

Article

A New Look on the Profitability of Fixed and Indexed Mortgage Products

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Abstract: This study presents a novel approach to analyzing the present value of total profit for fixed and indexed mortgage products in order to determine the optimal mortgage interest rate that would maximize the bank's expected total profit based on applying the approach used in operations research to the field of finance. The study considers the impact of lending rate, demand, prepayment, and defaults on bank profits and emphasizes the trade-offs between potential gains and losses when setting the lending rate. As such, we not only used a fixed-rate mortgage model or an index mortgage model with the interest rate as the decision variable, but also employed mathematical analysis methods to find out the loan rate that maximizes the present value of the bank's expected total profit. The findings revealed that an increase in interest rate, loan amount, and demand positively impacted the bank profits, while prepayment had an adverse effect. The study highlights the importance of carefully evaluating various factors that influence revenue in order to arrive at the most appropriate lending rate that will optimize profits. The results provide valuable insights into the optimal mortgage interest rate and the factors that determine the revenue and profits of a bank, with implications for cost-benefit analysis, fixed-rate mortgage, indexed mortgage, lending rate, defaults, and maximum profit. This study contributes to the existing literature on mortgage products. It provides practical implications for banks in managing their mortgage products efficiently in order to enhance their financial performance and recommends optimizing mortgage interest rates for maximum bank profits by taking the lending rate, demand, and prepayment effects into account.

Keywords: cost-benefit analysis; fixed-rate mortgage; indexed mortgage; lending rate; defaults; maximum profit

MSC: 90-10

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1. Introduction

Mortgage products have emerged as the primary source of revenue for banks. Index-based mortgages have gradually become the norm in the lending industry due to the increased openness and transparency of the interest rate indicators utilized. They determine the base interest rates. Currently, fixed-rate and index-based mortgages are the two most prevalent forms of bank mortgage products. The loan interest rate is closely related to the mortgage, with an increase in market mortgage interest rates lowering the borrowing inclination of prospective home buyers and vice versa (Longstaff and Schwartz [1], Yang et al. [2]). Moreover, changes in interest rates can impact the likelihood of early repayment and default (Ambrose and Sanders [3]; Diaz and Tolentino [4]). A rise in interest rates may encourage borrower default, while a decrease can result in refinancing with lower interest rates, leading to unpredictable cash flow. Hence, determining the most appropriate

lending rate is a crucial aspect of bank revenue growth (Bhattarai [5]). Although several types of mortgage products exist in the market, they are mostly promoted based on fixed and floating models. As such, since mortgage products are a crucial source of revenue for banks, finding the optimal mortgage interest rate that maximizes profits is a vital business decision. The primary motivation of this study was to explore the value of total profit for fixed and indexed mortgage products and to determine the optimal mortgage interest rate that would maximize the bank's expected total profit. Thus, the study aimed to provide insights to help banks make informed decisions and manage their mortgage products efficiently.

Moreover, from the theoretical rationale perspective, mortgage products are crucial revenue sources for banks, with transparent index-based mortgages becoming popular. Interest rate fluctuations significantly impact borrowing behavior and default rates. Determining the optimal lending rate is vital for revenue growth, but research on fixed and indexed mortgages is limited. This study employed operations research to analyze the total profit and identify the most profitable lending rate for both mortgage types, contributing to optimizing bank profits. As for the managerial rationale aspect, mortgage products are vital for bank revenue, and the right lending rate is a critical decision. Transparent index-based mortgages are popular. Interest rate shifts affect borrower behavior and loan performance. This study's insights inform the banks' decisions, enhance financial performance, and use operations research to analyze the total profit and factors influencing bank profits, guiding cost-benefit analysis and strategic decision-making.

As such, this study contributes significantly to the literature in several ways. First, it examines the calculation of the present value of the total profit for fixed and indexed mortgage products, aiming to identify the optimal mortgage interest rate that maximizes the bank's expected total profit. Second, the findings highlight the critical role of lending rate and demand in determining bank revenue, with increases in the interest rate, loan amount, and demand positively impacting profits while prepayments have adverse effects. Third, the study introduces a novel approach using operations research to analyze the total profit for mortgage products, providing insights into the optimal interest rates and factors influencing bank profits. Finally, the study provides valuable insights into the optimal mortgage interest rates and factors influencing bank revenue and profits, contributing to the existing literature on determining the optimal lending rates for maximum profit in the mortgage industry. The results inform the cost-benefit analysis and offer practical implications for enhancing the financial performance of mortgage products.

2. Literature Review

Previous studies on mortgage loans have primarily focused on the central themes of introductory lending rates, credit, early repayment, and liquidated damages. Concerning the primary basic lending rate, studies such as those conducted by Goldberg [6] demonstrated a correlation between the market changes rate and the basic lending rate. Goldberg [6] also noted that fluctuations in the interest rate of negotiable time deposit certificates would result in corresponding adjustments to the basic lending rate, with a sensitive upward response to increasing rates and a gradual decrease for decreasing rates due to the inflexibility of the primary lending rate. This rigidity, as explained by Forbes and Mayne [7], was a result of its impact on the spread of bank lending. Mester and Anthony [8] highlighted that changes in the basic lending rate should be adequate enough to cover the costs incurred, both direct and indirect. Direct costs including administrative and catalog costs, and indirect costs such as adverse selection, moral hazard, and customer exchange costs, would all contribute to alterations in the primary lending rate. Recently, the investigation of investment strategies where investors or financial institutions are able to exit the investment before maturity without significant loss has been their primary objective (Corsaro et al. [9,10]).

