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Event-Triggered Consensus Control of Nonlinear Strict Feedback Multi-Agent Systems †

Jiaojiao Zhuang [1,‡], Zhenxing Li [2,3,*,‡], Zongxiang Hou [4,‡] and Chengdong Yang [5,‡]

[1] School of Mechanical and Vehicle Engineering, Linyi University, Linyi 276000, China; zhuangjiaojiao@lyu.edu.cn
[2] Department of Automation, Linyi University, Linyi 276000, China
[3] School of Mathematics, Southeast University, Nanjing 210096, China
[4] Computing Department, University Pendidikan Sultan Idris, Tanjung Malim 35900, Malaysia; hou_zongxiang@126.com
[5] School of Information Science and Engineering, Linyi University, Linyi 276000, China; Yangchengdong@lyu.edu.cn
* Correspondence: zhxingli@gmail.com

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‡ These authors contributed equally to this work.

Abstract: In this paper, we investigate the event-triggered consensus problems of nonlinear strict feedback MASs under directed graph. Based on the high-gain control technique, we firstly give a state-based event-triggered consensus algorithm and prove that Zeno behavior can be excluded. When the full state information is unavailable, a high-gain observer is given to estimate state information of each agent and an observer-based algorithm is developed. Finally, we give an example to verify the effectiveness of both state-based and observer-based event-triggered consensus algorithms.

Keywords: multi-agent systems; consensus control; event-triggered control; strict feedback systems; directed graph

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1. Introduction

Due to its practical applications in various control systems, such as satellite coordination [1], UAV formation [2], information infusion of sensor networks [3], traffic flow [4], and cooperation of multi robots [5], consensus control of MASs has received increasing attention from engineering. In fact, most MASs are composed of mobile agents, which are equipped with embedded systems and limited energy resource. In recent years, many researchers have been devoted to event-triggered consensus control of MASs, which can effectively reduce continuous information transmission and save limited energy.

The past decade witnessed a rapid development of event-triggered consensus control of MASs, including single-integrator MASs [6–8], double-integrator MASs [9,10], and general linear MASs [11–13]. However, papers mentioned above only considered linear models. Many dynamical systems, such as Chua's circuit and Lagrange systems, are depicted by nonlinear systems. Therefore, it is necessary to study event-triggered consensus problems of nonlinear MASs. In [14], Adaldo et al. studied event-triggered pinning synchronization for first-order nonlinear MASs with time-varying undirected topology. Event-triggered consensus problems for first-order nonlinear MASs under directed graph were studied by using combinational state measurements in [15]. By using event-triggered and intermittent control mechanisms, Hu and Cao studied an event-triggered tracking control algorithm for first-order nonlinear MASs under directed graph [16]. Based on neural network weight estimation, event-triggered consensus control for second-order uncertain nonlinear MASs with undirected graph was studied in [17,18]. In [19], an event-triggered semi-global robust consensus problem for second-order uncertain nonlinear MASs was studied. By using periodic data sampling framework, an event-triggered synchronization controller with time varying control gain was designed for nonlinear multi-agent systems under directed graph in [20]. By estimating the states of neighboring agents, an event-triggered controller was developed for a type of nonlinear leader–follower MAS in [21]. Consensus control problems of nonlinear coupled parabolic PDE-ODE-based multi-agent systems were studied in [22].

Most of the nonlinear MASs mentioned above considered the first/second-order nonlinear MASs. However, first/second-order nonlinear systems do not contain many nonlinear systems, such as high-order nonlinear systems and output-feedback nonlinear systems. Nonlinear strict feedback systems are typical nonlinear dynamics, which include first/second/high-order nonlinear systems and output feedback nonlinear systems. Wang and Ji studied the distributed tracking problem for nonlinear strict feedback leader–follower MASs in [23]. S.J. Yoo studied the adaptive containment control for uncertain nonlinear strict feedback MASs under a directed topology in [24]. In [25], Shen and Shi considered the distributed tracking problem of uncertain nonlinear strict feedback MASs under a weighted undirected graph. Li and Ji studied the finite-time coordination control problems for nonlinear strict feedback MASs with directed topologies in [26]. Event-triggered tracking problems of nonlinear strict feedback MASs under undirected topologies were also considered in some recent papers. For uncertain nonlinear strict feedback MASs, an adaptive distributed event-based tracking algorithm was studied in [27]. Event/self-triggered leader–following tracking algorithms for stochastic nonlinear MASs were investigated in [28]. However, event-triggered consensus problems of leaderless nonlinear strict feedback MASs with directed topology are seldom considered.

From papers mentioned above, we can clearly see that papers [6–13] only consider event-triggered consensus problems of linear MASs, and papers [14–22] consider event-triggered consensus problems of first or second order nonlinear MASs. Although nonlinear strict feedback systems are typical nonlinear dynamics, papers [23–26] only consider cooperative control of nonlinear strict feedback MASs, and papers [27,28] only study event-triggered tracking problems for nonlinear strict feedback MASs. To overcome the disadvantages of the above papers, we will study event-triggered consensus problems of nonlinear MASs in strict feedback form under directed graph. The main contributions are listed as follows:

(1) Since nonlinear strict feedback systems are typical and general, MASs studied in [6–13] are special cases of our paper. (2) Different from papers [27,28], we study event-triggered consensus problems for leaderless MASs. The Laplacian matrix for leaderless MASs with directed topology is singular and asymmetric; it is a hard task to design Lyapunov function for such MASs. Moreover, our results can be extended to event-triggered tracking problems for leader–follower MASs easily. (3) Backstepping design technique is the traditional control method for strict feedback nonlinear systems, and its disadvantage is the tedious design of virtual control law. However, based on the high-gain control technique, the design of our controller can be easily achieved by solving the Riccati equation.

Notation 1. *Denote I_N as the identity matrix of $R^{N\times N}$, $1_N \in R^N$ as a vector with each entry being 1, $\otimes$ as the Kronecker product, and $\mathrm{diag}\{\mathrm{a}_1,\ldots,\mathrm{a}_\mathrm{N}\}$ as a diagonal matrix. For square matrix $P \in R^{n\times n}$, $P > 0$ means that P is positive definite. For a symmetric matrix $A \in R^{N\times N}$, $\lambda_{1A} \le \lambda_{2A} \le \cdots \le \lambda_{NA}$ denote its N eigenvalues. For a vector $x = [x_1,\ldots,x_n]^T \in R^n$, $x_{\max} = \max_{i=1,\ldots,N} x_i$, $x_{\min} = \min_{i=1,\ldots,N} x_i$, and $\vec{x}_l = [x_1,\ldots,x_l]^T \in R^l, l = 1,\ldots,n$.*

2. Problem Statement and Preliminaries

2.1. Problem Statement

We investigate event-triggered consensus problems of nonlinear strict feedback MASs with directed graph in this paper. Each agent is described by the following dynamics:

$$\begin{cases} \dot{x}_{i,l}(t) = x_{i,l+1}(t) + f_l(\vec{x}_{i,l}(t)), l = 1, \dots, n-1, \\ \dot{x}_{i,n}(t) = u_i(t) + f_n(x_i(t)), \\ y_i(t) = x_{i,1}(t), i = 1, \dots, N, \end{cases} \tag{1}$$

where $x_i(t) = [x_{i,1}(t), \dots, x_{i,n}(t)]^T \in R^n, u_i(t), y_i(t) \in R$ are the state, control input, and output of agent i, respectively. $f_l(\cdot), l = 1, \dots, n$, are nonlinear functions.

