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Fixed-Time Formation Tracking Control of Nonlinear Multi-Agent Systems with Directed Topology and Disturbance

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Abstract: In this paper, we study the fixed-time formation (FixF) control problem for the nonlinear second-order multi-agent systems (MASs) with directed graph, where all agents are subject to communication disturbances. To overcome the bounded disturbance of communication and to guarantee the realization of predesignated formation within a fixed-time, a suitable FixF control protocol based on the backstepping method is proposed. Furthermore, to eliminate the dependence of the control parameters on global information, a new adaptive FixF control protocol is provided to combine with some mild conditions of control gain such that the formation with acceptable bounded error of MASs can be achieved within a fixed-time. Finally, the theoretical results are verified by two simulation examples, one of the simulation examples is that we set the desired formation as a regular octagon and the desired formation motion trajectory as a circle, the results show that all agents can form formation motion within a fixed-time.

Keywords: multi-agent systems; formation control; backstepping method; adaptive control

MSC: 82C22; 92D50; 92C15

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1. Introduction

Formation control is one of the hot topics in the field of MASs due to its broad application in the military, aerospace, industry, entertainment and other fields. Roughly speaking, the subject of formation control is to design a control protocol, which is based on local interactions among agents, such that all agents in MASs move towards a specific target or direction and maintain the predetermined geometric formation over time [1]. Initially, researchers focused on time-invariant formation control; however, many practical tasks in engineering such as maneuvering targets tracking and collision prevention need to be modeled as time-varying formation control problems. Hence, the study of time-varying formation is of great practical significance [2].

Generally, the convergence speed is a critical factor to evaluate the efficiency of the proposed control protocol in the study of the formation control problem. Moreover, from a practical point of view, the finite-time stability is more practical than the asymptotic stability. Consequently, most of the current research focuses on finite-time formation control [3–6]. Recall some references, e.g., saying in [7], that the authors investigated the finite-time formation control problem of nonlinear first-order MASs under directed graphs. For leader-following formation control of MASs, the authors in [8] studied the framework of finite-time non-smooth formation control for nonlinear first-order MASs under digraphs. The authors in [9] studied the event-triggered finite-time formation control of leader–follower MASs with unknown velocities. In addition, there are also a lot of reported works on the finite-time formation control of second-order MASs. For example, the practical finite-time consensus problem for the second-order heterogeneous switched nonlinear MASs is investigated in [10]. The authors of [11] proposed a finite-time

adaptive formation control protocol for heterogeneous second-order perturbed MASs under directed graphs.

Although the finite-time formation control has practical advantages over the asymptotic case in practice, the settling time of finite-time stability is difficult to calculate accurately because it depends heavily on the initial values of systems, which are not known in advance at some points. Therefore, in order to overcome the disadvantage of finite-time stability, the fixed-time stability (FixS) is improved by [12]. The advantage of the fixed-time stability is that the upper bound of the settling time just depends on the parameter of systems and control protocol. Based on this advantage, the fixed-time stability has attracted wide attention from scholars, and it is also widely used in MASs formation control problems. For instance, the fixed-time stability of singular systems has been extensively studied in [13,14]. Furthermore, the authors in [15] investigated FixF control for heterogeneous MASs with partial unknown control directions. The fast fixed-time nonsingular terminal sliding-mode formation control for autonomous underwater vehicles based on a disturbance observer was considered in [16]. The fixed-time distributed formation control for a class of nonlinear MASs subject to internal uncertainties and external disturbances was investigated in [17]. Several control methods have been applied in the existing literature, such as leader–follower approaches [18,19] and event-triggered control [20,21]. In [18], the authors studied distributed fixed-time leader-following formation control for second-order MASs via output feedback and the authors in [19] provide a novel fixed-time control approach for the leader–follower formation control of surface vehicles. Combining the event-triggered control, the event-triggered fixed-time adaptive neural formation control for underactuated autonomous surface vessels with connectivity constraints and prescribed performance was studied in [20], and the authors of [21] provide a novel hierarchical event-triggered scheme for the formation control of networked autonomous surface vehicles with fully discontinuous communication. In [22–25], the fixed-time formation tracking control problem for second-order MASs is studied in a directed topological network, second-order integral system with bounded uncertainty, nonlinear dynamic disturbance, and input delay. The authors in [26–29] mainly studied the fixed-time formation tracking control problem of leader–follower MASs. In fact, the study of control for leaderless flocking is more challenging since the predetermined trajectory cannot be known; therefore, it is very difficult for agents to form a stable formation. The references [30,31] summarized the main research achievements and latest progress of distributed cooperative anti-jamming formation control for MASs. Compared with the anti-jamming formation control of a single system, the formation control of MASs is more challenging due to the coupling problem.

However, to the best of our knowledge, the fixed-time stability theory is rarely used in formation control of MASs, which arouse our research interest. Notice that most of the above literature studies the MASs under undirected communication topology, and many of them adopt the leader–follower approaches [9,18,19,26–28]. The control protocol is usually designed by using global information, such as [3,6,15,20,24,32]. In addition, most of the above literature does not consider realistic disturbance factors [2,4,8,22]. As a result, there are three aspects that are worth considering: Firstly, how to design an adaptive control protocol that does not rely on global information to ensure the FixF of MASs? Secondly, how to conquer the challenge of the disturbance in communication? Thirdly, how to design a control protocol that enables MASs to accomplish formation control without a leader? This paper will give the positive answers to the above issues. Therefore, compared with the related works, the main contributions of this paper are as follows:

(i) To conquer the difficulty of bounded disturbance, a new FixF control protocol is proposed by combining the backstepping method with the distributed cooperative variable, which has the advantages of a simple and effective structure.

(ii) To overcome the problem that some control parameters depend on the global information, a new simple adaptive FixF control protocol is proposed, which combines with a control gain condition to enable the MASs to achieve a fixed-time formation within bounded error.

(iii) The two control protocols designed in this paper can make MASs achieve the formation control goal without a leader.

The structure of the current paper is arranged as follows. Preliminaries and problem statement are presented in Section 2. Section 3 presents the main results of this paper. Section 4 verifies the theoretical results by two numerical simulations, while Section 5 summarizes the full paper.

2. Preliminaries and Problem Statement

Denote $\mathcal{I}_N = \{1, 2, \ldots, N\}$ and $\lambda_{max}(\cdot)$ is the maximal real eigenvalues of a real matrix. For any $s \in \mathbb{R}$, sign(s) is the signum function. Define $s^{[k]} = sign(k)|s|^k$ and $A^{[k]} = (a_{ij}^{[k]})_{n\times m}$, where $A = (a_{ij})_{n\times m}$. Let $\mathbb{R}^N$ be the N-dimensional Euclidean space. Let 1_m be the $m \times 1$ column vector of all ones. For real x, $|x|$ is the absolute value of x. Define $x = (x_1, ..., x_N)^T \in \mathbb{R}^N$ as a positive vector if all $x_i > 0$. For any real $p > 0$, define $\|x\|_p = \left(\sum_{i=1}^N |x_i|^p\right)^{\frac{1}{p}}$.

