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Optimization of Asset and Liability Management of Banks with Minimum Possible Changes

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Abstract: Asset-Liability Management (ALM) of banks is defined as simultaneous planning of all bank assets and liabilities under different conditions and its purpose is to maximize profits and minimize the risks in banks by optimizing the parameters in the balance sheet. Most of the studies 'and proposed models in the ALM field are based on an objective function that maximizes bank profit. It is not easy to apply changes in these models in order to reach the optimal values of the parameters in the balance sheet. In this article, an attempt has been made to propose a linear model using constraints to achieve optimal values of balance sheet parameters using ALM objectives and considering balance sheet, system and regulatory constraints. It has also been tried to design the model according to the most possible mode and with the least changes and to minimize the size of the balance sheet. The analysis of the model presented in this article has been conducted using the parameters of the balance sheet and income statement of one of the famous Iranian banks. The results obtained from the proposed model show that the values of cash and receivables from banks and other credit institutions have decreased by 30% and increased by 200%, respectively, compared to the actual values of these parameters. Also, Total Income, Operating Income and Non-Operating Income have grown by 30% compared to the actual values of these parameters. Also, the values of a number of parameters are estimated to be zero after optimization. According to the results, it is obvious that the performance of bank managers, especially in the management of bank assets, is significantly different from the optimal values of the balance sheet, and the results obtained from the proposed model can help the management of banks as much as possible.

Keywords: Asset-Liability Management; banking industry; mathematical optimization; financial statements; balance sheet; revenue and cost

MSC: 90-10; 90B90; 90C05; 90C90; 91-05; 91-10

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1. Introduction

The banking system is one of the most important economic sectors that play a very important role in the overall performance of economic systems in different countries [1–11]. The economic stability of a country is directly related to the stability and operational efficiency of a country's banking system, and the result of proper economic management is sustainable economic growth [12–20]. So, banks are not just looking to assess potential risks, but are always trying to find ways to deal with the effects of the risks in the banking system or even the shocks that occur [21–43]. Investigating the financial risks in the banking system is very complex due to the existence of many different variables in the financial statements of banks and should use advanced mathematical methods and numerical methods [44–67].

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Managers and policy makers of banks are desperately looking for the best or optimal way to manage the assets and liabilities of banks because the banking industry has a very important role in the economic system of different countries and also the need to pay constant attention to this sector has increased with major financial crises in the last few years and its impact on the banking system [68–86].

Asset-Liability Management (ALM) is a field that maximizes the wealth of shareholders and tries to increase the efficiency of banks and reduce the risks in banks. In fact, risk and return are two important and influential factors in maximizing shareholders' wealth. It should be said that today banks try to invest their assets in safe places so that they can protect themselves from major economic crises and uncertainties and earn a reasonable profit and the field that can answer the question is ALM. ALM includes a set of tools and technical methods that seek to guarantee the profits of shareholders and reduce the risks in banks [87–91].

Financial indicators, items in the financial statements and the relationships between them are factors that can analyze the performance of a bank. Therefore, the performance of banks and the current situation of their assets and liabilities are examined using different models. Numerous studies have been conducted to optimize the basic items of banks' balance sheets, all of which have been able to achieve reasonable results by considering the same objective function, i.e., maximizing the bank's profit or maximizing the difference between the bank's revenue and cost [92–100].

As mentioned, the most important goals of ALM are maximizing profit and minimizing risk in banks by optimizing the parameters in the balance sheet and finally increasing the efficiency of the bank. Therefore, many studies have been conducted to manage the assets and liabilities of banks. One of the conventional objective functions in the field of ALM is the objective function based on the maximization of bank profit, and in the proposed models based on this objective function, it is difficult to apply changes in the existing parameters to reach the optimal values of the balance sheet and income statement parameters. Also, in the conducted studies, no special effort has been made to reduce the size of the balance sheet [101–112]. But, this article has tried to eliminate this gap in the studies conducted in the past. The aim of this article is to achieve the optimal values of balance sheet and income statement parameters according to balance sheet constraints, system constraints and regulatory constraints according to the most feasible state and the least changes using a proposed model and various constraints. It has also been tried to minimize the size of the balance sheet of the bank under review, which is one of the largest banks in Iran.

In the rest of the article, the studies carried out related to the raised topic, the equations related to the proposed model and the considered limitations, the results obtained from the model and the conclusions, limitations and suggestions are stated in Sections 2–5, respectively.

2. Literature Review

In this section, a review of applied research is presented about ALM of banks using different models.

The development of simple and usable models to optimize asset and liability management of banks ensures stable profitability and minimizes the risks in this financial institution. Also, this issue makes the bank resources to be balanced and the management of liquidity and resources is carried out accurately. In research, Oguzsoy and Guven et al. [113] have tried to optimize the portfolio of assets and liabilities of Turkish banks using a multi-period stochastic linear model. In order to achieve this goal, the parameters available in the balance sheet such as investment returns, loan costs, liquidity and reserves and financial information of Turkish banks during the years 1987 to 1990 have been used. Using this information, the authors have sought to find out what effect banking policies and regulations, environmental factors, risks and restrictions have on the elements of the balance sheet and sensitivity analysis has also been performed. The obtained results show that the proposed model is more realistic than the deterministic model.

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Kosmidou and Zopounidis [114] have sought to manage assets and liabilities using a goal programming model and by imposing constraints on liquidity, solvency and average returns on assets and liabilities. By examining the sensitivity of balance sheet variables, this model tries to provide an optimal strategy for managing the assets and liabilities of a Greek bank. This model has been developed with important conditional applications, and that condition is that the model is evaluated in a stochastic interest-rate environment. This article states that organizations need the proposed models for ALM due to the instability in market rates and the difficulty of analyzing balance sheet data. The results show that the obtained strategy from the model is much more efficient than the present strategy of the bank in terms of risk and present value.

Papi and Sbaraglia [115] have proposed a dynamic programming model for ALM where constraints are imposed, and transaction costs are taken into account. In addition, this study has stated that stochastic optimization is very useful in research that aims at multiple subject periods. This model optimizes the portfolio and balance sheet of financial institutions according to the constraints and the objective function. One of the goals of this model is to find a way to reduce overhead, while one of the positive features of this model is not considering artificial boundary conditions. Finally, by presenting numerical examples, the performance of the model is evaluated and it is concluded that it can be generalized to a higher level.

Yan [116] has proposed a mean-variance model used for ALM and works based on a process that is continuously monitored in time. One of the problems with ALM is that financial institutions are trying to achieve a dynamic portfolio. This article has tried to provide a suitable and optimal strategy by solving a HJB stochastic equation. One of the features of this model is the optimal strategy, which can be achieved using the stochastic quadratic linear control technique. There are also limitations to the model presented in this paper including VaR (value at risk).

Chiu and Li [117] presented a model based on an important principle called the first principle of safety, which represents the amount of portfolio risk and measures the difference between the marginal risk limit and the value applied to the objective function for ALM. This model seeks to investigate the relationship between ALM in the framework of the safety-first principle and ALM in the context of mean variance and provides graphs in this context. Finally, this study differentiates between high-risk investors who seek high returns and low-risk investors who are cautious and provides an optimal strategy for risk investors.

