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Analysis and Applications of Artificial Intelligence in Digital Education Based on Complex Fuzzy Clustering Algorithms

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Abstract: Digital education is very important and valuable because it is a subpart of artificial intelligence, which is used in many real-life problems. Digital education is the modern utilization of digital techniques and tools during online purchasing, teaching, research, and learning and is often referred to as technology-enhanced learning or e-learning programs. Further, similarity measures (SM) and complex fuzzy (CF) logic are two different ideas that play a very valuable and dominant role in the environment of fuzzy decision theory. In this manuscript, we concentrate on utilizing different types of dice SM (D-SM) and generalized dice SM (GD-SM) in the environment of a CF set (CFS), called CF dice SM (CFD-SM), CF weighted dice SM (CFWD-SM), CF generalized dice SM (CFGD-SM), and CF weighted generalized dice SM (CFWGD-SM), and also derived associated outcomes. Furthermore, to evaluate or state the supremacy and effectiveness of the derived measures, we aim to evaluate the application of artificial intelligence in digital education under the consideration of derived measures for CF information and try to verify them with the help of several examples. Finally, with the help of examples, we illustrate the comparison between the presented and existing measures to show the supremacy and feasibility of the derived measures.

Keywords: artificial intelligence; digital education; complex fuzzy logic; generalized dice similarity measures; decision-making

MSC: 03B52; 68T27; 94D05; 03E72; 28E10

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1. Introduction

In recent years, artificial intelligence has emerged as an indispensable tool utilized by a multitude of companies, such as YouTube, Amazon, and Netflix, as well as within search engines, translation companies, online marketing and advertisements, and education. Scholars and businesses are no strangers to using it. In essence, artificial intelligence acts as a substitute for the lengths with which human intelligence can reach and exists to complete the same tasks that humans can. The term "artificial intelligence" concerns technology that can imitate "human" cognitive skills, ranging from "learning" to "problem solving". Its wide-ranging possibilities have altered the future landscape of education, learning, and teaching as techniques change to incorporate such advanced technology. Artificial intelligence can be utilized to improve aspects of education as well as aid in the future understanding of the brain and cognition.

Clustering analysis, or clustering algorithms, which allow companies to mine invaluable discrete data to pool information on customers, the population, and transactions, amongst other targets, is one of these powerful tools within artificial intelligence. Clustering analysis can also take the form of an evaluative statistical technique. Information is organized into collectives, or clusters, centered on careful associations, which would prove an extraordinarily difficult activity for scholars to arrange such masses of data based on similar attributes.

Much inspection of clustering analysis has utilized classical information, but various problems are yet to be resolved. One major issue involves the loss of masses of information in the process of forming clusters. Addressing this, Zadeh [1] established the main theory of fuzzy sets (FS) in 1965 by altering the function of the classical set and inventing a novel function termed the membership grade, where the derived values fit into the unite interval [0, 1]. Furthermore, Mahmood and Ali [2] modified the theory of FS and derive the theory of fuzzy superior mandelbrot information. Akram et al. [3] merged the theory of FS with N-soft information and originated the novel theory of fuzzy N-soft information. The theory of multi-fuzzy N-soft information emerged from work conducted by Fatimah and Alcantud [4] that combined the multi-fuzzy set theory with the N-soft set theory. Furthermore, a non-iterative reasoning technique under the consideration of fuzzy cognitive function was invented by Al Farsi et al. [5], Karimi et al. [6] examined the perceptual computer for hierarchical portfolio consideration in the presence of type-2 fuzzy information, and Tian et al. [7] evaluated the canonical triangular interval type-2 fuzzy linguistic distribution assessment.

Through the various usages of fuzzy information across multiple fields and scholars, the important question of what occurs when the range of FS is altered requires an answer. Ramot et al. [8] hence manipulated the range of FS and, in using the unit disc in lieu of the unit interval, generated the notion of complex fuzzy set (CFS). CFS possesses only one grade, but the resultant value is in the form of a complex number where real and unreal parts are contained in the unit interval [0, 1]. Further, Liu et al. [9] established methods to ascertain distances and values, which were titled cross-entropy measures based on CFS. Newly created complex fuzzy N-soft sets by Mahmood and Ali [10] fused CFS with N-soft information. Following on from this, the theory of a complex multi-fuzzy set was discovered by A-Qudah and Hassan [11], while Tamir et al. [12] created the theory of similarity measures reliant on complex fuzzy logic and CFSs. Moreover, Ghorbani et al. [13] exposed the semantic interoperability based on type-2 fuzzy sets; Jan et al. [14] evaluated the evaluation of digital systems for complex fuzzy soft information and their applications; Yahya et al. [15] derived the S-box based on image encryption for complex fuzzy frank operators; and Zeeshan et al. [16] evaluated the distance function for complex fuzzy soft information and their application in signals.

Similarity measures (SMs) play an essential role in evaluating the closeness between any two pieces of information, and based on their supremacy and advantages, certain people have utilized them in the environment of different fields. For instance, Lee-Kwang et al. [17] derived SMs for fuzzy information. In addition, Xuecheng [18] evaluated entropy, distance, and SMs for fuzzy information, while Wang [19] proffered two new SMs regarding fuzzy information. Beg and Ashraf [20] identified SMs for FSs; Chen et al. [21] contrasted a multitude of SMs for fuzzy information; Couso et al. [22] assessed the SMs and dis-SMs for fuzzy information; and Zhang and Fu [23] gleaned SMs based on three types of FSs. Moreover, Guo et al. [24] exposed the cosine SMs for CFS and their importance in robustness, whereas Hu et al. [25] conducted work on the theory of distance, similarity, and continuity with regard to complex fuzzy information and its application in digital education. In response to the aforementioned discoveries, it became evident that it was a complex challenge to develop new SMs based on CFS, and thus the core theme of the following analysis should be to discern the various forms of D-SMs [26] and GD-SMs for CFS. Before proposing the derived techniques, we have three major queries:

1. How do we invent new similarity or distance measures based on complex fuzzy logic;
2. How do we evaluate the problem of digital education based on distance or similarity measures for complex fuzzy logic;
3. How do we evaluate the best optimal form of the collection of finite values.

Evaluating or addressing the above queries is a very challenging task for young researchers because the structure of a complex fuzzy set is very complicated and ambiguous. Many scholars have derived the different types of measures based on fuzzy sets, but the dice and generalized dice similarity measures are more formal and valuable for depicting uncertain and unreliable information in genuine life problems. Therefore, the major advantages and disadvantages of the proposed work are stated in Table 1.

Table 1. Advantages and disadvantages of the proposed work.

Advantages of the Proposed Measures	Disadvantages of the Proposed Measures
1. Similar measures based on fuzzy sets are the special cases of the proposed theory. 2. Similarity measures based on complex fuzzy sets are the special cases of the proposed theory. 3. With the help of proposed measures, we can easily find the interrelationship between any two fuzzy or complex fuzzy numbers. 4. Dice similarity measures based on fuzzy sets and complex fuzzy sets are the special cases of the proposed work. 5. The presented measures can also use for evaluating two-dimension information.	1. The proposed measures have failed to depict the data which contained membership and non-membership grades. 2. The proposed measures have failed to aggregate the collection of information into a singleton set. 3. The proposed measures will be the special cases of these measures which will be proposed based on complex intuitionistic fuzzy sets and their extensions.

