different way compared to constraint (7h) in the aggregated DEA production problem. Namely, as seen in constraint (9g), the sum of  $\lambda_{\tau,d}^{k,t}$  for  $\tau = 1, \dots, T$  and  $d = 1, \dots, D_\tau$  is one, and it holds for each factory  $k$  and planning period  $t$ .

#### 2.4. Discussion on Multiperiod Input–Output Data

As mentioned in the previous subsections, we can obtain an optimal plan for multiple periods by solving the two-level multiperiod production and distribution planning problem including the aggregated DEA production problem (7a)–(7i) or the disaggregated DEA production problem (9a)–(9h) as the lower level problem. Such an optimal plan is implemented, and then the operation data should be accumulated as the period progresses. We can incorporate the latest operation data into the observed input–output data set.

Moreover, during the observation periods, the production activities may be disrupted for some reasons in a certain period, and the input–output data from such a period may not always be accurate for estimating the production possibility set. It is natural to consider that the accuracy, reliability, and importance of the input–output data differ depending on the observation periods. From this viewpoint, instead of treating all the input–output data in the same way, we give some idea for handling the input–output data differently according to their accuracy, reliability, and importance.

The paper proposes a method for incorporating the latest operation data into the observed input–output data set, and evaluates the input–output data for each observation period differently. Specifically, the weighting variables for all the data must satisfy certain constraints, which ensure that the data are evaluated appropriately based on the accuracy, reliability, and importance.

For example, suppose that the input–output data for the first three observation periods is less accurate or reliable than the data for the remaining periods. In this case, the weighting variables for the first three periods can be set to a lower value than the weighting variables for the remaining periods, to reflect the lower accuracy and reliability of the data. This allows the DEA approach to give more weight to the more accurate and reliable data, and produce more accurate and reliable results. Another example is when the input–output data for certain products or factories is more important than the data for other products or factories. In this case, the weighting variables for the important data can be set to a higher value than the weighting variables for the less important data, to reflect the greater importance of the data. This allows the DEA approach to prioritize the important data and produce more meaningful results. Overall, the proposed method for handling input–output data of differing accuracy and importance allows the DEA approach to produce more accurate, reliable, and meaningful results, by appropriately evaluating the input–output data based on its characteristics.

First, we consider a method for incorporating the latest operation data into the observed input–output data set. Let  $\tau \in \{1, \dots, T\}$  be an observation period. As shown in the left-hand side of Figure 3, using the input–output data for the  $T$  observation periods, the two-level multiperiod DEA production and distribution problems (1a)–(1e) and (7a)–(7i) with aggregated input–output data and (1a)–(1e) and (9a)–(9h) with disaggregated input–output data are formulated and solved, respectively.

Suppose that the distributor starts to operate according to the optimal plan for periods  $t = 1, \dots, T$ , and the first three periods  $t = 1, 2, 3$  have been carried out as shown in Figure 3, for example. Then, after the input–output data observed at periods  $t = 1, 2, 3$  are added to the input–output data set as newer, more accurate and reliable data for the remaining planning periods  $t = 4, 5, \dots, T$ , the optimal plan can be recalculated with reference to the congregated input–output data set. It is natural for the distributor to incorporate the production activities of the manufacturer which can be expressed more accurately by solving the problem including new sets of data.

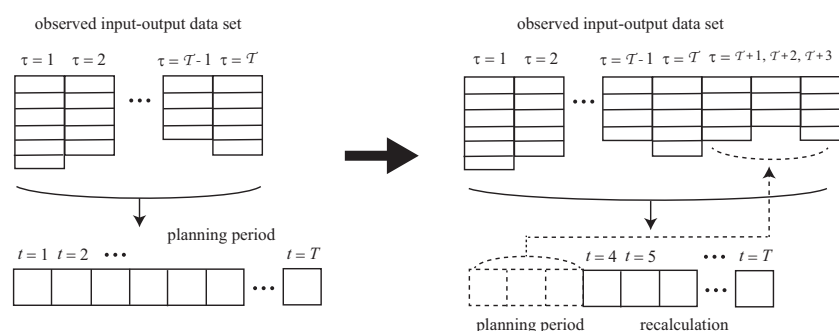


Figure 3. Enlarged input–output data set.

Next, we formulate the DEA production problem taking into account the accuracy, reliability, and importance of input–output data. In this paper, we deal with production and distribution planning for multiple periods, and the input–output data used for the DEA production problem are also observed during multiple periods. Therefore, it is considered that the accuracy, reliability, and importance of the input–output data differs depending on the observation. In order to accommodate such a situation successfully, we evaluate the input–output data for each observation period differently. When we use the aggregated input–output data, the weighting variables for each data must satisfy the following constraints:

$$\sum_{k=1}^K \sum_{\tau=1}^T \sum_{d=1}^{D_{\tau}} \lambda_{\tau,d}^{k,t} = 1, \quad t = 1, \dots, T \quad (10)$$

$$\lambda_{\tau,d}^{k,t} \geq 0, \quad k = 1, \dots, K, \quad t = 1, \dots, T, \quad \tau = 1, \dots, T, \quad d = 1, \dots, D_{\tau}. \quad (11)$$

Since the aggregated input–output data (5) is represented by the sum of data in all factories  $k = 1, \dots, K$ , the sum,  $\sum_{k=1}^K$ , for all factories in constraint (10) is needed in order to determine the production possibility set by combining the aggregated input–output data.

Taking into account situations where differences exist among the observed input–output data with respect to accuracy, reliability, and importance of the data, depending on each observation period  $\tau = 1, \dots, \mathcal{T}$  and the relationship between observation periods and planning periods, we introduce a parameter  $\pi_{\tau t}$  which is correlated with observation period  $\tau$  and planning period  $t$ . In order to utilize the input–output data observed at period  $\tau$  leading to production plan of period  $t$  to a limited extent, we give the following constraint with parameter  $\pi_{\tau t}$ :

$$\sum_{k=1}^K \sum_{d=1}^{D_{\tau}} \lambda_{\tau,d}^{k,t} \leq \pi_{\tau t}, \quad \tau = 1, \dots, \mathcal{T}, \quad t = 1, \dots, T. \quad (12)$$

The parameter  $\pi_{\tau t}$  is equal to or smaller than one due to constraint (10). By setting the value of  $\pi_{\tau t}$  to be smaller than one, it is possible to reduce the rate of referencing the input–output data observed at period  $\tau$  for the production plan at period  $t$ . For example, if it is determined that the accuracy, reliability, and importance of the input–output data observed at period  $\tau$  is quite low for the production plan at period  $t$ , its effect can be considerably reduced by setting the value of  $\pi_{\tau t}$  to be close to zero.

For the disaggregated input–output data (8), the weighting variables for each data must satisfy the following constraints:

$$\sum_{\tau=1}^{\mathcal{T}} \sum_{d=1}^{D_{\tau}} \lambda_{\tau,d}^{k,t} = 1, \quad t = 1, \dots, T, \quad k = 1, \dots, K \quad (13)$$

$$\lambda_{\tau,d}^{k,t} \geq 0, \quad k = 1, \dots, K, \quad t = 1, \dots, T, \quad \tau = 1, \dots, \mathcal{T}, \quad t = d, \dots, D_{\tau}. \quad (14)$$

Showing similarity to constraint (12) for the aggregated input–output data, from constraint (13), we give the following constraint that the input–output data observed at period  $\tau$  has a limited effect on the production plan at period  $t$  by utilizing the parameter  $\pi_{\tau t}$ .

$$\sum_{d=1}^{D_{\tau}} \lambda_{\tau,d}^{k,t} \leq \pi_{\tau t}, \quad \tau = 1, \dots, \mathcal{T}, \quad t = 1, \dots, T. \quad (15)$$

Specifying the parameter  $\pi_{\tau t}$ ,  $\tau = 1, \dots, \mathcal{T}$ ,  $t = 1, \dots, T$  appropriately, and solving problems (7a)–(7i) and (9a)–(9h) including constraints (12) and (15), respectively, the observed input–output data can be handled but accuracy, reliability, and importance must be taken into account.

