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An Information Recognition and Time Extraction Method of Tracking a Flying Target with a Sky Screen Sensor Based on Wavelet Modulus Maxima Theory

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Abstract: Aiming at the problems of big noise, lots of false targets, and accurate time extraction while tracking a flying target in the signal from a sky screen sensor, a flying target recognition and time extraction method is proposed, based on wavelet transformation. The noisy signal output by the sky screen sensor is filtered with wavelet transformation to filter out some high-frequency components; the filter is designed to handle the signal time frequency characteristics of the flying target and noise. To improve the recognition efficiency of whether the signal includes tracking of the flight target, based on a two-class discriminant model, the wavelet Fisher discriminant method is used to construct the feature vector of the false target and the flying target signal, and the recognition method of the flying target signal is studied. According to the wavelet modulus maxima theory, the single target signal is isolated, and the time moment of the flying target passing through the detection screen is calculated. The velocities calculated based on the flying target signal recognition method proposed in this paper and based on the least-mean-squares algorithm of the traditional sky screen sensor velocity measurement system are compared with the net target velocity measurement system. The results show that the velocity data obtained by the method in this paper are closer to the true value of the target flight velocity, and the average error between the velocity value obtained by the method in this paper and the standard net target velocity measurement system is less than 0.954 m/s, which verifies the superiority of the method proposed in this paper.



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MSC: 68T10; 65T60

1. Introduction

In the weapon range test, the velocity, co-ordinates and intensity of the flying target are the basic parameters to measure the performance of the gun weapon. The high-precision acquisition of these parameters is a constraint to improve the development of the gun weapon. The sky screen sensor is a detection instrument based on photoelectric conversion, which is used to detect the time of the moment when the flying target reaches a predetermined position in space [1–3]. Reference [4] studied the four-screen array intersection test device, and reported the velocity and coordinates of the flying target passing through the detection screens. This test device requires the flying target to enter the detection screens vertically, otherwise measurement error of the velocity and co-ordinate parameters is introduced. References [5–8] have calculated the velocity and co-ordinate parameters of a flying target through the six-screen array, which suppose that the flying target is operating in linear motion. Reference [9] studied the velocity and position parameters of the flying target with the seven-screen array intersection. This screen array is mainly comprised of an imaging lens, a slit diaphragm, a photoelectric conversion device, and a signal processing

circuit. Due to the effect of the slit diaphragm, the field of view of the imaging lens is fan-shaped with a certain thickness, which is usually called the sky screen sensor [10]. Once the flying target enters the detection screen, it covers part of the light projected into the slit by the sky screen sensor to change the photocurrent on the photoelectric conversion device. The change signal outputs a pulse signal after processing and amplifying the circuit and shaping it. The output pulse signal is used as the start and stop counting signal of the sky screen sensor test system. The sky screen sensor has many advantages, such as a large target surface; long-distance detection; the ability to repeat work continuously; no need for an artificial light source; and great advantages for the velocity measurement of elevation shooting [11]. For the signal acquired by the sky screen sensor, there are two processing modes for calculating parameters: the hardware mode of signal processing circuits and the software mode of acquiring the acquisition card.

For the hardware mode of signal processing circuits, it is difficult to accurately obtain the dynamic parameters of the flying target due to the constraints of environmental factors and the test equipment. This is mainly seen in two aspects. First, the attitude of the flying target is in a random dispersion state, and there is a large difference in the path and trajectory of each flying target. The target information output by the sensor with the photoelectric detection equipment as the core is, obviously, different. It leads to the time error of the same flying target at the time of the multiple sensor outputs. Second, there are differences in the detection sensitivity of photoelectric detection sensors, and there are also some differences in the characteristics of the flying target. These differences cause the amplitude of the output signal of the flying target to have obvious differences. Especially in the measurement of the velocity and co-ordinate parameters of the target by the array sensors with the sky screen sensor as the core, due to the difference between the attitude of the flying target and the sensitivity of the photoelectric detection sensor, the flying target parameters calculated by the hardware mode have obvious errors. As for the software mode, because the false signal is mainly caused by mosquitoes, strong light, and other environmental sources of interference with the photoelectric detector, there is a certain difference in signal amplitude and width with the real target signal, and most of the false signal can be removed during processing by the software. To some extent, this measure can improve the parameters calculation precision of the flying target.

The recognition and extraction of the time moment method of the target signal outputted by the sky screen sensor has been studied in some references, mainly using conventional signal processing methods, such as wavelet analysis, Kalman filtering, correlation function, etc. In [12], Di et al. used wavelet transformation to analyze the electromotive force generated when a target passes through the detection screen, and established a calculation method for the time interval and initial velocity of the target passing through the detection screen. In [13], based on Kalman filter principles and methods, the correlation between the target signals is analyzed after filtering the continuous flying target signal. With the tools of MATLAB, they deal with the filtered signal and the original signal, and then found the coherence function, confirming the effectiveness of filtering and analyzing pulse bomb among them. In [14], Huang et al. proposed a fast cross-correlation recognition algorithm of the target signal in the transonic target velocity measurement system. However, the research methods of target signal identification with the influence of environmental strong light change and gun vibration interference are still few. In [15], Tian et al. used a data acquisition instrument to collect signals of the target passing through the detection screen, and, then, used 30 criteria to remove singular points. Then, they designed a third-order low-pass filter to effectively filter out high-frequency noise and preserve the elastic signal. Finally, the triggering moment of the half peak could be selected correctively by means of the four-power Gaussian curve fitting for the sampling data to achieve high-precision measurement with the lower sampling rate after the time frame of the target signal was determined by threshold comparison. In [16], Lou et al. proposed a signal recognition method based on the Hopfield auto-associative neural network for sky screen sensor, which identifies and eliminates typical factor interference. In [17], Chen et al.

proposed a method using Bayesian generalized likelihood ratio tests (BGLRT) to detect the dynamic signal of sky screen sensor based velocity measurement system under poor signal-to-noise ratio (SNR). The characteristics of the dynamic signal were systematically analyzed, and, then, a Bayesian classification model was formulated based on BGLRT.

The recognition and extraction of the time moment method of the target signal in these references is mainly based on the information of the target in a certain light environment. The characteristics of the target signal have certain regularity and clarity. The target signal processing method in the existing references can accurately extract the time information of the target. However, when the environment background changes or the target characteristics change, the target signal output by the sky screen sensor will change according to the change of the environment and the target characteristics. If the existing conventional signal processing method is used, it will cause the misidentification and the time difference of the target information of each photoelectric sensor in the sky screen sensor array test system [18].

Due to the complexity of the out-of-field environment, the measurement accuracy of the sky screen sensor test system is usually affected, such as the inconsistent thickness of multiple sky screen sensors in the test system; the flying target does not pass through the detection screen vertically; and the position of the target signal extraction is inconsistent. Among the many influencing factors, the time moment extraction method of the target signal is the key to restricting the measurement accuracy. The traditional time moment extraction method of the target signal adopts the warhead trigger, the target tail trigger, and the target trigger. These methods have inconsistent conversion trigger points of the different output signals, which lead to timing errors between different screens. In order to improve the shortcomings of the existing target signal processing algorithm, for the flying target signal acquired by the sky screen sensor with noise of large amplitude, wide frequency range, the classification and recognition of false target signal, the flying target detection in the same frequency component noise as the flying target, and time moment extraction in a non-smooth flying target signal, this paper proposes a new high-velocity flying target information denoising, recognition, detection, and time moment extraction methods of the output signal of the sky screen sensor; the aim is to improve the recognition rate of the real target, obtain the precise time moment of the target passing through the detection screen, and calculate the real parameters of the flying target.