In the presence of the above information, it is clear that the proposed measures have many advantages. Moreover, we used the proposed measures and tried to evaluate the problem of digital education in northern areas based on their attributes. Therefore, the aim is to utilize them within the realms of digital education and artificial intelligence. Some important theories presented are listed below:

1. To expose the theories of CFD-SM and CFWD-SM and evaluate their valuable conclusions;
2. To examine the theory of CFGD-SM and CFWGD-SM and discuss the overarching themes;
3. To evaluate the problems of education with the help of artificial intelligence in digital education in the presence of the presented measures;
4. To contrast the proposed discussions with numerical evidence to conclude the superiority and effectiveness of the suggested methods.

The main structure of this analysis is of the form:

1. In Section 2, the concepts of D-SM and CFS and their operational laws will be reviewed;
2. In Section 3, the theories of CFD-SM, CFWD-SM, CFGD-SM, and CFWGD-SM will be introduced;
3. In Section 4, the discussion will tackle the problems of artificial intelligence in digital education, relying on assessed methods for complex fuzzy set theory;
4. In Section 5, the suggested measures will be evaluated using numerical evidence alongside the existing measures;
5. In Section 6, the conclusion will be featured.

2. Preliminaries

The main influence of this section is to revise the existing theory of dice similarity measures for the collection of positive integers, which is used for evaluating the interrelationship between any two positive numbers. Moreover, we stated the idea of fuzzy sets and complex fuzzy information and their operational laws. We also explained all parameters in Table 2.

Table 2. Meaning of all parameters used in this manuscript.

Parameter	Meanings	Parameter	Meanings
$\boldsymbol{V_1}$	Representation of vector	$\xi\xi_f$	Fuzzy sets
$\boldsymbol{x_j, y_j}$	Element of vectors	X	Universal set
$\boldsymbol{\xi\xi_{cf}}$	Complex fuzzy sets	x	Element of a universal set
$\boldsymbol{\Xi_R(x)+i\Xi_I(x)}$	Complex membership grade (Cartesian form)	$\Xi_R(x)$	The real shape of membership grade
$\boldsymbol{\Xi'_R(x)e^{i2\pi(\Xi'_I(x))}}$	Complex membership grade (Polar form)	$\Xi_I(x)$	The imaginary shape of membership grade
$\boldsymbol{\Xi'_R(x)}$	Amplitude term of membership grade	$\Xi'_I(x)$	Phase term of membership grade
$\boldsymbol{\overline{\overline{\theta}}\geq 1}$	Scaler		

Definition 1 [19]**.** *For any two vectors $V_1 = (x_1, x_2, \ldots, x_n)$ and $V_2 = (y_1, y_2, \ldots, y_n)$, D-SM is given by*

$$D - SM(V_1, V_2) = \frac{2V_1 \cdot V_2}{\|V_1\|_2^2 + \|V_2\|_2^2} = \frac{2\sum_{j=1}^{n} x_j y_j}{\sum_{j=1}^{n} x_j^2 + \sum_{j=1}^{n} y_j^2} \tag{1}$$

where $V_1 \cdot V_2 = \sum_{j=1}^{n} x_j y_j$ and $\|V_1\|_2^2 = \sum_{j=1}^{n} x_j^2, \|V_2\|_2^2 = \sum_{j=1}^{n} y_j^2$ represent the inner product and Euclidean norm under the consideration of vectors V_1 and V_2. The theory of D-SM is undefined for $x_j = y_j = 0$.

Further, we also need to recall the basic idea of FS.

Definition 2 [1]**.** *Under the presence of universal set X, the FS is given by*

$$\xi\xi_f = \{(\Xi_R(x)) : x \in X\} \tag{2}$$

where the term $\Xi_R(x)$ represent the membership function and defined it from universal set X to unit-interval [0, 1]. Furthermore, we are also able to write the short form and call it a fuzzy number (FN), $\xi\xi_{f-j} = \left(x, \Xi'_{R_j}\right), j = 1, 2, \ldots, n$. Furthermore, we discussed some operational laws for complex fuzzy information.

Further, we also need to recall the basic idea of CFS and their valuable laws under the consideration of algebraic information, such as algebraic t-norm and t-conorm.

Definition 3 [5]**.** *Under the presence of universal set X, the CFS is given by*

$$\xi\xi_{cf} = \{(\Xi_R(x) + i\Xi_I(x)) : x \in X\} = \left\{\left(\Xi'_R(x)e^{i2\pi(\Xi'_I(x))}\right) : x \in X\right\} \tag{3}$$

where the terms $\Xi_R(x)$ and $\Xi_I(x)$ represent the real and imaginary part of the complex-valued membership function and the terms $\Xi'_R(x)$ and $\Xi'_I(x)$ represent amplitude and phase grade of the membership function, and $\Xi'_R(x) = \left(\Xi_R^2(x) + \Xi_I^2(x)\right)^{\frac{1}{2}}$ and $\Xi'_I(x) = \tan^{-1}\frac{\Xi_I(x)}{\Xi_R(x)}$. Here, we are also able to write the short form and call it C-FN, $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j}e^{i2\pi(\Xi'_{I_j})}\right), j = 1, 2, \ldots, n$. Furthermore, we discussed some operational laws for complex fuzzy information.

Definition 4 [5]**.** *For C-FNs, $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j}e^{i2\pi(\Xi'_{I_j})}\right), j = 1, 2, \ldots, n$, we have*

$$\xi\xi_{cf-1} \cup \xi\xi_{cf-2} = \left(x, \max\left(\Xi'_{R_1}, \Xi'_{R_2}\right)e^{i2\pi(\max(\Xi'_{I_1}, \Xi'_{I_2}))}\right) \tag{4}$$

$$\xi\xi_{cf-1} \cap \xi\xi_{cf-2} = \left(x, \min\left(\Xi'_{R_1}, \Xi'_{R_2}\right) e^{i2\pi(\min(\Xi'_{I_1}, \Xi'_{I_2}))}\right) \tag{5}$$

$$\xi\xi^{c}_{cf-j} = \left(x, 1 - \Xi'_{R_j} e^{i2\pi(1-\Xi'_{I_j})}\right) \tag{6}$$

$$\xi\xi_{cf-1} \oplus \xi\xi_{cf-2} = \left(x, \left(\Xi'_{R_1} + \Xi'_{R_2} - \Xi'_{R_1}\Xi'_{R_2}\right) e^{i2\pi(\Xi'_{I_1} + \Xi'_{I_2} - \Xi'_{I_1}\Xi'_{I_2})}\right) \tag{7}$$

$$\xi\xi_{cf-1} \otimes \xi\xi_{cf-2} = \left(x, \left(\Xi'_{R_1}\Xi'_{R_2}\right) e^{i2\pi(\Xi'_{I_1}\Xi'_{I_2})}\right) \tag{8}$$

$$\overline{\overline{\theta}}\xi\xi_{cf-j} = \left(x, \left(1 - \left(1 - \Xi'_{R_j}\right)^{\overline{\overline{\theta}}}\right) e^{i2\pi(1-(1-\Xi'_{I_j})^{\overline{\overline{\theta}}})}\right) \tag{9}$$

$$\xi\xi^{\overline{\overline{\theta}}}_{cf-j} = \left(x, \left(\Xi'^{\overline{\overline{\theta}}}_{R_j}\right) e^{i2\pi(\Xi'^{\overline{\overline{\theta}}}_{I_j})}\right) \tag{10}$$

Now we aim to extend the theory of D-SM and GD-SM based on CFS and also try to describe their valuable results and properties.

3. Generalized Dice Similarity Measures for Complex Fuzzy Sets

In this section, we examined the ideas of CFD-SM, CFWD-SM, CFGD-SM, and CFWGD-SM. Moreover, some valuable results are also discussed in detail.

