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Almost Repdigit *k*-Fibonacci Numbers with an Application of *k*-Generalized Fibonacci Sequences

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Abstract: In this paper, we define the notion of almost repdigit as a positive integer whose digits are all equal except for at most one digit, and we search all terms of the *k*-generalized Fibonacci sequence which are almost repdigits. In particular, we find all *k*-generalized Fibonacci numbers which are powers of 10 as a special case of almost repdigits. In the second part of the paper, by using the roots of the characteristic polynomial of the *k*-generalized Fibonacci sequence, we introduce *k*-generalized tiny golden angles and show the feasibility of this new type of angles in application to magnetic resonance imaging.

Keywords: *k*-Fibonacci numbers; repdigits; almost repdigits; linear forms in logarithms; MR imaging; tiny golden angles; *k*-generalized tiny golden angles

MSC: 11B37; 11B39; 11J86

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1. Introduction

Let $k \geq 2$ be an integer. The *k*-generalized Fibonacci sequence or, for simplicity, the *k*-Fibonacci sequence is a sequence given by the recurrence relation

$$F_n^{(k)} = F_{n-1}^{(k)} + \cdots + F_{n-k}^{(k)} \quad \text{for all} \quad n \geq 2,$$

with the initial values $F_i^{(k)} = 0$ for $i = 2-k, \ldots, 0$ and $F_1^{(k)} = 1$. For $k = 2$, this sequence is the well-known Fibonacci sequence and, in this case, we may omit the superscript (k) in the notation.

Recall that, a positive integer whose all digits are equal is called a repdigit. In many cases, the relations between repdigits and *k*-Fibonacci numbers have already been settled by a number of authors in many papers, see for example [1–12]. In this study, we shall consider the numbers similar to the repdigits. Our motivation to study this kind of numbers comes from the terms of classical Fibonacci sequences.

Three consecutive Fibonacci numbers $F_{12} = 144$, $F_{13} = 233$ and $F_{14} = 377$ have a similar property that all digits are equal except only one digit. Thus, we call a positive integer whose digits are all equal except for at most one digit is an *almost repdigit*. These are the numbers of the form

$$a\left(\frac{10^{d_1}-1}{9}\right) + (b-a)10^{d_2}, \quad 0 \leq d_2 < d_1 \quad \text{and} \quad 0 \leq a, b \leq 9$$

The square and perfect power almost repdigits were examined in [13,14], without being attributed a specific name. In this paper, we search all almost repdigits in *k*-Fibonacci numbers for all $k \geq 2$. In particular, as a special case of almost repdigits, we search all

k-Fibonacci numbers that are powers of 10. In other words, we consider the Diophantine equation

$$F_n^{(k)} = a\left(\frac{10^{d_1}-1}{9}\right) + (b-a)10^{d_2}, \quad 0 \le d_2 < d_1 \quad \text{and} \quad 0 \le a, b \le 9 \tag{1}$$

in non-negative integers d_1, d_2, a and b. We state the main results of this paper as follows.

Theorem 1. *The Diophantine Equation* (1) *has solutions only in the cases* $F_{12}^{(2)} = 144$, $F_{13}^{(2)} = 233$, $F_{14}^{(2)} = 377$, $F_{12}^{(4)} = 773$, $F_{11}^{(5)} = 464$, $F_{13}^{(7)} = 2000$, $F_{10}^{(8)} = 255$ *and* $F_{11}^{(9)} = 511$ *when* $F_n^{(k)}$ *has at least three digits.*

To eliminate the trivial cases, the above theorem is stated for numbers with at least three digits, since all integers having one or two digits are trivially almost repdigits. Thus, we also take $d_1 \ge 3$ and $n > 5$.

The proofs of the above theorems come from two effective methods for Diophantine equations. One of them is linear forms in logarithms of algebraic numbers due to Matveev [15], whereas the other one is a version of the reduction algorithm due to Dujella and Pethő [16], which was in fact originally introduced by Baker and Davenport in [17]. In the application of these methods, we frequently need some calculations and computations. For all computations, we use the software Mathematica. Some details of the tools used in this study will be given in the next section.

In addition to all theoretical calculations, in the last section, we give some results which invite the researchers to use the roots of the characteristic polynomial of the k-generalized Fibonacci sequences in application, especially for magnetic resonance imaging. It is well known that, when $k = 2$, the positive root of the characteristic polynomial of the sequence $F_n^{(k)}$ is $\phi = \dfrac{1+\sqrt{5}}{2}$, that is the famous golden ratio. The golden angle is defined by $\psi_{\text{gold}} = \pi/\phi$, which is an angle that is calculated by dividing the semicircle by the golden ratio. Among other things, in [18], tiny golden angles are introduced and the authors showed the advantages of these angles for dynamic magnetic resonance imaging. In the last section of this paper, we introduce the *k-generalized tiny golden angles* which are based on k-generalized Fibonacci sequences and remark that these newly introduced angles are closely correlated with tiny golden angles. Thus, these new angles are also potentially applicable for magnetic resonance imaging. As a result, we open a new approach for researchers who are working in the healthcare field to apply this in MRI for diagnosing heart diseases, cancer, etc.

2. The Tools

Let θ be an algebraic number, and let

$$c_0 x^d + c_1 x^{d-1} + \cdots + c_d = c_0 \prod_{i=1}^{d} (x - \theta^{(i)})$$

be its minimal polynomial over $\mathbb{Z}$, with degree d, where the c_i's are relatively prime integers with $c_0 > 0$, and the $\theta^{(i)}$'s are conjugates of θ.

The logarithmic height of θ is defined by

$$h(\theta) = \frac{1}{d}\left(\log c_0 + \sum_{i=1}^{d} \log\Big(\max\{|\theta^{(i)}|, 1\}\Big)\right).$$

If $\theta = r/s$ is a rational number with relatively prime integers r and s and $s > 0$, then $h(r/s) = \log \max\{|r|, s\}$. The following properties are very useful in calculating a logarithmic height :

- $h(\theta_1 \pm \theta_2) \leq h(\theta_1) + h(\theta_2) + \log 2$.
- $h(\theta_1 \theta_2^{\pm 1}) \leq h(\theta_1) + h(\theta_2)$.
- $h(\theta^s) = |s| h(\theta), s \in \mathbb{Z}$.

