

Article

New Modified Couple Stress Theory of Thermoelasticity with Hyperbolic Two Temperature

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Abstract: This paper deals with the two-dimensional deformation in fibre-reinforced composites with new modified couple stress thermoelastic theory (nMCST) due to concentrated inclined load. Lord Shulman heat conduction equation with hyperbolic two temperature (H2T) has been used to form the mathematical model. Fourier and Laplace transform are used for obtaining the physical quantities of the mathematical model. The expressions for displacement components, thermodynamic temperature, conductive temperature, axial stress, tangential stress and couple stress are obtained in the transformed domain. A mathematical inversion procedure has been used to obtain the inversion of the integral transforms using MATLAB software. The effects of hyperbolic and classical two temperature are shown realistically on the various physical quantities.

Keywords: length scale parameters; fibre-reinforced composites; hyperbolic two-temperature; inclined load; new modified couple stress theory

MSC: 4F05; 44A10; 74F20; 74H15; 74S40



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1. Introduction

Low weight and high strength make fibre-reinforced composites an ideal material for many structures. The mechanical properties of such materials can be explained using a continuum model. Manmade fibre-reinforced composites are proving to be particularly effective in this regard. The materials (natural and synthetic) in which the properties like thermal conductivity vary with orientation are called anisotropic materials. For example, crystals, sedimentary rocks, metals with heavy cold pressing, laminated sheets and heat shielding materials used for space vehicles, wood, cables, fibre-reinforced composite structures, etc. are anisotropic materials. By introducing strong reinforcement fibers in one or more particular directions into a relatively weak, isotropic matrix, a matrix with certain desirable properties is strengthened throughout. An elastic solid reinforced with parallel fibers is usually assumed to be transversely isotropic. Additionally, the response of these composites is typically highly anisotropic, therefore isotropic theory would not be able to accurately predict their behavior under the majority of loading scenarios. For linear constitutive relations, there are five material constants relating to infinitesimal components of stress and strain. For the last three decades, solid mechanics has been studying stress and deformation in fibre-reinforced composite materials.

A revolutionary work was done on fibre-reinforced composite materials by Pipkin and Rogers [1]. Spencer [2] described the deformations of fibre-reinforced materials. Pipkin [3] studied the ideal fiber-reinforced materials that exhibit generalized plane deformations. Testa and Bolay [4] discussed the two key issues with fiber-reinforced materials' thermoelastic behavior. The maximum stress in each phase of the composite as well as boundaries

for the stress at every point in a beam are created in the first. The second issue takes into account the buckling of reinforcing fibres when subjected to thermal loading. The deformations of an idealized incompressible fiber-reinforced material in the finite plane are obtained by Craig and Hart [5]. A revolutionary work was done on fibre-reinforced composite materials by Rogers [6]. Bayones and Hussein [7] studied an anisotropic semi-infinite solid media with fiber-reinforced generalized thermoplastic with respect to thermal relaxation time and gravity effects. Said [8] investigated the effect of hydrostatic initial stress and the gravity field on a fibre-reinforced thermoelastic medium due to its own heat and constant motion using the three-phase-lag model (TPL) and GN theory of thermoelasticity of Type-II.

A macro-scale analysis of materials can be made with the help of classical continuum mechanics theory, which ignores the microstructure size-dependency. For a more complete continuum theory, new deformation measures are needed. This implies that couple stresses must also be introduced into such a theory. Firstly, in 1887, Voigt [9] proposed the asymmetric theory of elasticity, and afterward in 1909, a couple stress theory was presented by Cosserat and Cosserat [10], but examiners deemed the theory not important due to its failure to establish the constitutive interactions. A standard couple stress theory with constitutive relations $t_{ij} = \lambda e_{kk} \delta_{ij} + 2G e_{ij}$, $m_{ij} = 4l^2 G \chi_{ij}$ was introduced by Mindlin [11] for isotropic elastic materials. In this theory, Mindlin [11] used a single length scale parameter (LSP). Moreover, the strain tensor in Mindlin's theory is represented as $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$, and curvature tensors is $\chi_{ij} = \omega_{i,j}$, where the rotation vector is given by $\omega_i = \frac{1}{2} \epsilon_{ijk} u_{k,j}$. Due to the asymmetric nature of the couple and curvature stress tensors, this theory is also designated as asymmetric couple stress theory. In addition, Koiter [12] introduced constitutive relationships for anisotropic materials based on Cosserat couple stress theory as $t_{ij} = c_{ijkl} e_{kl}$; $m_{ij} = l_i^2 G_i \chi_{ij}$, where l_i ($i = 1, 2, 3$) is LSP. Here, t_{ij} is a symmetric tensor, and m_{ij} is an asymmetric tensor. Yang et al. [13] presented a modified couple stress theory (MCST) for isotropic elastic materials with an additional LSP as $t_{ij} = \lambda e_{kk} \delta_{ij} + 2G e_{ij}$, $m_{ij} = 4l^2 G \chi_{ij}$ and $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$, $\chi_{ij} = \frac{1}{2}(\omega_{i,j} + \omega_{j,i})$, and all the tensors are symmetric. MCST should be extended from isotropic to anisotropic media due to the importance of anisotropic mediums in microstructures. Hence, Chen et al. [14] proposed an nMCST that considers three length-scale parameters for anisotropic materials as

$$\begin{aligned} t_{ij} &= c_{ijkl} e_{kl}, \\ m_{ij} &= l_i^2 G_i \chi_{ij} + l_j^2 G_j \chi_{ji}, \\ \chi_{ij} &= \omega_{i,j}, \\ \omega_i &= \frac{1}{2} \epsilon_{ijk} u_{k,j}, \\ e_{ij} &= \frac{1}{2} (u_{i,j} + u_{j,i}). \end{aligned}$$

Here, the χ_{ij} is asymmetric, but the m_{ij} , t_{ij} , e_{ij} are symmetric tensors.

Chen and Gurtin [4] and Chen et al. [5,6] gave the two-temperature theory of thermoelasticity of deformable bodies for heat conduction, subject to two kinds of temperatures: The thermodynamic temperature T and the conductive temperature φ . According to this theory, we have:

$$T = \varphi - b_{ij} \varphi_{,ij}.$$

For the time-independent cases, the difference between T and φ is proportional to the supply of heat, and in the absence of heat supply, these temperatures are equal. For time dependent problems, the difference in two temperatures is non zero and does not depend on heat supply. When the two-temperature factor is zero, i.e., $\varphi = T$ and then the coupled thermoelasticity can be derived from the two-temperature theory of thermoelasticity. Youssef [15] proved uniqueness theorem for a homogeneous and isotropic media with two

temperature generalized thermoelasticity. Youssef [16] proposed a two-temperature model without energy dissipation for an elastic half-space subjected to moving heat source and Youssef [17] constructed a fractional order generalized theory of thermoelasticity based on Duhamel-Neumann stress–strain relation in the context of one temperature and two-temperature. The classical two temperature theory of thermoelasticity, however, does not introduce finite speed of the thermal wave propagation, which is physically unacceptable. Youssef and El Bary [18] improved two temperature theory of thermoelasticity and introduced hyperbolic two temperatures (H2T) theory of thermoelasticity for an isotropic body and proved uniqueness and derived its variational principle and showed finite speed of thermal and mechanical wave propagation. According to this theory, we have:

$$\ddot{\varphi} - \ddot{T} = b_{ij}\varphi_{,ij}$$

Regardless of this, numerous investigators, such as Kaur et al. [19,20], Craciun et al. [21], Khan et al. [22], Marin and Öchsner [23], Singh et al. [24], Zhang et al. [25], Singh et al. [26], Marin et al. [27], Bhatti et al. [28–30], Golewski [31–33], and Kaur et al. [34,35] worked on various thermoelastic problems. However, until now, no efforts have been made to study the fiber-reinforced thermoelastic medium using nMCST for H2T.