## 2.5. Computational Aspect for Solving Two-Level Multiperiod Production and Distribution Planning Problem

In a way that is similar to the single period production and distribution planning problem [12], we can solve the two-level multiperiod production and distribution planning problem including the technological coefficient problem (1a)–(1m), the aggregated DEA production problem (7a)–(7i) or disaggregated DEA production problem (9a)–(9h).

Specifically, since the technological coefficient problem (1a)–(1m), the aggregated DEA production problem (7a)–(7i), and the disaggregated DEA production problem (9a)–(9h) are linear programming problems, the necessary and sufficient condition for their optimality is the Karush–Kuhn–Tucker (KKT) condition. From this property, we can transform the two-level multiperiod production and distribution planning problems into single-level problems by replacing the lower-level problems with the corresponding KKT condition.

These single-level problems are nonlinear programming problems because they contain the complementarity conditions in the KKT condition. However, by using the method of [29], we can transform them into mixed 0-1 linear programming problems which can be solved by using a commercial solver such as Gurobi Optimizer.

### 3. Computational Experiment

In this section, we apply the DEA approaches to a numerical example of two-level multiperiod production and distribution planning problems, and demonstrate the validity of the proposed method.

The numbers of warehouses, factories, products, resources, retailers, planning periods, observation periods, and scenarios are  $L = 3$ ,  $K = 3$ ,  $P = 3$ ,  $M = 3$ ,  $J = 3$ ,  $T = 6$ ,  $D_\tau = 6$ , and  $S = 5$ , respectively. The details of parameters, i.e., transportation costs from factories to warehouses, transportation costs from warehouses to retailers, holding costs of one unit volume, storage volumes of products, purchase prices of products, ordering costs of products, penalties of supply shortage and oversupply, purchase prices of resources, production costs, technological coefficients, capacities of warehouses, and production capacities at factories, which are determined based on random values, aiming to reflect realistic situations, are shown in the tables of the Appendix A.

It should be noted that each cost is set to increase slightly as the period progresses. The penalties for supply shortage and oversupply are specified based on the expected profit per unit product, and the penalty for supply shortage is set at a larger value than the value for oversupply.

In Table 1, we show the computational result of solving the two-level multiperiod production and distribution planning problem with the technological coefficient problem (1a)–(1m) by using Gurobi Optimizer. We show the productions, inventories, supply amounts, and expected demands in columns of Table 1 for each of the three products, and rows of the table correspond to the planning periods.

In this section, we simply call problem (1a)–(1m) the technological coefficient model, and also call the two-level multiperiod production and distribution planning problems with the aggregated DEA production problem (7a)–(7i) and the disaggregated DEA production problem (9a)–(9h) the aggregated DEA model and the disaggregated DEA model, respectively.

**Table 1.** Result of technological coefficient model.

| $t \backslash p$ | Production |      |      | Inventory |   |     | Supply |      |      | Expected Demand |        |        |
|------------------|------------|------|------|-----------|---|-----|--------|------|------|-----------------|--------|--------|
|                  | 1          | 2    | 3    | 1         | 2 | 3   | 1      | 2    | 3    | 1               | 2      | 3      |
| 1                | 180        | 190  | 180  | 0         | 0 | 0   | 180    | 190  | 180  | 180.0           | 192.0  | 180.0  |
| 2                | 240        | 199  | 240  | 53        | 0 | 53  | 187    | 199  | 187  | 187.2           | 199.2  | 187.2  |
| 3                | 250        | 203  | 250  | 112       | 0 | 112 | 191    | 203  | 191  | 190.8           | 202.8  | 190.8  |
| 4                | 250        | 210  | 250  | 164       | 0 | 164 | 198    | 210  | 198  | 198.0           | 210.0  | 198.0  |
| 5                | 250        | 217  | 250  | 209       | 0 | 209 | 205    | 217  | 205  | 205.2           | 217.2  | 205.2  |
| 6                | 0          | 221  | 0    | 0         | 0 | 0   | 209    | 221  | 209  | 208.8           | 220.8  | 208.8  |
| sum              | 1170       | 1240 | 1170 | 538       | 0 | 538 | 1170   | 1240 | 1170 | 1170.0          | 1242.0 | 1170.0 |

When comparing supply amounts to expected demands, we can see that supply amounts to retailers are almost equivalent to the values of expected demands. The total amounts of supply for products 1 and 3 are the same with the corresponding expected demands, and the amount of supply for product 2 is 2 units less than the expected demands. Regarding the production amounts from period 1 to period 3, the products are produced near the threshold of production capacities and are more than the supply amounts. The difference of products between production and supply are stored in warehouses as inventory. At period 6 which is the final period, products 1 and 3 are not produced but are supplied from inventory. From the computational results shown in Table 1, we can utilize an appropriate production and transportation plan.

We verify whether the proposed aggregated and disaggregated DEA models are appropriate as alternatives to the technological coefficient model through computational experiment. Assuming that the optimal plan obtained from the technological coefficient model is true, we compare it with the computational results of the aggregated and disaggregated DEA models. We generate the input–output data for the aggregated and disaggregated DEA models based on the optimal plan of the technological coefficient

model (1a)–(1m). The planning periods in the technological coefficient model are from period 1 to period  $T$ ,  $t = 1, \dots, T$ , and the demand  $d_{j,p,t,s}^R$  at each planning period  $t$  is given for any retailer  $j$ , product  $p$ , and scenario  $s$ .

Based on the given stochastic demands  $d_{j,p,t,s}^R$  at planning periods  $t = 1, \dots, T$ , we prepare the demand data  $\hat{d}_{j,p,\tau,s,d}^R$  at observation periods  $\tau = 1, \dots, \mathcal{T}$ . Specifically, using uniform random numbers  $\rho_{j,p,\tau,s,d}^1 \in [0.8, 1.2]$ , we generate demand data  $\hat{d}_{j,p,\tau,s,d}^R$  at observation period  $\tau$  for each retailer  $j$ , product  $p$ , and scenario  $s$  in the following way.

$$\hat{d}_{j,p,\tau,s,d}^R = \rho_{j,p,\tau,s,d}^1 d_{j,p,\tau,s}^R, \quad \tau = 1, \dots, \mathcal{T}, \quad d = 1, \dots, D_\tau. \quad (16)$$

First, we compute the production amount  $z_{k,p,t}^F$  by solving the technological coefficient model (1a)–(1m) corresponding to the stochastic demand data  $\hat{d}_{j,p,\tau,s,d}^R$ . Using the production amount  $z_{k,p,t}^F$  and random numbers  $\rho_{k,\tau,d}^2$  and  $\rho_{j,p,\tau,s,d}^3$ , we generate the  $d$ th input data  $\alpha_{m,k}^{\tau,d}$  and output data  $\beta_{p,k}^{\tau,d}$  at observation period  $\tau$  as follows:

$$\beta_{p,k}^{\tau,d} = \rho_{k,\tau,d}^2 \rho_{j,p,\tau,s,d}^3 z_{k,p,t}^F, \quad p = 1, \dots, P, \quad k = 1, \dots, K, \quad \tau = 1, \dots, \mathcal{T}, \quad d = 1, \dots, D_\tau, \quad (17)$$

$$\alpha_{m,k}^{\tau,d} = \rho_{k,\tau,d}^2 \sum_{p=1}^P a_{m,p,t}^F z_{k,p,t}^F, \quad m = 1, \dots, M, \quad k = 1, \dots, K, \quad \tau = 1, \dots, \mathcal{T}, \quad d = 1, \dots, D_\tau. \quad (18)$$