Theorem 2 (Matveev's Theorem)**.** *Assume that $\alpha_1, \ldots, \alpha_t$ are positive real algebraic numbers in a real algebraic number field $\mathbb{K}$ of degree $d_{\mathbb{K}}$ and let $b_1, \ldots, b_t$ be rational integers, such that*

$$\Lambda := \alpha_1^{b_1} \cdots \alpha_t^{b_t} - 1,$$

is not zero. Then

$$|\Lambda| > \exp\Big(K(t) d_{\mathbb{K}}^2 (1 + \log d_{\mathbb{K}})(1 + \log B) A_1 \cdots A_t\Big),$$

where

$$K(t) := -1.4 \times 30^{t+3} \times t^{4.5} \quad \textit{and} \quad B \geq \max\{|b_1|, \ldots, |b_t|\},$$

and

$$A_i \geq \max\{d_{\mathbb{K}} h(\alpha_i), |\log \alpha_i|, 0.16\}, \quad \textit{for all} \quad i = 1, \ldots, t.$$

For a real number θ, we put $||\theta|| = \min\{|\theta - n| : n \in \mathbb{Z}\}$, which represents the distance from θ to the nearest integer. Now, we cite the following lemma which we will use to reduce some upper bounds on the variables.

Lemma 1 ([19] (Lemma 1))**.** *Let M be a positive integer, and let p/q be a convergent of the continued fraction of the irrational τ such that $q > 6M$. Let A, B, μ be some real numbers with $A > 0$ and $B > 1$. If $\epsilon := ||\mu q|| - M||\tau q|| > 0$, then there is no solution to the inequality*

$$0 < |u\tau - v + \mu| < AB^{-w},$$

in positive integers u, v and w with

$$u \leq M \quad \textit{and} \quad w \geq \frac{\log(Aq/\epsilon)}{\log B}.$$

3. Properties of k-Fibonacci Numbers

From its defining recurrence relation, the characteristic polynomial of k-Fibonacci sequence is

$$\Psi_k(x) = x^k - x^{k-1} - \cdots - x - 1,$$

which is an irreducible polynomial over $\mathbb{Q}[x]$. The polynomial $\Psi_k(x)$ has exactly one real distinguished root $\alpha(k)$ outside the unit circle [20–22]. The other roots of $\Psi_k(x)$ are strictly inside the unit circle [21]. This root $\alpha(k)$, say for simplicity α, is located in the interval

$$2(1 - 2^{-k}) < \alpha < 2 \quad \text{for all} \quad k \geq 2.$$

Let

$$f_k(x) = \frac{x - 1}{2 + (k + 1)(x - 2)}.$$

It is known that the inequalities

$$1/2 < f_k(\alpha) < 3/4 \quad \text{and} \quad |f_k(\alpha_i)| < 1, \quad 2 \leq i \leq k, \tag{2}$$

hold, where $\alpha := \alpha_1, \cdots, \alpha_k$ are all the roots of $\Psi_k(x)$ [19] (Lemma 2). In particular, we deduce that $f_k(\alpha)$ is not an algebraic integer. In the same lemma, it is also proven that

$$h(f_k(\alpha)) < 3 \log k \quad \text{holds} \quad \forall k \geq 2, \tag{3}$$

which will be useful in our study.

In [23], Dresden and Du showed that

$$F_n^{(k)} = \sum_{i=1}^{k} f_k(\alpha_i)(\alpha_i)^{n-1} \quad \text{and} \quad \left| F_n^{(k)} - f_k(\alpha)\alpha^{n-1} \right| < 1/2 \tag{4}$$

for all $k \geq 2$. In this section, we finally note that, as in the classical $k = 2$ case, we have the similar bounds as

$$\alpha^{n-2} \leq F_n^{(k)} \leq \alpha^{n-1} \tag{5}$$

for all $n \geq 1$ and $k \geq 2$ [24].

4. Proof of Theorem 1

First, we may directly derive some relations between the variables that will be useful in our subsequent study. From (1) and (5), we obtain

$$10^{d_1-2} < a\left(\frac{10^{d_1}-1}{9}\right) + (b-a)10^{d_2} = F_n^{(k)} \leq \alpha^{n-1} < 2^{n-1},$$

and

$$((1+\sqrt{5})/2)^{n-2} \leq \alpha^{n-2} \leq F_n^{(k)} = a\left(\frac{10^{d_1}-1}{9}\right) + (b-a)10^{d_2} \leq 2 \times 10^{d_1},$$

which implies that

$$d_1 < \frac{\log 2}{\log 10}(n-1) + 2 < 0.31n + 1.8 < n - 1 \tag{6}$$

and

$$0.2n - 0.8 < \frac{\log((1+\sqrt{5})/2)}{\log 10}(n-2) - \frac{\log 2}{\log 10} < d_1 \tag{7}$$

for all $n > 5$.

We will treat the case $a = 0$ separately in the last part of this section, in which case Equation (1) turns into $F_n^{(k)} = b10^{d_2}$.

4.1. The Case $n \leq k+1$ and Almost Repdigits of the Form 2^n

Assume that $n \leq k+1$. Then, $F_n^{(k)} = 2^{n-2}$, and hence Equation (1) can be written as

$$9 \times 2^{n-2} = a\left(10^{d_1} - 1\right) + 9(b-a)10^{d_2}.$$

From (6),

$$0 \equiv -a \pmod{2^{d_2}}.$$

Thus, $d_2 \leq 3$. By modulo 2^{d_1}, we find

$$0 \equiv -a + (b-a)10^{d_2} \pmod{2^{d_1}},$$

which means $2^{d_1} < 10^4$, that is $d_1 \leq 13$. Hence, from (7), we see that $n < 70$. Computations with Mathematica show that, when $n < 70$, there is no almost repdigits of the form 2^{n-2} with at least three digits.

Thus, from now on, we take $n \geq k+2$.