Definition 5. *For C-FNs $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi(\Xi'_{I_j})}\right), j = 1, 2, \ldots, n$ and $\xi\xi^{*}_{cf-j} = \left(x, \Xi^{*\prime}_{R_j} e^{i2\pi(\Xi^{*\prime}_{I_j})}\right), j = 1, 2, \ldots, n$, D-SM[1] is denoted and defined as:*

$$CFD-SM^{1}\left(\xi\xi_{cf-j}, \xi\xi^{*}_{cf-j}\right) = \frac{1}{n}\sum_{j=1}^{n} \frac{2\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j} + \Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\left(\Xi'^{2}_{R_j} + \Xi'^{2}_{I_j}\right) + \left(\Xi^{*\prime 2}_{R_j} + \Xi^{*\prime 2}_{I_j}\right)} \tag{11}$$

Further, D-SM [1] given in Equation (11) satisfies the following properties:

1. $0 \leq CFD-SM^{1}\left(\xi\xi_{cf-j}, \xi\xi^{*}_{cf-j}\right) \leq 1$.
2. $CFD-SM^{1}\left(\xi\xi_{cf-j}, \xi\xi^{*}_{cf-j}\right) = CFD-SM^{1}\left(\xi\xi^{*}_{cf-j}, \xi\xi_{cf-j}\right)$.
3. $CFD-SM^{1}\left(\xi\xi_{cf-j}, \xi\xi^{*}_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^{*}_{cf-j}$.

Example 1. *For C-FNs $\xi\xi_{cf} = \left(x, 0.7e^{i2\pi(0.9)}\right)$ and $\xi\xi^{*}_{cf} = \left(x, 0.3e^{i2\pi(0.5)}\right)$, then by using the theory in Equation (11) that is D-SM [1], we have*

$$\begin{aligned} CFD-SM^{1}\left(\xi\xi_{cf}, \xi\xi^{*}_{cf}\right) &= \tfrac{1}{1}\sum_{j=1}^{1} \tfrac{2(0.7*0.3+0.9*0.5)}{(0.7^2+0.9^2)+(0.3^2+0.5^2)} \\ &= \tfrac{2(0.21+0.45)}{(0.49+0.81)+(0.09+0.25)} \\ &= \tfrac{2(0.66)}{(1.30)+(0.34)} \\ &= \tfrac{1.32}{1.64} \\ &= 0.8048. \end{aligned}$$

Theorem 1. $CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)$ *satisfies the properties of similarity measures.*

Proof. Recall that

$$CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j}+\Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)+\left({\Xi^{*\prime}_{R_j}}^2+{\Xi^{*\prime}_{I_j}}^2\right)}$$

By definition, it is clear that $CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)\geq 0$, and

$$\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)+\left({\Xi^{*\prime}_{R_j}}^2+{\Xi^{*\prime}_{I_j}}^2\right)\geq 2\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j}+\Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)$$

But according to the inequality, we know that $a^2+b^2\geq 2ab$, thus, $0\leq CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)\leq 1$.

Further, we prove that $CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)=CFD-SM^1\left(\xi\xi^*_{cf-j},\xi\xi_{cf-j}\right)$; for this, we use Equation (11), such as:

$$CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j}+\Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)+\left({\Xi^{*\prime}_{R_j}}^2+{\Xi^{*\prime}_{I_j}}^2\right)}$$

$$=\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\Xi^{*\prime}_{R_j}\Xi'_{R_j}+\Xi^{*\prime}_{I_j}\Xi'_{I_j}\right)}{\left({\Xi^{*\prime}_{R_j}}^2+{\Xi^{*\prime}_{I_j}}^2\right)+\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)}=CFD-SM^1\left(\xi\xi^*_{cf-j},\xi\xi_{cf-j}\right).$$

Finally, we prove that $CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)=1\Leftrightarrow\xi\xi_{cf-j}=\xi\xi^*_{cf-j}$; for this, we consider if $\xi\xi_{cf-j}=\xi\xi^*_{cf-j}$, then we prove that $CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)=1$. Let $\xi\xi_{cf-j}=\xi\xi^*_{cf-j}$, which means that $\Xi'_{R_j}=\Xi^{*\prime}_{R_j}$ and $\Xi'_{I_j}=\Xi^{*\prime}_{I_j}$. Then, by using the information in Equation (11), we have

$$CFD-SM^1\left(\xi\xi_{cf-j},\xi\xi^*_{cf-j}\right)=1$$

$$\Leftrightarrow\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j}+\Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)+\left({\Xi^{*\prime}_{R_j}}^2+{\Xi^{*\prime}_{I_j}}^2\right)}=1$$

$$\Leftrightarrow\frac{1}{n}\sum_{j=1}^{n}\frac{2\left(\Xi'_{R_j}\Xi'_{R_j}+\Xi'_{I_j}\Xi'_{I_j}\right)}{\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)+\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)}=1$$

$$\Leftrightarrow\frac{1}{n}\sum_{j=1}^{n}\frac{2\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)}{\left(2{\Xi'_{R_j}}^2+2{\Xi'_{I_j}}^2\right)}=1$$

$$\Leftrightarrow\frac{1}{n}\sum\nolimits_{j=1}^{n}\frac{2\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)}{2\left({\Xi'_{R_j}}^2+{\Xi'_{I_j}}^2\right)}=1\Leftrightarrow\xi\xi_{cf-j}=\xi\xi^*_{cf-j}.\square$$

Definition 6. *For C-FNs* $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1,2,\ldots,n$ *and* $\xi\xi^*_{cf-j} = \left(x, {\Xi^*}'_{R_j} e^{i2\pi\left({\Xi^*}'_{I_j}\right)}\right), j = 1,2,\ldots,n$, *WD-SM* ^{1} *is denoted and defined as:*

$$CFWD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \sum_{j=1}^{n} \overline{\overline{\beth}}_j \frac{2\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + \left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \quad (12)$$

where $\overline{\overline{\beth}}_j \in [0,1]$. *Further, WD-SM* ^{1} *is defined in Equation (12) satisfies the following properties:*

1. $0 \le CFWD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \le 1$.
2. $CFWD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFWD - SM^1\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right)$.
3. $CFWD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}$.

Theorem 2. $CFWD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ *satisfies the properties of similarity measures.*

Proof. Straightforward.□

Here, it is worth noticing that the theory given in Equation (11) is a special case of the theory given in Equation (12), because if we put the value of $\overline{\overline{\beth}}_j = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ in Equation (12), then we can easily derive the theory in Equation (11). Moreover, from the information in Equations (11) and (12), we can easily derive the theory of distance measures such as $CFD - DM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ and $CFWD - DM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFWD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$.

Definition 7. *For C-FNs* $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1,2,\ldots,n$ *and* $\xi\xi^*_{cf-j} = \left(x, {\Xi^*}'_{R_j} e^{i2\pi\left({\Xi^*}'_{I_j}\right)}\right), j = 1,2,\ldots,n$, *the notion of D-SM*^{2} *is denoted and defined as:*

$$CFD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{2\sum_{j=1}^{n}\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\sum_{j=1}^{n}\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + \sum_{j=1}^{n}\left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \quad (13)$$

Further, D-SM ^{2} defined in Equation (13) satisfies the following properties:

1. $0 \le CFD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \le 1$.
2. $CFD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFD - SM^2\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right)$.
3. $CFD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}$.