The control goal of MASs (1) is state consensus, which is defined as follows.

Definition 1. *Suppose that there exists a controller $u_i(t)$, such that*

$$\lim_{t \to \infty} ||x_i(t) - x_j(t)|| = 0, \forall i, j \in \{1, \dots, N\}, \tag{2}$$

then, system (1) *reaches consensus.*

To design the event-triggered controller, we need nonlinear functions $f_l(\cdot), l = 1, \dots, n$, to satisfy the following assumption.

Assumption 1. *For any n-dimensional vectors $y = [y_1, \dots, y_n]^T, z = [z_1, \dots, z_n]^T$, there exists a known nonnegative real ρ, such that*

$$|f_l(\vec{y}_{i,l}(t)) - f_l(\vec{z}_{i,l}(t))| \leq \rho \sum_{j=1}^{l} |y_j(t) - z_j(t)|, l = 1, \dots, n. \tag{3}$$

Remark 1. *Nonlinear strict feedback systems are typical and general enough [26], and nonlinear systems, such as first/second/high-order nonlinear systems and output feedback nonlinear systems, are special cases of nonlinear strict feedback systems. In the literature [29], one can find the geometric conditions for how to translate a nonlinear system into strict feedback form.*

2.2. Preliminaries

The topology of MASs (1) is depicted by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V} = \{1, \dots, N\}, \mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ and $\mathcal{A} = [a_{ij}] \in R^{N \times N}$ are the agent set, edge set, and adjacent matrix, respectively. If agent i can receive local information from its neighbor agent j, then there exists a directed edge $(i, j) \in \mathcal{E}$ and $a_{ij} > 0$; otherwise $a_{ij} = 0$. This paper does not consider the self-loop case, i.e., $a_{ii} = 0$. Edge sequence $(i_{k_2}, i_{k_1}), (i_{k_3}, i_{k_2}), \dots, (i_{k_m}, i_{k_{m-1}})$ depicts a directed path from agent i_{k_1} to agent i_{k_m}. If there exists at least one directed path from agent i to agent j ($\forall i, j \in \mathcal{V}$), $\mathcal{G}$ is said to be strongly connected. Denote $\mathcal{L} = [\mathcal{L}_{ij}] \in R^{N \times N}$ as the Laplacian matrix of $\mathcal{G}$ with $\mathcal{L}_{ii} = \sum_{j=1}^{N} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}, i \neq j$.

Assumption 2. *The topology of MASs* (1) *is strongly connected.*

Lemma 1 ([30])**.** *If directed graph $\mathcal{G}$ is strongly connected, there exists a real vector $\gamma = [\gamma_1, \dots, \gamma_N]$ satisfying $\gamma_i > 0, i = 1, \dots, N$ and $\gamma 1_N = 1$ such that $\gamma \mathcal{L} = 0$. Denote $\Gamma = \mathrm{diag}\{\gamma_1, \dots, \gamma_N\}$ and $L = \frac{1}{2}(\Gamma \mathcal{L} + \mathcal{L}^T \Gamma)$. The second smallest eigenvalue of L is a positive number.*

Lemma 2 ([31])**.** *(Barbalat's Lemma) Let $\phi : \mathbb{R} \to \mathbb{R}$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t \to \infty} \int_0^t \phi(\tau) \mathrm{d}\tau$ exists and is finite. Then, $\lim_{t \to \infty} \phi(t) = 0$.*

Lemma 3 ([31]). *(Comparison Lemma) Consider the scalar differential equation*

$$\dot{u} = f(t,u),\ u(t_0) = u_0$$

where $f(t,u)$ is continuous in t and locally Lipschitz in u, for all $t \geq 0$ and all $u \in J \subset R$. Let $[t_0, T)$ (T could be infinity) be the maximal interval of existence of the solution $u(t)$, and suppose $u(t) \in J$ for all $t \in [t_0, T)$. Let $v(t)$ be a continuous function whose upper right-hand derivative $D^+v(t)$ satisfies the differential inequality

$$D^+v(t) \leq f(t, v(t)),\ v(t_0) \leq u_0$$

with $v(t) \in J$ for all $t \in [t_0, T)$. Then, $v(t) \leq u(t)$ for all $t \in [t_0, T)$.

3. State-Based Event-Triggered Algorithm

In this section, we will develop an event-triggered consensus algorithm for MASs (1) by using local relative state information.

For simplicity, we rewrite system (1) into a compact form:

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + f(x_i(t)) + Bu_i(t), \\ y_i(t) &= Cx_i(t), i = 1, \ldots, N, \end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \ldots & 1 \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C^T = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

and $f(x_i(t)) = [f_1(\vec{x}_{i,1}(t)), f_2(\vec{x}_{i,2}(t)), \ldots, f_n(x_i(t))]^T$.

Denote $D_{\kappa_1} = \mathrm{diag}\{\kappa_1, \ldots, \kappa_1^n\}$ with $\kappa_1 \geq 1$ being a constant to be determined later. Let $\xi_i(t) = D_{\kappa_1}^{-1} \sum_{j=1}^N a_{ij}(x_i(t) - x_j(t))$ be the local relative state information of agent i. Then, we obtain the dynamics of $\xi_i(t)$ as

$$\dot{\xi}_i(t) = \kappa_1 A\xi_i(t) + D_{\kappa_1}^{-1}\bar{f}_i(x(t)) + \frac{1}{\kappa_1^n} B \sum_{j=1}^N a_{ij}(u_i(t) - u_j(t)), \tag{5}$$

where $\bar{f}_i(x(t)) = \sum_{j=1}^N a_{ij}(f(x_i(t)) - f(x_j(t)))$.

Let $\xi(t) = [\xi_1^T(t), \ldots, \xi_N^T(t)]^T$ and $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$. We have $\xi(t) = (\mathcal{L} \otimes D_{\kappa_1}^{-1})x(t)$. Moreover, according to the definition of $\mathcal{L}$, we know that $\xi(t) = 0$ indicates $x_i(t) = x_j(t), \forall i, j \in \{1, \ldots, N\}$.