4.2. A Bound for n Depending on k

Now, assume that $n \geq k+2$. First, we rewrite (1) as

$$F_n^{(k)} + a/9 - (b-a)10^{d_2} = a10^{d_1}/9,$$

and, by using (4), we obtain

$$\begin{aligned} \left| f_k(\alpha)\alpha^{n-1} - a10^{d_1}/9 \right| &= \left| F_n^{(k)} - f_k(\alpha)\alpha^{n-1} + a/9 - (b-a)10^{d_2} \right| \\ &\leq (1/2) + \left| a/9 - (b-a)10^{d_2} \right|. \end{aligned}$$

By dividing both sides by $a10^{d_1}/9$, we obtain

$$|\Lambda_1| \leq \frac{9/2}{10^{d_1}} + \frac{1}{10^{d_1}} + \frac{|b-a|(9/a)}{10^{d_1-d_2}} \leq \frac{78}{10^{d_1-d_2}}, \tag{8}$$

where

$$\Lambda_1 := \alpha^{n-1}10^{-d_1} f_k(\alpha)9/a - 1.$$

Let $\eta_1 := \alpha$, $\eta_2 := 10$, $\eta_3 := f_k(\alpha)9/a$ and $b_1 := n-1$, $b_2 := -d_1$, $b_3 := 1$ where η_1, η_2 and η_3 belong to the real number field $\mathbb{K} = \mathbb{Q}(\alpha)$ with degree $d_{\mathbb{K}} = k$. By (6), we take $B := n-1 > d_1$.

Since $h(\eta_1) = (1/k)\log(\alpha)$ and $h(\eta_2) = \log 10$, we take $A_1 = \log\alpha$ and $A_2 = k\log(10)$. Furthermore, from (3), $h(\eta_3) \leq h(9/a) + h(f_k(\alpha)) \leq \log(9) + 3\log k < 7\log k$ holds for all $k \geq 2$. Thus, we take $A_3 = 7k\log k$.

We also have $\Lambda_1 \neq 0$. Indeed, if $\Lambda_1 = 0$, then we obtain

$$a10^{d_1}/9 = f_k(\alpha)\alpha^{n-1}.$$

Conjugating both sides of this relation by any one of the automorphisms $\sigma_i : \alpha \to \alpha_i$ for any $i \geq 2$ and by taking the absolute values, by (2), we find that

$$100 < 10^3/9 \leq |a10^{d_1}/9| = |f_k(\alpha_i)||\alpha_i|^{n-1} < 1,$$

which is clearly false. Thus, $\Lambda_1 \neq 0$. With these notations, by Theorem 2, we obtain that

$$\log|\Lambda_1| > -1.4 \times 30^6 \times 3^{4.5} \times k^2(1+\log k)(1+\log(n-1))\log\alpha \times k\log 10 \times 7k\log k.$$

On the other hand, from (8), we have that $\log|\Lambda_1| < \log 78 - (d_1-d_2)\log 10$. From the last two inequalities, we obtain

$$d_1 - d_2 < 4.2 \times 10^{12} \times k^4(\log k)^2\log(n-1), \tag{9}$$

where we used the facts that $\log(\alpha) \leq \log(2)$, $1+\log k < 3\log k$ and $(1+\log(n-1)) < 2\log(n-1)$ hold for all $k \geq 2$ and $n \geq 5$.

Now, we turn back to Equation (1) and rewrite it as follows

$$F_n^{(k)} + a/9 = a10^{d_1}/9 + (b-a)10^{d_2}.$$

Again, from (4), we write

$$\begin{aligned} \left| f_k(\alpha)\alpha^{n-1} - 10^{d_1}((a/9) + (b-a)10^{d_2-d_1}) \right| &\leq \left| F_n^{(k)} - f_k(\alpha)\alpha^{n-1} + a/9 \right| \\ &\leq (1/2) + (a/9) \leq 3/2. \end{aligned}$$

This time, we divide both sides by $f_k(\alpha)\alpha^{n-1}$ to obtain

$$|\Lambda_2| \leq \frac{3}{2}\frac{1}{f_k(\alpha)\alpha^{n-1}} \leq \frac{3}{\alpha^{n-1}}, \tag{10}$$

where

$$\Lambda_2 := \alpha^{-(n-1)}10^{d_1}f_k(\alpha)^{-1}((a/9)+(b-a)10^{d_2-d_1})-1.$$

Since

$$\frac{1}{90} \leq \frac{1}{9}-\frac{1}{10} \leq \frac{a}{9}-\frac{a}{10^{d_1-d_2}}+\frac{b}{10^{d_1-d_2}} \leq (a/9)+(b-a)10^{d_2-d_1},$$

we have that

$$11 < 10^3\frac{1}{90} \leq 10^{d_1}((a/9)+(b-a)10^{d_2-d_1}).$$

Thus, the similar argument that has been used before for Λ_1, shows that Λ_2 is not zero too.

Let $\eta_1 := \alpha$, $\eta_2 := 10$ and $\eta_3 := f_k(\alpha)^{-1}((a/9)+(b-a)10^{d_2-d_1})$ with $b_1 := -(n-1)$, $b_2 := d_1$, $b_3 := 1$. All η_1, η_2 and η_3 belong to the real number field $\mathbb{K} = \mathbb{Q}(\alpha)$, and therefore we take $d_{\mathbb{K}} = 2$, to be the degree of the number field $\mathbb{K}$.

Since $h(\eta_1) = (1/k)\log(\alpha)$ and $h(\eta_2) = \log 10$ we take $A_1 = \log(\alpha)$ and $A_2 = k\log(10)$. Using the properties of logarithmic height, we obtain:

$$\begin{aligned} h(\eta_3) &\leq h(f_k(\alpha)^{-1}) + h((a/9)+(b-a)10^{d_2-d_1}) \\ &\leq 3\log k + h(a/9) + h(b-a) + h(10^{d_2-d_1}) + \log(2) \\ &\leq 3\log k + \log(144) + |d_2-d_1|\log(10) \\ &< 11\log k + |d_2-d_1|\log(10). \end{aligned}$$

By applying Theorem 2, we get a bound for $\log|\Lambda_2|$. Then by combining this bound with the one comes from (10), we get

$$n-1 < 2\times 10^{12}k^4\log k\log(n-1)(11\log k + |d_1-d_2|\log(10)).$$

From (9), we may write

$$\begin{aligned} 11\log k + (d_1-d_2)\log(10 &< 4.2\times 10^{12}\times k^4(\log k)^2\log(n-1)\log(10) + 11\log k \\ &< 10^{13}\times k^4(\log k)^2\log(n-1). \end{aligned}$$

Now, by substituting this estimate into the above equation, we obtain

$$\begin{aligned} n-1 &< 2\times 10^{12}k^4\log k\log(n-1)10^{13}\times k^4(\log k)^2\log(n-1) \\ &< 2\times 10^{25}k^8(\log k)^3(\log(n-1))^2 \end{aligned}$$

From this relation, we may obtain a bound on n, depending on k. To do this, we need the following lemma from ([25] Lemma 7).