Theorem 3. $CFD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ *satisfies the properties of similarity measures.*

Proof. Straightforward. □

Definition 8. *For C-FNs* $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1,2,\ldots,n$ *and* $\xi\xi^*_{cf-j} = \left(x, \Xi^{*\prime}_{R_j} e^{i2\pi\left(\Xi^{*\prime}_{I_j}\right)}\right), j = 1,2,\ldots,n$*, the notion of WD-SM*^{2} *is denoted and defined as:*

$$CFWD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{2\sum_{j=1}^{n} \overline{\overline{\beth}}_j^2 \left(\Xi'_{R_j}\Xi^{*\prime}_{R_j} + \Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\sum_{j=1}^{n} \overline{\overline{\beth}}_j^2 \left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + \sum_{j=1}^{n} \overline{\overline{\beth}}_j^2 \left({\Xi^{*\prime}_{R_j}}^2 + {\Xi^{*\prime}_{I_j}}^2\right)} \quad (14)$$

where $\overline{\overline{\beth}}_j \in [0,1]$*. Further, WD-SM*^{2} *defined in Equation (14) satisfies the following properties*

1. $0 \le CFWD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \le 1.$
2. $CFWD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFWD - SM^2\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right).$
3. $CFWD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}.$

Theorem 4. $CFWD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ *satisfies the properties of similarity measures.*

Proof. Straightforward. □

Furthermore, the information in Equation (13) is the particular case of the information in Equation (14), if we use the value of $\overline{\overline{\beth}}_j = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$. Moreover, from the information in Equations (13) and (14), we can easily derive the theory of distance measures such as $CFD - DM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ and $CFWD - DM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFWD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$.

Definition 9. *For C-FNs* $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1,2,\ldots,n$ *and* $\xi\xi^*_{cf-j} = \left(x, \Xi^{*\prime}_{R_j} e^{i2\pi\left(\Xi^{*\prime}_{I_j}\right)}\right), j = 1,2,\ldots,n$*, the notion of GD-SM* ^{1} *is denoted and defined as:*

$$CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j} + \Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\Phi\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + (1-\Phi)\left({\Xi^{*\prime}_{R_j}}^2 + {\Xi^{*\prime}_{I_j}}^2\right)}, \ \Phi \in [0, 0.5] \quad (15)$$

GD-SM ^{1} defined in Equation (15) satisfies the following properties:

1. $0 \le CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \le 1.$
2. $CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFGD - SM^1\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right).$
3. $CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}.$

Furthermore, we can easily derive two different types of results by using the information in Equation (15); for instance, by putting the value of $\Phi = 0, 1$, we obtain

$$CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\Xi'_{R_j}\Xi^{*\prime}_{R_j} + \Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\left({\Xi^{*\prime}_{R_j}}^2 + {\Xi^{*\prime}_{I_j}}^2\right)} \quad (16)$$

$$CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{1}{n}\sum_{j=1}^{n} \frac{\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right)} \tag{17}$$

Which represent the asymmetric and projection similarity measures.

Theorem 5. *$CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ satisfies the properties of similarity measures.*

Proof. Straightforward. □

Definition 10. *For C-FNs $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1, 2, \ldots, n$ and $\xi\xi^*_{cf-j} = \left(x, {\Xi^*}'_{R_j} e^{i2\pi\left({\Xi^*}'_{I_j}\right)}\right), j = 1, 2, \ldots, n$, the notion of WGD-SM ^{1} is denoted and defined as:*

$$CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \sum_{j=1}^{n} \bar{\bar{\beth}}_j \frac{\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\Phi\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + (1-\Phi)\left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \tag{18}$$

where $\bar{\bar{\beth}}_j \in [0, 1]$. Further, the notion defined in Equation (18) satisfies the following properties:

1. $0 \leq CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \leq 1$.
2. $CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFWGD - SM^1\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right)$.
3. $CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}$.

Furthermore, it is easy to derive two special types of results by using the information in Equation (18); for instance, by putting the value of $\Phi = 0, 1$, we obtain

$$CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \sum_{j=1}^{n} \bar{\bar{\beth}}_j \frac{\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \tag{19}$$

$$CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \sum_{j=1}^{n} \bar{\bar{\beth}}_j \frac{\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right)} \tag{20}$$

Which represent the asymmetric and projection similarity measures.

Theorem 6. *$CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ satisfies the properties of similarity measures.*

Proof. Straightforward. □

By putting $\bar{\bar{\beth}}_j = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$ in Equation (18), we obtain the notion given in Equation (15). Moreover, from the information in Equations (15) and (18), we can easily derive the theory of distance measures such as $CFGD - DM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ and $CFWGD - DM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFWGD - SM^1\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$.

Definition 11. *For C-FNs* $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1,2,\ldots,n$ *and* $\xi\xi^*_{cf-j} = \left(x, {\Xi^*}'_{R_j} e^{i2\pi\left({\Xi^*}'_{I_j}\right)}\right), j = 1,2,\ldots,n$*, the notion of GD-SM*^{2} *is denoted and defined as:*

$$CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{\sum_{j=1}^{n}\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\Phi\sum_{j=1}^{n}\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + (1-\Phi)\sum_{j=1}^{n}\left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \tag{21}$$

And it satisfies the following properties:

1. $0 \le CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \le 1$;
2. $CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFGD - SM^2\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right)$;
3. $CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}$.

Theorem 7. $CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ *satisfies the properties of similarity measures.*

Proof. Straightforward. □

Furthermore, we can easily derive two different types of results by using the information in Equation (21); for instance, by putting the value of $\Phi = 0,1$, we obtain

$$CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{\sum_{j=1}^{n}\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\sum_{j=1}^{n}\left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \tag{22}$$

$$CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{\sum_{j=1}^{n}\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\sum_{j=1}^{n}\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right)} \tag{23}$$

Which represents asymmetric and projection similarity measures.

Definition 12. *For C-FNs* $\xi\xi_{cf-j} = \left(x, \Xi'_{R_j} e^{i2\pi\left(\Xi'_{I_j}\right)}\right), j = 1,2,\ldots,n$ *and* $\xi\xi^*_{cf-j} = \left(x, {\Xi^*}'_{R_j} e^{i2\pi\left({\Xi^*}'_{I_j}\right)}\right), j = 1,2,\ldots,n$*, the notion of WGD-SM*^{2} *is given by:*

$$CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{\sum_{j=1}^{n}\overset{=}{\beth}_j^2\left(\Xi'_{R_j}{\Xi^*}'_{R_j} + \Xi'_{I_j}{\Xi^*}'_{I_j}\right)}{\Phi\sum_{j=1}^{n}\overset{=}{\beth}_j^2\left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right) + (1-\Phi)\sum_{j=1}^{n}\overset{=}{\beth}_j^2\left({{\Xi^*}'_{R_j}}^2 + {{\Xi^*}'_{I_j}}^2\right)} \tag{24}$$

where $\overset{=}{\beth}_j \in [0,1]$ *and it satisfies the following properties.*

1. $0 \le CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) \le 1$;
2. $CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = CFWGD - SM^2\left(\xi\xi^*_{cf-j}, \xi\xi_{cf-j}\right)$;
3. $CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 \Leftrightarrow \xi\xi_{cf-j} = \xi\xi^*_{cf-j}$.

Theorem 8. $CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ *must be justified by the property of similarity measures.*

Proof. Straightforward. □

Furthermore, we can easily derive two different types of the result by using the information in Equation (24); for instance, by putting the value of $\Phi = 0, 1$, we obtain

$$CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{\sum_{j=1}^{n} \overset{=}{\beth}_j^2 \left(\Xi'_{R_j}\Xi^{*\prime}_{R_j} + \Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\sum_{j=1}^{n} \overset{=}{\beth}_j^2 \left({\Xi^{*\prime}_{R_j}}^2 + {\Xi^{*\prime}_{I_j}}^2\right)} \tag{25}$$

$$CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = \frac{\sum_{j=1}^{n} \overset{=}{\beth}_j^2 \left(\Xi'_{R_j}\Xi^{*\prime}_{R_j} + \Xi'_{I_j}\Xi^{*\prime}_{I_j}\right)}{\sum_{j=1}^{n} \overset{=}{\beth}_j^2 \left({\Xi'_{R_j}}^2 + {\Xi'_{I_j}}^2\right)} \tag{26}$$

Which represent the asymmetric and projection similarity measures.