We use $\epsilon_i(t)$ to denote the sampled error of $\xi_i(t)$ relative to its latest measurement

$$\epsilon_i(t) = \xi_i(t_k^i) - \xi_i(t), i = 1, \ldots, N. \tag{6}$$

By using $\xi_i(t_k)$, we give the following event-triggered consensus algorithm for agent i:

$$u_i(t) = -c\kappa_1^{n+1}B^T P\xi_i(t_k^i), \tag{7}$$

where $c \geq \frac{\gamma_{\max}}{2(\lambda_{2L} - \delta\lambda_{N\Xi}/2)}$ is the feedback gain and $P > 0$ is the solution of Riccati equation below.

$$PA + A^T P - PBB^T P + \frac{1}{\gamma_{\min}} I_n = 0, \tag{8}$$

with $\Xi = \Gamma\mathcal{L}\mathcal{L}^T\Gamma, \delta \in (0, \frac{2\lambda_{2L}}{\lambda_{N\Xi}})$. Triggering instant t_k^i is determined by the following trigger function:

$$h_i(t) = \epsilon_i^T(t) PBB^T P\epsilon_i(t) - \mu e^{-\sigma t}, \tag{9}$$

with μ and σ being positive constants. The method for calculating the event-triggered controller $u_i(t)$ is presented as the following state-based event-triggered Algorithm 1.

Algorithm 1: State-based event-triggered algorithm

Input: $A, B, \mu, \sigma, \kappa_1, \mathcal{L}, x_i(t)(i = 1, \ldots, N)$
Output: Controller $u_i(t)$ and trigger instants t_k^i

Calculate left eigenvector γ for Laplacian matrix $\mathcal{L}$, and obtain parameters c and $\gamma_{\min}$;
Calculate P by solving Riccati equation $PA + A^T P - PBB^T P + \frac{1}{\gamma_{\min}} I_n = 0$;
Let $t_0^i = 0$, $\xi_1^i(t_0^i) = D_{\kappa_1}^{-1} \sum_{j=1}^N a_{ij}(x_i(t_0^i) - x_j(t_0^i))$ and calculate $\epsilon_i(t) = \xi_i(t_0^i) - \xi_i(t)$;
Design event-triggered consensus controller: $u_i(t) = -c\kappa_1^{n+1} B^T P \xi_i(t_0^i)$;
while $h_i(t) = \epsilon_i^T(t) PBB^T P \epsilon_i(t) - \mu e^{-\sigma t} \geq 0$ **do**
  Calculate $t_{k+1}^i (k = 0, 1, 2, 3, \ldots)$, i.e. $t_{k+1}^i = \min\{t | t > t_k^i, h_i(t) \geq 0\}$;
  Update $\epsilon_i(t) = \xi_i(t_{k+1}^i) - \xi_i(t)$ and $u_i(t) = -c\kappa_1^{n+1} B^T P \xi_i(t_{k+1}^i)$;
end

Remark 2. *According to the definition of Laplacian matrix $\mathcal{L}$, if Assumption 2 holds, 0 is a simple eigenvalue of $\mathcal{L}$ and other eigenvalues of $\mathcal{L}$ have positive real parts. Moreover, 1_N is an associated eigenvector of eigenvalue 0. Since $D_{\kappa_1}^{-1}$ is nonsingular and $\xi(t) = (\mathcal{L} \otimes D_{\kappa_1}^{-1})x(t)$, $\xi(t) = 0$ means $x_i(t) = x_j(t), \forall i, j \in \{1, \ldots, N.\}$ According to system (4), (A, B) is controllable, i.e., there exists a positive definite matrix P that satisfies the Ricccati Equation (8). Theorem 1 below shows that the system (5) can be stabilized by the event-triggered controller (7).*

Theorem 1. *If Assumptions 1 and 2 hold, then there is a constant $\kappa_1^* \geq 1$, such that, for $\kappa_1 > \kappa_1^*$, the event-triggered consensus algorithm (7)–(9) solves the consensus problem of MASs (1). Moreover, Zeno behavior can be ruled out.*

Proof. Take Lyapunov function candidate as follows:

$$V_1 = \sum_{i=1}^N \gamma_i \xi_i^T(t) P \xi_i(t). \tag{10}$$

Along (5) and (7), we obtain

$$\begin{aligned} \dot{V}_1 = &\sum_{i=1}^N 2\kappa_1 \gamma_i \xi_i^T(t) PA\xi_i(t) - 2c\kappa_1 \xi^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\xi(t_k) \\ &+ 2\xi^T(t)(\Gamma\mathcal{L} \otimes PD_{\kappa_1}^{-1})F(x(t)), \end{aligned} \tag{11}$$

where $\xi(t_k) = [\xi_1^T(t_k^1), \ldots, \xi_N^T(t_k^N)]^T$ and $F(x(t)) = [f^T(x_1(t)), \ldots, f^T(x_N(t))]^T$.
Denote $\epsilon(t) = [\epsilon_1^T(t), \ldots, \epsilon_N^T(t)]^T$. Since $\epsilon(t) = \xi(t_k) - \xi(t)$, we have

$$\begin{aligned} &\xi^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\xi(t_k) \\ =&\xi^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\xi(t) + \xi^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\epsilon(t). \end{aligned} \tag{12}$$

According to Lemma 1, one obtains

$$\begin{aligned} -\xi^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\xi(t) &= -\xi^T(t)(L \otimes PBB^T P)\xi(t) \\ &\leq -\sum_{i=1}^N \lambda_{2L} \xi_i^T(t) PBB^T P \xi_i(t). \end{aligned} \tag{13}$$

From Young's inequality, one obtains

$$\begin{aligned}
&\xi^T(t)(\Gamma\mathcal{L}\otimes PBB^TP)\epsilon(t)\\
\leq&\frac{\delta}{2}\xi^T(t)(\Xi\otimes PBB^TP)\xi(t)+\frac{1}{2\delta}\epsilon^T(t)(I_N\otimes PBB^TP)\epsilon(t)\\
\leq&\frac{\delta}{2}\lambda_{N\Xi}\sum_{i=1}^{N}\xi_i^T(t)PBB^TP\xi_i(t)+\frac{1}{2\delta}\sum_{i=1}^{N}\epsilon_i^T(t)PBB^TP\epsilon_i(t),
\end{aligned}\tag{14}$$

where $\delta\in(0,\frac{\lambda_{2L}}{\lambda_{N\Xi}})$.

Due to $\mathcal{L}1_N=0$, for the last term of (11), one obtains

$$\begin{aligned}
&\xi^T(t)(\Gamma\mathcal{L}\otimes PD_{\kappa_1}^{-1})F(x(t))\\
=&\xi^T(t)(\Gamma\mathcal{L}\otimes PD_{\kappa_1}^{-1})(F(x(t))-1_N\otimes f(\bar{x}(t)))\\
\leq&||(\mathcal{L}^T\Gamma\otimes P)\xi(t)||\cdot||(I_N\otimes D_{\kappa_1}^{-1})(F(x(t))-1_N\otimes f(\bar{x}(t)))||,
\end{aligned}$$

where $\bar{x}(t)=\sum_{i=1}^{N}\gamma_i x_i(t)$. Denote $z_i(t)=D_{\kappa_1}^{-1}(x_i(t)-\bar{x}(t))$ and $z(t)=[z_1^T(t),\ldots,z_N^T(t)]^T$. It is easy to verify that