By using the random numbers  $\rho_{k,\tau,d}^2$  and  $\rho_{j,p,\tau,s,d}^3$ , duplication of the input–output data is prevented, and the random number  $\rho_{j,p,\tau,s,d}^3$  weakens the strong correlation between the input data and the output data. In this computational experiment, the number of observation periods is  $D_\tau = 6$  for all  $\tau = 1, \dots, 6$ . Suppose that the data are newer as observation period  $\tau$  increases, and the random numbers  $\rho_{k,\tau,d}^2$  and  $\rho_{j,p,\tau,s,d}^3$  are specified as follows:

$$\begin{aligned} \rho_{k,\tau,d}^2 &\in [0.75, 1.25], \quad \tau = 1, 2; \quad \rho_{k,\tau,d}^2 \in [0.80, 1.20], \quad \tau = 3, 4; \quad \rho_{k,\tau,d}^2 \in [0.85, 1.15], \quad \tau = 5, 6; \\ \rho_{j,p,\tau,s,d}^3 &\in [0.95, 1.05], \quad \tau = 1, \dots, 6. \end{aligned}$$

The input data and the output data

$$A^{\tau,d} = \begin{bmatrix} \alpha_{1,1}^{\tau,d} & \dots & \alpha_{1,K}^{\tau,d} \\ & \ddots & \\ \alpha_{M,1}^{\tau,d} & \dots & \alpha_{M,K}^{\tau,d} \end{bmatrix}, \quad B^{\tau,d} = \begin{bmatrix} \beta_{1,1}^{\tau,d} & \dots & \beta_{1,K}^{\tau,d} \\ & \ddots & \\ \beta_{P,1}^{\tau,d} & \dots & \beta_{P,K}^{\tau,d} \end{bmatrix}$$

Obtained from (17) and (18), respectively, are disaggregated data. The aggregated data are generated as follows:

$$\alpha_m^{\tau,d} = \sum_{k=1}^K \alpha_{m,k}^{\tau,d}, \quad \beta_p^{\tau,d} = \sum_{k=1}^K \beta_{p,k}^{\tau,d}.$$

Even if the two-level multiperiod production and distribution planning problems with the DEA production problem is solved using the input–output data generated as described above, we cannot find a solution such that the resources are not purchased and the products are not manufactured in the final period such as the result of the technological coefficient model. This is because the resource usage is a convex combination of the input data, and therefore it cannot become zero unless the input data contains a zero vector. To avoid this, it is necessary to add zero vectors for the input and output data such that both the resource purchase amount and the product production amount are zeros in order to enable not producing any product at some period.

Let the number of the data observed at any observation period  $\tau = 1, \dots, T$  be 5, i.e.,  $D_\tau = 5$ . After the input–output data are generated, we solved both the aggregated DEA model and the disaggregated DEA model. First, we compare the production amounts of the manufacturer. Although the production amount of product  $p$  in factory  $k$  at period  $t$  is represented by a decision variable  $z_{k,p,t}^F$  in the technological coefficient model, there are no variables that explicitly represent the production amounts in the DEA models. However, the production amount of product  $p$  in factory  $k$  at period  $t$  can be represented by the transportation amount  $\sum_{l=1}^L y_{k,l,p,t}^{FW}$  from factory  $k$  to all warehouses. We show the production amounts in the three models in Table 2.

**Table 2.** Production amounts in technological coefficient model, aggregated DEA model, and disaggregated DEA model.

| $k$                             | 1  |    |    | 2  |    |    | 3  |    |    |
|---------------------------------|----|----|----|----|----|----|----|----|----|
| $t \setminus p$                 | 1  | 2  | 3  | 1  | 2  | 3  | 1  | 2  | 3  |
| technological coefficient model |    |    |    |    |    |    |    |    |    |
| 1                               | 60 | 60 | 70 | 60 | 60 | 50 | 60 | 70 | 60 |
| 2                               | 80 | 80 | 90 | 80 | 80 | 70 | 80 | 39 | 80 |
| 3                               | 80 | 80 | 90 | 90 | 80 | 80 | 80 | 43 | 80 |
| 4                               | 80 | 80 | 90 | 90 | 80 | 80 | 80 | 50 | 80 |
| 5                               | 80 | 80 | 90 | 90 | 80 | 80 | 80 | 57 | 80 |
| 6                               | 0  | 80 | 0  | 0  | 80 | 0  | 0  | 61 | 0  |
| aggregated DEA model            |    |    |    |    |    |    |    |    |    |
| 1                               | 57 | 60 | 61 | 70 | 60 | 59 | 60 | 70 | 60 |
| 2                               | 80 | 80 | 88 | 73 | 77 | 80 | 80 | 85 | 80 |
| 3                               | 80 | 80 | 87 | 90 | 80 | 80 | 80 | 0  | 80 |
| 4                               | 80 | 66 | 87 | 90 | 80 | 80 | 80 | 64 | 80 |
| 5                               | 80 | 58 | 87 | 90 | 80 | 80 | 80 | 79 | 80 |
| 6                               | 0  | 51 | 0  | 0  | 80 | 0  | 0  | 90 | 0  |
| disaggregated DEA model         |    |    |    |    |    |    |    |    |    |
| 1                               | 60 | 60 | 69 | 70 | 60 | 60 | 54 | 70 | 60 |
| 2                               | 80 | 80 | 90 | 90 | 80 | 80 | 66 | 90 | 80 |
| 3                               | 80 | 80 | 90 | 90 | 80 | 80 | 80 | 90 | 80 |
| 4                               | 80 | 80 | 90 | 90 | 80 | 65 | 80 | 90 | 80 |
| 5                               | 80 | 80 | 90 | 90 | 80 | 76 | 80 | 90 | 80 |
| 6                               | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

It is found that the production amounts of the technological coefficient model and those of the aggregated DEA model or the disaggregated DEA model are often the same, and then the production plans obtained from the technological coefficient model, the aggregated DEA model, and the disaggregated DEA model are generally close to each other. The clear difference between the technological coefficient model and the aggregated DEA model is that product 2 is not manufactured in factory 3 at period 3 in the aggregated DEA model, while it is manufactured in the technological coefficient model. In the technological coefficient model, product 2 is produced at period 6, but not in the disaggregated DEA model.

Moreover, between the technological coefficient model and the aggregated DEA model or the disaggregated DEA model, most of the transportation amounts  $y_{k,l,p,t}^{FW}$  from factories to warehouses and  $x_{l,j,p,t}^{WR}$  from warehouses to retailers are the same. To compare the aggregated or disaggregated DEA model to the technological coefficient model, we calculate the concordance rates of production and transportation between the two models, and show them in Table 3.