Lemma 2. *Let $m \geq 1$ and $T > (4m^2)^m$. Then, we have*

$$\frac{x}{(\log x)^m} < T \Rightarrow x < 2^m T(\log(T))^m.$$

We take $T := 2\times 10^{25}k^8(\log k)^3$, so that

$$\begin{aligned} (\log(T))^2 &< (\log(2)+25\log(10)+8\log k+3\log(\log k))^2 \\ &< (\log k+100\log k+11\log k)^2 < 112^2(\log k)^2. \end{aligned}$$

Thus, from Lemma 2, we may end this subsection with the following bound of n, which is the aim of this part.

$$n < 1.1 \times 10^{30} k^8 (\log k)^5. \tag{11}$$

Now, we treat the cases $k \leq 470$ and $k > 470$ separately.

4.3. The Case $k \leq 470$

Let $2 \leq k \leq 470$. Then, from (11), n is also bounded above. Let

$$\Gamma_1 := (n-1)\log\alpha - d_1 \log 10 + \log(f_k(\alpha) \times 9/a).$$

Then

$$|\Lambda_1| := |\exp(\Gamma_1) - 1| < 78/10^{d_1 - d_2}.$$

We claim that $d_1 - d_2 < 145$. Suppose that $d_1 - d_2 > 3$. Then, $78/10^{d_1-d_2} < 1/2$ and therefore $|\Gamma_1| < \dfrac{156}{10^{d_1-d_2}}$. Thus, we have

$$0 < \left| (n-1)\frac{\log\alpha}{\log 10} - d_1 + \frac{\log(f_k(\alpha) \times 9/a)}{\log 10} \right| < 156/10^{d_1-d_2} \log 10. \tag{12}$$

For all $2 \leq k \leq 470$, we take $M_k := 1.1 \times 10^{30} k^8 (\log k)^5 > n$ and $\tau_k = \dfrac{\log\alpha}{\log 10}$. For each k, we find a convergent p_i/q_i of the continued fraction of irrational τ_k, such that $q_i > 6M_k$. Then, we calculate $\epsilon_{(k,a)} := ||\mu_{(k,a)} q_i|| - M_k ||\tau_k q_i||$ for each $a \in \{1, 2, \ldots, 9\}$, where

$$\mu_{(k,a)} := \frac{\log(f_k(\alpha) \times 9/a)}{\log 10}.$$

If $\epsilon_{(k,a)} < 0$, then we repeat the same calculation for q_{i+1}. For each k, we found such a denominator of τ_k, such that $\epsilon_{(k,a)} > 0$, in particular, which also implies that $\mu_{(k,a)} \notin \mathbb{Z}$. In fact, we have $0.7 \times 10^{-42} < \epsilon_{(k,a)}$. Thus, from Lemma 1, we find an upper bound on $d_1 - d_2$ for each $2 \leq k \leq 470$ and none of these bounds are greater than 142. Thus, we conclude that $d_1 - d_2 < 145$, as we claimed previously.

Let

$$\Gamma_2 := -(n-1)\log\alpha + d_1 \log 10 + \log(f_k(\alpha)^{-1} \times ((a/9) + (b-a)10^{d_2 - d_1})),$$

so that

$$|\Lambda_2| := |\exp(\Gamma_2) - 1| < 3/\alpha^{n-1} < 1/2.$$

$\Gamma_2 \neq 0$, since Λ_2. Hence, we obtain

$$\left| (n-1)\frac{\log\alpha}{\log 10} - d_1 - \frac{\log(f_k(\alpha)^{-1}(\frac{a}{9} + \frac{b-a}{10^{d_1-d_2}})}{\log 10} \right| < \frac{6}{\alpha^{n-1}\log 10}. \tag{13}$$

This time, we calculate $\epsilon_{(k,d_1-d_2,a,b)} := ||\mu_{(k,d_1-d_2,a,b)} q_i|| - M_k ||\tau_k q_i||$ for each $d_1 - d_2 \in \{1, 2, \ldots, 145\}$, $a \in \{1, 2, \ldots, 9\}$ and $b \in \{0, 1, \ldots, 9\}$, where

$$\mu_{(k,d_1-d_2,a,b)} := -\frac{\log(f_k(\alpha)^{-1} \times ((a/9) + (b-a)10^{d_2-d_1}))}{\log 10}.$$

If we encounter $\epsilon_{(k,d_1-d_2,a,b)} < 0$, for any values of $d_1 - d_2, a$ or b, then, we take the denominator q_{i+1} instead of q_i, as we did previously. For each k, we find such a denominator of τ_k such that $\epsilon_{(k,d_1-d_2,a,b)} > 0$. Thus, applying Lemma 1 to Equation (13), we obtain an

upper bound on $n-1$ for each $2 \leq k \leq 470$. Let us denote this upper bound by $n(k)$. Some of these bounds are $n(2) < 176$, $n(3) < 149$, $n(10) < 151$, $n(100) < 180$, $n(200) < 197$, $n(300) < 296$, $n(400) < 396$ and $n(470) < 465$, which show that, for some values of k , there is only a few values of n satisfying $n \geq k+2$. We use this estimate to shorten the runtime in the following computer search.

With the help of a computer program in Mathematica, and by using the bounds given in (6), we search all the variables in the range $2 \leq k \leq 470$, $k+2 \leq n \leq n(k)$, $0 \leq d_2 < d_1 < 0.31n + 1.8$, $1 \leq a \leq 9$ and $0 \leq b \leq 9$ satisfying (1). We find that $F_{12}^{(2)} = 144$, $F_{13}^{(2)} = 233$, $F_{14}^{(2)} = 377$, $F_{12}^{(4)} = 773$, $F_{11}^{(5)} = 464$, $F_{10}^{(8)} = 255$ and $F_{11}^{(9)} = 511$ are the only solutions of (1) when $k \leq 470$ and $a \neq 0$, with at least three digits, as we claimed in Theorem 1, see also Table A1 in the Appendix A. Now, we turn our focus to the case $k > 470$.