$$\xi(t)=(\mathcal{L}\otimes I_n)z(t).$$

Since $\kappa_1\geq 1$, one has

$$\begin{aligned}
&||(I_N\otimes D_{\kappa_1}^{-1})(F(x(t))-1_N\otimes f(\bar{x}(t)))||^2\\
=&\sum_{i=1}^{N}||D_{\kappa_1}^{-1}(f(x_i(t))-f(\bar{x}(t)))||^2\\
\leq&\sum_{i=1}^{N}\sum_{j=1}^{n}[\kappa_1^{-j}\rho(|x_{i,1}-\bar{x}_1|+\cdots+|x_{i,n}-\bar{x}_j|)]^2\\
\leq&\sum_{i=1}^{N}\sum_{j=1}^{n}\rho^2(|z_{i,1}|+\cdots+|z_{i,j}|)^2\\
\leq&n^2\rho^2z^T(t)z(t)\leq\frac{n^2\rho^2}{\lambda_{2W}}\xi^T(t)\xi(t),
\end{aligned}$$

where $W=\mathcal{L}^T\mathcal{L}$. Hence, we obtain the following inequality

$$\xi^T(t)(\Gamma\mathcal{L}\otimes PD_{\kappa_1}^{-1})F(x(t))\leq n\rho\lambda_{NP}\sqrt{\frac{\lambda_{N\Xi}}{\lambda_{2W}}}\sum_{i=1}^{N}\xi_i^T(t)\xi_i(t).\tag{15}$$

Substituting (12)–(15) into (11), we obtain

$$\begin{aligned}
\dot{V}_1\leq&2\kappa_1\sum_{i=1}^{N}\gamma_i\xi_i^T(t)PA\xi_i(t)-2\kappa_1\sum_{i=1}^{N}c(\lambda_{2L}-\delta\lambda_{N\Xi}/2)\\
&\times\xi_i^T(t)PBB^TP\xi_i(t)+\frac{c\kappa_1}{\delta}\sum_{i=1}^{N}\epsilon_i^T(t)PBB^TP\epsilon_i(t)\\
&+2n\rho\lambda_{NP}\sqrt{\frac{\lambda_{N\Xi}}{\lambda_{2W}}}\sum_{i=1}^{N}\xi_i^T(t)\xi_i(t).
\end{aligned}\tag{16}$$

Since $\delta \in (0, \frac{2\lambda_{2L}}{\lambda_{N\Xi}})$ and $c \geq \frac{\gamma_{\max}}{2(\lambda_{2L} - \delta\lambda_{N\Xi}/2)}$, Riccati Equation (8) ensures that

$$\begin{aligned}\dot{V}_1 \leq &-\sum_{i=1}^{N}(\kappa_1 - 2n\rho\lambda_{NP}\sqrt{\lambda_{N\Xi}/\lambda_{2W}})\xi_i^T(t)\xi_i(t)\\&+\frac{c\kappa_1}{\delta}\sum_{i=1}^{N}(\epsilon_i^T(t)PBB^TP\epsilon_i(t) - \mu e^{-\sigma t}) + \frac{c\kappa_1 N\mu}{\delta}e^{-\sigma t}.\end{aligned} \tag{17}$$

Denote $\kappa_1^* = \max\{1, 2n\rho\lambda_{NP}\sqrt{\lambda_{N\Xi}/\lambda_{2W}}\}$. Trigger function (9) and choice of $\kappa_1 > \kappa_1^*$ guarantee that

$$\dot{V}_1 \leq -(\kappa_1 - \kappa_1^*)\xi^T(t)\xi(t) + \frac{c\kappa_1 N\mu}{\delta}e^{-\sigma t}. \tag{18}$$

Integrating both left and right sides of (18) from 0 to t yields

$$\begin{aligned}V_1(t) \leq &-(\kappa_1 - \kappa_1^*)\int_0^t \xi^T(\tau)\xi(\tau)d\tau\\&+\frac{c\kappa_1 N\mu}{\delta\sigma}(1 - e^{-\sigma t}) + V_1(0).\end{aligned} \tag{19}$$

From definition of V_1, we know $V_1(t) \geq 0$. Inequality (19) indicates that both $V_1(t)$ and $\int_0^t \xi^T(\tau)\xi(\tau)d\tau$ are bounded for $\forall t \geq 0$. From (10) and (18), it is easy to obtain that $\xi(t)$ and $\dot{\xi}(t)$ are bounded too.

From the boundedness of $\dot{\xi}(t)$, we know that $\xi^T(t)\xi(t)$ is uniformly continuous. Since $\xi^T(t)\xi(t) \geq 0$, $\int_0^t \xi^T(\tau)\xi(\tau)d\tau$ is monotonously increasing. Then, we obtain that $\int_0^\infty \xi^T(\tau)\xi(\tau)d\tau$ exists and is finite.

According to Barbalat's Lemma, we obtain $\lim_{t\to\infty}\xi(t) = 0$, i.e., MASs (1) reaches consensus.

Besides consensus analysis, we should prove that Zeno behavior can be ruled out.

For $t \in [t_k^i, t_{k+1}^i)$, the Dini derivative of $||\epsilon_i(t)||$ satisfies the following inequalities

$$D^+||\epsilon_i(t)|| = \frac{d}{dt}\sqrt{\epsilon_i^T(t)\epsilon_i(t)} \leq ||\dot{\epsilon}_i(t)||. \tag{20}$$

From (5) and (6), for $t \in [t_k^i, t_{k+1}^i)$,we have

$$\begin{aligned}\dot{\epsilon}_i(t) =& -\kappa_1 A\xi_i(t) - D_{\kappa_1}^{-1}\bar{f}_i(x(t))\\&+c\kappa_1 BB^TP\sum_{j=1}^{N}a_{ij}(\xi_i(t_k^i) - \xi_j(t_k^j))\\=&\kappa_1 A\epsilon_i(t) - \kappa_1 A\xi_i(t_k^i) - D_{\kappa_1}^{-1}\bar{f}_i(x(t))\\&+c\kappa_1 BB^TP\sum_{j=1}^{N}a_{ij}(\xi_i(t_k^i) - \xi_j(t_k^j))\end{aligned} \tag{21}$$

Denote

$$\begin{aligned}\psi_k^i = \max_{t\in[t_k^i, t_{k+1}^i)} &||\kappa_1 A\xi_i(t_k^i) + D_{\kappa_1}^{-1}\bar{f}_i(x(t))\\&-c\kappa_1 BB^TP\sum_{j=1}^{N}a_{ij}(\xi_i(t_k^i) - \xi_j(t_k^j))||.\end{aligned}$$

Notice that $||A|| = 1$. Hence, we obtain

$$D^+||\epsilon_i(t)|| \leq \kappa_1||\epsilon_i(t)|| + \psi_k^i, \forall t \in [t_k^i, t_{k+1}^i). \tag{22}$$

At triggering instant t_k^i, $\epsilon_i(t)$ will be reset. According to the comparison Lemma, one obtains

$$||\epsilon_i(t)|| \leq \frac{1}{\kappa_1}\psi_k^i(\mathrm{e}^{\kappa_1(t-t_k^i)}-1), \forall t \in [t_k^i, t_{k+1}^i). \tag{23}$$

From trigger function (9), one can check that the $(k+1)$th event will not be triggered if $h_i(t) \leq 0$, and $h_i(t) \leq 0$ can be guaranteed by the following inequality:

$$||\epsilon_i(t)|| \leq \frac{\sqrt{\mu \mathrm{e}^{-\sigma t}}}{||PB||}. \tag{24}$$

