Article

# Can a Restaurant Benefit from Joining an Online Take-Out Platform? 

Peng Zhang ${ }^{1,2, *}$, Sisi Ju ${ }^{1}$ and Hongfu Huang ${ }^{3}$<br>1 Business School, Yangzhou University, Yangzhou 225127, China; jusisi_g@163.com<br>2 Jiangsu Modern Logistics Research Base, Yangzhou University, Yangzhou 225127, China<br>3 School of Economics and Management, Nanjing University of Science and Technology, Nanjing 210094, China; huanghf@njust.edu.cn<br>* Correspondence: 007400@yzu.edu.cn

Citation: Zhang, P.; Ju, S.; Huang, H. Can a Restaurant Benefit from Joining an Online Take-Out Platform? Mathematics 2022, 10, 1392.
https://doi.org/10.3390/ math10091392

Academic Editor: David Carfi

Received: 22 March 2022
Accepted: 19 April 2022
Published: 21 April 2022
Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.


Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/).


#### Abstract

In this paper, we study a restaurant's take-out model choice and the coordination of an online take-out supply chain. To this end, we first derive the restaurant's optimal price and/or platform's commission rate under the restaurant's three possible take-out models: do not provide online take-out service (NTO model), provide take-out service by joining an online take-out platform (TOF model), or provide online take-out service by itself (TOH model). We investigate the restaurant's optimal take-out model choice. We then derive the optimal decisions of price and the take-out model under centralization, and study the online take-out supply chain coordination problem. We find that, first, the restaurant may not always benefit from providing online take-out service. It will be beneficial only if the incremental demand generated by take-out service is high. Second, under the centralized supply chain, the TOF model is always better than the TOH model. Meanwhile, when the incremental demand is high, the restaurant should choose the TOF model; otherwise, the NTO model is better. Third, we find that the restaurant's take-out price and model choice decisions under a decentralized supply chain are both inconsistent with that under the centralized supply chain. Last, we design a sales reward contract which can achieve the price and model choice coordination as well as win-win outcomes for all supply chain members.


Keywords: online take-out service; take-out model choice; coordination; platform
MSC: 90B06

## 1. Introduction

With the popularity of mobile Internet, the market scale of online take-out is growing rapidly. According to the statistics of the ZhiYan consulting company [1], the market scale of China's online take-out grew to $\$ 88.6$ billion in 2019, an increase of $30.8 \%$ compared with 2018. By 2020, the number of online take-out users in China had reached 397.8 million. More and more restaurants started to provide online take-out service by joining some online take-out platforms (TOF model in short), such as Meituan, Ele.me and Delivery Hero. According to a financial report by Meituan, the number of restaurants that have joined the Meituan platform had reached 6.2 million by 2019. It is also reported that Delivery Hero has 150,000 restaurant partners across 40 countries [2].

In recent years, with the improvement of the industry concentration of online take-out platforms, some platforms have gradually increased their commission rate (or revenue share ratio). For example, Meituan's commission rate has increased from 5 to $22 \%$. At the high commission rate, some restaurants find it difficult to benefit from online take-out service. In addition, with the reduction of a restaurant's take-out profit margin, the quality of food materials will be difficult to guarantee, which may cause food safety problems. Subsequently, some of them choose not to provide online take-out service (NTO model in short), while others choose to provide online take-out service by themselves (TOH model
in short). As some restaurants leave the online take-out platform, the potential revenue of the platform will also be negatively affected.

Therefore, the following questions are of great practical importance. For online takeout platforms, how to set the commission rate and coordination mechanisms to attract more restaurants and to achieve win-win outcomes are key questions that need to be addressed. For restaurants, how to choose an optimal take-out model according to its own situation is equally important. To address these questions, we model a take-out supply chain with one restaurant and one online take-out platform (platform in short). We first derive the restaurant's optimal price (dine-in and take-out) and/or platform's commission rate under the TOF model, NTO model and TOH model. Then, we compare the restaurant's profit under the three models and analyze the restaurant's optimal take-out model under decentralization. Subsequently, we derive the optimal decisions of price and take-out model under centralization. By comparing these optimal decisions in the centralized and decentralized situations, we propose a sales reward contract and prove that it can coordinate the price and model choice simultaneously and achieve a win-win outcome.

