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Discrete Model for a Multi-Objective Maintenance Optimization Problem of Safety Systems

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Abstract: The aim of this article was to solve a multi-objective maintenance optimization problem by minimizing both unavailability and cost through the use of an optimal maintenance strategy. The problem took into account three different system designs upon which the objective functions are dependent, and the time to start preventive maintenance (PM) was used as a decision variable. This variable was optimized for all system components using a discrete maintenance model that allows for the specification of several discrete values of the decision variable in advance to find the optimal one. The optimization problem was solved using innovative computing methodology and newly updated software in MATLAB, which was used to quantify the unavailability of a complex system represented through a directed acyclic graph. A cost model was also developed to compute the cost of different maintenance configurations, and the optimal configuration was found. The results for a selected real system (a real fluid injection system adopted from references) showed that unavailability was less sensitive to variations in maintenance configurations, while cost variations were more noticeable in relation to different maintenance configurations. Applying PM, the increasing value of the decision variable increased cost because it led to more frequent corrective maintenance (CM) actions, and recovery times due to CM were more expensive than recovery times due to PM.

Keywords: multi-objective optimization; unavailability; cost; maintenance; acyclic graph; alternating renewal process

MSC: 60K10; 90B25

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1. Introduction

A real complex system can either be in a functioning or failed state. The probability of a functioning state under specified conditions over an intended period of time is usually defined [1] as system reliability $R(t)$. System maintainability is defined to be the probability the system can be restored to a functional state within a specified period of time known as downtime. To model reliability and maintainability, we define two relevant random variables: time to system failure and time to repair. Maintainability and reliability are the two most important factors that must be considered while designing the system. They directly affect the availability $A(t)$ of the system, which is defined as the probability that the system will continue to operate satisfactorily at any given point of time when it is being used under the specified conditions. The term reliability is often associated with systems that cannot be repaired; availability, however, is a term associated with repairable systems because it encompasses the full failure–recovery cycle over the mission time of the system. Thus, repairable systems operate during a time to failure (random variable) until a failure occurs. To recover the system's operating state, time to repair (random variable) is needed. Both random variables are modelled by appropriate continuous probability distributions.

Various maintenance strategies have been subjected to intense study and research in order to improve the reliability, availability, and usability of relevant industrial systems.

Unexpected failures can endanger human lives, cause unplanned production outages, etc., and this is why systems must be protected against them. A relevant tool to significantly improve system reliability is maintenance, both preventive and corrective. The authors in [2] state that maintenance is no longer a necessary evil and that production companies should invest in maintenance to maximize profit. To find the most suitable maintenance strategy for every component or subsystem, one must also consider the economic impact of the strategy employed to ensure profitable production. Therefore, different strategies must be evaluated in terms of their performance [2,3].

One of the common maintenance policies applied is a CM strategy, quite often denoted as a repair policy. It is launched at the time of failure when the system is already broken. Thereafter, the system is repaired into a functioning state through applying a CM action. The aim of a PM policy is to prevent the system from undesired breakdowns. PM is usually carried out while the system is still operational—it reduces the ageing processes, correlating to a decreased probability of system failure. Maintenance modeling is an emerging scientific discipline with rapid developments, and we do not want to present here a general overview of references on maintenance as this article is oriented towards the optimization of PM policy in context of system design. Several papers oriented towards both aspects are discussed below.

Many techniques can be utilized to either maximize the system availability or minimize the unavailability. Unavailability is the complement of system availability to one. The present article is focused on two of them. First, this is done by modifying the design process for the system to ensure enough redundancies are present to reduce the system's unavailability. For a system with a series-parallel configuration, redundancy can be thought of as a modification to the configuration to increase the number of parallel paths [4]. Second, an overall decrease in system unavailability is possible through PM [5]. The unavailability of a system due to its failure can occur at any time, requiring a significant endeavor to revert it back to the operating state. Contrarily, a planned shutdown to perform a PM task can represent a controlled situation with materials, spares, and human teams available, resulting in a reduced period of unavailability.

It is clear now that the unavailability of repairable systems can be improved in two ways, either by modifying the design or applying PM strategies. Only a few research articles are devoted to looking at the simultaneous optimization of both ways from a multi-objective perspective. In [6], a methodology for integrated safety system design and maintenance optimization based on a bi-level evolutionary process was demonstrated. The authors tried to find both the optimum maintenance strategy and the optimum system design by applying genetic algorithms (GAs) as the optimization method and cost and unavailability as objective functions. The simultaneous optimization of design and maintenance during the life cycle was also presented in [7]. Optimization was performed using GAs, and objective functions were used for system reliability, redundancy, and life-cycle cost. A new approach to parallel optimization of maintenance and design of complex systems using reliability and cost as objective functions was demonstrated in [8].

The authors in [9] implemented optimization of both system design and PM strategy by coupling a multi-objective evolutionary algorithm and discrete simulation. System design can be optimized for reliability using redundant components, whereas PM strategy optimizes the PM times of each system component. The system availability and operation cost were the objective functions that were maximized and minimized, respectively. They applied a simulation approach in which each solution generated by the multi-objective evolutionary algorithm was evaluated through the use of a discrete simulation. This method explains the evolution of the system as it varies depending on operation and recovery times. This technique enables the analysis of complex real systems. Several configurations of the multi-objective evolutionary algorithm non-dominated sorting genetic algorithm II (NSGA-II) were explored in [10]. The authors in [11] realized an exhaustive encoding comparative study, wherein some binary encoding alternatives were investigated. The authors were able to determine the optimal time to start a PM activity. All these approaches were based on a simulation technique that suffers from uncertainty, although the evolutionary processes can be improved in different ways, as for example, the effect of several chromosome lengths.

The system unavailability and operation cost are the objective functions as well in this article, but their optimization was solved by an alternative method. The effective analytical method based on modelling was developed, proceeding from our previous findings in renewal theory and alternating renewal processes [12]. The theorem, called a recurrent linear integral equation, was modified to implement the new decision variable, i.e., time to start a PM activity, in the context of problem formulation. In addition, a new cost model for the system configuration corresponding to the desired PM strategy is defined in this paper. The innovative theorem, cost model, and optimization algorithm were numerically modeled using the high-performance programming language MATLAB.