Ferstl and Weissensteiner [118] sought to understand how much the value of assets in the market now differs from the value of future liabilities. This article sought to manage the assets and liabilities of a pension fund using a stochastic linear programming model and applying time-varying investment conditions. This model is intended to provide a suitable strategy for efficient investment and increase the return on assets and liabilities by calibrating the basic items of the fund's balance sheet. The results show that the applied assumptions are reasonable and results are consistent with economic logic, and one of the reasons for presenting the model in this article is the predictability of the calculated and evaluated returns.

Wen-ze et al. [119] have sought to present a segmented and dynamic optimization model for the liquidity management of commercial banks in China, considering its regulatory aspect. The proposed model is designed in such a way that by applying favorable conditions in the future, optimization operations can be performed to manage the assets and liabilities of commercial banks. In the rest of this article, it is pointed out that in order to achieve the goals of commercial banks, their activities should be considered for a long time, and the presented model should be responsive for a long time because it cannot be considered only in the short term, and it may be in A long period of time when the proposed model has this feature will threaten different shocks and crises of commercial banks.

Halaj [120] has examined the various liquidity and solvency shocks in banks and their strategies for responding to these shocks. The proposed model is dynamic stochastic and

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some of its key parameters are calibrated. This model has been evaluated in several periods using the liquidity and solvency constraints in a bank's balance sheet. In this study, it is stated that the proposed model is a general model of the bank's optimal balance sheet and that this model can help in how to respond to shocks, return management in which the risk is considered and the regulatory aspects of the bank. To make the model easier to solve, the Monte-Carlo method is used, and Value-at-Risk (VaR) restrictions are facilitated. The research has evaluated the effects of macroeconomic shocks, the effects of shocks on the bank's financial resources and the effects of shocks on the bank's credit risk on the amounts of key balance sheet items based on a stochastic dynamics process model in the form of graphs.

Pan and Xiao [121] have optimized ALM with respect to stochastic interest rates and inflation risks and for this optimization operation, it is assumed that inflation risk exists as a random process. This article has tried to sort out the allocations contiguously and maximize the expected utility. The obtained results present the optimal investment strategy and the effect of model parameters on it using mathematical tools for managing assets and liabilities in the bank in the form of quantitative cases.

When it comes time to invest, one of the major concerns of investors is how and with what structure to choose their portfolio.

Cui et al. [122] in research will help investors choose the right multi-period portfolio when they encounter this problem and do not know when to exit. In this regard, this article has presented a model that is based on mean-field and suggests the necessary optimal strategy for investors. Finally, the sensitivity of the variables has been measured in the form of quantitative problems. The results show that when investors do not leave the market in time or the liability variance or correlation coefficient is growing, they try to expand their higher-risk assets.

Liquidity and risk are key components of ALM. Ahmadian and Shahchera [123] have tried to reduce the liquidity risk of the Iranian banking system during the years 2006 to 2018 by using a proposed model and information extracted from the financial statements of the Iranian banking system. Another goal of this study is to answer the question of whether liquidity risk is reduced if banks move towards ALM. The results show that the ALM and moving towards it has a positive effect on liquidity risk and reduces the risk in the banking system and one of the solutions to reduce liquidity risk is to increase the capital adequacy ratio in banks. Another result of this study is that liquidity risk and profitability are directly related. This means that with decreasing profitability, liquidity risk decreases and with increasing profitability, liquidity risk increases.

Oliveira et al. [124] have used Multistage Stochastic Planning for ALM and presented two different approaches. In the following, this paper has evaluated the impact of these approaches on the goal function using classical Monte Carlo Sampling and Moment Matching methods. The model is presented in such a way that it shows a realistic environment considering the technological restrictions. The results show that the goal function using Moment Matching and the Resampled Average Approximation method is much more stable than the goal function using the Monte Carlo Sampling and Monte Carlo method with naive allocation. Based on all the results, it is inferred that the best methods for the scenarios proposed for ALM are Moment Matching and the Resampled Average Approximation.

Pan et al. [125] have used ALM models to obtain the optimal investment strategy and minimize its risk. This research has tried to use the optimal strategy for investors to invest in different assets with different risks using the price process by the Heston model. The model presented in this paper is based on the Hamilton–Jacobi–Bellmann equation. The results show that the optimal strategy for optimal investment (lowest available risk) and the corresponding value function are obtained using quantitative problems and what effect do the different variables in the model have on the strategy. In the end, this proves whether the strategy derived from the model is really optimal. The results confirm that the various variables in the model have a significant impact on the optimal strategy.

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Li et al. [126] have evaluated the probability of bankruptcy of a financial institution using a multi-period mean variance model. One of the applications of this issue is that the investor can continuously analyze the investment risk of his portfolio. Managers can also use this model to adopt better and more accurate strategies and with less risk in investments. In the model presented in this paper, the return on assets and liabilities and the cash flow on the balance sheet are used to manage assets and liabilities. Finally, by evaluating the sensitivity of each component of the balance sheet, it identifies the degree of dependence and sensitivity of the financial institution to that component. The results show that an optimal strategy in the form of complex problems has been extracted using mean-field formulation that can show us the level of risk and efficiency of an investment.

Evaluating the basic items of the balance sheet and profit and loss statement of banks using the proposed models for ALM is one of the main ways to assess the amount of risk in the bank. Abdollahi [127] has presented a model using multi-objective planning method that suggests strategies for managers and investors to move towards investing with less risk and higher returns. This model has used the information of an Iranian bank called Mellat Bank during the period from 2009 to 2016. The results show that the condition of Mellat Bank's assets and liabilities is not in good condition, and it must redesign its balance sheet structure to reach the desired condition. At a glance at the results, we find that one of the best ways to get the bank out of the current situation is to reduce the amount of cash and bonds on the right side of the balance sheet.

Chunxiang et al. [128] have shown that investors who seek to invest in a variety of high or low-risk financial markets may incur contingent liability. This article has tried to examine this challenge in ALM using the mean of variance. Equations related to ALM have been evaluated using the Hamilton–Jacobi–Bellmann equation system and assuming continuity of time and existence of delay. In the following, the paper seeks to extract the investment strategy and the appropriate value function for ALM and then test these strategies using sensitivity analysis. Finally, the research results show that the investor's historical wealth and debt affect his investment strategy.

In research, Min et al. [129] proposed a hybrid robust portfolio model to solve one of the biggest problems of portfolio optimization, which is conservatism. To achieve this goal, two algorithms Long Short-Term Memory (LSTM) and eXtreme Gradient Boosting (XGBoost) and portfolio information of 12 industries in the United States have been used. The stated algorithms as well as two concepts of variance and value at risk have been used to monitor and predict market movements. Also, clustering has been used to reduce conservatism. The obtained results show that the suggested portfolios based on variance have higher effectiveness and Sharpe ratios, and the suggested portfolios based on the value at risk have higher returns.

Alshehri and Tayachi [130] have evaluated the management of banks' assets and liabilities using financial ratios (Average Total Assets, Total Equity at Average, Return on Assets and Return on Equity) extracted from the banking system in Saudi Arabia and Kuwait during the period 2012 to 2015. The reason for examining this issue using banking system information is to compare the ALM of the banking system assets between these two countries. The results show that the Saudi banking system performed better than the Kuwaiti banking system in terms of financial ratios. Finally, the research examines the gap ratio in the banking system of these two countries and compares them with each other, and the results show that both banks have a negative value, which means that both banks are more sensitive to liabilities.

