

Article

Wave Propagation and Stability Analysis for Ostrovsky and Symmetric Regularized Long-Wave Equations

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Abstract: This work focuses on the propagation of waves on the water's surface, which can be described via different mathematical models. Here, we apply the generalized exponential rational function method (GERFM) to several nonlinear models of surface wave propagation to identify their multiple solitary wave structures. We provide stability analysis and graphical representations for the considered models. Additionally, this paper compares the results obtained in this work and existing solutions for the considered models in the literature. The effectiveness and potency of the utilized approach are demonstrated, indicating their applicability to a broad range of nonlinear partial differential equations in physical phenomena.

Keywords: Ostrovsky equation; (1 + 1)-dimensional SRLW equation; exact solutions; symbolic computation

MSC: 35C07; 35Q51; 37K40; 68W30



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1. Introduction

Researchers in the scientific community are presented with nonlinear partial differential equations (NPDEs) capable of explaining various physical events from science to engineering [1–3]. Nonlinear partial differential equations models in mathematics and physics play an important role in theoretical sciences. The understanding of these nonlinear partial differential equations is also crucial to many applied areas such as meteorology, oceanography, and aerospace industry. A prominent area in nonlinear theory is the hunt for exact solutions to NPDEs [4,5]. Raza et al. employed a truncated Painleve approach for three coupled nonlinear Maccari's models in complex form [6]. Yel et al. considered a chiral nonlinear Schrödinger equation via the rational sine-Gordon expansion method [7]. Fadhal et al. utilized two techniques for some NFDEs with beta derivative [8]. Abuashad and Hashim applied the homotopy decomposition procedure [9]. Ahmad et al. considered a unified method [10]. Zafar et al. performed three distinct techniques for the considered NFDE [11]. Arshed et al. considered the Kraenkel–Manna–Merle model with the betaderivative [12]. Zhang and Si used the new generalized algebraic method for the (1 + 2)-dimensional nonlinear Schrödinger equation [13]. Akbulut and Islam studied the Biswas–Arshed equation with the beta derivative [14]. Ismael et al. probed the Schrodinger–Boussinesq system with the beta derivative [15]. Gurefe et al. employed the trial equation technique to the KdV equation with dual-power-lawnonlinearity [16]. Martínez et al. employed the in sub-equation technique [17]. Hosseini et al. turned to Jacobi techniques [18]. Kazmi et al. employed bifurcation-phase portraits of the q-deformed Sinh–Gordon equation [19]. Akar and Ozkan handled the sub-equation procedure [20]. Ouahid considered the unified solver technique [21]. Islam et al. used a modified simple equation method [22].

This paper will examine the Ostrovsky and (1 + 1)-dimensional symmetric regularized long wave (SRLW) equations. The Ostrovsky equation can be written as

$$uu_{xxt} - u_x u_{xt} + u^2 u_t = 0. \tag{1}$$

This equation provides a model of nonlinear waves in a rotating ocean [23–25]. The SRLW equation is given as

$$u_{tt} + u_{xx} + uu_{xt} + u_t u_x + u_{xxt} = 0. \tag{2}$$

Scientists use this equation to research physical oceanography, river floods, wave propagation in tsunami estimation, breakwater construction and control, dam-breaking problems, and coastal engineering. The modified extended tanh-function approach was employed in this equation to attain the soliton and periodic wave solutions of this equation [23].

The primary purpose of this work is to analyze wave propagations for the Ostrovsky and (1 + 1)-dimensional SRLW equations. The manuscript is structured as follows: Section 2 provides preliminary information. Section 3 briefly discusses the GERFM and includes a mathematical analysis of the suggested models and solutions. Section 4 presents the solitary wave solutions of the scrutinized equations. Section 5 provides some discussions and graphical representations of several analytical solutions. Finally, the conclusion in Section 7 summarizes the estimated findings.

2. Preliminary Information for Reduced ODE

The general form of the differential equation is given as follows:

$$P(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \dots) = 0, \tag{3}$$

where P is a polynomial of $u(x, t)$ and its partial derivatives. If we employ wave transformation in the following form,

$$u(x, t) = U(\varepsilon), \quad \varepsilon = kx - vt, \tag{4}$$

to Equation (3), we obtain a nonlinear ordinary differential equation (ODE) as follows:

$$Q(U, U', U'', U''', \dots) = 0. \tag{5}$$

3. Explanation of the GERFM

One possible way to describe the GERFM approach is as follows [26]:

- Step 1: We assume the exact solution of Equation (3):

$$u(\varepsilon) = \alpha_0 + \sum_{n=1}^m \alpha_n \psi^n(\varepsilon) + \sum_{n=1}^m \beta_n \psi^{-n}(\varepsilon), \tag{6}$$

$$\psi(\varepsilon) = \frac{\tau_1 e^{\zeta_1 \varepsilon} + \tau_2 e^{\zeta_2 \varepsilon}}{\tau_3 e^{\zeta_3 \varepsilon} + \tau_4 e^{\zeta_4 \varepsilon}}, \quad \varepsilon = kx - vt, \tag{7}$$

where τ_k and ζ_k ($1 \leq k \leq 4$) exhibit the real (or complex) numbers which will be later obtained, and $\alpha_0, \alpha_n, \beta_n$, ($1 \leq n \leq m$), the solution of Equation (6), will hold for Equation (5).

- Step 2: m can be found in the concept of the balancing principle;
- Step 3: Putting (6) into Equation (5) and collecting the like terms, we attain the polynomial equation $A(B_1, B_2, B_3, B_4) = 0$ in terms of $\beta_i = e^{\zeta_i \varepsilon}$ for $i = 1, \dots, 4$;
- Step 4: Equating the coefficients of A to zero, a set of algebraic expressions in τ_k, ζ_k ($1 \leq k \leq 4$), and $k, \omega, \alpha_0, \alpha_n, \beta_n$ ($1 \leq n \leq 4$) is reached;
- Step 5: We obtain the solutions to Equation (3) by evaluating the obtained expressions.