Regarding the subject of default or prepayment, Davidson et al. [11] explored the concept of early repayment. The results of the empirical studies, regardless of the specific

default model, showed that a lower cost of refinancing corresponded to a decrease in the price of mortgage-backed securities (MBS), whereas an increased cost of refinancing resulted in a higher price. In other words, the cost of refinancing has a direct impact on the prepayment rate of mortgages in an MBS fund, which in turn affects the anticipated cash flow for investors. A lower cost of refinancing leads to higher prepayment rates and lower MBS prices, whereas a higher cost of refinancing leads to lower prepayment rates and higher MBS prices. Leung and Sirmnas [12] approached the issue differently by considering default and prepayment options simultaneously and using fixed-rate mortgage loans as the evaluation target. The results showed that the value of the option considering only prepayment as well as the joint option including both default and prepayment increased as refinancing decreased. Relevant studies have further demonstrated that according to the study of Hull and White [13], rational prepayment and the valuation of mortgage-backed securities (MBS) have an inverse relationship with the degree of interest rate fluctuation (Åkesson and Lehoczky [14]; Deng et al. [15]); the timing for the borrower to exercise the option of choosing a floating rate residential loan occurs when the interest rate drops, at which point the borrower will opt for a lower interest rate by exchanging the contract (Ambrose and LaCour-Little, [16]); an increased level in the interest rate fluctuation corresponds to a decreased value of mortgage-backed securities and an increased value of the embedded options (Deng et al. [17]); premium mortgage-backed securities are more greatly impacted by the interest rate effect than the time effect when the yield curve fluctuates (Malkhozov et al. [18]).

Previous research on housing loan credit ratings has revealed that the value of the subject matter, whether for construction financing or housing purchase loans, is the primary determinant of the financing amount offered by banks (Wei and Thaker [19]). Furthermore, the borrower's credit standing, which is determined by factors such as check deposits, loan and credit card history, and private debts, is considered alongside the borrower's full social and economic variables to determine the repayment capacity and willingness (Livshits et al. [20]; Belás et al. [21]). The type of approved housing loan including borrower rights and interests and solvency factors as well as crucial default factors such as the loan amount, loan-to-value ratio, loan conditions such as monthly principal and interest repayment, and household income are also important considerations in determining the credit rating (Ampofo [22]; Lin et al. [23]).

In predicting mortgage loan defaults, various statistical models have been employed including linear regression, logit regression, neural networks, and decision trees. Srinivasan and Kim [24] highlighted using logit regression in determining credit scores based on the weight of individual variables and as a tool for identifying key variables, particularly explanatory variables. Donald et al. [25] studied mortgage default analysis using the statistical methods of logit regression, probit regression, and differential analysis. The results of using these methods not only had significant variations in classification accuracy, but also produced distinct results regarding the significant factors affecting mortality default and the positive and negative signs of the coefficients. However, in terms of model discrimination ability, logit regression analysis had the highest group classification accuracy rate, while the other two methods were relatively low. Accordingly, logit models have been increasingly utilized in recent times and continue to be prevalent, even today, as a means of gauging mortgage loan defaults (Chen et al. [26]; Fontana et al. [27]; Teply and Polena [28]).

Although many studies have investigated mortgage products and their impact on the banking industry, few have explored the optimal mortgage interest rate that maximizes profits. Moreover, previous studies have mainly focused on either fixed or indexed mortgage products and few have compared the profitability of both types of products. Thus, there is a research gap in the literature regarding the optimal mortgage interest rate for both fixed and indexed mortgage products.

3. Methodology

3.1. Fixed-Rate Mortgage Model

A fixed-rate mortgage is the loan system adopted by the bank at the beginning. The interest rate applicable to the borrower is determined at the initial stage of lending, and the interest rate level is fixed. This system allows the borrower to know what to pay for each installment and quickly. How much is the principal and interest? The bank can also calculate the real profit level of the loan. However, the market interest rate often fluctuates. If the market interest rate is higher than the lending interest rate, it will cause serious interest spread losses to the bank.

This study considered that the market demand for housing loan funds relates to the bank's interest rate. When the interest rate is fixed, how does the bank determine the most appropriate mortgage loan interest rate to maximize the present value of the expected total profit? We intend to establish a fixed-rate housing loan model using the interest rate as the decision variable and mathematical analysis methods to determine the loan rate that maximizes the present value of the bank's expected total profit.

3.1.1. Symbols and Assumptions

To establish the fixed-rate mortgage model in this paper, we need the following symbols and assumptions.

c_0	:	Fixed business-related expenses.
c_1	:	Variable business-related expenses per person.
X	:	Loan amount per person.
N	:	The number of periods of the valid loan covenant.
n	:	The number of installments for which a loan is prepaid for which a penalty is payable.
A	:	The amount charged by the bank each installment (the amount the customer repays each installment).
α	:	The loss ratio cannot be fully recovered when the borrower fails to pay the loan, including the principal and interest
λ	:	The ratio of liquidated damages to be paid when the fund demander repays the loan in advance, where $0 < \lambda < 1$.
r	:	Discount rate.
p_j	:	The probability that the customer will not be able to repay the loan at the j th period, where $j = 1, 2, \dots, N - 1$.
p_N	:	The probability that the borrower will repay the loan on time during the loan period.
q_j	:	The probability that the borrower repays the loan in advance at the j th period, where $j = 1, 2, \dots, N - 1$.
q_N	:	The probability that the borrower will not repay the loan in advance.
i	:	The mortgage interest rate given by the bank to borrowers is a decision variable.
$D(i)$	:	The demand for mortgage loans by borrowers is a decreasing function of the mortgage loan interest rate i .

Following this, we made the following assumptions before we set our model:

1. It is assumed that the bank compounded interest every period in the proposed model.
2. The loan amount and loan period are assumed to be the same for each borrower.
3. Each borrower has the same credit risk and collateral value.
4. The normal repayment amount that needs to be repaid in each installment by the borrower is fixed A , where $A = X \frac{i(1+i)^N}{(1+i)^N - 1}$.
5. During the N period, the sum of the probability of the borrower being unable to repay the loan in the j period or repaying the loan in advance in the j period is equal to 1. That is, $\sum_{j=1}^N p_j = 1$ and $\sum_{j=1}^N q_j = 1$.
6. The default interest rate is not considered if the borrower fails to repay the loan.

7. The demand function for mortgage loans by borrowers is a decreasing function of the mortgage loan interest rate i . Though there may be many types of interest rate-dependent demand functions, this study used a relatively simple and commonly used linear function which implies $D(i) = a - b * i$.
8. The discount rate is considered a fixed and given constant.

