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A Supply Chain Model with Carbon Emissions and Preservation Technology for Deteriorating Items under Trade Credit Policy and Learning in Fuzzy

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Abstract: In this study, a supply chain model is proposed with preservation technology under learning fuzzy theory for deteriorating items where the demand rate depends on the selling price and also treats as a triangular fuzzy number. The deterioration rate of any item cannot be eliminated due to its natural process, but it can be controlled with the help of preservation technology. Some harmful gases are emitted during the preservation process due to deteriorating items that harm the environment. In general, it can be easily seen that most of the sellers offer a trade credit policy to their regular buyers. In this paper, the retailer's inventory stock reduces due to demand and deterioration. It is also assumed that some units are defective due to machine defects or delivery inefficiency. The retailer accepted the policy of trade credit offered by the seller. The aim of this paper is to enhance the profit of the supply chain partners. We proposed a theorem to get the optimal values of the selling price and cycle length. The retailer's total profit is a function of selling price and cycle length, and the retailer's total profit is optimized with respect to selling price and cycle length under trade-credit. Numerical examples are also presented for the validation of the present study, and sensitivity analysis is also discussed to know the robustness of the supply chain model. Managerial insight and observation have been given in the sensitivity section. Limitations and future work of this paper have been presented in the conclusion section.

Keywords: learning effect; fuzzy environment; trade credit policy; preservation technology; carbon emissions; deteriorating items; supply chain management

MSC: 90-XX

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1. Basic Introduction

Naturally, the structure of any item decreases day by day due to deterioration, and it is a natural phenomenon and cannot be ignored from the items, but it can be controlled with the help of a preservation system. The players (buyer, seller and customer) of the supply chain have to face more problems regarding profit due to the presence of deterioration in the items. The reservation system is required for controlling the deterioration of items when deteriorating items are being used in the supply chain of various industries or firms. The preservation system is one of the mechanical systems that run with the help of electricity and a generator using fuel. Carbon units emit electricity and fuel and are harmful to the environment. Nowadays, some firms, industries and stockers of items offer some polices like trade credit policies, discount order quantity policies, partial trade credit policies and so many others to earn more profit from their buyers. The buyers also use this policy and get more profit during the business. In this paper, the seller offers the credit financing strategy to his regular buyer, and the buyer accepts the policy of the trade credit for the dealing of the business under the supply chain. Our objective is that develop a supply chain model with carbon emissions and preservation technology for deteriorating items under trade credit policy and learning in fuzzy. The contribution of this paper is that optimize the buyer's total fuzzy profit with respect to selling price and cycle length, where the demand

rate is a function of selling price and also imprecise in nature under trade credit policy, preservation technology, carbon emissions and the application of learning fuzzy theory for deteriorating items. The present study suggests to the practitioner of different industries or business organizers about the trade credit period, lower and upper deviation of the demand rate, the impotence of the preservation technology, the role of carbon emissions as well as the application of the learning fuzzy theory through supply chain when deteriorating items use for dealing of business. The effect of trade credit, the rate of learning, lower and upper deviation of the demand rate, some carbon emission cost and other inventory cost on the buyer's total fuzzy profit have shown in the sensitivity section, and managerial and observation covers more significant results. The future application and limitations have been in the conclusions section and subsection. The selected highlights of this manuscript are given below.

- ➢ The present paper develops a supply chain model with carbon emissions and preservation technology for deteriorating items under trade credit policy and learning in fuzzy.
- ➢ As per demand, the seller delivers lots to the buyer under a trade credit policy where the demand rate follows the triangular fuzzy number. The delivered lot reduces into two phases. In one phase, the lots decrease due to demand and deterioration without preservation technology, and in the other phase, lots decrease due to demand with the preservation technology. Preservation technology controls the deterioration rate of items. In this point of view, preservation technology is needed, and the preservation cost is included in this model. The impact of preservation cost has been shown in this model.
- ➢ The trade-credit period affects the buyer's total profit, and its variation has been presented in this model.
- ➢ A lower and upper deviation of the demand rate directly affects the buyer's total fuzzy profit and is briefly explained in this supply chain model.
- ➢ Learning rate affects the lower and upper demand as per consideration, and learning rate changes the buyer's total profit, and its effect has been shown in this proposed supply chain model.
- ➢ Managerial implications have been concluded and clearly stated too.
- ➢ Results reveal that the credit finance scheme, learning in fuzzy and preservation technology, will be beneficial for the buyer.

The seller and buyer play as game players in the supply chain through the policy of trade credit, and both benefited in this study. Further, it has deliberated a lot of literature reviews for developing our proposed model and has discussed it in the literature review section.

1.1. Literature Review

In this section, we included some selected literature reviews which are based on deterioration, trade-credit, preservation technology, carbon emissions, fuzzy theory and learning theory. In this point of view, Goyal (1985) [1] developed an inventory model with various strategies for deteriorating items. Aggarwal and Jaggi (1995) [2] gave a mathematical model with a new strategy using a trade-credit policy for single items. Tiwari et al. (2016) [3] proposed an inventory with one level credit policy for deteriorating items under various situations. Tiwari et al. (2018) [4], improved the previous model using two-level credit policies for deteriorating items under a different policy. Jagg et al. (2011) [5] worked on an inventory model with trade credit under an inflationary situation for imperfect items.

Sarkar (2016) [6] generalized a supply chain model with some rebate strategies under shortages for lot size. Tiwari et al. (2018) [7] provided a manufacture form with a backorder situation under trade-credit for a green environment. Jaggi et al. (2013) [8] assumed a model with credit-credit under inspection process for imperfect quality items. Tiwari et al. (2017) [9] presented carbon emissions based on a supply chain system where lots have

some imperfect quality items. Yadav et al. (2018) [10] urbanized a representation which is based on game theory for imperfect quality items where demand rate depends on selling price. Kumar et al. (2003) [11] improved fuzzy based inventory model for deteriorating items. Some authors have worked on inspection process, Salameh and Jaber (2000) [12] considered an EOQ model for imperfect quality items under screening process.

In order views, some authors have tried the concept of learning, Wright (1936) [13] developed a model for minimizing the inventory cost by using the learning theory. Jaber and Bonney (1997) [14] proposed imperfect quality-based inventory model under the learning theory. Balkhi (2003) [15] generalized a model with partial backorders where the demand rate depends on time under learning effect. Sangal et al. (2016) [16] considered fuzzy based inventory model for decaying items using learning effect. Aggarwal et al. (2017) [17] improved a fuzzy model under learning effect for deteriorating items. Patro et al. (2018) [18] generalized a fuzzy based inventory model for imperfect items under the effect of learning. Jayaswal et al. (2019) [19] presented a trade-credit based stock model for decaying items with the learning concept. To move in the direction of fuzzy theory, Jayaswal et al. (2019) [20] considered a fuzzy representation with the effect of learning process and shortages for deficient quality things under trade-credit. Jayaswal et al. (2022) [21] gave an inventory model with cloudy fuzzy concept under trade-credit for imperfect quality items. Alamri et al. (2022) [22], presented carbon emissions-based inventory model with the effect of inflation under learning effect. Finally, we have developed a mathematical model with carbon emissions, preservation technology and trade credit policy under learning in fuzzy for deteriorating items through supply chain. The present paper has been divided into sections such as notations and assumptions, mathematical formulation with and without fuzzy environment, fuzzy model has been defuzzified with help of singed distance method and using learning in fuzzy, used some theorem for optimization, numerical example, sensitivity analysis, conclusion and references.

1.2. Problem Definition

We maximized the buyer's total fuzzy profit per unit cycle for a supply chain model with carbon emissions and preservation technology for deteriorating items under trade credit policy and learning in fuzzy where buyer's selling price and cycle length considered as decision variable. The selling price of items and cycle length are the most important inventory parameters for the buyer. The selling of items depends on the demand rate. In this paper, we considered the demand rate is a function of selling price and also imprecise in nature. The buyer demanded Q units of deteriorating item from the seller under trade credit policy. At time $t = 0$, the inventory level is Q which decreases due to deterioration (θ) during carrying of item from the stock. In the buyer's sock level decreases due to fuzzy demand rate $D(p)$ and reduced deterioration rate by investment of preservation technology. The seller provides the buyer a fixed trade credit period to stimulate sale of item. During trade credit, no interest is paid by the seller. After this credit period, interest is charged under terms and conditions they agreed upon. We want to calculate what should be the values of selling price and cycle length for the buyer to sell the items for more profit under such model circumstances. Table 1 contains some selected authors who did not contribute such type of problems. The contribution of this paper has been shown at bottom of Table 1.

Table 1. The analysis among the previous contribution, recent contribution and present contribution.

Authors	Deterioration during Carrying	Deteriorating Items	Preservation Technology	Carbon Emissions	SCM	Trade Credit	Leaning in Fuzzy
Zhang et al. (2014) [23]		✓	✓				
Teng et al. (2014) [24]						✓	
He & Huang (2013) [25]		✓	✓				
Jaggi et al. (2017) [26]		✓				✓	
Li et al. (2019) [27]		✓	✓				

Table 1. *Cont.*

Authors	Deterioration during Carrying	Deteriorating Items	Preservation Technology	Carbon Emissions	SCM	Trade Credit	Leaning in Fuzzy
Khanna et al. (2020) [28]		✓	✓				
Jayaswal et al. (2019) [29]							
Mashud et al. (2021) [30]		✓			✓		
Alamri et al. (2022) [22]		✓		✓			
Jayaswal et al. (2022) [21]						✓	
Mahata and Debnath (2022) [31]	✓	✓	✓				
Alamari (2023) [32]				✓	✓	✓	
Mittal and Sarkar (2023) [33]				✓	✓		
Abbasi and Choukolaei (2023) [34]				✓	✓		
Wang et al. (2023) [35]			✓	✓	✓		
Alsaedi et al. (2023) [36]				✓	✓		✓
Present study	✓	✓	✓	✓	✓	✓	✓

1.3. Research Gap and Our Presently Contribution

We studied much renewed research study with the help of literature review who worked in related work on deterioration, preservation technology, carbon emissions fuzzy environment, trade credit policy and other recent with various policies for different kind of items. We are discussing some most renowned authors who worked in this direction, He & Huang (2013) [25], Zhang et al. (2014) [23], Li et al. (2019) [27] and Khanna et al. (2020) [28] developed an EOQ model with preservation technology for deteriorating items under preservation different approach and did not consider the concept of trade credit scheme, carbon emissions and learning in fuzzy. Motivated by the previous work of these authors. Jayaswal et al. (2021) [21], proposed a mathematical model without preservation technology for deteriorating items under trade credit policy and optimized profit but have not considered the concept of carbon emissions and learning in fuzzy. Mashud et al. (2021) [30] considered a supply chain model with different realistic situations for deteriorating items. Alamri et al. (2022) [22], developed a carbon emissions-based inventory model for deteriorating items under inspection process. Mahata and Debnath (2022) [31], generalized a model for deteriorating items under preservation technology where deterioration beings during carrying of items. Jayaswal et al. (2022) [21], derived a trade credit policy-based inventory model for the imperfect quality items. Alamri (2023) [32], developed a sustainable supply chain model for defective growing items (Fishery) with trade Credit policy and fuzzy learning effect and also optimized buyer's total profit with respect to order quantity. Mittal and Sarkar (2023) [33], develop a stochastic supply chain model under random energy price. Abbasi, and Choukolaei (2023) [34], provided a lot of literature reviews of green supply chain network design literature focusing on carbon taxation policy. Wang et al. (2023) [35], developed a gold supply chain model with preservation technology and carbon emissions under low carbon effect policy. Alsaedi et al. (2023) [36], presented a sustainable green supply chain model with carbon emissions for defective items under learning in fuzzy theory. We presented the comparative study with the previous contribution, recent contribution, and our present contribution in Table 1. Our contribution has been shown at bottom of Table 1 and tried to fill up the research gap among some selected research studies discussed in Table 1.

2. Mathematical Notations of Inventory Parameters and the Initial Considerations for Proposed Model

2.1. Mathematical Notations of Inventory Parameters

The notations and assumptions are given below:

T_2 = The cycle length for next replenishment (Decision variable) (year)

p = Unit selling price for the buyer (Decision variable) ($ per unit)

$D = \delta_1 - \delta_2 p$ = Selling price dependent demand

$\widetilde{D}$ = Fuzzy demand rate

δ_1 = The fixed component of the fuzzy demand rate (fixed)
δ_2 = The positive coefficient of price in the fuzzy demand rate (Fixed)
∇_h^D = Upper deviation of the fuzzy demand rate
∇_l^D = Lower deviation of the fuzzy demand rate
M = Seller's credit period credited to his buyer (in year)
$I_1(t)$ = The level of inventory in $[0, T_1]$
$I_2(t)$ = The level of inventory $[T_1, T_2]$
Y = Lot size (unit)
G = Good quality products in the lot (unit)
B = Defective quality products in the lot (unit)
I_e = Interest gained ($ per year)
I_p = Interest charged ($ per year)
l = Learning parameter
n = The number of shipments
E_c = Carbon emissions due to electricity (ton CO_2 per kWh)
F_c = Carbon emissions due to generate fuel (ton CO_2 per Liter)
e_c = The consumption of electricity in buyer's preservation technology (kWh per unit per year)
T_x = The unit tax for carbon emissions ($ per ton)
γ = Preservation cost ($ per unit time)
δ = The analyzed parameter of the investment to the decaying rate.
c = Purchasing cost ($ per unit)
P = The selling price for defective units ($ per unit)
$h(t)$ = Time dependent carrying cost ($ per unit per year)
g = Fixed part of carrying cost ($ per unit per year)
θ = The rate of deterioration during carrying (fixed)
$\rho(\gamma)$ = The fraction of reduced decaying rate (fixed)
$\overline{\gamma}$ = The optimal investment cost during preservation technology ($ per unit time)
p^* = Optimal selling price for the buyer under supply chain ($ per unit)
$T_2{}^*$ = Optimal cycle length for the supply chain system (year)
$\Delta_{111}(p, T_2)$ = The buyer's total fuzzy profit per unit time under learning effect for Case 1 ($)
$\Delta_{122}(p, T_2)$ = The buyer's total fuzzy profit per unit time under learning effect for Case 2 ($)
$\Delta_{133}(p, T_2)$ = The buyer's total fuzzy profit per unit time under learning effect for Case 3 ($)

2.2. Assumptions

- The fuzzy demand has been considered and it is treated as triangular fuzzy number.
- The learning effect involves upper and lower deviation of fuzzy demand.
- It is considered that carbon dioxide is emitted due to electricity and fuel.
- Carbon emissions are incorporated due to some preservation technology.
- The preservation cost is also included in this model due to some investment in preservation technology.
- The seller makes a good supply chain by offering the credit period to earn more profit from his regular buyer to maintain the dealing business.
- The seller and buyer play as game players during credit period.
- The buyer accepts the policy of the credit period offered by the seller.
- Time depended on holding cost has been considered.
- The fraction of decaying rate has been taken in the range, $0 \leq \rho(\gamma) \leq 1$.
- Suppose that γ/unit time invests in preservation system and also fulfils under such restriction, $0 \leq \gamma \leq \overline{\gamma}$, where $\overline{\gamma}$ is a highest investment of money concerning preservation system.

3. Mathematical Model

It is considered that the buyer orders Y units deteriorating items from his seller. Therefore, the stock of inventory at $t = 0$ is Y. After reaching the whole lot from the seller's stock to the buyer's stock. The buyer checks the whole lots and gets some defective items say B and G is none defective items during carrying. Finally, buyer's whole lot is the sum of defective and none defective items, i.e., $Y = (G + B)$ units. It assumed that $I_1(t)$ represents the stock of inventory in $[0,\ T_1]$ which decreases due to the rate of demand and decaying of items and there is no preservation technology in this phase. The preservation technology reduces the deterioration of deteriorating items and has applied preservation technology in this phase interval $[T_1,\ T_2]\cdot$ It considered that $I_2(t)$ represents the stock of inventory $[T_1,\ T_2]$. As per assumption (Mahata et al. 2022) [31], the deterioration rate is free from initial deterioration rate (θ) under preservation technology $\left(\rho(\gamma) = e^{-\delta\gamma},\ \rho(\gamma) \ll \theta\ \right)$ where δ is the sensitive parameter for investment and γ $(\$/\text{time})$ is the preservation technology cost. We calculated the amount of B and G under this situation (Figure 1).