4.4. The Case $k > 470$

We use the following lemma.

Lemma 3 ([3] (Lemma 3))**.** *If $n < 2^{k/2}$, then the following estimates hold:*

$$F_n^{(k)} = 2^{n-2}(1+\zeta(n,k)), \quad \textit{where} \quad |\zeta(n,k)| < \frac{2}{2^{k/2}}.$$

For $k > 470$, the inequality $n < 1.1 \times 10^{30} k^8 (\log k)^5 < 2^{k/2}$, holds and hence from Lemma 3, we have

$$\left|2^{n-2} - F_n^{(k)}\right| < \frac{2^{n-1}}{2^{k/2}}. \tag{14}$$

Now, we turn back to (1), one more time to rewrite it as

$$\left|F_n^{(k)} - (a/9)10^{d_1}\right| < (a/9) + |b-a|10^{d_2}. \tag{15}$$

Thus, combining (14) and (15), we obtain

$$\left|2^{n-2} - (a/9)10^{d_1}\right| < \frac{2^{n-1}}{2^{k/2}} + (a/9) + |b-a|10^{d_2}.$$

By multiplying both sides by $(9/a)10^{-d_1}$, we find

$$\left|2^{n-2}10^{-d_1}9/a - 1\right| < \frac{2^{n-1}}{2^{k/2}}\frac{9}{10^{d_1}a} + \frac{1}{10^{d_1}} + \frac{72}{10^{d_1-d_2}}.$$

Note that, the estimates

$$\begin{aligned}\frac{9\times 2^{n-1}}{a10^{d_1}} < \frac{2}{(1+\zeta)}\frac{9\times F_n^{(k)}}{a10^{d_1}} &< \frac{2}{0.999}\left(1 + \frac{1}{10^{d_1}} + \frac{72}{10^{d_1-d_2}}\right)\\ &< \frac{2}{0.999}\left(1 + \frac{1}{10^3} + \frac{72}{10}\right) < 17\end{aligned}$$

hold for all $k > 470$. Therefore, we have

$$|\Lambda_3| := \left|2^{n-2}10^{-d_1}9/a - 1\right| < \frac{1}{2^{\lambda}}, \tag{16}$$

where $\lambda := \min\{(k/2) - 6, (d_1 - d_2)\dfrac{\log(10)}{\log(2)} - 8\}$.

Let $\eta_1 := 2$, $\eta_2 := 10$, $\eta_3 := 9/a$ and $b_1 := n-2$, $b_2 := -d_1$, $b_3 := 1$. We take $t = 3$ if $a \neq 9$ and $t = 2$ if $a = 9$. We take $\mathbb{K} = \mathbb{Q}$, $d_{\mathbb{K}} = 1$ and $B := n$. Clearly, $\Lambda_3 \neq 0$. Thus, from Theorem 2, we obtain

$$\log|\Lambda_3| > -1.4 \times 30^6 \times 3^{4.5}(1+\log n)\log 2 \times \log 9 \times \log 10,$$

if $a \neq 9$, and

$$\log|\Lambda_3| > -1.4 \times 30^5 \times 2^{4.5}(1+\log n)\log 2 \times \log 10,$$

if $a = 9$. Then, in either case, by using the fact $\log(\Lambda_3) < -\lambda \log 2$ from (16), we find

$$\begin{aligned}\lambda &< 1.5 \times 10^{12} \log n \\ &< 1.5 \times 10^{12} \times 45 \log k < 6.8 \times 10^{13} \log k.\end{aligned}$$

In the above, we used the fact that

$$\begin{aligned}\log n &< \log(1.1 \times 10^{30} k^8 (\log k)^5) \\ &< \log(1.1) + 30\log(10) + 8\log k + 5\log\log k \\ &< 45 \log k.\end{aligned}$$

Thus, if $\lambda := (k/2) - 6$, then we obtain a bound $k < 5 \times 10^{15}$.
If $\lambda := (d_1 - d_2)\dfrac{\log(10)}{\log(2)} - 8$, then we obtain

$$d_1 - d_2 < 2.1 \times 10^{13} \log k. \tag{17}$$

Even in this case, we may obtain a bound for k with a little bit more effort. For this purpose, we rewrite (1) as follows

$$\left| F_n^{(k)} - (a/9)10^{d_1} - (b-a)10^{d_2} \right| \leq (a/9) \leq 1. \tag{18}$$

Combining (18) and (14), we have

$$\left| 2^{n-2} - 10^{d_1}((a/9) + (b-a)10^{d_2-d_1}) \right| < 1 + \frac{2^{n-1}}{2^{k/2}},$$

and from this relation, we obtain

$$|\Lambda_4| := \left| 2^{-(n-2)} 10^{d_1}((a/9) + (b-a)10^{d_2-d_1}) - 1 \right| < \frac{2}{2^{k/2}} + \frac{1}{2^{n-2}} \leq \frac{3}{2^{k/2}}. \tag{19}$$

Let $\eta_1 := 2$, $\eta_2 := 10$, $\eta_3 := (a/9) + (b-a)10^{d_2-d_1}$ and $b_1 := -(n-2)$, $b_2 := -d_1$, $b_3 := 1$. Then, we take $\mathbb{K} = \mathbb{Q}$, $d_{\mathbb{K}} = 1$, $B := n > n-2$. $h(\eta_1) = \log 2$, $h(\eta_2) = \log 10$ and

$$h(\eta_3) = h(a/9) + h(b-a) + |d_2 - d_1|\log 10 + \log 2 < \log 144 + (d_1 - d_2)\log 10.$$

Moreover, $\Lambda_4 \neq 0$. Indeed, $2^{n-2} = 10^{d_1}(a/9) + (b-a)10^{d_2}$ implies that $a = 9$ and $d_2 = 0$. For $d_1 = 3$, the equation $2^{n-2} = 10^{d_1} + b - 9$ clearly has no solution in integers. Therefore, $d_1 > 3$, and the congruence consideration modulo 2^4, shows that this equation has no integer solutions for $0 \leq b \leq 9$. Thus, $\Lambda_4 \neq 0$.