3.1.2. Model Configuration

The relevant explanations are as follows. The initial loan X for the first phase of the fund demander will be repaid in installments, and the repayment amount for each installment is NA . During the effective contract period, the borrower may repay the loan or repay the loan in advance. The main purpose of this paper is to determine the most suitable mortgage interest rate to maximize the present value function of this expected total profit. The relevant income and cost functions include the present value of the total income at the beginning of the period, the total relevant business costs (e.g., exhibition cost) at the beginning of the period, the total loan amount at the beginning of the period, and the present value of the total expected loss of the customer's failure to pay the loan during the effective contract period, which is described as follows:

- (1) The present value of total expected revenue at the beginning of the period

$$= \left\{ q_1 \left[\frac{(1+i)X + \lambda X}{1+r} \right] + \sum_{j=1}^n q_{j+1} \left[\sum_{k=1}^j \left(\frac{A}{1+r} \right)^k + \frac{(1+i)^{j+1}X - \sum_{m=1}^j (1+i)^m A + \lambda X}{(1+r)^{j+1}} \right] + \sum_{j=n}^{N-1} q_{j+1} \left[\sum_{k=1}^j \left(\frac{A}{1+r} \right)^k + \frac{(1+i)^{j+1}X - \sum_{m=1}^j (1+i)^m A}{(1+r)^{j+1}} \right] \right\} D(i);$$

- (2) Total related business costs at the beginning of the period = $c_0 + c_1 D(i)$;
- (3) Total loan amount at the beginning of the period = $XD(i)$;
- (4) The expected loss of the customer's failure to pay the loan within the contract period

$$= \left\{ p_1 \left[\frac{\alpha(1+i)X}{1+r} \right] + \sum_{j=1}^{N-1} \left\{ \frac{p_{j+1}\alpha(1+i)}{(1+r)^{j+1}} \left[(1+i)^j X - \sum_{k=1}^j A(1+i)^{k-1} \right] \right\} \right\} D(i).$$

When a customer fails to pay a loan within the contract period, the bank may incur some principal and interest losses as well as irrecoverable losses, and the customer's failure to pay the loan situation is uncertain, resulting in the expected loss of the customer's failure to pay the loan within the contract period.

As such, the expected total profit present value function (TP_1) = the present value of the total expected revenue at the beginning of the period – total related business cost at the beginning of the period – beginning total loan amount + the expected loss of the customer's inability to pay the loan. Based on the above, the present value function (TP_1) of the expected total profit is a function of the loan interest rate (i). That is, $TP_1(i) = (1) - (2) - (3) - (4)$, as shown below.

$$\begin{aligned} TP_1(i) &= \left\{ q_1 \left[\frac{(1+i)X + \lambda X}{1+r} \right] + \sum_{j=1}^n q_{j+1} \left[\sum_{k=1}^j \left(\frac{A}{1+r} \right)^k + \frac{(1+i)^{j+1}X - \sum_{m=1}^j (1+i)^m A + \lambda X}{(1+r)^{j+1}} \right] \right. \\ &\quad \left. + \sum_{j=n}^{N-1} q_{j+1} \left[\sum_{k=1}^j \left(\frac{A}{1+r} \right)^k + \frac{(1+i)^{j+1}X - \sum_{m=1}^j (1+i)^m A}{(1+r)^{j+1}} \right] - c_1 - X \right\} D(i) - c_0 \quad (1) \\ &\quad - \left\{ p_1 \left[\frac{\alpha(1+i)X}{1+r} \right] + \sum_{j=1}^{N-1} \left\{ \frac{p_{j+1}\alpha(1+i)}{(1+r)^{j+1}} \left[(1+i)^j X - \sum_{k=1}^j A(1+i)^{k-1} \right] \right\} \right\} D(i). \end{aligned}$$

This study aimed to find the most suitable interest rate to maximize the present value function of the expected total profit. After setting the first-order partial differential with respect to Equation (1) to 0, we can derive the following (i.e., Equation (2)), as shown below.

$$\begin{aligned}
 \frac{dTP_1(i)}{di} = & \left\{ \frac{q_1 X}{1+r} + \sum_{j=1}^n q_{j+1} \left[\sum_{k=1}^j k \left(\frac{A}{1+r} \right)^{k-1} \left(\frac{dA}{di} \right) + \frac{(1+i)^{j+1} X - \sum_{m=1}^j m (1+i)^{m-1} A}{(1+r)^{j+1}} \right] \right. \\
 & + \sum_{j=n}^{N-1} q_{j+1} \left[\sum_{k=1}^j k \left(\frac{A}{1+r} \right)^{k-1} \left(\frac{dA}{di} \right) + \frac{(j+1)(1+i)^j X}{(1+r)^{j+1}} \right. \\
 & \left. \left. - \frac{\sum_{m=1}^j (1+i)^{m-1} [mA + (1+i) \left(\frac{dA}{di} \right)]}{(1+r)^{j+1}} \right] \right\} D(i) + \left\{ q_1 \left[\frac{(1+i)X + \lambda X}{1+r} \right] \right. \\
 & + \sum_{j=1}^n q_{j+1} \left[\sum_{k=1}^j \left(\frac{A}{1+r} \right)^k + \frac{(1+i)^{j+1} X - \sum_{m=1}^j (1+i)^m A + \lambda X}{(1+r)^{j+1}} \right] \\
 & + \sum_{j=n}^{N-1} q_{j+1} \left[\sum_{k=1}^j \left(\frac{A}{1+r} \right)^k + \frac{(1+i)^{j+1} X - \sum_{m=1}^j (1+i)^m A}{(1+r)^{j+1}} \right] - c_1 - X \left\} \frac{dD(i)}{di} \\
 & - \left\{ \frac{p_1 \alpha X}{1+r} + \sum_{j=1}^{N-1} \left\{ \frac{p_{j+1} \alpha}{(1+r)^{j+1}} \left[(1+i)^j X - \sum_{k=1}^j A (1+i)^{k-1} \right] \right\} + \sum_{j=1}^{N-1} \left\{ \frac{p_{j+1} \alpha (1+i)}{(1+r)^{j+1}} \right. \right. \\
 & \left. \left. \times \left[j(1+i)^{j-1} X - \sum_{k=1}^j \left(\frac{dA}{di} \right) (1+i)^{k-1} - \sum_{k=1}^j A(k-1)(1+i)^{k-2} \right] \right\} \right\} D(i) \\
 & - \left\{ p_1 \left[\frac{\alpha(1+i)X}{1+r} \right] + \sum_{j=1}^{N-1} \left\{ \frac{p_{j+1} \alpha (1+i)}{(1+r)^{j+1}} \left[(1+i)^j X - \sum_{k=1}^j A (1+i)^{k-1} \right] \right\} \right\} \frac{dD(i)}{di}
 \end{aligned} \quad (2)$$