This article describes a new method to find the optimal PM policy to solve the designed optimization problem. The time to start PM was used as a decision variable in the optimization problem. This variable, which determines different maintenance modes of a system component, was optimally selected from a set of possible realistic maintenance modes. Optimization was performed for all system components. Thus, the discrete maintenance model was considered, wherein each component can function in one of several maintenance modes. The fixed value of the decision variable determined one maintenance mode of the component that predetermines both the evolution of unavailability and cost. Different maintenance modes of system components resulted in different system configurations, with each having a specific unavailability course as well as cost. The optimization process often demands plenty of computation time because a complex system can have several maintenance configurations. The discrete maintenance optimization is demonstrated on a real system selected from practice: the fluid injection system was adopted from the reference authorized by Cacereño et al. [11].

Literature on Maintenance Optimization Approaches

There are different methods to solve the multi-objective maintenance optimization problem. A recent thorough classification can be found in [13]. In our article, we try to optimize the parameters of an a priori selected maintenance strategy. This problem can be solved by different approaches:

- Our approach can be classified into mathematical approaches, wherein the optimization problem is formulated by means of mathematical equations, which are then solved by means of differential calculus to identify the optimal parameters of the maintenance strategy. In [14], a mathematical approach was used for optimizing maintenance profitability. Mathematical approaches were used in other references: for example, a single unit was optimized in [15] to optimize the scheduled maintenance strategy and the inventory management, optimal maintenance in the context of uncertainty is solved in [16,17], etc. Mathematical approaches can be used for the systems for which the optimization problem can be solved analytically or numerically.
- Mixed integer programming is the field of optimization that addresses optimization problems with continuous and integer variables in the objective or in the constraints. Linear or nonlinear problems can be solved by means of the method in [18]. If the method is used for maintenance optimization, the possible maintenance optima are represented by integer variables. Examples of application of the method are the following: in [19], the authors optimized the maintenance schedule of a wind farm, in [20], a power distribution system was optimized; etc. In [21], the authors developed a mixed integer programming model for integration production and scheduled maintenance planning that considers the system's manufacturing capacity and its reliability. The use of the method for maintenance optimization is mainly limited to simple systems because the computation time rapidly increases with the intricacy of systems [22].
- Dynamic programming is a method for solving multi-stage decision problems. The basic idea of the method is that complex problems are decomposed into simpler subproblems to be solved recursively at each time step. Examples of using the method

are the following: the optimal maintenance strategy of road networks under budget constraints was solved in [23], the optimization of the maintenance check schedules in the aeronautical industry was determined in [24], the optimal maintenance strategy for power cables was solved in [25], etc. The main problems with dynamic programming are the curse of dimensionality and the need to explicitly define the transition probabilities among all the possible system states, which makes it inapplicable for complex systems [26].

- Metaheuristic search algorithms are computational processes where the solution of an optimization problem is found approximately by iteratively improving the candidate solutions [27]. For example, GAs are based on the principles of genetics and natural selection. GAs have been used in many situations to solve the maintenance optimization problems: the scheduled maintenance strategy of a wind farm was optimized in [28], the scheduled maintenance strategy of a multi-unit system was optimized in [29], the PM plan was optimized by means of multi-objective GAs in [30], etc. Moreover the maintenance optimization problems can be solved by other metaheuristic search algorithms: the particle swarm optimization algorithm was used to optimize the predictive maintenance interval of a manufacturing system [31], the harmony search algorithm was applied to find the best maintenance strategy for bridge infrastructures [32], simulated annealing was used to find the optimal scheduled maintenance plan of bridge networks [33], ant colony optimization was applied to optimize the maintenance scheduling of multi-unit systems [34], etc. These algorithms are easy to understand and easily adaptable to different optimization problems. A drawback of these algorithms is that they are slow to converge and do not guarantee convergence towards the global optimum.

2. Formulation of a Multi-Objective Optimization Problem

Any multi-objective optimization problem works on the assumption that in general m-objective functions $f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots f_{\mathrm{m}}(\mathbf{x})$, each one varying in a given range has to be optimized, i.e., either maximized or minimized, constrained by several restrictions imposed on the decision variables that are mostly related to system components. The optimization problem in this article is formulated using the two following objective functions: $f_1(\mathbf{x})$, which represents the cost function C_S, and $f_2(\mathbf{x})$, which represents the stabilized unavailability function U_S. Thus, we searched for an optimal vector of decision variables $\mathbf{x}$, here considered as the vector of times to start PM all of k components $(TP_1, \ldots, TP_k)$ that minimize both $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$:

$$\min\,[f_1(\mathbf{x}) \cap f_2(\mathbf{x})] \tag{1}$$

In our notation, we searched for the optimal decision vector $(TP_1, \ldots, TP_k)$, minimizing both objective functions:

$$\min\,[C_S(TP_1, \ldots, TP_k) \cap U_S(TP_1, \ldots, TP_k)] \tag{2}$$

$C_S(TP_1, \ldots, TP_k) \ldots$ total cost of maintenance of a system configuration;
$U_S(TP_1, \ldots, TP_k) \ldots$ stabilized (asymptotic) system unavailability at the end of a mission time $\mathrm{T_M}$;
$(TP_1, \ldots, TP_k) \ldots$ decision variable vector;
$k \ldots$ number of system components, each having the decision variable TP_i (time to start PM of i-th component), which is optimized in this article.

From the point of view of computational feasibility, the optimization process was realized under following constraints: each decision variable has a prescribed domain containing three possible values, namely, $TP_{min},\ TP_{max}$, and the middle point TP_{mid} between minimal and maximal possible values.

$$TP_i \epsilon [TP_{i,min}; TP_{i,mid}; TP_{i,max}]\ i = 1, \ldots, k \tag{3}$$

Different values of TP_i constitute different maintenance modes of the component, and for each maintenance mode, one can first compute the time evolution of unavailability function $U(t)$ using the methodology described in Section 4.2, followed by the cost C_{i,T_M} described in Section 4.3. Evolutions of $U(t)$ of individual modes are then aggregated by means of the methodology based on directed acyclic graph (AG) described in Section 4.1 in order to compute the unavailability evolution of one system configuration. One system configuration is characterized by an asymptotic value U_S and total cost C_S. All system configurations are finally ordered with respect to Equation (2). This is the main idea of the discrete maintenance model further introduced in Section 3.

In most cases, both C_S and U_S are usually complex linear or non-linear functions of the decision variable vector $(TP_1, \ldots, TP_k)$, representing variables for which optimal values have to be found.