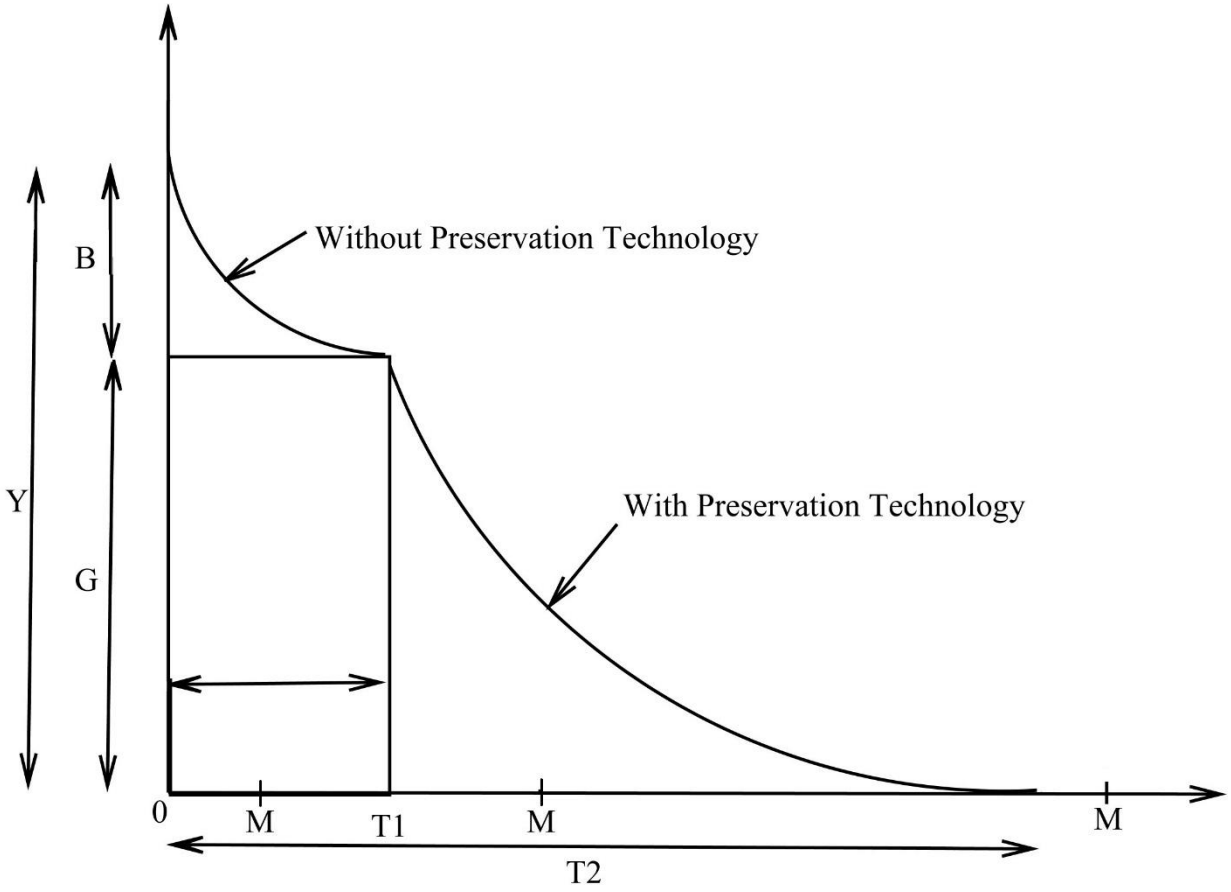


Figure 1. Representation of inventory model under trade credit.

The inventory stocks $I_1(t)$ and $I_2(t)$ are following linear differential equation which are given under below:

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = -D,\ \ t \in [0, T_2] \tag{1}$$

If the boundary restriction is that $I_1(T_1) = G(< Y)$. Now solving the linear differential Equation (1) for the inventory level $I_1(t)$ using the boundary condition. We get the value of $I_1(t)$ in the time interval $[0,\ T_1]$ where $0 \le t \le T_1$ and is shown by

$$I_1(t) = \left(-D\theta^{-1} + \theta^{-1}(G\theta + D)e^{\theta(T_1 - t)}\right) \tag{2}$$

Now the differential equation for the stock level of $I_2(t)$ under the boundary restriction $I_2(T_2) = 0$, and is represented below.

$$\frac{dI_2(t)}{dt} + I_2(t)\rho(\gamma) = -D,\ t \in [T_1, T_2] \tag{3}$$

The inventory level $I_2(t)$ in the interval $[T_1\ T_2]$ where $t \in [T_1\ T_2]$ and is shown below.

$$I_2(t) = \frac{D}{\rho(\gamma)}\left(e^{\theta(T_2-t)} - 1\right) \tag{4}$$

The amount of quantity which is not defected during carrying and using the condition of continuity $(I_1(T_1) = I_2(T_1))$ using the Equations (2) and (3), we get.

$$G = \frac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) \tag{5}$$

As per assumption, at initial $I_1(0) = Y$, then from Equations (2) and (5), we get

$$\begin{gathered} Y = \tfrac{1}{\theta}\left(-D + (G\theta + D)e^{\theta(T_1)}\right) \\ Y = \tfrac{1}{\theta}\left(-D + \left(\tfrac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\theta + D\right)e^{\theta(T_1)}\right) \end{gathered} \tag{6}$$

It is assume that, Y is the sum of defective and none effective items during carrying; $Y = B + G$, then the defective items $(B = Y - G)$, therefore

$$B = \frac{1}{\theta}\left(-D + (G\theta + D)e^{\theta(T_1)}\right) - \frac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) \tag{7}$$

Now, total profit function for buyer contains the following major parts.

(1) Set-up cost

$$\text{(OC)} = \text{K} \tag{8}$$

(2) Inventory holding cost (IHC);

$$IHC = \int_0^{T_1} [gc + hct] I_1(t)dt + \int_{T_1}^{T_2} [gc + hct] I_2(t)dt \tag{9}$$

The values of $I_1(t)$ and $I_2(t)$ put in the Equation (9) and after solving the integration, we get

$$\begin{aligned} IHC = \Bigg[cg\Bigg\{\frac{-DT_1}{\theta} &+ \left(G + \frac{D}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right)\Bigg\} + ch\Bigg\{\frac{-DT_1}{\theta} + \left(G + \frac{D}{\theta}\right)\left(-\frac{T_1}{\theta} - \frac{1}{\theta^2}\left(1 - e^{\theta T_1}\right)\right)\Bigg\}\Bigg] \\ &+ \Bigg[\frac{cgD}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) + (-T_2 + T_1)\right\} - \frac{DchT_2{}^2}{2\rho(\gamma)} + \frac{DchT_1{}^2}{2\rho(\gamma)} \\ &+ \left\{\frac{1}{\rho(\gamma)^3}\left(-1 + e^{-(\rho(\gamma)T_1 + \rho(\gamma)T_2)} - 1\right) + \left(-\frac{T_2}{\rho(\gamma)^2} + \frac{T_1}{\rho(\gamma)^2}e^{\rho(\gamma)(T_2-T_1)}\right)\right\} Dch\Bigg] \end{aligned} \tag{10}$$

(3) Purchasing cost for ordered units (PC); $PC = cY$;

$$PC = cY = \frac{1}{\theta}\left(-D + \left(\frac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\theta + D\right)e^{\theta(T_1)}\right) \tag{11}$$

(4) Total carbon units *(TCU)* due to holding units from 0 to T_2 which can be calculated (Alamri et al. (2022) [22]), $TCU = \int_0^{T_1} [I_1(t)] \cdot\, dt + \int_{T_1}^{T_2} [I_2(t)] \cdot dt$

$$TCU = \frac{-DT_1}{\theta} + \left(G + \frac{D}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right) + \frac{D}{\rho(\gamma)\theta}\left(-1 + e^{-\rho(\gamma)T_1 + (\rho(\gamma)T_2)} + \theta(T_1 - T_2)\right) \tag{12}$$

Now the total carbon emission cost due to electricity and generator's fuel (CEC) from Equation (12):

$$\begin{gathered} CEC = (e_c E_c\ T_x + e_c F_c\ )TCU \\ CEC = (e_c E_c\ T_x + e_c F_c\ )\left(\frac{-DT_1}{\theta} + \left(G + \frac{D}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right) + \frac{D}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2 - T_1)} - 1 + \theta(T_1 - T_2)\right)\right) \end{gathered} \tag{13}$$

(5) Preservation cost (PRC);

$$PRC = \gamma(T_2\ - T_1\ ) \tag{14}$$

The buyer earns from the good quality demand and defective items during carrying items.

(6) Buyer's total income (BTI); $TI = p\int_0^{T_2} Ddt + PB$

$$BTI = pDT_2 + P\left(\frac{1}{\theta}\left(-D + (G\theta + D)e^{\theta(T_1)}\right) - \frac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2 - T_1)} - 1\right)\right) \tag{15}$$

Buyer's entire yield function can be written from the Equations (8)–(15)

$$\begin{gathered} \text{Buyer's whole gain function}(\Delta(p, T_2)) \\ = \text{Buyer's whole revenue}(BTI) - \text{Set up cost}(\text{OC}) \\ -\text{Inventory holding cost}(\text{IHC}) - \text{Purchasing cost for ordered units}(PC) \\ -\text{Total carbon emission cost}(TEC) - \text{Preservation cost}(PRC) \end{gathered} \tag{16}$$

$$\begin{aligned} \Delta(p, T_2) = {} & pDT_2 + P\left(\tfrac{1}{\theta}\left(-D + (G\theta + D)e^{\theta(T_1)}\right) - \tfrac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2 - T_1)} - 1\right)\right) - K \\ & - \left[cg\left\{\tfrac{-DT_1}{\theta} + \left(G + \tfrac{D}{\theta}\right)\left(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\right)\right\}\right. \\ & \left. + ch\left\{\tfrac{-DT_1}{\theta} + \left(G + \tfrac{D}{\theta}\right)\left(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\left(1 - e^{\theta T_1}\right)\right)\right\}\right] \\ & + \left[\tfrac{cgD}{\rho(\gamma)}\left\{\tfrac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2 - T_1)} - 1\right) + (-T_2 + T_1)\right\} - \tfrac{DchT_2{}^2}{2\rho(\gamma)} + \tfrac{T_1{}^2 Dch}{2\rho(\gamma)}\right. \\ & \left. + \left\{\tfrac{1}{\rho(\gamma)^3}\left(-1 + e^{-\rho(\gamma)T_1 + (\rho(\gamma)T_2)}\right) - \tfrac{T_2}{\rho(\gamma)^2} + \tfrac{T_1}{\rho(\gamma)^2}e^{-\rho(\gamma)T_1 + (\rho(\gamma)T_2)}\right\}\right] Dch \\ & - \tfrac{1}{\theta}\left(-D + \left(\tfrac{D}{\rho(\gamma)}\left(e^{(\rho(\gamma)T_2 - \rho(\gamma)T_1)} - 1\right)\theta + D\right)e^{\theta(T_1)}\right) \\ & - (e_c E_c T_x + e_c F_c)\left(\tfrac{-DT_1}{\theta} + \left(G + \tfrac{D}{\theta}\right)\left(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\right)\right. \\ & \left. + \tfrac{D}{\rho(\gamma)\theta}\left(-1 + \tfrac{1}{e^{-\rho(\gamma)\cdot(T_2 - T_1)}} - \theta T_2 + T_1\theta\right)\right) - \gamma T_1 + T_2\gamma \end{aligned} \tag{17}$$

As considered, the buyer accepts the strategy of trade -credit and because of its profit function is also affected. Sometime buyer earns extra profit due to interest gain (IE) if he returns all credit money on or before credit period and interest gain increases the buyer's sale revenue. On the other hand, if he does not return all credit money up to a credit -period then the buyer will have to pay extra money to the seller and this extra money is known as interest paid (IP) and interest paid decreases the buyer's sale revenue. We can rewrite Equations (16) and (17) due to interest gain (IE) and interest paid (IP),

$$\begin{aligned}\text{Buyer's total profit function}(\Delta(p,T_2)) &= \text{Buyer's total income}(BTI) + \text{interest gain}(IE) - \\ &\text{Set up cost (OC)} - \quad \text{Inventory holding cost (IHC)} - \\ &\text{Purchasing cost for ordered units } (PC) - \\ &\text{Total carbon emission cost } (TEC) - \\ &\text{Preservation cost}(PRC) \; - \text{Interest paid } (IP)\end{aligned}$$

$$\begin{aligned}\Delta(p,T_2) = pDT_2 + P\Big(\tfrac{1}{\theta}\Big(-D+(G\theta+D)e^{\theta(T_1)}\Big) - \tfrac{D}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big) + IE - K \\ -\Big[cg\Big\{\tfrac{-DT_1}{\theta} + \Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\} \\ +ch\Big\{\tfrac{-DT_1}{\theta} + \Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\big(1-e^{\theta T_1}\big)\Big)\Big\}\Big] \\ +\Big[\tfrac{Dgc}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big) - T_2 + T_1\Big\} - \tfrac{{T_2}^2 Dch}{2\rho(\gamma)} + {T_1}^2\tfrac{Dch}{2\rho(\gamma)} \\ +\tfrac{Dhc}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(-1+e^{-\rho(\gamma)T_1+(\rho(\gamma)T_2)}\Big) - \tfrac{T_2}{\rho(\gamma)} + \tfrac{T_1}{\rho(\gamma)}e^{-\rho(\gamma)T_1+(\rho(\gamma)T_2)}\Big\}\Big] \\ -\tfrac{1}{\theta}\Big(-D+\Big(\tfrac{D}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\theta+D\Big)e^{\theta(T_1)}\Big) \\ -(e_cE_cT_x+e_cF_c)\Big(\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big) \\ +\tfrac{D}{\rho(\gamma)\theta}\Big(-1+e^{-\rho(\gamma)T_1+(\rho(\gamma)T_2)}-1-T_2\theta+\theta T_1\Big)\Big) + T_1\gamma - \gamma T_2 - IP\end{aligned} \tag{18}$$

Now, interest earned, and interest charged depend on the credit period and are calculated case wise with the help of figures.

Proposed model for the Case 1, $T_2 \geq T_1 \geq M$

In the Figure 2, the buyer gets the interest gain up to the trade–credit time M, which is equal to $\frac{pI_eDM^2}{2}$ for sold items and its interest paid after the trade-credit time from M up to T_2 for unsold items and given under below.

Buyer's interest earned

$$(IE) = \frac{pI_eDM^2}{2} \tag{19}$$

and buyer's interest charged

$$\begin{aligned}(IP) &= \int_M^{T_1} I_1(t)cI_p dt + \int_{T_1}^{T_2} I_2(t)cI_p dt \\ &= cI_p\Big[\tfrac{D}{\rho(\gamma)}\Big(e^{-M\theta+\theta T_1}\Big(-1+e^{-T_1\rho(\gamma)+T_2\rho(\gamma)}\Big) - e^{\rho(\gamma)(T_2-T_1)} - 1\Big) \\ &\quad -M\tfrac{D}{\theta} + T_1\tfrac{D}{\theta} + \Big(\tfrac{De^{\theta(T_1-M)}}{\theta^2}\Big) \\ &\quad +\tfrac{D}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)} - T_2 + T_1 \\ &\quad +\tfrac{1}{\rho(\gamma)}\Big(e^{-T_1\rho(\gamma)+T_2\rho(\gamma)}\Big)\Big)\Big]\end{aligned} \tag{20}$$

In this case, buyer's entire income per cycle using Equations (19) and (20), we get

$$\begin{aligned}\Delta(p,T_2) = \tfrac{1}{T_2}\Big(&pDT_2 + P\Big(\tfrac{1}{\theta}\Big(-D+(G\theta+D)e^{\theta(T_1)}\Big) - \tfrac{D}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big) + \tfrac{pI_eDM^2}{2}\\ &-K\\ &-\Big[cg\Big\{\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\ &+ch\Big\{\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\big(1-e^{\theta T_1}\big)\Big)\Big\}\Big]\\ &+\Big[\tfrac{cgD}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)-T_2+T_1\Big\}+\tfrac{DT_1{}^2ch}{2\rho(\gamma)}-\tfrac{Dch}{2\rho(\gamma)}T_2{}^2\\ &+\tfrac{D\cdot h\cdot c}{\rho(\gamma)}\cdot\Big\{\rho(\gamma)^{-2}\cdot\Big(-1+\tfrac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}\Big)-\tfrac{T_2}{\rho(\gamma)}+\tfrac{1}{\rho(\gamma)\cdot e^{\rho(\gamma)T_1-(\rho(\gamma)T_2)}}\cdot T_1\Big\}\Big]\\ &-\tfrac{1}{\theta}\Big(-D+\Big(\tfrac{D}{\rho(\gamma)}\Big(\tfrac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1\Big)\theta+D\Big)e^{\theta(T_1)}\Big)\\ &-(e_cE_cT_x+e_cF_c)\Big(\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\ &+\tfrac{D}{\rho(\gamma)\theta}\Big(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\Big)\Big)+\gamma T_2-T_1\gamma\\ &-cI_p\Big[\tfrac{D}{\rho(\gamma)}\Big(\tfrac{1}{e^{\theta M-\theta T_1}}\cdot\Big(-1+e^{-\rho(\gamma)T_1+(\rho(\gamma)T_2)}\Big)-e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big)\\ &-\tfrac{D}{\theta}((T_1-M))+\tfrac{D}{\theta^2}\Big(\tfrac{1}{e^{-\theta\cdot(T_1-M)}}\Big)\\ &+\tfrac{D}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)}-T_2+\tfrac{T_1}{\rho(\gamma)}+\tfrac{1}{\rho(\gamma)}e^{(\rho(\gamma)\cdot T_2-\rho(\gamma)\cdot T_1)}\Big)\Big]\Big)\end{aligned} \tag{21}$$

Proposed model for the Case 2, $T_2 \geq M \geq T_1$.