**Table 3.** Concordance rates of production and transportation.

| $t$     | Production<br>$z_{k,p,t}^F, \sum y_{k,l,p,t}^{FW}$ |      | Transportation<br>$y_{k,l,p,t}^{FW}$ |      |      |      | Inventory<br>$y_{p,l,t}^W$ |      |
|---------|----------------------------------------------------|------|--------------------------------------|------|------|------|----------------------------|------|
|         | A                                                  | DA   | A                                    | DA   | A    | DA   | A                          | DA   |
| 1       | 0.56                                               | 0.56 | 0.33                                 | 0.56 | 0.22 | 0.63 | 0.89                       | 0.67 |
| 2       | 0.44                                               | 0.56 | 0.44                                 | 0.56 | 0.41 | 0.56 | 0.33                       | 0.78 |
| 3       | 0.67                                               | 0.89 | 0.48                                 | 0.59 | 0.26 | 0.41 | 0.89                       | 0.78 |
| 4       | 0.67                                               | 0.78 | 0.48                                 | 0.41 | 0.70 | 0.56 | 0.33                       | 0.11 |
| 5       | 0.67                                               | 0.78 | 0.85                                 | 0.37 | 0.78 | 0.48 | 0.33                       | 0.22 |
| 6       | 0.78                                               | 0.67 | 0.81                                 | 0.89 | 0.59 | 0.52 | 1.00                       | 1.00 |
| average | 0.63                                               | 0.70 | 0.57                                 | 0.56 | 0.49 | 0.52 | 0.63                       | 0.59 |

A: aggregated, DA: disaggregated.

Since we deal with the numerical example with three products and three factories, the concordance ratios in Table 3 are calculated by dividing the number of production amounts that match the DEA model and the technological coefficient model by 9. As for the transportation, since there are 27 transportation routes for each of the three products, they are calculated by dividing the number of matched transportation amounts by 27. In a similar way, we also show the concordance rates about inventory.

As seen in Table 3, the averages of the concordance rates are at least 49%, and it seems that the DEA models sufficiently reproduce the result of the technological coefficient model which is interpreted as the true data. Comparing the aggregated DEA model and the disaggregated DEA model, we find that the disaggregated model shows higher rates of concordance in terms of the production amounts and the transportation amounts from warehouses to retailers. Since the transportation amounts from factories to warehouses in the two DEA models are almost the same, it leads us to believe that the disaggregate DEA model is more effective than the aggregated DEA model.

We show the amounts of resources purchased by the manufacturer for the three models in Table 4. For all the periods excluding period 6 which is the last period, the difference between the aggregated DEA model and the technological coefficient model is about 3% on average for the overall amounts, and for the disaggregated DEA model it is about 5%. Adding all purchase amounts of resources for all periods, we find that those of the aggregated and disaggregated DEA models are larger than that of the technological coefficient model. It is considered that this is attributed to the expression of resource constraints. In the technological coefficient model, the resource constraint (1g) is imposed on the sum of resources used in all factories. On the other hand, since in the aggregate and disaggregate DEA models the resource constraints (7b) and (9b) are imposed for each factory, it is thought that the purchase amounts of the DEA models are larger than that of the technological coefficient model. In Table 5, we show the sum of differences between each of the DEA models and the technological coefficient model, and the ratios of the differences to the purchase amounts of the technological coefficient model. The ratio is at most about 15%. We also find that the ratios of the disaggregated DEA model are slightly smaller than those of the aggregated DEA model. From the above facts, it shows that the DEA models work well with respect to the purchase amounts of resources.

**Table 4.** Purchase amounts of resources.

| $t \backslash m$ | Technological Coefficients |      |      | Aggregated DEA |      |      | Disaggregated DEA |      |      |
|------------------|----------------------------|------|------|----------------|------|------|-------------------|------|------|
|                  | 1                          | 2    | 3    | 1              | 2    | 3    | 1                 | 2    | 3    |
| 1                | 1650                       | 1660 | 1640 | 1682           | 1687 | 1615 | 1685              | 1695 | 1691 |
| 2                | 2038                       | 1997 | 2078 | 2120           | 2143 | 2225 | 2158              | 2247 | 2263 |
| 3                | 2108                       | 2061 | 2156 | 2248           | 2134 | 2229 | 2298              | 2230 | 2227 |
| 4                | 2131                       | 2092 | 2171 | 2248           | 2133 | 2231 | 2283              | 2222 | 2073 |
| 5                | 2150                       | 2117 | 2184 | 2250           | 2135 | 2229 | 2295              | 2230 | 2175 |
| 6                | 662                        | 883  | 442  | 1663           | 1985 | 1730 | 0                 | 0    | 0    |

**Table 5.** Differences and ratios on purchase amounts of resources.

|                    | Aggregated DEA |         |         | Disaggregated DEA |         |         |
|--------------------|----------------|---------|---------|-------------------|---------|---------|
|                    | $m = 1$        | $m = 2$ | $m = 3$ | $m = 1$           | $m = 2$ | $m = 3$ |
| sum of differences | 1471           | 1407    | 1638    | 1303              | 1581    | 855     |
| ratios             | 0.137          | 0.130   | 0.154   | 0.121             | 0.146   | 0.080   |

Finally, we show the values of all terms of the objective functions in Table 6. Since the objective function values of the upper and lower levels for the aggregated and disaggregated DEA models are close to the corresponding values of the technological coefficient model, the DEA models are considered to be sufficient alternatives to the technological coefficient model. Especially when comparing the aggregated and disaggregated DEA models, we find that both the objective function values of the upper and the lower levels in the disaggregated DEA model are close to those of the technological coefficient model.

**Table 6.** Objective functions.

|                   | Upper Level       |                   |            |          |
|-------------------|-------------------|-------------------|------------|----------|
|                   | Transportation #1 | Transportation #2 | Inventory  | Purchase |
| technological     | 134,839           | 119,727           | 6172       | 323,096  |
| aggregated DEA    | 131,685           | 104,645           | 7210       | 322,930  |
| disaggregated DEA | 127,459           | 117,105           | 8513       | 315,910  |
|                   | Order             | Supply Shortage   | Oversupply | Sum      |
| technological     | 253,000           | 82,496            | 27,099     | 946,430  |
| aggregated DEA    | 253,000           | 83,067            | 27,017     | 929,554  |
| disaggregated DEA | 234,000           | 105,522           | 23,809     | 932,318  |
|                   | Lower Level       |                   |            |          |
|                   | Resource          | Production        | Sum        |          |
| technological     | 1,114,944         | 161,457           | 1,276,401  |          |
| aggregated DEA    | 1,300,572         | 161,446           | 1,462,019  |          |
| disaggregated DEA | 1,067,845         | 158,950           | 1,226,795  |          |

The results obtained from this experiment can be summarized as follows:

- The DEA models sufficiently reproduce the outcomes of the technological coefficient model, which is interpreted as the true data.
- Comparing the aggregated DEA model with the disaggregated DEA model, the disaggregated model demonstrates a higher rate of concordance in terms of production amounts and transportation amounts from warehouses to retailers.
- Given that the transportation amounts from factories to warehouses are nearly identical in both DEA models, it is inferred that the disaggregated DEA model is more effective than the aggregated DEA model.
- On average, the difference between the aggregated DEA model and the technological coefficient model is about 3% for the overall amounts, and for the disaggregated DEA model, it is approximately 5%.
- Summing up the purchase amounts of resources across all periods reveals that the amounts from the aggregated and disaggregated DEA models exceed that of the technological coefficient model. This discrepancy is believed to stem from the manner in which resource constraints are expressed.
- The ratios of differences between each of the DEA models and the technological coefficient model relative to the purchase amounts of the technological coefficient model tops out at around 15%.
- Notably, when juxtaposing the aggregated and disaggregated DEA models, it becomes evident that both the upper and lower objective function values in the disaggregated DEA model align more closely with those of the technological coefficient model.