In light of (24), when the $(k+1)$th event is triggered, $||\epsilon_i(t)||$ is bigger than $\sqrt{\mu \mathrm{e}^{-\sigma t}}/||PB||$, i.e.,

$$\frac{1}{\kappa_1}\psi_k^i(\mathrm{e}^{\kappa_1(t_{k+1}^i-t_k^i)}-1) \geq \frac{\sqrt{\mu \mathrm{e}^{-\sigma t_{k+1}^i}}}{||PB||}.$$

Then, we have

$$t_{k+1}^i - t_k^i \geq \frac{1}{\kappa_1}\ln\left(\frac{\kappa_1\sqrt{\mu \mathrm{e}^{-\sigma t_{k+1}^i}}}{||PB||\psi_k^i}+1\right). \tag{25}$$

Note that $\mu \mathrm{e}^{-\sigma t}$ approaches zero only when $t \to \infty$. Hence, we obtain that $t_{k+1}^i - t_k^i$ is strictly positive for any finite time, and Zeno behavior is ruled out. □

4. Observer-Based Event-Triggered Algorithm

In last section, we studied the event-triggered consensus problem by using local relative state information. In this part, we consider another case where local relative state information is unavailable. A high-gain observer is designed for each agent to estimate its state.

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + f(\hat{x}_i(t)) + Bu_i(t) + D_{\kappa_2}F(C\hat{x}_i(t) - y_i(t)), \tag{26}$$

where $D_{\kappa_2} = \mathrm{diag}\{\kappa_2, \dots, \kappa_2^n\}$ is a matrix to be determined and $F = -SC^T$ with $S > 0$ being the solution of the following Riccati equation:

$$SA^T + AS - SC^TCS + I_n = 0. \tag{27}$$

Let $\tilde{\xi}_i(t) = D_{\kappa_2}^{-1}\sum_{j=1}^N a_{ij}(\hat{x}_i(t) - \hat{x}_j(t))$ be the local relative information of agent i and $\tilde{\epsilon}_i(t) = \tilde{\xi}_i(t_k^i) - \tilde{\xi}_i(t)$ be the sample error of $\tilde{\xi}_i(t)$ relative to its latest measurement $\tilde{\xi}_i(t_k^i)$. Then, we give agent i the following observer-based consensus algorithm

$$u_i(t) = -c\kappa_2^{n+1}B^TP\tilde{\xi}_i(t_k^i). \tag{28}$$

The associated trigger function is given as

$$h_i(t) = \tilde{\epsilon}_i^T(t)PBB^TP\tilde{\epsilon}_i(t) - \mu \mathrm{e}^{-\sigma t}. \tag{29}$$

Algorithm 2 shows how to calculate the observer-based event-triggered controller $u_i(t)$.

Algorithm 2: Observer-based event-triggered algorithm

Input: $A, B, C, \mu, \sigma, \mathcal{L}, x_i(t)(i = 1, \ldots, N)$
Output: Controller $u_i(t)$ and trigger instants t_k^i
Solve Riccati equation $SA^T + AS - SC^TCS + I_n = 0$ and obtain matrix S;
Calculate eigenvector γ for $\mathcal{L}$, and obtain parameters c, κ_2 and $\gamma_{\min}$;
Design high-gain observer
$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + f(\hat{x}_i(t)) + Bu_i(t) + D_{\kappa_2}F(C\hat{x}_i(t) - y_i(t))$;
Calculate P by solving Riccati equation $PA + A^TP - PBB^TP + \frac{1}{\gamma_{\min}}I_n = 0$;
Let $t_0^i = 0$, $\tilde{\xi}_1^i(t_0^i) = D_{\kappa_2}^{-1}\sum_{j=1}^N a_{ij}(\hat{x}_i(t_0^i) - \hat{x}_j(t_0^i))$ and calculate
$\tilde{e}_i(t) = \tilde{\xi}_i(t_0^i) - \tilde{\xi}_i(t)$;
Design event-triggered consensus controller: $u_i(t) = -c\kappa_2^{n+1}B^TP\tilde{\xi}_i(t_0^i)$;
while $h_i(t) = \tilde{e}_i^T(t)PBB^TP\tilde{e}_i(t) - \mu e^{-\sigma t} \geq 0$ **do**
  Calculate $t_{k+1}^i (k = 0, 1, 2, 3, \ldots)$, i.e. $t_{k+1}^i = \min\{t | t > t_k^i, h_i(t) \geq 0\}$;
  Update $\tilde{e}_i(t) = \tilde{\xi}_i(t_{k+1}^i) - \tilde{\xi}_i(t)$ and $u_i(t) = -c\kappa_2^{n+1}B^TP\tilde{\xi}_i(t_{k+1}^i)$;
end

Theorem 2. *If Assumptions 1 and 2 hold, then there is a constant $\kappa_2^* \geq 1$, such that, for $\kappa_2 > \kappa_2^*$, observer-based event-triggered consensus algorithm (28) and (29) solve the consensus problem of MASs (1). In addition, Zeno behavior is also excluded.*

Proof. Let $e_i(t) = \hat{x}_i(t) - x_i(t), i = 1, \ldots, N$, be the estimation error. Then, we have

$$\dot{e}_i(t) = (A + D_{\kappa_2}FC)e_i(t) + (f(\hat{x}_i(t)) - f(x_i(t))).$$

Denote $\hat{e}_i(t) = D_{\kappa_2}^{-1}e_i(t)$, and one obtains

$$\dot{\hat{e}}_i(t) = \kappa_2(A + FC)\hat{e}_i(t) + D_{\kappa_2}^{-1}(f(\hat{x}_i(t)) - f(x_i(t))). \tag{30}$$

Since (A, C) is detectable, Riccati Equation (27) guarantees that $A + FC$ is Hurwitz. Hence, there exists a positive definite matrix Q such that

$$Q(A + FC) + (A + FC)^TQ = -I_n. \tag{31}$$

For high-gain observer (26) of agent i, take the following Lyapunov function candidate

$$V_{2i} = \hat{e}_i^T(t)Q\hat{e}_i(t). \tag{32}$$

Along error dynamics (30), one obtains

$$\dot{V}_{2i} = -\kappa_2\hat{e}_i^T(t)\hat{e}_i(t) + 2\hat{e}_i^T(t)QD_{\kappa_2}^{-1}(f(\hat{x}_i(t)) - f(x_i(t))).$$

Note that

$$\begin{aligned}
&2\hat{e}_i^T(t)QD_{\kappa_2}^{-1}(f(\hat{x}_i(t)) - f(x_i(t))) \\
\leq& 2\lambda_{NQ}||\hat{e}_i(t)|| \cdot ||D_{\kappa_2}^{-1}(f(\hat{x}_i(t)) - f(x_i(t)))|| \\
\leq& 2\lambda_{NQ}||\hat{e}_i(t)|| \cdot [\sum_{j=1}^n(\rho/\kappa_2^j(|e_{i,1}| + \cdots + |e_{i,j}(t)|))^2]^{1/2} \\
\leq& 2\lambda_{NQ}||\hat{e}_i(t)||[\sum_{j=1}^n(\rho(|\hat{e}_{i,1}(t)| + \cdots + |\hat{e}_{i,j}(t)|))^2]^{1/2} \\
\leq& 2n\rho\lambda_{NQ}\hat{e}_i^T(t)\hat{e}_i(t).
\end{aligned}$$