2.1. Graph Theory

In this paper, the communication network of a MASs with N agents can be regarded as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{p_1, p_2, \ldots, p_N\}$ presents the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} = \{(p_i, p_j)|p_i, p_j \in \mathcal{V}\}$ is edge set, $\mathcal{A} = (a_{ij})_{N\times N}$ is the adjacency matrix of the graph $\mathcal{G}$ with non-negative elements. An edge e_{ij} rooted at spacecraft j and ending at spacecraft i is denoted by (i, j). So, the weight $a_{ij} > 0 \Leftrightarrow (i, j) \in \mathcal{E}$. Furthermore, it is assumed that no self-loop exists (i.e., $a_{ii} = 0$) and multiple edges between agents will not be allowed in the current paper. The corresponding Laplacian matrix $L = (l_{ij})_{N\times N}$ can be defined as

$$l_{ij} = \begin{cases} \sum\limits_{\substack{k=1 \\ k\neq i}}^{N} a_{ik}, & j = i, \\ -a_{ij}, & j \neq i, \end{cases} \quad i, j \in \mathcal{I}_N. \tag{1}$$

Lemma 1 ([11,33,34]). *If $\mathcal{G}$ is a strongly connected graph and its Laplacian matrix is denoted by L, then $L1_N = 0$, and there exists a vector $\xi = [\xi_1, \xi_2, \ldots, \xi_N]^T \in \mathbb{R}^N$ with $\xi_i > 0$, $\forall\, i \in \mathcal{I}_N$ and $\sum_{i=1}^N \xi_i = 1$, such that $\xi^T L = 0$. Furthermore, define $\tilde{L} = \Xi L + L^T \Xi$, where $\Xi = diag(\xi_1, \xi_2, \ldots, \xi_N)$. In addition, let $\varsigma \in \mathbb{R}^N$ be any positive vector and $\vartheta \in \mathbb{R}^N$. Then, the following inequality holds*

$$\min_{\substack{\vartheta^T\varsigma=0 \\ \vartheta\neq 0}} \vartheta^T \tilde{L} \vartheta \geqslant \frac{\lambda_2(\tilde{L})}{N} \vartheta^T \vartheta, \tag{2}$$

where $\lambda_2(\tilde{L})$ is the second smallest eigenvalue of $\tilde{L}$. In particular, if $\vartheta^T\vartheta = 1$, then $\min\limits_{\substack{\vartheta^T\varsigma=0 \\ \vartheta\neq 0}} \vartheta^T \tilde{L} \vartheta > \frac{\lambda_2(\tilde{L})}{N} \vartheta^T \vartheta$.

2.2. Fixed-Time Stability Theory

Consider the following system:

$$\dot{x} = f(t, x), x(0) = x_0, \tag{3}$$

where $x \in \mathbb{R}^n$ and $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ is nonlinear function, which can be discontinuous. Assume the origin is an equilibrium point of (3).

Definition 1 ([12]). *The origin of (3) is said to be globally finite-time stable if it is globally asymptotically stable and any solution $x(t, x_0)$ of (3) reaches the equilibria at some finite time moment, i.e., $x(t, x_0) = 0$, $\forall\, t \geqslant T(x_0)$, where $T : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ is the settling-time function.*

Moreover, if the settling-time function $T(x_0)$ is bounded, i.e., $\exists\ T^* > 0 : T(x_0) \leqslant T^*$, for all $x_0 \in \mathbb{R}^n$. Then, the origin of (3) is said to be FixS.

Lemma 2 ([12]). *If there exists a continuous radially unbounded function $V : \mathbb{R}^n \to \mathbb{R}_+ \cup \{0\}$ such that $V(x(t)) = 0 \Leftrightarrow x(t) = 0$ and $\dot{V}(x(t)) \leqslant -\alpha V^{r_1}(x(t)) - \beta V^{r_2}(x(t))$ for some positive constants α, β, r_1, r_2 satisfying $0 < r_1 < 1 < r_2$, then, the origin of system (3) is FixS with the settling time estimated by $T(x_0) \leqslant T^* = 1/(\alpha(1-r_1)) + 1/(\beta(r_2-1))$, for all $x_0 \in \mathbb{R}^n$.*

Lemma 3 ([35]). *Consider the system (3). Suppose that there exists a Lyapunov function $V(x)$ such that $\dot{V}(x(t)) \leqslant -\alpha V(x(t))^{r_1} - \beta V(x(t))^{r_2} + s$ for some positive constants α, β, r_1, r_2 satisfying $0 < r_1 < 1 < r_2$. Then, the system (3) is practical FixS. Moreover, the residual set of the solution of system (3) can be given by $\left\{\lim_{t\to T} x(t) \middle| V(x(t)) \leqslant \min\left\{1/\alpha^{1/r_1}(s/(1-\theta))^{1/r_1},\ 1/\beta^{1/r_2}(s/(1-\theta))^{1/r_2}\right\}\right\}$, where θ is a scalar and satisfies $0 < \theta < 1$. The time needed to reach the residual set is bounded as $T(x_0) \leqslant T^* = 1/(\alpha\theta(1-r_1)) + 1/(\beta\theta(r_2-1))$.*

2.3. Several Supporting Inequalities

Lemma 4 ([36]). *For any x, $x' \in \mathbb{R}$, $\alpha \in (0,1]$, it holds*

$$\left|x^{[\alpha]} - x'^{[\alpha]}\right| \leqslant 2^{1-\alpha}|x - x'|^{\alpha}. \tag{4}$$

Lemma 5 ([37]). *Let $a_1, a_2, \ldots, a_N \geqslant 0$ and $0 < p \leqslant 1$. Then*

$$\sum_{i=1}^{N} a_i^p \geqslant \left(\sum_{i=1}^{N} a_i\right)^p. \tag{5}$$

Lemma 6 ([37]). *Let $a_1, a_2, \ldots, a_N \geqslant 0$ and $p > 1$. Then*

$$\sum_{i=1}^{N} a_i^p \geqslant N^{1-p}\left(\sum_{i=1}^{N} a_i\right)^p. \tag{6}$$

Lemma 7 ([38] (Young's Inequality)). *Assuming a and b are non-negative real, $v \in (0,1)$, let $p = \frac{1}{1-v}$ and $q = \frac{1}{v}$, then*

$$ab \leqslant \frac{1}{p}a^p + \frac{1}{q}b^q$$

with equality if and only if $a^p = b^q$. Moreover, for any $\epsilon > 0$, let $x = a\epsilon^{\frac{1}{p}}$ and $y = b\epsilon^{-\frac{1}{p}}$, then

$$ab = xy \leqslant \frac{1}{p}x^p + \frac{1}{q}y^q = \frac{\epsilon}{p}a^p + \frac{\epsilon^{-\frac{q}{p}}}{q}b^q \tag{7}$$

with equality if and only if $a^p\epsilon = b^q\epsilon^{-\frac{q}{p}}$.

2.4. Problem Formulation

Consider the second-order MASs containing N agents with disturbances, whose dynamics can be described as

$$\begin{cases} \dot{x}_i(t) = v_i(t), \\ \dot{v}_i(t) = u_i(t) + d_i(t), \end{cases} \tag{8}$$

with the initial data $(x_i, v_i)|_{t=0} = (x_i(0), v_i(0))$, $i \in \mathcal{I}_N$, where $x_i(t)$, $v_i(t) \in \mathbb{R}$ represent the position and velocity states of ith agent at time t, respectively. $u_i(t)$ is the control input of ith agent. $d_i(t) \in \mathbb{R}$ stands for the disturbance.

Definition 2 ([39]). *For $i, j \in \mathcal{I}_N$, let a time-dependent vector $X^* = (x_1^*, x_2^*, \dots, x_N^*)^T \in \mathbb{R}^N$ be formation information. Then, for any initial state $\big(x(0), v(0)\big)$, the FixS formation for the second-order MASs (8) is said to be achieved if there exists a fixed constant T^*, which is unrelated to the initial state $\big(x(0), v(0)\big)$, such that the following conditions are simultaneously satisfied*
(1) $\lim\limits_{t \to T^*} \Big\| \big(x_i(t) - x_i^*(t)\big) - \big(x_j(t) - x_j^*(t)\big) \Big\|_2 = 0$, $\Big\| \big(x_i(t) - x_i^*(t)\big) - \big(x_j(t) - x_j^*(t)\big) \Big\|_2 \equiv 0$, *for $t \geqslant T^*$;*
(2) $\lim\limits_{t \to T^*} \Big\| \big(v_i(t) - \dot{x}_i^*(t)\big) - \big(v_j(t) - \dot{x}_j^*(t)\big) \Big\|_2 = 0$, $\Big\| \big(v_i(t) - \dot{x}_i^*(t)\big) - \big(v_j(t) - \dot{x}_j^*(t)\big) \Big\|_2 \equiv 0$, *for $t \geqslant T^*$.*

To study the desired results, the following assumptions are employed.