The work and innovation of this paper are as follows:

(1) An approximate frequency-domain model is constructed for the output signal of the flying target using Fourier transform. Based on the signal filtering of the wavelet transformation, the target signal output by the detection circuit of the sky screen sensor is attributed to a two-class discriminant model. The wavelet Fisher discriminant method is proposed to construct the feature vector of the signal, and the wavelet Fisher target signal recognition model is established;

(2) On the basis of the constructed signal of the flying target after filtering and recognition, the single point of the target signal is isolated according to the wavelet modulus maximum theory, and the starting time moment of the target passing through the detection screen is extracted.

The remainder of this paper is organized as follows: Section 2 establishes the block diagram of the signal processing algorithm. Section 3 explains the construction and wavelet filtering method of the output signal of the sky screen sensor. Section 4 studies the recognition method of the output signal of the detecting screen based on wavelet Fisher. Section 5 provides the extraction method of the target signal time. The signal detection analysis and test are provided in Section 6. Finally, Section 7 concludes this paper.

Through the study of the above methods, the high-frequency noise of the signal obtained by the sky screen sensor is effectively suppressed with wavelet transformation, the included flying target signal is identified using the Fisher discriminant method, the flying target position is located from the approximate low-frequency interference noise

using wavelet modulus maximum theory, and the accurate time moment of the flying target is extracted based on signal point singularity.

2. Signal Processing Method

Because there are non-uniformity and time-varying brightness of the sky-screen illumination, and the interference of insects and particles, many uncertainties produce noise and false targets in the signal. It is difficult to use the traditional signal processing method to recognition the flying target and extract the time moment accurately.

For the time-varying noise signal generated by the illumination, there are both high-frequency white noise and low-frequency background components. At the same time, the signal of high-speed flying target belongs to a frequency component. In order to keep the frequency component of the flying target signal when the noise is removed, a denoising algorithm is designed according to the time-frequency local analysis characteristics of wavelet transformation. Regarding the false target signal generated by insects and particles, in order to ensure the correctness, reliability, and high efficiency of the subsequent detection and extraction of the flying target, the signal recognition method containing the flying target based on wavelet Fisher is designed. In view of the frequency component similarity between the background and the flying target signal, the modulus maxima of wavelet transformation is used to reliably detect the flying target, and the Lipschitz exponent is used to accurately extract the arrival time of the flying target. According to the above analysis, the signal processing algorithm is shown in Figure 1:

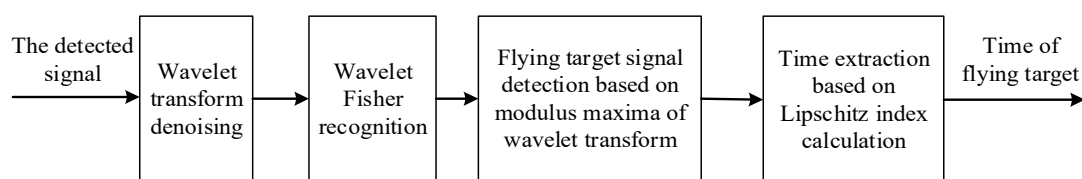


Figure 1. The block diagram of the signal processing algorithm.

3. High-Frequency Noise Filtering Method

3.1. Construction of the Output Signal of the Sky Screen Sensor

The flying target signal is the light change in the sky screen sensor when the flying target passes through the sky screen sensor. Taking the 7.62 mm flying target as an example, its length is 39 mm, and the velocity of the flying target is about 800 m/s, so the period of the flying target signal is about 48 μ s. If the signal sampling frequency of the sky screen sensor is 10 MHz, then the flying target signal will last 480 sampling points. Due to the influence of the detection circuit design, after the flying target passes through the detection screen, there is negative recoil. Coupled with the circuit design, the flying target signal period is approximately 1000 μ s, the frequency is about 1 kHz. The additive white Gaussian noise signal caused by the sky's refraction and the sun's light change are uniformly distributed in frequency, but the maximum amplitude is generally less than the flying target's signal. Assuming that the flying target signal collected by the sky screen sensor is $y(t)$, its spectrum density is $F(j\omega)$. According to the Fourier transform, the output signal of the sky screen sensor is expressed by Formula (1).

$$\begin{cases} F(j\omega) = \int_{-\infty}^{\infty} y(t)e^{-j\omega t} dt \\ y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega)e^{j\omega t} d\omega \end{cases} \quad (1)$$

where the Fourier spectrum $|F(j\omega)|$ of the flying target signal can be obtained. If n frequency points $\omega_1, \omega_2, \dots, \omega_n$ of larger amplitude found from the Fourier spectrum, the corresponding amplitudes are $A_1, A_2, \dots, A_n$, then flying target signal $y(t)$ can be superposed approximately as a series of frequency:

$$y(t) = A_1 \cdot \cos(\omega_1 t + \varphi_1) + A_2 \cdot \cos(\omega_2 t + \varphi_2) + \dots + A_n \cdot \cos(\omega_n t + \varphi_n) \quad (2)$$

The output signal of the sky screen sensor contains low-frequency signal, high-frequency signal, and so on. Usually, the low-frequency part can be eliminated by the signal processing circuit, and the high-frequency component larger than the transient of the flying target passing through the detection screen can also be eliminated by the band-pass filtering principle in the circuit. However, the signal of high-velocity target passing through the detection screen is also a high-frequency signal, so it is necessary to detect and extract the signal in a certain band-pass output signal. Therefore, according to the principle of wavelet transformation, extracting the characteristics of the flying target signal in the detection screen only needs to pay attention to the band-pass frequency characteristics.

The flying target signal acquired by the sky screen sensor always includes additive white Gaussian noise, and this noise is expressed as Formula (2).

$$f(t) = y(t) + n(t) \quad (3)$$

where $y(t)$ is the flying target signal and $n(t)$ is the noise signal.

3.2. Signal Filtering Based on Discrete Wavelet Transformation

The noise in the flying target signal affects the accurate recognition and time moment extraction of the flying target passing through the detection screen, so it is necessary to suppress the noise before further signal processing. Because part of the noise and the high-speed flying target signal belong to the same frequency component, in order to remove the high-frequency noise and retain the flying target signal, according to the time-frequency local analysis characteristics of the wavelet transformation, a denoising algorithm based on wavelet transformation is designed.

3.2.1. Principle of Wavelet Transformation

Wavelet transformation is used to perform the output signal of the sky screen sensor, and there is [19]:

$$W(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \varphi^*\left(\frac{t-\tau}{a}\right) dt \quad (4)$$

where a is the scaling factor, τ is the time-shift factor, and $\varphi^*\left(\frac{t-\tau}{a}\right)$ is the conjugate function of the wavelet generating function $\varphi(t)$. The signal $f(t)$ is discretized as $f(n)$. In the algorithm, n is selected as the discrete point of the signal sampling, and the wavelet scale coefficient $b_{i,j}$ and the wavelet coefficient $c_{i,j}$ are obtained by orthogonal wavelet decomposition, and their expressions are shown in Formula (5).

$$\begin{cases} b_{j,k} = \sum_n \bar{h}(2k-n) c_{j-1,n} \\ c_{j,k} = \sum_n \bar{g}(2k-n) c_{j-1,n} \end{cases} \quad (5)$$

where $h(n)$ and $g(n)$ are a pair of orthogonal mirror filter banks; $h(n)$ is the low-pass filter coefficient, which acts on the signal to obtain a low-frequency smooth profile c_j ; $g(n)$ is the high-pass filter coefficient, which acts on the signal to obtain the high-frequency detail part b_j , and $j(j = 1, 2, 3, \dots)$ is the wavelet decomposition level.