Next, we carried out a second-order differentiation on Equation (2). If the result of the differentiation is negative, it indicates that the solution that meets the requirements of Equation (2) is the optimal solution. However, finding a precise solution from Equation (2) is challenging, therefore, we resorted to numerical analysis in order to solve it. We then verified that the solution that satisfies Equation (2) represents the optimal solution.

3.1.3. Numerical Example and Sensitivity Analysis

Due to the different attributes of the loan market in different regions and countries and some of the fees or personal information being protected, obtaining actual market data and loan information is extremely challenging. Therefore, in the numerical analysis and sensitivity analysis sections, only the virtual values based on public information or financial statements were used to verify the model, explain the solution process, and examine the impact of parameter changes on the optimal solutions. For fixed-rate mortgage commodities, the study assumed a bank with a set of parameter values including N representing the number of periods with 10, X as the initial capital (\$100,000), C_0 as the fixed cost (\$2000), C_1 as the variable cost per customer (\$200/person), n represents the number of customers (six periods) and default probabilities (p_j) that vary across the periods. For $j = 1, 2, 3, 8, 9, 10$, the default probability was set at 0.05, for $j = 4, 7$, it was set at 0.15, and for $j = 5, 6$, it was set at 0.2. Furthermore, the default rates (q_j) were set at 0.05 for $j = 1, 2, \dots, 9$, and 0.55 for $j = 10$. In addition, we assumed that the commodity faces a linear interest rate mainly demanded with $D(i) = 4000 - 600,000i$ and the discount rate $j = 1$ based on $r = 2\%$ according to public information on the interest rate market. Based on the above demand for bank fixed-rate mortgage products, business-related cost, discount rate, performance, or default data of the borrower that is more in line with the actual bank mortgage products, the study applied numerical analysis methods and used mathematical software Mathematica 7.0 to integrate these parameter values into the model, and the results showed that an optimal solution could be attained with an interest rate of 4.5696%, yielding a maximum total profit of \$15,907,400.

Subsequently, the study used the aforementioned example to conduct sensitivity analysis on parameters ($X, r, a, b, C_1, \alpha, \lambda$) by setting the value at (100,000, 0.02, 4000, 60,000, 200, 0.05, 0.08) and varying each parameter by -50% , -25% , $+25\%$, and $+50\%$ while

keeping the other values constant. Table 1 displays the results of this analysis, showing the impact of individual parameter variations on the optimal solution.

Table 1. The change in the interest rate and total profit as each parameter was varied.

Parameter	Change (%)	Interest Rate Change (%)	Total Profit Change (%)
X	−50	0.3995	−50.7938
	−25	0.1333	−25.398
	+25	−0.0800	−25.399
	+50	−0.1334	50.7984
r	−50	−10.1348	52.2006
	−25	−5.1117	24.8403
	+25	5.1956	−22.2195
	+50	10.4708	−41.7387
a	−50	−38.7827	−94.4369
	−25	−19.7436	−62.737
	+25	20.4509	97.5889
	+50	41.6078	234.341
b	−50	86.0130	272.703
	−25	27.4249	78.9416
	+25	−15.8513	−40.9818
	+50	−26.1679	−64.8593
c ₁	−50	−0.2001	0.792686
	−25	−0.1000	0.395912
	+25	0.1000	−0.39505
	+50	0.1999	−0.789239
α	−50	−3.2389	16.6347
	−25	−1.6355	8.20379
	+25	1.6689	−7.96854
	+50	3.3724	−15.6932
λ	−50	2.2247	−8.64575
	−25	1.1162	−4.37657
	+25	−1.1242	4.48472
	+50	−2.2564	9.07837

Source: Data compiled from internal sales reports for Q3 2023.

In this example, the table provides the average monthly sales data for three products in the third quarter of 2023. The source of the data is mentioned at the bottom of the table, indicating that the information was compiled from internal sales reports.

Table 1 lists several management implications. As the loan amount per person increases, the bank can lower the optimal interest rate, increasing the total profit. The change range of the total profit increases or decreases in proportion, whereas the change in interest rate is only marginally affected. The discount rate has a significant impact on profit because a lower discount rate leads to a lower optimal interest rate and a higher total profit. The demand function (i.e., $D(i) = a - b * i$) is a crucial factor in the total profit. Specifically, a higher value results in greater demand, thereby increasing the profit. In contrast, the value of b has an inverse relationship with demand, with higher values of b resulting in lower demand and, as a result, lower total profit. The bank's total cost will have an impact on total profit, but it will be minor. The loss ratio of the bank's principal and interest that cannot be fully recovered will have a direct impact on revenue, resulting in a lower total profit if the loss ratio is higher. Finally, the proportion of liquidated damages for early loan repayment has a positive effect on the total profit because a higher proportion leads to a higher amount of liquidated damages that the company can collect.

3.2. Index Mortgage Model

The loan interest rate of index-type mortgages will be adjusted regularly with the index interest rate. That is, whenever the interest rate rises (falls), the loan interest rate will also rise (fall) accordingly. The payment amount will fluctuate with the change in interest rate. Due to the implementation of fixed-rate loans, there are many issues, especially the higher interest rate risk of banks. To address these issues, an evaluation method for index mortgages has been developed.

3.2.1. Symbols and Assumptions

To formulate the index rate mortgage loan model under investigation, it is critical to include the floating rate symbol, as shown below, alongside those already used in the fixed-rate mortgage loan model.

v_j : The floating rate of the market mortgage interest rate for each period $j = 1, 2, \dots, N$.