Sometimes, the optimization problem is complicated by other constraints that apply for a given scope (e.g., a limitation for system performance). Such matters must be solved by finding the global optimum not violating any constraint. Thus, optimization, objective functions, and constraints cannot be managed independently because they influence each other. Solution of such optimization issues require advanced numerical algorithms [35–37]. The optimization problem with one objective function and restrictions including a specified limitation of maximal permissible value of the unavailability function was solved in our previous research work, for example, in [38]. In this article, we solved the optimization problem minimizing two objective functions (see Equation (2)) and respecting k restrictions given by Equation (3).

Another tool to solve similar optimization problems results from GAs that have been frequently used in previous research, for example in [39], where GAs were used to optimize surveillance testing and maintenance, or in [11], which provided a real application example to demonstrate the innovative methodology presented in this article.

As mentioned above, concerning the optimization outcome and decision variables, the discrete maintenance model can be classified as a process that finds optimized parameter values defining a single maintenance strategy selected a priori, e.g., in this paper, the time to start a PM activity must be found optimally, in the context of the problem formulation. To solve the multi-objective optimization problem, we selected a mathematical approach for the following reasons:

- The search algorithm of the optimization problem requires good conditions for computing unavailability. We have the long-term experience to generate effective numerical algorithms for quantifying instantaneous unavailability of different types of maintained components, as well as complex systems. For example, in [12], we created and numerically elaborated the recurrent linear integral equation proceeding from alternating renewal processes. In this paper, the algorithms were further developed, i.e., properly modified and adopted to solve the formulated optimization problem.
- We have long-term experience with the high-performance programming language MATLAB that was effectively used for the development of all numerical algorithms in the paper that are absolutely necessary to compute the optimization problem.
- In the future, we intend to continue the research work in close collaboration with power industry experts, who are oriented towards the optimization of complex distribution networks that are hard to solve by the other above-mentioned alternative approaches.

3. Discrete Maintenance Model

In [40], we introduced the discrete maintenance model for complex real systems with non-identical components. The model investigates systems with repairable components and latent failures to find optimal maintenance strategies to minimize cost and maintain unavailability under restriction. Latent failures are indicated using adaptable periods of inspections as a decision variable.

A real complex system consists of a multitude of components that can be maintained in different ways, both by corrective and preventive interventions. Any component can function in different maintenance modes. One discrete maintenance mode of the ith component is determined by a selected value of the decision variable (in our application $x_i = TP_i$) that immediately affects the maintenance cost of the mode. On the condition that a system consists of k components, wherein each component can have five maintenance modes, in total, we have to explore 5^k maintenance configurations of the system. Each configuration is characterized by a typical unavailability value (mostly maximal or asymptotical) and total cost C_S, which is commonly computed as a sum of the costs of all component modes constituting the configuration. It is necessary to find the optimal system configuration that meets requirements (2). We call this maintenance model the discrete maintenance model in this article.

4. Methodology for Computing Unavailability and Cost of a System Configuration

4.1. Graph Structure as a System Representation

Any system structure can be represented with the help of a directed acyclic graph. Figure 1 demonstrates the AGs of a real system from practice—a fluid injection system, which is later analyzed and optimized in detail in the next section. AGs are frequently used as systematic schemes to quantify the unavailability of complex systems [41] because they enable a reflective description of a system's functionality. Obviously, AGs contain nodes and edges, wherein the exceptional node is the TOP node that describes the functionality of all systems depending on the functionality of its inferior subsystems and components that constitute internal and terminal nodes. Nodes are interconnected by edges, and AGs are acyclic, meaning that feedback loops are inadmissible. Terminal nodes—for example, V1 or P3—are denoted by blue squares, and they represent system components. Failure time as well as repair time or time necessary for PM of a system component are described by a suitable probability distribution. Applying these distributions, the time evolution of unavailability for each component can be computed using advanced renewal theory [40].

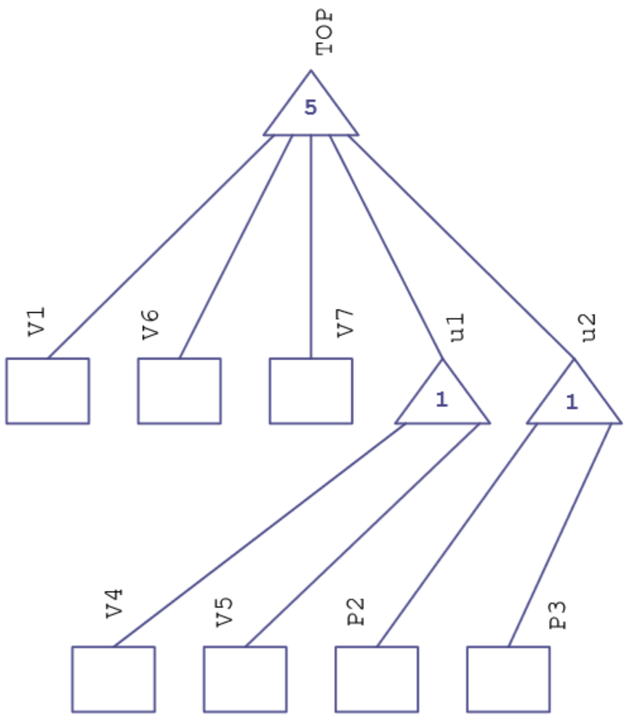

Figure 1. Directed acyclic graph of a system from practice.

Internal nodes are denoted by blue triangles, e.g., u1 or u2, respresenting subsystems of the fluid system. Subsystems as well as components (i.e., internal respectively terminal nodes) at a given time are either correctly working or in a failed state (under restoration). A node is in a functioning state when the number of child nodes is greater or equal to the number of nodes inside the triangle, otherwise it is in a failed state. For example, the node u1 is correctly working if the number of correctly working directly inferior nodes is either 1 or 2.

The knowledge of the unavailability functions of terminal nodes (components) can be used to quantify the unavailability function of internal nodes (subsystems), with both serving as inputs for computing the unavailability function of the TOP node $U(t)$, which represents all systems. The unavailability function $U(t)$ demonstrates the time-dependent probability that the system is unavailable at time t due to a failure or due to a still ongoing repair process.

4.2. Model for Unavailability Exploration of a Terminal Node with Both CM and PM

To find the unavailability function of TOP node $U(t)$, it is necessary to find a model and algorithm for the unavailability quantification of terminal nodes that undergo both PM and CM. At first, the model with CM will be introduced, which will be further generalized to enable the implementation of both PM and CM.