The wavelet reconstruction function is [20]:

$$B_{i,j} = \sum_{n'} b_{i+1,n} h(j-2n) + \sum_{n'} c_{i+1,n} g(j-2n) \quad (6)$$

where $h(j-2n)$ is the low-pass filter coefficient, which acts on the signal to obtain the low-frequency smooth profile K_i , $g(j-2n)$ is the high-pass filter coefficient, which acts on the signal to obtain the high-frequency detail part G_i , and $i(i = 1, 2, 3, \dots)$ is the wavelet decomposition level. They are one part of $B_{i,j}$.

3.2.2. Parameters Determination of Wavelet Decomposition

For the denoising of the output signal of the sky screen sensor, it is necessary to select suitable wavelet bases according to the signal characteristics and processing purposes. The wavelet bases include the Haar wavelet system, the Daubechies wavelet system, the Biorthogonal wavelet system, the Coiflet wavelet system, the Symlets wavelet system, the Morlet wavelet system, and so on. In order to improve the denoising efficiency and reconstruction accuracy of the output signal of the sky screen sensor, the wavelet bases are required to have fast wavelet attenuation and tight support. Wavelets symmetry is required to ensure that the filter has a linear phase and avoids phase distortion for signal processing. Wavelets are also required to have orthogonality to reduce the correlation and redundancy of wavelet subbands. Among these wavelets, Daubechies (Db) wavelet has the characteristics of being compactly supported, being approximately symmetric, and having orthogonality, so Db wavelet is chosen.

The wavelet order is positive correlation with vanishing moments, but vanishing moments restrict each other in efficiency and smoothness of the reconstructed signal. For Daubechies series wavelets, the higher the order and vanishing moment, the better the frequency band division, but the compactness of time domain would be weakened, and the amount of computation will be greatly increased. Therefore, for choosing the wavelet order, we should not only pay attention to good results, but also the efficiency of the algorithm. The vanishing moment of the wavelet is also related to the signal processing purposes. The processing purpose of the flying target signal is to measure the singularity, to reduce the oscillation of the wavelet, and to enhance the singularity of the signal being processed, hence, the wavelet with low order and vanishing moments is selected. In addition, in order to remove the noise and retain the flying target signal to the greatest extent, the wavelet which is close to the change trend of the flying target signal shape is selected. So, the db02 wavelet basis is selected to filter the target signal output by the sky screen sensor [21].

For different signals and different signal-to-noise ratios, there exist appropriate decomposition layers with the best result. The number of decomposition layers has a great influence on the denoising result. Usually, if there are too many decomposition layers and all the coefficients of each layer are processed with a threshold, the loss of signal information is serious, and the signal-to-noise ratio decreases, meanwhile, this leads to the increase in computation. If the number of decomposition layers is too small, the denoising result is not obvious, and the signal-to-noise ratio does not increase much, but the signal-to-noise ratio will not decrease. So, according to the characteristics of the collected signal of the flying target passing through the detection screen, Daubechies wavelet is used to decompose the detected signal in six layers in the algorithm. The Daubechies wavelet is called db wavelet for short. It is an orthogonal wavelet base and has good compact support.

3.2.3. Wavelet Threshold Denoising

By adjusting the signal of different scales, the low-frequency smooth profile K_6 of the sixth layer is observed, and K_6 is set to 0. The high-frequency component of the target signal is obtained by dividing the details of the 1–6 layers into $G_1 - G_6$ reconstructed signals.

In order to eliminate the high-frequency component of the target signal, the wavelet coefficient $w_{i,j}$ of the target signal is divided into two parts. One is the wavelet coefficient $y_{i,j}$ which characterizes the target signal and the wavelet coefficient $n_{i,j}$ which characterizes the noise signal, $w_{i,j} = y_{i,j} + n_{i,j}$. The wavelet threshold method is used to remove the noise signal. The Daubechies wavelet basis is selected to decompose six layers, and the noise signal is transformed into the wavelet domain by orthogonal discrete wavelet transformation to obtain a set of wavelet coefficients $w(i, j)$.

Because the flying target signal is continuous in space, the wavelet coefficient modulus of the flying target signal is larger in the wavelet domain. The additive white Gaussian noise is not continuous in space, so the noise is still a strong random Gaussian white noise after wavelet transformation, and the noise is smaller in the wavelet domain, which can be filtered out.

There are two methods of wavelet threshold denoising: hard threshold denoising and soft threshold denoising. For the hard threshold denoising method, if the wavelet coefficient absolute value of the point is less than the threshold, which is set to zero, the other remains unchanged; for the soft threshold method, if the wavelet coefficient absolute value of a point is greater than or equal to the threshold, it shrinks toward zero and becomes the difference value between the wavelet coefficient and the threshold, and the others are set to zero. The hard threshold method can preserve local information such as edge and detail of the signal, and the reconstructed signal has better fidelity, but the signal will have local distortion and additional vibration, while soft threshold processing can reconstruct the signal with relative smoothness, but the error is big, the edge is fuzzy, and the signal is distorted. So, the hard threshold function is selected, and the threshold is assumed to be ε . If $|w| > \varepsilon$, the signal feature is retained, if $|w| \leq \varepsilon$, the signal feature is not retained, and $w = 0$.

Then, the $w(i, j)$ of the threshold processed and updated is used for wavelet reconstruction to obtain the estimated signal $f(t)$ after denoising, and the filtered signal containing target information is approximately $y(t)$.

3.2.4. Threshold Calculation of Wavelet Denoising

For the wavelet denoising threshold ε calculation, firstly, the variance σ is calculated according to the Gaussian distribution of the noise characteristic in the wavelet domain. Then, according to the 3σ criteria: most (99.99%) of the noise coefficients are in the interval of $[-3\sigma, +3\sigma]$ and the wavelet denoising threshold was calculated with $\varepsilon = 3\sigma$.

3.2.5. The Denoising Algorithm

According to the above analysis and wavelet selection, the denoising algorithm of the output signal of the sky screen sensor is designed as follows:

- (1) According to the characteristics of the output signal of the sky screen sensor and the processing purpose of the signal, the db2 wavelet is selected;
- (2) The signal obtained by the sky screen sensor is decomposed with six-layer wavelet transformation with Formula (5);
- (3) According to the 3σ criteria, the denoising threshold value of each layer wavelet decomposition coefficient is calculated;
- (4) Based on the hard threshold denoising method, the wavelet coefficients are filtered;
- (5) The denoised flying target signal is reconstructed with Formula (5).

Figure 2 shows the collected target signal of any shot and the result of filtering through discrete wavelet transformation.