In the present investigation, the two-dimensional deformation in fibre-reinforced composites with nMCST and H2T due to concentrated inclined load and L-S thermoelasticity has been investigated. Laplace and Fourier transform methods are used for obtaining the solutions of the mathematical model. The expressions for displacement components, thermodynamic temperature, conductive temperature, axial stress, tangential stress and couple stress are obtained in the transformed domain. The effects of hyperbolic and classical 2T are illustrated graphically on the various physical quantities.

2. Basic Equations

Following Chen and Li [14], Kumar et al. [36] and Youssef and El Bary [18], the field equations for fibre-reinforced thermoelastic medium with respect to a preferred direction $a_i = (a_1, a_2, a_3)$, where $a_1^2 + a_2^2 + a_3^2 = 1$, with nMCST and L-S theory of Thermoelasticity with H2T are given by the following equations:

1. Constitutive Relations

$$t_{ij} = c_{ijkl}e_{kl} - \gamma_{ij}T, \quad (1)$$

For fibre reinforced material with respect to a preferred direction $a_i = (a_1, a_2, a_3)$, where $a_1^2 + a_2^2 + a_3^2 = 1$, we have $c_{ijkl}e_{kl}$ in Equation (1), as mentioned below:

$$\begin{aligned} c_{ijkl}e_{kl} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) \\ &\quad + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j \\ m_{ij} &= l_i^2 G_i \chi_{ij} + l_j^2 G_j \chi_{ji}, \end{aligned} \quad (2)$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad (3)$$

$$\chi_{ij} = \omega_{i,j}, \quad (4)$$

$$\omega_i = \frac{1}{2}\epsilon_{ijk}u_{k,j}. \quad (5)$$

$$\ddot{\varphi} - \ddot{T} = b_{ij}\varphi_{,ij}, \quad (6)$$

$$\gamma_{ij} = c_{ijkl}\alpha_{kl}, \quad (7)$$

2. Equation of Motion

$$\begin{aligned} & \left(\lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha (a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T) (a_i a_k e_{kj} + a_j a_k e_{ki}) + \beta a_k a_m e_{km} a_i a_j \right)_{,j} \\ & + \frac{1}{4} \left(l_k^2 G_k (\delta_{im} \delta_{jn} - \delta_{in} \delta_{jm}) u_{n,mllj} + l_l^2 G_l \epsilon_{ijk} u_{n,mklj} \right) - \gamma_{ij} T_{,j} = \rho \ddot{u}_i, \end{aligned} \quad (8)$$

3. Heat Conduction Equation Proposed by Youssef [15]

$$K_{ij} \varphi_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) (\rho C_E T + \gamma_{ij} T_0 e_{ij}), \quad (9)$$

3. Formulation and Solution of the Problem

Consider a 2-D homogeneous fiber-reinforced thermoelastic medium (Figure 1) in the half-space $x_3 \geq 0$, primarily at a uniform temp T_0 . We choose the fiber reinforcement direction as $\vec{a} = (0, 0, 1)$ i.e., along z -axis. The rectangular coordinate system (x_1, x_2, x_3) with the origin on the surface $x_2 = 0$ is considered.

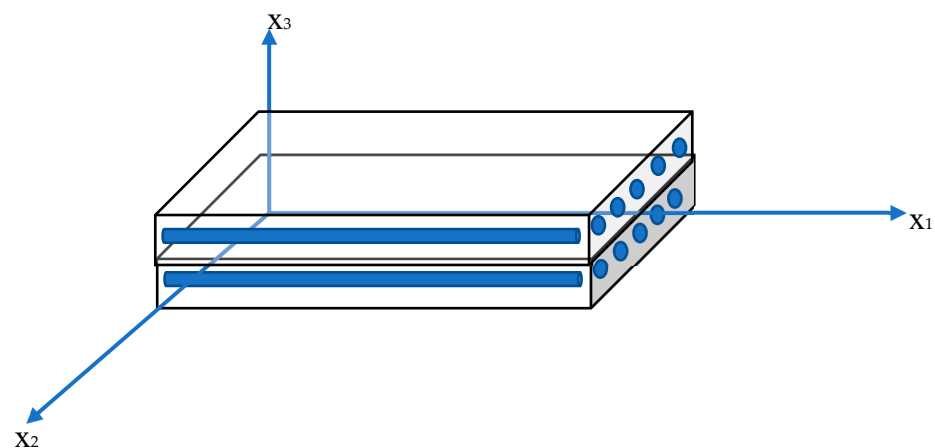


Figure 1. Fiber-reinforced thermoelastic medium.

Consider plane strain problem parallel to $x_1 x_3$ -plane with displacement vector $u = (u_1, 0, u_3)(x_1, 0, x_3)$. The field equations become.

$$\begin{aligned} & \left(\lambda + 2\mu_T \right) \frac{\partial^2 u_1}{\partial x_1^2} + \left(\mu_L - \frac{1}{4} l_2^2 G_2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) \right) \frac{\partial^2 u_1}{\partial x_3^2} \\ & + \left(\lambda + \alpha + \mu_L + \frac{1}{4} l_2^2 G_2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) \right) \frac{\partial^2 u_3}{\partial x_1 \partial x_3} - \gamma_{13} \frac{\partial T}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \end{aligned} \quad (10)$$

$$\begin{aligned} & \left(\lambda + 4\mu_L - 2\mu_T + 2\alpha + \beta \right) \frac{\partial^2 u_3}{\partial x_3^2} + \left(\mu_L + \lambda + \alpha + \frac{1}{4} l_2^2 G_2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) \right) \frac{\partial^2 u_1}{\partial x_1 \partial x_3} \\ & + \left(\mu_L - \frac{1}{4} l_2^2 G_2 \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2} \right) \right) \frac{\partial^2 u_3}{\partial x_1^2} - \gamma_3 \frac{\partial T}{\partial x_3} = \rho \frac{\partial^2 u_3}{\partial t^2}, \end{aligned} \quad (11)$$

$$K_1 \frac{\partial^2 \varphi}{\partial x_1^2} + K_3 \frac{\partial^2 \varphi}{\partial x_3^2} - \rho C_E \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T = T_0 \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left(\gamma_1 \frac{\partial u_1}{\partial x_1} + \gamma_3 \frac{\partial u_3}{\partial x_3} \right), \quad (12)$$

$$\frac{\partial^2 T}{\partial t^2} = \frac{\partial^2 \varphi}{\partial t^2} - b_1 \frac{\partial^2 \varphi}{\partial x_1^2} - b_3 \frac{\partial^2 \varphi}{\partial x_3^2}. \quad (13)$$

where $c_{11} = c_{22} = (\lambda + 2\mu_T)$, $c_{23} = c_{13} = (\lambda + \alpha)$, $c_{33} = (\lambda + 4\mu_L - 2\mu_T + 2\alpha + \beta)$, $c_{66} = \frac{c_{11} - c_{12}}{2} = \mu_T$, $c_{55} = c_{44} = \mu_L$, $c_{12} = \lambda$, we have used the notations $11 \rightarrow 1, 22 \rightarrow 2, 33 \rightarrow 3, 23 \rightarrow 4, 13 \rightarrow 5, 12 \rightarrow 6$ for the material constants.