From these observations, it can be inferred that the DEA models are functioning effectively, especially concerning the procurement amounts of resources.

The proposed DEA approach addresses the limitations of existing models like the Bayesian game approach by utilizing observed input–output data to identify incomplete information on production technologies of the manufacturer at the lower level. Unlike the Bayesian game approach, the DEA approach does not require full understanding of technological coefficients expressing the relationship between resources and products. Instead, the distributor can observe the input–output data such as the amount of resources purchased and the amount of products produced.

By using observed input–output data, the DEA approach can estimate the unknown parameters of the production planning problem without requiring complete information about the opponent player’s production technologies. This is because the input–output data provides information about the production process that can be used to estimate the unknown parameters. In contrast, existing models like the Bayesian game approach rely on estimating the unknown parameters directly, which can be difficult or impossible without complete information about the opponent player’s production technologies.

Overall, the DEA approach provides a practical and effective alternative to existing models for solving production planning problems in supply chain management, particularly when complete information about the opponent player’s production technologies is not available.

#### 4. Conclusions

In this paper, from importance of the multiperiod problem, we investigate a two-level multiperiod production and distribution planning problem with uncertain demands. We assume that although the distributor which is the leader in the two-level production and distribution planning problem does not have sufficient information about the parameters of the production planning problem of the manufacturer which is the follower, the distributor can observe the input–output data of the manufacturer. We formulate the lower level problem as the DEA production problems by utilizing input–output data, and employ the two-stage model with simple recourse in order to manipulate the uncertain demands. To demonstrate the validity and effectiveness of the proposed model, we carry out numerical examinations by using a numerical example.

Our proposed DEA approaches offer a practical solution to a complex production and distribution planning problem but have limitations regarding data accuracy and external factors like market conditions. Future research should address these by refining data handling methods and incorporating external influences to enhance the model’s effectiveness in supply chain management.

**Author Contributions:** Conceptualization, T.H. and I.N.; methodology, T.H. and I.N.; software, J.O.; validation, T.H., I.N., S.S. and J.O.; formal analysis, T.H. and I.N.; investigation, I.N.; resources, I.N. and J.O.; data curation, I.N. and J.O.; writing—original draft preparation, I.N.; writing—review and editing, T.H.; visualization, T.H.; supervision, T.H.; project administration, T.H. and I.N.; funding acquisition, I.N. All authors have read and agreed to the published version of the manuscript.

**Funding:** This work was supported by JSPS KAKENHI Grant Number: 21H01565.

**Informed Consent Statement:** Not applicable.

**Data Availability Statement:** All data generated for this study are stored in our laboratory and are not publicly available. Researchers who wish to access the data should contact the corresponding author directly.

**Conflicts of Interest:** The authors declare no conflict of interest.

## Appendix A. Data of the Numerical Example

**Table A1.** Transportation cost: from factories to warehouses.

| $k = 1$ | $l = 1$ |         |         | $l = 2$ |         |         | $l = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 40      | 42      | 43      | 30      | 32      | 33      | 20      | 22      | 23      |
| $t = 2$ | 42      | 44      | 45      | 32      | 34      | 35      | 22      | 24      | 25      |
| $t = 3$ | 43      | 45      | 46      | 33      | 35      | 36      | 23      | 25      | 26      |
| $t = 4$ | 45      | 47      | 48      | 35      | 37      | 38      | 25      | 27      | 28      |
| $t = 5$ | 46      | 48      | 49      | 36      | 38      | 39      | 26      | 28      | 29      |
| $t = 6$ | 47      | 49      | 50      | 37      | 39      | 40      | 27      | 29      | 30      |

| $k = 2$ | $l = 1$ |         |         | $l = 2$ |         |         | $l = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 30      | 32      | 33      | 25      | 27      | 28      | 30      | 32      | 33      |
| $t = 2$ | 32      | 34      | 35      | 27      | 29      | 30      | 32      | 34      | 35      |
| $t = 3$ | 33      | 35      | 36      | 28      | 30      | 31      | 33      | 35      | 36      |
| $t = 4$ | 35      | 37      | 38      | 30      | 32      | 33      | 35      | 37      | 38      |
| $t = 5$ | 36      | 38      | 39      | 31      | 33      | 34      | 36      | 38      | 39      |
| $t = 6$ | 37      | 39      | 40      | 32      | 34      | 35      | 37      | 39      | 40      |

| $k = 3$ | $l = 1$ |         |         | $l = 2$ |         |         | $l = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 30      | 32      | 33      | 30      | 32      | 33      | 20      | 22      | 23      |
| $t = 2$ | 32      | 34      | 35      | 32      | 34      | 35      | 22      | 24      | 25      |
| $t = 3$ | 33      | 35      | 36      | 33      | 35      | 36      | 23      | 25      | 26      |
| $t = 4$ | 35      | 37      | 38      | 35      | 37      | 38      | 25      | 27      | 28      |
| $t = 5$ | 36      | 38      | 39      | 36      | 38      | 39      | 26      | 28      | 29      |
| $t = 6$ | 37      | 39      | 40      | 37      | 39      | 40      | 27      | 29      | 30      |

**Table A2.** Transportation cost: from warehouses to retailers.

| $k = 1$ | $l = 1$ |         |         | $l = 2$ |         |         | $l = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 30      | 33      | 35      | 40      | 43      | 45      | 30      | 33      | 35      |
| $t = 2$ | 32      | 35      | 37      | 42      | 45      | 47      | 32      | 35      | 37      |
| $t = 3$ | 33      | 36      | 38      | 43      | 46      | 48      | 33      | 36      | 38      |
| $t = 4$ | 34      | 37      | 39      | 44      | 47      | 49      | 34      | 37      | 39      |
| $t = 5$ | 35      | 38      | 40      | 45      | 48      | 50      | 35      | 38      | 40      |
| $t = 6$ | 36      | 39      | 41      | 46      | 49      | 51      | 36      | 39      | 41      |

| $k = 2$ | $l = 1$ |         |         | $l = 2$ |         |         | $l = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 40      | 43      | 45      | 30      | 33      | 35      | 30      | 33      | 35      |
| $t = 2$ | 42      | 45      | 47      | 32      | 35      | 37      | 32      | 35      | 37      |
| $t = 3$ | 43      | 46      | 48      | 33      | 36      | 38      | 33      | 36      | 38      |
| $t = 4$ | 44      | 47      | 49      | 34      | 37      | 39      | 34      | 37      | 39      |
| $t = 5$ | 45      | 48      | 50      | 35      | 38      | 40      | 35      | 38      | 40      |
| $t = 6$ | 46      | 49      | 51      | 36      | 39      | 41      | 36      | 39      | 41      |

| $k = 3$ | $l = 1$ |         |         | $l = 2$ |         |         | $l = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 30      | 33      | 35      | 30      | 33      | 35      | 40      | 43      | 45      |
| $t = 2$ | 32      | 35      | 37      | 32      | 35      | 37      | 42      | 45      | 47      |
| $t = 3$ | 33      | 36      | 38      | 33      | 36      | 38      | 43      | 46      | 48      |
| $t = 4$ | 34      | 37      | 39      | 34      | 37      | 39      | 44      | 47      | 49      |
| $t = 5$ | 35      | 38      | 40      | 35      | 38      | 40      | 45      | 48      | 50      |
| $t = 6$ | 36      | 39      | 41      | 36      | 39      | 41      | 46      | 49      | 51      |