In order to ensure that the signal acquired by the sky screen sensor contains the signal of the flying target when it passes through the detection screen, the pre-triggered signal acquisition method is used and the acquisition duration time is longer. The signal containing the flying target signal is shown in Figure 2a. The amplitude of the noise is relatively large and the frequency distribution is relatively wide, and there exist high-frequency components and frequency components which are similar to the flying target signal. The wavelet denoising method is used for the noisy signal, and the result is shown in Figure 2b. The high-frequency component of the noise is suppressed effectively in Figure 2b, but the frequency component of the noise which is approximately matched to the flying target signal is still present. The change trend of the flying target signal waveform is similar to the db2 wavelet, the db2 wavelet is selected, and the waveform of the flying target signal is changed in Figure 2b. The processing result verified the correctness of choosing the wavelet, determining the vanishing moment and the wavelet decomposition level.

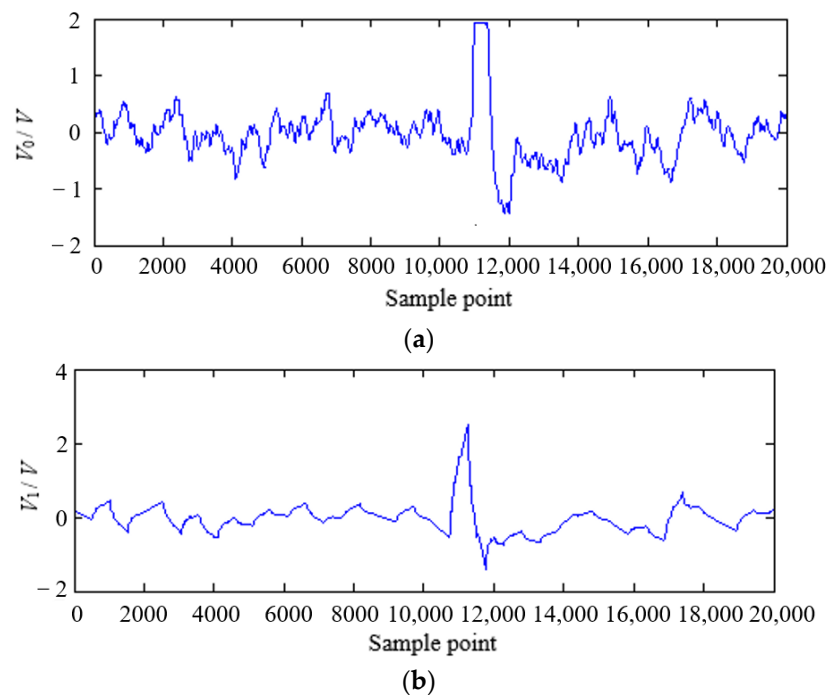


Figure 2. Collected original signal and processing result. (a) The collected original target signal. (b) The denoising result of signal.

4. Flying Target Recognition Method

When the flying target passes through the detection screen, the identification of the signal output by the sky screen sensor can be regarded as two kinds of discriminant problems. In order to quickly identify the target signal, the Fisher discriminant method is selected. The idea of the Fisher discriminant model is to project two groups of d -dimensional samples into a certain direction, and it uses the idea of variance analysis to separate the projection groups as much as possible, so as to obtain the linear discriminant function under Fisher discriminant criteria [22,23]. The function is shown in Formula (7).

$$y = Q^T X = Q_1 x_1 + Q_2 x_2 + \cdots + Q_d x_d \quad (7)$$

where $X = (x_1, x_2, \dots, x_d)^T$ is the feature vector and $Q = (Q_1, Q_2, Q_3, \dots, Q_d)^T$ is the discriminant coefficient vector, which is calculated as follows:

Step 1: The class centers of false target and flying target signal are calculated, respectively:

$$m_{n_1} = \frac{1}{n_1} \sum_{i=1}^{n_1} X_i^{(1)}, m_{n_2} = \frac{1}{n_2} \sum_{i=1}^{n_2} X_i^{(2)};$$

Step 2: The class centers are projected using the discriminant coefficient vector Q and the class center of the projected sample is calculated: $\bar{m}_{n_1} = Q m_{n_1}$, $\bar{m}_{n_2} = Q m_{n_2}$;

Step 3: The most effective direction of the discriminant coefficient vector Q of distinguishing the two classes is to maximize the center distance $|\bar{m}_{n_1} - \bar{m}_{n_2}|$ between classes after projection, minimize the intra-class dispersion distance $\bar{s}_{n_1} = \sum_{i=1}^{n_1} (X_i^{(1)} - \bar{m}_{n_1})^2$ and $\bar{s}_{n_2} = \sum_{i=1}^{n_2} (X_i^{(2)} - \bar{m}_{n_2})^2$ after projection, namely, to maximize the following equation:

$$J(Q) = \frac{|\bar{m}_{n_1} - \bar{m}_{n_2}|}{\bar{s}_{n_1}^2 + \bar{s}_{n_2}^2} \quad (8)$$

According to the calculated parameter Q from Equation (8), the feature vector of the false target and the flying target signal is projected in the direction Q , which can

make the sample points distance larger between the two classes, while smaller in the class after projection.

How to judge that the signal output by the detection circuit of the sky screen sensor contains the information of the target passing through the detection screen, can be summed up as a two-class discriminant model. The problem is described as follows: there are two populations C_1 and C_2 . Let C_1 be the background signal population, C_2 be the target signal population, and their feature vectors are d -dimensional vectors X . For a given new sample, it is necessary to determine whether it belongs to the overall C_1 or the overall C_2 , that is, to determine whether it is a background signal or contains a target signal. Let the mean values of the two populations be r_1 and r_2 , and the corresponding discriminant function values be $Q^T r_1$ and $Q^T r_2$, respectively. If $Q^T r_1 < Q^T r_2$, then the discriminant rule is $Q^T r_1 < y_0$, then it belongs to C_1 ; if $Q^T r_1 \geq y_0$, it is judged to belong to C_2 . Where y_0 is the threshold point, it can be a simple average or a weighted average of $Q^T r_1$ and $Q^T r_2$.

Although the extraction of some wavelet coefficients cannot accurately restore the original signal of the target passing through the detection screen, it is not necessary to restore the original signal for the identification of the target signal, but only to accurately and effectively extract the characteristics of the target signal for identification. As the time duration of the target passing through the detection screen is very short, the real-time performance of the algorithm is particularly important. In this way, the feature dimension is reduced under the effective wavelet decomposition level, and the real-time recognition of the target signal is improved.

According to the theory of wavelet transformation, only the wavelet coefficients c_3, c_4, c_5, d_5, c_6 , and d_6 are extracted from the six-layer wavelet decomposition, and the energy of the signal frequency band is represented as the feature to construct the feature vector: $X = (E_0, E_1, E_2, E_3, E_4, E_5)$. Let the feature vectors of n_1 times observation data of the background signal of the sky screen sensor be $X_1^{(1)}, X_2^{(1)}, \dots, X_{n_1}^{(1)}$, and the feature vectors of n_2 times observation data containing the target signal be $X_1^{(2)}, X_2^{(2)}, \dots, X_{n_2}^{(2)}$. The linear discriminant function is obtained by the Fisher criterion:

$$y = Q^T X = Q_0 E_0 + Q_1 E_1 + Q_2 E_2 + Q_3 E_3 + Q_4 E_4 + Q_5 E_5 \quad (9)$$

Combined with the discriminant rules, the collected n_1 observation data of the background signal of the sky screen sensor and the n_2 observation data containing the target signal are discriminated one by one to realize the recognition of the target signal.