And we obtain

$$\dot{V}_{2i} \leq -(\kappa_2 - 2n\rho\lambda_{NQ})\hat{e}_i^T(t)\hat{e}_i(t). \tag{33}$$

Now, we take the following Lyapunov function candidate for MASs (1)

$$V_2 = \sum_{i=1}^{N} \gamma_i \tilde{\xi}_i^T(t) P \tilde{\xi}_i(t) + \sum_{i=1}^{N} \beta V_{2i}. \tag{34}$$

Notice that

$$\begin{aligned} \dot{\tilde{\xi}}_i(t) =& \kappa_2 A \tilde{\xi}_i(t) + D_{\kappa_2}^{-1} \hat{f}_i(\hat{x}(t)) + \frac{1}{\kappa_2^n} B \sum_{j=1}^{N} a_{ij}(u_i(t) - u_j(t)) \\ &+ FC \sum_{j=1}^{N} a_{ij}(e_i(t) - e_j(t)), \end{aligned} \tag{35}$$

where $\hat{f}_i(\hat{x}(t)) = \sum_{j=1}^{N} a_{ij}(f(\hat{x}_i(t) - f(\hat{x}_j(t)))$. Then, we obtain

$$\begin{aligned} \dot{V}_2 \leq& \sum_{i=1}^{N} 2\kappa_2 \gamma_i \tilde{\xi}_i^T(t) PA \tilde{\xi}_i(t) - 2c\kappa_2 \tilde{\xi}^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\tilde{\xi}(t_k) \\ &+ 2\tilde{\xi}^T(t)(\Gamma\mathcal{L} \otimes PD_{\kappa_2}^{-1})\hat{F}(\hat{x}(t)) + 2\tilde{\xi}^T(t)(\Gamma\mathcal{L} \otimes PFC)e(t) \\ &- \sum_{i=1}^{N} \beta(\kappa_2 - 2n\rho\lambda_{NQ})\hat{e}_i^T(t)\hat{e}_i(t), \end{aligned} \tag{36}$$

where $\hat{F}(\hat{x}(t)) = [f^T(\hat{x}_1(t)), \ldots, f^T(\hat{x}_N(t))]^T$ and $e(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$.

Following from (12)–(15), we have

$$\begin{aligned} &- \tilde{\xi}^T(t)(\Gamma\mathcal{L} \otimes PBB^T P)\tilde{\xi}(t_k) \\ \leq& - \sum_{i=1}^{N} (\lambda_{2L} - \delta_1 \lambda_{N\Xi}/2) \tilde{\xi}_i^T(t) PBB^T P \tilde{\xi}_i(t) \\ &+ \frac{1}{2\delta_1} \sum_{i=1}^{N} \tilde{\varepsilon}_i^T(t) PBB^T P \tilde{\varepsilon}_i(t), \end{aligned} \tag{37}$$

and

$$\tilde{\xi}^T(t)(\Gamma\mathcal{L} \otimes PD_{\kappa_2}^{-1})\hat{F}(\hat{x}(t)) \leq n\rho\lambda_{NP} \sqrt{\frac{\lambda_{N\Xi}}{\lambda_{2W}}} \sum_{i=1}^{N} \tilde{\xi}_i^T(t)\tilde{\xi}_i(t), \tag{38}$$

where $\delta_1 \in (0, \frac{2\lambda_{2L}}{\lambda_{N\Xi}})$. For the fourth term of right side of (36), we have

$$\begin{aligned} &\tilde{\xi}^T(t)(\Gamma\mathcal{L} \otimes PFC)e(t) \\ \leq& \frac{\delta_2}{2} \tilde{\xi}^T(t)(\Xi \otimes I_n)\tilde{\xi}(t) + \frac{1}{2\delta_2} e^T(t)(I_N \otimes \Pi)e(t) \\ \leq& \frac{\delta_2}{2} \lambda_{N\Xi} \sum_{i=1}^{N} \tilde{\xi}_i^T(t)\tilde{\xi}_i(t) + \frac{\lambda_{N\Pi}}{2\delta_2} \sum_{i=1}^{N} \hat{e}_i^T(t) D_{\kappa_2}^2 \hat{e}_i(t) \\ \leq& \frac{\delta_2}{2} \lambda_{N\Xi} \sum_{i=1}^{N} \tilde{\xi}_i^T(t)\tilde{\xi}_i(t) + \frac{\lambda_{N\Pi}\kappa_2^{2n}}{2\delta_2} \sum_{i=1}^{N} \hat{e}_i^T(t)\hat{e}_i(t), \end{aligned} \tag{39}$$

where $\delta_2 > 0$, $\Pi = C^T F^T PPFC$.

Substituting (37)–(39) into (36), we obtain

$$\begin{aligned}\dot{V}_2 \leq & 2\kappa_2 \sum_{i=1}^{N} \gamma_i \tilde{\xi}_i^T(t) PA \tilde{\xi}_i(t) - 2\kappa_2 \sum_{i=1}^{N} c(\lambda_{2L} - \delta_1 \lambda_{N\Xi}/2) \\ & \times \tilde{\xi}_i^T(t) PBB^T P \tilde{\xi}_i(t) + \frac{c\kappa_2}{\delta_1} \sum_{i=1}^{N} \tilde{\epsilon}_i^T(t) PBB^T P \tilde{\epsilon}_i(t) \\ & + (2n\rho\lambda_{NP} \sqrt{\frac{\lambda_{N\Xi}}{\lambda_{2W}}} + \delta_2 \lambda_{N\Xi}) \sum_{i=1}^{N} \tilde{\xi}_i^T(t) \tilde{\xi}_i(t) \\ & - \sum_{i=1}^{N} [\beta(\kappa_2 - 2n\rho\lambda_{NQ}) - \frac{\lambda_{N\Pi}\kappa_2^{2n}}{\delta_2}] e_i^T(t) e_i(t).\end{aligned} \tag{40}$$

Denote $\kappa_2^* = \max\{1, 2n\rho\lambda_{NQ}, 2n\rho\lambda_{NP}\sqrt{\lambda_{N\Xi}/\lambda_{2W}} + \delta_2\lambda_{N\Xi}\}$. Choose $\kappa_2 > \kappa_2^*$ and $\beta \geq \frac{\lambda_{N\Pi}\kappa_2^{2n}}{\delta_2(\kappa_2 - 2n\rho\lambda_{NQ})}$, and we obtain

$$\dot{V}_2 \leq -(\kappa_2 - \kappa_2^*) \tilde{\xi}^T(t) \tilde{\xi}(t) + \frac{c\kappa_2 N\mu}{\delta_1} \mathrm{e}^{-\sigma t}. \tag{41}$$

From the last proof of Theorem 1, we can draw the conclusion that $\lim_{t\to\infty} \tilde{\xi}(t) = 0$, i.e., $\lim_{t\to\infty} \hat{x}_i(t) = \hat{x}_j(t), \forall i, j \in \{1, \dots, N\}$. Note that $\kappa_2 > 2n\rho\lambda_{NQ_2}$. Inequality (33) indicates that $\lim_{t\to\infty} \hat{e}_i(t) = 0, i = 1, \dots, N$, that is, $\lim_{t\to\infty} x_i(t) = x_j(t), \forall i, j \in \{1, \dots, N\}$. Then, MAS (1) reaches consensus.