Remark 1. By setting $\tau_1 = \tau_3 = \tau_4 = 1$ and $\tau_2 = \zeta_1 = \zeta_2 = \zeta_3 = 0, \zeta_4 = 1$ and β_n ($n = 1, \dots, m$) are equal to zero in Equations (6) and (7), and the ERF approach is thus obtained [26].

$$\varphi(\varepsilon) = \frac{1}{1 + e^\varepsilon} \tag{8}$$

$$u(\varepsilon) = \alpha_0 + \sum_{n=1}^m \frac{\alpha_n}{(1 + e^\varepsilon)^n} \tag{9}$$

4. Applications of the GERFM

4.1. Applications to the Ostrovsky Equation

In this section, by plugging Equation (4) into Equation (1), the following equation is retrieved:

$$k^2(uu''' - u'u'') + u^2u' = 0. \tag{10}$$

The balancing number can be calculated as $m = 2$.

$$u(\varepsilon) = a_0 + a_1\psi(\varepsilon) + a_2\psi^2(\varepsilon) + \frac{b_1}{\psi(\varepsilon)} + \frac{b_2}{\psi^2(\varepsilon)} \tag{11}$$

where $\psi(\zeta)$ is exhibited by Equation (7). By plugging Equation (11) into Equation (10) and collecting all the same terms, they are turned into polynomials $A(B_1, B_2, B_3, B_4) = 0$ k and v .

Family 1: Setting

$$\begin{aligned} \tau_1 = i, \quad \tau_2 = i, \quad \tau_3 = 1, \quad \tau_4 = -1, \\ \zeta_1 = i, \quad \zeta_2 = -i, \quad \zeta_3 = i, \quad \zeta_4 = -i, \end{aligned} \tag{12}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{\cos(\varepsilon)}{\sin(\varepsilon)}. \tag{13}$$

From the surrogation of Equation (13) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming (Maple 14, Maple Inc., Waterloo, ON, Canada) software, and we find the following results:

1.1:

$$a_0 = -2k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{14}$$

Surrogating Equations (13) and (16) into Equation (11), we find the solitary wave solution of the considered equation as follows:

$$u(\varepsilon) = -2k^2 - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)} \tag{15}$$

1.2:

$$a_0 = -6k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{16}$$

Surrogating Equations (13) and (16) into Equation (11), we find the solitary wave solution of the considered equation as follows:

$$u(\varepsilon) = -6k^2 - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)} \tag{17}$$

1.3:

$$a_0 = -12k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = -6k^2. \tag{18}$$

Surrogating Equations (13) and (18) into Equation (11), we find the trigonometric exact solution of the examined model as follows:

$$u(\varepsilon) = -12k^2 - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)} - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{19}$$

1.4:

$$a_0 = -2k^2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -6k^2. \tag{20}$$

Surrogating Equations (13) and (20) into Equation (11), we find the solitary solution of the Ostrovsky model as follows:

$$u(\varepsilon) = -2k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{21}$$

1.5:

$$a_0 = -6k^2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -6k^2. \tag{22}$$

Surrogating Equations (13) and (22) into Equation (11), we find the solitary solution of the scrutinized model as follows:

$$u(\varepsilon) = -6k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{23}$$

1.6:

$$a_0 = 4k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = -6k^2. \tag{24}$$

Surrogating Equations (13) and (24) into Equation (11), we find the solitary solution of the examined model as follows:

$$u(\varepsilon) = 4k^2 - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)} - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{25}$$

Family 2: Setting

$$\begin{aligned} \tau_1 = \tau_2 = \tau_3 = 1, \tau_4 = -1, \\ \zeta_1 = \zeta_3 = 1, \zeta_2 = \zeta_4 = -1, \end{aligned} \tag{26}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{\cosh(\varepsilon)}{\sinh(\varepsilon)}. \tag{27}$$

From the surrogation of Equation (27) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

2.1:

$$a_0 = 6k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{28}$$

Surrogating Equations (27) and (28) into Equation (11), we find the solitary solution of the scrutinized model as follows:

$$u(\varepsilon) = 6k^2 - \frac{6k^2 \cosh^2(\varepsilon)}{\sinh^2(\varepsilon)}. \tag{29}$$

2.2:

$$a_0 = 2k^2, a_2 = -6k^2, a_1 = b_1 = b_2 = 0. \tag{30}$$

Surrogating Equations (27) and (30) into Equation (11), we find the solitary solution of the examined model as follows:

$$u(\varepsilon) = 2k^2 - \frac{6k^2 \cosh^2(\varepsilon)}{\sinh^2(\varepsilon)}. \tag{31}$$

2.3:

$$a_0 = -4k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = -6k^2. \tag{32}$$

Surrogating Equations (27) and (32) into Equation (11), we find the solitary solution of the examined model as follows:

$$u(\varepsilon) = -4k^2 - \frac{6k^2 \cosh^2(\varepsilon)}{\sinh^2(\varepsilon)} - \frac{6k^2 \sinh^2(\varepsilon)}{\cosh^2(\varepsilon)}. \tag{33}$$

2.4:

$$a_0 = 12k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = -6k^2. \tag{34}$$

Surrogating Equations (27) and (34) into Equation (11), we find the solitary wave solution of the examined equation as follows:

$$u(\varepsilon) = 12k^2 - \frac{6k^2 \cosh^2(\varepsilon)}{\sinh^2(\varepsilon)} - \frac{6k^2 \sinh^2(\varepsilon)}{\cosh^2(\varepsilon)}. \tag{35}$$

2.5:

$$a_0 = 2k^2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -6k^2. \tag{36}$$