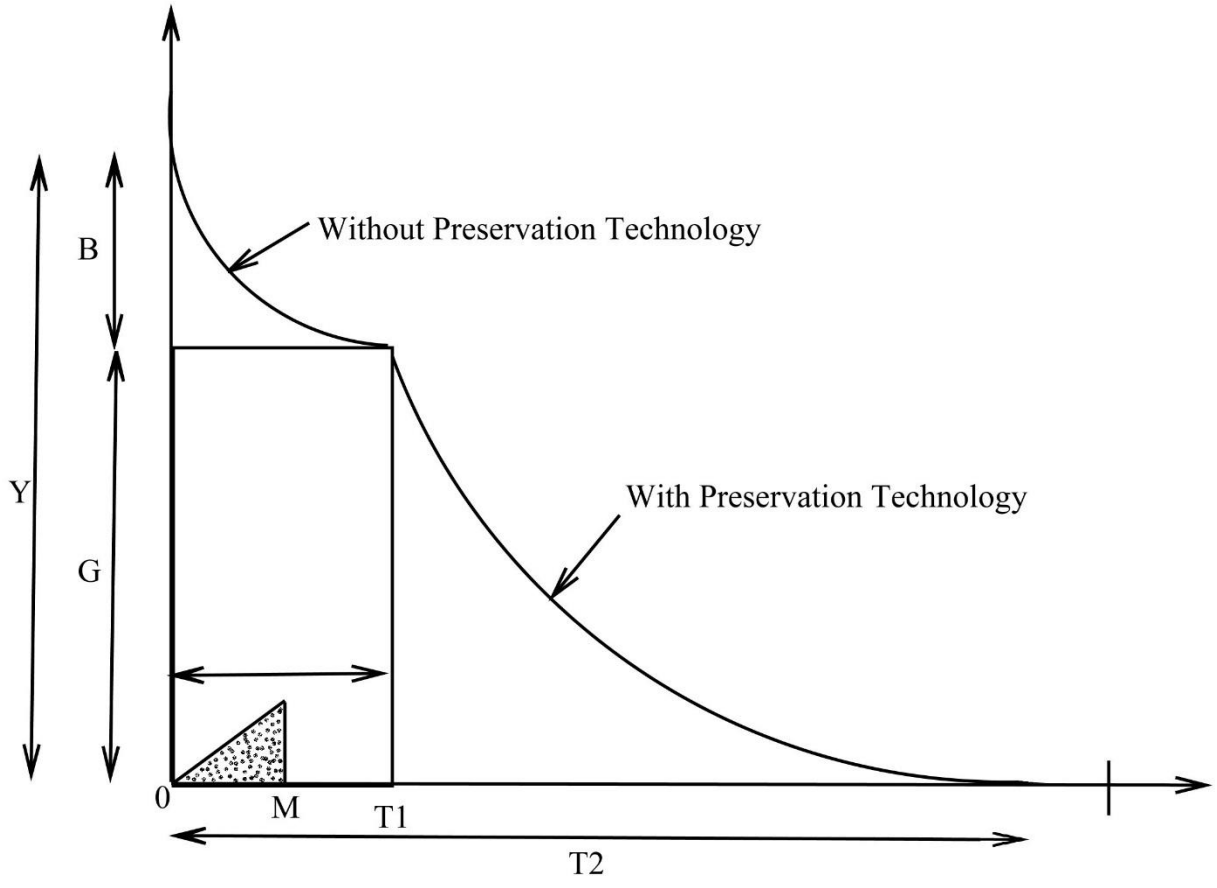


Figure 2. Proposed model under Case 1.

From the Figure 3, the buyer earns the interest gain up to the credit time M, which is more than and equal to T_1 and its value is equal to $\frac{pI_eDM^2}{2}$ for the sold items and its rate of interest paid will be after T_1 and numerically can be write $cI_p\left[\int_M^{T_2} I_2(t)dt\right]$ for unsold items.

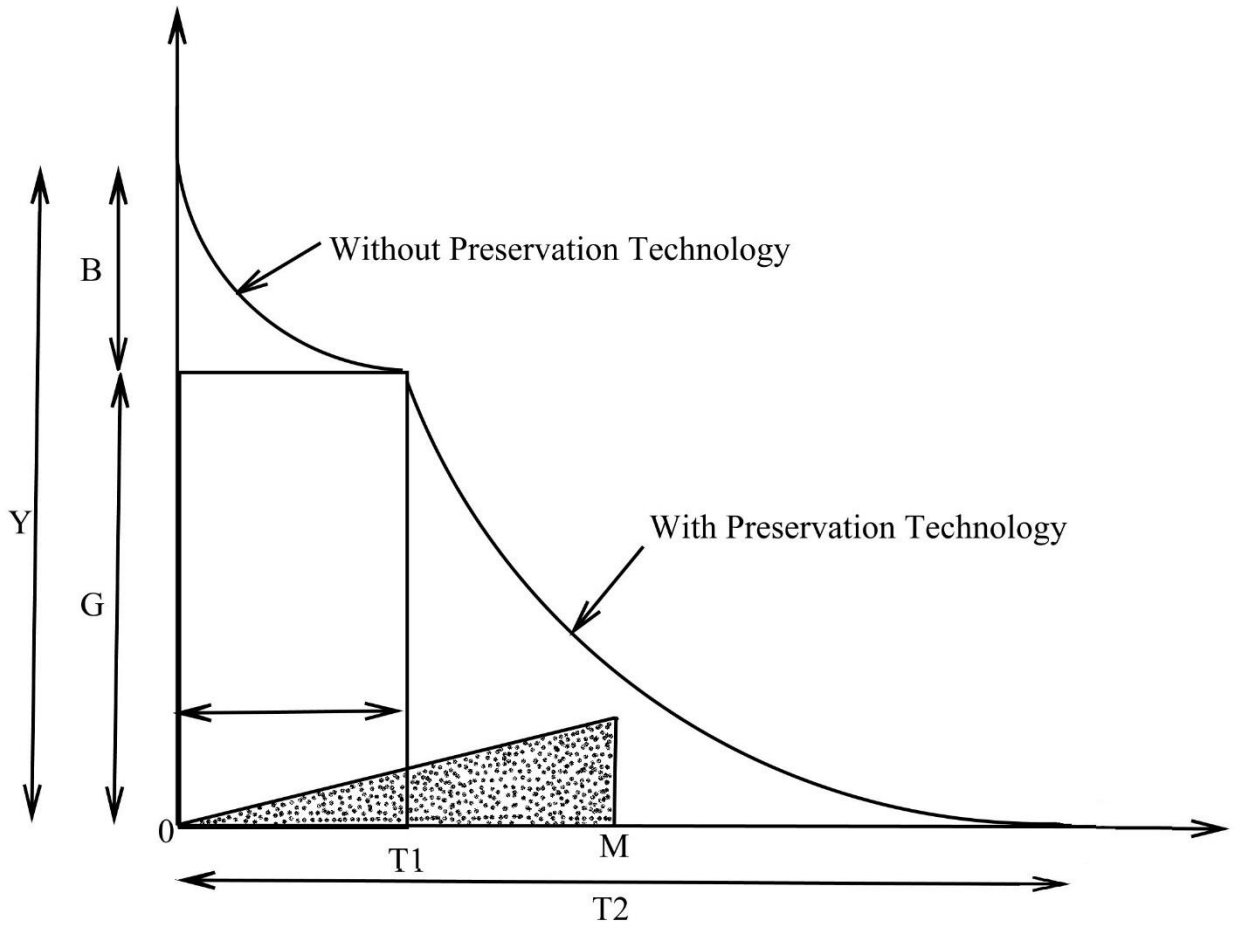


Figure 3. Proposed model under Case 2.

Buyer's interest gain

$$(IE) = \frac{pI_eDM^2}{2} \tag{22}$$

and buyer's interest charged

$$\begin{aligned}(IP) &= \int_M^{T_2} I_2(t)cI_p dt \\ &= cI_p\left[-\tfrac{1}{\rho(\gamma)} - T_2 + \tfrac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-M)}\right) + M\right]\end{aligned} \tag{23}$$

In this case, buyer's total profit per cycle from Equations (22) and (23), we get

$$\begin{aligned}\Delta(p,T_2) = \tfrac{1}{T_2}\Big(&pDT_2 + P\Big(\tfrac{1}{\theta}\Big(-D+(G\theta+D)e^{\theta(T_1)}\Big) - \tfrac{D}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big) + \tfrac{pI_eDM^2}{2}\\ &-K\\ &-\Big[cg\Big\{\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\ &+ch\Big\{\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\cdot(1-e^{\theta\cdot T_1})\Big)\Big\}\Big]\\ &+\Big[\tfrac{gDc}{\rho(\gamma)}\Big\{\Big(\tfrac{e^{\rho(\gamma)(T_2-T_1)}}{\rho(\gamma)}-\tfrac{1}{\rho(\gamma)}\Big)+T_1-T_2\Big\}+\tfrac{Dch}{2\rho(\gamma)}\Big(-{T_2}^2+{T_1}^2\Big)\\ &+\tfrac{Dhc}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big)+\Big(\tfrac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\ &-\tfrac{1}{\theta}\Big(-D+\Big(\tfrac{D}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\theta+D\Big)e^{\theta(T_1)}\Big)\\ &-(e_cE_cT_x+e_cF_c)\Big(\tfrac{-DT_1}{\theta}+\Big(G+\tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\ &+\tfrac{1}{\rho(\gamma)\theta}D\Big(-1+e^{-T_1\rho(\gamma)+T_2\rho(\gamma)}+\theta(T_1-T_2)\Big)\Big)-\gamma(T_2-T_1)\\ &-cI_p\Big[\tfrac{D}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)}-T_2+\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-M)}\Big)+M\Big)\Big]\Big)\end{aligned} \tag{24}$$

Proposed model for the Case 3, $M \geq T_2 \geq T_1$.

As of Figure 4, the buyer gets the interest gain up to the credit time M, which is more than T_1 and T_2 and its value is equal to $\frac{pI_eD{T_2}^2}{2} + pI_eD(M-T_2)$ for the sold items and its interest paid will be 0.

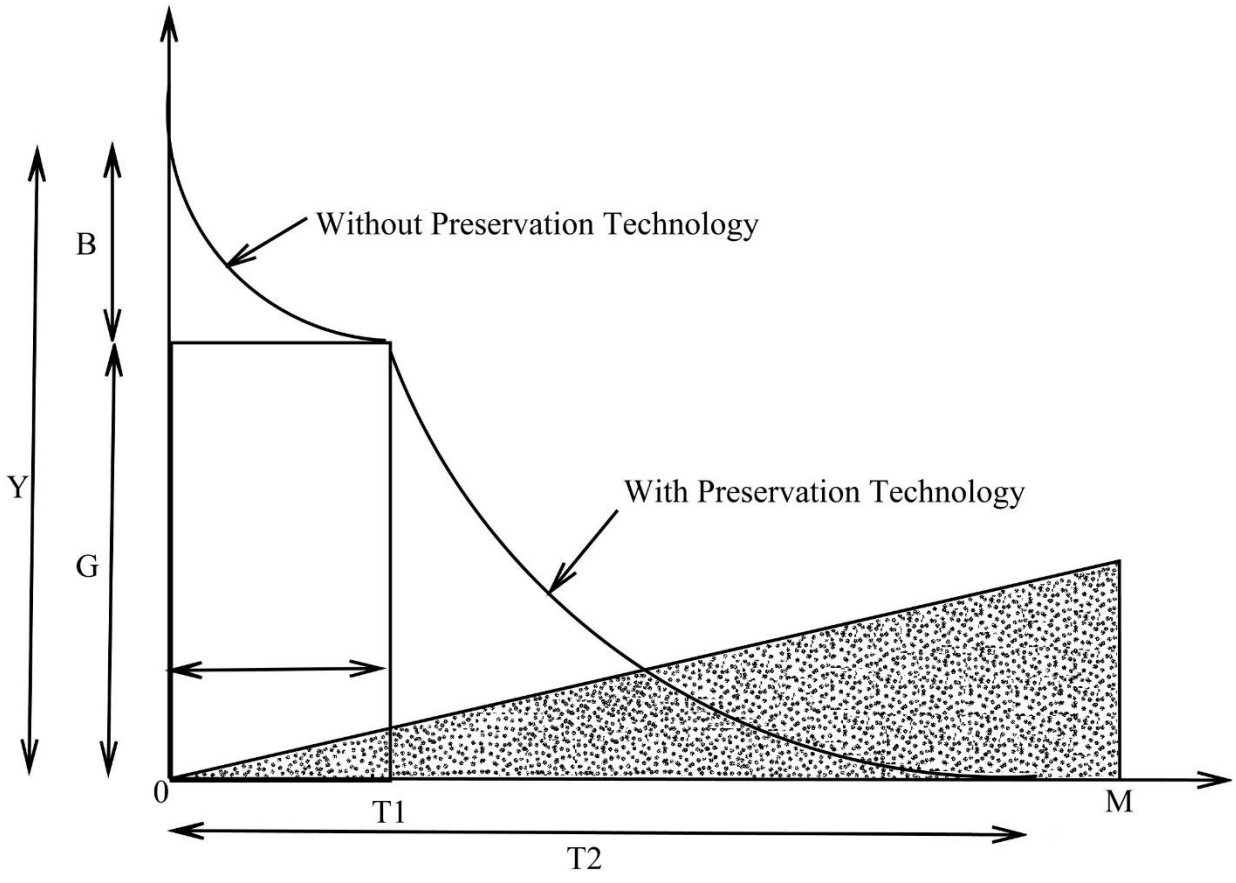


Figure 4. Proposed model under Case 3.

Buyer's interest gain

$$(IE) = \frac{pI_eDT_2^2}{2} + pI_eD(M - T_2) \tag{25}$$

Buyer's interest paid

$$(IP) = 0 \tag{26}$$

In this case, buyer's entire yield per cycle from Equations (25) and (26), we get

$$\begin{aligned}\Delta(p, T_2) = \tfrac{1}{T_2}\Big(&pDT_2 + P\Big(\tfrac{1}{\theta}\Big(-D + (G\theta + D)e^{\theta(T_1)}\Big) - \tfrac{D}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2 - T_1)} - 1\Big)\Big) + \tfrac{pI_eDT_2^2}{2}\\ &+pI_eD(M - T_2) - K\\ &-\Big[cg\Big\{\tfrac{-DT_1}{\theta} + \Big(G + \tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\ &+ch\Big[\tfrac{-DT_1}{\theta} + \Big(G + \tfrac{D}{\theta}\Big)\Big(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\Big(1 - \tfrac{1}{e^{-\theta T_1}}\Big)\Big)\Big]\\ &+\Big[\tfrac{D\cdot g\cdot c}{\rho(\gamma)}\Big\{\Big(\tfrac{1}{\rho(\gamma)\cdot e^{-\rho(\gamma)(T_2 - T_1)}} - \tfrac{1}{\rho(\gamma)}\Big) - T_2 + T_1\Big\} + \tfrac{2D\cdot c\cdot h}{4\cdot\rho(\gamma)}\Big(-{T_2}^2 + {T_1}^2\Big)\\ &+\tfrac{Dhc}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(e^{(\rho(\gamma)T_2 - \rho(\gamma)T_1)} - 1\Big) + \Big(\tfrac{T_1}{e^{-(\rho(\gamma)T_2 + \rho(\gamma)T_1)}\cdot\rho(\gamma)} - \tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\ &-\theta^{-1}\Big(-D + \Big(\tfrac{D}{\rho(\gamma)}\Big(-1 + e^{-T_1\rho(\gamma) + T_2\rho(\gamma)}\Big)\theta + D\Big)e^{\theta\cdot(T_1)}\Big)\\ &-(e_cE_cT_x + e_cF_c)\Big(\tfrac{-DT_1}{\theta} + \Big(G + \tfrac{D}{\theta}\Big)\Big(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\Big)\\ &+\tfrac{D}{\rho(\gamma)\theta}\Big(e^{\rho(\gamma)(T_2 - T_1)} - 1 + \theta(T_1 - T_2)\Big)\Big) - \gamma(-T_1 + T_2)\Big)\end{aligned} \tag{27}$$

3.1. Mathematical Model under Fuzzy Environment

Now, the buyer's total profit function of Case 1, Case 2 and Case 3 from the Equations (21), (24) and (27) are reduced into the buyer's fuzzy total profit function of Case 1, Case 2 and Case 3 and shown case wise in below subsections (Jayaswal et al., 2021) [37].

