

Editorial

Preface to “Applications of Partial Differential Equations in Engineering”

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Many problems in the broad spectrum of science require the solution of a partial differential equation. Obtaining an analytical solution to the problem is not always possible, either because we are not yet able to find it or because it cannot be expressed by means of elementary functions. Numerical methods are used to approach the solution of this type of problem.

In recent decades, due to advances in computer technology, it has been possible to improve existing methods and develop new ones.

The aim of this Special Issue is to extend the applicability of numerical methods for the resolution of PDEs arising in different engineering problems. The response from researchers in this field has been very important both from the point of view of the number of papers submitted and the quality of each of the papers (28) that were finally accepted, after a thorough peer-review process based on quality and novelty criteria.

The paper by Valbuena et al. [1] proposes the derivation of a separable entropy for a one-dimensional model of reduced blood flow, which will be used to address the symmetrizability of the model in its generality and to construct entropy-conservative flows. The work by Mahariq et al. [2] on acoustic wave scattering in a homogeneous media by an obstacle examines plane wave excitation, and the formation of acoustic jets is explored. The spectral element method (SEM) is employed for the approximate solution of scattered acoustic.

In the paper authored by Vidal et al. [3], the authors explore the extraction of some analytical solutions to the nonlinear perturbed sine-Gordon equation with long Josephson-junction properties. The model studied was formed to observe the long Josephson-junction properties separated by two superconductors.

The work by Zheng et al. [4] proposes a new unsteady numerical model to investigate the unsteady flow field generated by the magnetorheological (MR) fluid of a high-velocity unsteady laminar boundary layer flow in a narrow-long magnetorheological absorber (MRA) gap, which is the gap with regions activated and inactivated by the magnetic field.

The paper by Benito et al. [5] introduces a meshless method derived by considering the time variable as a spatial variable without the need to extend further conditions to the solution of linear and non-linear parabolic PDEs.

The paper authored by Wu [6] proposes one-stage multiple comparison procedures for several treatment groups compared with several control groups in terms of exponential mean lifetimes.

The work by Wu et al. [7] studies the improvement of the lifetime performance of products. In this research, the lifetime performance index is used to evaluate the lifetime performance of products following the Rayleigh distribution.

In the paper by García et al. [8], the existence and uniqueness of the discrete solutions of a porous medium equation with diffusion are demonstrated. The Cauchy problem



Citation: Ureña, F.; García, Á.; Vargas, A.M. Preface to “Applications of Partial Differential Equations in Engineering”. *Mathematics* **2023**, *11*, 199. <https://doi.org/10.3390/math11010199>

Received: 22 December 2022

Revised: 27 December 2022

Accepted: 27 December 2022

Published: 30 December 2022



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contains a fractional Laplacian and it is equivalent to the extension formulation in the sense of trace and harmonic extension operators.

The paper authored by Dostál et al. [9] presents a finite volume method formulated on a mixed Eulerian–Lagrangian mesh for highly advective 1D hyperbolic systems, with its application in a plug-flow heat exchanger modeling/simulation.

The work of Murillo-García et al. [10] presents an Adaptive Boundary Control for a Certain Class of Reaction–Advection–Diffusion System that allows us to predict the behaviour in the interaction of various physical parameters.

The paper by Aly et al. [11] studies the flow of a hybrid nanofluid (copper-titanium dioxide/water) over a nonlinear stretching surface with suction and radiation effects. The governing partial differential equations are then converted to nonlinear ordinary differential equations by using appropriate similarity transformations.

In the paper of [12] Wu et al. show, as a consequence of the constant change of technology, production techniques that are sophisticated and complicated are found to emerge; therefore, manufacturers are dedicated to improving the quality of products by increasing the shelf life in order to reach the quality standards demanded by consumers.

The work of Lai et al. [13] considers numerical solutions for fractional partial differential equations in Riesz space with a second-order time derivative. They propose a Galerkin finite element scheme for the temporal and spatial discretisations.

The work of Treanță et al. [14] uses scalar multiple integral cost functions and the notion of convexity associated with a multiple integral function controlled by a multitemporally controlled uncertain second-order Lagrangian; they develop a new mathematical framework for multidimensional scalar variational control problems.

The paper authored by Benito et al. [15] shows the application of a meshless numerical method called “Generalized Finite Difference Method” (GFDM) for solving a model for tumor growth with nutrient density, extracellular matrix and matrix-degrading enzymes.