Surrogating Equations (27) and (36) into Equation (11), we find the solitary solution of the Ostrovsky model as follows:

$$u(\varepsilon) = 2k^2 - \frac{6k^2 \sinh^2(\varepsilon)}{\cosh^2(\varepsilon)}. \tag{37}$$

2.6:

$$a_0 = 6k^2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -6k^2. \tag{38}$$

Surrogating Equations (27) and (38) into Equation (11), we find the solitary solution of the scrutinized model as follows:

$$u(\varepsilon) = 6k^2 - \frac{6k^2 \sinh^2(\varepsilon)}{\cosh^2(\varepsilon)}. \tag{39}$$

Family 3: Setting

$$\begin{aligned} \tau_1 = 2, \tau_2 = 3, \tau_3 = \tau_4 = 1, \\ \zeta_1 = \zeta_3 = 1, \zeta_2 = \zeta_4 = 0, \end{aligned} \tag{40}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{3 + 2e^\varepsilon}{1 + e^\varepsilon}. \tag{41}$$

From the surrogation of Equation (41) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

3.1:

$$a_0 = -36k^2, a_1 = 30k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{42}$$

Surrogating Equations (41) and (42) into Equation (11), we find the solitary solution of the Ostrovsky model as follows:

$$u(\varepsilon) = \frac{3k^2}{1 + \cosh(\varepsilon)}. \tag{43}$$

3.2:

$$a_0 = -36k^2, a_1 = 0, a_2 = 0, b_1 = 180k^2, b_2 = -216k^2. \tag{44}$$

Surrogating Equations (41) and (44) into Equation (11), we find the solitary wave solution of the examined equation as follows:

$$u(\varepsilon) = \frac{36k^2(5 \sinh(\varepsilon) + 13 \cosh(\varepsilon) + 12)}{144 \cosh^2(\varepsilon) + 169 + 312 \cosh(\varepsilon)}. \tag{45}$$

3.3:

$$a_0 = -37k^2, a_1 = 30k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{46}$$

Surrogating Equations (41) and (46) into Equation (11), we find the solitary solution of the Ostrovsky model as follows:

$$u(\varepsilon) = -\frac{k^2(\cosh(\varepsilon) - 2)}{1 + \cosh(\varepsilon)}. \tag{47}$$

3.4:

$$a_0 = -37k^2, a_1 = 0, a_2 = 0, b_1 = 180k^2, b_2 = -216k^2. \tag{48}$$

Surrogating Equations (41) and (48) into Equation (11), we find the solitary solution of the examined model as follows:

$$u(\varepsilon) = -\frac{k^2(-5 \sinh(\varepsilon) - 24 + 13 \cosh(\varepsilon))}{-5 \sinh(\varepsilon) + 13 \cosh(\varepsilon) + 12}. \tag{49}$$

Family 4: Setting

$$\begin{aligned} \tau_1 = i, \tau_2 = -i, \tau_3 = \tau_4 = 1, \\ \zeta_1 = \zeta_3 = i, \zeta_2 = \zeta_4 = -i, \end{aligned} \tag{50}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = -\frac{\sin(\varepsilon)}{\cos(\varepsilon)}. \tag{51}$$

If we substitution of Equation (51) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

4.1:

$$a_0 = -2k^2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -6k^2. \tag{52}$$

Surrogating Equations (51) and (52) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -2k^2 - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{53}$$

4.2:

$$a_0 = -6k^2, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = -6k^2. \tag{54}$$

Surrogating Equations (51) and (54) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -6k^2 - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{55}$$

4.3:

$$a_0 = 4k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = -6k^2. \tag{56}$$

Surrogating Equations (51) and (56) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = 4k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)} - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{57}$$

4.4:

$$a_0 = -12k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = -6k^2. \tag{58}$$

Surrogating Equations (51) and (58) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -12k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)} - \frac{6k^2 \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{59}$$

4.5:

$$a_0 = -2k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{60}$$

Surrogating Equations (51) and (60) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -2k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{61}$$

4.6:

$$a_0 = -6k^2, a_1 = 0, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{62}$$

Substituting Equations (51) and (62) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -6k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{63}$$

Family 5: Setting

$$\begin{aligned} \tau_1 = 3, \tau_2 = 2, \tau_3 = \tau_4 = 1, \\ \zeta_1 = \zeta_3 = 1, \zeta_2 = \zeta_4 = 0, \end{aligned} \tag{64}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = -\frac{1}{e^\varepsilon + 1}. \tag{65}$$

From the surrogation of Equation (65) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

5.1:

$$a_0 = -k^2, a_1 = -6k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{66}$$

Surrogating Equations (65) and (66) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -k^2 + \frac{6k^2}{1 + \sinh(\varepsilon) + \cosh(\varepsilon)} - \frac{6k^2}{(1 + \sinh(\varepsilon) + \cosh(\varepsilon))^2}. \tag{67}$$

5.2:

$$a_0 = 0, a_1 = -6k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{68}$$

Surrogating Equations (65) and (68) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = \frac{6k^2}{1 + \sinh(\varepsilon) + \cosh(\varepsilon)} - \frac{6k^2}{(1 + \sinh(\varepsilon) + \cosh(\varepsilon))^2}. \tag{69}$$

Family 6: Setting

$$\begin{aligned} \tau_1 = 2 - i, \tau_2 = 2 + i, \tau_3 = \tau_4 = 1, \\ \zeta_1 = \zeta_3 = i, \zeta_2 = \zeta_4 = -i, \end{aligned} \tag{70}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{2 \cos(\varepsilon) + \sin(\varepsilon)}{\cos(\varepsilon)}. \tag{71}$$

From the surrogation of Equation (71) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