3.1.1. Mathematical Model for the Case 1 in Fuzzy Environment

The buyer's fuzzy entire yield function per cycle

$$\begin{aligned}\Delta_{11}(p, T_2) = \tfrac{1}{T_2}\Big(&p\widetilde{D}T_2 + P\Big(\tfrac{1}{\theta}\Big(-\widetilde{D} + \Big(G\theta + \widetilde{D}\Big)e^{\theta(T_1)}\Big) - \tfrac{\widetilde{D}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2 - T_1)} - 1\Big)\Big) + \tfrac{pI_e\widetilde{D}M^2}{2}\\ &-K\\ &-\Big[cg\Big\{\tfrac{-T\widetilde{D}_1}{\theta} + \Big(G + \tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\ &+ch\Big\{\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G + \tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\Big(1 - \tfrac{1}{e^{-\theta T_1}}\Big)\Big)\Big\}\Big]\\ &+\Big[\tfrac{\widetilde{D}gc}{\rho(\gamma)}\Big\{\Big(\tfrac{e^{-T_1\rho(\gamma) + T_2\rho(\gamma)}}{\rho(\gamma)} - \tfrac{1}{\rho(\gamma)}\Big) - T_1 + T_2\Big\} + \tfrac{\widetilde{D}hc}{2\rho(\gamma)}\big(-T_2^2 + T_1^2\big)\\ &+\tfrac{ch\widetilde{D}}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(e^{(\rho(\gamma)T_2 - \rho(\gamma)T_1)} - 1\Big) + \Big(\tfrac{T_1e^{(\rho(\gamma)T_2 - \rho(\gamma)T_1)}}{\rho(\gamma)} - \tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\ &-\tfrac{1}{\theta}\Big(-\widetilde{D} + \Big(\tfrac{\widetilde{D}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2 - T_1)} - 1\Big)\theta + \widetilde{D}\Big)e^{\theta(T_1)}\Big)\\ &-(e_cE_cT_x + e_cF_c)\Big(\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G + \tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\Big)\\ &+\tfrac{\widetilde{D}}{\rho(\gamma)\theta}\Big(e^{\rho(\gamma)(T_2 - T_1)} - 1 + \theta(T_1 - T_2)\Big)\Big) - T_2\gamma - \gamma T_1\\ &-cI_p\Big[\tfrac{\widetilde{D}}{\rho(\gamma)}\Big(\tfrac{\left(e^{\rho(\gamma)(T_2 - T_1)} - 1\right)}{e^{-\theta(T_1 - M)}} - 1 - e^{-T_1\rho(\gamma) + T_2\rho(\gamma)}\Big) - \tfrac{\widetilde{D}}{\theta}((T_1 - M))\\ &+\tfrac{\widetilde{D}}{\theta^2}\Big(e^{\theta(T_1 - M)}\Big) + \tfrac{\widetilde{D}}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)} - T_2 + \tfrac{T_1}{\rho(\gamma)} + \tfrac{e^{\rho(\gamma)(T_2 - T_1)}}{\rho(\gamma)}\Big)\Big]\Big)\end{aligned} \tag{28}$$

3.1.2. Mathematical Model for the Case 2 in Fuzzy Environment

The buyer's fuzzy entire yield function per cycle

$$\begin{aligned}\Delta_{12}(p,T_2) = {} & \tfrac{1}{T_2}\Big(p\widetilde{D}T_2 + P\Big(\tfrac{1}{\theta}\Big(-\widetilde{D} + \Big(G\theta+\widetilde{D}\Big)e^{\theta(T_1)}\Big) - \tfrac{\widetilde{D}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big) + \tfrac{pI_e\widetilde{D}M^2}{2}\\ & -K\\ & -\Big[cg\Big\{\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G+\tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\ & +ch\Big\{\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G+\tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\Big(1-\tfrac{1}{e^{-\theta T_1}}\Big)\Big)\Big\}\Big]\\ & +\Big[\tfrac{\widetilde{D}cg}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big) - T_2 + T_1\Big\} + \tfrac{hc\widetilde{D}}{2\rho(\gamma)}\big(-T_2^2+T_1^2\big)\\ & +\tfrac{ch\widetilde{D}}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big) + \Big(\tfrac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)} - \tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\ & -\tfrac{1}{\theta}\Big(-\widetilde{D} + \Big(\tfrac{\widetilde{D}}{\rho(\gamma)}\Big(e^{\rho(\gamma)\cdot(T_2-T_1)}-1\Big)\theta+\widetilde{D}\Big)e^{\theta\cdot(T_1)}\Big)\\ & -(e_cE_cT_x + e_cF_c)\Big(\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G+\tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\ & +\tfrac{\widetilde{D}}{\rho(\gamma)\theta}\Big(-1+e^{\rho(\gamma)(T_2-T_1)}+\theta(T_1-T_2)\Big)\Big) - \gamma(T_2-T_1)\\ & -cI_p\Big[\tfrac{\widetilde{D}}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)} - T_2 + \tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-M)}\Big)+M\Big)\Big]\Big)\end{aligned} \tag{29}$$

3.1.3. Mathematical Model for the Case 3 in Fuzzy Environment

The buyer's fuzzy entire yield function per cycle

$$\begin{aligned}\Delta_{13}(p,T_2) = {} & \tfrac{1}{T_2}\Big(p\widetilde{D}T_2 + P\Big(\tfrac{1}{\theta}\Big(-\widetilde{D} + \Big(G\theta+\widetilde{D}\Big)e^{\theta(T_1)}\Big) - \tfrac{\widetilde{D}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big) + \tfrac{pI_e\widetilde{D}T_2^2}{2}\\ & +pI_e\widetilde{D}(M-T_2) - K\\ & -\Big[cg\Big\{\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G+\tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\ & +ch\Big\{\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G+\tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\Big(1-\tfrac{1}{e^{-\theta T_1}}\Big)\Big)\Big\}\Big]\\ & +\Big[\tfrac{\widetilde{D}}{\rho(\gamma)}gc\Big\{\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big) - T_2 + T_1\Big\} + \tfrac{\widetilde{D}}{2\rho(\gamma)}\big(-hT_2^2c+hT_1^2\big)c\\ & +\tfrac{ch\widetilde{D}}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big) + \Big(\tfrac{T_1}{\rho(\gamma)\cdot e^{-(\rho(\gamma)T_2+\rho(\gamma)T_1)}} - \tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\ & -\theta^{-1}\cdot\Big(-\widetilde{D} + \Big(\tfrac{\widetilde{D}}{\rho(\gamma)}\Big(-1+e^{-T_1\rho(\gamma)+T_2\rho(\gamma)}\Big)\theta+\widetilde{D}\Big)e^{\theta(T_1)}\Big)\\ & -(e_cE_cT_x + e_cF_c)\Big(\tfrac{-\widetilde{D}T_1}{\theta} + \Big(G+\tfrac{\widetilde{D}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\ & +\tfrac{\widetilde{D}}{\rho(\gamma)\theta}\cdot\Big(-1+e^{\rho(\gamma)(T_2-T_1)}+\theta(T_1-T_2)\Big)\Big) - \gamma(T_2-T_1)\Big)\end{aligned} \tag{30}$$

In this order, we combined the buyer's fuzzy entire yield function of Case 1, Case 2 and Case 3, which are given below in the Equation (31).

$$\widetilde{\Delta}(p,T_2)\begin{cases}\Delta_{11}(p,T_2) \text{ for Case 1}\\ \Delta_{12}(p,T_2) \text{ for Case 2}\\ \Delta_{13}(p,T_2) \text{ for Case 3}\end{cases} \tag{31}$$

Now, defuzzify the Equation (31) with the help of signed distance method, we get

$$d\Big(\widetilde{\Delta}(p,T_2),0\Big)\begin{cases} d(\Delta_{11}(p,T_2),0) \text{ for Case 1} \\ d(\Delta_{12}(p,T_2),0) \text{ for Case 2} \\ d(\Delta_{13}(p,T_2),0) \text{ for Case 3} \end{cases} \tag{32}$$

After that the defuzzification for the Case 1,

$$\begin{aligned}
d(\Delta_{11}(p,T_2),0) &= \Delta_{111}(p,T_2) \\
&= \tfrac{1}{T_2}\Big(pd\Big(\widetilde{D},0\Big)T_2 \\
&+P\Big(\tfrac{1}{\theta}\Big(-d\Big(\widetilde{D},0\Big)+\Big(G\theta+d\Big(\widetilde{D},0\Big)\Big)e^{\theta(T_1)}\Big)-\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big) \\
&+\tfrac{pI_e d\left(\widetilde{D},0\right)M^2}{2}-K \\
&-\Big[cg\Big\{\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\} \\
&+ch\Big\{\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\Big)\Big\}\Big] \\
&+\Big[\tfrac{cg}{\rho(\gamma)}d\Big(\widetilde{D},0\Big)\Big\{\Big(\tfrac{e^{-T_1\rho(\gamma)+T_2\rho(\gamma)}}{\rho(\gamma)}-\tfrac{1}{\rho(\gamma)}\Big)+T_1-T_2\Big\} \\
&+\tfrac{d\left(\widetilde{D},0\right)}{2}hc\Big(\tfrac{-T_2^2}{\rho(\gamma)}+\tfrac{T_1^2}{\rho(\gamma)}\Big) \\
&+\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}hc\Big\{\Big(-\tfrac{1}{\rho(\gamma)^2}+\tfrac{1}{\rho(\gamma)^2}e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big) \\
&+\tfrac{1}{\rho(\gamma)^2}\Big(\tfrac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big] \\
&-\tfrac{1}{\theta}\Big(-d\Big(\widetilde{D},0\Big)+\Big(\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\theta+d\Big(\widetilde{D},0\Big)\Big)e^{\theta(T_1)}\Big) \\
&-(e_cE_cT_x+e_cF_c)\Big(\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big) \\
&+\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)\theta}\Big(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}+(-T_2\theta+\theta T_1)\Big)\Big)-\gamma T_1+\gamma T_2 \\
&-cI_p\Big[\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{(\theta T_1-\theta M)}\Big(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big)-1-e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big) \\
&-\tfrac{d\left(\widetilde{D},0\right)}{\theta}((T_1-M))+\tfrac{d\left(\widetilde{D},0\right)}{\theta^2}\left(e^{-\theta M+T\theta_1}\right) \\
&+\tfrac{1}{\rho(\gamma)}\Big(-d\Big(\widetilde{D},0\Big)\cdot T_2-d\Big(\widetilde{D},0\Big)\rho(\gamma)^{-1}+d\Big(\widetilde{D},0\Big)\rho(\gamma)^{-1}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big) \\
&+T_1)\Big]\Big)
\end{aligned} \tag{33}$$

Here, the demand function is considered as triangular fuzzy number and used its definition, we get from Equation (33)

$$
\begin{aligned}
d(\Delta_{11}(p,T_2),0) &= \Delta_{111}(p,T_2)\\
&= \tfrac{1}{T_2}\cdot\Big(T_2 p\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\\
&+P\left(\tfrac{1}{\theta}\Big(-\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}+\Big(G\theta+\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\Big)e^{T_1\theta}\Big)-\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{2}}{2\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\right)\\
&+\tfrac{pl_e\frac{4D+\nabla_K^D-\nabla_1^D}{4}M^2}{2}-K\\
&-\left[cg\left\{\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\left(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_1^D}{4}}{\theta}\right)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\right\}\right.\\
&\left.+ch\left\{\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\left(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\right)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\Big(1-\tfrac{1}{e^{-6T_1}}\Big)\Big)\right\}\right)\\
&+\left[\tfrac{1}{\rho(\gamma)}gc\tfrac{4D+\nabla_h^D-\nabla_1^D}{4}\left\{\tfrac{1}{\rho(\gamma)}\cdot\Big(\tfrac{1}{e^{-\rho(\gamma)(T_2-T_1)}}-1\Big)-T_2+T_1\right\}+\tfrac{c\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{2\rho(\gamma)}h(-T_2^2+T_1^2)\right.\\
&\left.+\tfrac{\frac{4D+\nabla_h^\rho-\nabla_l^D}{4}}{\rho(\gamma)}hc\left\{\tfrac{1}{\rho(\gamma)^2}\Big(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big)+\Big(\tfrac{9T_1e^{(\rho(y)T_2-\rho(\gamma)T_1)}}{9\rho(\gamma)}-\tfrac{T_2}{\rho(\gamma)}\Big)\right\}\right)\\
&-\tfrac{1}{\theta}\left(-\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}+\left(\tfrac{\frac{4D+\nabla_h^D-\nabla_1^D}{4}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\theta+\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\right)e^{\theta(T_1)}\right)\\
&-(e_cE_cT_x+e_cF_c)\left(\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\left(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\right)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\right.\\
&\left.+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)\theta}\Big(e^{\rho(y)(\gamma_2-T_i)}-1+\theta(T_1-T_2)\Big)\right)-\gamma(T_2-T_1)\\
&-cl_p\left[\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}\Big(e^{(\theta T_1-\theta M)}\Big(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big)-1-e^{\rho(\gamma)(T_2-T_1)}\Big)-\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}((T_1-M))\right.\\
&\left.+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta^2}\left(\tfrac{1}{e^{-\theta\cdot(T_1-M)}}\right)+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}\Big(-\rho(\gamma)^{-1}-T_2+\tfrac{1}{\rho(\gamma)}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big)+T_1\Big)\right]
\end{aligned}
\tag{34}
$$

The defuzzification for the Case 2 from equation

$$
\begin{aligned}
d(\Delta_{12}(p,T_2),0) &= \Delta_{122}(p,T_2)\\
&= \tfrac{1}{T_2}\Big(pd\Big(\widetilde{D},0\Big)T_2\\
&+P\Big(\tfrac{1}{\theta}\Big(-d\Big(\widetilde{D},0\Big)+\Big(G\theta+d\Big(\widetilde{D},0\Big)\Big)e^{\theta(T_1)}\Big)-\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big)\\
&+\tfrac{pI_e d\left(\widetilde{D},0\right)M^2}{2}-K\\
&-\Big[cg\Big\{\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\
&+ch\Big\{\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\Big)\Big\}\Big]\\
&+\Big[\tfrac{cg}{\rho(\gamma)}d\Big(\widetilde{D},0\Big)\Big\{T_1+\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)-T_2\Big\}\\
&+\tfrac{d\left(\widetilde{D},0\right)}{2\rho(\gamma)}c\left(-hT_2^2+hT_1^2\right)\\
&+\tfrac{7cd\left(\widetilde{D},0\right)h}{7\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\Big)+\Big(\tfrac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\
&-\tfrac{1}{\theta}\Big(-d\Big(\widetilde{D},0\Big)+\Big(\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\theta+d\Big(\widetilde{D},0\Big)\Big)e^{\theta(T_1)}\Big)\\
&-(e_cE_cT_x+e_cF_c)\Big(\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\
&+\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)\theta}\Big(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\Big)\Big)-\gamma(T_2-T_1)\\
&-cI_p\Big[\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)}-T_2+\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-M)}\Big)+M\Big)\Big]\Big)
\end{aligned}
\tag{35}
$$

After that same procedure as Case 1 and used the definition of triangular fuzzy number, we get from Equation (35)

$$
\begin{aligned}
d(\Delta_{12}(p,T_2),0) &= \Delta_{122}(p,T_2)\\
&= \tfrac{1}{T_2}\cdot\Big(\tfrac{p4D+p\nabla_h^D-p\nabla_l^p}{4}T_2\\
&+P\Big(\tfrac{1}{\theta}\Big(-\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}+\Big(G\theta+\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\Big)e^{\theta(T_1)}\Big)-\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big)\\
&+\tfrac{pI_e\frac{4D+\nabla_h^D-\nabla_1^D}{4}M^2}{2}-K\\
&-\Big[cg\Big\{\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\Big(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\
&+ch\Big\{\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\Big(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\Big)\Big\}\Big]\\
&+\Big[\tfrac{cg\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)+(-T_2+T_1)\Big\}+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{2\rho(\gamma)}ch\left(-T_2^2+T_1^2\right)\\
&+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}hc\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(\tfrac{1}{e^{-(\rho(\gamma)T_2+\rho(\gamma)T_1)}}-1\Big)+T_1\cdot\Big(\tfrac{e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\tfrac{T_2}{T_1\rho(\gamma)}\Big)\Big\}\Big]\\
&-\theta^{-1}\Big(\tfrac{-4D-\nabla_h^D+\nabla_l^D}{4}+\Big(\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{2}}{2\rho(\gamma)}\Big(\theta e^{\rho(\gamma)\cdot(T_2-T_1)}-\theta\Big)+\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\Big)\cdot e^{T_1\theta}\Big)\\
&-(e_{c\cdot}E_{c\cdot}T_x+\theta_{c\cdot}F_{c\cdot})\Big(\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\Big(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\
&+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)\theta}\Big(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\Big)\Big)-\gamma(T_2-T_1)\\
&-cI_p\Big[\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}\Big(-\tfrac{1}{\rho(\gamma)}-T_2+\tfrac{1}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-M)}\Big)+M\Big)\Big]\Big)
\end{aligned}
\tag{36}
$$