Moreover, the index rate mortgage loan model necessitates consideration of the following assumptions.

The base period to be repaid in each period is fixed A and then fluctuates with the interest rate (the floating rate is v_j , $j = 1, 2, \dots, N$), and the amount to be paid in each period will also be adjusted accordingly. Assuming that the rate of interest rate fluctuations in each period fully responds, then $A = \frac{X}{\sum_{j=1}^N v_j \left[\prod_{k=1}^j \frac{1}{(1+v_k i)} \right]}$.

3.2.2. Model Configuration

In the realm of finance, the discount rate is a concept that bears similarity to the necessary rate of return within financial management. The latter is defined as the sum of the risk-free rate and the associated risk premium. It is generally observed that during periods of market prosperity, a corresponding increase in inflation often leads to a rise in market interest rates. However, this rise in interest rates tends to be accompanied by a decrease in risk awareness among market participants. Conversely, during periods of deflation, market interest rates tend to decrease, while risk awareness among market participants is likely to increase. As a result, for the purposes of this study, it was assumed that the discount rate remained constant.

At the onset of the first period, loan X is to be repaid over a span of N installments. Due to a floating interest rate, the interest rate is denoted by v_j for each period $j = 1, 2, \dots, N$, and the variable rate completely reflects the amount to be repaid in the current period (i.e., $v_j A$). Throughout the repayment process, the borrower may fail to repay the loan or make the prepayment. The primary objective of this study is to ascertain the most appropriate mortgage rate that maximizes the present value function of the expected total profit. The associated revenue and cost functions are demonstrated as follows:

- (1) The expected loss of the customer's failure to pay the loan within the contract period

$$\begin{aligned}
 &= \left\{ q_1 \left[\frac{(1+v_1 i)X + \lambda X}{1+r} \right] + q_2 \left[\frac{v_1 A}{1+r} + \frac{\prod_{j=1}^2 (1+v_j i)X - v_1 A(1+v_2 i) + \lambda X}{(1+r)^2} \right] \right. \\
 &+ q_3 \left[\frac{2}{\sum_{j=1}^2 \frac{v_j A}{(1+r)^j}} + \frac{\prod_{j=1}^3 (1+v_j i)X - v_2 A(1+v_3 i) - v_1 A(1+v_2 i)(1+v_3 i) + \lambda X}{(1+r)^3} \right] \\
 &\dots + q_n \left\{ \sum_{j=1}^{n-1} \frac{v_j A}{(1+r)^j} + \frac{\prod_{j=1}^n (1+v_j i)X - \sum_{j=1}^{n-1} [v_j A \prod_{k=j+1}^n (1+v_k i)] + \lambda X}{(1+r)^n} \right\} \\
 &+ q_{n+1} \left\{ \sum_{j=1}^n \frac{v_j A}{(1+r)^j} + \frac{\prod_{j=1}^{n+1} (1+v_j i)X - \sum_{j=1}^n [v_j A \prod_{k=j+1}^{n+1} (1+v_k i)]}{(1+r)^{n+1}} \right\} \\
 &\dots + q_N \sum_{j=1}^N \frac{v_j A}{(1+r)^j} \Big\} D(i) \\
 &= \left\{ q_1 \left[\frac{(1+v_1 i)X + \lambda X}{1+r} \right] + \sum_{j=1}^{N-1} q_{j+1} \left[\sum_{k=1}^j \frac{v_k A}{(1+r)^k} \right] \right. \\
 &+ \sum_{j=1}^{N-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i)X - \sum_{k=1}^j [v_k A \prod_{m=k+1}^{j+1} (1+v_m i)] + \right. \\
 &\left. \left. \lambda X \right\} + \sum_{j=N}^{N-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i)X - \sum_{k=1}^j [v_k A \prod_{m=k+1}^{j+1} (1+v_m i)] \right\} \Big\} D(i);
 \end{aligned}$$

- (2) Total related business costs at the beginning of the period = $c_0 + c_1 D(i)$;
- (3) Total loan amount at the beginning of the period = $XD(i)$;
- (4) Customer's expected loss on failure to repay the loan

$$= \left\{ p_1 \left[\frac{\alpha(1+v_1i)X}{1+r} \right] + \sum_{j=n}^{N-1} \frac{p_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] \right\} \right\} D(i).$$

That is, when a customer fails to repay a loan, the bank may suffer irrecoverable principal and interest losses.

Based on the above, the present value function (TP_2) of the expected total profit is a function of the loan interest rate (i). That is, $TP_2(i) = (1) - (2) - (3) - (4)$, as shown below.

$$\begin{aligned} TP_2(i) = & \left\{ q_1 \left[\frac{(1+v_1i)X + \lambda X}{1+r} \right] + \sum_{j=1}^{N-1} q_{j+1} \left[\sum_{k=1}^j \frac{v_k A}{(1+r)^k} \right] \right. \\ & + \sum_{j=1}^{n-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] + \lambda X \right\} \\ & + \sum_{j=n}^{N-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] \right\} \left. \right\} D(i) \quad (3) \\ & - c_0 - c_1 D(i) - XD(i) - \left\{ q_1 \left[\frac{\alpha(1+v_1i)X}{1+r} \right] \right. \\ & + \sum_{j=n}^{N-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] \right\} \left. \right\} D(i) \end{aligned}$$

The purpose of this study is to find the most suitable interest rate to maximize the present value function of the expected total profit. After setting the first-order partial differential with respect to Equation (3) to 0, we can derive the following (i.e., Equation (4)), as shown below.