Our main findings are as follows. First, we derive the optimal commission rate for the take-out platform. Second, the restaurant can benefit from providing the online takeout service only when the incremental demand generated by take-out service is high. In addition, the TOF model is always better than the TOH model under the centralized supply chain. Lastly, we find that the take-out price and the model choice under the centralized situation are not consistent with that under the decentralized situation. To coordinate the supply chain, we then propose a sales reward contract.

This paper makes three major contributions. First, to the best of our knowledge, there is scant literature on online take-out supply chain coordination. It is important that we fill this research gap, as this kind of supply chain structure is common in practice. Second, the existing relevant literature mainly focuses on the price and service effort level decisions of the platform. However, in practice, the commission rate has important effects on the platform and a restaurant's profits and take-out model choice. This paper not only considers the optimal commission rate decision, but also analyzes the impact of parameters on it. Lastly, although some scholars study the restaurant's take-out model choice, they mainly analyze whether the restaurants join the platform or not. But in real life, some restaurants provide online sales service through instant messaging software (e.g., QQ, WeChat and Skype) and online payment tools (e.g., Alipay, PayPal and WeChat), and then deliver the food to consumers by themselves. Therefore, we also consider this model when we study a restaurant's take-out model choice.

The rest of this paper is organized as follows. Section 2 provides a literature review. Section 3 describes the models and notations. Section 4 derives the optimal price and/or commission rate under the three take-out models. Section 5 studies the restaurant's take-out model choice. Section 6 proposes a sales reward contract which can coordinate the supply chain. Section 7 provides concluding remarks and suggests future research directions.

## 2. Literature Review

We classify the relevant literature into two streams. The first research stream includes papers on the restaurant industry; The second research stream includes papers on supply chain coordination.

The recently published papers in the first research stream are reviewed as follows. With the rapid development of e-commerce and the popularity of mobile Internet, many restaurants begin to open online sales channels or O2O channels by joining some platforms. Whether to join and how to price after joining has become a major issue for restaurants. Heo [3] studied the impact of joining in group-buying platforms on restaurant profits. Through an empirical study, he found that if restaurants can attract many new patrons by participating in group-buying platforms, they should cooperate with the platform; if many regular patrons shift to discount customers, joining the platform will damage restaurants' profits. Zheng and Guo [4] further consider a number of competing restaurants
and examine the optimal pricing strategy of them. They also find that joining the platform is not always beneficial to the restaurant. They suggest that if the number of offline loyal customers is relatively small, the restaurants can join the platform and provide a discount for the online price. The above studies do not consider the game relationship between the restaurant and the platform.

In practice, the price subsidy will be provided in the initial stage to attract restaurants to join the platform. Xu et al. [5] study online price strategies of a restaurant, as well as the optimal service effort level of a take-out platform and further propose optimal cooperative strategies in different scenarios. Some scholars further analyzed whether restaurants should join the take-out platform. Zhang et al. [2] study the impacts of adding a take-out platform channel on firms' offline and total sales and profits based on the data of a Chinese fast-food restaurant chain. They find that although joining the platform will hurt offline and total profits in the short run, it improves offline and total sales and profits in the long run. In addition, they also find that joining more take-out platforms is not necessarily conducive to the improvement of a restaurant's profit. Zhang et al. [6] establish a Stackelberg game model between a take-out platform and a restaurant and study the optimal price subsidy of the platform and/or take-out price of the restaurant under the price subsidy and no price subsidy. By comparing the benefits in different cases, they propose the optimal strategy for the restaurant. In some of the above-mentioned papers, although the game relationship between restaurants and platforms is considered, the decision-making problem of platform commission is not studied. However, in practice, the commission is an important factor to affect the profit of platform and restaurant. This paper studies the commission decision of the platform. Meanwhile, in addition to study whether the restaurant should join the take-out platform, this paper also considers the situation that the restaurant provides online take-out service by itself. In addition, some scholars study the optimization of delivery network [7-9], quality of food supply chain [10,11], the purchasing behavior of consumers [12,13], and the recommendation system of take-out platform [14,15].