The proof of how to rule out Zeno behavior is similar to that of the last Theorem. We omit it. □

5. Simulations

To illustrate the proposed event-triggered consensus algorithms, we consider a nonlinear MAS with four agents:

$$\begin{aligned}\dot{x}_{i1}(t) =& x_{i2}(t) + 0.1\arctan(x_{i1}(t)), \\ \dot{x}_{i2}(t) =& -0.2x_{i1}(t) + 0.1\ln(1 + x_{i2}^2(t)) + u_i(t), \\ y_i(t) =& x_{i1}(t), i = 1, \dots, 4,\end{aligned} \tag{42}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T \in R^2, u_i(t), y_i(t) \in R$, are the state, control input, and measurement output of agent i, respectively.

Figure 1 shows the block diagram of (42).

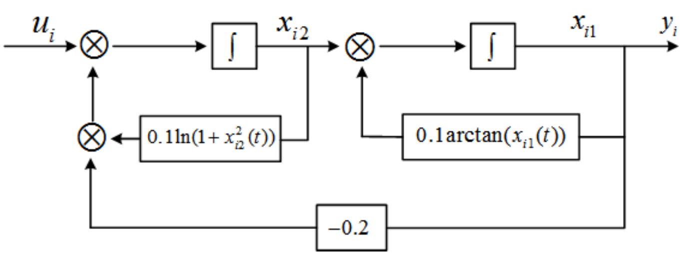

Figure 1. Block diagram of system (42).

The topology of MAS (42) is depicted by the following adjacent matrix:

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

5.1. State-Based Consensus

By solving Riccati Equation (8), we obtain the feedback matrix $B^T P = [2.4495, 3.3014]$. For controller (7), we set $\delta = 0.1$ and select $c = 1$, $\kappa_1 = 3.4$, $\mu = 0.5$ and $\sigma = 0.8$. The initial states of MAS (42) are randomly chosen. Under event-triggered consensus Algorithm 1, simulation results are shown in Figures 2–4. From Figure 2, one can see that the first state of each agent reaches consensus in five seconds. Figure 3 shows that the second state of each agent achieves consensus in nine seconds. Figure 4 displays that the time interval between two sequential events will not approach to zero, and Zeno behavior will not occur. Hence, our algorithm (7) is valid for the event-triggered consensus problem of (42).

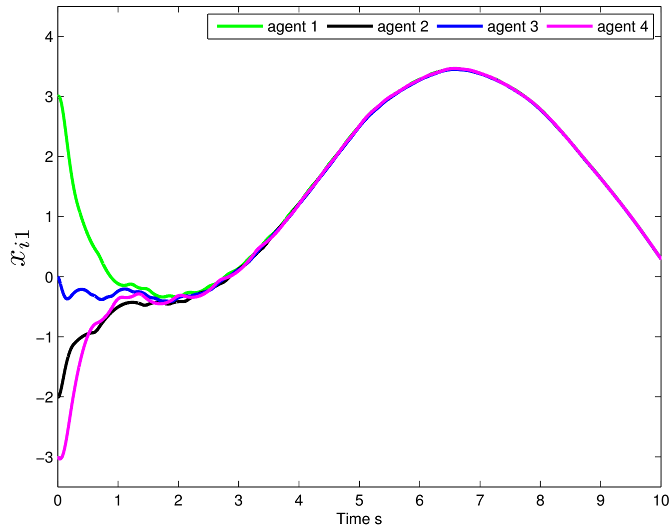

Figure 2. $x_{i1}(t)$ trajectories of MAS (42) with state-based algorithm.

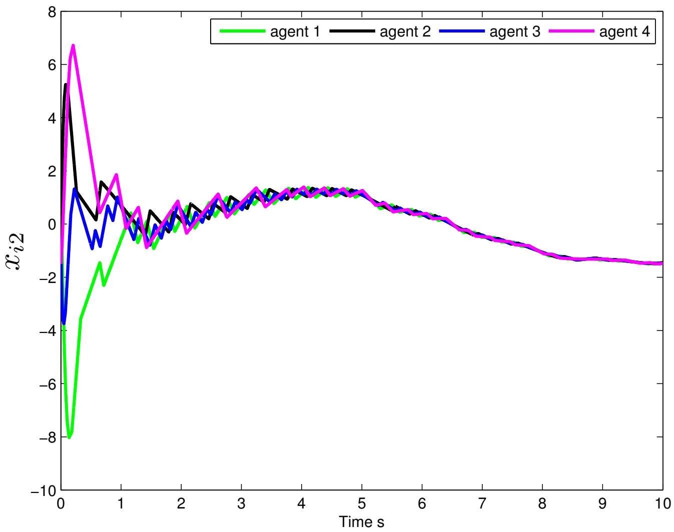

Figure 3. $x_{i2}(t)$ trajectories of MAS (42) with state-based algorithm.

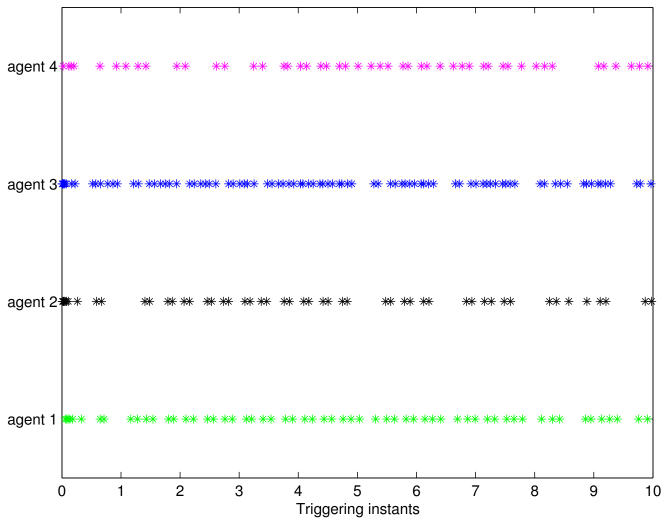

Figure 4. Triggering instants of MAS (42) with state-based algorithm.