In the detection, through signal filtering processing, the Fisher discriminant is used to determine the target signal, and, then, the wavelet modulus maximum theory is used to find the singular point of the Fisher discriminant output signal. Using the singular point of signal change, the initial moment of the flying target passing through the detection screen is found. Combined with the spatial geometric relationship of the sky screen sensor, the corresponding parameters of the flying target are calculated.

5. Time Moment Extraction Method

5.1. The Principle of Time Moment Extraction

Because of the unsmoothness of the flying target signal, the wavelet modulus maxima along the scale is not sufficient for flying target singularity detection, and the Lipschitz regularity of the signal at one point needs to be calculated from the attenuation of the modulus maxima, thus, we can judge whether the modulus maxima is noise or flying target signal time moment.

(1) The modulus maximum of wavelet transformation

Assuming that the filtered target estimation signal is $y'(t)$, and $y'(t) \in L^2(R)$, the smoothing function is [24]:

$$\begin{cases} \delta(t) = O(\frac{1}{1+t^2}) \\ \int_R \delta(t) dt \neq 0 \end{cases} \quad (10)$$

If $\delta_a(t) = \frac{1}{a}\delta(\frac{t}{a})$ is a scaling function, then the edge of $y'(t)$ at scale a is defined as the local break point after $y'(t)$ is smoothed, that is, $y'(t) * \delta(t)$ plays the role of smoothing $y'(t)$. According to the length and the sampling frequency of the sky screen sensor, and the frequency band of the flying target signal, the value of the wavelet transformation scale a is determined. Because the frequency of the flying target signal is about 1 K, the sky screen sensor signal acquisition frequency is 10 MHz, and the signal acquisition length is 20 k, so the binary scale $a = 2^n$, $n = 1, 2, \dots, 10$ is used.

Supposing that $\psi(t) = d\delta(t)/dt$, the multi-scale edge of the output signal of the sky screen sensor is:

$$Wy(a, u) = a^{1/2}y * \bar{\psi}_a(t) = a^{1/2} \frac{d}{dt}(y * \bar{\delta}_a(t))(t) \quad (11)$$

For the edge point, the judgment of modulus maximum is as follows:

Step 1: Suppose $T > 0$ is a threshold value, at the scale $s > 0$, if discrete wavelet transformation coefficients $|Wy(s, t)| \geq T$ is met, then $Wy(s, t)$ gets the local maximum at point of t , the point t is an edge point at scale s ;

Step 2: For discrete wavelet transform coefficients $Wy(s, t)$, if the edge point is satisfied with the following conditions:

$|Wy(s, t)| \geq |Wy(s, t-1)|$ and $|Wy(s, t)| \geq |Wy(s, t+1)|$, they cannot take the equal sign at the same time.

Then, the edge point is said to obtain the modulus maximum at the point t .

(2) The singularity point judgment

If the singularity of $y'(t)$ at point t_0 can be described by the Lipschitz exponent σ [25], let k' be a nonnegative integer, and $k' \leq \sigma \leq k' + 1$, if and only if there exist two constants M and $u_0 > 0$, and Taylor polynomials of order k' , such that for any $u \leq u_0$, there is:

$$|y(t_0 + u) - P_{k'}(u)| \leq M|u|^\sigma \quad (12)$$

where σ is the Lipschitz index of the target signal passing through the detection screen at point t_0 . The higher the derivative order of $y'(t)$ at point t_0 is, the larger the corresponding σ is, and the smoother the target signal is here. If $y'(t)$ is at the Lipschitz index $\sigma < 1$ of point t_0 , then $y'(t)$ is said to be singular at point t_0 .

5.2. Time Moment Extraction Based on Modulus Maximum and Lipschitz Index

The flying target signal has a modulus maximum, and the noise generated in some environments also has a modulus maximum, which is always superimposed on the flying target signal, and they can not be distinguished with the modular maxima. For the flying target signal, the residual noise components have a different changing trend with the wavelet transformation layers, and the Lipschitz exponent can be used to describe the different singularity of a signal and to determine the true singularity points along the modular maximal curves.

Supposing that $\delta(t)$ is a Gaussian function, wavelet $\varphi(t)$ has a vanishing moment of order k' , and order k' is differentiable, k' is a positive integer, $\sigma \leq k'$. There exists a constant M in the neighborhood of t_0 such that the wavelet transformation of target signal satisfies:

$$|W(a, t)| \leq M(a^\sigma + |t - t_0|^\sigma) \quad (13)$$

It can be seen from Formula (13) that the singular points of the flying target signal are distributed on the modulus extremum line of the signal wavelet transformation. The Lipschitz index $\sigma < 1$, the sudden change signal of the flying target shows singularity, and the Lipschitz index $\sigma > 0$. Therefore, the wavelet transformation can be used to detect the singularity of the fragment signal. If it is not a local singular point of signal $y'(t)$, then the point satisfies Formula (14).

$$|W(a_0, t)| \leq |W(a_0, t_0)| \quad (14)$$

where (a_0, t_0) is the modulus maximum point of $|W(a, t)|$ at a_0 scale. $W(a_0, t_0)$ is the corresponding modulus maximum, and the curve formed by the modulus maximum

points on the scale time (a, t) plane is the modulus maximum line. The discrete dyadic wavelet transformation is introduced, and Formula (14) becomes:

$$|W_{2^{m'}}(a, t)| \leq H(2^{m'})^\sigma (1 + |t - t_0|^\sigma) \quad (15)$$

where m' is a binary scale parameter and t is a discrete value. If the signal of the flying target passing through the detection screen is greater than 0 at the index σ of Lipschitz, then the modulus maximum of the wavelet transformation increases with the increase in the scale m' . Therefore, for the collected output signal of the detection screen, the singularity caused by the target is located, and the wavelet transformation can be used to perform multi-scale analysis on it. The singularity is determined by detecting the modulus maxima of the target signal. At the same time, when the discrete wavelet is used for signal transformation, the minimum scale a should be correctly selected according to the characteristics of the signal. Assuming that the singular point determined by the wavelet transformation modulus maximum point of the flying target signal is t_0 , that is, the starting time value $t = t_0$ is the corresponding time value of flying target passing through the detection screen, the filtering and time value extraction of the target signal of any detection screen is completed.

5.3. The Algorithm of Time Moment Extraction

The target signal recognition algorithm of the sky screen sensor is shown in Algorithm 1.

Algorithm 1. The target signal recognition algorithm of sky screen sensor

- Step 1: Input the target signal $y(t)$ collected by sky screen sensor;
 - Step 2: The Daubechies wavelet is used to decompose the target signal into six layers and reconstruct the wavelet to obtain the estimated signal $y'(t)$ after denoising;
 - Step 3: The Fisher discriminant criterion is used to determine whether $y'(t)$ belongs to population C_1 or population C_2 ;
 - If $Q^T r_1 < y_0$, it is judged to belong to C_1 , it is the background signal.
 - If $Q^T r_1 \geq y_0$, it is judged to belong to C_2 , it contains target signal.
 - Step 4: Combined with the discriminant rules, the collected n_1 times observation data of the background signal of the detection screen and the n_2 times observation data containing the target signal are discriminated one by one by using the Formula (7);
 - Step 5: Find the singular point of the Fisher discriminant output signal by wavelet modulus maximum theory;
 - Step 6: According to the singular point of the signal change, find out the time value of the target passing through the detection screen.
-

6. Signal Detection Analysis and Test

6.1. Signal Detection Analysis

The primary output signal of the detection circuit is simulated and analyzed by loading different background noise signals. Figures 3 and 4 are the original signal of the flying target when the loaded noise ratio is 18.3 dB and 14.7 dB, respectively.