Assumption 1. *For $i \in \mathcal{I}_N$, $d_i(t)$ is bounded, i.e., there is a positive M such that $|d_i(t)| \leqslant M$.*

Assumption 2. *For $i \in \mathcal{I}_N$, $\dot{x}_i^*$ is bounded, i.e., there is a positive $\bar{x}^*$ such that $|\dot{x}_i^*| \leqslant \bar{x}^*$.*

Remark 1. *In fact, Assumptions 1 and 2 are reasonable due to the energies of the disturbance and the acceleration of the agents are all finite in practice.*

3. Main Results

Letting $\tilde{x}_i = x_i - x_i^*$ and $\tilde{v}_i = v_i - \dot{x}_i^*$, then the error dynamics can be given as:

$$\begin{cases} \dot{\tilde{x}}_i(t) = \dot{x}_i(t) - \dot{x}_i^*(t) = v_i(t) - \dot{x}_i^*(t) = \tilde{v}_i(t), \\ \dot{\tilde{v}}_i(t) = \dot{v}_i(t) - \ddot{x}_i^*(t) = u_i(t) + d_i(t) - \ddot{x}_i^*(t) = u_i(t) + \tilde{d}_i(t), \end{cases} \tag{9}$$

where $\tilde{d}_i(t) = d_i(t) - \ddot{x}_i^*(t)$. According to Assumptions 1 and 2, $\tilde{d}_i(t)$ satisfies $\big|\tilde{d}_i(t)\big| \leqslant M + \bar{x}^*$. For convenience, define $\phi_i = \sum_{j=1}^N a_{ij}(\tilde{x}_i - \tilde{x}_j)$, $\phi = (\phi_1, \phi_2, \dots, \phi_N)^T$ and $\psi_i = \sum_{j=1}^N a_{ij}(\tilde{v}_i - \tilde{v}_j)$, $\psi = (\psi_1, \psi_2, \dots, \psi_N)^T$. Moreover, let $\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_N)^T$, $\tilde{v} = (\tilde{v}_1, \tilde{v}_2, \dots, \tilde{v}_N)^T$. Consequently, $\phi = L\tilde{x}$ and $\psi = L\tilde{v} = \dot{\phi}$.

In what follows, the backstepping method is employed. Firstly, consider the following system

$$\dot{\phi} = L\tilde{v}, \tag{10}$$

and design a virtual control input for it as follows

$$v_i^* = \tilde{v}_i + l_1 \phi_i^{[\beta]}, i \in \mathcal{I}_N, \tag{11}$$

where $\frac{1}{2} < \beta < 1$ and $l_1 > 0$. Denote $v^* = (v_1^*, v_2^*, \dots, v_N^*)^T$. Furthermore, motivated by [11], we set

$$G(x) = \begin{cases} 1, & x = 0, \\ \frac{1}{l_1 |x|^{\beta - 1}}, & x \neq 0, \end{cases} \tag{12}$$

where $x \in \mathbb{R}$. It follows from Lemma 1 that $\exists\, \xi^T > 0$ satisfies $\xi^T L = \xi^T L\tilde{x} = \xi^T \phi = 0$. Then we have

$$\xi^T \phi = \sum_{k=1}^N \xi_k \phi_k = \sum_{\substack{k=1 \\ \phi_k \neq 0}}^N \frac{\xi_k l_1 \phi_k^{[\beta]}}{l_1 |\phi_k|^{\beta - 1}} + \sum_{\substack{k=1 \\ \phi_k = 0}}^N \xi_k l_1 \phi_k^{[\beta]} = \sum_{k=1}^N \xi_k G(\phi_k) l_1 \phi_k^{[\beta]} = 0. \tag{13}$$

Let $\hat{\xi} = \big(\xi_1 G(\phi_1), \xi_2 G(\phi_2), \dots, \xi_N G(\phi_N)\big)^T$. Obviously, $\hat{\xi}$ is a positive vector. Then, (13) can be rewritten as

$$\sum_{k=1}^N \xi_k G(\phi_k) l_1 \phi_k^{[\beta]} = \Big(l_1 \phi^{[\beta]}\Big)^T \hat{\xi} = 0. \tag{14}$$

Thus, by Lemma 1, it holds that

$$\left(l_1\phi^{[\beta]}\right)^T\tilde{L}l_1\phi^{[\beta]} \geqslant \frac{\lambda_2(\tilde{L})}{N}\left(l_1\phi^{[\beta]}\right)^T l_1\phi^{[\beta]}. \tag{15}$$

Secondly, deducing from (9) and (11), we have

$$\dot{v}_i^*(t) = \dot{\tilde{v}}_i(t) + \beta l_1|\phi_i(t)|^{\beta-1}\psi_i(t) = u_i(t) + \tilde{d}_i(t) + \beta l_1|\phi_i(t)|^{\beta-1}\psi_i(t). \tag{16}$$

Then, the control protocol $u_i(t)$ is designed as follows:

$$u_i = -k_1\eta_i^{[2\beta-1]} - k_2 sign(\eta_i) - \beta l_1|\phi_i|^{\beta-1}\psi_i, \ \ i \in \mathcal{I}_N, \tag{17}$$

where l_1 and β are defined in (11), k_1 and k_2 will be determined later. In addition, denote $\eta = (\eta_1, \eta_2, \ldots, \eta_N)^T$ with

$$\eta_i = v_i^{*[\frac{1}{\beta}]}, \ \ i \in \mathcal{I}_N. \tag{18}$$

Remark 2. *It should be pointed out that the controller designed in (17) is just confined to four parameters, that is k_1, k_2, l_1, β. On one hand, compared with [40], the controller relies on fewer parameters and is more convenient in practical applications. On the other hand, controller (17) is proposed by the backstepping method, which can be extended to adaptive fixed-time consensus controller and deal with communication disturbances.*

Theorem 1. *For $\frac{1}{2} < \beta < 1$, $l_1 > 0$ and $0 < \varepsilon < \frac{\lambda_2(\tilde{L})}{N\lambda_{max}(\Xi LL^T\Xi)}$, under the Assumptions 1 and 2, assume that the network topology of MASs (8) is strongly connected, then the FixF of MASs (8) can be achieved under control protocol (17) if $k_1 > 2^{1-2\beta}\varepsilon^{-2}$ and $k_2 > M + \bar{x}^*$.*

Proof. Consider the Lyapunov function candidate

$$H(t) = H_0(t) + \varepsilon\sum_{k=1}^{N} H_k(t) \tag{19}$$

with

$$H_0(t) = \sum_{k=1}^{N}\frac{l_1\xi_k|\phi_k|^{1+\beta}}{1+\beta} \tag{20}$$

and

$$H_k(t) = \int_0^{v_k^*}\left(s^{[\frac{1}{\beta}]}\right)\mathrm{d}s, \tag{21}$$

where $k \in \mathcal{I}_N$, l_1, β, v_k^* are given in (11), ξ_k is defined in Lemma 1.