Data in Equation (21) is the subpart of the theory in Equation (24) if we consider the value of $\overset{=}{\beth}_j = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$. Moreover, we have also the following ideas: $CFGD - DM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$ and $CFWGD - DM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right) = 1 - CFWGD - SM^2\left(\xi\xi_{cf-j}, \xi\xi^*_{cf-j}\right)$.

4. Artificial Intelligence in Digital Education

With the aid of suggested methods for CF information, the section below will detail the real-life utilization of artificial intelligence in the context of digital education. Schools, colleges, and universities experienced vast problems throughout the pandemic; to assess these problems, algorithms can be employed to evaluate the position of digital education in such problems in the context of a global pandemic.

4.1. Algorithm-1

The process of clustering analysis for assessing the problems of digital education within reality is outlined below. It aims to discern valuable evaluations from variable information about genuine problems in life. The process below attempts to manage complex and awkward data.

Step 1: Arrange information and compute a decision matrix, whose very information will be written in the shape of a complex fuzzy number.

Step 2: In the presence of CFD-SM^1, we aim to find the closeness between any two possible pieces of information and write it in a matrix $\xi\xi_{cf-m} = [r_{ij}]_{m\times m}$.

Step 3: Evaluate the composition of the matrix $\xi\xi_{cf-m}$ such as: $\xi\xi^2_{cf-m} = \xi\xi_{cf-m}{}^\circ\xi\xi_{cf-m}$, where

$$\xi\xi_{cf-m}{}^\circ\xi\xi_{cf-m} = (r_{ij})_{m\times m} = \max_k\left\{\min\left\{r_{ik}, r_{kj}\right\}\right\} \tag{27}$$

If the $\xi\xi_{cf-m}{}^\circ\xi\xi_{cf-m} \subseteq \xi\xi_{cf-m}$, then continues the above process and finds the value of $\xi\xi^4_{cf-m} = \xi\xi^2_{cf-m}{}^\circ\xi\xi^2_{cf-m}$, if the $\xi\xi^2_{cf-m}{}^\circ\xi\xi^2_{cf-m} \subseteq \xi\xi^2_{cf-m}$, similarly continues this procedure even if we cannot receive the information in the form:

$$\xi\xi^{2k}_{cf-m}{}^\circ\xi\xi^{2k}_{cf-m} = \xi\xi^k_{cf-m} \tag{28}$$

Step 4: After obtaining the $\xi\xi^{2k}_{cf-m}{}^\circ\xi\xi^{2k}_{cf-m} = \xi\xi^k_{cf-m}$, we find the $\alpha-$ cutting based on the information in step 3 based on the below information, such as:

$$\xi\xi^\alpha_{cf-m} = \begin{cases} 0 & r_{ik} \le \alpha \\ 1 & r_{ik} > \alpha \end{cases} \tag{29}$$

Therefore, we try to utilize this information in practical application to enhance the worth and stability of the derived measures.

Example 2. *Here we aim to resolve the education systems of Saudi Arabia and categorize provinces based on the parameters of evaluation. We consider a clustering approach to categorize/cluster provinces based on five education-related parameters:*

x_1: Gross enrollment ratio.
x_2: Drop-out rate.
x_3: Schools with boys' toilets.
x_4: Percentage of schools with computers.
x_5: Percentage of schools with electricity.

The main theme of this demonstration is to utilize digital education in the environments of different provinces in Saudi Arabia. For this, we consider five different regions: $\xi\xi_{cf-1}, \xi\xi_{cf-2}, \xi\xi_{cf-3}, \xi\xi_{cf-4}$ and $\xi\xi_{cf-5}$. Then, using the proposed algorithm, we evaluate the problems below.

Step 1: Arrange information and compute a decision matrix whose very information will be written in the shape of a complex fuzzy number such as:

$$\xi\xi_{cf-1} = \left\{ \left(x_1, 0.9e^{i2\pi(0.7)}\right), \left(x_2, 0.7e^{i2\pi(0.8)}\right), \left(x_3, 0.8e^{i2\pi(0.1)}\right), \left(x_4, 0.5e^{i2\pi(0.3)}\right), \left(x_5, 0.6e^{i2\pi(0.7)}\right) \right\}$$

$$\xi\xi_{cf-2} = \left\{ \left(x_1, 0.2e^{i2\pi(0.3)}\right), \left(x_2, 0.1e^{i2\pi(0.3)}\right), \left(x_3, 0.1e^{i2\pi(0.2)}\right), \left(x_4, 0.3e^{i2\pi(0.3)}\right), \left(x_5, 0.2e^{i2\pi(0.4)}\right) \right\}$$

$$\xi\xi_{cf-3} = \left\{ \left(x_1, 0.1e^{i2\pi(0.2)}\right), \left(x_2, 0.2e^{i2\pi(0.3)}\right), \left(x_3, 0.3e^{i2\pi(0.5)}\right), \left(x_4, 0.4e^{i2\pi(0.4)}\right), \left(x_5, 0.5e^{i2\pi(0.1)}\right) \right\}$$

$$\xi\xi_{cf-4} = \left\{ \left(x_1, 0.7e^{i2\pi(0.3)}\right), \left(x_2, 0.6e^{i2\pi(0.3)}\right), \left(x_3, 0.5e^{i2\pi(0.4)}\right), \left(x_4, 0.4e^{i2\pi(0.5)}\right), \left(x_5, 0.2e^{i2\pi(0.6)}\right) \right\}$$

$$\xi\xi_{cf-5} = \left\{ \left(x_1, 0.5e^{i2\pi(0.8)}\right), \left(x_2, 0.6e^{i2\pi(0.7)}\right), \left(x_3, 0.7e^{i2\pi(0.6)}\right), \left(x_4, 0.8e^{i2\pi(0.5)}\right), \left(x_5, 0.9e^{i2\pi(0.4)}\right) \right\}$$

Step 2: Under the presence of CFD-SM[1], we aim to find the closeness between any two possible pieces of information and write it in a matrix $\xi\xi_{cf-m} = \left[r_{ij}\right]_{m\times m}$, which is listed below:

$$\xi\xi_{cf-m} = \begin{bmatrix} 1 & 0.604 & 0.6332 & 0.8713 & 0.9069 \\ 0.604 & 1 & 0.8162 & 0.7214 & 0.5935 \\ 0.6332 & 0.8162 & 1 & 0.7083 & 0.7267 \\ 0.8713 & 0.7214 & 0.7083 & 1 & 0.8212 \\ 0.9069 & 0.5935 & 0.7267 & 0.8212 & 1 \end{bmatrix}$$

Step 3: Evaluate the composition of the matrix $\xi\xi_{cf-m}$ such as: $\xi\xi^2_{cf-m} = \xi\xi_{cf-m} \circ \xi\xi_{cf-m}$, where

$$\xi\xi_{cf-m} \circ \xi\xi_{cf-m} = \left(r_{ij}\right)_{m\times m} = \max_k \left\{ \min\left\{ r_{ik}, r_{kj} \right\} \right\}$$