$\gamma_1 = (\lambda + 2\mu_T + \lambda)\alpha_1 + (\lambda + \alpha)\alpha_3$, $\gamma_3 = 2(\lambda + \alpha)\alpha_1 + (\lambda + 4\mu_L - 2\mu_T + 2\alpha + \beta)\alpha_3$. Additionally, the stress components and couple stress are

$$t_{33} = (\lambda + \alpha)\frac{\partial u_1}{\partial x_1} + (\lambda + 4\mu_L - 2\mu_T + 2\alpha + \beta)\frac{\partial u_3}{\partial x_3} - \gamma_3 T, \quad (14)$$

$$t_{31} = \mu_L \left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1} \right), \quad (15)$$

$$m_{32} = m_{23} = \frac{1}{2} l_2^2 G_2 \left(\frac{\partial^2 u_1}{\partial x_3^2} - \frac{\partial^2 u_3}{\partial x_1 \partial x_3} \right), \quad (16)$$

$$m_{12} = m_{21} = \frac{1}{2} l_2^2 G_2 \left(\frac{\partial^2 u_1}{\partial x_1 \partial x_3} - \frac{\partial^2 u_3}{\partial x_1^2} \right), \quad (17)$$

$$m_{11} = m_{13} = m_{22} = m_{31} = m_{33} = 0 \quad (18)$$

The preliminary and regularity conditions are given by

$$u_1(x_1, x_3, 0) = 0 = \dot{u}_1(x_1, x_3, 0) \quad (19)$$

$$u_3(x_1, x_3, 0) = 0 = \dot{u}_3(x_1, x_3, 0) \quad (20)$$

$$T(x_1, x_3, 0) = 0 = \dot{T}(x_1, x_3, 0) \quad (21)$$

$$\varphi(x_1, x_3, 0) = 0 = \dot{\varphi}(x_1, x_3, 0) \text{ for } x_3 \geq 0, -\infty < x_1 < \infty \quad (22)$$

$$u_1(x_1, x_3, t) = u_3(x_1, x_3, t) = T(x_1, x_3, 0) = \varphi(x_1, x_3, 0) = 0 \text{ for } t > 0 \text{ when } x_3 \rightarrow \infty \quad (23)$$

Dimensionless quantities are given by

$$x'_1 = \frac{x_1}{L}, x'_3 = \frac{x_3}{L}, u'_1 = \frac{\rho c_1^2}{L\gamma_1 T_0} u_1, u'_3 = \frac{\rho c_1^2}{L\gamma_1 T_0} u_3, T' = \frac{T}{T_0}, \varphi' = \frac{\varphi}{T_0}, t' = \frac{c_1}{L} t, t'_{11} = \frac{t_{11}}{\gamma_1 T_0}, t'_{33} = \frac{t_{33}}{\gamma_1 T_0}, m'_{32} = \frac{m_{32}}{L\gamma_1 T_0}, l'_i = \frac{l_i}{L}, G'_i = \frac{G_i}{\rho c_1^2}, c_1^2 = \frac{(\lambda + 2\mu_T)}{\rho} \quad (24)$$

The Laplace transforms is defined by

$$\mathcal{L}[f(x_1, x_3, t)] = \int_0^\infty e^{-st} f(x_1, x_3, t) dt = \tilde{f}(x_1, x_3, s) \quad (25)$$

with basic properties

$$L\left(\frac{\partial f}{\partial t}\right) = s\tilde{f}(x_1, x_3, s) - f(x_1, x_3, 0), \quad (26)$$

$$L\left(\frac{\partial^2 f}{\partial t^2}\right) = s^2\tilde{f}(x_1, x_3, s) - sf(x_1, x_3, 0) - \left(\frac{\partial f}{\partial t}\right)_{t=0}. \quad (27)$$

Fourier transforms of a function $\tilde{f}$ w.r.t. variable x with ξ as a Fourier variable is

$$\hat{f}(\xi, x_3, s) = \int_{-\infty}^\infty \tilde{f}(x_1, x_3, s) e^{i\xi x_1} dx_1. \quad (28)$$

Such that if $f(x_1) \rightarrow 0$ and $f^{n-1}(x_1) \rightarrow 0$, as $x_1 \rightarrow \pm\infty$, then

$$\mathcal{F}\left\{\frac{\partial^n f(x_1, x_3, s)}{\partial x_1^n}\right\} = (i\xi)^n \hat{f}(\xi, x_3, s). \quad (29)$$

Using the dimensionless quantities defined by (17) into Equations (10)–(16) and suppressing the primes, the Equations (9)–(14) and then applying Laplace and Fourier transform defined by (25) and (28) becomes

$$(\zeta_1 + \zeta_2 D^2 + \zeta_3 D^4) \hat{u} + (\zeta_4 D + \zeta_5 D^3) \hat{w} + \zeta_6 \hat{T} = 0, \quad (30)$$

$$(\zeta_4 D + \zeta_5 D^3) \hat{u} + (\zeta_7 + \zeta_8 D^2) \hat{w} + \zeta_9 D \hat{T} = 0, \quad (31)$$

$$\zeta_{10} \hat{u} + \zeta_{11} D \hat{w} + \zeta_{12} \hat{T} + (\zeta_{13} + \delta_3 D^2) \hat{\phi} = 0, \quad (32)$$

$$(\zeta_{14} - \zeta_{15} D^2) \hat{\phi} + \zeta_{16} \hat{T} = 0, \quad (33)$$

$$\hat{t}_{33} = \zeta_{17} \hat{u} + \delta_4 D \hat{w} + \zeta_9 \hat{T}, \quad (34)$$

$$\hat{t}_{31} = \delta_1 D \hat{u} + \zeta_{18} \hat{w}, \quad (35)$$

$$m_{32} = -2(\zeta_3 D^2 \hat{u} + \zeta_5 \hat{w}). \quad (36)$$

where

$$\begin{aligned} \delta_1 &= \frac{\mu_L}{\rho c_1^2}, \delta_2 = \frac{(\lambda + \alpha + \mu_L)}{\rho c_1^2}, \delta_4 = \frac{(\lambda + 4\mu_L - 2\mu_T + 2\alpha + \beta)}{\rho c_1^2}, \delta_3 = \frac{-K_3}{K_1}, \delta_5 = \frac{\rho C_E C_1 L}{K_1} \left(s + \frac{\tau_0 C_1 s^2}{L} \right), \\ \delta_6 &= \frac{L T_0 \gamma_1}{\rho C_1 K_1} \left(s + \frac{\tau_0 C_1 s^2}{L} \right), \delta_8 = \frac{(\lambda + \alpha)}{\rho c_1^2}, \zeta_1 = -\zeta^2 - s^2, \zeta_2 = \delta_1 + \frac{1}{4} l_2^2 G_2 \zeta^2, \\ \zeta_3 &= -\frac{1}{4} (l_2^2 G_2), \zeta_4 = \left(\delta_2 - \frac{1}{4} l_2^2 G_2 \zeta^2 \right) i \zeta, \zeta_5 = \frac{1}{4} (l_2^2 G_2) i \zeta, \zeta_6 = -i \zeta \\ \zeta_7 &= -s^2 + \delta_1 \zeta^2 - \frac{1}{4} (l_2^2 G_2) \zeta^4, \zeta_8 = \delta_4 + \frac{1}{4} (l_2^2 G_2) \zeta^2, \zeta_9 = -\frac{\gamma_3}{\gamma_1}, \zeta_{10} = \delta_6 \gamma_1 i \zeta, \zeta_{11} = \delta_6 \gamma_3 \\ \zeta_{12} &= \delta_5, \zeta_{13} = \zeta^2, \zeta_{14} = s^2 + \frac{b_1}{c_1^2} \zeta^2, \zeta_{15} = -\frac{b_3}{c_1^2}, \zeta_{16} = -s^2, \zeta_{17} = \delta_8 i \zeta, \zeta_{18} = \delta_1 i \zeta, D = \frac{d}{dx_3}. \end{aligned} \quad (37)$$