On one hand, from (10), (11) and (15), the time derivative of $H_0(t)$ can be computed as

$$\begin{aligned}\frac{dH_0(t)}{dt} &= \sum_{k=1}^{N} l_1\xi_k\phi_k^{[\beta]}\dot{\phi}_k = \left(l_1\phi^{[\beta]}\right)^T\Xi\dot{\phi} = \left(l_1\phi^{[\beta]}\right)^T\Xi L\tilde{v}\\ &= \left(l_1\phi^{[\beta]}\right)^T\Xi L\left(v^* - l_1\phi^{[\beta]}\right)\\ &= \left(l_1\phi^{[\beta]}\right)^T\Xi Lv^* - \frac{1}{2}\left(l_1\phi^{[\beta]}\right)^T\tilde{L}l_1\phi^{[\beta]}\\ &\leqslant \left(l_1\phi^{[\beta]}\right)^T\Xi Lv^* - \frac{\lambda_2(\tilde{L})}{2N}\left(l_1\phi^{[\beta]}\right)^T l_1\phi^{[\beta]}\\ &\leqslant \left(l_1\phi^{[\beta]}\right)^T\Xi Lv^* - \frac{\lambda_2(\tilde{L})}{2N}\left\|l_1\phi^{[\beta]}\right\|_2^2,\end{aligned} \tag{22}$$

where $\Xi, L, \tilde{L}$ are given in Lemma 1 and (1). On the other hand, based on Lemma 4, we obtain from (18) that

$$|v_k^*| = \left| \left(v_k^{*[\frac{1}{\beta}]} \right)^{[\beta]} - 0^{[\beta]} \right| \leqslant 2^{1-\beta} \left| v_k^{*[\frac{1}{\beta}]} - 0 \right|^{\beta} = 2^{1-\beta} |\eta_k|^{\beta}. \tag{23}$$

Furthermore, employing (23) and Lemma 7 with $\varepsilon > 0$ yields that

$$\begin{aligned} \left(l_1 \phi^{[\beta]} \right)^T \Xi L v^* &\leqslant \frac{\varepsilon}{2} \left(l_1 \phi^{[\beta]} \right)^T \Xi L L^T \Xi l_1 \phi^{[\beta]} + \frac{1}{2\varepsilon} \sum_{k=1}^{N} (v_k^*)^2 \\ &\leqslant \frac{\varepsilon}{2} \lambda_{max}(\Xi L L^T \Xi) \left\| l_1 \phi^{[\beta]} \right\|_2^2 + \frac{2^{1-2\beta}}{\varepsilon} \sum_{k=1}^{N} |\eta_k|^{2\beta}. \end{aligned} \tag{24}$$

Thus, combining (22), (23) and (24) yields that

$$\dot{H}_0(t) \leqslant -\frac{1}{2} \left(\frac{\lambda_2(\tilde{L})}{N} - \varepsilon \lambda_{max}(\Xi L L^T \Xi) \right) \left\| l_1 \phi^{[\beta]} \right\|_2^2 + \frac{2^{1-2\beta}}{\varepsilon} \sum_{k=1}^{N} |\eta_k|^{2\beta}. \tag{25}$$

It is deduced from (18) that

$$\frac{dH_k(t)}{dt} = \dot{v}_k^* \left(v_k^{*[\frac{1}{\beta}]} - 0^{[\frac{1}{\beta}]} \right) = \dot{v}_k^* \eta_k, \tag{26}$$

which, combing with (19), implies that

$$\begin{aligned} \dot{H}(t) &= \dot{H}_0(t) + \varepsilon \sum_{k=1}^{N} \dot{H}_k(t) \\ &\leqslant -c_1 \sum_{k=1}^{N} \left| l_1 \phi_k^{[\beta]} \right|^2 + \sum_{k=1}^{N} \frac{2^{1-2\beta}}{\varepsilon} |\eta_k|^{2\beta} + \varepsilon \eta^T \dot{v}^*, \end{aligned} \tag{27}$$

where $c_1 = \frac{\lambda_2(\tilde{L})}{2N} - \frac{1}{2}\varepsilon \lambda_{max}(\Xi L L^T \Xi)$. Substituting (16) and (17) into (27), one obtains that

$$\dot{H}(t) \leqslant -c_1 \sum_{k=1}^{N} \left| l_1 \phi_k^{[\beta]} \right|^2 - c_2 \sum_{k=1}^{N} |\eta_k|^{2\beta} - \varepsilon \sum_{k=1}^{N} (k_2 - |\tilde{d}_k|) |\eta_k|, \tag{28}$$

where $c_2 = \varepsilon k_1 - \frac{2^{1-2\beta}}{\varepsilon}$. Note that $0 < \varepsilon < \frac{\lambda_2(\tilde{L})}{N \lambda_{max}(\Xi L L^T \Xi)}$, $k_1 > 2^{1-2\beta} \varepsilon^{-2}$ and $k_2 > M + \bar{x}^*$. Then, $c_1 > 0$, $c_2 > 0$, and $k_2 - |\tilde{d}_k| > 0$, for all $k \in \mathcal{I}_N$. From (28), one obtains that

$$\begin{aligned} \dot{H}(t) &\leqslant -c_1 \sum_{k=1}^{N} \left| l_1 \phi_k^{[\beta]} \right|^2 - c_2 \sum_{k=1}^{N} |\eta_k|^{2\beta} \\ &\leqslant -c_1 l_1^2 \sum_{k=1}^{N} |\phi_k|^{2\beta} - c_2 \sum_{k=1}^{N} |\eta_k|^{2\beta}. \end{aligned} \tag{29}$$

Let $e_1 = \max\limits_{k \in \mathcal{I}_N} \left\{ \frac{l_1 \xi_k}{1+\beta} \right\}$ and $e_2 = 2^{1-\beta}\varepsilon$, then one obtains from (23) that

$$\begin{aligned} H_0(t) &= \sum_{k=1}^{N} \frac{l_1 \xi_k |\phi_k|^{1+\beta}}{1+\beta} \leqslant e_1 \sum_{k=1}^{N} |\phi_k|^{1+\beta}, \\ \bar{H}_0(t) &= \varepsilon \sum_{k=1}^{N} H_k \leqslant \varepsilon \sum_{k=1}^{N} |v_k^* - 0| \left| v_k^{*[\frac{1}{\beta}]} \right| \leqslant \varepsilon \sum_{k=1}^{N} 2^{1-2\beta} |\eta_k|^{1+\beta} = e_2 \sum_{k=1}^{N} |\eta_k|^{1+\beta}. \end{aligned} \tag{30}$$

For any real $\alpha > 1$, let $r_1 = \frac{2\beta}{1+\beta}$ and $r_2 = \frac{\beta+\alpha}{1+\beta}$. Then, it is easy to get from $\alpha > 1$ and $\frac{1}{2} < \beta < 1$ that $0 < r_1 < 1$, $r_2 > 1$. Thus, by Lemma 5 and Lemma 6, one obtains

$$\begin{aligned} \left(\sum_{k=1}^{N} |\phi_k|^{1+\beta}\right)^{r_1} &\leqslant \sum_{k=1}^{N} |\phi_k|^{2\beta} \\ N^{1-r_2}\left(\sum_{k=1}^{N} |\phi_k|^{1+\beta}\right)^{r_2} &\leqslant \sum_{k=1}^{N} |\phi_k|^{\beta+\alpha}. \end{aligned} \tag{31}$$

It follows from (29) that ϕ_k and η_k are bounded, i.e., there are two positive constants K_1 and K_2 such that $|\phi_k| \leqslant K_1$ and $|\eta_k| \leqslant K_2$ for all $k \in \mathcal{I}_N$. Then we have

$$\begin{aligned} N^{1-r_2}\left(\sum_{k=1}^{N} |\phi_k|^{1+\beta}\right)^{r_2} &\leqslant \sum_{k=1}^{N} |\phi_k|^{\beta+\alpha} = \sum_{k=1}^{N} |\phi_k|^{2\beta}|\phi_k|^{\alpha-\beta} \\ &\leqslant \left(\sum_{k=1}^{N} |\phi_k|^{\alpha-\beta}\right)\left(\sum_{k=1}^{N} |\phi_k|^{2\beta}\right) \\ &\leqslant NK_1^{\alpha-\beta}\sum_{k=1}^{N} |\phi_k|^{2\beta}. \end{aligned} \tag{32}$$