Applying CM, we have to consider two mutually cooperating random variables: the lifetime X, described by either distribution function $F(t)$ or probability density function (pdf) $f(t)$, and a random time necessary for completing the repair, actually repair, or recovery time Y, described by either distribution function $G(t)$ or pdf $g(t)$. Resulting from renewal theory and alternating renewal processes, availability $A(t)$ can be computed as follows [12]:

$$A(t) = 1 - F(t) + \int_0^t h(x)[1 - F(t-x)]\mathrm{dx} = R(t) + \int_0^t h(x)R(t-x)\mathrm{dx}, \quad (4)$$

where $R(t) = 1 - F(t)$ is the reliability function and $h(x)$ is the renewal density of the corresponding alternating renewal process.

The unavailability $U(t)$ is given as follows:

$$U(t) = 1 - A(t) = F(t) - \int\limits_0^t h(x)[1 - F(t-x)]dx \quad (5)$$

To compute the unavailability from the Formula (5), $h(x)$ must be known, which could be a problem in practice, bringing about a large number of numerical complications because the renewal density is numerically represented as an infinite sum of probability densities—here, each being computed as a convolution. Fortunately, Formula (5) can be superseded by the equivalent Formula (6), which brings the following theorem, called the recurrent linear integral equation, which was first mentioned and proven in [12].

Theorem 1. *The unavailability U(t) in Formula (5) is equivalent to the unavailability U(t) in the following Formula (6)*

$$U(t) = \int_0^t f(x)\,[1 - G(t-x)]\mathrm{dx} + \int_0^t (f * g)(x)\,U(t-x)\mathrm{dx} \quad (6)$$

where $*$ means convolution.

Following this, we consider the PM strategies to maintain the operating status of the node.

These operations performed before failure are usually less complex than CM activities that must be undertaken in the case of failure. Obviously, each PM activity starting at time TP that serves as a decision variable in our discrete maintenance model requires some recovery time, which we define to be a new random variable Z. Thus, the operational time of a component is interrupted either by the time to failure X (lifetime) or by the time to start a PM activity TP, whichever occurs first. A recovery time can be realized either by the recovery time Y due to CM or recovery time Z due to PM. In other words, PM activities are scheduled shutdowns, and recovery times are usually shorter and less expensive than the same due to CM. Examples of PM include ensuring the availability of spare parts and the training of human personnel. PM activities should be conducted optimally before the failure but as close as possible to it. From an optimization point of view, it is necessary to minimize the system unavailability as well as cost due to recovery times.

To compute component unavailability $U(t)$ respecting both CM and implemented PM activities, our Formula (6) can be employed with the modification that the random variable X will be substituted by the random variable $V = \min(X, TP)$, which represents interruption of the operation time. The distribution function F_V of V can be easily found:

$$F_V\ (t) = P(\min(X,\ TP) < t) = \mathrm{P}(X < t) + \mathrm{P}(TP < t) - \mathrm{P}(X < t).\,\mathrm{P}(TP < t) = 1 - \mathrm{P}(\min(X,\ TP) \geq t) = 1 - \mathrm{P}(X \geq t).\,\mathrm{P}(TP \geq t) \tag{7}$$

Consequently, the following formulas hold:

$$\begin{aligned} F_V\ (t) &= F\ (t)\ \text{ for } t < TP \\ F_V\ (t) &= 1 \quad \text{for } t \geq TP \end{aligned} \tag{8}$$

Thus, in the latter case, theorem (6) is modified as follows:

$$U(t) = \int\limits_0^{TP} f(x)\cdot[1 - G(t-x)]dx + \int\limits_0^{TP} (f * g)(x)\cdot U(t-x)dx + \int\limits_{TP}^{t-TP} w(x)\ U(t - TP - x)dx \tag{9}$$

where the last integral of (9) mathematically represents the remaining contribution to unavailability function $U(t)$ for $t \geq TP$, provided that at time TP the recovery time started due to PM. $w(x)$ is the pdf of recovery time Z.

The expected value μ_V of V can be found according to the following formula:

$$\mathrm{E\ V} = \mu_V = \int_0^{\mathrm{TP}} (1 - F_V(t))\mathrm{dt} \tag{10}$$

Computing possibilities of the method for unavailability quantification of complex multi-component and highly reliable systems were successfully demonstrated in a comparison study in [42], as well as in [12].

4.3. Cost Model of a System Configuration

The cost model of a system configuration can be obtained by adding up all contributions resulting from both CM and PM replacement interventions of a mode over all of the system components. Components can operate in different maintenance modes; the cost of one maintenance mode consists of two main contributions generated by CM on the one hand and PM on the other. The cost of CM further depends on the mean number of all recovery times due to both CM and PM during mission time $\mathrm{T_M}$ and CM parameters. The cost of PM depends on the decision variable TP and PM's parameters. In practical situations, the cost contributions result from a year database to gain an average yearly cost for system configurations in a monitored period. In the remainder of this article, the cost computed in non-identified cost units on the basis of the summation principle is provided.

To obtain the cost of one system configuration, we simply add up the costs of all maintenance modes of all system components. The cost of one maintenance mode of i-th component C_{i,T_M} can be computed as follows:

$$C_{i,T_M} = n_{i,R}.F(TP_i).C_{i,R} + n_{i,R}.R(TP_i).C_{i,PM} \tag{11}$$

where

$n_{i,R} = \frac{T_M}{MTTI_i + MRT_i}$ ... the mean number of recovery actions of the i-th component per mission T_M;

$MTTI_i = \mu_V$ ... is the mean time to intervention caused by either CM or PM;

MRT_i ... mean recovery time of the i-th component, which is due to either PM or CM.

$$MRT_i = F(TP_i).MTRCM_i + R(TP_i).MTRPM_i \tag{12}$$

$MTRCM_i \ldots$ mean recovery time due to CM;
$MTRPM_i \ldots$ mean recovery time due to PM;
$TP_i \ldots$ decision variable of the i-th component determining PM strategy;
$C_{i,R} \ldots$ CM cost = cost of one CM intervention of the i-th component in cost units;
$C_{i,PM} \ldots$ PM cost = cost of one PM intervention of the i-th component in cost units.