For the non-trivial solution of Equations (30)–(33) the determinant of the coefficients $(\hat{u}_1, \hat{u}_3, \hat{T}, \hat{\phi})$ must vanish, which thus yields

$$AD^8 + BD^6 + CD^4 + ED^2 + F)(\hat{u}_1, \hat{u}_3, \hat{T}, \hat{\phi}) = 0, \quad (38)$$

where

$$\begin{aligned} A &= -\zeta_3 \zeta_8 \zeta_{12} \zeta_{15} + \zeta_5 (\zeta_5 \zeta_{12} \zeta_{15} - \zeta_5 \zeta_{16} \delta_3), \\ B &= -\zeta_2 \zeta_8 \zeta_{12} \zeta_{15} + \zeta_3 (-\zeta_7 \zeta_{12} \zeta_{15} + \zeta_8 \zeta_{12} \zeta_{14} - \zeta_{16} \delta_3) + \zeta_4 (\zeta_5 \zeta_{12} \zeta_{15} + \zeta_5 \zeta_{16} \delta_3) \\ &\quad + \zeta_5 (\zeta_5 \zeta_{12} \zeta_{14} - \zeta_4 \zeta_{12} \zeta_{15} - \zeta_4 \zeta_{16} \delta_3 - \zeta_5 \zeta_{16} \zeta_{13} - \zeta_9 \zeta_{10} \zeta_{15}) - \zeta_6 \zeta_{11} \zeta_5 \zeta_{15}, \\ C &= \zeta_3 (\zeta_7 \zeta_{12} \zeta_{14} - \zeta_{16} \zeta_{13}) + \zeta_2 (-\zeta_7 \zeta_{12} \zeta_{15} + \zeta_8 \zeta_{12} \zeta_{14} - \zeta_{16} \delta_3) - \zeta_1 \zeta_{12} \zeta_8 \zeta_{15} \\ &\quad + \zeta_5 (\zeta_4 \zeta_{12} \zeta_{14} - \zeta_4 \zeta_{16} \zeta_{13} - \zeta_9 \zeta_{10} \zeta_{14}) \\ &\quad - \zeta_4 (\zeta_5 \zeta_{12} \zeta_{14} - \zeta_4 \zeta_{12} \zeta_{15} - \zeta_4 \zeta_{16} \delta_3 - \zeta_5 \zeta_{16} \zeta_{13} - \zeta_9 \zeta_{10} \zeta_{15}) + \zeta_6 \zeta_{11} (-\zeta_4 \zeta_{15} + \zeta_5 \zeta_{14}), \\ E &= \zeta_2 (\zeta_7 \zeta_{12} \zeta_{14} - \zeta_{16} \zeta_{13}) + \zeta_1 (-\zeta_7 \zeta_{12} \zeta_{15} + \zeta_8 \zeta_{12} \zeta_{14} - \zeta_{16} \delta_3) - \zeta_4 (\zeta_4 \zeta_{12} \zeta_{14} - \zeta_4 \zeta_{16} \zeta_{13} - \zeta_9 \zeta_{10} \zeta_{14}) + \zeta_6 \zeta_{11} \zeta_4 \zeta_{14}, \\ F &= \zeta_1 (\zeta_7 \zeta_{12} \zeta_{14} - \zeta_{16} \zeta_{13}). \end{aligned}$$

Consider that the roots of the Equation (38) are $\pm \lambda_i (i = 1, 2, 3, 4)$, and hence using the radiation conditions that $\hat{u}_1, \hat{u}_3, \hat{\phi}, \hat{T} \rightarrow 0$ as $x_3 \rightarrow \infty$ the soln. of Equations (30)–(33) are

$$\hat{u}_1 = A_1 e^{-\lambda_1 x_3} + A_2 e^{-\lambda_2 x_3} + A_3 e^{-\lambda_3 x_3} + A_4 e^{-\lambda_4 x_3}, \quad (39)$$

$$\hat{u}_3 = d_1 A_1 e^{-\lambda_1 x_3} + d_2 A_2 e^{-\lambda_2 x_3} + d_3 A_3 e^{-\lambda_3 x_3} + d_4 A_4 e^{-\lambda_4 x_3}, \quad (40)$$

$$\hat{T} = g_1 A_1 e^{-\lambda_1 x_3} + g_2 A_2 e^{-\lambda_2 x_3} + g_3 A_3 e^{-\lambda_3 x_3} + g_4 A_4 e^{-\lambda_4 x_3}, \quad (41)$$

$$\hat{\phi} = h_1 A_1 e^{-\lambda_1 x_3} + h_2 A_2 e^{-\lambda_2 x_3} + h_3 A_3 e^{-\lambda_3 x_3} + h_4 A_4 e^{-\lambda_4 x_3}. \quad (42)$$

where

$$d_j = \frac{\lambda_j[\zeta_4(\zeta_{12}\zeta_{14} - \zeta_{16}\zeta_{13}) - \zeta_9\zeta_{10}\zeta_{14}] + \lambda_j^3[\zeta_5(\zeta_{12}\zeta_{14} - \zeta_{16}\zeta_{13}) + \zeta_9\zeta_{10}\zeta_{15}] - \zeta_5(-\zeta_{12}\zeta_{15} - \zeta_{16}\delta_3)\lambda_j^5}{\zeta_7\zeta_{12}\zeta_{14} + \lambda_j^2[\zeta_8(\zeta_{12}\zeta_{14} - \zeta_{16}\zeta_{13}) - \zeta_9\zeta_{11}\zeta_{14}] + \lambda_j^4[\zeta_8(-\zeta_{12}\zeta_{15} - \zeta_{16}\delta_3) + \zeta_9\zeta_{11}\zeta_{15}] + \zeta_8\zeta_{11}\zeta_9\zeta_{15}\lambda_j^6},$$

$$g_j = \frac{-\zeta_7\zeta_{10}\zeta_{14} + \lambda_j^2[\zeta_4\zeta_{11}\zeta_{14} - \zeta_8\zeta_{10}\zeta_{14} - \zeta_7\zeta_{10}\zeta_{15}] + \lambda_j^4[\zeta_5\zeta_{11}\zeta_{14} - \zeta_4\zeta_{11}\zeta_{15} + \zeta_8\zeta_{10}\zeta_{15}] - \zeta_{11}\zeta_5\zeta_{15}\lambda_j^6}{\zeta_7\zeta_{12}\zeta_{14} + \lambda_j^2[\zeta_8(\zeta_{12}\zeta_{14} - \zeta_{16}\zeta_{13}) - \zeta_9\zeta_{11}\zeta_{14}] + \lambda_j^4[\zeta_8(-\zeta_{12}\zeta_{15} - \zeta_{16}\delta_3) + \zeta_9\zeta_{11}\zeta_{15}] + \zeta_8\zeta_{11}\zeta_9\zeta_{15}\lambda_j^6},$$

$$h_j = \frac{-\zeta_7\zeta_{10}\zeta_{16} + \lambda_j^2[\zeta_4\zeta_{11}\zeta_{16} - \zeta_8\zeta_{10}\zeta_{16}] + \lambda_j^4\zeta_{11}\zeta_5\zeta_{11}}{\zeta_7\zeta_{12}\zeta_{14} + \lambda_j^2[\zeta_8(\zeta_{12}\zeta_{14} - \zeta_{16}\zeta_{13}) - \zeta_9\zeta_{11}\zeta_{14}] + \lambda_j^4[\zeta_8(-\zeta_{12}\zeta_{15} - \zeta_{16}\delta_3) + \zeta_9\zeta_{11}\zeta_{15}] + \zeta_8\zeta_{11}\zeta_9\zeta_{15}\lambda_j^6}.$$