6.1:

$$a_0 = -30k^2, a_1 = 0, a_2 = 0, b_1 = 120k^2, b_2 = -150k^2. \tag{72}$$

Surrogating Equations (71) and (72) into Equation (11), we find the solitary solution of the Ostrovsky model as follows:

$$u(\varepsilon) = -30k^2 + \frac{120k^2 \cos(\varepsilon)}{2 \cos(\varepsilon) + \sin(\varepsilon)} - \frac{150k^2 \cos^2(\varepsilon)}{(2 \cos(\varepsilon) + \sin(\varepsilon))^2}. \tag{73}$$

6.2:

$$a_0 = -26k^2, a_1 = 0, a_2 = 0, b_1 = 120k^2, b_2 = -150k^2. \tag{74}$$

Surrogating Equations (71) and (74) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -26k^2 + \frac{120k^2 \cos(\varepsilon)}{2 \cos(\varepsilon) + \sin(\varepsilon)} - \frac{150k^2 \cos^2(\varepsilon)}{(2 \cos(\varepsilon) + \sin(\varepsilon))^2}. \tag{75}$$

6.3:

$$a_0 = -30k^2, a_1 = 24k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{76}$$

Surrogating Equations (71) and (76) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -6k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{77}$$

6.4:

$$a_0 = -30k^2, a_1 = 24k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{78}$$

Surrogating Equations (71) and (78) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -2k^2 - \frac{6k^2 \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{79}$$

Family 7: Setting

$$\begin{aligned} \tau_1 = 2, \tau_2 = \tau_3 = \tau_4 = 1, \\ \zeta_1 = \zeta_3 = 1, \zeta_2 = \zeta_4 = 0, \end{aligned} \tag{80}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{2e^\varepsilon + 1}{1 + e^\varepsilon}. \tag{81}$$

From the surrogation of Equation (81) into Equations (6) and (11), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

7.1:

$$a_0 = -12k^2, a_1 = 18k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{82}$$

Surrogating Equations (81) and (82) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = \frac{3k^2}{1 + \cosh(\varepsilon)}. \tag{83}$$

7.2:

$$a_0 = -12k^2, a_1 = 0, a_2 = 0, b_1 = 36k^2, b_2 = -24k^2. \tag{84}$$

Surrogating Equations (81) and (84) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = \frac{12k^2(-3 \sinh(\varepsilon) + 5 \cosh(\varepsilon) + 4)}{16 \cosh^2(\varepsilon) + 40 \cosh(\varepsilon) + 25}. \tag{85}$$

7.3:

$$a_0 = -13k^2, a_1 = 18k^2, a_2 = -6k^2, b_1 = 0, b_2 = 0. \tag{86}$$

Surrogating Equations (81) and (84) into Equation (11), we find the solitary solution of the Ostrovsky model as follows:

$$u(\varepsilon) = -\frac{k^2(\cosh(\varepsilon) - 2)}{1 + \cosh(\varepsilon)}. \tag{87}$$

7.4:

$$a_0 = -13k^2, a_1 = 0, a_2 = 0, b_1 = 36k^2, b_2 = -24k^2. \tag{88}$$

Surrogating Equations (81) and (88) into Equation (11), we find the solitary wave solution of the examined equation:

$$u(\varepsilon) = -\frac{k^2(5 \cosh(\varepsilon) + 3 \sinh(\varepsilon) - 8)}{5 \cosh(\varepsilon) + 3 \sinh(\varepsilon) + 4}. \tag{89}$$

In this part, fresh traveling wave solutions to Equation (1) via the GERFM will be examined. As a result, we begin with a mathematical study of the considered equation.

4.2. Applications to the SRLW Equation

Using the chain rule and travelling wave transformation Equation (4), we find:

$$(k^2 + v^2)u'' - kvuu'' - kv(u')^2 + k^2v^2u'''' = 0. \tag{90}$$

The balancing number can be calculated as $m = 2$.

$$u(\varepsilon) = a_0 + a_1\psi(\varepsilon) + a_2\psi^2(\varepsilon) + \frac{b_1}{\psi(\varepsilon)} + \frac{b_2}{\psi^2(\varepsilon)}, \tag{91}$$

where $\psi(\zeta)$ is exhibited by Equation (7). By plugging Equation (91) into Equation (90) and combining all like terms, they are turned into polynomials $A(B_1, B_2, B_3, B_4) = 0$ k and v .

Family 1: Setting

$$\begin{aligned} \tau_1 = 1, \quad \tau_2 = 1, \quad \tau_3 = 1, \quad \tau_4 = -1, \\ \varsigma_1 = 1, \quad \varsigma_2 = -1, \quad \varsigma_3 = 1, \quad \varsigma_4 = -1, \end{aligned} \tag{92}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{\cosh(\varepsilon)}{\sinh(\varepsilon)}. \tag{93}$$

We surrogate Equation (93) into Equations (90) and (91), and we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

1.1:

$$a_0 = \frac{k^2 + v^2 - 8k^2v^2}{kv}, a_1 = 0, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{94}$$

Then, we surrogate Equations (93) and (94) into Equation (91), and we find the following exact solution for the (1 + 1)-dimensional SRLW equation:

$$u(\varepsilon) = \frac{k^2 + v^2 - 8k^2v^2}{kv} + \frac{12kv \cosh^2(\varepsilon)}{\sinh^2(\varepsilon)}. \tag{95}$$

1.2:

$$a_0 = \frac{k^2 + v^2 - 8k^2v^2}{kv}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = 12kv. \tag{96}$$

When we surrogate Equations (93) and (96) into Equation (91), we obtain the following exact solution for the (1 + 1)-dimensional SRLW equation:

$$u(\varepsilon) = \frac{k^2 + v^2 - 8k^2v^2}{kv} + \frac{12kv \sinh^2(\varepsilon)}{\cosh^2(\varepsilon)}. \tag{97}$$