The total cost of one system configuration C_S is given by summing up these contributions described by Formula (11) over all of the system components k:

$$C_S = \sum_{i=1}^{k} C_{i,T_M} \tag{13}$$

5. Results with the Real Complex System—Fluid Injection System and Discussion

The discrete maintenance optimization was demonstrated on the industrial fluid injection system adopted by authors in [11], wherein the authors created a massive simulation to study the system using GA to achieve both maximum availability and minimum cost, paying attention to the possible impact on solutions as a result of different encodings, chromosome lengths, and parameter configurations. This article differs in computing methodology—it is based on probabilistic modelling resulting from alternating renewal processes.

It was necessary to optimize both the design and the PM policy, i.e., to find the optimal decision variable vector **x** = ($TP_1, \ldots, TP_\mathrm{k}$) minimizing both unavailability and cost. The system consisted of valves (V) and pumps (P) and is depicted in its full version in Figure 2. Two other partial versions (designs) were taken into account: Version 1 intended as the simplest version without parallel ordered components P2 and V4, and Version 2, which was intended as the full version without pump P2. The functionality of the system (full version) is described and analyzed within the framework of Section 4.1, utilizing the AG depicted in Figure 1. Unavailability of the system can be decreased by increasing investment in the PM on the one hand, but it signifies the growth of cost on the other hand.

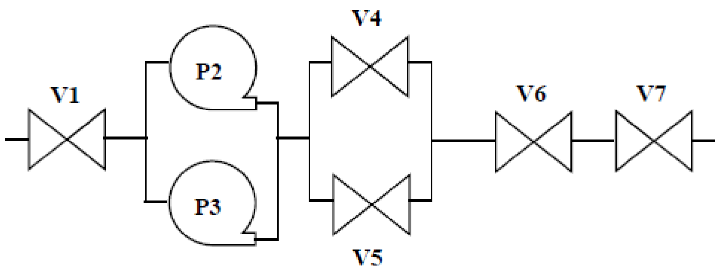

Figure 2. Real fluid injection system adopted from [11].

Other assumptions:

- only two component states are admissible: operational and faulty state;
- the components are mutually independent;
- if failure of a component comes, the repair starts immediately;
- if a repair is completed, the component's state is equivalent to that of a new component.

Table 1 brings about the data used in the analysis. Additional information on the input parameters is found in the Notations. The data were adopted from the special source for reliability analysis OREDA (offshore reliability data handbook [43]), further from expert appraisal (originating from the experience of the Machinery and Reliability Institute (MRI), Alabama, USA) and corresponding reliability mathematics. The OREDA has traditionally been focused on the reliability and availability of production systems and equipment. As we declared above, the discrete maintenance optimization process was carried out, resulting from the minimization of both unavailability and cost due to maintaining system stages including both PM and CM. To realize this, one must

Table 1. Reliability and cost data for system components.

Parameters for Pump	**Value**	**pdf**
Mission time T_M	2920 days	-
CM cost C_R	0.5 units	-
PM cost C_{PM}	0.125 units	-
Failure rate λ	159.57×10^{-6} h^{-1}	Exponential
Parameters of Y (TR_{min}; TR_{max})	(5.23; 16.77) h	Rectangular
Pump TP × (TP_{min}; TP_{max})	(240; 365) days	-
Parameters of Z (TRP_{min}; TRP_{max})	(4; 8) h	Rectangular
Parameters for Valve	**Value**	**pdf**
Mission time	2920 days	-
CM cost C_R	0.5 units	-
PM cost C_{PM}	0.125 units	-
Failure rate λ	44.61×10^{-6} h^{-1}	Exponential
Parameters of Y (TR_{min}; TR_{max})	(4.6; 14.4) h	Rectangular
Valve TP × (TP_{min}; TP_{max})	(830; 1460) days	-
Parameters of Z (TRP_{min}; TRP_{max})	(1; 3) h	Rectangular

× Decision variable for optimization.

- find optimal times TP to start PM actions for the system components, and
- make a decision related to the system version, i.e., which version should be applied (full version, Versions 1 and/or 2). For this reason, it was necessary to evaluate all three versions. It is clear that if redundant components are included, system unavailability will decline on the one hand but system operation cost will be increased on the other hand.

From the point of view of computational feasibility, the optimization process was realized by applying only three maintenance modes of each component related to the decision variable TP: factually TP_{min}, TP_{max}, and the middle point TP_{mid} between minimal and maximal possible values. This means that it is necessary to explore in total $3^7 = 2187$ configurations of the full system version, $3^6 = 729$ configurations of the system version 2, and $3^5 = 243$ configurations of the system version 1. All computations were performed for the mission time T_M = 2920 days = 8 years.

5.1. Maintenance Optimization of the System Version 1

Figure 3 demonstrates the unavailability evolution $U(t)$ of a simple basic configuration of the fluid injection system considered as Version 1, having the decision variable of all components in the mode $TP = TP_{mid}$. We can see a clear decreasing trend, which is stabilized to the value $U_S(\mathbf{x}) = 3.1923 \times 10^{-3}$ at the end of mission time T_M = 8 years. As we mentioned above, PM activities should optimally be performed before the failure but as close as possible to it. This idea corresponds to the choice of TP_{mid}. Since TP_{min} was placed between median and MTTF, TP_{mid} was placed closer to MTTF, mildly exceeding it. For example, pump TP_{min} = 240 days, median = 180 days, MTTF = 261 days, and TP_{mid} = 302, which was similar for valves. In the evolution, in Figure 3, maintenance actions are clearly recognizable. The first unavailability jump came at 302 days, which was the PM time of pump P3. Moreover, the decreasing of the height of unavailability jumps showed that the effect of PM was dampened step by step, and the unavailability was stabilized at the end of the mission time.

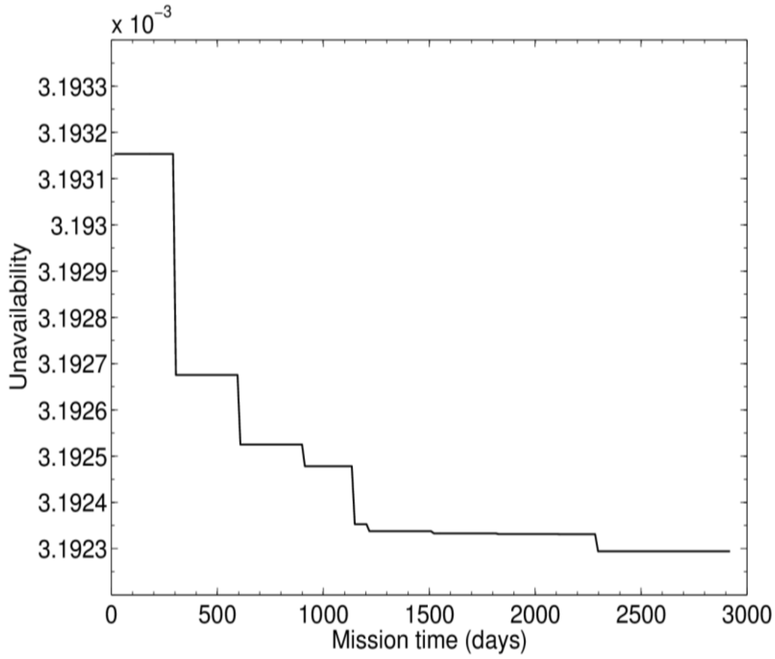

Figure 3. Unavailability evolution of system Version 1 with $TP = TP_{mid}$ (all components).