Moreover, applying Theorem 2 to Λ_4, together with (19) gives that

$$\log 3 - (k/2)\log 2 < -1.4 \times 30^6 \times 3^{4.5}(1+\log n)\log 2 \times \log 10 \times (\log 144 + (d_1 - d_2)\log 10).$$

By substituting the upper bound of $d_1 - d_2$ given in (17) into the above inequality and using the estimate $\log 144 < \log k$ and $\log n < 45 \log k$, we obtain an upper bound for k as follows

$$k < 2 \times 10^{31}.$$

Thus, by (11), we have also a bound for n as

$$n < 5.5 \times 10^{289}.$$

4.5. Reducing the Bound on k

The above upper bounds are far from being able to directly search for the solutions. Thus, this subsection is devoted to reducing these bounds. Let

$$|\Gamma_3| := (n-2)\log 2 - d_1 \log 10 + \log(9/a). \tag{20}$$

Then, $\Lambda_3 := |\exp(\Gamma_3) - 1| < \frac{1}{2^\lambda}$. Suppose that $\lambda > 2$. Then, $\frac{1}{2^\lambda} < \frac{1}{2}$ and hence we obtain $|\Gamma_3| < \frac{2}{2^\lambda}$. Now, we work on the Γ_3 according to the case $a = 9$ and $a \neq 9$, separately.

Assume that $a = 9$. Then, from (20)

$$\left| \frac{\log 2}{\log 10} - \frac{d_1}{n-2} \right| < \frac{2}{2^\lambda (n-2)\log 10}. \tag{21}$$

If $\frac{2}{2^\lambda (n-2)\log 10} < \frac{1}{2(n-2)^2}$, then $\frac{d_1}{n-2}$ is a convergent of continued fraction expansion of irrational $\log 2/\log 10$, say $\frac{p_i}{q_i}$. Since p_i and q_i are relatively prime, we deduce that $q_i \leq n - 2 < 5.5 \times 10^{289}$. A quick search with Mathematica shows that $i < 585$. Let $[a_0, a_1, a_2, a_3, a_4, \ldots] = [0, 3, 3, 9, 2, 2, \ldots]$ be the continued fraction expansion of $\log 2/\log 10$. Then, $\max\{a_i\} = 5393$ for $i = 0, 1, 2, \ldots, 589$. Thus, from the well-known property of continued fractions, see for example ([26] Theorem 1.1.(iv)), we write

$$\frac{1}{5395 \times (n-2)^2} \leq \frac{1}{(a_i+2)(n-2)^2} < \left| \frac{\log 2}{\log 10} - \frac{d_1}{n-2} \right| < \frac{2}{2^\lambda (n-2)\log 10}.$$

Thus, from the inequality

$$2^\lambda < \frac{2 \times 5395 \times 5.5 \times 10^{289}}{\log 10} < 2.58 \times 10^{293} < 2^{975},$$

we find $\lambda < 975$. If

$$\frac{2}{2^\lambda (n-2)\log 10} > \frac{1}{2(n-2)^2},$$

then this bound clearly holds.

Assume that $a \neq 9$. Then, from (20), we write

$$0 < \left| (n-2)\frac{\log 2}{\log 10} - d_1 + \frac{\log(9/a)}{\log 10} \right| < \frac{2}{2^\lambda \log 10}.$$

Let $M := 5.5 \times 10^{289} > n$, $\tau = \frac{\log 2}{\log 10}$, and $\mu_a := \log(9/a)/\log 10$. By letting the parameters $A := \frac{2}{\log 10}$, $B := 2$ and

$$\epsilon := 0.159626 \leq \epsilon_a := ||\mu_a q_{587}|| - M||\tau q_{587}||$$

for all $a \in \{1, 2, \ldots, 8\}$, from Lemma 1, we find that $\lambda < 970$. Thus, regardless of whether $a = 9$, we have that $\lambda < 975$.

If $\lambda = k/2 - 6$, then $k < 1962$. If $\lambda = (d_1 - d_2)\frac{\log(10)}{\log(2)} - 8$, then

$$d_1 - d_2 < 295.92 < 300.$$

We show that this case also leads to an upper bound for k as $k < 1985$. Let

$$\Gamma_4 = \left|(n-2)\log 2 - d_1 \log 10 - \log((a/9) + (b-a)10^{d_2-d_1})\right|.$$

Then

$$|\Lambda_4| := |\exp(\Gamma_4) - 1| < \frac{6}{2^{k/2}} < \frac{1}{2}.$$

So

$$0 < \left|\frac{\Gamma_4}{\log 10}\right| < \frac{3}{2^{k/2}\log 10}. \tag{22}$$

Let M and τ be as above and $\mu_{(a,b,d_1-d_2)} := -\dfrac{\log(a/9) + (b-a)10^{d_2-d_1}}{\log 10}$. We apply Lemma 1 to (22) with the parameters

$A := \dfrac{6}{\log 10}$, $B := 2$, $\omega := k/2$ and $\epsilon_{(a,b,d_1-d_2)} := ||\mu_{(a,b,d_1-d_2)}q_{593}|| - M||\tau q_{593}||$.

By calculation with Mathematica, we find that

$$0.000059 < \epsilon_{(6,7,59)} \leq \epsilon_{(a,b,d_1-d_2)}$$

holds for all $a \in \{1,2,\ldots,8\}$, $b \in \{1,2,\ldots,9\}$ and $d_1 - d_2 \in \{1,2,\ldots,300\}$. Thus, by Lemma 1, we deduce that $k < 1985$. Hence, from (11), $n < 6.7 \times 10^{60}$.

With this new and better bound on k, we repeat the same steps starting from the beginning of this subsection, but we take $M := 6.7 \times 10^{60}$. Regardless of whether $a = 9$, similar calculations on Γ_3 show that $\lambda < 215$. Thus, if $\lambda = (k/2) - 6$, then $k < 443$, whereas $\lambda = (d_1 - d_2)\dfrac{\log(10)}{\log(2)} - 8$ gives

$$d_1 - d_2 < 68.$$

We work on Γ_4 as we did before but with q_{135}. Thus, we find that

$$0.000072 < \epsilon_{(5,6,2)} \leq \epsilon_{(a,b,d_1-d_2)},$$

for all $a, b, d_1 - d_2$. With these parameters, by Lemma 1, we find $k/2 < 228.93$, which means that $k < 458$, which contradicts our assumption that $k > 470$. This completes the proof for $a \neq 0$.