5.2. Observer-Based Consensus

When the full state information $x_i(t), i = 1, \ldots, 4$, is unavailable, we design high-gain observers to estimate full state information by using measurement outputs $y_i(t)$. First, we calculate gain matrix $F = [-1.7321, -1]^T$ by solving Riccati Equation (27). Then, we use the following observers to estimate the full states of MAS (42):

$$\begin{aligned}\dot{\hat{x}}_{i1}(t) =& \hat{x}_{i2}(t) + 0.1\arctan(\hat{x}_{i1}(t)) \\ &- 1.7321\kappa_2(\hat{x}_{i1}(t) - x_{i1}(t)), \\ \dot{\hat{x}}_{i2}(t) =& -0.2\hat{x}_{i1}(t) + 0.1\ln(1 + \hat{x}_{i2}^2(t)) \\ &- \kappa_2^2(\hat{x}_{i1}(t) - x_{i1}(t)) + u_i(t),\end{aligned} \tag{43}$$

with $\hat{x}_{i1}(0) = 0, \hat{x}_{i2}(0) = 0, i = 1, \ldots, 4$.

We set $\delta_1 = \delta_2 = 0.1$ and select $c = 1$, $\kappa_2 = 3.5$, $\mu = 0.5$, and $\sigma = 0.8$. Under observer-based Algorithm 2 simulation results of MAS (42) are displayed in Figures 5–8. From Figures 5 and 6, we can see that the first and second states of each agent reach consensus in ten seconds. Figure 7 shows that estimation errors of high-gain observers (43) converge to zero exponentially. Figure 8 displays that the time interval between two sequential events will not approach to zero, and Zeno behavior is ruled out.

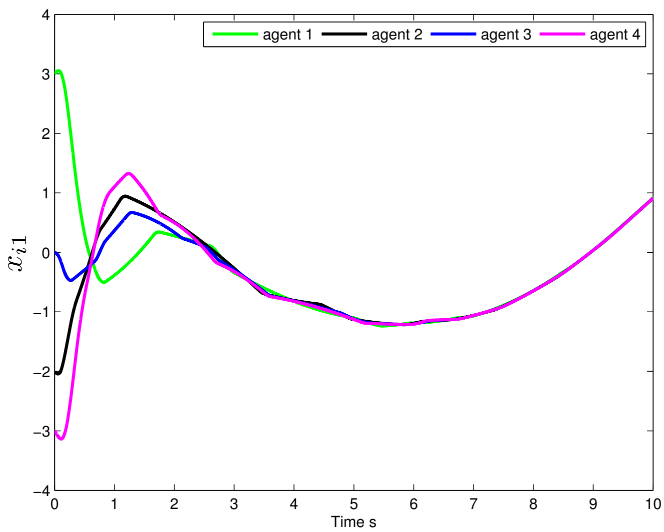

Figure 5. $x_{i1}(t)$ trajectories of MAS (42) with observer-based algorithm.

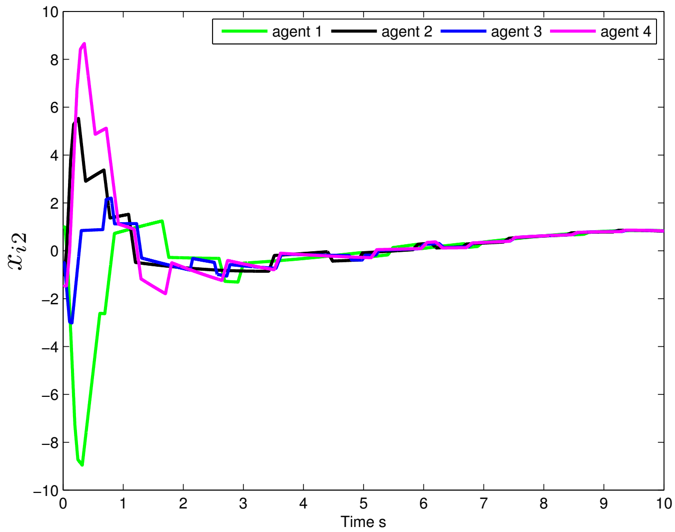

Figure 6. $x_{i2}(t)$ trajectories of MAS (42) with observer-based algorithm.

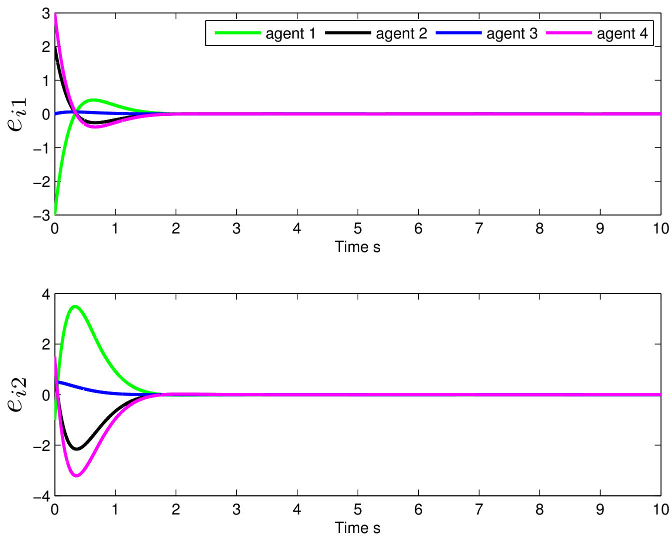

Figure 7. Observer errors of (43).

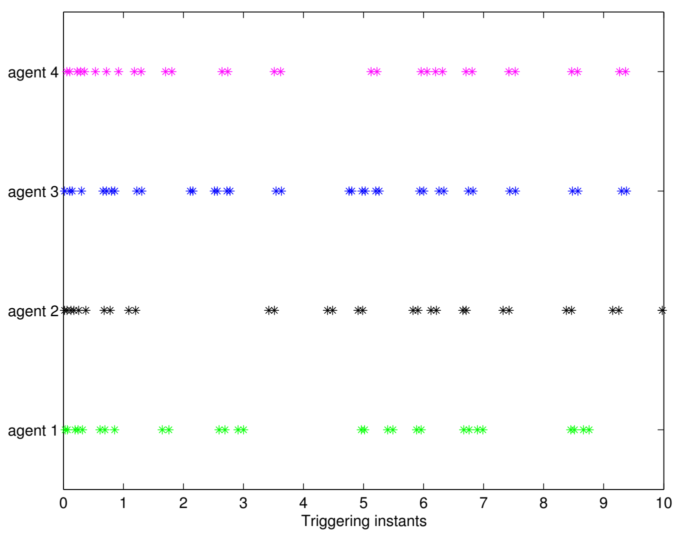

Figure 8. Triggering instants of MAS (42) with observer-based algorithm.

6. Conclusions

In this paper, we investigated the event-triggered consensus problems of nonlinear strict feedback MASs. Based on high-gain control technique and high-gain observer, we propose both state-based and observer-based event-triggered consensus algorithms to solve consensus problems of nonlinear strict feedback MASs. In addition, we proved that those two consensus algorithms are free from Zeno behavior. Theoretical analysis shows that the presented algorithms can solve the event-triggered consensus problems of nonlinear strict feedback MASs. Moreover, we also give a numerical example to verify the effectiveness of our event-triggered algorithms; the simulation results show that the given algorithms reach the objectives.

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Abbreviations

The following abbreviations are used in this manuscript:

MASs	Multi-agent Systems
UAV	Unmanned Aerial Vehicle
PDE-ODE	Partial Differential Equations and Ordinary Differential Equations

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