1.3:

$$a_0 = \frac{k^2 + v^2 - 8k^2v^2}{kv}, a_1 = 0, a_2 = 12kv, b_1 = 0, b_2 = 12kv. \tag{98}$$

Then, we surrogate Equations (93) and (98) into Equation (91) to find the exact solution for the (1 + 1)-dimensional SRLW equation as follows:

$$u(\varepsilon) = \frac{k^2 + v^2 - 8k^2v^2}{kv} + \frac{12kv \cosh^2(\varepsilon)}{\sinh^2(\varepsilon)} + \frac{12kv \sinh^2(\varepsilon)}{\cosh^2(\varepsilon)}. \tag{99}$$

Family 2: Setting

$$\begin{aligned} \tau_1 = 2, \quad \tau_2 = 3, \quad \tau_3 = 1, \quad \tau_4 = 1, \\ \zeta_1 = 1, \quad \zeta_2 = 0, \quad \zeta_3 = 1, \quad \zeta_4 = 0, \end{aligned} \tag{100}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{3 + 2e^\varepsilon}{1 + e^\varepsilon}. \tag{101}$$

When we surrogate Equation (101) into Equations (90) and (91), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

2.1:

$$a_0 = \frac{73k^2v^2 + k^2 + v^2}{kv}, a_1 = -60kv, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{102}$$

When we surrogate Equation (102) and the obtained results into Equation (91), we find:

$$u(\varepsilon) = \frac{-5k^2 \sinh(\varepsilon)(v^2 + 1) + 13v^2 \cosh(\varepsilon)(k^2 + 1) - 5v^2 \sinh(\varepsilon) + 12(v^2 + k^2) + 13k^2 \cosh(\varepsilon) - 60k^2v^2}{kv(-5 \sinh(\varepsilon) + 13 \cosh(\varepsilon) + 12)}. \tag{103}$$

2.2:

$$a_0 = \frac{73k^2v^2 + k^2 + v^2}{kv}, a_1 = -60kv, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{104}$$

When we surrogate Equation (104) and the obtained results into Equation (91), we find:

$$u(\varepsilon) = \frac{(k^2 + v^2)(\cosh(\varepsilon) + 1) + k^2v^2(\cosh(\varepsilon) - 5)}{kv(1 + \cosh(\varepsilon))}. \tag{105}$$

Family 3: Setting

$$\begin{aligned} \tau_1 = i, \quad \tau_2 = -i, \quad \tau_3 = 1, \quad \tau_4 = 1, \\ \zeta_1 = i, \quad \zeta_2 = -i, \quad \zeta_3 = i, \quad \zeta_4 = -i, \end{aligned} \tag{106}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = -\frac{\sin(\varepsilon)}{\cos(\varepsilon)}. \tag{107}$$

When we surrogate Equation (107) into Equations (90) and (91), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

3.1:

$$a_0 = \frac{v^2 + k^2 + 8k^2v^2}{kv}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = 12kv. \tag{108}$$

We surrogate Equation (108) and the found results into Equation (91) to find:

$$u(\varepsilon) = \frac{v^2 + k^2 + 8k^2v^2}{kv} + \frac{12kv \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{109}$$

3.2:

$$a_0 = \frac{v^2 + k^2 + 8k^2v^2}{kv}, a_1 = 0, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{110}$$

Then, we surrogate Equation (110) and the found results into Equation (91) to find:

$$u(\varepsilon) = \frac{v^2 + k^2 + 8k^2v^2}{kv} + \frac{12kv \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{111}$$

3.3:

$$a_0 = \frac{v^2 + k^2 + 8k^2v^2}{kv}, a_1 = 0, a_2 = 12kv, b_1 = 0, b_2 = 12kv. \tag{112}$$

We surrogate Equation (112) and the found results into Equation (91) to find:

$$u(\varepsilon) = \frac{v^2 + k^2 + 8k^2v^2}{kv} + \frac{12kv \cos^2(\varepsilon)}{\sin^2(\varepsilon)} + \frac{12kv \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{113}$$

Family 4: Setting

$$\begin{aligned} \tau_1 = 2, \quad \tau_2 = 1, \quad \tau_3 = 1, \quad \tau_4 = 1, \\ \zeta_1 = 1, \quad \zeta_2 = 0, \quad \zeta_3 = 1, \quad \zeta_4 = 0, \end{aligned} \tag{114}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{2e^\varepsilon + 1}{1 + e^\varepsilon}. \tag{115}$$

When we surrogate Equation (115) into Equations (90) and (91), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

4.1:

$$a_0 = \frac{25k^2v^2 + k^2 + v^2}{kv}, a_1 = -36kv, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{116}$$

We surrogate Equation (116) and the found results into Equation (91) to obtain

$$u(\varepsilon) = \frac{(k^2 + v^2)(\cosh(\varepsilon) + 1) + k^2v^2 \cosh(\varepsilon) - 5k^2v^2}{kv(1 + \cosh(\varepsilon))}. \tag{117}$$

4.2:

$$a_0 = \frac{25k^2v^2 + k^2 + v^2}{kv}, a_1 = 0, a_2 = 0, b_1 = -72kv, b_2 = 48kv. \tag{118}$$

We surrogate Equation (118) and the found results into Equation (91) to obtain

$$u(\varepsilon) = \frac{(3 \sinh(\varepsilon) + 5 \cosh(\varepsilon))(k^2 + v^2) + k^2v^2(5 \cosh(\varepsilon) + 3 \sinh(\varepsilon)) + 4(k^2 + v^2 - 5k^2v^2)}{kv(3 \sinh(\varepsilon) + 4 + 5 \cosh(\varepsilon))}. \tag{119}$$

Family 5: Setting

$$\begin{aligned} \tau_1 = i, \quad \tau_2 = i, \quad \tau_3 = 1, \quad \tau_4 = -1, \\ \zeta_1 = i, \quad \zeta_2 = -i, \quad \zeta_3 = i, \quad \zeta_4 = -i, \end{aligned} \tag{120}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{\cos(\varepsilon)}{\sin(\varepsilon)}. \tag{121}$$