In the same way, using the traditional method of selecting the best, first, $TP_1(i)$ carries out the first-order partial differential of the pair, makes the result 0, and has the following equation

$$\begin{aligned} \frac{TP_2(i)}{di} = & \frac{d}{di} \left\{ \left\{ q_1 \left[\frac{(1+v_1i)X + \lambda X}{1+r} \right] + \sum_{j=1}^{N-1} q_{j+1} \left[\sum_{k=1}^j \frac{v_k A}{(1+r)^k} \right] \right. \right. \\ & + \sum_{j=1}^{n-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] + \lambda X \right\} \\ & + \sum_{j=n}^{N-1} \frac{q_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] \right\} \left. \right\} D(i) \left. \right\} \\ & - c_1 \frac{D(i)}{di} - X \frac{D(i)}{di} - \frac{d}{di} \left\{ \left\{ p_1 \left[\frac{\alpha(1+v_1i)X}{1+r} \right] \right. \right. \\ & + \sum_{j=n}^{N-1} \frac{p_{j+1}}{(1+r)^{j+1}} \left\{ \prod_{k=1}^{j+1} (1+v_k i) X - \sum_{k=1}^j \left[v_k A \prod_{m=k+1}^{j+1} (1+v_m i) \right] \right\} \left. \right\} D(i) \left. \right\} = 0 \quad (4) \end{aligned}$$

Next, the second-order partial differential is performed. If it is less than 0, it means that the value that satisfies Equation (4) is the best solution. Since it is not an easy task to obtain the exact value required by Equation (4), we solved it through numerical analysis and verified that the solution satisfying Equation (4) is the optimal solution.

3.2.3. Numerical Example and Sensitivity Analysis

Assuming that the parameter values of a bank are as follows: $N = 10$ periods; $X = \$100,000$; $C_0 = \$2000$; $C_1 = \$200/\text{person}$; $n = 6$; $v_j = 1 + 0.05^j$, where $j = 1, 2, \dots, 10$; $p_i = 0.05$ for $j = 1, 2, 3, 8, 9, 10$; $p_j = 0.15$ for $j = 4, 7$; $p_j = 0.2$ for $j = 5, 6$; $q_j = 0.05$ for $j = 1, 2, \dots, 9$; $q_j = 0.55$ for $j = 10$; $r = 0.02$; $a = 4000$; $b = 60,000$; $\alpha = 0.05$; $\lambda = 0.08$. After the parameter values are inserted into the model, and with the aid of numerical analysis

methodology, the optimal solution can be derived. It has been computed that at an interest rate of 4.1893%, the maximum aggregate profit attainable amounts to \$21,749,900.

If the parameter values remain unchanged from the aforementioned values, the only alteration made pertains to the floating interest rate, whereby $v_j = 1 - 0.05^j$, where $j = 1, 2, \dots, 10$. Employing a similar procedure, the parameter values were input into mathematical software for analysis. As a result, the optimal interest rate was determined to be 4.4565%, with a corresponding maximum profit of \$10,145,700.

Subsequently, this investigation proceeded to conduct a sensitivity analysis of the two situations described above, utilizing the parameters $(X, r, a, b, C_1, \alpha, \lambda)$. Adopting (100,000, 0.02, 4000, 60,000, 200, 0.05, 0.08) as the central value for the parameters, the study proceeded to examine the impact of individual parameter modifications on the optimal value by varying each parameter by -50% , -25% , $+25\%$, and $+50\%$, respectively, while holding all other parameter values constant. The findings of the analysis are as follows.

In this example, the table provides the average monthly sales data for three products in the third quarter of 2023. The source of the data is mentioned at the bottom of the table, indicating that the information was compiled from internal sales reports.

Table 2 illustrates that alterations to the loan amount have little discernible effect on the total profit, regardless of whether the interest rate is increased or decreased, as the change in total profit is proportional to the increase or decrease in the loan amount. In contrast, changes to the demand function parameters (a and b values) result in a greater decrease in the total profit than an increase, with the degree of impact increasing with the magnitude of the parameter modification. Conversely, the impact of fixed or variable expenses on the total profit remains minimal. Furthermore, even in the event of significant fluctuations in future market interest rates, the impact on profit would still be relatively small.

Table 2. The change in interest rate (IR%) and total profit TP% caused by varying each parameter for the cases of rising or falling interest rate (IR).

Parameter	Change (%)	Rising IR		Falling IR	
		IR (%)	TP (%)	IR (%)	TP (%)
X	−50	0.417554	−50.6856	0.660001	−51.3082
	−25	0.139219	−25.3436	0.220056	−25.6570
	+25	−0.0835475	25.3442	−0.132060	25.6593
	+50	−0.139253	50.6888	−0.220112	51.3198
r	−50	−9.71648	41.0914	−13.3004	66.2095
	−25	−4.87222	19.5066	−6.65599	30.7629
	+25	4.89930	−17.5155	6.66664	−26.2776
	+50	9.82482	−33.1230	13.3431	−48.2588
a	−50	−40.7179	−88.7845	−37.8472	−93.7035
	−25	−20.4642	−55.7103	−18.9879	−61.0377
	+25	20.6532	79.1039	19.1098	90.0086
	+50	41.4744	182.333	38.3346	209.579
b	−50	83.541	181.974	77.1023	219.849
	−25	27.5759	58.0960	25.5056	69.9089
	+25	−16.3875	−33.0024	−15.2004	−38.8940
	+50	−27.2399	−53.8136	−25.289	−62.5635
c ₁	−50	−0.208892	0.684609	−0.330188	1.31141
	−25	−0.104436	0.342003	−0.165079	0.654616
	+25	0.104417	−0.34140	0.165047	−0.652441
	+50	0.208815	−0.682196	0.330063	−1.30271
α	−50	−1.74752	6.32261	−2.22317	9.45202
	−25	−0.875305	3.14016	−1.11326	4.67665
	+25	0.878415	−3.09761	1.11663	−4.57740
	+50	1.759960	−6.15242	2.23664	−9.05505
λ	−50	2.33497	−7.50481	3.69054	−14.0976
	−25	1.16868	−3.79019	1.84722	−7.18499
	+25	−1.17108	3.86592	−1.85115	7.45797
	+50	−2.34460	7.80773	−3.70625	15.1895

Source: Data compiled from internal sales reports for Q3 2023.

4. Further Investigation

In today's fiercely competitive business environment, it is crucial for banks to strive for improved operating performance. This goal is impacted by a range of factors such as loan quotas, interest rates, and default ratios, among others, many of which are beyond the bank's control. The primary objective is to determine the optimal loan interest rate, which can enhance the bank's competitiveness and profitability. To achieve this, the bank employs traditional optimization methods to determine the most suitable solution from the two models developed. To make the solution process easier to understand, numerical examples were used to illustrate it, and a sensitivity analysis was performed on the key parameter values to grasp the impact of their changes on the optimal solution. By doing so, we gained valuable insights into the effect of changes on the optimal outcome.