In research, Owusu and Alhassan [131] analyzed the Ghanaian banking system and used a model to assess the relationship between profit and ALM. This model is Statistical Cost Accounting (SCA) and has examined the data of 27 banks in Ghana during the period 2007 to 2015. This article considers six groups of basic items on the right side of the balance sheet and seven groups of basic items on the left side of banks' balance sheets to evaluate the model and obtain the mentioned relationship and it has categorized this information based on the amount of profit and the type of banks. The results show that there is a

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relationship between profitability and balance sheet items because the regression of the composition of the groups obtained to the right and left of the balance sheet is significant which shows the importance of Asset-Liability Management (ALM) to earn more profit and implement better strategies. Finally, the study examines the differences between domestic and foreign banks and how they operate. Based on the results, domestic banks performed better in terms of return on assets, cash and foreign banks performed better in terms of return on liabilities and fixed assets.

Today, one of the biggest concerns of bank managers is planning to deal with the risks in banks that have been caused by various global crises. Lysiak et al. [68] have proposed a model that analyzes banking risks and responds to these concerns. This model is an economic-mathematical optimization model and mostly evaluates operational risk, currency risk and credit risk. To achieve this goal, the financial information of a Polish bank has been used. The obtained results show how much of the bank's profit value should be considered for this bank, and finally, it suggests important strategies to make more accurate and correct decisions for bank managers.

Braiek et al. [132] have tried to optimize an Islamic portfolio. To achieve this goal, ARMA-FIAPARCH and ARMA-FIGARCH models have been used. By using these models and Mean CoVaR and according to VaR limitation, downside and upside risk, market return and optimized portfolio have been obtained. Also, the Mean CoVaR model and the mean-variance model have been compared. The obtained results show that systematic risk affects the optimized portfolio. The comparison between the Mean CoVaR model and the mean-variance model shows that the performance of the Mean CoVaR model is better than the performance of the mean-variance model.

Due to the big financial crises that occurred in 2008 and 2019, the importance of optimizing the portfolio of financial institutions has multiplied. In research, Li et al. [133] seek to optimize a multi-period portfolio. In order to achieve this goal, a variable called the securities return rate variable and restrictions such as bankruptcy, liquidity, diversification and financing have been used. Also, by using normalization, it has been tried to obtain maximum return and minimum risk. Finally, the modified root system growth model is analyzed and expanded.

3. Methodology

In this section, the equations and constraints of the proposed model are described in detail. The present research is applied research in which an attempt has been made to present the optimal amounts of assets, liabilities and cash in proportion to the structure of the balance sheet. Given that financial data is very diverse and extensive, it is possible to provide many other variables in addition to balance sheet data for this study, but it should be noted that first, the number of balance sheet variables is very large. Second, incorporating more variables into the proposed model makes data collection difficult and also complicates modeling. Therefore, it has been tried to use only the balance sheet as a criterion for calculations, except for a few special cases. Based on this, the internal relations between balance sheet variables and the relationship between balance sheet items and other bank data are first evaluated. The methodology of the article is shown in Figure 1.

This research is conducted with a descriptive and analytical approach and will use mathematical modeling to explain the main variables of the problem and the relationships between them as well as optimizing the structure of the bank balance sheet. Using the main factors affecting the structure of the bank's balance sheet and managing various related areas such as granting facilities, attracting deposits, profitability, liquidity, risk, etc. are among the items that will be considered in the model. The methodology of this article has been followed by important steps. The first step to present the model of this research is to analyze the data in the financial documents of the bank. The main part of this research is to understand the structure of assets and liabilities in a bank. For this purpose, past data related to balance sheet components such as assets, liabilities, profit and loss and other

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items should be examined to formulate the problem to predict the structure of the bank's balance sheet using them in next years.

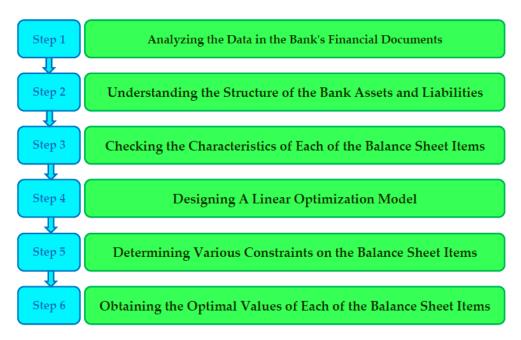


Figure 1. The schematic summary of all steps in the proposed linear optimization model.

Recognizing and examining the characteristics of each item of the balance sheet separately and how it relates to other items can be considered the main step in designing models related to balance sheet management. Accordingly, the present study follows the analysis of balance sheet items in various formats such as reviewing the trend of previous years and reviewing the share of each item in the overall composition of the balance sheet. The second step is to design a mathematical model of optimization. The goals and constraints of the first step will be presented in the form of a mathematical optimization model with the aim of finding the best possible answers. The next steps to present the model in this article are to solve the model and extract the optimal solutions and check the validity of the model.

The model presented in this research is a linear optimization model. The objective function of this model seeks to make the least changes in the assets and liabilities structure of the balance sheet. One of the most important goals of this model is to obtain the most feasible solution for managing the bank's assets and liabilities by using the assets and liabilities in the bank's balance sheet, which can reduce the size of the balance sheet as much as possible and achieve the best possible state.

Many items with different impact rates can be considered in the management of the bank's balance sheet structure, so that each of these items alone and in interaction with each other will affect the bank's profitability and risk. In this case and taking into account the changing environmental conditions, it seems possible to provide an optimal structure. Therefore, this study seeks to provide a model that the least changes in the structure of assets and liabilities occur using the results of the analysis. In the past, there have been conventional models that achieve the highest level of profitability of the bank at an appropriate level of risk and taking into account the existing constraints, which causes the balance sheet size to be very large. However, the model presented in this paper, with the objective function based on the least changes, significantly reduces the size of the balance sheet and examines the optimal possible case.

As shown in Figure 2, by entering the inputs into the model, which are the current values of the bank's balance sheet items, optimal outputs are obtained based on the constraints and according to the risks in the bank, which are the optimal values of the bank's balance sheet items.

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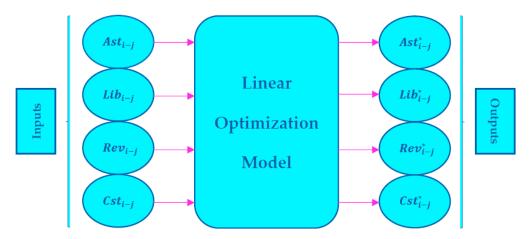


Figure 2. The presentation of inputs and outputs of linear optimization model. * means the optimal state of primary variables.

In this paper, any parameter that has an asterisk above it means its optimal state such as Ast_k^* which is the optimal state obtained from the model provided for the k-parameter of assets on the balance sheet.