When we surrogate Equation (121) into Equations (90) and (91), we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

5.1:

$$a_0 = \frac{v^2 + 8v^2k^2 + k^2}{kv}, a_1 = 0, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{122}$$

Surrogating Equation (122) and the found results into Equation (91) leads to

$$u(\varepsilon) = \frac{v^2 + 8v^2k^2 + k^2}{kv} + \frac{12kv \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{123}$$

5.2:

$$a_0 = \frac{v^2 + 8v^2k^2 + k^2}{kv}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = 12kv. \tag{124}$$

Surrogating Equation (124) and the found results into Equation (91) leads to

$$u(\varepsilon) = \frac{v^2 + 8v^2k^2 + k^2}{kv} + \frac{12kv \sin^2(\varepsilon)}{\cos^2(\varepsilon)}. \tag{125}$$

5.3:

$$a_0 = \frac{v^2 + 8v^2k^2 + k^2}{kv}, a_1 = 0, a_2 = 12kv, b_1 = 0, b_2 = 12kv. \tag{126}$$

Surrogating Equation (126) and the found results into Equation (91) leads to

$$u(\varepsilon) = \frac{v^2 + 8v^2k^2 + k^2}{kv} + \frac{12kv \sin^2(\varepsilon)}{\cos^2(\varepsilon)} + \frac{12kv \cos^2(\varepsilon)}{\sin^2(\varepsilon)}. \tag{127}$$

Family 6: Setting

$$\begin{aligned} \tau_1 &= -1, & \tau_2 &= 0, & \tau_3 &= 1, & \tau_4 &= 1, \\ \zeta_1 &= 0, & \zeta_2 &= 0, & \zeta_3 &= 1, & \zeta_4 &= 0, \end{aligned} \tag{128}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = -\frac{1}{e^\varepsilon + 1}. \tag{129}$$

Then, we surrogate Equation (129) into Equations (90) and (91), and we find an equation system. Then, we solve this system using mathematical programming software, and we find the following results:

6.1:

$$a_0 = \frac{k^2 + v^2 + k^2v^2}{kv}, a_1 = 12kv, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{130}$$

Surrogating Equation (130) and the found results into Equation (91) leads to

$$u(\varepsilon) = \frac{(k^2 + v^2) \cosh(\varepsilon) + k^2v^2 \cosh(\varepsilon) + v^2 + k^2 - 5k^2v^2}{kv(1 + \cosh(\varepsilon))}. \tag{131}$$

Family 7: Setting

$$\begin{aligned} \tau_1 &= 2 - i, & \tau_2 &= 2 + i, & \tau_3 &= 1, & \tau_4 &= 1, \\ \zeta_1 &= i, & \zeta_2 &= -i, & \zeta_3 &= i, & \zeta_4 &= -i, \end{aligned} \tag{132}$$

in Equation (7), one acquires

$$\psi(\varepsilon) = \frac{\sin(\varepsilon) + 2 \cos(\varepsilon)}{\cos(\varepsilon)}. \tag{133}$$

Substituting Equation (133) and the found results into Equation (91) leads to

7.1:

$$a_0 = \frac{56k^2v^2 + k^2 + v^2}{kv}, a_1 = 0, a_2 = 0, b_1 = -240kv, b_2 = 300kv. \tag{134}$$

Substituting Equation (134) and the found results into Equation (91) leads to

$$u(\varepsilon) = \frac{k^2v^2(-240 \sin(\varepsilon) \cos(\varepsilon) - 100 \cos^4(\varepsilon) + 220 \cos^2(\varepsilon) + 56) - (10 \cos^2(\varepsilon) - 1 - 25 \cos^4(\varepsilon))(v^2 + k^2)}{kv(-10 \cos^2(\varepsilon) + 25 \cos^4(\varepsilon) + 1)}. \tag{135}$$

7.2:

$$a_0 = \frac{56k^2v^2 + k^2 + v^2}{kv}, a_1 = -48kv, a_2 = 12kv, b_1 = 0, b_2 = 0. \tag{136}$$

Substituting Equation (136) and the found results into Equation (91) leads to

$$u(\varepsilon) = \frac{(k^2 + v^2) \cos^2(\varepsilon) + k^2v^2(12 - 4 \cos^2(\varepsilon))}{kv \cos^2(\varepsilon)}. \tag{137}$$

5. Review on Stability Analysis and Graphical Representations of the Results

The stability property of the solutions is closely related to momentum in the Hamilton system. From this point of view, the following formula is given for the Hamiltonian system of the solution:

$$\eta_H = \frac{1}{2} \int_{-\varepsilon}^{\varepsilon} u^2(\xi) d\xi,$$

where $u(\xi)$ is the solution of the model; then, we calculate the momentum of the Hamilton system as follows:

$$\frac{\partial \eta}{\partial \omega} |_{\omega=\sigma} > 0,$$

where σ is optional constant [27–29].

If we substitute $k = 0.5$ in Equation (15), we obtain the Hamilton system in the square area of $[-1, 1]$ and do the necessary operations and find the condition as follows:

$$\frac{\partial \eta}{\partial v} |_{v=2} = 0.7903143020 > 0.$$

According to the result, we can say that our solution (15) is stable for the assumed conditions. If we chose the square area of $[-2, 2]$ by the same constants, we obtain the unstable result.

With the same procedure above, when we substitute $k = 0.5, v = 2$ Equation (47) in the square area of $[-2, 2]$, the stable condition is obtained as follows:

$$\frac{\partial \eta}{\partial v} |_{v=2} = 0.1161257398 - 3.10^{-10}i.$$

This value is located in the 4th region of the coordinate system, so the result is stable.