**Table A3.** Holding cost of one unit volume.

|         | $l = 1$ | $l = 2$ | $l = 3$ |
|---------|---------|---------|---------|
| $t = 1$ | 2       | 2       | 3       |
| $t = 2$ | 3       | 2       | 2       |
| $t = 3$ | 2       | 3       | 2       |
| $t = 4$ | 3       | 2       | 2       |
| $t = 5$ | 3       | 2       | 2       |
| $t = 6$ | 3       | 3       | 3       |

**Table A4.** Storage volume of one unit of product.

| $p = 1$ | $p = 2$ | $p = 3$ |
|---------|---------|---------|
| 2       | 3       | 4       |

**Table A5.** Purchase price of one unit of product.

|         | $p = 1$ | $p = 2$ | $p = 3$ |
|---------|---------|---------|---------|
| $t = 1$ | 60      | 80      | 100     |
| $t = 2$ | 62      | 81      | 110     |
| $t = 3$ | 63      | 82      | 113     |
| $t = 4$ | 72      | 93      | 124     |
| $t = 5$ | 75      | 95      | 126     |
| $t = 6$ | 77      | 98      | 129     |

**Table A6.** Ordering cost of product.

|         | $p = 1$ | $p = 2$ | $p = 3$ |
|---------|---------|---------|---------|
| $t = 1$ | 14,000  | 15,000  | 16,000  |
| $t = 2$ | 15,000  | 16,000  | 14,000  |
| $t = 3$ | 16,000  | 14,000  | 15,000  |
| $t = 4$ | 16,000  | 15,000  | 16,000  |
| $t = 5$ | 18,000  | 17,000  | 17,000  |
| $t = 6$ | 19,000  | 19,000  | 18,000  |

**Table A7.** Penalties of the supply shortage and oversupply.

| Supply Shortage | $j = 1$ |         |         | $j = 2$ |         |         | $j = 3$ |         |         |
|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|                 | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$         | 450     | 630     | 810     | 510     | 600     | 780     | 480     | 660     | 750     |
| $t = 2$         | 480     | 660     | 840     | 540     | 630     | 810     | 510     | 690     | 780     |
| $t = 3$         | 495     | 675     | 855     | 555     | 645     | 825     | 525     | 705     | 795     |
| $t = 4$         | 510     | 690     | 870     | 570     | 660     | 840     | 540     | 720     | 810     |
| $t = 5$         | 525     | 705     | 885     | 585     | 675     | 855     | 555     | 735     | 825     |
| $t = 6$         | 540     | 720     | 900     | 600     | 690     | 870     | 570     | 750     | 840     |
| Oversupply      | $j = 1$ |         |         | $j = 2$ |         |         | $j = 3$ |         |         |
|                 | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$         | 150     | 210     | 270     | 170     | 200     | 260     | 160     | 220     | 250     |
| $t = 2$         | 160     | 220     | 280     | 180     | 210     | 270     | 170     | 230     | 260     |
| $t = 3$         | 165     | 225     | 285     | 185     | 215     | 275     | 175     | 235     | 265     |
| $t = 4$         | 170     | 230     | 290     | 190     | 220     | 280     | 180     | 240     | 270     |
| $t = 5$         | 175     | 235     | 295     | 195     | 225     | 285     | 185     | 245     | 275     |
| $t = 6$         | 180     | 240     | 300     | 200     | 230     | 290     | 190     | 250     | 280     |

**Table A8.** Stochastic demands of products.

| $j = 1$ | $p = 1$ |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 40      | 42      | 43      | 45      | 47      | 48      |
| $s = 2$ | 44      | 46      | 47      | 50      | 52      | 53      |
| $s = 3$ | 48      | 50      | 52      | 54      | 56      | 58      |
| $s = 4$ | 52      | 55      | 56      | 59      | 61      | 62      |
| $s = 5$ | 56      | 59      | 60      | 63      | 66      | 67      |
|         | $p = 2$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 50      | 52      | 53      | 55      | 57      | 58      |
| $s = 2$ | 55      | 57      | 58      | 61      | 63      | 64      |
| $s = 3$ | 60      | 62      | 64      | 66      | 68      | 70      |
| $s = 4$ | 65      | 68      | 69      | 72      | 74      | 75      |
| $s = 5$ | 70      | 73      | 74      | 77      | 80      | 81      |
|         | $p = 3$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 60      | 62      | 63      | 65      | 67      | 68      |
| $s = 2$ | 66      | 68      | 69      | 72      | 74      | 75      |
| $s = 3$ | 72      | 74      | 76      | 78      | 80      | 82      |
| $s = 4$ | 78      | 81      | 82      | 85      | 87      | 88      |
| $s = 5$ | 84      | 87      | 88      | 91      | 94      | 95      |
| $j = 2$ | $p = 1$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 60      | 62      | 63      | 65      | 67      | 68      |
| $s = 2$ | 66      | 68      | 69      | 72      | 74      | 75      |
| $s = 3$ | 72      | 74      | 76      | 78      | 80      | 82      |
| $s = 4$ | 78      | 81      | 82      | 85      | 87      | 88      |
| $s = 5$ | 84      | 87      | 88      | 91      | 94      | 95      |
|         | $p = 2$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 50      | 52      | 53      | 55      | 57      | 58      |
| $s = 2$ | 55      | 57      | 58      | 61      | 63      | 64      |
| $s = 3$ | 60      | 62      | 64      | 66      | 68      | 70      |
| $s = 4$ | 65      | 68      | 69      | 72      | 74      | 75      |
| $s = 5$ | 70      | 73      | 74      | 77      | 80      | 81      |
|         | $p = 3$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 40      | 42      | 43      | 45      | 47      | 48      |
| $s = 2$ | 44      | 46      | 47      | 50      | 52      | 53      |
| $s = 3$ | 48      | 50      | 52      | 54      | 56      | 58      |
| $s = 4$ | 52      | 55      | 56      | 59      | 61      | 62      |
| $s = 5$ | 56      | 59      | 60      | 63      | 66      | 67      |
| $j = 3$ | $p = 1$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 50      | 52      | 53      | 55      | 57      | 58      |
| $s = 2$ | 55      | 57      | 58      | 61      | 63      | 64      |
| $s = 3$ | 60      | 62      | 64      | 66      | 68      | 70      |
| $s = 4$ | 65      | 68      | 69      | 72      | 74      | 75      |
| $s = 5$ | 70      | 73      | 74      | 77      | 80      | 81      |
|         | $p = 2$ |         |         |         |         |         |
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 60      | 62      | 63      | 65      | 67      | 68      |
| $s = 2$ | 66      | 68      | 69      | 72      | 74      | 75      |
| $s = 3$ | 72      | 74      | 76      | 78      | 80      | 82      |
| $s = 4$ | 78      | 81      | 82      | 85      | 87      | 88      |
| $s = 5$ | 84      | 87      | 88      | 91      | 94      | 95      |

**Table A8.** *Cont.*

|         | $p = 3$ |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
|         | $t = 1$ | $t = 2$ | $t = 3$ | $t = 4$ | $t = 5$ | $t = 6$ |
| $s = 1$ | 50      | 52      | 53      | 55      | 57      | 58      |
| $s = 2$ | 55      | 57      | 58      | 61      | 63      | 64      |
| $s = 3$ | 60      | 62      | 64      | 66      | 68      | 70      |
| $s = 4$ | 65      | 68      | 69      | 72      | 74      | 75      |
| $s = 5$ | 70      | 73      | 74      | 77      | 80      | 81      |