Defuzzification of Case 3 is similar as Case 1 and Case 2, and then we can write Equation (30)

$$
\begin{aligned}
d(\Delta_{13}(p,T_2),0) &= \Delta_{133}(p,T_2)\\
&= \tfrac{1}{T_2}\Big(pd\Big(\widetilde{D},0\Big)T_2\\
&+P\Big(\tfrac{1}{\theta}\Big(-d\Big(\widetilde{D},0\Big)+\Big(G\theta+d\Big(\widetilde{D},0\Big)\Big)e^{\theta(T_1)}\Big)-\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big)\\
&+\tfrac{pI_ed\left(\widetilde{D},0\right)T_2^2}{2}+pI_ed\Big(\widetilde{D},0\Big)(M-T_2)-K\\
&-\Big[cg\Big\{\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\
&+ch\Big\{\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\Big)\Big\}\Big]\\
&+\Big[\tfrac{cgd\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big)+T_1-T_2\Big\}\\
&+\tfrac{d\left(\widetilde{D},0\right)}{2\rho(\gamma)}ch\left(-cT_2^2h+cT_1^2h\right)\\
&+\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}hc\Big\{\tfrac{1}{\rho(\gamma)^2}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1\Big)+\Big(\tfrac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\tfrac{T_2}{\rho(\gamma)}\Big)\Big\}\Big]\\
&-\tfrac{1}{\theta}\Big(-d\Big(\widetilde{D},0\Big)+\Big(\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\theta+d\Big(\widetilde{D},0\Big)\Big)e^{\theta(T_1)}\Big)\\
&-(e_cE_cT_x+e_cF_c)\Big(\tfrac{-d\left(\widetilde{D},0\right)T_1}{\theta}+\Big(G+\tfrac{d\left(\widetilde{D},0\right)}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\
&+\tfrac{d\left(\widetilde{D},0\right)}{\rho(\gamma)\theta}\Big(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}-1+\theta(-T_2+T_1)\Big)\Big)-\gamma(T_2-T_1)\Big)
\end{aligned}
\tag{37}
$$

Now, using the definition of triangular fuzzy number for demand rate in the Equation (37) then, we get

$$
\begin{aligned}
d(\Delta_{13}(p,T_2),0) &= \Delta_{133}(p,T_2)\\
&= \tfrac{1}{T_2}\Big(p\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}T_2\\
&+P\Big(\tfrac{1}{\theta}\Big(-\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}+\Big(G\theta+\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\Big)e^{\theta(T_1)}\Big)-\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)\Big)\\
&+\tfrac{pI_e\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_2^2}{2}+pI_e\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}(M-T_2)-K\\
&-\Big[cg\Big\{\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\Big(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\Big\}\\
&+ch\Big\{\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\Big(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\Big)\Big(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\Big(1-\tfrac{1}{e^{-\theta T_1}}\Big)\Big)\Big\}\Big]\\
&+\Big[\tfrac{cg\frac{16D+4\nabla_h^D-4\nabla_l^D}{16}}{\rho(\gamma)}\Big\{\rho(\gamma)^{-1}\cdot\Big(e^{\rho(\gamma)(T_2-T_1)}-1\Big)+T_1-T_2\Big\}+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{2\rho(\gamma)}\left(-cT_2{}^2h+cT_1{}^2h\right)\\
&+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)}hc\Big\{-1+\rho(\gamma)^{-2}\Big(\tfrac{1}{e^{-(\rho(\gamma)T_2+\rho(\gamma)T_1)}}\Big)+\Big(\tfrac{T_1}{\rho(\gamma)\cdot e^{-(\rho(\gamma)T_2+\rho(\gamma)T_1)}}-\tfrac{1}{T_2^{-1}\rho(\gamma)}\Big)\Big\}\Big]\\
&-\theta^{-1}\Big(-\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}+\Big(\tfrac{\frac{16D+4\nabla_h^D-4\nabla_l^D}{16}}{\rho(\gamma)}\Big(\tfrac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1\Big)\theta+\tfrac{4D+\nabla_h^D-\nabla_l^D}{4}\Big)e^{\theta(T_1)}\Big)\\
&-(e_cE_cT_x+e_cF_c)\Big(\tfrac{-\frac{4D+\nabla_h^D-\nabla_l^D}{4}T_1}{\theta}+\Big(G+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\theta}\Big)\Big(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\Big)\\
&+\tfrac{\frac{4D+\nabla_h^D-\nabla_l^D}{4}}{\rho(\gamma)\theta}\Big(\tfrac{1}{e^{-(\rho(\gamma)T_2*\rho(\gamma)T_1)}}-1+T_1\theta-\theta T_2\Big)\Big)-\gamma(-T_1+T_2)\Big)
\end{aligned}
\tag{38}
$$

where demand rate D is a function of p.

3.2. Crisp Model under Learning in Fuzzy Environment

In this sequence, we are moving in the direction of learning shape and governed by Wright (1936) [13], mathematically shown below.

$$S_n = S_{n1} n^{-l}, \tag{39}$$

where S_n time for nth order is, S_{n1} is the initial time and l is the factor of learning. Using the Equation (38) and defined learning in upper and lower triangular fuzzy number of demand rate, we get

$$\nabla^D_{h\cdot i} = \begin{cases} \nabla^D_{h.1}, \ \ i = 1 \\ \left((i-1)\frac{365}{n}\right)^{-l} \nabla^D_{h\cdot i}, i > 1 \end{cases} \tag{40}$$

$$\nabla^D_{l\cdot i} = \begin{cases} \nabla^D_{l.1}, \ \ i = 1 \\ \nabla^D_{l\cdot i}\left((i-1)\frac{365}{n}\right)^{-l} \nabla^D_{l\cdot i}, i > 1 \end{cases} \tag{41}$$

From the Equations (40), (41) and (35), we get

$$\begin{aligned}
&d(\Delta_{11}(p,T_2),0) = \Delta_{111}(p,T_2) \\
&= \tfrac{1}{T_2}\Bigg(p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4} T_2 \\
&+P\Bigg(\tfrac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg) \\
&-\frac{(\delta_1-\delta_2 p)4+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\Bigg)+\frac{pIe\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M^2}{2}-K \\
&-\Bigg[cg\Bigg\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\right)\Bigg\} \\
&+ch\Bigg\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\tfrac{T_1}{\theta}-\theta^{-2}\left(1-\tfrac{1}{e^{-\theta T_1}}\right)\right)\Bigg\}\Bigg] \\
&+\Bigg[\frac{cg}{\rho(\gamma)}\frac{16(\delta_1-\delta_2 p)+4\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{16}\left\{\tfrac{1}{\rho(\gamma)}\left(\tfrac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1\right)+(T_1-T_2)\right\}+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left(c{T_1}^2 h-c{T_2}^2 h\right) \\
&+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\left(\rho(\gamma)^{-2}\cdot e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\tfrac{T_1}{e^{-(\rho(\gamma)T_2-\rho(\gamma)T_1)}\cdot\rho(\gamma)}-\tfrac{1}{T_2^{-1}\rho(\gamma)}\right)\right\}\Bigg] \\
&-\tfrac{1}{\theta}\cdot\Bigg(-\frac{16(\delta_1-\delta_2 p)+4\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{16} \\
&+\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg) \\
&-(e_c E_c\, T_x+e_c F_c)\Bigg(-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\right) \\
&+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(\tfrac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1+\theta\cdot(T_1-T_2)\right)\Bigg)-\gamma(T_2-T_1) \\
&-cI_p\Bigg[\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{-M\theta+\theta T_1}\left(e^{\rho(\gamma)\cdot(T_2-T_1)}-1\right)-e^{\rho(\gamma)(T_2-T_1)}-1\right)-\frac{(4\delta_1-4\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\cdot\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\theta}}((T_1-M)) \\
&+\frac{(4\delta_1-4\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\theta^2}}\left(e^{\theta(T_1-M)}\right) \\
&+\frac{(4\cdot\delta_1-4\cdot\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(-\tfrac{1}{\rho(\gamma)}-T_2+\tfrac{1}{\rho(\gamma)}\left(T_1+e^{-T_1\rho(\gamma)+\rho(\gamma T_2)}\right)\right)\Bigg]\Bigg)
\end{aligned} \tag{42}$$

From Equations (36), (40) and (41), we get

$$
\begin{aligned}
d(\Delta_{12}(p,T_2),0) = \Delta_{122}(p,T_2) = \frac{1}{T_2}\Bigg(& p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2 \\
& +P\left(\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right) \\
& +\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M^2}{2}-K \\
& -\Bigg[cg\left\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right\} \\
& +ch\left\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\cdot\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{T_1}{\theta}-\cdot\theta^{-2}\left(1-\frac{1}{e^{-\theta T_1}}\right)\right)\right\}\Bigg] \\
& +\Bigg[\frac{\frac{(18\delta_1-16\delta_2 p)+4\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{4\cdot\rho(\gamma)}gc\left\{\rho(\gamma)^{-1}\left(\frac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1\right)-T_2+T_1\right\}+\frac{\frac{(16\delta_1-16\delta_2 p)+4\cdot\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{16}}{2\cdot\rho(\gamma)}c\cdot\left(hT_1{}^2-hT_2{}^2\right) \\
& +\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\left(\rho(\gamma)^{-2}\cdot e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1}{e^{-(\rho(\gamma)T_2+\rho(\gamma)T_1)}\cdot\rho(\gamma)}-\frac{1}{T_2^{-1}\rho(\gamma)}\right)\right\}\Bigg] \\
& -\frac{1}{\theta}\left(-\frac{4(16\delta_1-16\cdot\delta_2 p)+4\cdot\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{16}+\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(\frac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right) \\
& -(e_cE_c\ T_x+e_cF_c\ )\left(\frac{-\frac{(4\delta_1-4\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\cdot\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right. \\
& \left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(\frac{1}{e^{-\rho(\gamma\cdot)(T_2-T_1)}}-1+\theta(T_1-T_2)\right)\right)-\gamma(T_2\ -T_1\ ) \\
& -cI_p\left[\frac{\frac{(4\delta_1-4\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\cdot\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-M)}\right)+M\right)\right]\Bigg)
\end{aligned}
\tag{43}
$$

From Equations (38), (40) and (41), we get

$$
\begin{aligned}
d(\Delta_{13}(p,T_2),0) = \Delta_{133}(p,T_2) = \tfrac{1}{T_2}\Bigg(& p\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2 \\
& +P\left(\tfrac{1}{\theta}\left(-\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)-\tfrac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right) \\
& +\tfrac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}{T_2}^2}{2}+pI_e\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}(M-T_2)-K \\
& -\Bigg[cg\Bigg\{\tfrac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\tfrac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\right)\Bigg\} \\
& +ch\Bigg\{\tfrac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\tfrac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\tfrac{T_1}{\theta}-\tfrac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\Bigg\}\Bigg] \\
& +\Bigg[\tfrac{cg\frac{(16\delta_1-16\delta_2 p)+4\left((i-1)\frac{365}{n}\right)^{-l}\cdot\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{16}}{\rho(\gamma)}\left\{\rho(\gamma)^{-1}\cdot\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)-T_2+T_1\right\}+\tfrac{hc\frac{(16\delta_1-16\delta_2 p)+4\left((i-1)\frac{365}{n}\right)^{-l}\cdot\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{16}}{2\rho(\gamma)}\left(-c{T_2}^2h+c{T_1}^2h\right) \\
& +\tfrac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\rho(\gamma)^{-2}\cdot e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}+\left(\tfrac{T_1}{e^{-(\rho(\gamma)T_2+\rho(\gamma)T_1)}\cdot\rho(\gamma)}-\tfrac{1}{T_2^{-1}\rho(\gamma)}\right)\right\}\Bigg] \\
& -\tfrac{1}{\theta}\left(-\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(\tfrac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right) \\
& -\left(e_cE_c\ T_x+e_cF_c\ \right)\Bigg(\tfrac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\tfrac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\tfrac{1}{\theta}+\tfrac{e^{\theta T_1}}{\theta}\right) \\
& +\tfrac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\Bigg)-\gamma(T_2\ -T_1\ ))
\end{aligned}
\tag{44}
$$

3.3. Optimal Solution Procedure

In this part, we have discussed the concavity of buyer's profit function for the Case 1, Case 2 and Case 3. We followed the useful results of concavity from Combini and Martien (2009) [38] for the present model. Combini and Martien (2009) [38] have applied some theorems (3.2.9 and 3.2.10) for the optimization of the objective function in his article. Any function of type, $L(x) = \frac{L_1(x)}{L_2(x)}$ is strictly pseudo-concave if $L_1(x)$ and $L_2(x)$ are differentiable and strictly concave function as well as non-negative function. We will use these results and represent the optimality of our aim. We calculated first the maximum value of the selling price (p^*), and then we determined the maximum value (p^*) of replenishment cycle length $(T_2{}^*)$ which optimizes the buyer's total fuzzy profit per cycle with the help of maximum value of (p^*).

Theorem 1. *For any fixed positive value of* $p(>0)$, $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ *and* $\Delta_{133}(p, T_2)$ *are the pseudo concave function of* T_2, *then the unique value of* T_2 *exists in each cases (Case 1, Case 2 and Case 3) and this unique value is consider that* $T_2{}^*$. *Then,* $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ *and* $\Delta_{133}(p, T_2)$ *attain maximum value.*

Optimal Solution procedure for Case 1, Case 2 and Case 3 have been shown in Appendix A.1.

Theorem 2. *If* $T_2(> 0)$ *is the positive specific value of cycle time,* $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ *and* $\Delta_{133}(p, T_2)$ *are the pseudo concave function of* p, *then the unique value of* p *exists in each cases (Case 1, Case 2 and Case 3) and suppose that the unique value of selling price is* p^*. *Then,* $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ *and* $\Delta_{133}(p, T_2)$ *attain maximum value and we have used this result for each case.*

Proof for Case 1, Case 2 and Case 3 have been shown in the Appendix A.2.

3.4. Numerical Example

The input parameters have been taken from the Alamri et al. (2022) [22], Mahata and Debnath (2022) [31] and Jayaswal et al. (2021) [37]. Notations and inventory parameters with its numerical values of this model have been presented in Table 2.

Table 2. Notations and inventory parameters.

Input Parameters for the Proposed Model	Numerical Values of Input Parameters	Input Parameters for the Proposed Model	Numerical Values of Input Parameters
δ_1	80 fixed	c	40 \$/unit
δ_2	0.9 fixed	K	10 \$/set up
g	0.2 \$/unit	T_1	0.0136 years
h	0.05 \$/unit/year	l	0.151
P	35 \$/unit	e_c	1.44 kWh/unit/year
T_x	75 \$/ton CO_2	F_c	2.6×10^{-3}ton$\frac{CO_2}{L}$
E_c	0.5×10^{-3} ton CO_2/kWh	I_e	0.14 c/unit
θ	0.05 fixed	I_p	0.15 \$/unit
γ	$5 \in [0, \overline{\gamma}]$	$\overline{\gamma}$	10
$\rho(\gamma)$	0.0183	n	6
∇_h^D	20	∇_l^D	5

The optimal selling price, cycle length and the fuzzy entire profit function have been calculated, in the Case 1 ($M \leq T_1 \leq T_2$), the value of T_1 is mentioned in the above Table 2 and the value of trade credit $M = 0.055$ year. After using the algorithm and optimization process, we get the optimal values of the decision variables which are $p^* = 45$ \$, $T^*{}_2 = 1.80$ year, $G^* = 85.87$ unit, $B^* = 35.98$ unit, $Y^* = 121.85$ unit, and total fuzzy profit $\nabla_{111}(p^*, T^*{}_2) = \$1605$. The Case 1 holds the condition $M \leq T_1 \leq T_2$, $0.0055 \leq 0.0136 \leq 1.90$. If the value of trade credit (M = 0.0055 year) increases for more

trade credit period, then the Case 1 does not satisfy. Therefore, we moved for Case 2 and using the same algorithm, we get the optimal values of the decision variables which are, $p^* = 60$ \$, $T^*{}_2 = 2.48$ year, $G^* = 105.17$ unit, $B^* = 45.37$ unit, $Y^* = 150.54$ unit, and total fuzzy profit $\nabla_{122}(p^*, T^*{}_2) = \$1684$. The Case 2 holds the condition $T_1 \leq M \leq T_2$, numerically we can write $0.0219 \leq 0.0547 \leq 2.48$. In this case, Case 2 does not hold when the value of trade credit being more than 0.0547 years. Similarly we go to Case 3 and using algorithm process, we get the optimal values of the decision variables which are, $p^* = 72$ \$, $T^*{}_2 = 2.89$ year, $G^* = 121.23$, unit, $B^* = 52.32$ unit, $Y^* = 173.55$ unit and total fuzzy profit $\nabla_{133}(p^*, T^*{}_2) = \$1805$. After using algorithm, we get the value of the trade credit (M = 2.98 year). The Case 1 is not considered for sensitivity analysis due to the less credit period and it is not beneficial for the buyer. The Case 3 is also not considered because the buyer does not deposit extra money to the seller and for this reason, the seller does not allow longer trade credit period for the buyer. The Case 2 is good for both players. We have considered Case 2 for sensitivity analysis Case 2. Figure 5 explains the proof for the concavity of the fuzzy total profit with respect to cycle time and selling price.

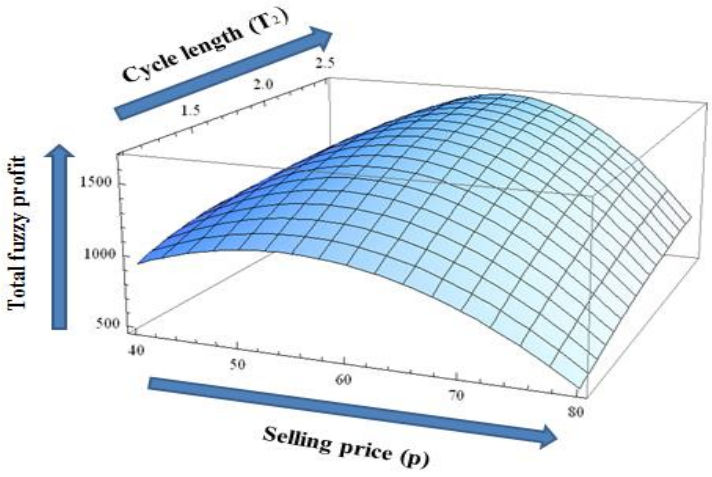


Figure 5. The concavity of total fuzzy profit functions against selling price and cycle length.

