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Correction

Correction: Tolmachev et al. Algorithmic Aspects of Simulation of Magnetic Field Generation by Thermal Convection in a Plane Layer of Fluid. *Mathematics* 2023, *11*, 808

Daniil Tolmachev [1], Roman Chertovskih [2,*] and Vladislav Zheligovsky [1]

[1] Institute of Earthquake Prediction Theory and Mathematical Geophysics, Russian Academy of Sciences, 84/32 Profsoyuznaya St., 117997 Moscow, Russia
[2] Faculty of Engineering, University of Porto, Rua Dr. Roberto Frias, s/n, 4200-465 Porto, Portugal
* Correspondence: roman@fe.up.pt

There was an error in the original publication [1], and we make a number of corrections.

(1) The wrong formula (20e) on p. 16 is as follows:

$$\Gamma'_k = 8k(k+1)(k+2)\pi/\Gamma^2. \tag{20e}$$

The correct formula is

$$\Gamma'_k = 4k(k+1)(k+2)/\Gamma^2. \tag{20e}$$

(2) Section 4. The wrong equations on p. 21 are as follows:

$$T^{\mathbf{b}}\Big|_{x_3=\pm1} = 0, \tag{27a}$$

$$\nabla P^{\mathbf{b}}\Big|_{x_3=1} = \nabla h|_{x_3=1'} \tag{27b}$$

$$\frac{\partial^2 P^{\mathbf{b}}}{\partial x_3^2}\Big|_{x_3=-1} = P^{\mathbf{b}}\Big|_{x_3=-1} = 0, \tag{27c}$$

$$\mathbf{M}^{\mathbf{b}}\Big|_{x_3=\pm1} = 0. \tag{27d}$$

The corrected equations are:

$$T^{\mathbf{b}}\Big|_{x_3=1} = \frac{\partial T^{\mathbf{b}}}{\partial x_3}\Big|_{x_3=-1} = 0, \tag{27a}$$

$$\nabla P^{\mathbf{b}}\Big|_{x_3=1} = \nabla h|_{x_3=1'} \tag{27b}$$

$$\frac{\partial^2 P^{\mathbf{b}}}{\partial x_3^2}\Big|_{x_3=-1} = P^{\mathbf{b}}\Big|_{x_3=-1} = 0, \tag{27c}$$

$$\mathbf{M}^{\mathbf{b}}\Big|_{x_3=1} = \frac{\partial \mathbf{M}^{\mathbf{b}}}{\partial x_3}\Big|_{x_3=-1} = 0. \tag{27d}$$

(3) Section 4. The first paragraph on p. 22 is as follows:

and the mean-field component of the magnetic field from the equation derived by averaging over horizontal variables of the horizontal component of (1b). Since the boundary

check for updates

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conditions for the toroidal and mean-field components of the magnetic field are the same as for the respective flow components, we use analogous expansions

$$T^{\mathbf{b}}(\mathbf{x},t)=\sum_{|n_1|\le N,\ |n_2|\le N,\ 0\le n\le N-3} t^{\mathbf{b}}_{\mathbf{n}}(t)\mathrm{e}^{\mathrm{i}(\alpha_1 n_1 x_1+\alpha_2 n_2 x_2)}T^{\star}_n(x_3),$$

$$\mathbf{M}^{\mathbf{b}}(x_3,t)=\sum_{0\le n\le N}\mathbf{M}^{\mathbf{b}}_n(t)T^{\star}_n(x_3),$$

where $\mathbf{n}=(n_1,n_2,n)$, and equations for the coefficients in the expansions are obtainedsimilarly by orthogonal projection on the polynomials $T^{\star}_m(x_3)$(10).

The second sentence here is deleted, i.e., the corrected paragraph is as follows:
and the mean-field component of the magnetic field from the equation is derived by averaging over horizontal variables of the horizontal component of (1b).

(4) Section 4. The text in the middle of p. 22 is as follows:

$$\begin{aligned}T^{\star\star\star}_n(x_3)&=(4(n+1)^4-2(n+1)^2+1+2\Gamma(2(n+1)^2+1))\Pi_n\\&\quad-(4n^4-2n^2+1+2\Gamma(2n^2+1))\Pi_{n+1}\end{aligned}\tag{30}$$

for $n\ge 1$. Thus, $P^{\mathbf{b}}_{n_1,\,n_2}(x_3,t)$ can be expanded as

The revised text is:

$$\begin{aligned}T^{\star\star\star}_n(x_3)&=((4(n+1)^4-2(n+1)^2+1+\Gamma(4(n+1)^2+2))\Pi_n\\&\quad-\left(4n^4-2n^2+1+\Gamma(4n^2+2)\right)\Pi_{n+1})/(2n+1)\\&=\kappa_{n+1}T_{n-1}+(\kappa_n+\lambda_n)T_n-(\kappa_{n+1}-\lambda_n)T_{n+1}-\kappa_nT_{n+2}\end{aligned}\tag{30}$$

for $n\ge 1$, where

$$\kappa_n=n(4n^4-2n^2+1+\Gamma(4n^2+2)),\qquad \lambda_n=16n(n+1)(n(n+1)+1).$$

Thus, $P^{\mathbf{b}}_{n_1,\,n_2}(x_3,t)$ can be expanded as

(5) The following text is added at the end of Section 4:

The boundary conditions (27a) and (27d) for the toroidal potential and mean-field components of the magnetic field are satisfied by the polynomials

$$T^{\star\star\star\star}_n(x_3)=\mu_nT_{n-1}-4nT_n-\mu_{n-1}T_{n+1},\text{ where }\mu_n=2n(n+1)+1,\tag{32}$$

and hence we use the expansions

$$T^{\mathbf{b}}(\mathbf{x},t)=\sum_{|n_1|\le N,\ |n_2|\le N,\ 1\le n\le N-2} t^{\mathbf{b}}_{\mathbf{n}}(t)\mathrm{e}^{\mathrm{i}(\alpha_1 n_1 x_1+\alpha_2 n_2 x_2)}T^{\star\star\star\star}_n(x_3),$$

$$\mathbf{M}^{\mathbf{b}}(x_3,t)=\sum_{0\le n\le N}\mathbf{M}^{\mathbf{b}}_n(t)T^{\star\star\star\star}_n(x_3).$$

The coefficients in the expansion of a function in this basis can be determined by solving a linear system of equations with a pentadiagonal Gram matrix G, whose non-zero entries are as follows:

$$G_{k,k-2}=-\mu_{k-3}\mu_k,\quad G_{k,k-1}=-4(k-1)\mu_k+4k\mu_{k-2},\quad G_{k,k}=\mu_k^2\left(1+\delta_1^k\right)+16k^2+\mu_{k-1}^2,$$

$$G_{k,k+1}=-4k\mu_{k+1}+4(k+1)\mu_{k-1},\quad G_{k,k+2}=-\mu_{k-1}\mu_{k+2}.$$

Apparently, the system has no special properties enabling us to develop specialised methods for solving it numerically; it can be solved by the shuttle method (see Algorithm 1 in Section 3.2).

(6) The erroneous second paragraph of Section 5 Conclusions is:

The Galerkin method is applied for computation of the toroidal and poloidal components of physical vector fields and their mean components. They are expanded in the basic functions that are products of linear combinations ($T_n^\star(10)$, $T_n^{\star\star}(18)$ and $T_n^{\star\star\star}(30)$) of the Chebyshev polynomials in the vertical coordinate and Fourier harmonics in the horizontal coordinates. The basic functions involving the polynomials $T_n^\star$ are used for the spatial discretisation of the unknown functions that take zero values on the horizontal boundaries $x_3 = \pm 1$, namely, in the problem under discussion, the toroidal potentials of the flow and magnetic field, their mean fields, and the deviation of temperature from the steady-state linear profile. The basic functions involving $T_n^{\star\star}$ are used for discretising the poloidal potential of the flow, which, for the no-slip boundary conditions, must vanish on the horizontal boundaries together with the first derivative in the vertical direction. Finally, the basic functions involving $T_n^{\star\star\star}$ are used for discretising the poloidal potential of the magnetic field in the presence of the dielectric over the fluid layer and an electric conductor below it; while the no-slip boundary conditions for a solenoidal field are often considered in different problems, the latter boundary conditions are rather special.

It is substituted by the following correct paragraph:

The Galerkin method is applied for computing the toroidal and poloidal components of physical vector fields and their mean components. We exploit the bases of functions that are products of linear combinations ($T_n^\star(10)$, $T_n^{\star\star}(18)$, $T_n^{\star\star\star}(30)$, and $T_n^{\star\star\star\star}(32)$) of Chebyshev polynomials in the vertical coordinate and Fourier harmonics in the horizontal coordinates. The basic functions involving the polynomials $T_n^\star$ are used for the spatial discretisation of the unknown functions that take zero values on the horizontal boundaries $x_3 = \pm 1$, namely, in the problem under discussion, the toroidal potential ofthe flow and its mean fields, and the deviation of temperature from the steady-state linear profile. The poloidal potential of the flow, which, for the no-slip boundary conditions, must vanish on the horizontal boundaries together with the first derivative in the vertical direction, is expanded in a series of functions involving the polynomial factors $T_n^{\star\star}$. The functions involving $T_n^{\star\star\star}$ are used for discretising the poloidal potential of the magnetic field in the presence of the dielectric over the fluid layer and an electric conductor below it; while the no-slip boundary conditions for a solenoidal field are often considered in different problems, the latter boundary conditions are rather special. Finally, the toroidal potential of the magnetic field, which, for the boundary conditions considered here, is zero on the upper fluid boundary, and whose first derivative in x_3 vanishes on the lower boundary, is expanded in the series of the basic functions involving the polynomials $T_n^{\star\star\star\star}$.

(7) The second sentence of the third paragraph of the Section 5 Conclusions is:

Specialised algorithms for determining the coefficients in expansion of an arbitrary function in a series of the basic functions are proposed and analysed.

It is now substituted with the following expanded sentence:

Specialised algorithms for determining the coefficients in the expansion of an arbitrary function in a series of the basic functions involving polynomials $T_n^\star$ and $T_n^{\star\star}$ are proposed and analysed.

The authors state that the scientific conclusions are unaffected. This correction was approved by the Academic Editor. The original publication has also been updated.

Reference

1. Tolmachev, D.; Chertovskih, R.; Zheligovsky, V. Algorithmic Aspects of Simulation of Magnetic Field Generation by Thermal Convection in a Plane Layer of Fluid. *Mathematics* **2023**, *11*, 808. [CrossRef]

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