If the $\xi\xi_{cf-m} \circ \xi\xi_{cf-m} \subseteq \xi\xi_{cf-m}$, then continues the above process and finds the value of $\xi\xi^4_{cf-m} = \xi\xi^2_{cf-m} \circ \xi\xi^2_{cf-m}$, if the $\xi\xi^2_{cf-m} \circ \xi\xi^2_{cf-m} \subseteq \xi\xi^2_{cf-m}$, similarly continues this procedure even if we cannot receive the information in the shape: $\xi\xi^{2k}_{cf-m} \circ \xi\xi^{2k}_{cf-m} = \xi\xi^{k}_{cf-m}$. Therefore, the procedure is continuing as:

$$\xi\xi^{2}_{cf-m} = \xi\xi_{cf-m} \circ \xi\xi_{cf-m} = \begin{bmatrix} 1 & 0.7214 & 0.7267 & 0.8713 & 0.9069 \\ 0.7214 & 1 & 0.8162 & 0.7214 & 0.8162 \\ 0.7267 & 0.8162 & 1 & 0.7267 & 0.7267 \\ 0.8713 & 0.7214 & 0.7267 & 1 & 0.8713 \\ 0.9069 & 0.8162 & 0.7267 & 0.8713 & 1 \end{bmatrix}$$

$$\xi\xi^{4}_{cf-m} = \xi\xi^{2}_{cf-m} \circ \xi\xi^{2}_{cf-m} = \begin{bmatrix} 1 & 0.8162 & 0.7267 & 0.8713 & 0.9069 \\ 0.8162 & 1 & 0.8162 & 0.8162 & 0.8162 \\ 0.7267 & 0.8162 & 1 & 0.7267 & 0.8162 \\ 0.8713 & 0.8162 & 0.7267 & 1 & 0.9069 \\ 0.9069 & 0.8162 & 0.8162 & 0.9069 & 1 \end{bmatrix}$$

$$\xi\xi^{8}_{cf-m} = \xi\xi^{4}_{cf-m} \circ \xi\xi^{4}_{cf-m} = \begin{bmatrix} 1 & 0.8162 & 0.8162 & 0.9069 & 0.9069 \\ 0.8162 & 1 & 0.8162 & 0.8162 & 0.8162 \\ 0.8162 & 0.8162 & 1 & 0.8162 & 0.8162 \\ 0.9069 & 0.8162 & 0.8162 & 1 & 0.9069 \\ 0.9069 & 0.8162 & 0.8162 & 0.9069 & 1 \end{bmatrix}$$

$$\xi\xi^{16}_{cf-m} = \xi\xi^{8}_{cf-m} \circ \xi\xi^{8}_{cf-m} = \begin{bmatrix} 1 & 0.8162 & 0.8162 & 0.9069 & 0.9069 \\ 0.8162 & 1 & 0.8162 & 0.8162 & 0.8162 \\ 0.8162 & 0.8162 & 1 & 0.8162 & 0.8162 \\ 0.9069 & 0.8162 & 0.8162 & 1 & 0.9069 \\ 0.9069 & 0.8162 & 0.8162 & 0.9069 & 1 \end{bmatrix} = \xi\xi^{8}_{cf-m}$$

Therefore, we proceed to the next steps.

Step 4: After obtaining the $\xi\xi^{2k}_{cf-m} \circ \xi\xi^{2k}_{cf-m} = \xi\xi^{k}_{cf-m}$, we find the $\alpha-$ cutting based on the information in step 3 based on the below information, such as:

$$\xi\xi^{\alpha}_{cf-m} = \begin{cases} 0 & r_{ik} \leq \alpha \\ 1 & r_{ik} > \alpha \end{cases}$$

Therefore, the cutting matrix is given in the shape of Table 3.

Table 3. Classifications of different places.

Value of Parameter α	Classifications
$\alpha \in [0, 0.7267]$	$\{\xi\xi_{cf-1}, \xi\xi_{cf-2}, \xi\xi_{cf-3}, \xi\xi_{cf-4}, \xi\xi_{cf-5}\}$
$\alpha \in [0.7267, 0.8162]$	$\{\xi\xi_{cf-1}, \xi\xi_{cf-2}, \xi\xi_{cf-3}, \xi\xi_{cf-4}\}, \{\xi\xi_{cf-5}\}$
$\alpha \in [0.8162, 0.8713]$	$\{\xi\xi_{cf-2}\}, \{\xi\xi_{cf-1}, \xi\xi_{cf-3}, \xi\xi_{cf-4}\}, \{\xi\xi_{cf-5}\}$
$\alpha \in [0.8713, 0.9069]$	$\{\xi\xi_{cf-1}\}, \{\xi\xi_{cf-2}\}, \{\xi\xi_{cf-3}, \xi\xi_{cf-4}\}, \{\xi\xi_{cf-5}\}$
$\alpha \in [0.9069, 1]$	$\{\xi\xi_{cf-1}\}, \{\xi\xi_{cf-2}\}, \{\xi\xi_{cf-3}\}, \{\xi\xi_{cf-4}\}, \{\xi\xi_{cf-5}\}$

4.2. Algorithm-2

In this subsection, we considered five unknown pieces of information on the source of digital education and one known type of digital education information and tried to evaluate them with the help of derived measures.

Example 3. *For instance, five valuable sources for digital education will be addressed $\xi\xi_{cf-1}$: Mobile phone apps; $\xi\xi_{cf-2}$: virtual reality glasses; $\xi\xi_{cf-3}$: Holograms; $\xi\xi_{cf-4}$: website and blogs;*

and$\xi\xi_{cf-5}$: digital books. In assessing such sources, sources will be prioritized by effectiveness according to various factors, such as x_1: feasibility; x_2: not time vesting; x_3: inexpensive; x_4: time-efficient; andx_5: money-saving.

Under the consideration of five features of all digital education, five unknowns are listed below:

$$\xi\xi_{cf-1} = \left\{ \left(x_1, 0.9e^{i2\pi(0.7)}\right), \left(x_2, 0.7e^{i2\pi(0.8)}\right), \left(x_3, 0.8e^{i2\pi(0.1)}\right), \left(x_4, 0.5e^{i2\pi(0.3)}\right), \left(x_5, 0.6e^{i2\pi(0.7)}\right) \right\}$$

$$\xi\xi_{cf-2} = \left\{ \left(x_1, 0.5e^{i2\pi(0.9)}\right), \left(x_2, 0.6e^{i2\pi(0.5)}\right), \left(x_3, 0.3e^{i2\pi(0.3)}\right), \left(x_4, 0.7e^{i2\pi(0.4)}\right), \left(x_5, 0.6e^{i2\pi(0.7)}\right) \right\}$$

$$\xi\xi_{cf-3} = \left\{ \left(x_1, 0.2e^{i2\pi(0.5)}\right), \left(x_2, 0.5e^{i2\pi(0.4)}\right), \left(x_3, 0.3e^{i2\pi(0.6)}\right), \left(x_4, 0.7e^{i2\pi(0.7)}\right), \left(x_5, 0.3e^{i2\pi(0.8)}\right) \right\}$$

$$\xi\xi_{cf-4} = \left\{ \left(x_1, 0.4e^{i2\pi(0.4)}\right), \left(x_2, 0.5e^{i2\pi(0.3)}\right), \left(x_3, 0.6e^{i2\pi(0.2)}\right), \left(x_4, 0.7e^{i2\pi(0.1)}\right), \left(x_5, 0.8e^{i2\pi(0.3)}\right) \right\}$$

$$\xi\xi_{cf-5} = \left\{ \left(x_1, 0.9e^{i2\pi(0.1)}\right), \left(x_2, 0.8e^{i2\pi(0.2)}\right), \left(x_3, 0.7e^{i2\pi(0.7)}\right), \left(x_4, 0.6e^{i2\pi(0.4)}\right), \left(x_5, 0.5e^{i2\pi(0.5)}\right) \right\}$$

For evaluating the above dilemma, we use one known information, which is stated below:

$$\xi\xi_{cf}^{*} = \left\{ \left(x_1, 1e^{i2\pi(1)}\right), \left(x_2, 1e^{i2\pi(1)}\right), \left(x_3, 1e^{i2\pi(1)}\right), \left(x_4, 1e^{i2\pi(1)}\right), \left(x_5, 1e^{i2\pi(1)}\right) \right\}$$

Then, by using the derived measures based on weight vectors 0.2, 0.3, 0.1, 0.3, and 0.1 with $\Phi = 0.4$, the evaluated information is listed in Table 4.