According to the definition of SNR:

$$SNR = 10\lg\left(\frac{\sum_{i=1}^N s(i)^2}{\sum_{i=1}^N n(i)^2}\right) \quad (16)$$

where $s(i)$ is the output signal of the detection circuit of the sky screen sensor; $n(i)$ is noise, and N is the sampling point.

According to the Fisher target signal extraction method, the energy of the corresponding frequency band is extracted as the feature. The feature vectors are $X = (H_0, H_1, H_2, H_3, H_4, H_5)^T$, and $H_0, H_1, H_2, H_3, H_4, H_5$ are taken as the values of the flying target signal and the noise

signal, respectively, as shown in Table 1. For the two sets of characteristic values, the discriminant function is obtained by Fisher discriminant analysis.

$$y = 0.0008H_0 + 0.0011H_1 + 0.0013H_2 + 0.0015H_3 + 0.001H_4 + 0.0006H_5 \quad (17)$$

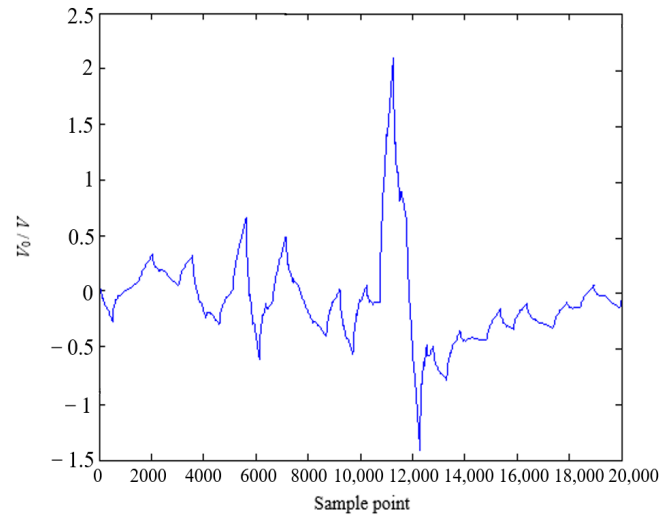


Figure 3. The original signal of the flying target with a signal-to-noise ratio of 18.3 dB.

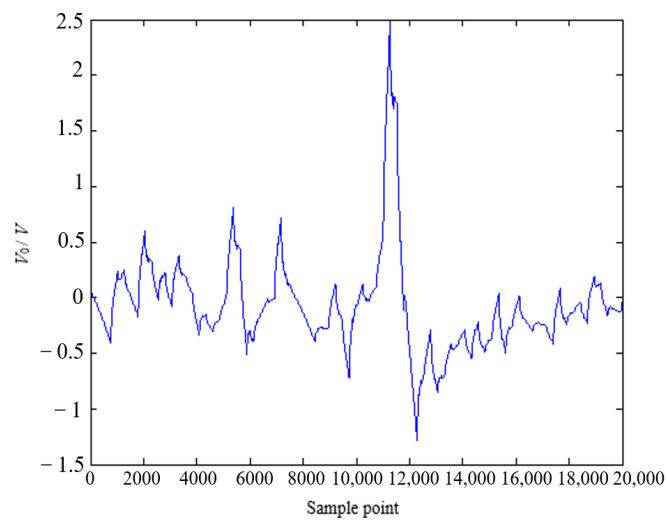


Figure 4. The original signal of the flying target with a signal-to-noise ratio of 14.7 dB.

Table 1. The feature vectors of flying target signal and background signal.

	H_0	H_1	H_2	H_3	H_4	H_5
Target signal	296.13	314.25	347.21	372.19	417.65	583.21
	304.17	319.16	359.65	385.45	429.17	589.61
	286.21	305.71	336.54	361.21	408.62	579.98
	301.39	316.58	352.81	378.37	423.48	585.81
	H_0	H_1	H_2	H_3	H_4	H_5
Background signal	190.84	227.05	253.77	299.64	316.95	403.71
	181.99	212.72	239.83	281.51	292.59	397.31
	198.62	239.14	268.92	311.32	325.64	408.20
	186.43	218.13	245.37	293.43	308.19	401.73

According to the parameters of Table 1, combined with the discriminant rules, it can be inferred that the Fisher discriminant method can process the signal-to-noise ratio of 18.3 dB. This method is superior to the traditional detection method, and the calculation is simple and the operation speed is faster than other complex detection methods.

6.2. Test and Analysis

A double parallel sky screen sensor velocity measurement system is used to measure and verify the target velocity. The principle of velocity measurement refers to the reference [26]. The basic principle is a test method developed on the basis of the detection principle of the sky screen sensor. The principle of the double parallel sky screen sensor velocity measurement system is shown in Figure 5.

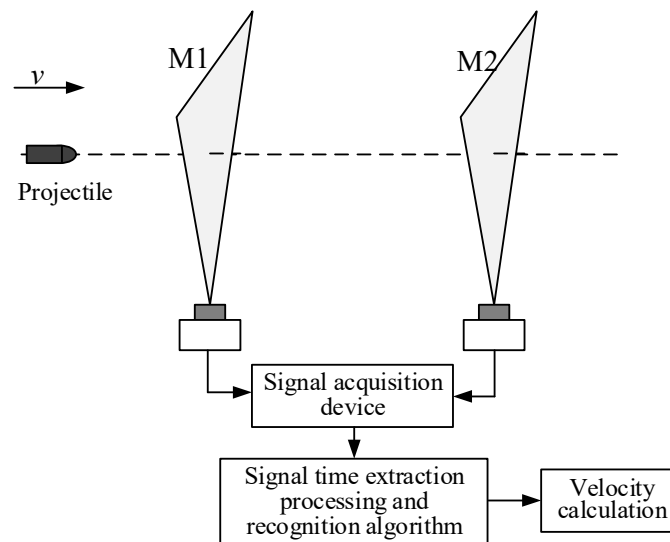


Figure 5. Principle diagram of the double parallel sky screen sensor velocity measurement system.

In Figure 5, M1 represents the sky screen sensor I, M2 represents the sky screen sensor II, and the detection screens of the two sky screen sensors are parallel to each other, and the distance between the two is S , which is about 5 m in the test, and the two sky screen sensors are orthogonal to the ballistic line, that is, the ballistic line is perpendicular to the detection screen plane. The signal acquisition device is used to collect the target signal output by two sky screen sensors, the sampling frequency of the acquisition card is 10 MHz, and the flight time value of the target is obtained by the signal time extraction processing and recognition algorithm. The diameter of the flying target is 7.62 mm, and the shooting method is that the target is shot vertically through the detection screen at a distance of about 3 m from the first detection screen M1. The initial velocity of the target is:

$$v = \frac{S}{(n_4 - n_1)/f} \quad (18)$$

Figure 6 is the original signal of a target passing through the sky screen sensor collected in the experiment.