Furthermore,

$$\frac{N^{1-r_2}}{NK_1^{\alpha-\beta}}\left(\sum_{k=1}^{N} |\phi_k|^{1+\beta}\right)^{r_2} \leqslant \sum_{k=1}^{N} |\phi_k|^{2\beta}. \tag{33}$$

Simultaneously,

$$\begin{aligned} \left(\sum_{k=1}^{N} |\eta_k|^{1+\beta}\right)^{r_1} &\leqslant \sum_{k=1}^{N} |\eta_k|^{2\beta}, \\ \frac{N^{1-r_2}}{NK_2^{\alpha-\beta}}\left(\sum_{k=1}^{N} |\eta_k|^{1+\beta}\right)^{r_2} &\leqslant \sum_{k=1}^{N} |\eta_k|^{2\beta}. \end{aligned} \tag{34}$$

Therefore, from (30)–(34), the following inequality holds

$$\begin{aligned} e_1^{-r_1} H_0(t)^{r_1} &\leqslant \sum_{k=1}^{N} |\phi_k|^{2\beta}, \\ e_1^{-r_2}\frac{N^{1-r_2}}{NK_1^{\alpha-\beta}} H_0(t)^{r_2} &\leqslant \sum_{k=1}^{N} |\phi_k|^{2\beta}, \\ e_2^{-r_1} \bar{H}_0(t)^{r_1} &\leqslant \sum_{k=1}^{N} |\eta_k|^{2\beta}, \\ e_2^{-r_2}\frac{N^{1-r_2}}{NK_2^{\alpha-\beta}} \bar{H}_0(t)^{r_2} &\leqslant \sum_{k=1}^{N} |\eta_k|^{2\beta}. \end{aligned} \tag{35}$$

Based on Lemmas 5 and 6, and combining (29) with (35), we have

$$\dot{H} \leqslant -\frac{c_1 l_1^2}{2e_1^{r_1}} H_0^{r_1} - \frac{c_1 l_1^2 N^{1-r_2}}{2e_1^{r_2} NK_1^{\alpha-\beta}} H_0^{r_2} - \frac{c_2}{2e_2^{r_1}} \bar{H}_0^{r_1} - \frac{c_2 N^{1-r_2}}{2e_2^{r_2} NK_2^{\alpha-\beta}} \bar{H}_0^{r_2}. \tag{36}$$

Let $\bar{c}_1 = \min\left\{\frac{c_1 l_1^2}{2e_1^{r_1}}, \frac{c_2}{2e_2^{r_1}}\right\}$ and $\bar{c}_2 = \min\left\{\frac{c_1 l_1^2 N^{1-r_2}}{2e_1^{r_2} N K_1^{\alpha-\beta}}, \frac{c_2 N^{1-r_2}}{2e_2^{r_2} N K_2^{\alpha-\beta}}\right\}$. Then $\bar{c}_1 > 0$ and $2^{1-r_2}\bar{c}_2 > 0$. Employing (19), (30) and Lemmas 5 and 6 yields that

$$\begin{aligned} \dot{H} &\leqslant -\bar{c}_1 H_0^{r_1} - \bar{c}_2 H_0^{r_2} - \bar{c}_1 \bar{H}_0^{r_1} - \bar{c}_2 \bar{H}_0^{r_2} \\ &\leqslant -\bar{c}_1 H^{r_1} - 2^{1-r_2}\bar{c}_2 H^{r_2}. \end{aligned} \tag{37}$$

According to Lemma 2, it can be obtained that

$$\lim_{t\to T^*} H(t) = 0 \text{ and } H(t) \equiv 0, \text{ for } t \geqslant T^*,$$

which means that

$$\begin{aligned} &\lim_{t\to T^*} x_i(t) - x_i^*(t) = x_j(t) - x_j^*(t) \text{ and } x_i(t) - x_i^*(t) \equiv x_j(t) - x_j^*(t), \text{ for } t \geqslant T^*, \\ &\lim_{t\to T^*} v_i(t) - \dot{x}_i^*(t) = v_j(t) - \dot{x}_j^*(t) \text{ and } v_i(t) - \dot{x}_i^*(t) \equiv v_j(t) - \dot{x}_j^*(t), \text{ for } t \geqslant T^*, \end{aligned}$$

where $i, j \in \mathcal{I}_N$ and $T^* = \frac{1}{\bar{c}_1(1-r_1)} + \frac{1}{2^{1-r_2}\bar{c}_2(r_2-1)}$. Then the FixS formation of MASs (8) is achieved. The proof is completed. □

Remark 3. *The control input* (17) *is globally bounded. From* (29) *in the paper, one obtains that* $H(t)$ *is bounded, which implies that* ϕ_k, v_k^*, $k \in \mathcal{I}_N$ *are bounded. Further, we can obtain from* (18) *that* η_k *is bounded. Besides,* ψ_k *is bounded by noting that* $\dot{\phi}_k = \psi_k$. *By the expression of control input* (17), *it is known that the control input* $u(t)$ *is globally bounded.*

Remark 4. *It can be observed from Theorem 1 that the parameters* l_1 *can be chosen to be any positive and* k_2 *is a positive. However, the control gain* k_1 *is dependent on global information, viz., the spectrum of the Laplacian matrix* $\lambda_{max}(\Xi L L^T \Xi)$. *In practical applications, control gains may be ineffective due to some system information missing. Therefore, it is necessary to design a control gain that does not depend on global information. To solve this problem, a new adaptive practical fixed-time control protocol is proposed.*

Subsequently, we consider the adaptive FixF control protocol with control gain. We designed the following adaptive FixF control protocol

$$u_i = -w_i \eta_i^{[2\beta-1]} - k_2 sign(\eta_i) - \beta l_1 |\phi_i|^{\beta-1}\psi_i, \ \ i \in \mathcal{I}_N, \tag{38}$$

where l_1, β are shown in (11). Moreover, w_i are dynamic control gains satisfying

$$\begin{aligned} \dot{w}_i(t) &= -\ell w_i(t), \ w_i(0) = w_{i0}, \\ w_i(t) &\geqslant \check{w}_i = 2^{1-2\beta}\varepsilon^{-2} + \varepsilon^{-1}c_3, \forall\, t \geqslant 0, \end{aligned} \tag{39}$$

where $\ell > 0$, $i \in \mathcal{I}_N$, w_{i0} and c_3 are positive.

Theorem 2. *For* $\frac{1}{2} < \beta < 1$, $l_1 > 0$, *assume the Assumptions 1 and 2 hold and the network topology of MASs* (8) *is strongly connected. Then, the FixF of MASs* (8) *can be achieved under control protocol* (38) *if* $k_2 > M + \bar{x}^*$.