Table 4. Values of different types of measures for the data in Example 3.

Methods	$\xi\xi_{cf-1}$	$\xi\xi_{cf-2}$	$\xi\xi_{cf-3}$	$\xi\xi_{cf-4}$	$\xi\xi_{cf-5}$
$CFD-SM^1$	0.84069	0.81017	0.76768	0.69719	0.79775
$CFWD-SM^1$	0.16915	0.16624	0.15645	0.13646	0.15556
$CFD-SM^2$	0.90149	0.87207	0.85014	0.62494	0.79033
$CFWD-SM^2$	0.91634	0.88266	0.84685	0.60532	0.74813
$CFGD-SM^1$	0.77952	0.73862	0.69212	0.61963	0.72754
$CFWGD-SM^1$	0.15683	0.15171	0.14142	0.12082	0.14143
$CFGD-SM^2$	0.82285	0.78064	0.7457	0.57299	0.72327
$CFWGD-SM^2$	0.83456	0.79344	0.75018	0.55446	0.68778

Using the information in Table 4, the ranking results are given in Table 5.

Table 5. Ranking information for the data in Table 4.

Methods	Ranking Results
$CFD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$
$CFWD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFD-SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFWD-SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFGD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$
$CFWGD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$
$CFGD-SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFWGD-SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$

From the information in Table 5, we noticed that all derived measures are given the same ranking information, such as: $\xi\xi_{cf-1}$, where $\xi\xi_{cf-1}$ represented mobile phone apps, which are very beneficial and valuable for all students.

4.3. Algorithm-3

In this sub-section, we select five unknown places and one known place and try to evaluate the best place for digital education in Saudi Arabia under the consideration of derived measures.

Example 4. *Here, we considered five types of places, such as $\xi\xi_{cf-1}$: Place-1; $\xi\xi_{cf-2}$: Place-2; $\xi\xi_{cf-3}$: Place-3; $\xi\xi_{cf-4}$: Place-4; and $\xi\xi_{cf-5}$: Place-5. For evaluating the above problem, we decided the places needed which kind of digital education was best for their people. For this, based on the following information, we will be deciding which kind of place needs digital education, such as: x_1: quality of internet services; x_2: the price of laptops; x_3: quantity of people; x_4: feasibility of employment; and x_5: reliability of social impact. Under the consideration of five things in all places, five unknowns are listed below:*

$$\xi\xi_{cf-1} = \left\{ \left(x_1, 0.9e^{i2\pi(0.7)}\right), \left(x_2, 0.7e^{i2\pi(0.8)}\right), \left(x_3, 0.8e^{i2\pi(0.1)}\right), \left(x_4, 0.5e^{i2\pi(0.3)}\right), \left(x_5, 0.6e^{i2\pi(0.7)}\right) \right\}$$

$$\xi\xi_{cf-2} = \left\{ \left(x_1, 0.5e^{i2\pi(0.9)}\right), \left(x_2, 0.6e^{i2\pi(0.5)}\right), \left(x_3, 0.3e^{i2\pi(0.3)}\right), \left(x_4, 0.7e^{i2\pi(0.4)}\right), \left(x_5, 0.6e^{i2\pi(0.7)}\right) \right\}$$

$$\xi\xi_{cf-3} = \left\{ \left(x_1, 0.2e^{i2\pi(0.5)}\right), \left(x_2, 0.5e^{i2\pi(0.4)}\right), \left(x_3, 0.3e^{i2\pi(0.6)}\right), \left(x_4, 0.7e^{i2\pi(0.7)}\right), \left(x_5, 0.3e^{i2\pi(0.8)}\right) \right\}$$

$$\xi\xi_{cf-4} = \left\{ \left(x_1, 0.4e^{i2\pi(0.4)}\right), \left(x_2, 0.5e^{i2\pi(0.3)}\right), \left(x_3, 0.6e^{i2\pi(0.2)}\right), \left(x_4, 0.7e^{i2\pi(0.1)}\right), \left(x_5, 0.8e^{i2\pi(0.3)}\right) \right\}$$

$$\xi\xi_{cf-5} = \left\{ \left(x_1, 0.9e^{i2\pi(0.1)}\right), \left(x_2, 0.8e^{i2\pi(0.2)}\right), \left(x_3, 0.7e^{i2\pi(0.7)}\right), \left(x_4, 0.6e^{i2\pi(0.4)}\right), \left(x_5, 0.5e^{i2\pi(0.5)}\right) \right\}$$

For evaluating the above dilemma, we use one known information which is stated below:

$$\xi\xi^*_{cf} = \left\{ \left(x_1, 0.2e^{i2\pi(0.3)} \right), \left(x_2, 0.1e^{i2\pi(0.3)} \right), \left(x_3, 0.1e^{i2\pi(0.2)} \right), \left(x_4, 0.3e^{i2\pi(0.3)} \right), \left(x_5, 0.2e^{i2\pi(0.4)} \right) \right\}$$

Then by using the derived measures based on weight vectors 0.2,0.3,0.1,0.3, and 0.1 with $\Phi = 0.9$, the evaluated information is listed in Table 6.

Table 6. Values of different types of measures for the data in Example 4.

Methods	$\xi\xi_{cf-1}$	$\xi\xi_{cf-2}$	$\xi\xi_{cf-3}$	$\xi\xi_{cf-4}$	$\xi\xi_{cf-5}$
$CFD - SM^1$	0.60404	0.71062	0.67928	0.65555	0.58463
$CFWD - SM^1$	0.1284	0.13897	0.129	0.13702	0.11595
$CFD - SM^2$	0.68682	0.85532	0.95954	0.43427	0.53639
$CFWD - SM^2$	0.71499	0.81963	0.77573	0.43768	0.49166
$CFGD - SM^1$	0.41026	0.51404	0.46118	0.46157	0.39941
$CFWGD - SM^1$	0.08932	0.10015	0.0874	0.09768	0.07923
$CFGD - SM^2$	0.44364	0.6298	0.67094	0.27817	0.34476
$CFWGD - SM^2$	0.46535	0.59956	0.51394	0.28168	0.31436

Using the information in Table 6, the ranking results are given in Table 7.

Table 7. Ranking information for the data in Table 6.