4. Boundary Conditions

Let us consider that along +ve x_3 -axis a normal point load F_1 per unit length acts at the origin and along +ve x_1 -axis a tangential load F_2 per unit length, acts at the origin and due to concentrated normal force on the half space, the dimensionless conditions are given by

i. Normal stress

$$t_{33}(x_1, x_3, t) = -F_1\delta(x_1)H(t) \quad (43)$$

ii. Tangential stress

$$t_{31}(x_1, x_3, t) = -F_2\delta(x_1)H(t) \quad (44)$$

iii. Vanishing of tangential couple stress

$$m_{32}(x_1, x_3, t) = 0 \quad (45)$$

iv. Condition of temperature change

$$\frac{\partial \varphi}{\partial x_3}(x_1, x_3, t) = 0 \quad (46)$$

where F_1 and F_2 are the vertical and horizontal magnitude of the forces applied, $\delta(x)$ specify the concentrated load distribution function along the x -axis, and $H()$ is given by

$$H(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0 \end{cases} \quad (47)$$

$$H(s) = L(H(t)) = \frac{1}{s}. \quad (48)$$

As an application along the positive boundary x_3 -axis, a normal point load F_1 is applied at the origin and along the positive x_1 -axis a load F_2 per unit length and are applied at the origin. Assume an inclined load, F_0 per unit length inclined at an angle θ with x_3 -axis is acting on the x_2 -axis (see Figure 2):

$$F_1 = F_0 \cos \theta \text{ and } F_2 = F_0 \sin \theta. \quad (49)$$

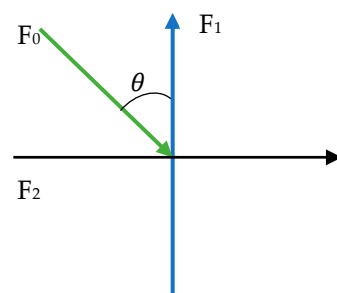


Figure 2. Inclined load over a fiber-reinforced thermoelastic solid.

Using (34)–(36) and (42) in Equations (43)–(46) and then applying the transforms (25) and (28) and hence these equations can be written as

$$\sum_{j=1}^4 M_j A_j = -\frac{F_1}{s}, \quad (50)$$

$$\sum_{j=1}^4 N_j A_j = -\frac{F_2}{s}, \quad (51)$$

$$\sum_{j=1}^4 O_j A_j = 0, \quad (52)$$

$$\sum_{j=1}^4 P_j A_j = 0, \quad (53)$$

$$M_j = (\zeta_{17} + \delta_4 \lambda_j d_j + \zeta_9 g_j),$$

$$N_j = (\delta_1 \lambda_j + \zeta_{18} d_j),$$

$$O_j = (\zeta_3 \lambda_j^2 + \zeta_5 d_j),$$

$$P_j = (\lambda_j h_j i \zeta).$$

Solving (50)–(53), by Cramer's rule we get A_1, A_2, A_3, A_4 as

$$A_1 = \frac{-F_1 Q_1 + F_2 Q_5}{\Lambda s}, \quad (54)$$

$$A_2 = \frac{F_1 Q_2 - F_2 Q_6}{\Lambda s}, \quad (55)$$

$$A_3 = \frac{-F_1 Q_3 + F_2 Q_7}{\Lambda s}, \quad (56)$$

$$A_4 = \frac{F_1 Q_4 - F_2 Q_8}{\Lambda s}, \quad (57)$$

where

$$Q_1 = N_2 R_1 - R_2 N_3 + R_3 N_4,$$

$$Q_5 = M_2 R_1 - R_2 M_3 + R_3 M_4,$$

$$Q_2 = N_1 R_1 - R_4 N_3 + R_5 N_4,$$

$$Q_6 = M_1 R_1 - R_4 M_3 + R_5 M_4,$$

$$Q_3 = N_1 R_2 - R_4 N_2 + R_6 N_4,$$

$$Q_7 = M_1 R_2 - R_4 M_2 + R_6 M_4,$$

$$Q_4 = N_1 R_3 - R_5 N_2 + R_6 N_3,$$

$$Q_8 = M_1 R_3 - R_5 M_2 + R_6 M_3,$$

$$\Lambda = M_1 Q_1 - Q_2 M_2 + Q_3 M_3 - Q_4 M_4,$$

$$R_1 = O_3 P_4 - O_4 P_3,$$

$$R_2 = O_2 P_4 - O_4 P_2,$$

$$R_3 = O_2 P_3 - O_3 P_2,$$

$$R_4 = O_1 P_4 - O_4 P_1,$$

$$R_5 = O_1 P_3 - O_3 P_1,$$

$$R_6 = O_1 P_2 - O_2 P_1.$$

The expressions for displacement components, conductive temperature, thermodynamic temperature, the normal, tangential and couple stress in transformed domain are obtained from (39)–(42) and (34)–(36) by placing the values of A_1, A_2, A_3, A_4 from (54)–(57) as

$$\hat{u}_1 = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 Q_{j+4} e^{-\lambda_j x_3} \right], \quad (58)$$

$$\hat{u}_3 = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 d_j Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 d_j Q_{j+4} e^{-\lambda_j x_3} \right], \quad (59)$$

$$\hat{T} = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 g_j Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 g_j Q_{j+4} e^{-\lambda_j x_3} \right], \quad (60)$$

$$\hat{\phi} = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 h_j Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 h_j Q_{j+4} e^{-\lambda_j x_3} \right], \quad (61)$$

$$\hat{t}_{13} = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 N_j Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 N_j Q_{j+4} e^{-\lambda_j x_3} \right], \quad (62)$$

$$\hat{t}_{33} = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 M_j Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 M_j Q_{j+4} e^{-\lambda_j x_3} \right], \quad (63)$$

$$\hat{m}_{32} = \frac{F_1}{s\Lambda} \left[\sum_{j=1}^4 O_j Q_j e^{-\lambda_j x_3} \right] + \frac{F_2}{s\Lambda} \left[\sum_{j=1}^4 O_j Q_{j+4} e^{-\lambda_j x_3} \right]. \quad (64)$$

5. Inversion of the Transforms

To get the result of the problem in physical domain, we will have to invert the transforms in Equations (58)–(64). To find the function $\hat{f}(\zeta, x_3, s)$, we first invert the Fourier transform using

$$\bar{f}(x_1, x_3, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\zeta x} \hat{f}(\zeta, x_3, s) d\zeta = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cos(\zeta x_1) f_e - i \sin(\zeta x_1) f_o| d\zeta \quad (65)$$

where f_e and f_o are, respectively, the odd and even parts of $\hat{f}(\zeta, x_3, s)$. Following Honig and Hirdes [37], the Laplace transform function $\tilde{f}(x_1, x_3, s)$ can be inverted to $f(x_1, x_3, t)$ by

$$f(x_1, x_3, t) = \frac{1}{2\pi i} \int_{e^{-i\infty}}^{e^{+i\infty}} \tilde{f}(x_1, x_3, s) e^{-st} ds. \quad (66)$$

The last step is to calculate the integral in Equation (65). The method for evaluating this integral is described in Press et al. [38]. It involves the use of Romberg's integration with adaptive step size.

6. Numerical Findings and Interpretation

For numerical computations, computer programming using the software MATLAB has been used and the material constants for a fiber reinforced material are taken from Kumar et al. [39].

$$\lambda = 5.65 \times 10^{10} \text{ Nm}^{-2}, \mu_T = 2.46 \times 10^{10} \text{ Nm}^{-2}, \mu_L = 5.66 \times 10^{10} \text{ Nm}^{-2},$$

$$\beta = 220.90 \times 10^{10} \text{ Nm}^{-2}, C_E = 0.787 \times 10^3 \text{ Jkg}^{-1} \text{ K}^{-1}, K_1 = 0.0921 \times 10^3 \text{ Jm}^{-1} \text{ K}^{-1} \text{ s}^{-1},$$

$$K_3 = 0.0963 \times 10^3 \text{ Jm}^{-1} \text{ K}^{-1} \text{ s}^{-1}, \alpha_1 = 0.017 \times 10^{-4}, \alpha_3 = 0.015 \times 10^{-4},$$

$$\alpha = -1.28 \times 10^{10} \text{ Nm}^{-2}, T_0 = 293 \text{ K}, b_1 = 0.02 \text{ m}^2, b_3 = 0.03 \text{ m}^2,$$

$$G_2 = 0.2, \tau_0 = 0.1, L = 1, l_2 = 0.843 \text{ }\mu\text{m}.$$

The graphs are plotted for $t = 0.15$. The values of the displacement component u_1 , displacement component u_3 , thermodynamic temperature T , normal stress t_{13} , conductive temperature φ , tangential stress t_{33} , and coupled stress m_{32} , for nMCST and H2T due to inclined load.