3.5. Sensitivity Analysis

We have analyzed the impact of inventory parameters of our present model on the selling price, cycle time and fuzzy total profit from Tables 3–10. The graphical representation is also shown in Figures 6–14.

Table 3. Effect of credit time on selling price, cycle length and fuzzy profit.

Trade Credit Period M (Year)	Selling Price (\$) p^*	Cycle Time (Year) T_2	Profit (\$) $\nabla_{122}(p, T_2)$
0.0136	60	2.41	1674
0.0273	60	2.45	1677
0.0410	60	2.46	1681
0.0547	60	2.48	1684

Table 4. Effect of learning parameter of learning rate on selling price, cycle length and fuzzy profit.

Learning Rate l	Selling Price (\$) p	Cycle Time (Year) T_2	Profit (\$) $\nabla_{122}(p, T_2)$
0.100	49	2.44	1648
0.111	51	2.45	1656
0.121	55	2.46	1664
0.131	57	2.47	1671

Table 4. *Cont.*

Learning Rate l	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
0.141	58.5	2.45	1677
0.151	60	2.48	1684
0.161	60.01	2.48	1684
0.171	60	2.48	1684

Table 5. Effect of shipments on selling price, cycle length and fuzzy profit.

Number of Shipments n	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
1	59.04	2.53	1722
2	59.05	2.51	1709
3	59.06	2.50	1700
4	59.7	2.49	1694
5	59.8	2.49	1689
6	60	2.48	1684

Table 6. Impact of deterioration rate on fuzzy profit.

Deterioration Rate θ	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
0.05	60.04	2.48	1684
0.06	60.05	2.41	1679
0.06	60.08	2.50	1673
0.07	60.09	2.49	1670

Table 7. Impact of fraction of reduced decaying rate on fuzzy profit.

Fraction of Reduced Decaying Rate $\rho(\gamma)$	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
0.0183	60.02	2.48	1684
0.0147	60.07	2.40	1682
0.0165	60.08	2.39	1680
0.0201	60.09	2.37	1679

Table 8. Impact of purchasing cost on selling price, cycle length fuzzy profit.

Purchasing Price c	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
32	60.02	2.65	1695
36	60.07	2.55	1690
40	60.09	2.48	1684

Table 9. Impact of holding cost on selling price, cycle length fuzzy profit.

Purchasing Price h	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
0.05	60.00	2.48	1684
0.06	60.08	2.41	1674
0.07	60.09	2.39	1669

Table 10. Effect of upper and lower deviation of fuzzy demand rate on fuzzy profit.

Upper Deviation of Demand Rate ∇_h^D	Lower Deviation of Demand Rate ∇_l^D	Selling Price ($) p	Cycle Time (Year) T_2	Profit ($) $\nabla_{122}(p, T_2)$
20	5	60	2.48	1684
40	10	59	2.31	1538
60	15	58	2.15	1410
80	20	57	2.05	1295
100	25	57	1.90	1195

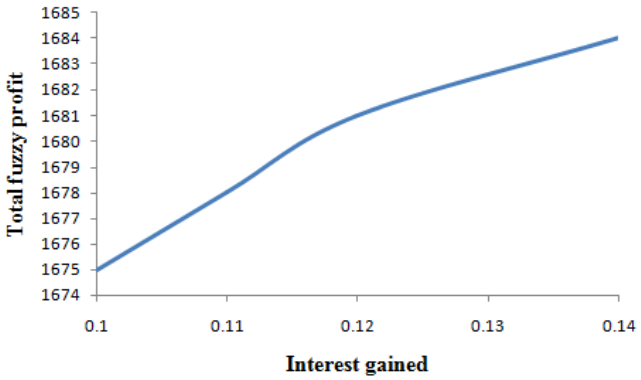


Figure 6. Effect of interest gained on total fuzzy profit.

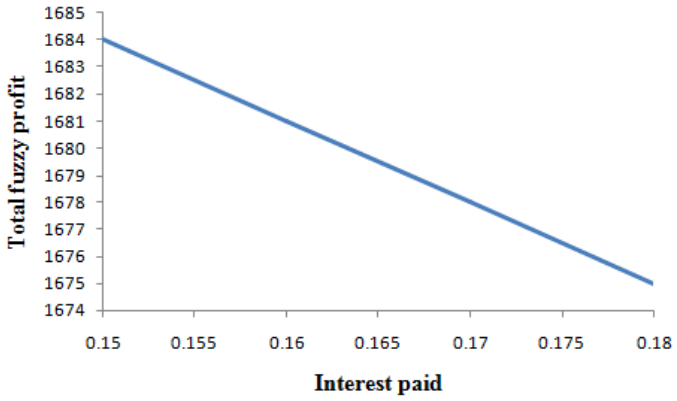


Figure 7. Effect of interest paid on total fuzzy profit.

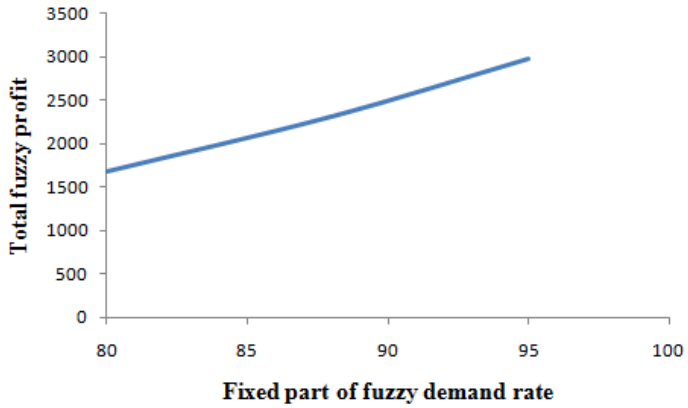


Figure 8. Effect of fixed part in the fuzzy demand on total fuzzy profit.

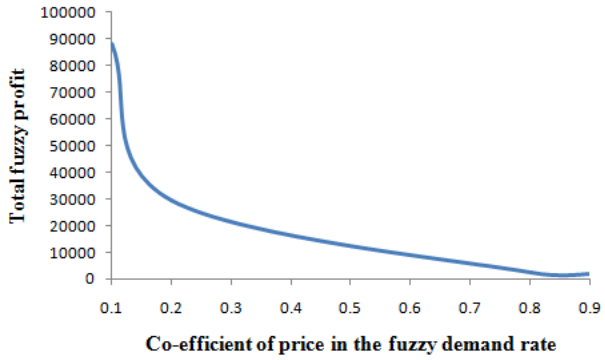


Figure 9. Effect of co-efficient of price in the fuzzy demand on total fuzzy profit.

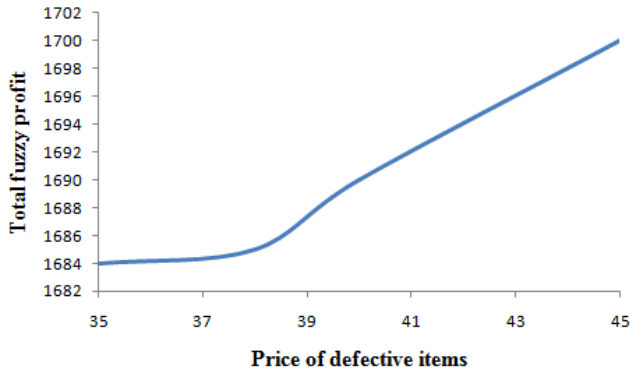


Figure 10. Effect of price of defective items in the fuzzy demand on total fuzzy profit.

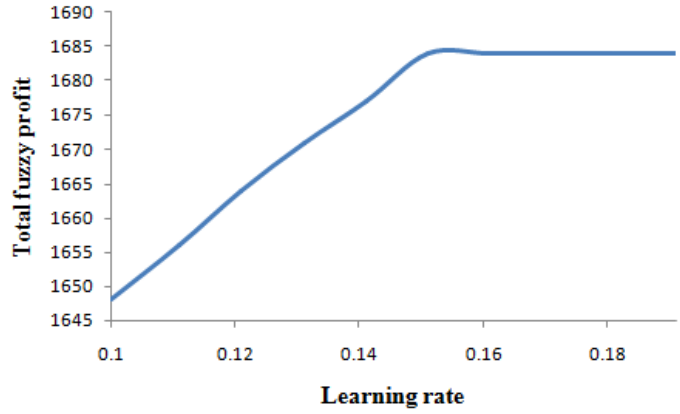


Figure 11. Effect of learning rate on total fuzzy profit.

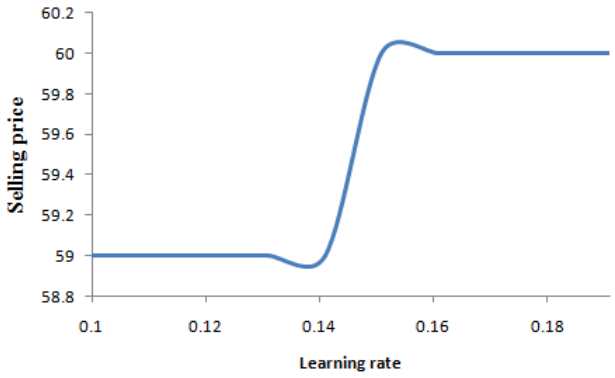


Figure 12. Effect of learning rate on selling price.

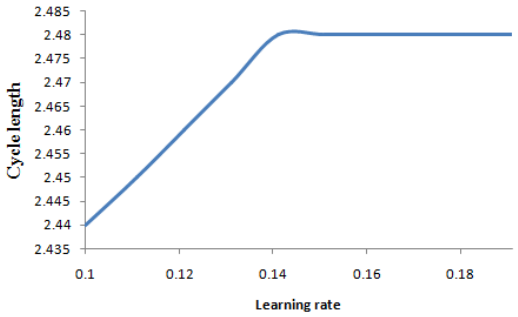


Figure 13. Effect of learning rate on cycle length.

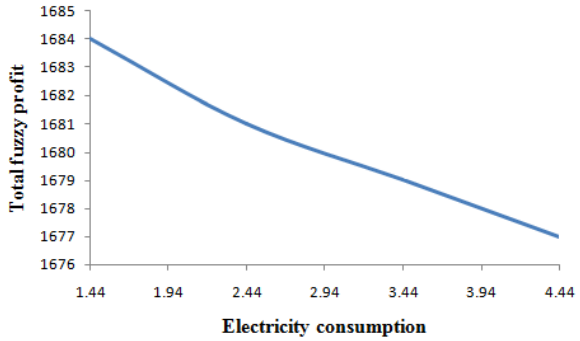


Figure 14. Effect of electricity consumption on total fuzzy profit.

Managerial insight and observation

- From Table 3, the cycle duration and entire fuzzy profit increase when the value of the trade credit increases whereas the selling price remains almost fixed. Both players should be aware of this point during supply chain.
- From Table 4, the selling price of the items, cycle time and buyer's total fuzzy profit increase when learning rate increases from the 0.100 to 0.151 but the selling price of items, cycle length and buyer's total fuzzy profit remain constant if the rate of learning increases from 0.161 to up to so on.
- From Table 5, if the number of shipments increases then the cycle length and the buyer's total fuzzy profit decrease while that selling price increases.
- From Table 6 when the deterioration rate increases then the cycle length and the buyer's total fuzzy profit decrease but the selling price increases.
- From Table 7, if the fraction of reduced deterioration rate under preservation technology increases, then cycle length and buyer's total fuzzy profit decrease as compared to the Table 6 but the selling price increases.
- If the purchasing cost and holding cost increase, then the cycle duration and entire profit decrease from Tables 8 and 9 but the selling price increases.
- From Table 10, the upper values and the lower values of the fuzzy demand deviation increase then the selling price, cycle duration and entire fuzzy profit decrease.
- From Figure 6, when the interest gained increases then the cost of the function decreases and the profit of buyer increases. The buyer and seller both should be aware of it before dealing with business because it affects the buyer's profit.
- From Figure 7, when the interest paid increases then the cost of the function increases and has a negative effect on the buyer's profit.
- From Figure 8, if the fixed part of the demand rate increases then the buyer's profit increases and has a positive effect on the buyer's profit.
- From Figure 9, if the co-efficient of the selling price increases in the fuzzy demand the then the buyer's total fuzzy profit decreases and gave negative effect on the total fuzzy profit.

- From Figure 10, if the price of defective items increases then the buyer's total fuzzy profit increases because the buyer gets extra benefit due to selling defective items and gave positive effect on the buyer's profit.
- From Figure 11, if the learning rate increases initially, buyer's profit increases up to 0.15 after this value if the learning rate increases but the buyer's profit remains constant and got the saturation position.
- From Figure 12, if the learning rate increases initially, buyer's selling price remains constant before the value of 0.14 after this value if the learning rate increases, the buyer's selling price rapidly increases before the value of 0.16 and when the value of learning rate increases after 0.16 then buyer's selling price got the saturation position.
- From Figure 13, if the learning rate increases initially, buyer's cycle time increases before the value of 0.16 after this value if the learning rate increases, the buyer's cycle time remains constant after the value of 0.16 and got the saturation position.
- From Figure 14, if the electricity consumption increases then the buyer's profit decreases and has negative effect on the buyer's profit.

4. Conclusions

In this study, we developed an inventory model with carbon emissions and preservation technology for deteriorating items under trade credit policy where the demand rate has taken a triangular fuzzy number in the supply chain. There are many fruitful results got from the managerial insight and observation and these results are very useful for many industries where deteriorating items are involved and demand rate is imprecise in nature. Our model tried to solve thorough the mathematical model for such type of problems using fuzzy theory and have shown in Table 10. The effect of learning manages the range of selling price, cycle length and the buyer's total fuzzy profit and has been discussed in the managerial insight with Table 4. The trade credit policy is more effective. The interest gained, interest paid, fixed part in the fuzzy demand, electricity consumption, co-efficient of selling price and price of defective items affect the buyer' total fuzzy profit and as shown in the figures from Figures 6–14. When the buyer purchases very highly deteriorating items from the seller then some quantities damage due the deterioration and buyer bear more loss during transportation of deteriorating items. The proposed model tried to solve such loss due to use of preservation technology and improved the buyer's profit. Trade credit policy is more beneficial from the selling point of view and trade credit policy is included for this reason also benefited both player during supply chain through interest gain and interest paid. In the general way, if the demand rate is well known in any business firm or industry sector as an omni channel then the owner of the business firm or industry sector gets more profit and also an easy to manage supply chain. The present paper deals with selling price dependent demand rate where demand rate is imprecise in nature as per consideration and in this situation, how will handle the situation of demand rate. Imprecise nature of the demand rate tried to manage with the help of defuzzyfication of demand rate. The lower and upper deviation of demand rate can be taken according to data manager. The role of learning rate is more useful where any task repeats many times and, in this manuscript, got the saturation of order quantity with the help of learning theory. The saturation of order quantity means size of order quantity for ordering to seller for the business. This proposed model is very help full where demand rate is unknown, and demand depends on selling price as well as item is deteriorating item. The application of this paper can be used in the field of fruits business, food factory as well as supply of vegetables for business. The present study can be improved for supply chain under partial trade credit policy among the buyer, seller and customer with the various strategies.

Limitations of the Present Model

The limitations of the present study are given below in some bullet points.

1. We used hypothetical data from some renowned literature reviews already mentioned in numerical example instead of the real data because the companies do not share their real data as secret business policy.
2. Our proposed model is applicable only for those companies that work with deteriorating items.
3. The trade credit period, interest gain and interest paid should be according to the cases of the proposed model.
4. The lower and upper deviation of the demand rate should be according to the proposed model.
5. The rate of learning should be according to this proposed model.

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Appendix A

Appendix A.1

Optimal Solution procedure for Case 1, Case 2 and Case 3.