To find the optimal balance sheet of banks, it has been common for bank profits to be calculated at a maximum, meaning that the difference between income and expenses is maximized by the following equation:

$$Max\{Total\ Revenues^* - Total\ Costs^*\}.$$
 (1)

But, in this article, we are going to introduce a new method that can be used a lot in the future. In this paper, we have tried to use the following formula to minimize the changes in the parameters in the bank's financial statements to optimize it:

$$Min\left(\sum_{k=1}^{n} \left| \frac{Ast_k^* - Ast_k}{Ast_k} \right| + \sum_{j=1}^{m} \left| \frac{Lib_j^* - Lib_j}{Lib_j} \right| \right). \tag{2}$$

We know that the total assets on the balance sheet must be equal to the total liabilities.

$$Total \ Assets^* - Total \ Liabilities^* = 0. \tag{3}$$

The following constraints on obligations for guarantees and letters of credit and other obligations are

$$Ast_x^* = Lib_x^*.x = 13.....16.$$
 (4)

We are now examining the non-structural constraints of assets on the balance sheet. Cash is considered at least two percent of deposits:

$$Ast_1^* + (Lib_{1-2}^*)(-r_1) + (Lib_2^*)(-r_1) \ge 0.r_1 = 0.02.$$
 (5)

$$Ast_1^* \le 12 Ast_1. \tag{6}$$

Regarding the claims that the bank has from the central bank, it can be conducted in such a way that the rate of receivables from the central bank is assumed to be 10% ($r_2 = 0.1$).

$$Ast_{2-x}^* = Ast_{2-x}(1+r_2).x = 1....4.$$
(7)

In the case of receivables from banks, a reasonable rate should be considered, which can be assumed to increase the receivables by a maximum of 50% ($r_3 = 0.5$).

$$Ast_{2-5}(1-r_3) \le Ast_{2-5}^* \le Ast_{2-5}(1+r_3).$$
 (8)

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Regarding claims from the government, we can consider the maximum growth rate of claims from the government as zero ($r_4 = 0$).

$$Ast_3^* \le Ast_3(1+r_4). \tag{9}$$

In the case of facilities granted and claims from the public sector, the following is evident:

$$Ast_4^* = Ast_4. (10)$$

The maximum and minimum ratio of facilities to deposits in the bank should be as follows ($r_5 = 0.5$, $r_6 = 1$):

$$Ast_5^* + (Lib_{1-2}^*)(r_6) + (Lib_2^*)(-r_6) \le 0, \tag{11}$$

$$Ast_5^* + (Lib_{1-2}^*)(r_5) + (Lib_2^*)(-r_5) \ge 0.$$
 (12)

In this section, it should be mentioned that $Ast_{5-1-X}(x=1.2....13)$ are thirteen types of current facilities of the non-governmental sector and $Ast_{5-2-X}(x=1.2....13)$ are thirteen types of non-current facilities of the non-governmental sector in the bank balance sheet, which can be referred to facilities granted to foreign branches and repurchased participation bonds. Ast_{5-2-1} , Ast_{5-2-2} , Ast_{5-2-3} , Ast_{5-2-4} , Ast_{5-2-5} , Ast_{5-2-6} , Ast_{5-2-7} , Ast_{5-2-8} , Ast_{5-2-9} , Ast_{5-2-10} , Ast_{5-2-11} , Ast_{5-2-12} and Ast_{5-2-13} parameters are Installment Sale Contract, Ju'alah Contract, Hire Purchase Contract, Mudarabah Contract, Civil Partnership Contract, Forward Contract, Debt Buying Contract, Murabahah Contract, Interest-Free Loan Contract, Facilities Granted in Foreign Currency, Redeemed Musharaka Contract, Debtors for Letters of Credit and Guarantees and Facilities Granted to Branches Abroad, respectively. Also, Ast_{5-1-9} is the interest-free loan in the current facilities section of the non-governmental sector and Lib_{1-2-2} is the savings account in the liability section.

$$(Ast_{5-1-X}^*)(Ast_{5-1}) + (Ast_{5-1}^*)(Ast_{5-1-X})(-0.9) \ge 0.x = 1.2....13,$$
 (13)

$$\left(Ast_{5-1-X}^{*}\right)\left(Ast_{5-1}\right) + \left(Ast_{5-1}^{*}\right)\left(Ast_{5-1-X}\right)(-1.2) \le 0.x = 1.2....13,\tag{14}$$

$$\left(Ast_{5-2-X}^* \right) \left(Ast_{5-2} \right) + \left(Ast_{5-2}^* \right) \left(Ast_{5-2-X} \right) (-0.9) \ge 0.x = 1.2....11,$$
 (15)

$$Ast_{5-1-9}^* - Lib_{1-2-2}^* + 0.0001 \le 0, (16)$$

$$Lib_{1-2-2}^* \le 1.5 Lib_{1-2-2}.$$
 (17)

We now evaluate the relationship between customer liabilities for guarantees and letters of credit and bank liabilities arising from letters of guarantee and letters of credit. Ast_{5-1-12} and Ast_{5-2-12} is the liability of others to the bank for letters of credit and guarantees in the current and non-current facilities section. Also, Ast_{5-2-13} is facility granted to branches abroad.

$$Ast_{5-1-12}^*(Ast_{13} + Ast_{14}) + Ast_{5-2-12}^*(Ast_{13} + Ast_{14}) + Ast_{13}^*(-Ast_{5-2-12} - Ast_{5-1-2}) + Ast_{14}^*(-Ast_{5-2-12} - Ast_{5-1-2}) = 0,$$
(18)

$$Ast_{5-1-12}^* + Ast_{5-2-12}^* - Ast_{13}^* - Ast_{14}^* + 0.0001 \le 0, (19)$$

$$\left(Ast_{5-2-13}^{*}\right)\left(Ast_{5-2}\right) + \left(Ast_{5-2}^{*}\right)\left(-0.1Ast_{5-2-13}\right) \ge 0.$$
 (20)

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Then, the minimum and maximum of ratios of non-current non-governmental facilities to non-governmental facilities are examined ($r_7 = 0.07$, $r_8 = 0.1$):

$$Ast_{5-2}^* + (Ast_5^*)(-r_8) \le 0, (21)$$

$$Ast_{5-2}^* + (Ast_5^*)(-r_7) \ge 0. (22)$$

In this article, the growth rate of corporate bonds is predicted to be -10% and the growth rate of investment in stocks and other securities is also predicted to be 10% ($r_9 = -0.1$, $r_{10} = 0.1$):

$$Ast6 - 3 * = Ast6 - 3(1 + r9), \tag{23}$$

$$Ast_6^* \le (Ast_6)(1+r_{10}),$$
 (24)

$$Ast_6^* \ge (Ast_6)(1 - r_{10}).$$
 (25)

Restrictions on claims from subsidiaries and affiliates and the minimum and maximum of this parameter as well as other accounts receivable are stated as follows (r_{11} = 0.1, r_{12} = 0.8, r_{13} = 0.1):

$$Ast_7^* \ge Ast_7(1 - r_{11}),$$
 (26)

$$Ast_7^* \le Ast_7(1+r_{12}),$$
 (27)

$$Ast_8^* = Ast_8(1 + r_{13}). (28)$$

In the section of tangible and intangible assets and other assets, the relevant constraints are provided as follows, and the coefficients of increase of tangible and intangible assets are 0.3 and the coefficient increase of parameter of other assets is one ($r_{14} = 0.3$, $r_{15} = 0.3$, $r_{16} = 1$):

$$Ast_x^* \ge Ast_x.x = 9.10.13.14.15.16,$$
 (29)

$$Ast_9^* \le Ast_9(1+r_{14}),$$
 (30)

$$Ast_{10}^* \le Ast_{10}(1+r_{15}),$$
 (31)

$$Ast_{12}^* \le Ast_{12}(1+r_{16}).$$
 (32)

We now turn to the non-structural liabilities constraints and assess the bank's liabilities constraints to banks and other credit institutions, as well as the growth of deposits, which is assumed to be negligible and then a parameter called OldDeposit is defined ($r_{17} = 0$).