In Figures 3–9 the effect of H2T and Classical 2T on the various quantities have been shown.

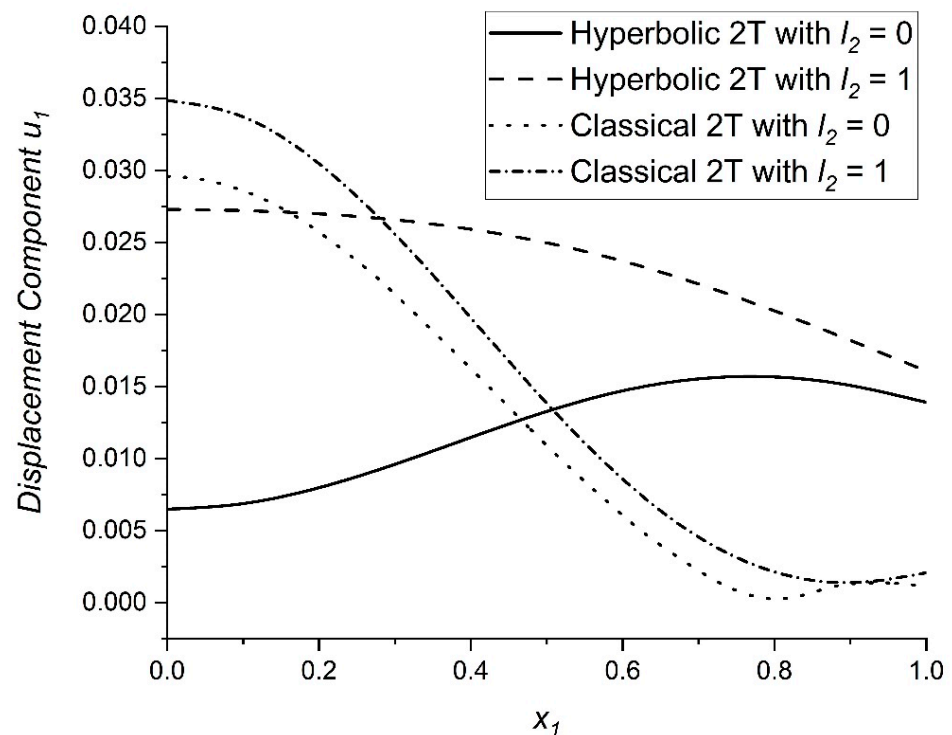


Figure 3. Variations in dimensionless displacement component u_1 .

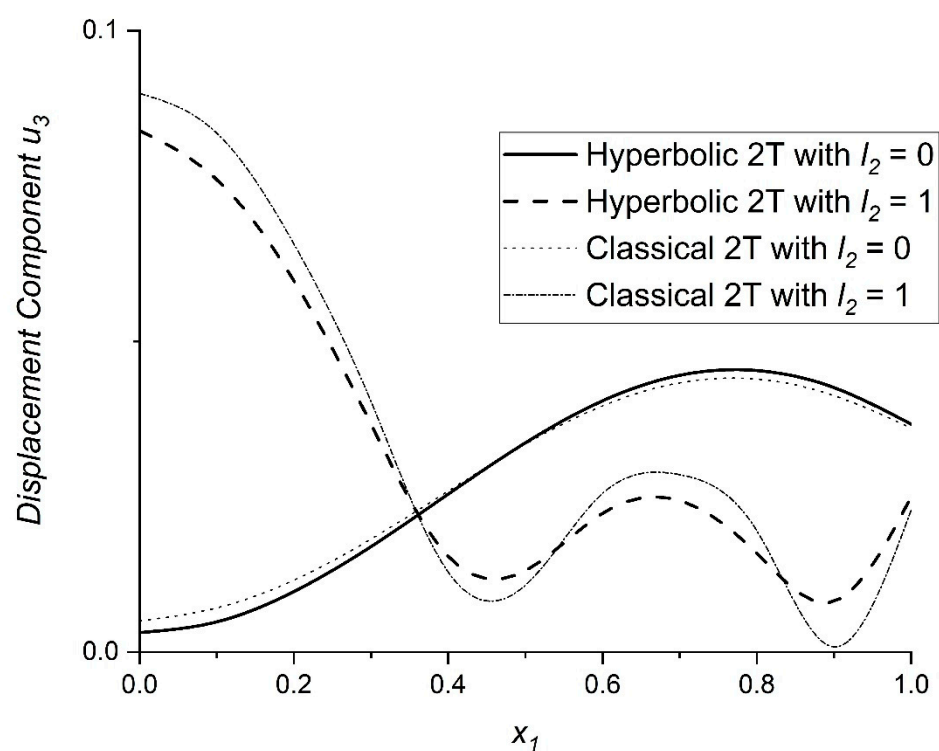


Figure 4. Variations in dimensionless displacement component u_3 .

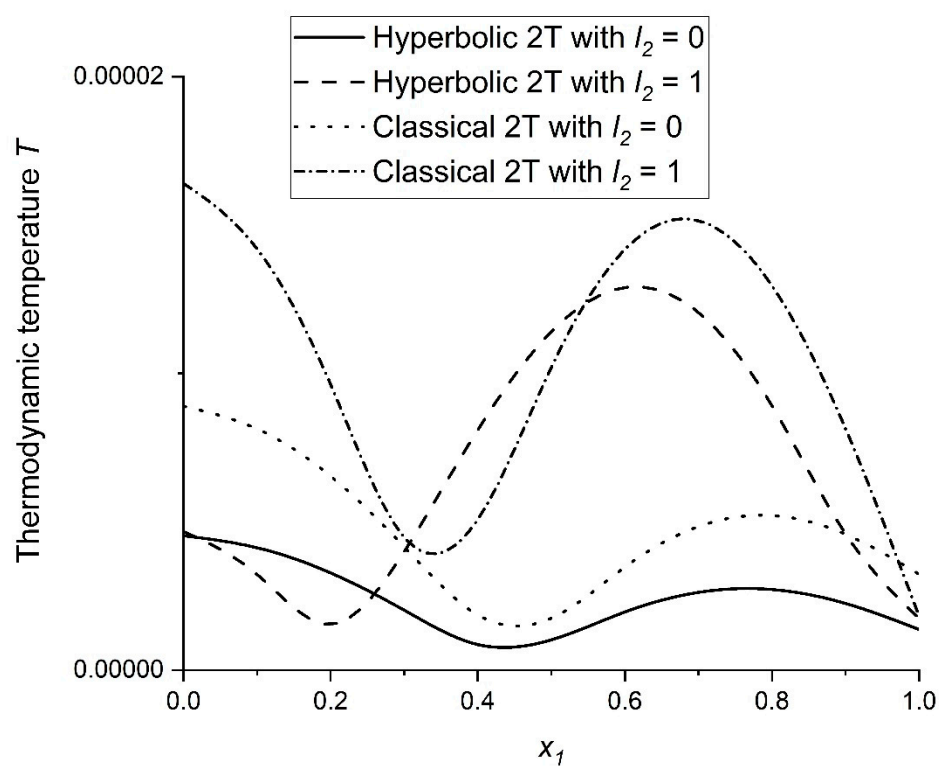


Figure 5. Variations in dimensionless thermodynamic temperature T .