4.6. The Case $a = 0$ and k-Fibonacci Numbers as Powers of 10

Let $a = 0$. Then, Equation (1) is of the form

$$F_n^{(k)} = b10^{d_2}. \tag{23}$$

Clearly, we take $b \neq 0$. In fact, our previous work contains most of the material to solve this equation, with some small manipulation on the variables. So, in any applicable case, we follow the previous notation to prevent the recalculation.

By (23), Λ_2 which was given in (10) is valid as

$$|\Lambda_2| := |\alpha^{-(n-1)}10^{d_2}bf_k(\alpha)^{-1} - 1| \leq \frac{1}{\alpha^{n-1}},$$

and $\Lambda_2 \neq 0$. This time, we set $\eta_1 := \alpha$, $\eta_2 := 10$, $\eta_3 := bf_k(\alpha)^{-1}$ with $b_1 := -(n-1)$, $b_2 := d_2$, $b_3 := 1$. Therefore,

$$h(\eta_3) \leq h(b) + h(f_k(\alpha)^{-1}) \leq \log 9 + 3\log k < 7\log k.$$

Using the bound given in (5) together with (23), we see that $d_2 \log 10 < (n-1)\log \alpha$, which means $d_2 < n-1$. Thus, $B := n-1$. Applying Theorem 2, as we did before for Λ_2, we obtain that

$$n - 1 < 1.4 \times 10^{13} k^4 \log^2 k \log(n-1).$$

We take $T := 1.4 \times 10^{13} k^4 \log^2 k$. Then $\log T < 60 \log k$ for all $k \geq 2$. Thus, from Lemma 2, we find

$$n < 2.1 \times 10^{17} k^4 \log^4 k. \tag{24}$$

If $k \leq 470$, then $n < 3 \times 10^{29}$. By performing the previous calculations, as we did before for (13) to the inequality,

$$0 < \left| (n-1)\frac{\log \alpha}{\log 10} - d_2 - \frac{\log(b f_k(\alpha)^{-1}}{\log 10} \right| < \frac{6}{\alpha^{n-1} \log 10},$$

we see that the same bounds strictly hold for the case $a = 0$. Hence, a computer search shows that we have only one solution of (23) which is $F_{13}^{(7)} = 2000$.

For $k > 470$, from (14), we write

$$0 \neq \Lambda_4' := \left| 2^{-(n-2)} 10^{d_2} b - 1 \right| \leq \frac{2}{2^{k/2}}.$$

By taking $(\eta_1, |b_1|) := (2, n-2)$, $(\eta_2, |b_2|) := (10, d_2)$ and $(\eta_3, |b_3|) := (b, 1)$, from Theorem 2 together with (24), we find $k < 4 \times 10^{14}$ and hence, from (24), $n < 6.9 \times 10^{81}$. To reduce these bounds, we write

$$\Gamma_4' := |(n-2)\log 2 - d_2 \log 10 - \log b|,$$

so that, as we did before, we obtain

$$0 < \left| (n-2)\frac{\log 2}{\log 10} - d_2 - \frac{\log b}{\log 10} \right| < \frac{4}{2^{k/2} \log 10}. \tag{25}$$

Assume that $b \notin \{1, 2, 4, 5, 8\}$. Then, applying Lemma 1 by choosing the parameters as $M := 6.9 \times 10^{81}$, $\mu_b := -\log b / \log 10$, $\epsilon_b := ||\mu_b q_{170}|| - M||\tau q_{170}||$ and the others as in the previous section, we find that $k < 564$. If b is 1,2,4,5 or 8 then, from Γ_4', we have that

$$\left| \frac{\log 2}{\log 10} - \frac{u}{v} \right| < \frac{4}{2^{k/2} v s. \log 10},$$

where $\frac{u}{v}$ is $\frac{d_2}{n-2}$, $\frac{d_2}{n-3}$, $\frac{d_2}{n-4}$, $\frac{d_2+1}{n-1}$ and $\frac{d_2}{n-5}$, respectively. We use the theory of continued fractions as we did before for (21), to obtain that $k < 572$. Thus, from (24), we obtain a reduced bound as $n < 4 \times 10^{31}$. We repeat the same reduction algorithm with $M := 4 \times 10^{31}$ and as a result we obtain that $k < 440$, a contradiction. This completes the proof.

5. An Application of *k*-Generalized Tiny Golden Angles to MR Imaging

Studying the Fibonacci sequence and its properties has been an interesting point of research for many years. Indeed, the Fibonacci sequence which is associated with the golden ratio exists naturally in biological settings. This sequence appears in tree's branches, phyllotaxis, flowers, and the human body. Therefore, it has applications in the growth of living things [27]. Moreover, recent applications were introduced in several areas of research including healthcare and medical fields.

In [28], Jiancheng Zou et al. introduced a novel family of image scrambling transforms, which can be applied in medical imaging, based on the distinguished generalized Fibonacci sequence, and the experiments showed that the proposed methods have many advantages.

Carlos Davrieux and Juan Davrieux associated the anatomical distribution of the human biliary tree with the Fibonacci sequence. Furthermore, they carried out a bibliographic analysis of the relation of this sequence to medicine [29].

In [30], the multidimensional golden means were derived from modified Fibonacci sequences and used to introduce a tool that is useful for 3D adaptive imaging which leads to improve specificity in breast MRI. During the year 2021, a new diagnostic technique for breast cancer detection was introduced by applying Fibonacci sequence, golden ratio and predictive algorithm to mammography and ultrasonography [31].

In [18], the authors introduced a new sequence of angles (tiny golden angels) which is based on a generalized Fibonacci sequence [32]. They showed that the tiny golden angles exhibit properties that are very similar to the original golden angle, and the advantages of the new angles for MRI in combination with fully balanced steady-state free precession sequences. These were applied for dynamic imaging of the temporomandibular joint and the heart. In 2021, Alexander Fyrdahl et al. proposed a novel generalization which allows for whole-heart volumetric imaging with retrospective binning and reduced eddy current artifacts. They showed that the tiny golden angle scheme was successful in reducing the angular step in cardio-respiratory-binned golden-angle imaging [33]. In what follows, by using the roots of characteristic polynomial of k-generalized Fibonacci sequences, we give a generalization of the notion of tiny golden angle.