Based on the original signal of the target in Figure 6, combined with the wavelet filtering algorithm studied, the wavelet Fisher target signal recognition method and the target starting time extraction algorithm based on modulus maxima is used to process the signal. The calculation process of target waveform filtering and time extraction processing is as follows:

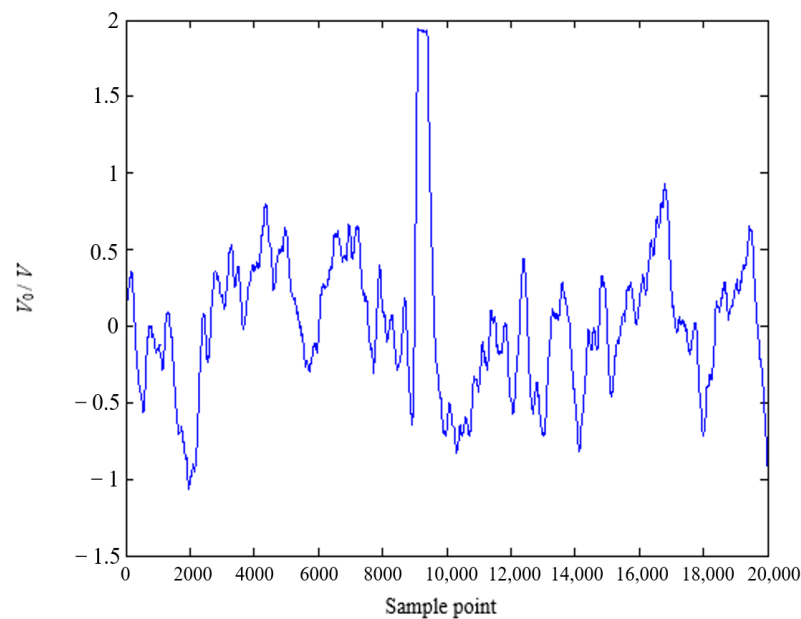


Figure 6. The original signal of a target collected in the test.

(1) According to the characteristics of the output target signal of the sky screen sensor, the original signal of the collected target is processed according to Formulas (3)–(5). The abscissa is the signal sampling point, and the ordinate represents the amplitude of the signal. In the low-pass filtering algorithm, Daubechies wavelet is used to decompose the target signal into six layers, and, then, the final filtering signal shown in Figure 7 is obtained according to the wavelet threshold function. Among them, the average noise of the target signal $E_\psi = 0.879V$, then $\varepsilon = 0.898V$. In Figure 7, CA1–CA6 denote the modal components.

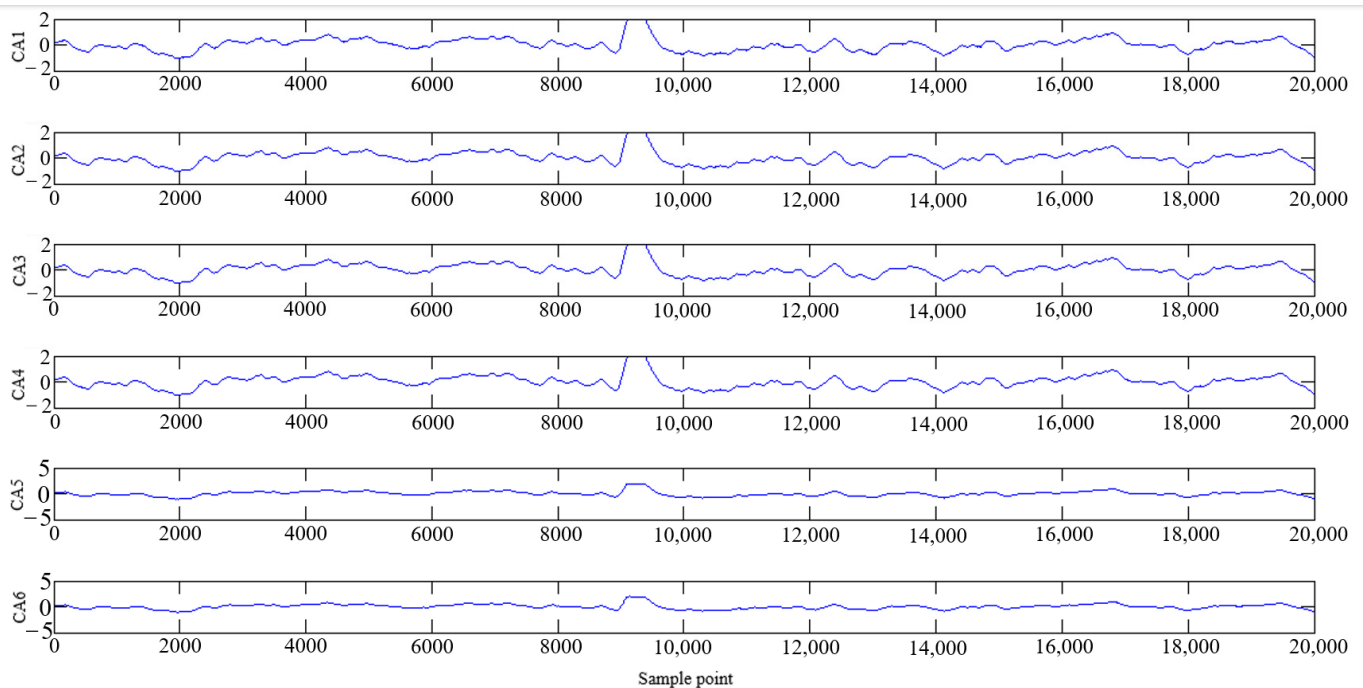


Figure 7. Signal filtering results.

(2) According to the Fisher discriminant method, the signal of Figure 7 is determined to be the target signal. The singular points of the target signal in Figure 7 are determined by

the modulus maxima of the target signal, and the time value of the target signal is extracted, as shown in Figure 8.

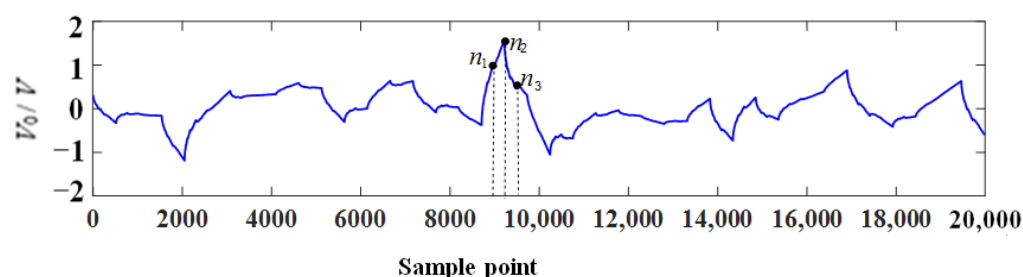


Figure 8. Maximum moment extraction.

After the output signal of the sky screen sensor is filtered, it is inevitable that there are some background noise signals. The amplitude is generally low, which is less than a certain amplitude u_0' . In Figure 8, u_0 is used as the reference voltage at a certain moment, and the corresponding sampling point is n_2 . According to the forward and backward search method, the sampling points n_1 and n_3 with the signal drop less than u_0' are found. According to the sampling time of the sampling points n_1 and n_3 , it is determined that n_1 is the initial change time of the target entering the screen, that is, the position of the singular point. The recorded abscissa 8700 is the sampling point. According to the sampling frequency selected by the acquisition system, it is at the abscissa 8700. The recorded time value is 8.7 ms, which is the corresponding time moment value of the target entering the screen.