In the following equations, $Lib_{1-2-x}(x = 1.....5)$ is the types of customer deposits in the bank.

$$0.8Lib_{1-1} \le Lib_{1-1}^* \le 1.2Lib_{1-1}, \tag{33}$$

$$OldDeposit = Lib_{1-2-1} + Lib_{1-2-2} + Lib_{1-2-3} + Lib_{1-2-4} + Lib_{1-2-5} + Lib_{2}.$$
 (34)

$$Lib_{1-2-1}^* + Lib_{1-2-2}^* + Lib_{1-2-3}^* + Lib_{1-2-4}^* + Lib_{1-2-5}^* + Lib_{2}^* \ge (OldDeposit)(1 + r_{17})$$

$$Lib_{1-x}^* (-OldDeposit) + Lib_{1-2-1}^* Lib_{1-x} + Lib_{1-2-2}^* Lib_{1-x} + Lib_{1-2-3}^* Lib_{1-x}$$

$$+ Lib_{1-2-4}^* Lib_{1-x} + Lib_{1-2-5}^* Lib_{1-x} + Lib_{2}^* Lib_{1-x} = 0.x = 3.....6.$$
(35)

It should be noted that the parameters under the Investment Deposits Equity parameter change on average and are almost homogeneous.

$$Lib_{2-x}^* Lib_2 - 0.95 Lib_2^* Lib_{2-x} \ge 0.x = 1....6.$$
 (36)

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Equity growth is assumed to be zero, and the constraints on the parameters of Capital, Legal Reserve, Other Reserves and Retained Earnings are as follows ($r_{18} = 0$):

$$Lib_{3-1}^* = Lib_{3-1}(1+r_{18}),$$
 (37)

$$Lib_{3-2} = Lib_{3-2}^* - 0.2Rev_1^* + 0.2Cst_1^*, (38)$$

$$Lib_{3-3}^* = Lib_{3-3}, (39)$$

$$Lib_{3-5} = Lib_{3-5}^* - Rev_1^* + Cst_1^*. (40)$$

Then, the ratio of other deposits to Letters of credits and Guarantees should be examined. Lib_{1-2-3} is Cash Deposit of Guarantees and Lib_{1-2-4} is Prepayment of Letters of Credit. Also, it should be remembered here that Lib_{1-2-1} , Lib_{1-2-2} and Lib_{1-2-5} are Current Accounts, Savings Accounts and Other, respectively.

$$Lib_{1-2-3}^*(-Ast_{14}) + Ast_{14}^*Lib_{1-2-3} = 0, (41)$$

$$Lib_{1-2-4}^*(-Ast_{13}) + Ast_{13}^*Lib_{1-2-4} = 0. (42)$$

Restrictions on bank income from new facilities based on a distribution factor of 7.2% are as follows ($r_{19} = 0.072$). $Rev_{1-1-1-x}(x = 1.....13)$ are thirteen types of granted facility income available in the Profit and Loss Statement of the bank. $Ast_{5-1-x}(x = 1.....13)$ are thirteen types of non-governmental sector current facilities are on the bank's balance sheet. $Rev_{1-1-2-1}$ is the legal deposit bonus (depositors and bank share) and $Rev_{1-1-2-2}$ is the interest on time deposits. $Rev_{1-1-3-x}(x = 1.....16)$ are sixteen types of fee income.

$$Rev_{1-1-1-x}^* + Ast_{5-1-x}^*(-r_{19}) = Rev_{1-1-1-x} - Ast_{5-1-x}(r_{19}).x = 1.....8,$$
 (43)

$$Rev_{1-1-1-9}^* Ast_{5-1-8} - Rev_{1-1-1-8} Ast_{5-1-8}^* = 0,$$
 (44)

$$-Rev_{1-1-1-10}^* + 0.2 Ast_3^* = 0, (45)$$

$$-Rev_{1-1-1-1}^* - Rev_{1-1-1-12}^* + 0.0775 Ast_{5-2-12}^* = 0, (46)$$

$$0.9 Rev_{1-1-1-13} \le Rev_{1-1-1-13}^* \le 0.11 Rev_{1-1-1-13}, \tag{47}$$

$$-Rev_{1-1-2-1}^* + \left(\frac{Ast_{11}^*Rev_{1-1-2-1}}{Ast_{11}}\right) = 0, \ -Rev_{1-1-2-2}^* + \left(\frac{Ast_2^*Rev_{1-1-2-2}}{Ast_2}\right) = 0, \tag{48}$$

$$-Rev_{1-1-3-1}^* + \left(\frac{Ast_{5-1-9}^*Rev_{1-1-3-1}}{Ast_{5-1-9}}\right) = 0, -Rev_{1-1-3-2}^* + \left(\frac{Ast_{13}^*Rev_{1-1-3-2}}{Ast_{13}}\right) = 0, \tag{49}$$

$$-Rev_{1-1-3-3}^* + \left(\frac{Ast_{14}^*Rev_{1-1-3-3}}{Ast_{14}}\right) = 0, -Rev_{1-1-3-4}^* + \left(\frac{Lib_{1-2-1}^*Rev_{1-1-3-4}}{Lib_{1-2-1}}\right) = 0$$
 (50)

$$-Rev_{1-1-3-5}^* + \left(\frac{Lib_{1-2}^*Rev_{1-1-3-5}}{Lib_{1-2}}\right) = 0, -Rev_{1-1-3-6}^* + \left(\frac{Lib_{1-2}^*Rev_{1-1-3-6}}{Lib_{1-2}}\right) = 0,$$
 (51)

$$-Rev_{1-1-3-7}^* + \left(\frac{Lib_{1-2}^*Rev_{1-1-3-7}}{Lib_{1-2}}\right) = 0, -Rev_{1-1-3-8}^* + \left(\frac{Ast_{5-1}^*Rev_{1-1-3-8}}{Ast_{5-1}}\right) = 0,$$
 (52)

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$$-Rev_{1-1-3-9}^* + \left(\frac{Ast_{5-1}^*Rev_{1-1-3-9}}{Ast_{5-1}}\right) = 0, -Rev_{1-1-3-10}^* + \left(\frac{Lib_{1-2}^*Rev_{1-1-3-10}}{Lib_{1-2}}\right) = 0,$$
 (53)

$$-Rev_{1-1-3-11}^* + \left(\frac{Ast_5^*Rev_{1-1-3-11}}{Ast_5}\right) = 0, -Rev_{1-1-3-12}^* + \left(\frac{Lib_{1-2-1}^*Rev_{1-1-3-12}}{Lib_{1-2-1}}\right) = 0, \tag{54}$$

$$-Rev_{1-1-3-13}^* + \left(\frac{Ast_{5-1}^*Rev_{1-1-3-13}}{Ast_{5-1}}\right) = 0, -Rev_{1-1-3-14}^* + \left(\frac{Ast_{6-3}^*Rev_{1-1-3-14}}{Ast_{6-3}}\right) = 0, \quad (55)$$

$$-Rev_{1-1-3-15}^* + \left(\frac{Lib_{1-2}^*Rev_{1-1-3-15}}{Lib_{1-2}}\right) = 0, -Rev_{1-1-3-16}^* + \left(\frac{Lib_{1-2}^*Rev_{1-1-3-16}}{Lib_{1-2}}\right) = 0.$$
 (56)