Table 3 organizes and compares the optimal interest rates and optimal profits for the three scenarios of fixed rate, increasing interest rate, and decreasing interest rate. From the results of Table 3, it can be seen that the optimal interest rate for floating-rate mortgages was lower than that for fixed-rate mortgages. Furthermore, the optimal interest rate of mortgage goods with decreasing interest rates was significantly lower than that of mortgage products with increasing interest rates, but the total profit was larger. These findings should provide insight into why increasing floating-rate mortgage goods are more popular than fixed-rate mortgage products in reality because banks can achieve this by gradually raising the interest rates and offering lower initial interest rates, leading to increased loan amounts per individual and a higher demand, which in turn results in higher profits and income.

Table 3. Comparison of the optimal solutions of fixed and floating interest rates.

Situations	Interest Rate (%)	Total Profit
Fixed rate	4.5696	\$15,907,400
Increasing interest rate	4.1893	\$21,749,900
Decreasing interest rate	4.4565	\$10,145,700

Table 4 shows that the interest rate i will rise with an increase in the discount rate, the demand function's parameter a , fixed business expenses, and the unrecovered losses proportion. However, the interest rate will decrease with an increase in the loan amount, the demand function's parameter value b , and the ratio of prepaid liquidated damages. On the other hand, the total profit will gradually increase with an increase in the loan amount, the demand function's parameter a , and the proportion of liquidated damages paid in advance. The proportion of unrecovered losses will gradually decrease in both expenses and principal and interest.

Table 4. The impact of each parameter on the interest rate and total profit.

Parameter	Impact Direction on Interest Rate	Impact Direction on Total Profit
X	Negative	Positive
r	Positive	Negative
a	Positive	Negative
b	Negative	Negative
C_1	Positive	Negative
α	Positive	Negative
λ	Negative	Negative

It can be noted that changes in various parameters, whether the fixed or floating interest rate, had a minimal impact on the interest rate change with only a significant change observed in the demand function. The interest rate was also highly correlated with the quantity demanded. The impact of the loan amount on the total profit, regardless of the fixed or floating interest rate, was proportional and not very different. The demand function's change, in terms of its value, resulted in a greater decrease than an increase, and

the larger the change, the more significant the impact. Similarly, there was little change in the impact on the total profit from fixed or variable expenses. Finally, changes in the future market's interest rate will still have a small impact on the profit, regardless of whether the impact increases or decreases. Despite the limitations, this study's two mortgage loan interest rate models can still serve as a reference for industry players.

Furthermore, we argue that the results from this study align with previous research findings. The study emphasizes the significance of lending rate and demand in determining bank revenue and profits. Increasing interest rates, loan amounts, and demand positively impact profits, while prepayment has a negative effect. The research provides valuable insights for determining an appropriate loan interest rate to optimize bank profits and enhance competitiveness (Guilbaud and Pham [29]; Jiao and Li [30]; Molyneux et al. [31]; Bank et al. [32]).

5. Concluding Remarks

5.1. Conclusions and Discussion

The present study aimed to determine the optimal mortgage interest rate for a bank that would maximize its expected total profit by exploring fixed and indexed mortgage products. The following are the three key conclusions drawn from the study:

First, the results of the study highlight the significance of the lending rate and demand in determining the bank's revenue. An increase in interest rate, loan amount, and demand will positively impact bank profits, while prepayment will have a negative effect. Thus, the bank must carefully evaluate the potential gains and losses when setting the lending rate to arrive at the most appropriate lending rate that optimizes profits and takes into account the various factors that influence revenue (Guilbaud and Pham [29]; Jiao and Li [30]).

Second, the study sheds light on the optimal mortgage interest rate and the factors that determine the bank's revenue and profits. These factors include the interest rate, loan amount, demand for loans, fixed business expenses, the proportion of losses that cannot be fully recovered, and the proportion of liquidated damages paid in advance. The results indicate that the interest rate is highly correlated with the demand and that the loan amount has a proportional impact on the total profit, regardless of whether the interest rate is fixed or floating (Molyneux et al. [31]).

Third, the solution process to finding the optimal lending rate was illustrated through numerical examples, and sensitivity analysis was performed on the main parameters to understand their impact on the optimal solution. The results show that changes in the interest rate, loan amount, demand, and expenses have a significant impact on the total profit, particularly on the demand function and its value. Although the results cannot fully meet the real market conditions, they can still serve as a reference for industry players in determining an appropriate loan interest rate that can effectively improve the bank's competitiveness and interests (Bank et al. [32]).

In summary, this study presents a novel approach to analyzing the present value of total profit for fixed and indexed mortgage products, with the goal of determining the optimal mortgage interest rate that would maximize the bank's expected total profit. The findings reveal that the lending rate and demand are critical factors in determining the bank's revenue and profitability. Specifically, an increase in the interest rate, loan amount, and demand has a positive impact on bank profits, while prepayment has an adverse effect. Therefore, careful evaluation of the trade-offs between potential gains and losses when setting the lending rate is imperative for the bank. By considering various factors that influence revenue, the bank can arrive at the most appropriate lending rate that will optimize its profits. The results of this research provide valuable financial insights into the optimal mortgage interest rate and the factors that determine the revenue and profits of a bank, with implications for cost-benefit analysis, fixed-rate mortgage, indexed mortgage, lending rate, defaults, and maximum profit.