Let $\phi = \dfrac{1+\sqrt{5}}{2}$ be the golden ratio. The golden angle is defined as the angle that is resulted from dividing the semicircle by the golden ratio, that is the angle $\psi_{\text{gold}} = \pi/\phi$. In [18], a new sequence of angles are constructed by the relation

$$\frac{\psi_N}{\pi - N\psi_N} = \phi.$$

Solving the above equation for ψ_N leads to the sequence of angles

$$\psi_N = \frac{\pi}{\phi + N - 1}.$$

For $N = 1$ and $N = 2$, these angles are golden angle and complementary small golden angles as $\psi_1 = \pi/\phi$ and $\psi_2 = \dfrac{\pi}{\phi+1} = \pi - \psi_1$. The tiny golden angles are defined to be the angles ψ_N, for $N > 2$. In [18], the advantages of using tiny golden angles instead of using the usual golden angle are examined by giving many experimental data including the real-time cardiac imaging ([18] Figure 7). In this paper, we define the *k-generalized tiny golden angles* $\psi_N^{(k)}$ as follows

$$\frac{\psi_N^{(k)}}{\pi - N\psi_N^{(k)}} = \alpha(k), \quad k \geq 2,$$

where $\alpha(k)$ is the unique root of the characteristic polynomial of $F_n^{(k)}$ which is placed outside the unit circle. Solving this equation for $\psi_N^{(k)}$, and using the fact that $\dfrac{1}{\alpha(k)} = \alpha(k)^{k-1} - \sum_{i=0}^{k-2} \alpha(k)^i$, we find that

$$\psi_N^{(k)} = \frac{\pi}{\alpha(k)^{k-1} - \sum_{i=0}^{k-2} \alpha(k)^i + N}.$$

If $k = 2$, then $\psi_N^{(2)}$ is just tiny golden angles ψ_N. Thus, we call all $\psi_N^{(k)}$ for $N > 2$ *k-generalized tiny golden angles*. In Table 1, we give some numerical values of $\psi_N^{(k)}$ for some distinct values of k to compare the results with tiny golden angles when $k = 2$. Table 1 shows that the values of tiny golden angles and k-generalized tiny golden angles are very close.

Thus, we believe that, because of this correlation, a more detailed study with experimental data will reveal the practical efficiency of this k-generalized tiny golden angles.

Table 1. The First Ten Elements of the Sequence $\psi_N^{(k)}$ for $k = 2, 3, 4, 7$ and 10 as degree.

N	$\psi_N^{(2)}$	$\psi_N^{(3)}$	$\psi_N^{(4)}$	$\psi_N^{(7)}$	$\psi_N^{(10)}$
1	111.24611...°	116.60379...°	118.51539...°	119.83884...°	119.98031...°
2	68.75388...°	70.76336...°	71.46288...°	71.94195...°	71.99291...°
3	49.75077...°	50.79452...°	51.15394...°	51.39894...°	51.42495...°
4	38.97762...°	39.61538...°	39.83367...°	39.98207...°	39.99781...°
5	32.03967...°	32.46935...°	32.61584...°	32.71527...°	32.72580...°
6	27.19840...°	27.50741...°	27.61248...°	27.68371...°	27.69125...°
7	23.62814...°	23.86100...°	23.94002...°	23.99354...°	23.99921...°
8	20.88643...°	21.06818...°	21.12976...°	21.17144...°	21.17585...°
9	18.71484...°	18.86063...°	18.90996...°	18.94334...°	18.94687...°
10	16.95229...°	17.07182...°	17.11223...°	17.13956...°	17.14245...°

6. Discussion

It is known that the largest repdigit in the Fibonacci sequence is 55 [34]. When we look at the subsequent terms of this sequence, one can see that the consecutive three terms $F_{12} = 144$, $F_{13} = 233$ and $F_{14} = 377$ of this sequence have the property that all digits are equal except for at most one digit, which we have called almost repdigits. Thus, it is natural to ask whether there are any other almost repdigits in the Fibonacci sequence? In this paper, we give an answer to this question not only for classical Fibonacci numbers but also for the order $k \geq 2$ generalization of this sequence. In particular, we show that $F_{13}^{(7)} = 2000$ is the largest almost repdigit in the k-Fibonacci sequences.

At the end of the paper, we also open the door for an application of k-generalized Fibonacci sequences for interested readers.

7. Recommendations

Recently, specific Fibonacci numbers with some special properties were calculated. Among the most popular numbers were Fibonacci numbers which were concatenations of two or three repdigits. These calculations and more were also performed on generalized Fibonacci sequence and other sequences [1,2,5,12,35,36]. In our paper, we defined almost repdigit Fibonacci numbers and found them in the generalized case. Since repdigit and almost repdigit numbers seem special, we recommend researchers who are interested in applications of Fibonacci numbers to take a closer look at these specific numbers and consider them in their studies.

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Appendix A

Table A1. The First 15 Elements of the Sequence $F_n^{(k)}$: for $2 \leq k \leq 9$ and $1 \leq n \leq 15$. The circled numbers are all almost repdigits given in Theorem 1.

k = 2	$F_n^{(2)}$:	1,	1,	2,	3,	5,	8,	13,	21,	34,	55,	89,	(144),	(233),	(377),	610
k = 3	$F_n^{(3)}$:	1,	1,	2,	4,	7,	13,	24,	44,	81,	149,	274,	504,	927,	1705,	3136
k = 4	$F_n^{(4)}$:	1,	1,	2,	4,	8,	15,	29,	56,	108,	208,	401,	(773),	1490,	2872,	5536
k = 5	$F_n^{(5)}$:	1,	1,	2,	4,	8,	16,	31,	61,	120,	236,	(464),	912,	1793,	3525,	6930
k = 6	$F_n^{(6)}$:	1,	1,	2,	4,	8,	16,	32,	63,	125,	248,	492,	976,	1936,	3840,	7617
k = 7	$F_n^{(7)}$:	1,	1,	2,	4,	8,	16,	32,	64,	127,	253,	504,	1004,	(2000),	3984,	7936
k = 8	$F_n^{(8)}$:	1,	1,	2,	4,	8,	16,	32,	64,	128,	(255),	509,	1016,	2028,	4048,	8080
k = 9	$F_n^{(9)}$:	1,	1,	2,	4,	8,	16,	32,	64,	128,	256,	(511),	1021,	2040,	4076,	8144

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