Because the decreasing trend was indicated for any of the considered configurations, the stabilized unavailability value U_S at the end of mission time was selected as the typical unavailability value for the optimization purpose because it appeared to be adequately conservative, i.e., it exceeded future (decreasing) unavailability course, in the context of the long-term time horizon.

When we compared the decreasing unavailability trend in Figure 3 with the system configuration without PM, apparently PM had a positive influence to the unavailability course, although the difference was not so relevant at the end of the mission time, as is evident from Figure 4, where asymptotic system unavailability without PM was 3.1949×10^{-3}.

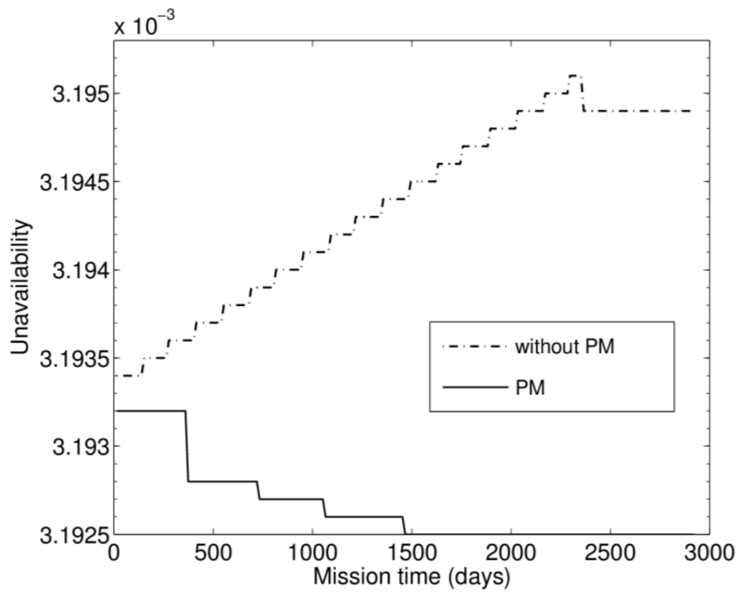

Figure 4. Effect of PM on the unavailability evolution of system Version 1.

Solutions to the above-mentioned optimization problem relating to Version 1 of the system are shown in Table 2. In total, 243 maintenance configurations of the system Version 1 were run, which were similar in shape to the evolution in Figure 3, and differences were only in locations of unavailability jumps that depended on maintenance configuration. The sequence of index digits in Table 2 in the column configuration corresponded to the following sequence of components: V7, V6, V5, P3, and V1 and specified component maintenance modes. For example, sequence 11111 described such a maintenance configuration wherein all components had the decision variable TP set in the mode TP_{min}, Index 2 signified a setting in the mode TP_{mid}, and similarly, Index 3 a setting in the mode TP_{max}. The optimal maintenance configuration respecting Formula (2) is found in the first row of Table 2, i.e., the decision variable of all components was set in the mode TP_{min}. The cost of the optimal configuration was 83.5 cost units, which was comparable with the optimal solution in the simulation study conducted by the authors in [11], which was 82.3 (after dividing the cost by 10 because the simulation study was carried out for a mission time that was 10× longer, time T_M = 80 years). Unavailability was somewhat greater (compare 3.1919×10^{-3} versus 2.720×10^{-3}), but as we showed previously, the unavailability had a decreasing trend and the simulation study was carried out for 80 years, whereas our optimization was realized for T_M = 8 years.

Table 2. Decision variable vectors $(TP_1, \ldots, TP_k)$ for the configurations of system Version 1.

Configuration	Unavailability U_S	Cost C_S (Units)	V1 (Days)	P3 (Days)	V55 (Days)	V6 (Days)	V7 (Days)
11111	3.1919×10^{-3}	83.5	830	240	830	830	830
33333	3.1926×10^{-3}	99.557	1460	365	1460	1460	1460
No PM	3.1949×10^{-3}	112	∞	∞	∞	∞	∞
22222	3.1923×10^{-3}	92.69	1145	302	1145	1145	1145
31111	3.1920×10^{-3}	85.92	830	240	830	830	1460

Figure 5 demonstrates the unavailability evolution $U(t)$ of Version 1, wherein all components have the decision variable set in the mode $TP = TP_{max}$. The first three unavailability jumps became smaller due to the time spent performing PM on the pump, and the fourth was due to the time spent on PM for all the valves. We can observe in Table 2 that the increasing value of the decision variable TP (time to start PM) resulted in slight unavailability growth, whereas cost increased much more quickly, from 83.5 to almost 100, a natural consequence of the PM process. Increasing time TP produced more frequented CM actions and recovery times due to CM being more expensive than recovery times due to PM. In real situations, operators of the system should be able to reach a compromise between the growth of unavailability and cost, as is, for example, configuration 31111 in Table 2 (not exceeding a hypothetical cost limit 86 units), wherein the valve V7 had the maximal value of the decision variable TP_{max} and other components had a minimal value TP_{min}.

5.2. Maintenance Optimization of System Version 2 and Full System Version

Computer processing of the maintenance optimization was quite difficult because it was necessary to explore a total of $3^7 = 2187$ maintenance configurations of the full system version and $3^6 = 729$ configurations of system Version 2. For both system modifications, we can make similar conclusions as for system Version 1 concerning unavailability and cost trends in the context of changes of decision variable TP. Not surprisingly, one can identify that absolute unavailability values were correspondingly less, whereas cost was correspondingly greater. The results on maintenance optimization are found in Table 3 for system Version 2 and in Table 4 for the full system version.