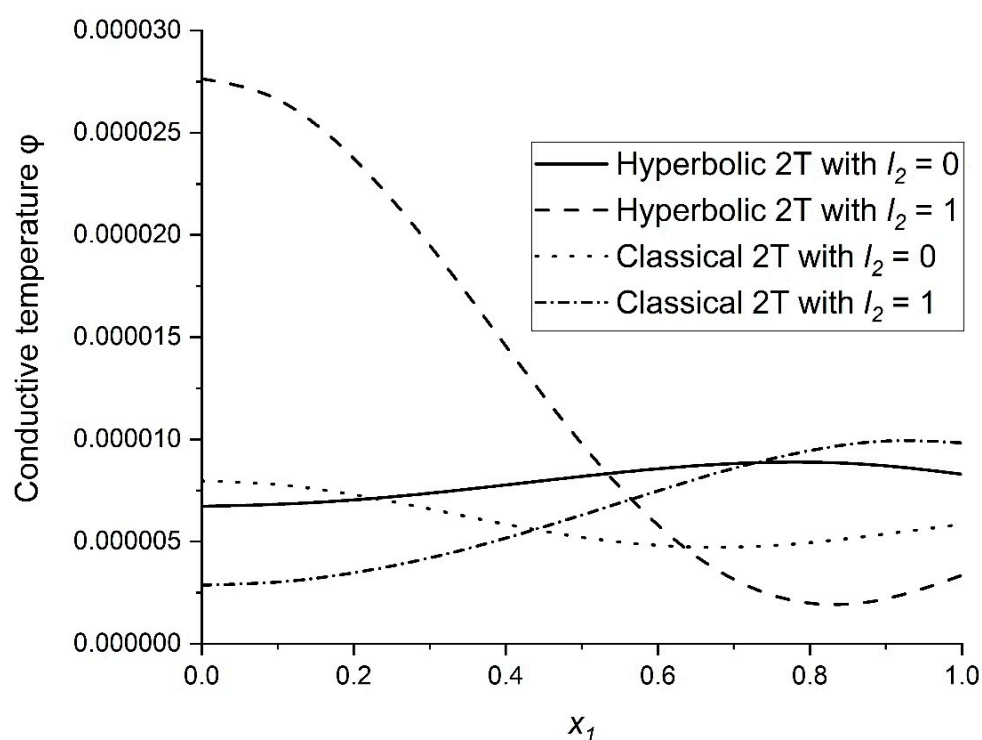


Figure 6. Variations in dimensionless conductive temperature ϕ .

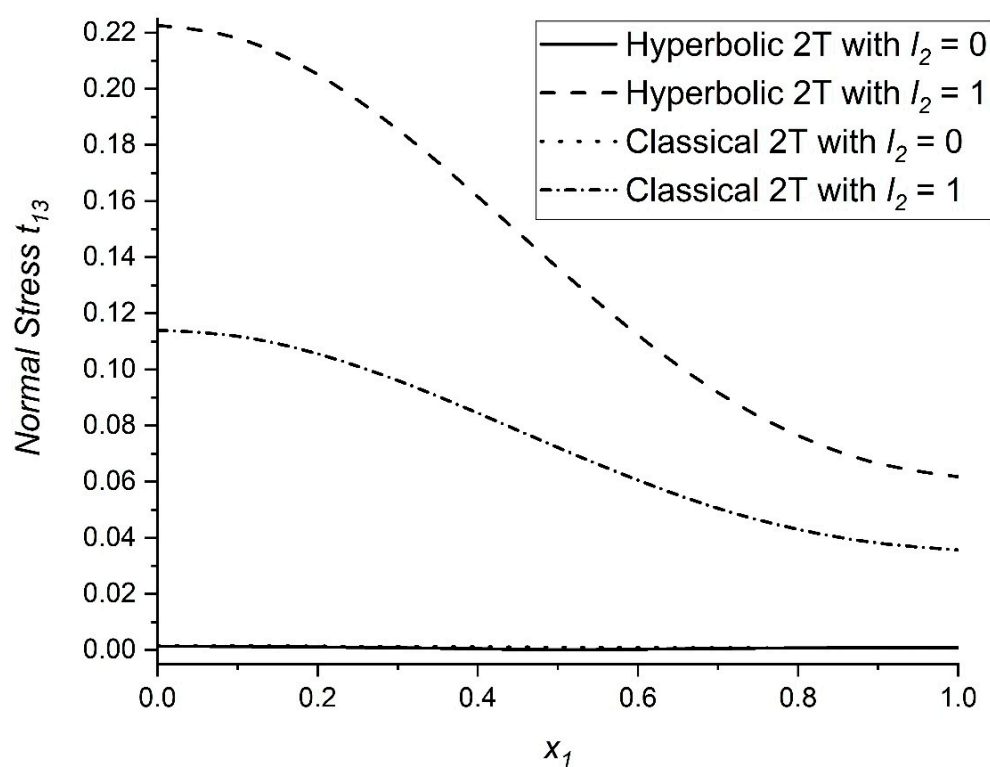


Figure 7. Variations in dimensionless Normal Stress t_{13} .

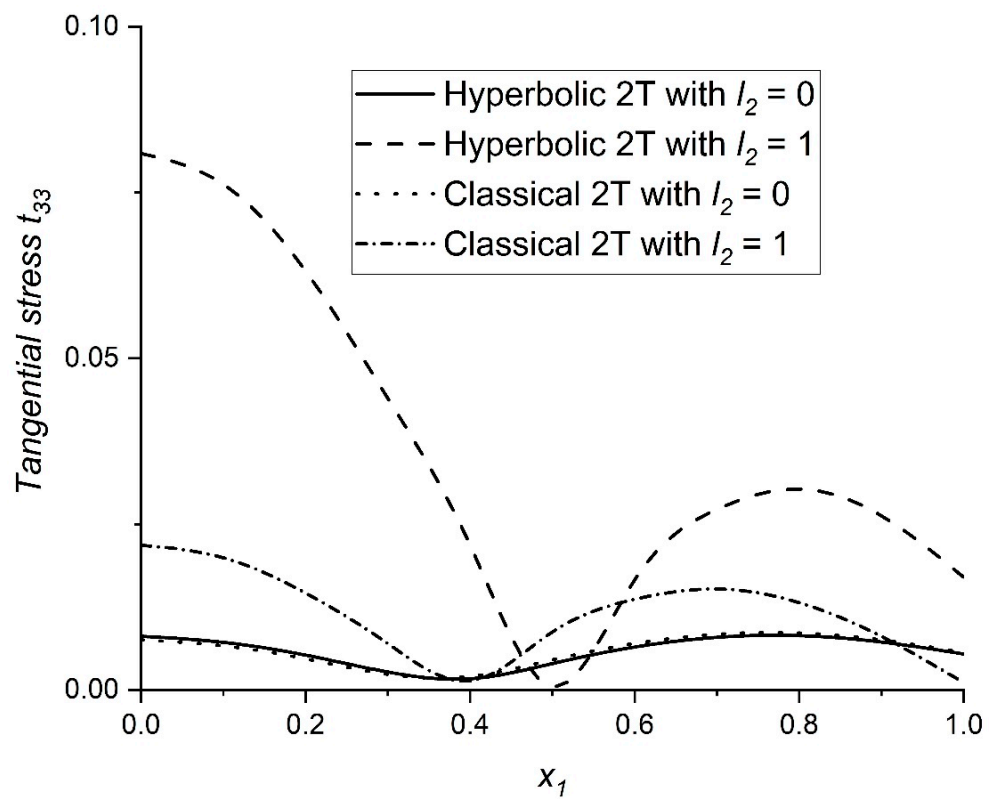


Figure 8. Variations in dimensionless tangential stress t_{33} .