It is assumed that p is a fixed value (only for proof) such that $p(>0)$, then $\Delta_{111}(p, T_2)$ is a function of T_2

$$
\begin{aligned}
\Delta_{111}(p,T_2) = \frac{1}{T_2}\Bigg(& p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2 \\
& +P\Bigg(\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg) \\
& -\frac{(\delta_1-\delta_2 p)4+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\Bigg)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M^2}{2}-K \\
& -\Bigg[cg\Bigg\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\Bigg\} \\
& +ch\Bigg\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\Bigg\}\Bigg] \\
& +\Bigg[\frac{cg}{\rho(\gamma)}\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right) \\
& +\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\frac{1}{\rho(\gamma)^2}\left(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\Bigg] \\
& -\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4} \\
& +\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg) \\
& -(e_cE_c\,T_x+e_cF_c)\Bigg(-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right) \\
& +\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(\frac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}-1+\theta(T_1-T_2)\right)\Bigg)-\gamma(T_2-T_1) \\
& -cI_p\Bigg[\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\theta(T_1-M)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)-e^{\rho(\gamma)(T_2-T_1)}-1\right)-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}((T_1-M)) \\
& +\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta^2}\left(e^{\theta(T_1-M)}\right)+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\rho(\gamma)^{-1}\cdot\left(\frac{1}{e^{-\rho(\gamma)\cdot(T_2-T_1)}}\right)+T_1\right)\Bigg])
\end{aligned}
$$

where

$$\phi_1(T_2) = p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2$$
$$+P\left(\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)\right.$$
$$\left.-\frac{(\delta_1-\delta_2 p)4+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M}{\frac{4}{2}}-K$$
$$-\left[cg\left\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right\}\right.$$
$$\left.+ch\left\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\right\}\right]$$
$$+\left[\frac{cg}{\rho(\gamma)}\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)\right.$$
$$\left.+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\frac{1}{\rho(\gamma)^2}\left(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\right]$$
$$-\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right.$$
$$\left.+\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)$$
$$-(e_cE_c\,T_x+e_cF_c\,)\left(\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.$$
$$\left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)-\gamma(T_2-T_1)$$
$$-cI_p\left[\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\theta(T_1-M)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)-e^{\rho(\gamma)(T_2-T_1)}-1\right)-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\frac{4}{\theta}}((T_1-M))\right.$$
$$\left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta^2}\left(e^{\theta(T_1-M)}\right)+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}\right)+T_1\right)\right]$$

and $\phi_2(T_2) = T_2$

The first derivative of $\phi_1(T_2)$

$$\phi_1'(T_2) = p\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)-P\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(e^{\rho(\gamma)(T_2-T_1)}\right)+\frac{cg}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-\right.$$
$$\left.1\right)\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)-\frac{ch\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)}{\rho(\gamma)}T_2+\frac{ch\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)}{\rho(\gamma)}\left[\frac{e^{\rho(\gamma)(T_2-T_1)}}{\rho(\gamma)}+T_1e^{\rho(\gamma)(T_2-T_1)}-\frac{1}{\rho(\gamma)}\right]-$$
$$\frac{1}{\theta}\left[\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\theta\left(e^{\rho(\gamma)(T_2-T_1)}\right)e^{\theta T_1}\right]-(e_cE_cT_x+e_cF_c)\left[\frac{1}{\theta}e^{\rho(\gamma)(T_2-T_1)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l,i}\right)}{4}\right)-\right.$$
$$\left.\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l,i}\right)}{4}\right)\right]-\gamma-cI_p\left[\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l,i}\right)}{4}\right)e^{\theta(T_1-M)}e^{\rho(\gamma)(T_2-T_1)}-\right.$$
$$\left.\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\rho(\gamma)(T_2-T_1)}-\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}\right)\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\right]$$

and its second derivative

$$\phi''_1 1(T_2) = -P\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(e^{\rho(\gamma)(T_2-T_1)}\right)+cg\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\rho(\gamma)(T_2-T_1)}-$$
$$\frac{ch}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)+\frac{ch}{\rho(\gamma)}e^{\rho(\gamma)(T_2-T_1)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)+$$
$$ch\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)T_1 e^{\rho(\gamma)(T_2-T_1)}-\frac{1}{\theta}\left[\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\rho(\gamma)\theta\left(e^{\rho(\gamma)(T_2-T_1)}\right)e^{\theta T_1}\right]-(e_c E_c T_x+$$
$$e_c F_c)\frac{\rho(\gamma)}{\theta}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\rho(\gamma)(T_2-T_1)}-cI_p\left[\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\rho(\gamma)e^{\theta(T_1-M)}e^{\rho(\gamma)(T_2-T_1)}-\right.$$
$$\left.\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}+\left(\rho(\gamma)\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\rho(\gamma)(T_2-T_1)}\right)\right]$$

Our objective is that $\phi_1(T_2)$ is strctly cancave and $\phi_2(T_2)$ is convex.

Here, we can take generally $c > p, \rho(\gamma)e^{\theta(T_1-M)}e^{\rho(\gamma)(T_2-T_1)} > 0$ and $\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4} > 0$. Then, we get $\phi_1(T_2) < o$, hence, we can say that $\phi_1(T_2)$ is showing the strict concavity property for each values of $p \geq 0$ and $\phi_2(T_2) = T_2$ is represents the convexity of the function. It can be used for the Case 2 and Case 3.

In the Case 2, it is assumed that $\phi_3(T_2)$

$$\Delta_{122}(p, T_2) = \frac{\phi_3(T_2)}{\phi_4(T_2)},$$

where

$$\phi_3(T_2) = \left(p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2\right.$$
$$+P\left(\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)\right.$$
$$\left.-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M^2}{2}-K$$
$$-\left[cg\left\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_1}{\theta}+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right\}\right.$$
$$\left.+ch\left\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_1}{\theta}+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\right\}\right]$$
$$+\left[\frac{cg\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)\right.$$
$$\left.+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\frac{1}{\rho(\gamma)^2}\left(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1 e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\right]$$
$$-\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right.$$
$$\left.+\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)$$
$$-(e_c E_c\ T_x+e_c F_c)\left(\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_1}{\theta}+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.$$
$$\left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)-\gamma(T_2\ -T_1)$$
$$\left.-cI_p\left[\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-M)}\right)+M\right)\right]\right)$$

The first derivative of this function $\phi_3(T_2)$

$$\phi_3'(T_2) = p\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{Li}\right)}{4}\right)$$
$$+P\left(-\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right)$$
$$+\left[\frac{cg}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)-1\right\}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{Li}\right)}{4}\right)\right.$$
$$+\frac{hc}{2\rho(\gamma)}\left(-2T_2\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{hi}-\nabla^D_{L\cdot i}\right)}{4}\right)\right)$$
$$\left.+\frac{ch}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{Li}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)^2}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)+\frac{1}{\rho(\gamma)}\left(T_1\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}-1\right)\right\}\right]$$
$$-\frac{1}{\theta}\left(\left(\frac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\theta\right)\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot t}-\nabla^D_{l\cdot t}\right)}{4}\right)e^{\theta(T_1)}\right)$$
$$-(e_cE_cT_x+e_cF_c)\left(\frac{1}{\rho(\gamma)\theta}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-t}\left(\nabla^D_{h\cdot t}-\nabla^D_{Li}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}-\theta\right)\right)-\gamma$$
$$-cI_p\left[\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{L\cdot i}\right)}{4}\right)\left(-1+\frac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-M)}\right)\right)\right]$$

And its second derivative of $\phi_3(T_2)$

$$\phi_3''(T_2) = P\left(-\frac{1}{\rho(\gamma)}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{L\cdot i}\right)}{4}\right)\right)$$
$$+\left[\frac{cg}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{L\cdot i}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right\}\right.$$
$$-\frac{hc}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{L\cdot t}\right)}{4}\right)$$
$$\left.+\frac{ch}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{L\cdot t}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)^2}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)+\frac{1}{\rho(\gamma)}\left(T_1\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right\}\right]$$
$$-\frac{1}{\theta}\left(\left(\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-i}\left(\nabla^D_{hi}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\theta\right)e^{\theta(T_1)}\right)$$
$$-(e_cE_cT_x+e_cF_c)\left(\frac{D}{\rho(\gamma)\theta}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right)$$
$$-cl_p\left[\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\frac{1}{\rho(\gamma)}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-M)}\right)\right)\right]$$

Here, it is already considered that above in Case 1.

$c > p, \rho(\gamma)e^{\theta(T_1-M)}e^{\rho(\gamma)(T_2-T_1)} > 0$ and $\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4} > 0$. Then, we get

$\phi_3(T_2) < o$, hence, we can say that $\phi_3(T_2)$ is showing the strict concavity property for each value of $p \geq 0$, and $\phi_4(T_2) = T_2$ represents the convexity of the function. It can be used for Case 3 similarly.

Now, for the Case 3,

$$\Delta_{133}(p, T_2) = \frac{\phi_5(T_2)}{\phi_6(T_2)},$$

where

$$\phi_5(T_2) = \Bigg(p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4} T_2$$
$$+P\Bigg(\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg)$$
$$-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\Bigg)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}{T_2}^2}{2}$$
$$+pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}(M-T_2)-K$$
$$-\Bigg[cg\Bigg\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\Bigg\}$$
$$+ch\Bigg\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\Bigg\}\Bigg]$$
$$+\Bigg[\frac{cg\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)$$
$$+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{-1+\frac{1}{\rho(\gamma)^2}\left(e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\Bigg]$$
$$-\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}$$
$$+\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg)$$
$$-(e_cE_c\ T_x+e_cF_c\ )\left(\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.$$
$$\left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)-\gamma(T_2\ -T_1\ ))$$

Now the derivative of $\phi_5(T_2)$

$$\phi_5'(T_2) = p\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{Li}\right)}{4}\right)$$
$$+P\left(-\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right)$$
$$+\frac{2pI_cT_2}{2}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)$$
$$-pI_e\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)$$
$$+\left[\frac{cg}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)-1\right\}\right.$$
$$+\frac{hc}{2\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-1}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)(-2T_2)$$
$$+\frac{ch}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)^2}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right.$$
$$\left.\left.+\frac{1}{\rho(\gamma)}\left(T_1\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}-1\right)\right\}\right]$$
$$-\frac{1}{\theta}\left(\left(\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\theta\right)e^{\theta(T_1)}\right)$$
$$-(e_cE_cT_x+e_cF_c)\left(\frac{1}{\rho(\gamma)\theta}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}-\theta\right)\right)$$
$$-\gamma(T_2)$$

And its second derivative is

$$\begin{aligned}\phi_5''(T_2) = P\Bigg(&-\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\Bigg)\\&+\frac{2pI_e\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)}{2}\\&+\left[\frac{cg}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right\}\right.\\&-\frac{hc}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)\\&+\frac{ch}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)\left\{\frac{1}{\rho(\gamma)^2}\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right.\\&\left.\left.+\frac{1}{\rho(\gamma)}\left(T_1\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right\}\right]\\&-\frac{1}{\theta}\left(\left(\frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\theta\right)e^{\theta(T_1)}\right)\\&-(e_cE_cT_x+e_cF_c)\left(\frac{1}{\rho(\gamma)\theta}\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4}\right)\left(\rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right)-\gamma\end{aligned}$$

Here, if it satisfies $c > p, \rho(\gamma)\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)} > 0$ and $\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla_{h\cdot i}^{D}-\nabla_{l\cdot i}^{D}\right)}{4} > 0$. Then, we get $\phi_5(T_2) < o$, hence, we can say that $\phi_5(T_2)$is showing the strict concavity property for each value of $p \geq 0$, and $\phi_6(T_2) = T_2$ represents the convexity of the function. In all three cases, we proved that for any positive values of p (≥ 0), the buyer's fuzzy profit in three cases (Case 1, Case 2 and Case 3) $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ and $\Delta_{133}(p, T_2)$ are the pseudo concave function of T_2 and hence gets the unique value of T_2 such that $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ and $\Delta_{133}(p, T_2)$ attain the optimal value. Incoming theorem we discussed case wise the optimal value of p such that $\Delta_{111}(p, T_2)$, $\Delta_{122}(p, T_2)$ and $\Delta_{133}(p, T_2)$ attain the optimal value.

Appendix A.2

Proof for Case 1, Case 2 and Case 3 are given below:

Proof for the Case 1. We suppose that model $T_2(> 0)$ is the fixed specific values of cycle length. Then, $\Delta_{111}(p, T_2)$ is the function of p which is presented by $\phi_7(p)$ such that

$$
\begin{aligned}
\Delta_{111}(p,T_2)=\phi_7(p)=\frac{1}{T_2}\Bigg(& p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2\\
&+P\left(\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)\right.\\
&\left.-\frac{(\delta_1-\delta_2 p)4+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M^2}{2}-K\\
&-\left[cg\left\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right\}\right.\\
&\left.+ch\left\{-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\right\}\right]\\
&+\left[\frac{cg}{\rho(\gamma)}\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)\right.\\
&\left.+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\right]\\
&-\frac{1}{\theta}\left(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right.\\
&\left.+\left(\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right)\\
&-(e_cE_c\,T_x+e_cF_c)\left(-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.\\
&\left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)-\gamma(T_2-T_1)\\
&-cI_p\left[\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\theta(T_1-M)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)-e^{\rho(\gamma)(T_2-T_1)}-1\right)-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}((T_1-M))\right.\\
&\left.+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta^2}\left(e^{\theta(T_1-M)}\right)+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}\right)+T_1\right)\right]\Bigg)
\end{aligned}
$$

The first derivative of $\phi_7(p)$ with respect to p, we get

$$
\begin{aligned}
\phi_7'(p)=\frac{1}{T_2}\Bigg(&(\delta_1-2\delta_2 p)T_2+\frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)T_2}{4}\\
&+P\left(\frac{1}{\theta}\left(\delta_2+(-\delta_2)e^{\theta(T_1)}\right)+\frac{\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right)-\frac{I_e(\delta_1-2\delta_2 p)M^2}{2}\\
&+\frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)I_eM^2}{8}\\
&-\left[cg\left\{\frac{\delta_2T_1}{\theta}+\left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right\}\right.\\
&\left.+ch\left\{\delta_2\frac{T_1}{\theta}+\left(\frac{-\delta_2}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\right\}\right]\\
&+\left[\frac{-cg\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{-hc\delta_2}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)\right.\\
&\left.+\frac{-ch\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+\frac{1}{\rho(\gamma)}\left(T_1e^{\rho(\gamma)(T_2-T_1)}-T_2\right)\right\}\right]\\
&-\frac{1}{\theta}\left(\delta_2+\left(\frac{-\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta-\delta_2\right)e^{\theta(T_1)}\right)\\
&-(e_cE_cT_x+e_cF_c)\left(\frac{\delta_2T_1}{\theta}+\left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.\\
&\left.+\frac{-\delta_2}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)\\
&-cI_p\left[\frac{-\delta_2}{\rho(\gamma)}\left(e^{\theta(T_1-M)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)-e^{\rho(\gamma)(T_2-T_1)}-1\right)+\frac{\delta_2}{\theta}((T_1-M))\right.\\
&\left.+-\frac{\delta_2}{\theta^2}\left(e^{\theta(T_1-M)}\right)+\frac{-\delta_2}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}\right)+T_1\right)\right]\Bigg)
\end{aligned}
$$

And its second derivative

$$\phi''_7(p) = \frac{-2\delta_2\ T_2}{T_2} - \frac{I_e(\delta_2)M^2}{T_2}$$

From the second derivative, we get $\phi''_7(p) < 0$ because as per consideration $T_2 > 0$ and hence, buyer's total fuzzy profit is a concave function. We can say that the unique value of p exits (it considers p^*) such that $\Delta_{111}(p, T_2)$ gets the optimal value.