**Table A9.** Purchase price of one unit of resource.

|         | $m = 1$ | $m = 2$ | $m = 3$ |
|---------|---------|---------|---------|
| $t = 1$ | 28      | 25      | 30      |
| $t = 2$ | 30      | 32      | 34      |
| $t = 3$ | 33      | 33      | 35      |
| $t = 4$ | 35      | 36      | 37      |
| $t = 5$ | 36      | 38      | 38      |
| $t = 6$ | 55      | 45      | 50      |

**Table A10.** Production cost.

|         | $k = 1$ |         |         | $k = 2$ |         |         | $k = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 30      | 42      | 54      | 34      | 40      | 52      | 32      | 44      | 50      |
| $t = 2$ | 32      | 44      | 56      | 36      | 42      | 54      | 34      | 46      | 52      |
| $t = 3$ | 33      | 45      | 57      | 37      | 43      | 55      | 35      | 47      | 53      |
| $t = 4$ | 34      | 46      | 58      | 38      | 44      | 56      | 36      | 48      | 54      |
| $t = 5$ | 35      | 47      | 59      | 39      | 45      | 57      | 37      | 49      | 55      |
| $t = 6$ | 36      | 48      | 60      | 40      | 46      | 58      | 38      | 50      | 56      |

**Table A11.** Technological coefficient.

|         | $p = 1$ | $p = 2$ | $p = 3$ |
|---------|---------|---------|---------|
| $m = 1$ | 2       | 3       | 4       |
| $m = 2$ | 3       | 4       | 2       |
| $m = 3$ | 4       | 2       | 3       |

**Table A12.** Capacities of warehouses.

| $l = 1$ | $l = 2$ | $l = 3$ |
|---------|---------|---------|
| 10,000  | 10,000  | 10,000  |

**Table A13.** Production capacity.

|         | $k = 1$ |         |         | $k = 2$ |         |         | $k = 3$ |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
|         | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ | $p = 1$ | $p = 2$ | $p = 3$ |
| $t = 1$ | 60      | 60      | 70      | 70      | 60      | 60      | 60      | 70      | 60      |
| $t = 2$ | 80      | 80      | 90      | 90      | 80      | 80      | 80      | 90      | 80      |
| $t = 3$ | 80      | 80      | 90      | 90      | 80      | 80      | 80      | 90      | 80      |
| $t = 4$ | 80      | 80      | 90      | 90      | 80      | 80      | 80      | 90      | 80      |
| $t = 5$ | 80      | 80      | 90      | 90      | 80      | 80      | 80      | 90      | 80      |
| $t = 6$ | 80      | 80      | 90      | 90      | 80      | 80      | 80      | 90      | 80      |

**Table A14.** Input–output data.

| Input   | $\tau \backslash d$ | $m = 1$ |     |     |     |     | $m = 2$ |     |     |     |     | $m = 3$ |     |     |     |     |
|---------|---------------------|---------|-----|-----|-----|-----|---------|-----|-----|-----|-----|---------|-----|-----|-----|-----|
|         |                     | 1       | 2   | 3   | 4   | 5   | 1       | 2   | 3   | 4   | 5   | 1       | 2   | 3   | 4   | 5   |
| $k = 1$ | 1                   | 651     | 629 | 597 | 453 | 658 | 471     | 581 | 592 | 675 | 579 | 491     | 434 | 650 | 635 | 451 |
|         | 2                   | 902     | 650 | 608 | 795 | 696 | 642     | 916 | 796 | 761 | 759 | 614     | 675 | 730 | 926 | 783 |
|         | 3                   | 901     | 751 | 755 | 882 | 706 | 653     | 723 | 770 | 811 | 692 | 678     | 689 | 617 | 829 | 792 |
|         | 4                   | 816     | 561 | 589 | 687 | 757 | 635     | 780 | 786 | 754 | 840 | 705     | 783 | 568 | 700 | 589 |
|         | 5                   | 855     | 759 | 716 | 767 | 734 | 838     | 755 | 765 | 660 | 662 | 656     | 668 | 757 | 738 | 832 |
|         | 6                   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   |
| $k = 2$ | 1                   | 436     | 612 | 516 | 603 | 578 | 664     | 573 | 477 | 548 | 652 | 443     | 685 | 613 | 524 | 515 |
|         | 2                   | 859     | 832 | 800 | 726 | 637 | 708     | 715 | 585 | 591 | 759 | 743     | 780 | 924 | 738 | 688 |
|         | 3                   | 645     | 708 | 745 | 757 | 640 | 651     | 666 | 646 | 632 | 606 | 709     | 614 | 846 | 810 | 811 |
|         | 4                   | 571     | 719 | 785 | 770 | 674 | 760     | 618 | 661 | 824 | 627 | 805     | 686 | 754 | 772 | 684 |
|         | 5                   | 800     | 792 | 757 | 743 | 634 | 834     | 736 | 640 | 766 | 862 | 715     | 809 | 720 | 719 | 747 |
|         | 6                   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   |
| $k = 3$ | 1                   | 522     | 593 | 498 | 504 | 428 | 487     | 528 | 527 | 622 | 498 | 605     | 670 | 601 | 562 | 578 |
|         | 2                   | 779     | 589 | 903 | 909 | 579 | 949     | 678 | 933 | 653 | 814 | 696     | 814 | 744 | 613 | 561 |
|         | 3                   | 856     | 712 | 867 | 777 | 657 | 900     | 719 | 617 | 819 | 645 | 632     | 869 | 788 | 682 | 756 |
|         | 4                   | 793     | 610 | 897 | 762 | 660 | 655     | 760 | 729 | 683 | 777 | 621     | 595 | 660 | 623 | 869 |
|         | 5                   | 732     | 729 | 710 | 848 | 773 | 858     | 831 | 709 | 790 | 826 | 697     | 811 | 847 | 822 | 736 |
|         | 6                   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   |
| Output  | $\tau \backslash d$ | $p = 1$ |     |     |     |     | $p = 2$ |     |     |     |     | $p = 3$ |     |     |     |     |
|         |                     | 1       | 2   | 3   | 4   | 5   | 1       | 2   | 3   | 4   | 5   | 1       | 2   | 3   | 4   | 5   |
| $k = 1$ | 1                   | 68      | 65  | 61  | 48  | 70  | 48      | 63  | 65  | 73  | 63  | 61      | 55  | 80  | 75  | 58  |
|         | 2                   | 92      | 69  | 67  | 87  | 71  | 66      | 102 | 84  | 85  | 82  | 72      | 77  | 88  | 111 | 98  |
|         | 3                   | 99      | 81  | 79  | 94  | 71  | 73      | 78  | 82  | 88  | 73  | 83      | 79  | 77  | 96  | 93  |
|         | 4                   | 95      | 64  | 70  | 80  | 88  | 72      | 88  | 87  | 82  | 92  | 71      | 81  | 58  | 70  | 63  |
|         | 5                   | 93      | 81  | 72  | 81  | 77  | 89      | 85  | 79  | 70  | 71  | 76      | 83  | 90  | 85  | 100 |
|         | 6                   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   |
| $k = 2$ | 1                   | 55      | 79  | 63  | 77  | 75  | 73      | 58  | 48  | 58  | 70  | 45      | 71  | 65  | 52  | 53  |
|         | 2                   | 109     | 103 | 94  | 91  | 80  | 77      | 74  | 60  | 62  | 78  | 78      | 79  | 93  | 80  | 72  |
|         | 3                   | 80      | 88  | 92  | 96  | 78  | 67      | 73  | 71  | 65  | 65  | 71      | 65  | 92  | 82  | 84  |
|         | 4                   | 56      | 76  | 83  | 80  | 65  | 92      | 69  | 77  | 91  | 72  | 93      | 77  | 85  | 94  | 83  |
|         | 5                   | 92      | 99  | 92  | 89  | 76  | 86      | 76  | 70  | 79  | 88  | 74      | 82  | 76  | 76  | 80  |
|         | 6                   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   |
| $k = 3$ | 1                   | 57      | 61  | 50  | 55  | 43  | 56      | 62  | 63  | 73  | 60  | 67      | 71  | 63  | 58  | 65  |
|         | 2                   | 79      | 62  | 101 | 100 | 63  | 114     | 79  | 109 | 80  | 97  | 76      | 85  | 79  | 67  | 62  |
|         | 3                   | 95      | 78  | 94  | 80  | 71  | 106     | 87  | 77  | 98  | 79  | 67      | 92  | 84  | 74  | 85  |
|         | 4                   | 83      | 68  | 97  | 79  | 69  | 75      | 91  | 86  | 82  | 97  | 68      | 62  | 74  | 68  | 95  |
|         | 5                   | 79      | 81  | 75  | 91  | 79  | 101     | 96  | 81  | 95  | 97  | 76      | 90  | 92  | 86  | 81  |
|         | 6                   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   | 0       | 0   | 0   | 0   | 0   |