In order to illustrate the effectiveness and superiority of the signal processing algorithm proposed in this paper, three sets of test equipment are set up and tests are conducted at the field range; three sets of test equipment are the sky screen sensor velocity measurement system based on the target signal recognition method proposed in this paper, the traditional sky screen sensor velocity measurement system based on the least-mean-squares algorithm, and the net target velocity measurement system. The position of the sky screen sensor velocity measurement system is the closest to the target firing position, the net target velocity measurement system is the farthest from the target firing position, the traditional sky screen sensor velocity measurement system is placed between the sky screen sensor velocity measurement system and the net target velocity measurement system, and the approximate interval between the three sets of equipment is 2 m. Three test systems are placed on the ballistic line, and firing of 10 targets with a diameter of 7.62 mm. Table 2 gives the test results of 10 targets firing at different positions in the effective field of view.

Table 2. Velocity test data.

No.	Sky Screen Sensor I		Sky Screen Sensor II		v_s (m/s)	v_t (m/s)	v_n (m/s)	Δv_1 (m/s)	Δv_2 (m/s)
	t_I (ms)	V_{s-I} (V)	t_{II} (ms)	V_{s-II} (V)					
1	3.8	2.14	10.7	2.05	724.88	729.35	725.63	0.75	3.72
2	5.3	1.45	12.2	1.49	725.23	687.92	726.17	0.94	38.25
3	5.9	2.04	12.7	1.86	726.25	778.34	724.88	1.37	53.46
4	8.1	1.68	14.9	1.72	726.39	726.58	725.41	0.98	1.17
5	12.4	2.15	19.2	2.35	727.2	728.34	725.75	1.45	2.59
6	10.5	2.27	17.4	1.95	723.78	726.63	725.71	1.73	0.92
7	6.5	2.01	13.3	1.89	725.71	726.66	726.02	0.31	0.64
8	7.1	1.77	13.9	1.61	726.78	725.91	724.96	0.87	0.95
9	4.9	2.49	11.7	2.39	726.61	727.34	725.68	0.93	1.66
10	12.4	2.21	19.2	2.12	725.38	729.59	725.33	0.21	4.26

In Table 2, we use $V_{s-I}(V)$ and $t_I(ms)$ to represent the peak value and extraction time of the output signal of the sky screen sensor I, $V_{s-II}(V)$ and $t_{II}(ms)$ to represent the peak value and extraction time of the output signal of the sky screen sensor II, v_s represents the velocity parameter obtained by the sky screen sensor velocity measurement system based on the target signal recognition method proposed in this paper, v_t represents the velocity parameter obtained by the traditional sky screen sensor velocity measurement system based on the least-mean-squares algorithm, and v_n represents the velocity parameter obtained by the net target velocity measurement system. Δv_1 represents the difference between the target velocity values obtained by the sky screen sensor velocity measurement system and the net target velocity measurement system. Δv_2 represents the difference between the target velocity values obtained by the traditional sky screen sensor velocity measurement system and the net target velocity measurement system.

From the data compared in Table 2, it can be seen that the average error between the sky screen sensor velocity measurement system and the net target velocity measurement system is less than 0.954 m/s. The net target velocity measurement system is a contact velocity measurement system; it is standard test equipment in target velocity testing of the weapon range, and uses the switch form of the coil to measure the velocity. The principle of net target velocity measurement is detailed in reference [27]. The results show that the velocity data obtained by the method in this paper is close to the true value of the target flight velocity. Meantime, we also give the target velocity value obtained by the target recognition method based on the least-mean-squares algorithm. For the second and third firing targets, there is a large error between the velocity measured by the traditional sky screen sensor velocity measurement system and the net target velocity measurement system, indicating that the traditional sky screen sensor velocity measurement system has problems in identifying when the target passes through the detection screens, and it is likely to identify false signals resulting in a large velocity difference, and it is not used as effective data for calculating the errors of the two methods. Therefore, the average error between the target velocity value obtained by the traditional sky screen sensor velocity measurement system and the net target velocity measurement system is 1.989 m/s. Obviously, the error is 1.035 m/s larger than the average error obtained by the method of this paper, and the velocity value obtained by the target recognition method based on the least-mean-squares algorithm can be seen in maximum or minimum cases. The main reason for this situation is that the traditional sky screen sensor test system uses the least-mean-squares algorithm. When there is a signal similar to the real signal output by the target through the infrared detection screen cannot eliminate the interference of similar signals when the dust particles or mosquito interference is similar to the real signal output by the target passing through the detection screen, which leads to the extraction of the time value of the target passing through the detection screen before or after the real time value. Therefore, the velocity values obtained by the two test methods are either too large or too small. By comparison, it is obvious that the method proposed in this paper offers obvious improvement.

7. Conclusions

In this paper, for the problem that the high-frequency noise of the signal acquired by the sky screen sensor is large, there are too many false targets, and there is noise superposed on the flying target, the paper studies a new high-velocity flying target information denoising method, flying target recognition, and time moment extraction method of flying target. Based on the signal filtering of wavelet transformation, the target signal output by the sky screen sensor is attributed to a two-class discriminant model. The wavelet Fisher discriminant method is proposed to construct the feature vector of the signal and to establish the target signal recognition model. According to the theory of wavelet modulus maxima, the singularity of the target signal is found. The time moment value of the target passing through the detection screen is calculated according to the singularity of the signal change. The main conclusions are as follows:

(1) The method based on wavelet hard threshold is suitable for the non-uniformity and time-variance of the sky-screen illumination, and the irregular changes of the flying target signal, which is advantageous to the subsequent signal recognition;

(2) The wavelet Fisher discriminant method can recognize the flying target from the false target, which avoided the error of time moment extraction;

(3) The flying target time extraction method based on wavelet modulus maxima and signal singularity can judge the properties of the modulus maximum according to the difference of the singularity exponent of the modulus maximum signal, therefore, it can remove the modulus maximum of the slowly varying background noise, preserve the modulus maxima of the mutative flying target signal, and it can accurately extract the time of the flying target;

(4) This method calculates the actual time value of the flying target passing through the detection screen by strictly looking for the starting point of the target passing through the detection screen, which reduces the time value error of the test system and improves the measurement accuracy of the system;

(5) This method is not a measure to determine whether a single point reaches a predetermined comparative voltage value that the chronograph used in the traditional sky screen target, which avoids the timing error caused by the inconsistency of the detection signal caused by the tilt of the sky screen sensor, the change of illumination, and the inconsistency of the detection circuit parameters;

(6) The time value is more accurate and can better reflect the starting time of the target passing through the detection screen. This method can also be applied to the transient signal extraction of the measurement system of the multi-screen vertical target co-ordinate and the coil target measurement system.

In this paper, the recognition and time moment extraction of the flying target in the sky screen sensor signal is studied and analyzed with the signal acquired by the collection card. Although a complete signal acquisition and processing system has been built with developed software, the experiment is expensive. We cannot do many real flying target signal acquiring and processing experiments, so we need to further accumulate experiment data to improve the efficiency and stability of the algorithm.

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