Now, the essential circumstance to get the maximum value of p , we set the first derivative of $\Delta_{111}(p, T_2)$ equal to zero.

$$\begin{aligned}
&\left((\delta_1 - 2\delta_2 p)T_2 + \frac{\left((i-1)\frac{365}{n}\right)^{-1}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)T_2}{4} + P\left(\frac{1}{\theta}\left(\delta_2 + (-\delta_2)e^{\theta(T_1)}\right) + \frac{\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\right) - \right.\\
&\frac{I_e(\delta_1 - 2\delta_2 p)M^2}{2} + \frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)I_e M^2}{8} - \left[cg\left\{\frac{\delta_2 T_1}{\theta} + \left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right)\right\} + ch\left\{\delta_2\frac{T_1}{\theta} + \left(\frac{-\delta_2}{\theta}\right)\left(-\frac{T_1}{\theta} - \frac{1}{\theta^2}(1-\right.\right.\right.\\
&\left.\left.\left.e^{\theta T_1})\right)\right\}\Big] + \left[\frac{-cg\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) + (T_1 - T_2)\right\} + \frac{-hc\delta_2}{2\rho(\gamma)}\left({T_1}^2 - {T_2}^2\right) + \frac{-ch\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(e^{\rho(\gamma)(T_2-T_1)} - \right.\right.\right.\\
&\left.\left.\left.1) + \frac{1}{\rho(\gamma)}\left(T_1 e^{\rho(\gamma)(T_2-T_1)} - T_2\right)\right\}\right] - \frac{1}{\theta}\left(\delta_2 + \left(\frac{-\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\theta - \delta_2\right)e^{\theta(T_1)}\right) - (e_c E_c T_x + \\
&e_c F_c)\left(\frac{\delta_2 T_1}{\theta} + \left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right) + \frac{-\delta_2}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)} - 1 + \theta(T_1 - T_2)\right)\right) - \\
&cI_p\left[\frac{-\delta_2}{\rho(\gamma)}\left(e^{\theta(T_1-M)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) - e^{\rho(\gamma)(T_2-T_1)} - 1\right) + \frac{\delta_2}{\theta}((T_1 - M)) - \frac{\delta_2}{\theta^2}\left(e^{\theta(T_1-M)}\right) + \frac{-\delta_2}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)} - \right.\right.\\
&\left.\left.T_2 + \frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}\right) + T_1\right)\right]\Big) = 0
\end{aligned}$$

Now the necessary condition to get the maximum value of T_2, we take the partial derivative of $\Delta_{111}(p, T_2)$ with respect to T_2 and set it equal to zero, we get

$$\begin{aligned}
&\frac{1}{T_2}\left(p\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right) - P\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\left(e^{\rho(Y)(T_2-T_1))}\right)\right.\\
&\quad + \frac{cg}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}\right.\\
&\quad \left.-1\right)\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right) - \frac{ch\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)}{\rho(r)}T_2\\
&\quad + \frac{ch\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)}{\rho(\gamma)}\left[\frac{e^{\rho(\gamma)(\tau_2-\tau_1)}}{\rho(\gamma)} + T_1 e^{\rho(\gamma)(\tau_2-\tau_1)} - \frac{1}{\rho(\gamma)}\right]\\
&\quad - \frac{1}{\theta}\left[\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\theta\left(e^{\theta(\gamma)(T_2-T_1)}\right)e^{\theta T_1}\right]\\
&\quad - (e_c E_c T_x + \theta_c F_c)\left[\frac{1}{\theta}e^{e^{\rho(\gamma)(T_2-T_1)}}\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-t}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\right.\\
&\quad \left.- \frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\right] - \gamma\\
&\quad - cI_p\left[\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1-\mu)}e^{\theta(\gamma)(\tau_2-T_1)}\right.\\
&\quad - \left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)e^{\rho(\gamma)(T_2-T_1)}\\
&\quad - \frac{1}{\rho(\gamma)}\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{hL} - \nabla^D_{L}\right)}{4}\right)\\
&\quad \left.+ \frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(\tau_2-\tau_1)}\right)\left(\frac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\mu} - \nabla^D_{L}\right)}{4}\right)\right] = 0
\end{aligned}$$

□

Proof for the Case 2. As per mentioned that T_2 is a fixed values of the cycle lenght then $\Delta_{122}(p, T_2)$ is the only function of p,

$$\begin{aligned}
\Delta_{122}(p,T_2)=\phi_8(p)=\frac{1}{T_2}\Bigg[\Bigg(&p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2\\
&+P\Bigg(\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg)\\
&-\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\Bigg)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}M}{2}-K\\
&-\Bigg[cg\Bigg\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_1}{\theta}+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\Bigg\}\\
&+ch\Bigg\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_1}{\theta}+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\Bigg\}\Bigg]\\
&+\Bigg[\frac{cg\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)\\
&+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\Bigg]\\
&-\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\\
&+\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg)\\
&-(e_cE_c\ T_x+e_cF_c\ )\Bigg(\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_1}{\theta}+\left(G+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\\
&+\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\Bigg)-\gamma(T_2\ -T_1\ )\\
&-cI_p\Bigg[\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-M)}\right)+M\right)\Bigg]\Bigg)\Bigg]
\end{aligned}$$

Now we take the first derivative of the function with respect to p.

$$\begin{aligned}
\phi_8'(p)=\frac{1}{T_2}\Bigg[&(\delta_1-2\delta_2 p)T_2+\frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)T_2}{4}\\
&+P\left(\frac{1}{\theta}\left(\delta_2+(-\delta_2)e^{\theta(T_1)}\right)+\frac{\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right)+\frac{(\delta_1-2\delta_2 p)I_eM^2}{2}\\
&+\frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)I_eM^2}{8}\\
&-\Big[cg\Big\{\frac{\delta_2T_1}{\theta}+\left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\Big\}\\
&+ch\Big\{\frac{\delta_2T_1}{\theta}+\left(\frac{\delta_2}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\Big\}\Big]\\
&+\Bigg[\frac{-cg}{\rho(\gamma)}\delta_2\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}-\frac{\delta_2}{2}\left(\frac{hT_1^2c}{\rho(\gamma)}-\frac{cT_2^2h}{\rho(\gamma)}\right)\\
&-\frac{\delta_2}{\rho(\gamma)}\cdot\left\{\rho(\gamma)^{-2}\left(ce^{\rho(\gamma)(T_2-T_1)}h-hc\right)+\left(T_1\rho(\gamma)^{-1}e^{\rho(\gamma)\cdot(T_2-T_1)}-T_2\cdot\rho(\gamma)\right)\right\}\Bigg]\\
&-\frac{1}{\theta}\left(\delta_2+\left(\frac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta-\delta_2\right)e^{\theta(T_1)}\right)\\
&-(e_cE_cT_x+e_cF_c)\left(\delta_2\frac{T_1}{\theta}+\left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.\\
&\left.-\frac{\delta_2}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)\\
&-cI_p\Big[-\frac{\delta_2}{\rho(\gamma)}\left(-\frac{1}{\rho(\gamma)}-T_2+\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-M)}\right)+M\right)\Big]\Big]
\end{aligned}$$

and its second derivative of function with respect to p

$$\phi''_8(p) = (-2\delta_2)T_2 + \frac{-2\delta_2 p I_e M^2}{2} -$$

It shows that $\phi''_8(p) < 0$, if $T_2 > 0$, then $\Delta_{122}(p, T_2)$ reveals the concavity with respect to the selling price and cycle time and there exists a unique value of p (assume p^*) such that $\Delta_{122}(p, T_2)$ gets the optimal value. Now the essential circumstance to get the maximum value of p, we set the first derivative of $\Delta_{122}(p, T_2)$ equal to zero.

$$\begin{aligned}
\phi'_8(p) = \tfrac{1}{T_2}\Bigg[& \phi'_8(p) = (\delta_1 - 2\delta_2 p)T_2 + \tfrac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)T_2}{4} \\
& + P\left(\tfrac{1}{\theta}\left(\delta_2 + (-\delta_2)e^{\theta(T_1)}\right) + \tfrac{\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\right) + \tfrac{(\delta_1 - 2\delta_2 p)I_e M^2}{2} \\
& + \tfrac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)I_e M^2}{8} \\
& - \Big[cg\Big\{\tfrac{\delta_2 T_1}{\theta} + \left(\tfrac{-\delta_2}{\theta}\right)\left(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\right)\Big\} \\
& + ch\Big\{\tfrac{\delta_2 T_1}{\theta} + \left(\tfrac{\delta_2}{\theta}\right)\left(-\tfrac{T_1}{\theta} - \tfrac{1}{\theta^2}\left(1 - e^{\theta T_1}\right)\right)\Big\}\Big] \\
& + \Big[\tfrac{-cg}{\rho(\gamma)}\delta_2\Big\{\tfrac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) + (T_1 - T_2)\Big\} - \tfrac{hc\delta_2}{2\rho(\gamma)}\left({T_1}^2 - {T_2}^2\right) \\
& - \tfrac{ch\delta_2}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)^2}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) + \tfrac{1}{\rho(\gamma)}\left(T_1 e^{\rho(\gamma)(T_2-T_1)} - T_2\right)\Big\}\Big] \\
& - \tfrac{1}{\theta}\left(\delta_2 + \left(\tfrac{D}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\theta - \delta_2\right)e^{\theta(T_1)}\right) \\
& - (e_c E_c T_x + e_c F_c)\left(\delta_2\tfrac{T_1}{\theta} + \left(\tfrac{-\delta_2}{\theta}\right)\left(-\tfrac{1}{\theta} + \tfrac{e^{\theta T_1}}{\theta}\right)\right. \\
& \left. - \tfrac{\delta_2}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)} - 1 + \theta(T_1 - T_2)\right)\right) \\
& - cI_p\Big[-\tfrac{\delta_2}{\rho(\gamma)}\left(-\tfrac{1}{\rho(\gamma)} - T_2 + \tfrac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-M)}\right) + M\right)\Big]\Big] = 0
\end{aligned}$$

Now, to obtain the necessary condition to get the maximum value of T_2, we take the partial derivative of $\Delta_{122}(p, T_2)$ with respect to T_2 and to put equal to zero, we get

$$\begin{aligned}
& p\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right) \\
& \quad + P\left(-\tfrac{1}{\rho(\gamma)}\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\right) \\
& \quad + \Big[\tfrac{cg}{\rho(\gamma)}\Big\{\tfrac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right) - 1\Big\}\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right) \\
& \quad + \tfrac{hc}{2\rho(\gamma)}\left(-2T_2\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\right) \\
& \quad + \tfrac{ch}{\rho(\gamma)}\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\Big\{\tfrac{1}{\rho(\gamma)^2}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right) \\
& \quad + \tfrac{1}{\rho(\gamma)}\left(T_1\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)} - 1\right)\Big\}] \\
& \quad - \tfrac{1}{\theta}\left(\left(\tfrac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\theta\right)\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\right) \\
& \quad - (e_c E_c T_x + e_c F_c)\left(\tfrac{1}{\rho(\gamma)\theta}\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)} - \theta\right)\right) - \gamma \\
& \quad - cI_p\left[\tfrac{1}{\rho(\gamma)}\left(\tfrac{4(\delta_1 - \delta_2 p) + \left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)}{4}\right)\left(-1 + \tfrac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-M)}\right)\right)\right] = 0
\end{aligned}$$

□

Proof for the Case 3. Theoretical explanation has been taken same as Case 1 and Case 2.

$$\Delta_{133}(p,T_2)=\phi_9(p)=\frac{1}{T_2}\Bigg(p\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}T_2$$
$$+P\Bigg(\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}+\left(G\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg)$$
$$-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)}}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\Bigg)+\frac{pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}{T_2}^2}{2}$$
$$+pI_e\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}(M-T_2)-K$$
$$-\Bigg[cg\Bigg\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\Bigg\}$$
$$+ch\Bigg\{\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\Bigg\}\Bigg]$$
$$+\Bigg[\frac{cg\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}+\frac{hc\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{2\rho(\gamma)}\left({T_1}^2-{T_2}^2\right)$$
$$+\frac{ch\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(-1+e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}\right)+\left(\frac{T_1e^{(\rho(\gamma)T_2-\rho(\gamma)T_1)}}{\rho(\gamma)}-\frac{T_2}{\rho(\gamma)}\right)\right\}\Bigg]$$
$$-\frac{1}{\theta}\Bigg(-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}$$
$$+\left(\frac{\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)e^{\theta(T_1)}\Bigg)$$
$$-(e_cE_c\,T_x+e_cF_c\,)\Bigg(\frac{-\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}}{\theta}T_1+\left(G+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)$$
$$+\frac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{\frac{4}{\rho(\gamma)\theta}}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\Bigg)-\gamma(T_2\,-T_1\,)\Bigg)$$

Now, we take first derivative of the function with respect to p

$$\begin{aligned}\phi'_9(p)=\frac{1}{T_2}\Bigg[&(\delta_1-2\delta_2 p)T_2+\frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)T_2}{4}\\&+P\left(\frac{1}{\theta}\left(\delta_2+(-\delta_2)e^{\theta(T_1)}\right)+\frac{\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\right)+\frac{I_e(\delta_1-2\delta_2 p){T_2}^2}{2}\\&+\frac{I_e\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right){T_2}^2}{8}-pI_e\delta_2(M-T_2)\\&-\Bigg[cg\left\{\delta_2\frac{T_1}{\theta}+\left(-\frac{\delta_2}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right\}\\&+ch\left\{\frac{\delta_2 T_1}{\theta}+\left(-\frac{\delta_2}{\theta}\right)\left(-\frac{T_1}{\theta}-\frac{1}{\theta^2}\left(1-e^{\theta T_1}\right)\right)\right\}\Bigg]\\&-\Bigg[\frac{cg\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+(T_1-T_2)\right\}-\frac{hc\delta_2}{2\rho(\gamma)}\left({T_1}^2-T_2^2\right)\\&-\frac{ch\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)+\frac{1}{\rho(\gamma)}\left(T_1e^{\rho(\gamma)(T_2-T_1)}-T_2\right)\right\}\Bigg]\\&-\frac{1}{\theta}\left(\delta_2+\left(\frac{-\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)}-1\right)\theta-\delta_2\right)e^{\theta(T_1)}\right)\\&-(e_cE_cT_x+e_cF_c)\left(\delta_2\frac{T_1}{\theta}+\left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta}+\frac{e^{\theta T_1}}{\theta}\right)\right.\\&\left.-\frac{\delta_2}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)}-1+\theta(T_1-T_2)\right)\right)\Bigg]\end{aligned}$$

and its second derivative with respect to p

$$\phi''_9(p)=-2\delta_2T_2-I_e\delta_2){T_2}^2$$

It reveals that $\phi''_9(p)<0$, if $T_2>0$, then $\Delta_{133}(p,T_2)$ presents the concavity with respect to p and T_2 and there exists a unique value of p (assume p^*) such that $\Delta_{133}(p,T_2)$

gets the highest value. Now the essential circumstance to get the maximum value of p, we set the first derivative of $\Delta_{133}(p, T_2)$ equal to zero.

$$\begin{aligned}
&\frac{1}{T_2}[(\delta_1 - 2\delta_2 p)\,T_2 + \frac{\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)T_2}{4} \\
&\quad + P\left(\frac{1}{\theta}\left(\delta_2 + (-\delta_2)e^{\theta(T_1)}\right) + \frac{\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\right) + \frac{I_e(\delta_1 - 2\delta_2 p)T_2^2}{2} \\
&\quad + \frac{I_e\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i} - \nabla^D_{l\cdot i}\right)T_2^2}{8} - pI_e\delta_2(M - T_2) \\
&\quad - \left[cg\left\{\delta_2\frac{T_1}{\theta} + \left(-\frac{\delta_2}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right)\right\}\right. \\
&\quad \left. + ch\left\{\frac{\delta_2 T_1}{\theta} + \left(-\frac{\delta_2}{\theta}\right)\left(-\frac{T_1}{\theta} - \frac{1}{\theta^2}\left(1 - e^{\theta T_1}\right)\right)\right\}\right] \\
&\quad - \left[\frac{cg\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) + (T_1 - T_2)\right\} - \frac{hc\delta_2}{2\rho(\gamma)}\left({T_1}^2 - T_2^2\right)\right. \\
&\quad \left. - \frac{ch\delta_2}{\rho(\gamma)}\left\{\frac{1}{\rho(\gamma)^2}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right) + \frac{1}{\rho(\gamma)}\left(T_1 e^{\rho(\gamma)(T_2-T_1)} - T_2\right)\right\}\right] \\
&\quad - \frac{1}{\theta}\left(\delta_2 + \left(\frac{-\delta_2}{\rho(\gamma)}\left(e^{\rho(\gamma)(T_2-T_1)} - 1\right)\theta - \delta_2\right)e^{\theta(T_1)}\right) \\
&\quad - (e_c E_c T_x + e_c F_c)\left(\delta_2\frac{T_1}{\theta} + \left(\frac{-\delta_2}{\theta}\right)\left(-\frac{1}{\theta} + \frac{e^{\theta T_1}}{\theta}\right)\right. \\
&\quad \left. - \frac{\delta_2}{\rho(\gamma)\theta}\left(e^{\rho(\gamma)(T_2-T_1)} - 1 + \theta(T_1 - T_2)\right)\right)\Bigg] = 0
\end{aligned}$$

Now, to obtain the necessary condition to get the maximum value of T_2, we take the partial derivative of $\Delta_{133}(p, T_2)$ with respect to T_2 and set equal to zero, we get

$$\begin{aligned}
&\tfrac{1}{T_2}\left[p\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right) + P\left(-\tfrac{1}{\rho(\gamma)}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^p_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1))}\right)\right. \\
&\quad + \tfrac{2pI_eT_2}{2}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right) - pI_e\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right) \\
&\quad + \left[\tfrac{cg}{\rho(\gamma)}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left\{\tfrac{1}{\rho(\gamma)}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right) - 1\right\}\right. \\
&\quad + \tfrac{hc}{2\rho(\gamma)}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-t}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)(-2T_2) \\
&\quad \left. + \tfrac{ch}{\rho(\gamma)}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{Li}\right)}{4}\right)\left\{\tfrac{1}{\rho(\gamma)^2}\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right) + \tfrac{1}{\rho(\gamma)}\left(T_1\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)} - 1\right)\right\}\right] \\
&\quad - \tfrac{1}{\theta}\left(\left(\tfrac{1}{\rho(\gamma)}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)}\right)\theta\right)e^{\theta(T_1)}\right) \\
&\quad \left. - (e_cE_cT_x + e_cF_c)\left(\tfrac{1}{\rho(\gamma)\theta}\left(\tfrac{4(\delta_1-\delta_2 p)+\left((i-1)\frac{365}{n}\right)^{-l}\left(\nabla^D_{h\cdot i}-\nabla^D_{l\cdot i}\right)}{4}\right)\left(\rho(\gamma)e^{\rho(\gamma)(T_2-T_1)} - \theta\right)\right) - \gamma(T_2)\right] = 0
\end{aligned}$$

After simplifying, we get the optimal values of cycle time of ${T_2}^*$ and selling price p^*. After that, we can find out the optimal values of $\Delta_{111}(p^*,\ {T_2}^*)$ $\Delta_{122}(p^*,\ {T_2}^*)$ and $\Delta_{133}(p^*,\ {T_2}^*)$ for the retailer's entire yield per unit time. □

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