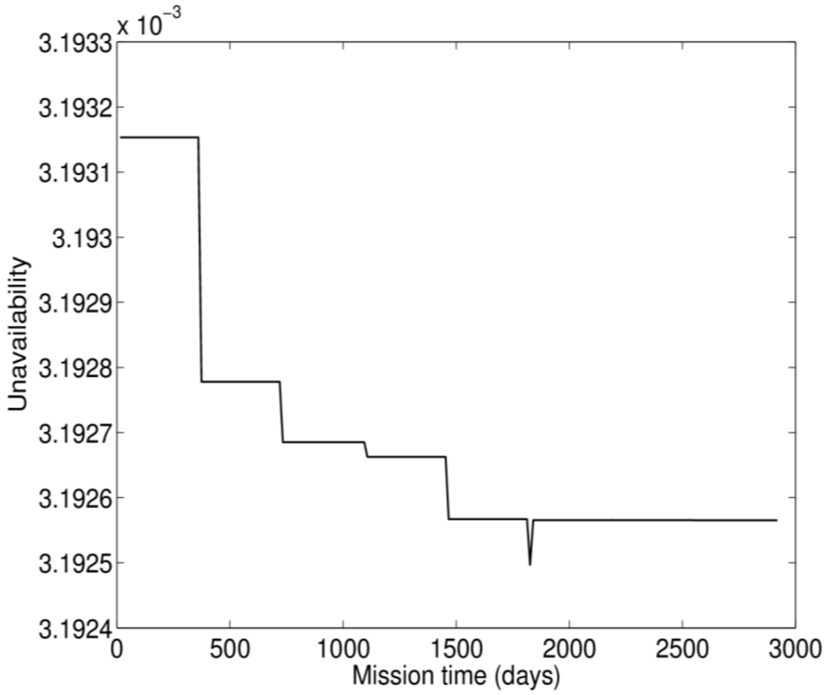

Figure 5. Unavailability evolution of system Version 1 with TP = TP_{max} (all components).

Table 3. Decision variable vectors $(TP_1, \ldots, TP_k)$ for the configurations of system Version 2.

Configuration	Unavailability U_S	Cost C_S (Units)	V4 (Days)	V5 (Days)	V7 (Days)	V6 (Days)	P3 (Days)	V1 (Days)
111111	2.7922×10^{-3}	94.6	830	830	830	830	240	830
222222	2.7926×10^{-3}	105.2	1145	1145	1145	1145	302	1145
333333	2.7929×10^{-3}	113.04	1460	1460	1460	1460	365	1460
No PM	2.7952×10^{-3}	136	∞	∞	∞	∞	∞	∞
113212	2.7934×10^{-3}	99.81	830	830	1460	1145	240	1145

Table 4. Decision variable vectors $(TP_1, \ldots, TP_k)$ for the configurations of the full system version.

Configu-ration	Unavailability U_S	Cost C_S (Units)	P2 (Days)	V4 (Days)	V55 (Days)	P3 (Days)	V7 (Days)	V6 (Days)	V1 (Days)
1111111 22	1.2051×10^{-3}	133.81	240	830	830	240	830	830	830
2222222	1.2052×10^{-3}	147.967	302	1145	1145	302	1145	1145	1145
3333333	1.2053×10^{-3}	158.7	365	1460	1460	365	1460	1460	1460
No PM	1.2054×10^{-3}	192	∞	∞	∞	∞	∞	∞	∞
2111311	1.20525×10^{-3}	139.8	302	830	830	240	1460	830	830

Similarly to system Version 1, optimal maintenance configuration respecting Formula (2) is found in the first row of Tables 3 and 4, i.e., the decision variable of all components had the value TP_{min}. Again, we can state that results in both tables are comparable with the optimal solutions found in the simulation study [11], even if markedly different methods were used for computing both cost and unavailability. The cost of the optimal configuration of Version 2 was 94.6 units (compared with 98.6, after dividing the cost by 10). Unavailability was somewhat greater (compare 2.7922×10^{-3} versus 2.591×10^{-3}), but as we showed previously, the unavailability had a decreasing trend, and the simulation study was carried out for 80 years, whereas our optimization was realized for T_M = 8 years. Similar comparisons can be obtained for the full system version.

Results of both system modifications showed that unavailability was less sensitive to a variation of maintenance configurations, whereas cost variations were much greater in the context of different maintenance configurations. For example, cost of all configurations ranged from 94.6 to 113.04 cost units for system Version 2 and from 133.8 to 158.7 for the full system version. Both intervals were significantly less than the cost of CM without PM (136 units for Version 2, 192 for the full version), and therefore PM significantly decreased cost.

The sequence of index digits in Table 3 in the column Configuration corresponds to the following sequence of components: V4, V5, V7, V6, P3, and V1 and specifies component maintenance modes. The last row of Table 3 specifies the still possible compromise configuration (i.e., series ordered valves V1 and V6 with decision variable set to TP_{mid}, V7 with the decision set to TP_{max}, and remaining components with the decision set to TP_{min}) not exceeding a hypothetical cost limit of 100 units.

Similarly, in Table 4, the sequence of index digits in the column Configuration corresponds to the following sequence of components: P2, V4, V5, P3, V7, V6, and V1 and specifies component maintenance modes. The last row of Table 4 specifies the still possible compromise configuration (i.e., parallel ordered pump P2 with the decision variable set to TP_{mid}, series ordered valve V7 with the decision set to TP_{max}, and remaining components with the decision set to the mode TP_{min}) not exceeding a hypothetical cost limit 140 units.

5.3. Comparison of all System Versions

Comparing all three system versions where all components have the decision variable set to the $TP = TP_{mid}$, one can see in Figure 6 that parallel ordered pumps in the full system version had a much greater influence on unavailability than parallel ordered valves (version 2). Unavailability jumps due to PM were hard to distinguish for all system versions, confirming our previous conclusion that PM influences the unavailability insignificantly. The unavailability of system version 1 was about 2.65 times worse than the unavailability of the full system, which was particularly recognizable in cost, increasing from 92.7 to almost 148 cost units for the full system.