## References

- Calvete, H.I.; Galé, C.; Iranzo, J.A. Planning of a decentralized distribution network using bilevel optimization. *Omega* **2014**, *49*, 30–41. [\[CrossRef\]](#)
- Harsanyi, J.C. Games with incomplete information played by “Bayesian” players, I–III, part I. The basic model. *Manag. Sci.* **1967**, *14*, 159–182. [\[CrossRef\]](#)
- Yao, Z.; Leung, S.C.H.; Lai, K.K. Manufacturer’s revenue-sharing contract and retail competition. *Eur. J. Oper. Res.* **2008**, *186*, 637–651. [\[CrossRef\]](#)
- Harsanyi, J.C. Games with incomplete information played by “Bayesian” players, part II. Bayesian equilibrium points. *Manag. Sci.* **1968**, *14*, 320–334. [\[CrossRef\]](#)
- Harsanyi, J.C. Games with incomplete information played by “Bayesian” players, part III. the basic probability distribution of the game. *Manag. Sci.* **1968**, *14*, 486–502. [\[CrossRef\]](#)
- Jain, M.; Tsai, J.; Pita, J.; Kiekintveld, C.; Rathi, S.; Tambe, M.; Ordóñez, F. Software assistants for randomized patrol planning for the LAX airport police and the federal air marshal service. *Interfaces* **2010**, *40*, 267–290. [\[CrossRef\]](#)
- Jiang, J.; Liu, X. Bayesian Stackelberg game model for water supply networks against interdictions with mixed strategies. *Int. J. Prod. Res.* **2021**, *59*, 2537–2557. [\[CrossRef\]](#)
- Page, F.H., Jr. Existence of optimal auctions in general environments. *J. Math. Econ.* **1998**, *29*, 389–418. [\[CrossRef\]](#)
- Paruchuri, P.; Pearce, J.P.; Marecki, J.; Tambe, M.; Ordóñez, F.; Kraus, S. Coordinating randomized policies for increasing security of agent systems. *Inf. Technol. Manag.* **2009**, *10*, 67–79. [\[CrossRef\]](#)
- Sarkar, S. Gratitude, conscience, and reciprocity: Models of supplier motivation when quality is non-contractible. *Eur. J. Oper. Res.* **2019**, *277*, 633–642. [\[CrossRef\]](#)
- Xu, Y.; Ligett, K. Commitment in first-price auctions. *Econ. Theory* **2018**, *66*, 449–489. [\[CrossRef\]](#)
- Nishizaki, I.; Hayashida, T.; Sekizaki, S.; Okabe, J. Data envelopment analysis approaches for two-level production and distribution planning problems. *Eur. J. Oper. Res.* **2022**, *300*, 255–268. [\[CrossRef\]](#)

13. Banker, R.D.; Charnes, A.; Cooper, W.W. Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Manag. Sci.* **1984**, *30*, 1078–1092. [[CrossRef](#)]
14. Charnes, A.; Cooper, W.W.; Rhodes, E. Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* **1978**, *2*, 429–444. [[CrossRef](#)]
15. Saberi, S.; Cruz, J.M.; Sarkis, J.; Nagurney, A. A competitive multiperiod supply chain network model with freight carriers and green technology investment option. *Eur. J. Oper. Res.* **2018**, *266*, 934–949. [[CrossRef](#)]
16. Lim, C.H.; How, B.S.; Ng, W.P.Q.; Lam, H.L. Debottlenecking of biomass element deficiency in a multiperiod supply chain system via element targeting approach. *J. Clean. Prod.* **2019**, *230*, 751–766. [[CrossRef](#)]
17. Rahim, M.K.I.A.; Zhong, Y.; Aghezzaf, E.-H.; Aouam, T. Modelling and solving the multiperiod inventory-routing problem with stochastic stationary demand rates. *Int. J. Prod. Res.* **2014**, *52*, 4351–4363. [[CrossRef](#)]
18. Shin, Y.; Lee, S.; Moon, I. Robust multiperiod inventory model considering trade-in program and refurbishment service: Implications to emerging markets. *Transp. Res. Part E* **2020**, *138*, 101932. [[CrossRef](#)]
19. Wan, N.; Chen, X. Multiperiod production and ordering policies for a retailer-led supply chain through option contracts. *Math. Probl. Eng.* **2018**, *2018*, 9097136. [[CrossRef](#)]
20. Yu, M.; Cruz, J.M.; Li, D.; Masoumi, A.H. A multiperiod competitive supply chain framework with environmental policies and investments in sustainable operations. *Eur. J. Oper. Res.* **2022**, *300*, 112–123. [[CrossRef](#)]
21. Zeballos, L.J.; Méndez, C.A.; Barbosa-Povoa, A.P. Design and planning of closed-loop supply chains: a risk-averse multistage stochastic approach. *Ind. Eng. Chem. Res.* **2016**, *55*, 6236–6249. [[CrossRef](#)]
22. Charnes, A.; Cooper, W.W.; Lewin, A.R.; Seiford, L.M. *Data Envelopment Analysis: Theory, Methodology and Applications*; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1994.
23. Sengupta, J.K. *Dynamics of Data Envelopment Analysis, Theory of Systems Efficiency*; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1995.
24. Färe, R.; Grosskopf, S. *Intertemporal Production Frontiers: With Dynamic DEA*; Kluwer Academic Publishers: Dordrecht, The Netherlands, 1996.
25. Färe, R.; Grosskopf, S. Network DEA. *Socio-Econ. Plan. Sci.* **2000**, *34*, 35–49. [[CrossRef](#)]
26. Halkos, G.E.; Tzeremes, N.G.; Kourtzidis, S.A. A unified classification of two-stage DEA models. *Surv. Oper. Res. Manag. Sci.* **2014**, *19*, 1–16. [[CrossRef](#)]
27. Lozano, S. DEA production games. *Eur. J. Oper. Res.* **2013**, *231*, 405–413. [[CrossRef](#)]
28. Owen, G. On the core of linear production games. *Math. Program.* **1975**, *9*, 358–370. [[CrossRef](#)]
29. Fortuny-Amat, J.; McCarl, B. A representation and economic interpretation of a two-level programming problem. *J. Oper. Res. Soc.* **1981**, *32*, 783–792. [[CrossRef](#)]

**Disclaimer/Publisher’s Note:** The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.