Then, constraints related to net investment profit and loss, other operating income and other net income and expenses in the bank's financial statements are evaluated.

$$-Rev_{1-1-4}^* + Ast_{6-3}^* \left(\frac{Rev_{1-1-4}}{Ast_{6-3} + Ast_6} \right) + Ast_6^* \left(\frac{Rev_{1-1-4}}{Ast_{6-3} + Ast_6} \right) = 0, \quad (57)$$

$$Rev_{1-2-1}^*(Ast_9 + Ast_{10}) + Ast_9^*(-Rev_{1-2-1}) + Ast_{10}^*(-Rev_{1-2-1}) = 0.$$
 (58)

One of the costs of a bank is the cost of interest on deposits. It should be noted that part of the interest on deposits is a continuation of last year's deposits and the part related to new deposits which are given the following average interest rates according to the deposit distribution structure ($r_{20} = 0.05$, $r_{21} = 0.075$, $r_{22} = 0.05$).

$$Cst_{1}^{*} - 1 - 1 - r_{21}Lib_{2-1}^{*} - r_{20}Lib_{2-2}^{*} - r_{20}Lib_{2-3}^{*} - r_{22}Lib_{2-4}^{*} - r_{22}Lib_{2-5}^{*} - r_{22}Lib_{2-6}^{*}$$

$$= Cst_{1-1-1} - r_{21}Lib_{2-1} - r_{20}Lib_{2-2} + Lib_{2-3} - r_{22}Lib_{2-4}.$$
(59)

In this section, the constraints related to other parameters related to income and expenses in the bank's financial statements are evaluated. $Cst_{1-2-5-1}$ is the cost of this year's tax.

$$-Cst_{1-1-2}^{*} + Lib_{17}^{*} \left(\frac{Cst_{1-1-2}}{Lib_{17}}\right) = 0, \quad -Cst_{1-2-1}^{*} + Lib_{17}^{*} \left(\frac{Cst_{1-2-1}}{Lib_{17}}\right) = 0$$

$$-Cst_{1-2-2}^{*} + Ast_{5-2}^{*} \left(\frac{Cst_{1-2-2}}{Ast_{5-2}}\right) = 0, \quad -Cst_{1-2-3}^{*} + Lib_{1-1}^{*} \left(\frac{Cst_{1-2-3}}{Lib_{1-1}}\right) = 0$$

$$Cst_{1-2-4}^{*} \left(-(Ast_{9} + Ast_{10})\right) + Ast_{9}^{*} Cst_{1-2-4} + Ast_{10}^{*} Cst_{1-2-4}$$

$$-Cst_{1-2-5-1}^{*} + \left(Rev_{1}^{*} \frac{Cst_{1-2-5-1}}{Rev_{1}}\right) = 0$$

$$(60)$$

$$Cst_{1-2-5-2}^* = Cst_{1-2-5}, (61)$$

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$$Rev_{1-1-x}^{*}Rev_{1} + Rev_{1}^{*}(0.8Rev_{1-1-x}) \ge 0.x = 1....6,$$
 (62)

$$Rev_{1-1-3}^* \ge 1.1 Rev_{1-1-3}.$$
 (63)

In 2017, the legal deposit rate notified to Bank Mellat was equal to 12.2 and 10 percent. A maximum of 2% of the deposit can be kept in cash instead of the legal deposit ($r_{23} = 0.122$, $r_{24} = 0.1$). The amount of cash in the bank fund is displayed with F and the amount of legal deposit with the central bank is displayed with D. $Lib_{1-2-x}(x=1,\ldots,5)$ are five types of customer deposits.

$$(1.06)(r_{23})Lib_{1-2-1}^* + (1.06)(r_{24})Lib_{1-2-2}^* + (1.06)(r_{23})Lib_{1-2-3}^*$$

$$+ (1.06)(r_{23})Lib_{1-2-4}^* + (1.06)(r_{23})Lib_{1-2-5}^* + (1.06)(r_{23})Lib_{2-1}^*$$

$$+ (1.06)(r_{23})Lib_{2-2}^* + (1.06)(r_{23})Lib_{2-3}^* + (1.06)(r_{23})Lib_{2-4}^*$$

$$+ (1.06)(r_{23})Lib_{2-5}^* + (1.06)(r_{23})Lib_{2-6}^* - Ast_{11}^* = F + D,$$

$$(64)$$

$$-0.9 Ast_{17}^* + Ast_2^* + Ast_3^* + Ast_4^* + Ast_5^* + Ast_6^* + Ast_7^* + Ast_8^* \le 0,$$
 (65)

$$-0.8 Ast_{17}^* + Ast_2^* + Ast_3^* + Ast_4^* + Ast_5^* + Ast_6^* + Ast_7^* + Ast_8^* \ge 0.$$
 (66)

The objective function is as follows:

$$Min\left(\sum_{k=1}^{n} \left| \frac{Ast_k^* - Ast_k}{Ast_k} \right| + \sum_{j=1}^{m} \left| \frac{Lib_j^* - Lib_j}{Lib_j} \right| \right). \tag{67}$$

The various variables in the balance sheet and profit and loss statement of the bank as well as the model are fully defined in Table 1.

Table 1. Description of Model Variables.

Variable	Description					
		Assets				
Ast_1		Cash				
Ast_2	Receivables from banks and other credit institutions	Ast_{2-1} Ast_{2-2} Ast_{2-3} Ast_{2-4} Ast_{2-5}	Central Bank—Past Currency Liabilities Central Bank—Overnight withdrawals from accounts Central Bank of Shaparak Transactions Central Bank—Government Deposits Facilities Granted to Other Banks and Credit Institutions (Overnight Interbank)			
		Ast_{2-6}	Other			
Ast_3		Claims from the Government				
Ast_4	Facilities	Granted and Cla	ims from the Public Sector			
Ast_5	Facilities Granted and Claims from the Non-Governmental Sector	$Ast_{5-1} \\ Ast_{5-2}$	Current Facilities of the Non-Governmental Sector Non-Current Facilities of the Non-Governmental Sector			
Ast_6	Investing in stocks and other securities	Ast_{6-1} Ast_{6-2} Ast_{6-3}	Stocks Other Non-Government Securities Other Government Securities			
Ast ₇	Claims on Subsidiaries and Affiliates					
Ast_8	Other Accounts Receivable					

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 Table 1. Cont.