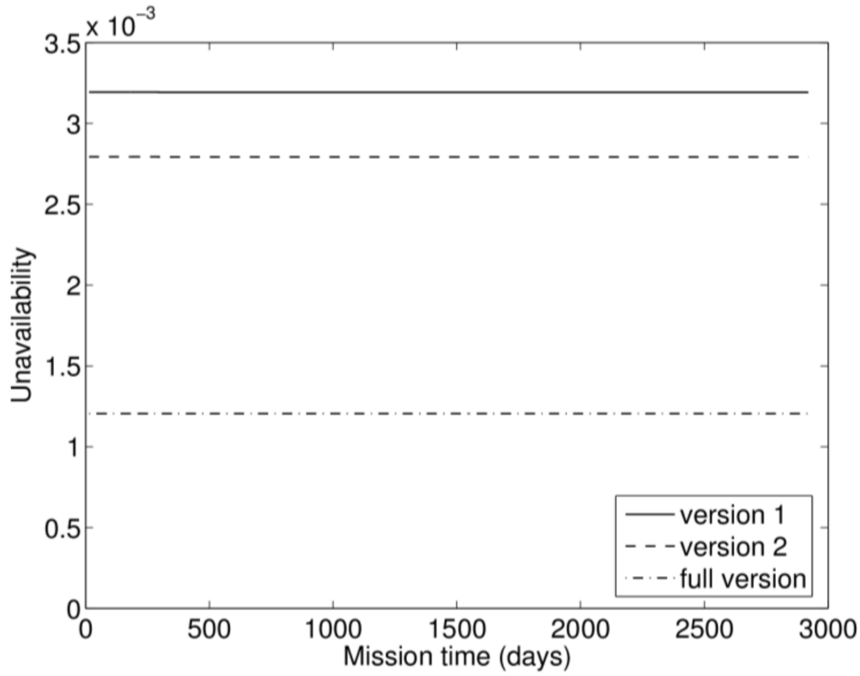

Figure 6. Comparison of unavailability evolutions for all system versions with $TP = TP_{mid.}$.

The system modifications in Version 1 and Version 2 were close to one another in terms of expected unavailability, and therefore in special cases when the cost has a prescribed limitation, both versions can be interchanged by applying the influence of PM on cost. For example, when the prescribed limitation on cost is 95 units and the difference in unavailability is not taken into account, system Version 2 with *TP* configuration 111111 (cost is 94.6 cost units) can be substituted by system Version 1 with *TP* configuration 13331 (94.72) because the cost of both configurations is somewhat below the limitation. Or, when the prescribed limitation on cost is 100 cost units, system Version 2 with *TP* configuration 322111 (cost is 99.8 cost units) can be substituted by system Version 1 with *TP* configuration 33333 (cost is 99.56 cost units).

6. Conclusions

This article formulated and solved the multi-objective optimization problem where two objective functions were minimized for cost and unavailability. The system design defines the objective functions, and three different designs were considered. All three versions were explored, compared, and discussed. The main decision variable in the optimization problem is the time to start PM (*TP*). To solve the optimization problem, we innovated on the methods we had developed in the past to quantify system and component unavailability as well as cost. Our original formula to compute unavailability had to be modified in order to account for a new situation where the operating time of a component is interrupted either by the time to failure X (lifetime) or by the time *TP*, whichever occurs first. This means that that we can substitute the random variable X with $V = \min(X, TP)$. A new formula to compute the cost of one system configuration was derived.

The results of all system modifications showed that unavailability was less sensitive to a variety of maintenance configurations, whereas cost variations were much more perceptible in context with different maintenance configurations. Comparing system modifications with and without PM showed that PM significantly decreased cost, whereas unavailability changes were mild. As *TP* increased for PM, cost increased, as increased *TP* leads to more frequented CM actions, and recovery times from CM are more expensive than recovery times due to PM. Comparing three system versions, we were able to conclude in Figure 6 that parallel ordered pumps in the full system version had a much greater influence on unavailability than parallel ordered valves (Version 2). In addition, for special cases, when the cost has a prescribed limitation, the system Versions 1 and 2 can be interchanged through applying a significant influence of PM on cost.

Numerical experiments applied on a real fluid injection system indicated that discrete maintenance optimization is a useful method to make the most optimal decision. Although the computing process may be complicated and time consuming for multi-component systems (depending on the number of possible system configurations), time taken by all of the optimization computations did not exceed 1 h. All computations were numerically computed using the high-performance programming language MATLAB on computing equipment with the following parameters: Intel (R) Core™ i7-3770 CPU @ 3.4 GHz 3.9 GHz, 8.00 GB RAM.

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Notations

Variables	
X	time to failure (the lifetime)
Y	repair (recovery) time after a failure occurs
Z	time to perform a PM
TP	decision variable—time to start a PM process
$V = \min(X, TP)$	time to interruption of the operation time caused by either failure or PM
$(TP_1, \ldots, TP_k)$	vector of decision variables of k components, days
TP_i	decision variable of i-th component determining its PM strategy, days
$TP_{i,min}$; $TP_{i,max}$	minimal and maximal possible values of TP_i
$TP_{i,mid}$	$(TP_{i,min} + TP_{i,max})/2$
$C_S(TP_1, \ldots, TP_k)$	total cost of maintenance of a system configuration, cost units
$U_S(TP_1, \ldots, TP_k)$	stabilized (asymptotic) system unavailability at the end of $\mathrm{T_M}$
$n_{i,R}$	mean number of recovery actions of i-th component per mission time $\mathrm{T_M}$
$MTTI_i$	mean time to intervention of i-th component caused by either CM or PM, days
MRT_i	mean recovery time of i-th component that is due to either PM or CM, days
$MTRCM_i$	mean recovery time of i-th component due to CM, days
$MTRPM_i$	mean recovery time i-th component due to PM, days
Indices	
$F(t)$	distribution function of a random variable X
$R(t) = 1 - F(t)$	reliability function of a random variable X
$f(t)$	probability density function (pdf) of a random variable X
$G(t)$	distribution function of a random variable Y
$g(t)$	probability density function (pdf) of a random variable Y
$W(t)$	distribution function of a random variable Z
$w(t)$	probability density function (pdf) of a random variable Z
$F_V(t)$	distribution function of a random variable V
$U(t)$	instantaneous time-dependent unavailability function
$A(t) = 1 - U(t)$	instantaneous availability function
$h(x)$	renewal density
Parameters	
$C_{i,R}$	CM cost = cost of one CM intervention of the i-th component in cost units
$C_{i,PM}$	PM cost = cost of one PM intervention of the i-th component in cost units
$\mathrm{T_M}$	mission time, days
λ	the failure rate: parameter of exponential distribution of the random variable X, h^{-1}
$[TR_{min}; TR_{max}]$	parameters of rectangular distribution of the random variable Y
$[TRP_{min}; TRP_{max}]$	parameters of rectangular distribution of the random variable Z

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