If we obtain the Hamilton system for Equation (109) with $k = 0.5$ in the square area of $[-2, 2]$, the condition is obtained as follows:

$$\frac{\partial \eta}{\partial v} |_{v=2} = -35833.30446 < 0.$$

So, the result is unstable.

Finally, the condition is obtained for Equation (131) as follows:

$$\frac{\partial \eta}{\partial v} |_{v=2} = 146.8890868 + 1.700745411.10^{-8}i,$$

where $k = 0.5$ and the square area of $[-2, 2]$ is assumed. So, the result is stable.

In this part of the paper, we provide graphical representations of some results. These representations are given by three-dimensional, two-dimensional, and contour plots. All three-dimensional and contour plots were drawn when $k = 0.5$ and $v = 2$. Moreover, the red line was drawn when $t = 0$, the green line was drawn when $t = 0.5$, and the blue

line was drawn when $t = 1$ for all two-dimensional plots. Figure 1 depicts graphical representations of Equation (15) when $k = 0.5$ and $v = 2$.

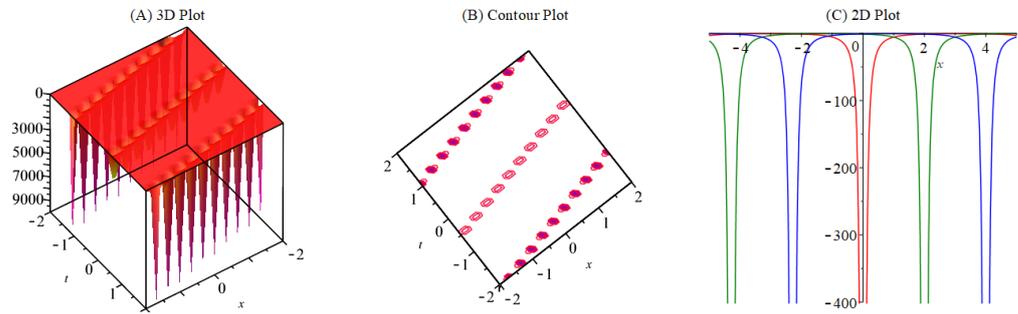


Figure 1. Plots of the Equation (15) when $k = 0.5$ and $v = 2$.

Figure 2 depicts graphical representations of Equation (19).

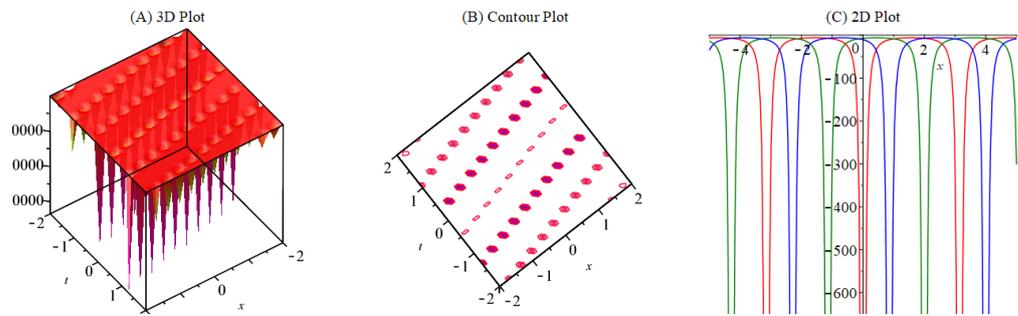


Figure 2. Plots of Equation (19) when $k = 0.5$ and $v = 2$.

Figure 3 depicts graphical representations of Equation (29).

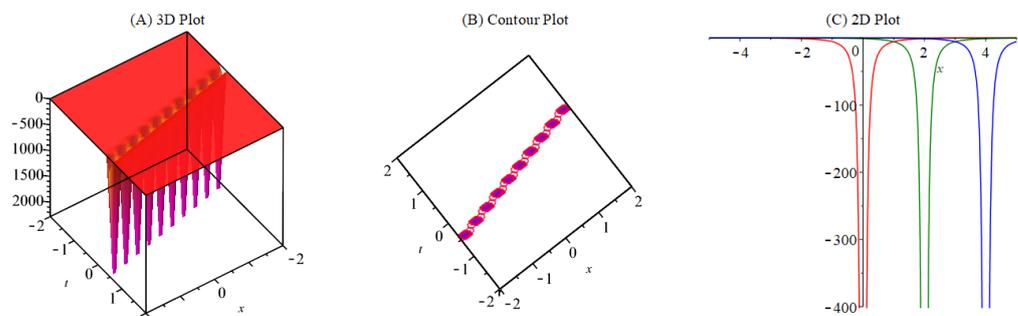


Figure 3. Plots of Equation (29) when $k = 0.5$ and $v = 2$.

Figure 4 depicts graphical representations of Equation (47).

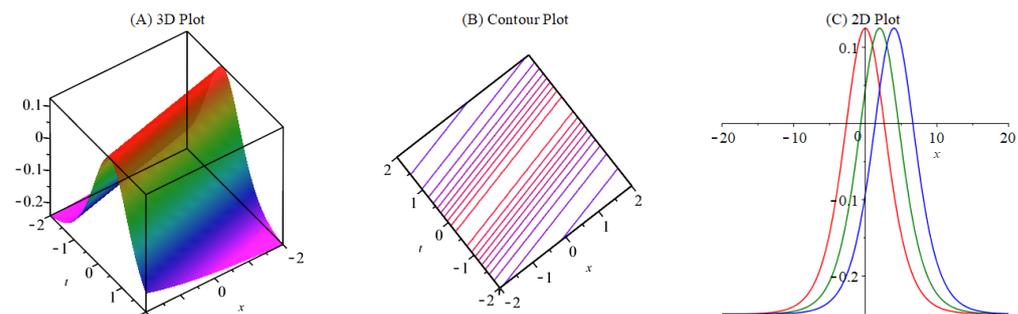


Figure 4. Plots of Equation (47) when $k = 0.5$ and $v = 2$.