Proof. Consider the Lyapunov function candidate

$$V(t) = H(t) + \frac{1}{2}\varepsilon \sum_{k=1}^{N} \tilde{w}_k^2, \tag{40}$$

where $H(t)$ is defined in (19). Defining $\tilde{w}_k = w_k - \check{w}_k$ and utilizing (39), one obtains

$$\frac{d}{dt}\left(\frac{1}{2}\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2\right) = \varepsilon\sum_{k=1}^{N}\tilde{w}_k\dot{\tilde{w}}_k = -\ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k w_k. \tag{41}$$

Similar to the derivation of (27), combining (16) and (38) with (39) shows that

$$\begin{aligned}
\dot{H}(t) &\leqslant -c_1\sum_{k=1}^{N}\left|l_1\phi_k^{[\beta]}\right|^2 + \sum_{k=1}^{N}\frac{2^{1-2\beta}}{\varepsilon}|\eta_k|^{2\beta} + \varepsilon\eta^T\dot{v}^* \\
&= -c_1\sum_{k=1}^{N}\left|l_1\phi_k^{[\beta]}\right|^2 - \sum_{k=1}^{N}\left(\varepsilon w_k - \frac{2^{1-2\beta}}{\varepsilon}\right)|\eta_k|^{2\beta} - \varepsilon\sum_{k=1}^{N}\left(k_2 - |\tilde{d}_k|\right)|\eta_k| \\
&\leqslant -c_1\sum_{k=1}^{N}\left|l_1\phi_k^{[\beta]}\right|^2 - \sum_{k=1}^{N}\left(\varepsilon w_k - \frac{2^{1-2\beta}}{\varepsilon}\right)|\eta_k|^{2\beta} \\
&\leqslant -c_1\sum_{k=1}^{N}\left|l_1\phi_k^{[\beta]}\right|^2 - \sum_{k=1}^{N}\left(\varepsilon\check{w}_k - \frac{2^{1-2\beta}}{\varepsilon}\right)|\eta_k|^{2\beta} \\
&= -c_1\sum_{k=1}^{N}\left|l_1\phi_k^{[\beta]}\right|^2 - c_3\sum_{k=1}^{N}|\eta_k|^{2\beta},
\end{aligned} \tag{42}$$

where c_1 is designed in (27). Similarly to (29), from (40)–(42), one has

$$\begin{aligned}
\dot{V}(t) &\leqslant -c_1\sum_{k=1}^{N}\left|l_1\phi_k^{[\beta]}\right|^2 - c_3\sum_{k=1}^{N}|\eta_k|^{2\beta} - \ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k w_k \\
&\leqslant -\bar{c}_1 H^{r_1} - 2^{1-r_2}\bar{c}_3 H^{r_2} - \ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k w_k,
\end{aligned} \tag{43}$$

where $\bar{c}_1 = \min\left\{\frac{c_1 l_1^2}{2e_1^{r_1}}, \frac{c_3}{2e_2^{r_1}}\right\}$ and $\bar{c}_3 = \min\left\{\frac{c_1 l_1^2 N^{1-r_2}}{2e_1^{r_2} N K_1^{\alpha-\beta}}, \frac{c_3 N^{1-r_2}}{2e_2^{r_2} N K_2^{\alpha-\beta}}\right\}$ are positive. Notice the fact that $-\tilde{w}_k w_k = -\tilde{w}(\tilde{w} + \check{w}_k) = -\tilde{w}_k^2 - \tilde{w}_k\check{w}_k \leqslant -\frac{1}{2}\tilde{w}_k^2 + \frac{1}{2}\check{w}_k^2$. Thus, (43) can be rewritten as

$$\begin{aligned}
\dot{V}(t) &\leqslant -\bar{c}_1 H^{r_1} - 2^{1-r_2}\bar{c}_3 H^{r_2} - \ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k w_k \\
&\leqslant -\bar{c}_1 H^{r_1} - 2^{1-r_2}\bar{c}_3 H^{r_2} - \frac{1}{2}\ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2 + \frac{1}{2}\ell\varepsilon\sum_{k=1}^{N}\check{w}_k^2.
\end{aligned} \tag{44}$$

From the definition of $\check{w}$ and (43), it is easy to obtain that $V(t)$ and $\tilde{w}_k$ are bounded, i.e., there exists a constant $\bar{w}$ such that $\sum_{k=1}^{N}\tilde{w}_k^2 \leqslant \bar{w}$. In addition, employing (44) and Lemmas 5 and 6, one obtains

$$\begin{aligned}
\dot{V}(t) &\leqslant -\bar{c}_1 H^{r_1} - 2^{1-r_2}\bar{c}_3 H^{r_2} - \ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k w_k \\
&\leqslant -\bar{c}_1 H^{r_1} - 2^{1-r_2}\bar{c}_3 H^{r_2} - \frac{1}{2}\ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2 + \frac{1}{2}\ell\varepsilon\sum_{k=1}^{N}\check{w}_k^2 \\
&= -\bar{c}_1 H^{r_1} - \bar{c}_1\left(\frac{1}{2}\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2\right)^{r_1} - 2^{1-r_2}\bar{c}_3 H^{r_2} - 2^{1-r_2}\bar{c}_3\left(\frac{1}{2}\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2\right)^{r_2} \\
&\quad - \frac{1}{2}\ell\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2 + \frac{1}{2}\ell\varepsilon\sum_{k=1}^{N}\check{w}_k^2 + \bar{c}_1\left(\frac{1}{2}\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2\right)^{r_1} + 2^{1-r_2}\bar{c}_3\left(\frac{1}{2}\varepsilon\sum_{k=1}^{N}\tilde{w}_k^2\right)^{r_2} \\
&\leqslant -\bar{c}_1 V^{r_1} - 2^{2-2r_2}\bar{c}_3 V^{r_2} + s,
\end{aligned} \tag{45}$$

where

$$s=\begin{cases}\frac{1}{2}\ell\varepsilon\sum\limits_{k=1}^{N}\breve{w}_k, & \text{if } \bar{c}_1\left(\frac{1}{2}\varepsilon\sum\limits_{k=1}^{N}\tilde{w}_k^2\right)^{r_1}+2^{1-r_2}\bar{c}_3\left(\frac{1}{2}\varepsilon\sum\limits_{k=1}^{N}\tilde{w}_k^2\right)^{r_2}-\frac{1}{2}\ell\varepsilon\sum\limits_{k=1}^{N}\tilde{w}_k^2\leqslant 0,\\ \bar{c}_1\varepsilon^{r_1}\bar{w}^{r_1}+2^{1-r_2}\bar{c}_3\varepsilon^{r_2}\bar{w}^{r_2}+\frac{1}{2}\ell\varepsilon\sum\limits_{k=1}^{N}\breve{w}_k^2, & \text{otherwise.}\end{cases}$$

Note that r_1, r_2 satisfying $0<r_1<1<r_2$ and $\bar{c}_1$, $2^{2-2r_2}\bar{c}_3$, s are positive. Then, employing Lemma 3 to (45) shows that the MASs (8) can achieve the practical FixS under control protocol (38) and the solution of system (8) converges to residual set

$$\left\{\lim_{t\to T}x(t)\middle|V(x(t))\leqslant\min\left\{\left(\frac{s}{\bar{c}_1(1-\theta)}\right)^{\frac{1}{r_1}},\left(\frac{s}{2^{2-2r_2}\bar{c}_3(1-\theta)}\right)^{\frac{1}{r_2}}\right\}\right\}$$

within a fixed T^* satisfying

$$T\leqslant T^*=\frac{1}{\bar{c}_1\theta(1-r_1)}+\frac{1}{2^{2-2r_2}\bar{c}_3\theta(r_2-1)},\quad \theta\in(0,1).$$

The proof is completed. □

4. Simulations

In this section, two examples are presented to demonstrate the theoretical results. Consider MASs (8) with eight agents, i.e., $N=8$. The position information and velocity information of the agents can be represented as $x_i=(x_{i1},x_{i2})^T$ and $v_i=(v_{i1},v_{i2})^T$, $i\in\mathcal{I}_8$. The information of the predetermined formation trajectory is $x_i^*=(x_{i1}^*,x_{i2}^*)^T$. The relative displacements of each agent to the formation center are set by $(-2,0)^T$, $(-\sqrt{2},\sqrt{2})^T$, $(0,2)^T$, $(\sqrt{2},\sqrt{2})^T$, $(2,0)^T$, $(\sqrt{2},-\sqrt{2})^T$, $(0,-2)^T$, $(-\sqrt{2},-\sqrt{2})^T$. Take the disturbance $d_i(t)=\left(2\sin(it+1)+\pi,2\cos(it)-\pi\right)^T$, $i\in\mathcal{I}_8$. A topology graph is a network structure graph composed of network node devices and communication media. In this paper, each multi-agent is regarded as a network node, and the communication situation between them is described by the topology graph. The specific communication topology graph of the two examples are given below.