In the paper [16] Hwang et al. considered a four-quadrant Riemann problem for a 2×2 hyperbolic system in which delta shock appears at the initial discontinuity without assuming that each jump of the initial data projects exactly one plane elementary wave.

The next paper [17] by Yuli D. Chashechkin studies the problem of generating periodic internal wave beams in a viscous, exponentially stratified fluid by a band oscillating along an inclined plane; it is considered using the methods of singular perturbation theory in the linear and weakly nonlinear approximations.

In the paper authored by Hwang et al. [18], a four-quadrant Riemann problem for a 2×2 system of hyperbolic conservation laws is considered in the case of delta shock appearing at the initial discontinuity.

In [19], Wu presents, under double censoring, the one-stage multiple comparison procedures with control in terms of exponential half-lives. The unbiased uniform minimum variance estimator for the median life is found.

In the work of Falcó et al., [20] a novel algorithm called Generalised Proper Decomposition (PGD) is widely used by the engineering community to compute the solution of high-dimensional problems.

The work of Saletė et al. [21] obtains a novel implementation for irregular clouds of nodes of the meshless method called the Generalized Finite Difference Method for solving the complex Ginzburg–Landau equation.

In this paper [22], Jurado et al. explore the state feedback regulator problem (SFRP) to achieve the goal of trajectory tracking with harmonic disturbance rejection to a one-dimensional (1-D) reaction–diffusion (R–D) equation.

In [23], Karmran et al. propose a localized transform-based meshless method for approximating the solution of the 2D multi-term partial integro-differential equation involving the time fractional derivative in Caputo’s sense with a weakly singular kernel.

The paper authored by Cruz-Quintero et al. [24] tests the backstepping design for the boundary control of a reaction–advection–diffusion (R–A–D) equation, i.e., a parabolic PDE, but with constant coefficients and Neumann boundary conditions, with action on one

of the latter. The heat equation with Neumann boundary conditions is considered as the target system. The dynamics of the open and closed loop solution of the PDE system is validated by performing numerical simulation.

The work of Jia et al [25] discusses the stable control of one class of chaotic systems, and a control method based on the accurate exponential solution of a differential equation is used. Compared with other methods, the advantages are that this method determines that the system can exponentially converge at the origin and the convergence rate can be easily regulated. The chaotic system with unknown parameters is also deduced and validated by using this method.

In this article Khalique et al. [26] examine a (3 + 1)-dimensional generalized breaking soliton equation which is highly applicable in the fields of engineering and nonlinear sciences. Closed-form solutions in the form of Jacobi elliptic functions of the underlying equation are derived by the method of Lie symmetry reductions together with direct integration.

The paper authored by Wu [27] provides, when an additional sample for the second stage may not be available, one-stage multiple comparisons for exponential median lifetimes with the control under heteroscedasticity, including one-sided and two-sided confidence intervals. This is because the median is a more robust measure of central tendency compared to the mean.

The paper authored by Alshomrani [28] describes the features of bio-convection and motile microorganisms in magnetized Burgers' nanoliquid flows with a stretchable sheet. The theory of Cattaneo–Christov mass and heat diffusion is also discussed.

As the Guest Editor of this Special Issue, I am grateful to all the authors for their articles. I would also like to express my gratitude to all the reviewers for their valuable comments and the improvement of the submitted papers. The goal of this Special Issue was to attract high-quality and novel papers in the field of “Applications of Partial Differential Equations in Engineering”. It is hoped that these selected research papers will be significant for the international scientific community and that these papers will motivate further research on applications of partial differential equations in engineering.

Author Contributions: Conceptualization, Á.G., F.U. and A.M.V.; methodology, Á.G., F.U. and A.M.V.; software, Á.G., F.U. and A.M.V.; validation, Á.G., F.U. and A.M.V.; formal analysis, Á.G., F.U. and A.M.V.; investigation, Á.G., F.U. and A.M.V.; resources, Á.G., F.U. and A.M.V.; data curation, Á.G., F.U. and A.M.V.; visualization, Á.G., F.U. and A.M.V.; supervision, Á.G., F.U. and A.M.V.; project administration, Á.G., F.U. and A.M.V.; funding acquisition, Á.G., F.U. and A.M.V. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Acknowledgments: A.M.V. acknowledges support from MICINN (Spain) under de project FJC2021-046953-I.

Conflicts of Interest: The authors declare no conflict of interest.

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