Variable	Description					
		Assets				
Ast ₉		Tangible Fix	red Assets			
Ast_{10}	Intangible Assets					
Ast_{11}	Legal Deposit	$Ast_{11-1} \ Ast_{11-2} \ Ast_{11-3}$	Savings Deposits Free Zone Branch Deposits Other			
Ast_{12}	Other Assets	$Ast_{12-1} \\ Ast_{12-2} \\ Ast_{12-3} \\ Ast_{12-4}$	Customer Liability for Long-term Letters of Credi Proprietary Documents Assets Ready for Sale Other			
Ast_{13}		Customer Liabilities f	for Letters of Credit			
Ast_{14}		Customer Obligatio	ns for Guarantees			
Ast_{15}		Other Obl				
Ast_{16}		Managed Funds at				
Ast_{17}		Total A				
1201/		Liabilities				
Lib_1	Total Liability Before Equity	$Lib_{1-1} \ Lib_{1-2} \ Lib_{1-3} \ Lib_{1-4} \ Lib_{1-5} \ Lib_{1-6} \ Lib_{1-7}$	Liability to Banks and Other Credit Institutions Customer Deposits Dividends Payout Performance Tax Reserve Reserves and Other Liabilities Allocation of Rial Sector Resources to Foreign Exchange Sector Provision for Staff Termination Benefits			
Lib ₂	Investment Deposits Equity	$Lib_{2-1} \ Lib_{2-2} \ Lib_{2-3} \ Lib_{2-4} \ Lib_{2-5} \ Lib_{2-6}$	Long-term Investment Deposits Short-term Investment Deposits Special Short-term Investment Deposits Investment Deposits Received from Banks Interest Payable on Short-term Deposits Interest Payable on Long-term Deposits			
Lib ₃	Equity	$Lib_{3-1} \ Lib_{3-2} \ Lib_{3-3} \ Lib_{3-4} \ Lib_{3-5} \ Lib_{3-6}$	Capital Legal Reserve Other Reserves Foreign Exchange Differences Retained Earnings Asset Revaluation Surplus			
Lib ₁₃		Bank Liabilities for	Letters of Credit			
Lib_{14}		Bank Obligations	for Guarantees			
Lib ₁₅		Other Obl				
Lib ₁₆		Managed Funds at				
Lib ₁₇	Total Liabilities					
1/		Revenues				
Rev_{1-1}	Operating Income	$Rev_{1-1-1} \ Rev_{1-1-2} \ Rev_{1-1-3} \ Rev_{1-1-4} \ Rev_{1-1-5} \ Rev_{1-1-6}$	Granted Facilities Income Deposit Income and Bonds Fee Income Net Investment Profit and Loss Net Foreign Exchange Profit and Loss Other Operating Income			
Rev_{1-2}	Non-Operating Income	Rev_{1-2-1}	Net Other Income and Expenses			

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Table 1. Cont.

Variable	Description			
		Costs		
Cst_{1-1}	Operational Cost	$Cst_{1-1-1} \\ Cst_{1-1-2}$	Deposit Interest Cost Fee Cost	
Cst_{1-2}	Non-Operating Costs	$Cst_{1-2-1} \ Cst_{1-2-2} \ Cst_{1-2-3} \ Cst_{1-2-4} \ Cst_{1-2-5} \ Cst_{1-2-6}$	Administrative and General Expenses Cost of Bad Debts Financial Expenses Depreciation Cost Tax Cost Other Costs	

4. Results

In this section, the results obtained from the proposed model are displayed based on the financial information of one of the largest Iranian banks.

The proposed and optimized amounts of each of the bank's balance sheet items, revenues and expenses are shown in Tables 2 and 3 based on the new model presented in this article and using the information of a famous bank in Iran in 2019. Table 2 shows the proposed balance sheet of the bank that model is presented. Based on this balance sheet, bank managers can pay more attention to the management of bank assets and liabilities, which by its nature is the management of the balance sheet.

Table 2. Actual and Optimal Values of Balance Sheet Items in Dollars.

	Assets			Liabilities	
Asset Variables	Optimal Values	Actual Values	Liability Variables	Optimal Values	Actual Values
Ast_1	214,521,652	310,229,372	Lib_1	9,825,662,400	3,624,873,156
Ast_2	1,277,251,120	474,827,460	Lib_{1-1}	1,523,196,200	130,097,592
Ast_{2-1}	159,046,252	0	Lib_{1-1-1}	0	0
Ast_{2-2}	11,163,266	10,148,424	Lib_{1-1-2}	0	16,723,760
Ast_{2-3}	281,701	256,092	Lib_{1-1-3}	0	78,127,308
Ast_{2-4}	17,065,673	15,514,248	Lib_{1-1-4}	0	25,412,684
Ast_{2-5}	671,967,000	447,978,000	Lib_{1-1-5}	0	9,767,496
Ast_{2-6}	417,727,240	930,696	Lib_{1-1-6}	1,523,196,200	66,344
Ast_3	638,046,720	638,046,712	Lib_{1-2}	4,826,737,200	3,146,697,924
Ast_4	6,403,239,200	23,338,672	Lib_{1-2-1}	0	1,918,006,484
Ast_5	5,544,497,200	4,531,427,724	Lib_{1-2-2}	1,097,930,800	588,519,528
Ast_{5-1}	5,156,382,400	4,334,705,352	Lib_{1-2-3}	98,715,844	61,378,536
Ast_{5-2}	388,114,816	196,722,372	Lib_{1-2-4}	244,550,368	9,069,424
Ast_6	317,767,000	256,771,028	Lib_{1-2-5}	3,385,540,160	569,723,952
Ast_{6-1}	0	88,013,264	Lib_{1-3}	146,056	146,056
Ast_{6-2}	172,728,608	7,603,992	Lib_{1-4}	93,996,416	93,996,416
Ast_{6-3}	145,038,396	161,153,772	Lib_{1-5}	3,381,586,360	107,123,372
Ast_7	203,070,304	111,670,728	Lib_{1-6}	0	-1,995,141,349
Ast_8	194,836,108	165,794,888	Lib_{1-7}	0	146,811,796
Ast_9	982,591,440	755,839,560	Lib_2	5,899,345,600	5,310,967,584
Ast_{10}	87,738,076	67,490,828	Lib_{2-1}	2,711,129,440	2,490,780,188
Ast_{11}	1,108,973,240	1,008,837,668	Lib_{2-2}	2,702,175,480	2,784,167,288
Ast_{11-1}	0	996,610,412	Lib_{2-3}	150	-51,368,756
Ast_{11-2}	0	12,227,256	Lib_{2-4}	119,109,664	21,281,912
Ast_{11-3}	1,108,973,240	0	Lib_{2-5}	18,695,200	17,504,168
Ast_{12}	241,701,736	-12,596,952	Lib_{2-6}	348,235,468	48,602,784
Ast_{12-1}	0	33,468,860	Lib_3	1,489,226,400	1,390,978,289
Ast_{12-2}	0	51,881,480	Lib_{3-1}	200,000,000	200,000,000
Ast_{12-3}	0	12,538,896	Lib_{3-2}	277,311,904	157,311,904
Ast_{12-4}	241,701,736	$-110,\!486,\!188$	Lib_{3-3}	320,351,400	320,351,400

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Table 2. Cont.

Assets			Liabilities		
Asset Variables	Optimal Values	Actual Values	Liability Variables	Optimal Values	Actual Values
Ast_{13}	1,393,507,880	208,838,912	Lib_{3-4}	0	0
Ast_{14}	1,452,764,480	756,409,460	Lib_{3-5}	691,563,120	85,143,533
Ast_{15}	1,485,988,280	759,366,252	Lib_{3-6}	0	628,171,452
Ast_{16}	1,318,836,720	8,492,500	Lib_{13}	1,393,507,880	208,838,912
Ast_{17}	17,214,234,000	8,331,677,688	Lib_{14}	1,452,764,480	756,409,460
			Lib_{15}	1,485,988,280	759,366,252
			Lib ₁₆	1,318,836,720	8,492,500

Table 3. Actual and optimal values of revenue and cost items in dollars.