Methods	Ranking Results
$CFD - SM^1$	$\xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-4} > \xi\xi_{cf-1} > \xi\xi_{cf-5}$
$CFWD - SM^1$	$\xi\xi_{cf-2} > \xi\xi_{cf-4} > \xi\xi_{cf-3} > \xi\xi_{cf-1} > \xi\xi_{cf-5}$
$CFD - SM^2$	$\xi\xi_{cf-3} > \xi\xi_{cf-2} > \xi\xi_{cf-1} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFWD - SM^2$	$\xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-1} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFGD - SM^1$	$\xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-4} > \xi\xi_{cf-1} > \xi\xi_{cf-5}$
$CFWGD - SM^1$	$\xi\xi_{cf-2} > \xi\xi_{cf-4} > \xi\xi_{cf-1} > \xi\xi_{cf-3} > \xi\xi_{cf-5}$
$CFGD - SM^2$	$\xi\xi_{cf-3} > \xi\xi_{cf-2} > \xi\xi_{cf-1} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFWGD - SM^2$	$\xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-1} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$

From the information in Table 7, we noticed that all derived measures are given two different ranking information, such as: $\xi\xi_{cf-2}$ and $\xi\xi_{cf-3}$, where $\xi\xi_{cf-2}$ represented Place 2 in Saudi Arabia, and $\xi\xi_{cf-3}$ represented Place-3 in Saudi Arabia, which is a very important and main feature for constructing any utilization of digital education.

5. Comparative Analysis

Comparative analysis will encompass the comparison between pervasive and recently introduced methods concerning CFS. In this context, pre-existing methods, including Lee-Kwang et al.'s [10] examination of the theory of SMs with fuzzy information, serve as the foundation for this. In addition, Xuecheng [11] established the three varying categories of methodology, such as entropy, distance, and SMs based on fuzzy information, while Beg and Ashraf [13] outlined the SMs for FSs, and Chen et al. [14] considered a multitude of SMs based on fuzzy information. Moreover, the work of Couso et al. [15] resulted in the theory of SMs and dis-SMs with their existing knowledge of fuzzy set theory, whereas Zhang and Fu [16] showcased their SMs based on FSs. Guo et al. [17], however, focused on the theory of cosine SMs based on CFS, while Hu et al. [18] evaluated the distance, similarity, and continuity information for CFS. The aforementioned comparative analysis will feature clearly in Table 8 by employing the information collated in Table 4.

Table 8. Comparison: values of different types of measures for the data in Example 4.

Methods	$\xi\xi_{cf-1}$	$\xi\xi_{cf-2}$	$\xi\xi_{cf-3}$	$\xi\xi_{cf-4}$	$\xi\xi_{cf-5}$
Lee-Kwang et al. [10]	× × ×	× × ×	× × ×	× × ×	× × ×
Xuecheng [11]	× × ×	× × ×	× × ×	× × ×	× × ×
Wang [12]	× × ×	× × ×	× × ×	× × ×	× × ×
Beg and Ashraf [13]	× × ×	× × ×	× × ×	× × ×	× × ×
Chen et al. [14]	× × ×	× × ×	× × ×	× × ×	× × ×
Couso et al. [15]	× × ×	× × ×	× × ×	× × ×	× × ×
Zhang and Fu [16]	× × ×	× × ×	× × ×	× × ×	× × ×
Guo et al. [17]	0.71432	0.68064	0.64483	0.40330	0.54611
Hu et al. [18]	0.35481	0.35070	0.34040	0.32080	0.34041
$CFD-SM^1$	0.84069	0.81017	0.76768	0.69719	0.79775
$CFWD-SM^1$	0.16915	0.16624	0.15645	0.13646	0.15556
$CFD-SM^2$	0.90149	0.87207	0.85014	0.62494	0.79033
$CFWD-SM^2$	0.91634	0.88266	0.84685	0.60532	0.74813
$CFGD-SM^1$	0.77952	0.73862	0.69212	0.61963	0.72754
$CFWGD-SM^1$	0.15683	0.15171	0.14142	0.12082	0.14143
$CFGD-SM^2$	0.82285	0.78064	0.7457	0.57299	0.72327
$CFWGD-SM^2$	0.83456	0.79344	0.75018	0.55446	0.68778

Using the information in Table 8, the ranking results are given in Table 9.

Table 9. Ranking information for the data in Table 8.

Methods	Ranking Results
Lee-Kwang et al. [10]	Not applicable
Xuecheng [11]	Not applicable
Wang [12]	Not applicable
Beg and Ashraf [13]	Not applicable
Chen et al. [14]	Not applicable
Couso et al. [15]	Not applicable
Zhang and Fu [16]	Not applicable
Guo et al. [17]	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
Hu et al. [18]	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$
$CFD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$
$CFWD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFD-SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFWD-SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFGD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$
$CFWGD-SM^1$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-5} > \xi\xi_{cf-3} > \xi\xi_{cf-4}$

Table 9. *Cont.*

Methods	Ranking Results
$CFGD - SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$
$CFWGD - SM^2$	$\xi\xi_{cf-1} > \xi\xi_{cf-2} > \xi\xi_{cf-3} > \xi\xi_{cf-5} > \xi\xi_{cf-4}$

It is important to bring to the reader's attention that the ranked results in Table 9 are identical for all suggested measures as well as for the research conducted by Guo et al. [17] and Hu et al. [18]. With all measures collected from work guided by complex fuzzy information, the most appropriate conclusion is $\xi\xi_{cf-1}$. However, methods originating from the research of Lee-Kwang et al. [10], Xuecheng [11], Wang [12], Beg and Ashraf [13], Chen et al. [14], Couso et al. [15], and Zhang and Fu [16] were unsuccessful as a result of the fact they emerged from computed data based on fuzzy information as per the unique situation of the obtained measures.

Hence, such measures are considerably more generalized than the methods derived from various other scholars [10–18]. Thus, the measures utilized in this paper encompass many employment opportunities in artificial intelligence, machine learning, and problem solving.

6. Conclusions and Future Studies

Fuzzy set theory has a lot of applications in artificial intelligence, machine learning, and neural networks, but in many situations, the theory of fuzzy set has not worked effectively. For instance, if someone provides two-dimensional information instead of one-dimensional information, then the theory of fuzzy set has failed. For evaluating such types of problems, the theory of complex fuzzy theory is very effective and dominant in managing such types of problems. The major influence of this manuscript is stated below:

1. We derived the theory of complex fuzzy dice similarity measures and assessed the significant outcomes;
2. We derived the theory of complex fuzzy weighted dice similarity measures and assessed the significant outcomes;
3. We evaluated the idea of complex fuzzy generalized dice similarity measures;
4. We evaluated the idea of complex fuzzy weighted generalized dice similarity measures;
5. The issues within digital education were assessed using the obtained measures;
6. The contrasts between existing and suggested methods were showcased to highlight the flexibility and viability of the obtained and derived measures.

In the upcoming times, we aim to utilize the above measures in the environment of artificial intelligence, machine learning, game theory, and road signals. Further research regarding these concepts should evolve the suggested themes using the work of various disciplines, including soft expert sets [27], complex multi-fuzzy soft expert sets [28], m-polar fuzzy soft expert sets [29], Q multi-fuzzy soft expert sets [30], fuzzy soft expert sets [31], generalized fuzzy soft expert sets [32], cubic soft expert sets [33], complex fuzzy soft sets [34], data-driven fuzzy active disturbance rejection control [35], and enhanced p-type control [36]. Combined efforts can only be effective in the fields of artificial intelligence, neural networks, problem-solving, software engineering, computer science, and game theory.

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Abbreviations

SM—similarity measures; CF—complex fuzzy; D-SM—dice similarity measure; GD-SM—generalized dice similarity measure; CFS—complex fuzzy set; CFD-SM—complex fuzzy dice similarity measure; CFWD-SM—complex fuzzy weighted dice similarity measure; CFGD-SM—complex fuzzy generalized dice similarity measure; CFWGD-SM—complex fuzzy weighted generalized dice similarity measure; FS—fuzzy sets.

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