Figure 5 depicts graphical representations of Equation (95).

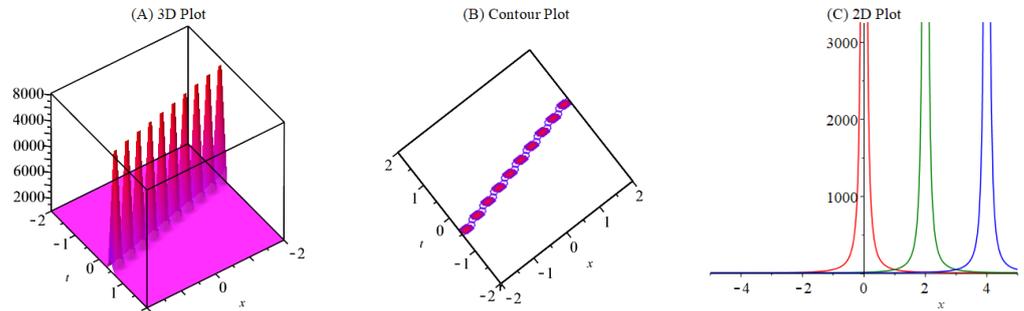


Figure 5. Plots of Equation (95) when $k = 0.5$ and $v = 2$.

Figure 6 depicts graphical representations of Equation (109).

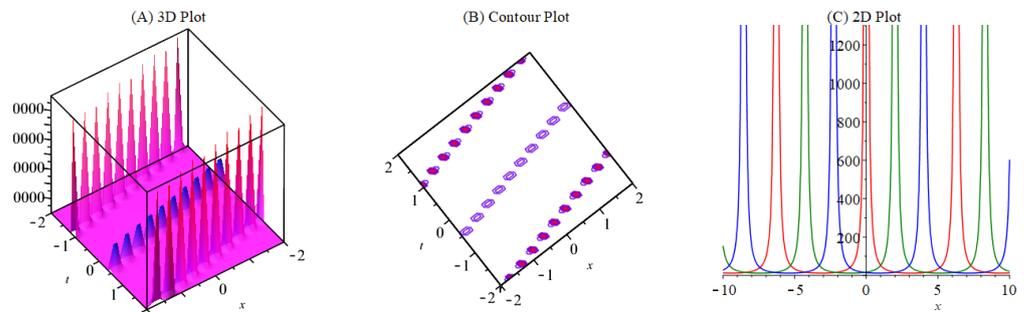


Figure 6. Plots of Equation (109) when $k = 0.5$ and $v = 2$.

Figure 7 depicts graphical representations of Equation (131).

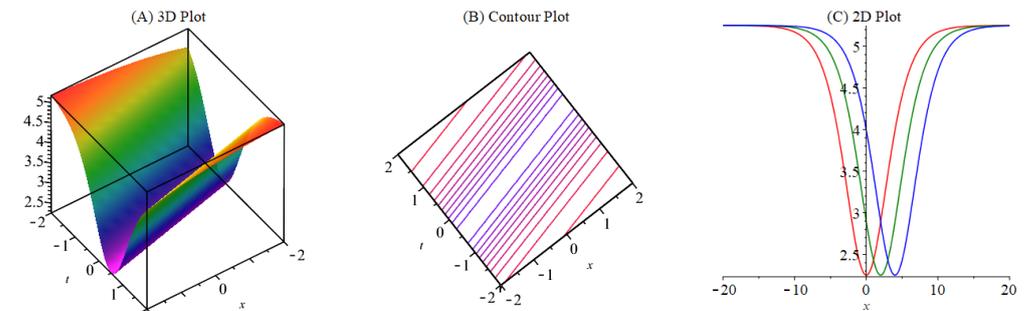


Figure 7. Plots of Equation (131) when $k = 0.5$ and $v = 2$.

6. Discussion

In this article, seven families of solutions were originated via the GERFM, each different from the other. The obtained exact solutions differ from those obtained in the literature [23–25]. Equations (15)–(89) offer a variety of different types of solutions by equating different parameter values. Arbitrary parameters are contained in the obtained results, and different solutions can be constructed by equating the parameters to unlike values. Moreover, the depictions of contour, 2D, and 3D plots are formed. Plots 1, 2, and 3 represent dark solitary waves, and Figure 4 represents bright waves. All figures inherit contour, 2D, and 3D plots for the related results. The red, green, and blue lines are plotted as $t = 0$, $t = 0.5$, and $t = 1$, respectively, in all 2D figures.

Figure 1 is plotted for the values $v = 1, k = 0.5$ and $\beta = 0.9999$ in Equation (15). This plot represents a dark soliton solution.

7. Conclusions

The current paper investigated new exact solutions and their stability analysis via the Ostrovsky and SRLW equations. The GERFM is utilized to find novel soliton solutions for the model under examination. We have conducted an analysis of the physical properties of the produced solutions. When we compared our results with earlier results in the literature, we obtained a wide family of solutions, and we observe that the movements of our results are different from one another. The resulting solutions are fresh and innovative, having not been reported in previous research, and they are treasured for describing nonlinear physical structures. To further explain the dynamic nature of the solutions, the obtained solutions are displayed in 3D and 2D graphs. Figures 1–4 illustrate the obtained optical solitons for various values. One future goal is to identify other exact solutions to this model utilizing similar integration strategies. We anticipate that the given results will be valuable in mathematical physics. We plotted 3D, contour, and 2D plots for some results. Graphical representations are useful for understanding wave motions. Results enhance the nonlinear dynamical behavior of a given system and demonstrate the effectiveness of the employed methodology, and they will be beneficial to a large number of engineering model specialists.

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