	Revenues			Costs	
Revenue Variables	Optimal Values	Actual Values	Cost Variables	Optimal Values	Actual Values
Rev_1	2,076,628,360	1,616,378,376	Cst_1	1,476,628,360	1,515,936,972
Rev_{1-1}	1,967,804,360	1,532,667,608	Cst_{1-1}	668,823,120	675,106,200
Rev_{1-1-1}	792,082,520	805,051,156	Cst_{1-1-1}	615,069,600	619,853,088
Rev_{1-1-2}	1,002,935,520	197,988,452	Cst_{1-1-2}	53,753,496	55,253,112
Rev_{1-1-3}	83,625,080	76,022,800	Cst_{1-2}	807,805,240	840,830,772
Rev_{1-1-4}	13,744,786	13,365,456	Cst_{1-2-1}	342,885,944	352,451,792
Rev_{1-1-5}	62,215,456	427,038,752	Cst_{1-2-2}	212,653,944	279,727,912
Rev_{1-1-6}	13,200,992	13,200,992	Cst_{1-2-3}	18,201,837	22,752,296
Rev_{1-2}	108,824,000	83,710,768	Cst_{1-2-4}	14,261,920	10,970,708
Rev_{1-2-1}	108,824,000	83,710,768	Cst_{1-2-5}	219,801,596	174,928,064
1 2 1			Cst_{1-2-6}	0	0

Given that one of the most important goals of assets and liabilities management is liquidity risk management, the optimal amount of Cash shown in Table 2 is the proposed amount of USD 214,521,652, which is about 30% less than the actual amount in the year under review. Reducing the amount of liquidity to this amount, in addition to showing the impact of the objectives of controlling the risks in the bank on the proposed model, also reminds the realities of the money market. Also, due to the special economic conditions of Iran, bank deposits have undergone structural changes and the status of deposits has become weaker. The Receivables from Banks and Other Credit Institutions parameter on the proposed balance sheet is USD 1,277,251,120 and has grown about three times the actual amount. The Claims from the Government parameter has not changed much in the proposed balance sheet and the actual value of this parameter is very close to the optimal value. The Facilities Granted and Claims from the Public Sector parameter and the Granted and Claims Facilities from the Non-Governmental Sector parameter in the proposed balance sheet are USD 6,403,239,200 and USD 5,544,497,200, respectively. Interestingly, the Facilities Granted and Claims from the Public Sector parameter has grown about thirty times its actual value. Another parameter that is growing much larger than its real value is the Non-Government Securities parameter, which is USD 172,728,608 in Table 2, which is about 23 times, and suggests that the bank's assets in the area of investment in non-government securities should increase 23 times next year.

In terms of liabilities, it should be noted that the Total Liability before Equity parameter should be about tripled and the Bank Liabilities for Letters of Credit parameter should be seven times the current amount to experience the best for managing the bank's assets and Liabilities. Another point that can be seen in Table 2 is that the parameters of Allocation of Rial Sector Resources to the Foreign Exchange Sector, Provision for Staff Termination Benefits, Foreign Exchange Differences and Asset Revaluation Surplus are optimally suggested to have zero values.

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Table 3 shows the actual and optimal values of the bank's revenues and costs, and the value of the total revenue parameter of the bank is proposed at USD 2,076,628,360, which is a growth of about 25% compared to the actual amount. The Operating Income and Non-Operating Income parameters are at USD 1,967,804,360 and USD 108,824,000 in Table 3, respectively, and both show growth of around 30%. It is noteworthy that important parameters such as Operational Cost and Non-Operating Costs in the cost section have not changed much and have remained almost constant.

5. Conclusions and Future Research Directions

Today, banks are looking for a way to invest their assets over time to gain a satisfactory level of profit by taking into account uncertainties, various bottlenecks and committed liabilities. Assets and liabilities management is the area that answers the above question. Assets and liabilities management includes a set of technical tools and methods that control risks according to value to shareholders and being in control. One of the most important tasks of banks is to manage assets and liabilities with the aim of maximizing returns and minimizing risk. Therefore, it is very important for banks to pay attention to domestic and foreign laws and regulations. One of the challenges facing banks in assets and liabilities management is managing liquidity and default risks. In this regard, this article seeks a new model that best defines and optimizes balance sheet items with respect to the subject of assets and liabilities management. A large number of constraints have been used to achieve the desired model, and according to these constraints and the new objective function defined in the model, the proposed balance sheet and the proposed profit and loss statement have been obtained where specific values are suggested for all variables. Today, the growing trend of change in world banking from the expansion of balance sheet items to a focus on return on capital and risk control has made the knowledge of assets and liabilities management a necessity for bank managers to respond to profit results. The importance of assets and liabilities management can be summarized in the fact that banks are financial institutions that must create a balance between resources and expenses or costs and income from their activities that can continue their financial life in the market while maintaining Asset value, increase efficiency and effectiveness of revenues and expenses. In this article, we have tried to evaluate the important items of the balance sheet and profit and loss statement of a reputable bank in Iran by presenting a new model, and we have sought to determine the optimal amounts of important items of the balance sheet and profit and loss statement of the bank and to achieve this goal, we have used Python software to analyze the model and financial information of the bank. In the results section, the actual values and the optimal values of the items are shown in Tables 2 and 3. According to the results, it is obvious that the performance of the bank managers is significantly different from the optimal balance sheet values, especially in the management of bank assets. The obtained results show that after optimizing the elements of the balance sheet, the amounts of cash, Receivables from Banks and Other Credit Institutions, Facilities Granted and Claims from the Public Sector, Granted and Claims Facilities from the Non-Governmental Sector and Non-Government Securities. It reached USD 214,521,652, USD 1,277,251,120, USD 6,403,239,200, USD 5,544,497,200 and USD 172,728,608, respectively. It is worth mentioning that the amounts of cash and Receivables from Banks and Other Credit Institutions have decreased by 30% and increased by 200% compared to the actual amount. Another thing that should be mentioned is that The Claims from the Government parameter have not changed significantly compared to the actual value. Also, the values of parameters of Allocation of Rial Sector Resources to Foreign Exchange Sector, Provision for Staff Termination Benefits, Foreign Exchange Differences and Asset Revaluation Surplus have reached zero after optimization. The results obtained from the profit and loss statement, Nissan says that after optimization, the amounts of total income, Operating Income and Non-Operating Income have reached USD 2,076,628,360, USD 1,967,804,360 and USD 108,824,000, respectively, which compared to the actual values, all three. The parameter has grown by about thirty percent. Although the coefficients used in the proposed model

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are based on the monetary policies of one of the largest Iranian banks. But, one of the limitations of optimizing the parameters of this article for asset and liability management is that the range of changes of some parameters are considered less and more reasonable. For future research directions, it is suggested that balance sheet items be optimized for each of the banks in the banking system or the entire banking system. At the end, for future research, the effects of data ambiguity and uncertainty in asset-liability management can be considered [134–142]. For this purpose, the popular and applicable uncertain programming approaches such as fuzzy optimization [143–164] and robust optimization [165–181] can be used to deal with uncertainty of data.

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