Example 1. *The topology network is represented by a strongly connected directed graph in Figure 1. Control parameters are chosen as* $\beta=0.9$, $l_1=1$, $\ell=0.5$, $k_1=10$, *the initial value of dynamic control gain is set to* $w_{i0}=1$. *The desired formation shape is regular octagon and the desired trajectory of the formation center is "O":* $\left(t,2\sin(\frac{\pi t}{4})\right)^T$. *Moreover, the initial data of positions are taken as* $(-4,-1)^T$, $(-3,-2)^T$, $(-3,2)^T$, $(-1,3)^T$, $(2,2)^T$, $(3,1)^T$, $(2,-3)^T$, $(1,-3)^T$, *and the initial data of velocities are taken as* $(3,1)^T$, $(1,3)^T$, $(2,2)^T$, $(3,1)^T$, $(2,1)^T$, $(1,2)^T$, $(1,3)^T$, $(3,1)^T$.

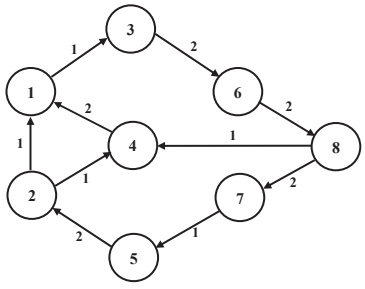


Figure 1. The communication digraph $\mathcal{G}$ of Example 1.

Furthermore, by Matlab, the following Figures 2–6 are obtained. Figure 2 shows that the desired formation can be achieved within a fixed-time, which demonstrates that

Theorem 2 holds. Furthermore, Figures 3 and 4 show that the error between the agents and the predetermined position can achieve convergence in fixed-time. Figures 5 and 6 show that the error between the agents and the predetermined velocity can converge to 0 within a fixed-time. Therefore, the Figures 2–6 indicate that the MASs (8) can achieve a relatively static state and form a stable formation within a fixed-time.

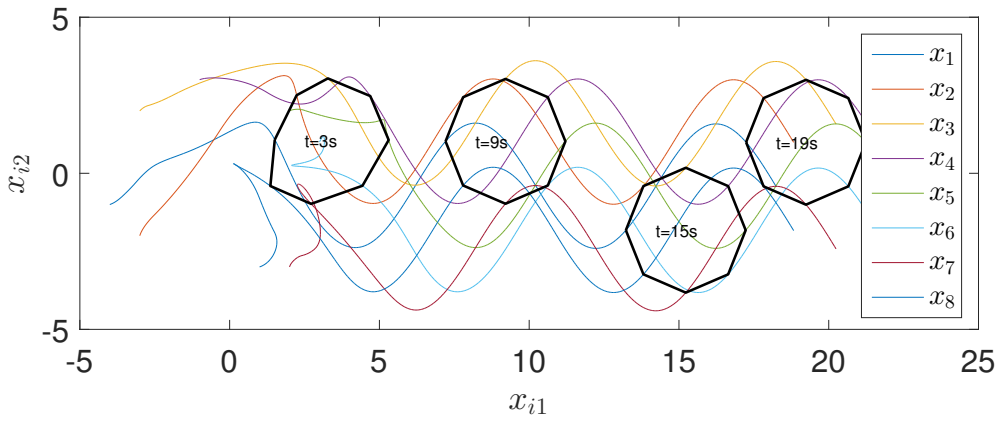


Figure 2. The trajectory of agents for Example 1.

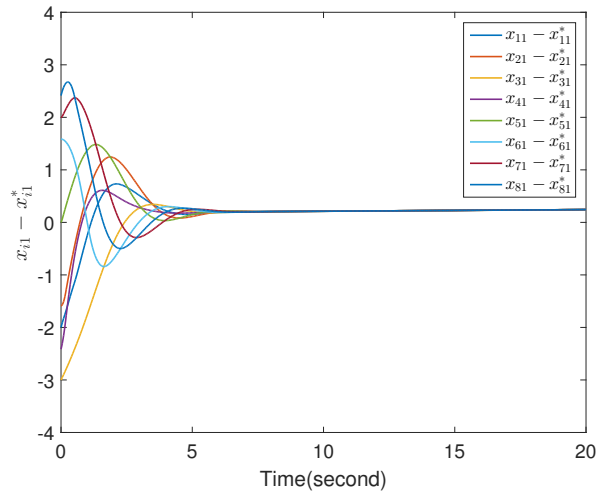


Figure 3. The position tracking error on horizontal direction.

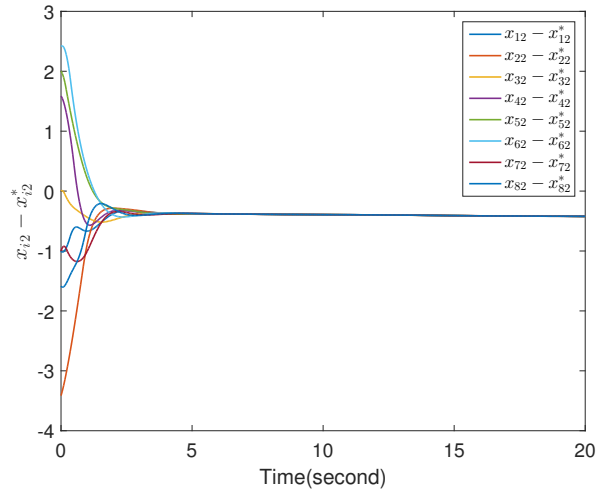


Figure 4. The position tracking error on vertical direction.

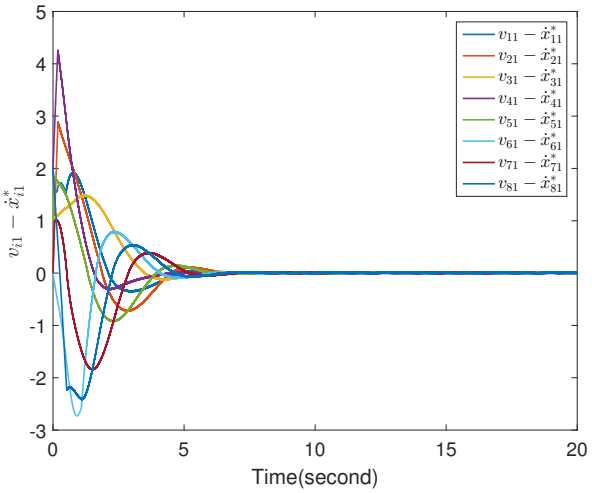


Figure 5. The velocity tracking error on horizontal direction.

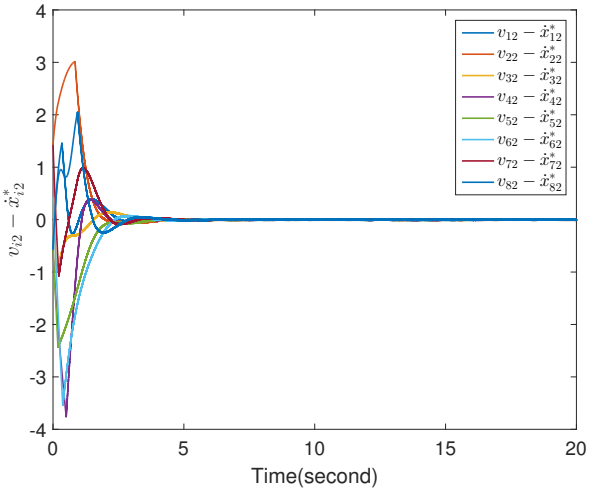


Figure 6. The velocity tracking error on vertical direction.