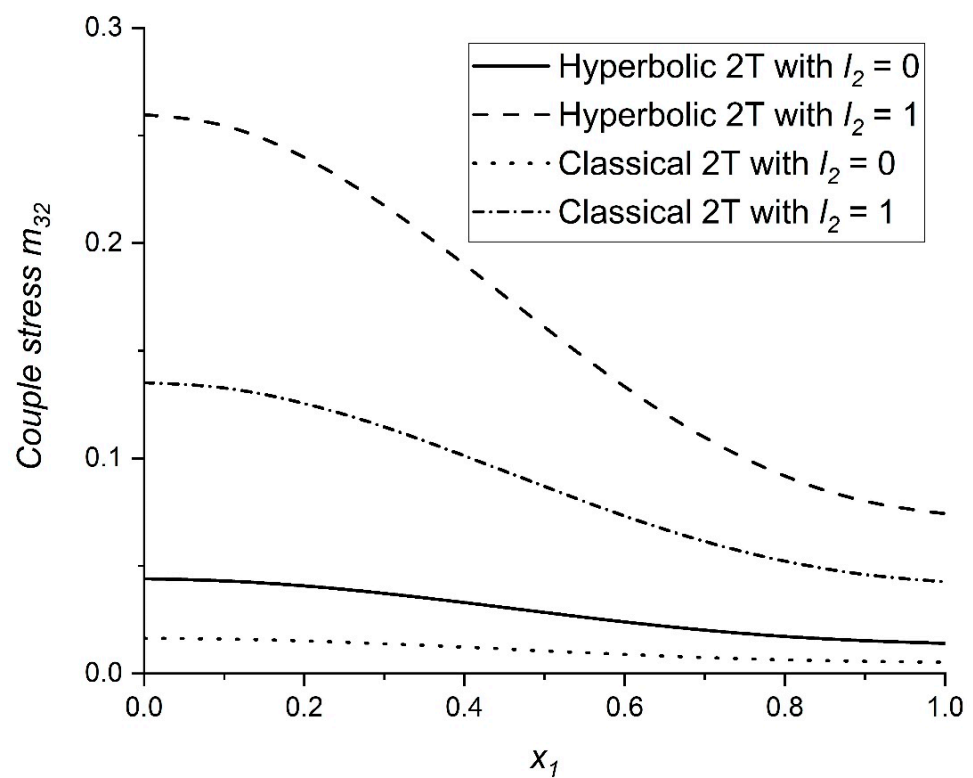


Figure 9. Variations in dimensionless couple stress m_{32} .

- i. The solid line relates to H2T theory with $l_2 = 0$
- ii. The dashed line relates to H2T theory with $l_2 = 1$

- iii. The dotted line relates to Classical 2T (C2T) theory with $l_2 = 0$
- iv. The dot-dashed line relates to Classical 2T (C2T) theory with $l_2 = 1$

Figure 3 shows the variations of the u_1 using nMCST for H2T and C2T with different values of length scale parameter l_2 . In the initial value of x_1 , the u_1 decreases sharply for H2T with $l_2 = 1$ and H2T with length scale parameter $l_2 = 0$ and 1 and then diminishes to zero for C2T theory. C2T shows a higher variation in the u_1 as compared to H2T. Figure 4 illustrates the variations of the u_3 using nMCST for H2T and C2T with different values of length scale parameter l_2 . In the initial value of x_1 , and length scale parameter $l_2 = 1$ the u_3 decreases sharply for both H2T and C2T. Moreover In the initial value of x_1 , and length scale parameter $l_2 = 0$ the u_3 increases for both H2T and C2T. C2T shows a higher variation in the u_3 as compared to H2T. Figure 5 displays the variations of thermodynamic temperature T using nMCST for H2T and C2T with different values of length scale parameter l_2 . In the initial value of x_1 , the T decreases sharply and then illustrates a sharp rise in the T for the rest of the value of x_3 . C2T shows a higher variation in the T as compared to H2T with the length scale parameter $l_2 = 1$. Figure 6 displays the variations of conductive temperature φ using nMCST for H2T and C2T with different values of length scale parameter l_2 . In the initial value of x_1 , the φ decreases sharply for H2T with the length scale parameter $l_2 = 1$ and then shows the oscillatory behavior for the rest of the value of x_3 . C2T shows a higher variation in the φ as compared to H2T.

Figure 7 displays the deviations of Normal Stress t_{13} using nMCST for H2T and C2T with different values of length scale parameter l_2 . In the initial value of x_1 , the t_{13} increases slowly for H2T and C2T for length scale parameter $l_2 = 0$ and decreases sharply for H2T and C2T for length scale parameter $l_2 = 1$. H2T with length scale parameter $l_2 = 1$ shows a higher variation in the t_{13} as compared to C2T. Figure 8 displays the deviations of tangential Stress t_{33} using nMCST for H2T and C2T with different values of length scale parameter l_2 . In the initial value of x_1 , the t_{33} decreases sharply and then shows a slight rise for the rest of the value of x_3 . H2T shows a higher variation in the t_{33} as compared to C2T. Figure 9 displays the deviations of couple stress m_{32} using nMCST for H2T and C2T with different values of length scale parameter l_2 . For H2T and C2T, the m_{32} for length scale parameter $l_2 = 1$ decreases sharply. H2T shows a higher variation in the m_{32} as compared to C2T with length scale parameter.

7. Conclusions

The new modified couple stress theory of thermoelasticity with hyperbolic two-temperature is studied in fibre-reinforced composites. The medium is subjected to a concentrated inclined load. It is a significant problem in solid mechanics to analyze stresses, hyperbolic conductive temperatures, displacement components and couple stresses caused by inclined loads in fibre-reinforced materials. The Lord Shulman heat conduction equation with hyperbolic two-temperature is used to form the model of the problem. Fourier and Laplace integral transforms method are employed to get the results of the mathematical model. Dynamic response of concentrated inclined load is investigated. The effects of hyperbolic and classical two temperature with length scale parameter are represented explicitly on the physical quantities. Classical two-temperature shows a higher variation in the resultant quantities such as hyperbolic conductive temperatures, displacement components as compared to hyperbolic two-temperature. Hyperbolic two-temperature shows less deformation in the displacement components as compared to C2T and shows better results than the C2T theory of thermoelasticity. The length scale parameter shows the major effect on the analyze stresses, hyperbolic conductive temperatures, displacement components and couple stresses with both theories of thermoelasticity. Researchers working in thermomechanical, sensors, resonators, medical science, accelerometers, as well as in future research, should benefit from the results of this research.

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Nomenclature

L	constant of the dimension of length
γ_{ij}	Thermal elastic coupling tensor,
T	thermodynamic temperature,
α, β	fiber reinforcement elastic parameters,
φ	conductive temperature,
t_{ij}	Stress tensors (Nm^{-2}),
$\delta()$	Dirac's delta function,
μ_L	shear modulus in longitudinal shear in the preferred direction
λ	elastic constants,
C_E	Specific heat at constant strain,
m_{ij}	couple stress moment tensor
G_i	elasticity constants
K_{ij}	Thermal conductivity,
μ_T	shear modulus in transverse shear across the preferred direction
χ_{ij}	curvature tensor,
$\mathbf{a}$	Direction of fibre-reinforcement
e_{ij}	Strain tensors,
T_0	Reference temperature
$H()$	Heaviside unit step function
ϵ_{ijk}	Permutation symbol,
b_{ij}	Two temperature parameters,
t	time,
ρ	Medium density (kgm^{-3}),
ρ	the density of the medium,
ω_i	rotation components,
u_i	Components of displacement (m),
δ_{ij}	Kronecker delta,
l_j	material length scale parameters
α_{ij}	Linear thermal expansion coefficient,
τ_0	Phase lag of temperature gradient

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