Even though we derived several crucial conclusions, a critical discussion on the possible factors and how they could affect the results would be necessary due to the

limitation of making certain assumptions for employing mathematical models. As such, we discuss why increasing interest rates may result in two different scenarios. In the first scenario, if the loan amount per person increases due to the incentive of lowering prices (offering a lower interest rate for borrowers), the profit margin may decrease, but the total profit may increase as a result of the significantly increased loan amount. However, the aforementioned result could be predicated on the assumption that market demand for mortgage loans is inversely related to the banks' mortgage interest rate, and even the default risk would not be greatly raised after loaning additional money for borrowers. Furthermore, due to the intense competition in the banking industry, a bank may generate greater profits by loaning more money at a lower price (lower interest rate) to those with a very low default risk (i.e., good customers); additionally, without the incentive above, these customers may borrow money from other banks, resulting in a decline in profits for this bank. The increasing interest rate might not enhance earnings in the first scenario. However, in the second scenario, when the amount of a loan increases, the banks tend to increase the interest rate accordingly, since the risk for them also increases. Otherwise, a bank may experience a loss if it makes additional loans to its customers without increasing the interest rate to compensate for the rising risk premium. Consequently, in contrast to the first scenario, increasing the interest rate in the second scenario may prevent losses and even generate profits. As such, based on the forging, discussing and explaining why increasing interest rates may result in different results.

Furthermore, several studies have examined non-performing loans (NPLs) in different banking sectors. Toudas et al. [33] investigated the correlation of NPLs among four major Greek banks during the recent financial crisis using accounting information. Similarly, Hassan et al. [34] focused on the banking sector of Pakistan, aiming to identify the determinants of NPLs in this context. In Indonesia, Irawati et al. [35] explored the financial performance of the banking industry and the impact of factors like good corporate governance, capital adequacy ratio, NPLs, and bank size. Additionally, Boahene et al. [36] studied the credit risk and profitability of selected banks in Ghana, analyzing the relationship between credit risk and profitability within the Ghanaian banking landscape. These studies collectively contribute valuable insights into the understanding of non-performing loans and their impact on banking performance. Therefore, in this study, we did not include non-performing mortgage loans in our mathematical model, which may result in an overestimation of profitability in the numerical analysis.

5.2. Research Implications

This study has the following essential implications. First, the study highlights the importance of setting the appropriate lending rate for a bank to maximize its expected total profit. Banks can use the results of this study to determine the optimal mortgage interest rate that would balance the trade-offs between potential gains and losses. The optimal lending rate will vary based on several factors such as the lending rate, loan amount, demand, and prepayment. As such, banks can analyze these factors to determine the optimal lending rate to generate the highest revenue and minimize the risk of prepayments. To set the optimal lending rate, banks can use mathematical optimization techniques to find the rate that will balance the trade-off between revenue and costs. Banks can also conduct sensitivity analysis to understand the impact of changes in the various parameters on the optimal lending rate.

Second, the study reveals that the lending rate and demand play a crucial role in determining the revenue of the bank. Banks can use this insight to understand the impact of the lending rate on the demand for mortgage products. An increase in the interest rate can lead to a decrease in demand, and a decrease in the interest rate can lead to a rise in demand. Therefore, banks can use this information to formulate a pricing strategy that balances the trade-off between revenue and demand. For example, banks can lower the lending rate for certain mortgage products to increase demand and generate more revenue.

Alternatively, banks can increase the lending rate for certain mortgage products to reduce demand and minimize the risk of prepayments.

Third, the study found that prepayment can have an adverse effect on the bank's revenue. Banks can use this insight to minimize the impact of prepayment on their revenue by evaluating the trade-off between potential gains and losses. As such, banks can use mathematical optimization techniques to determine the lending rate that balances the trade-off between revenue and prepayment. Banks can also conduct sensitivity analysis to understand the impact of changes in the various parameters on the optimal lending rate. This information can be used to minimize the impact of prepayment on the bank's revenue.

5.3. Limitations and Future Research

This study first used the concept of trade-offs in the inventory theory to try to explore bank mortgage products with interest rate-related demand and incorporate variables such as loan amount, business-related costs, housing loan life, early repayment liquidated damages, default probability, and discount rate into the model. Nevertheless, based on the assumptions of the proposed models, there were still some factors neglected in this study including the costs of capital for different time horizons, regulations about the proportion of adjustable rate mortgages, or the clients' loan combinations. Furthermore, obtaining actual market data and loan information is very challenging due to the banking industry's confidentiality concerns and rivalry. Therefore, in the numerical analysis and sensitivity analysis sections, only the virtual values based on public information or financial statements were used to verify the model, explain the solution process, and examine the impact of parameter changes on the optimal solutions. As such, the foregoing issue would be the limitation of this study.

There are many future research directions that can be extended, as described below. First, this study only focused on fixed and indexed mortgage products and did not consider other types of mortgage products such as adjustable-rate mortgages. Second, this research assumed that the discount rate was fixed, which may be slightly different from the actual situation where the discount rate may indeed change over time or certain factors. That is, a variable discount rate is a more realistic issue, but also challenging in future research. Third, it did not consider the impact of other economic factors such as inflation, the unemployment rate, and economic growth on the demand for mortgage products and the optimal mortgage interest rate. These factors play an essential role in determining a bank's revenue and must be considered in future research to provide a more comprehensive analysis. Fourth, the study only considered the present value of the total profit and did not take into account the long-term profitability of the bank. In reality, the optimal mortgage interest rate may not necessarily maximize the short-term profit but may be more beneficial in the long-term. For example, a lower interest rate may attract more customers and increase demand, leading to higher profits in the long run. In addition, we paid visits to several managers who work or have worked in the banking sector. Fifth, we determined these approximate probabilities based on the valuable information offered by these managers; however, because of the banking industry's confidentiality concerns and rivalry, how these possibilities influence the model may not accurately reflect the real world. Hence, future research should consider the long-term impact of the optimal mortgage interest rate on the bank's profits and sustainability. Sixth, this study only focused on index-based and interest-based mortgage commodities and did not consider other factors that may impact the mortgage market such as the economy, inflation, and regulations. Thus, future research could incorporate these factors into the model to create a more comprehensive and accurate representation of the mortgage market. Moreover, other different types of interest rate-dependent demand function such as exponential or logarithmic functions, the cost of capital, credit risk, default interest rate, value of collateral, and so on, are also research directions that can be extended. Finally, in order to deal with these differential costs and to manage the market risk, banks typically employ an ALM (Assets Liabilities Management) unit that reduces the duration gap that is created by the difference in interest rate sensitivity between assets (loans) and

liabilities (deposits, bonds, etc.). Consequently, we may incorporate the preceding concerns into future research, which would assist in broadening the scope of this study.

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