Example 2. *The topology network is represented by a strongly connected directed graph in Figure 7. Control parameters are chosen as* $\beta = 0.8$, $l_1 = 1.5$, $\ell = 0.5$, $k_1 = 8$, *the initial value of dynamic control gain is set as* $w_{i0} = 1$. *The desired formation shape is a regular octagon and the desired trajectory of the formation center is "O":* $\left(8\cos(\frac{\pi t}{8}), 8\sin(\frac{\pi t}{8})\right)^T$. *Moreover, the initial data of positions are taken as* $(-4,-3)^T$, $(-3,-2)^T$, $(-3,2)^T$, $(-1,3)^T$, $(2,2)^T$, $(3,1)^T$, $(2,-3)^T$, $(2,-4)^T$, *and the initial data of velocities are taken as* $(4,2)^T$, $(2,3)^T$, $(2,2)^T$, $(4,2)^T$, $(3,2)^T$, $(2,1)^T$, $(2,4)^T$, $(3,2)^T$.

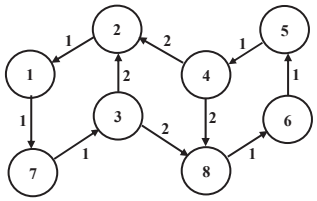


Figure 7. The communication digraph $\mathcal{G}$ of Example 2.

Similar to Example 1, by Matlab, the following Figures 8–12 are obtained. Figure 8 shows that the MASs (8) did not form a stable formation at t = 4 s, while the formation reaches stability at t = 8 s. Figures 9–12 show that the errors of agents with predetermined

velocity and predetermined position achieve convergence within a fixed-time, which indicates that the MASs (8) achieve a relatively static state and form a stable formation within a fixed-time.

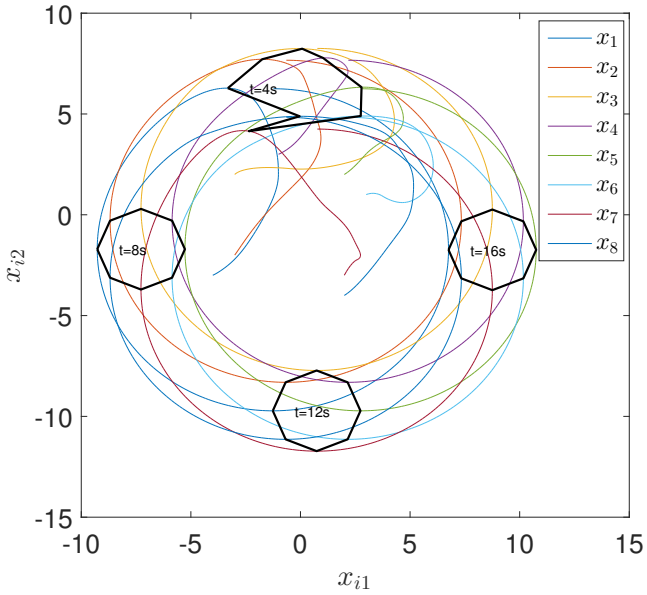


Figure 8. The trajectory of agents for Example 2.

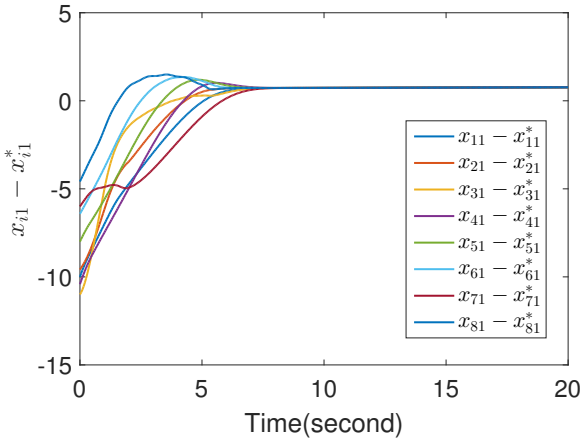


Figure 9. The position tracking error on horizontal direction.

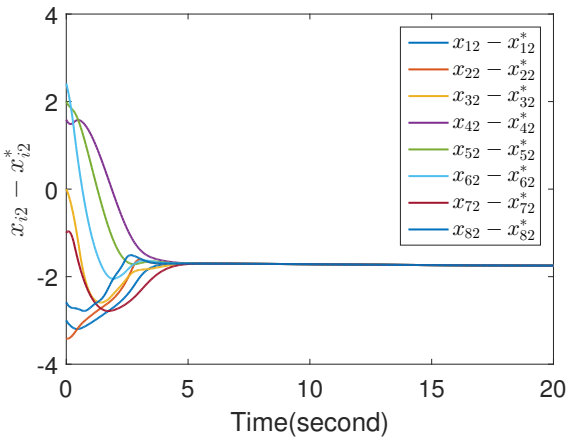


Figure 10. The position tracking error on vertical direction.

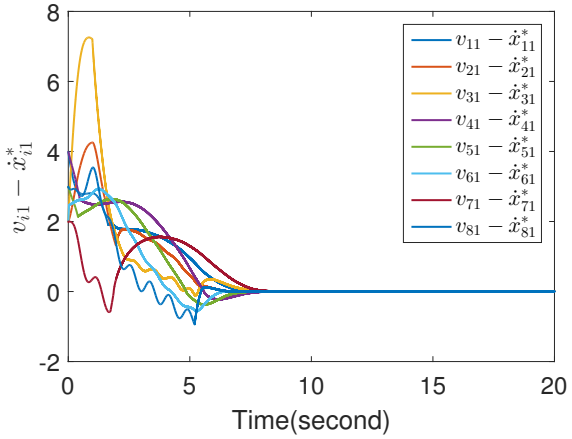


Figure 11. The velocity tracking error on horizontal direction.

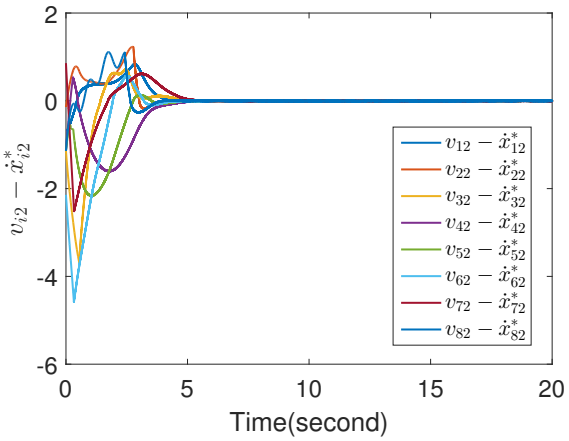


Figure 12. The velocity tracking error on vertical direction.

5. Conclusions

In this paper, the backstepping method is employed to design a new FixF control protocol for a kind of second-order MASs with disturbances. Compared with some relative existing works, the advantage of the protocol is that it depends on fewer parameters and can greatly reduce the control cost. Then, to remove the dependence of control gains on global conditions, a new adaptive practical FixF control protocol has been presented, which is fully distributed. The control gain of the protocol does not depend on any system information, which is more convenient in practical applications. Then, the desired formation can be achieved within a fixed-time under the control protocol described above. In fact, these results can be extended to more general directed graphs. In this paper, the main limitation of the simulation is that the accuracy is limited when solving the model. Although the shorter the time step, the higher the accuracy, the simulation calculation cost will be greatly increased. In addition, the work in this paper can be extended to MASs formation control under various attacks, such as [41]. In the future, we will try to combine this work with the neural network method [42] to study the formation control of MASs.

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