Papers in the second research stream are related to supply chain coordination. There is a large amount of literature in this field. The most literature mainly focuses on the traditional supply chain structure. In this field, scholars design a variety of contracts to coordinate the supply chain. For example, there are quantity discount contracts $[16,17]$, buyback contracts [18,19], revenue sharing contracts [20,21], and sales rebate contracts [22]. Some scholars study the supply chain coordination under dual-channels (online and offline). Chen et al. [23] find that the manufacturer's contract with a wholesale price and an online sale price can coordinate the dual-channel supply chain, but does not achieve the win-win situation. They then combine this contract with a complementary agreement to solve the problem. He et al. [24] further consider a unidirectional transshipment policy between the online channel and offline channel and develop a quantity-discount contract to the dual-channel supply chain. Zhu et al. [25] establish a dual-channel supply chain in which the Conditional Value-at-Risk criterion is considered. They design a buyback with a revenue sharing contract to coordinate the dual-channel supply chain. In the above literature, companies set up the online channels by themselves. In reality, most companies establish online channels through online platforms. Zha et al. [26] establish a model where a hotel established an online channel through Ctrip.com (online platform) and propose a cost sharing contract that achieves channel coordination. Different from ordinary online platforms, the take-out platform not only provides online sales service, but also provides delivery service. Meanwhile, in the above literature, the study of supply chain coordination does not consider the mode choice of companies, that is, to join or not to join the platform. In practice, the restaurant has three modes to choose: NTO, TOF and TOH. This paper will study how the take-out platform can coordinate take-out price and model choice simultaneously, i.e., it makes the take-out price and model choice decision of restaurant under both decentralized and centralized situations.

## 3. The Models and Notations

In this paper, we consider a supply chain with one take-out platform (she) and one restaurant (he). The restaurant provides a single catering product at per unit cost $c$. The take-out platform provides the take-out service (online sale and delivery) at per unit $\operatorname{cost} c_{t}$. Simultaneously, she charges a delivery fee $c_{d}$ from consumers for unit product and a commission from the restaurant based on a certain percentage of turnover $\lambda . \lambda$ is also called the commission rate. We assume the demand function is linear in price effects, and then analyze the demands under the NTO, TOF and TOH models.

Case 1: the demand under NTO model.
In this case, the restaurant just provides the dine-in service and sells to dine-in consumers at unit price $p_{r}$. Following Kurata et al. [27] and Hua et al. [28], we assume that the demand functions are linear in self-price effects. Hence, the dine-in demand $q_{r}$ under the NTO model is formulated as follows:

$$
\begin{equation*}
q_{r}=a-b_{r} p_{r} \tag{1}
\end{equation*}
$$

In (1), a represents the base demand. $b_{r}$ denotes the coefficient of price elasticity of dine-in demand $q_{r}$.

Case 2: the demand under TOF model.
In this case, the restaurant not only provides the dine-in service, but also provides the take-out service through an online take-out platform. In reality, take-out and dine-in are often applicable to different consumption situations. For example, when consumers order take-out, they often have no time to go to the restaurant or don't want to go out. Consumers who adopt dine-in often want to enjoy better food and environment, or have extra social needs. Therefore, this paper does not consider the price competition between take-out and dine-in. This is different from the previous studies (e.g., $[27,28]$ ), because the restaurant can reach more potential consumers through the platform, and the base demand increases from $a$ to $(a+s)$, where $s$ denotes the incremental demand. In addition, consumers need to pay a delivery fee $c_{d}$ for ordering meals through the take-out platform, so we further consider the impact of delivery fee on demand in our model. We then use $p_{r}^{j}$ and $p_{e}^{j}$ to denote the dine-in price and take-out price under the TOF model, respectively. The dine-in demand $q_{r}^{j}$ and take-out demand $q_{e}^{j}$ under TOF model are formulated as follows:

$$
\begin{align*}
q_{r}^{j} & =(1-\theta)(a+s)-b_{r} p_{r}^{j}  \tag{2}\\
q_{e}^{j} & =\theta(a+s)-b_{e} p_{e}^{j}-b_{e} c_{d} \tag{3}
\end{align*}
$$

In (2) and (3), $\theta(0 \leq \theta \leq 1)$ and $(1-\theta)$ denote the percentage of the base demand divided by take-out and dine-in when $p_{e}^{j}$ and $p_{r}^{j}$ are zero. $\theta$ can also be used to measure the preference degree of take-out. $b_{e}$ denotes the coefficient of price and delivery fee elasticity of dine-in demand $q_{e}^{j}$.

Case 3: the demand under TOH model.
In this case, the restaurant provides the take-out service by itself. Because the restaurant lacks the support of platform traffic, his incremental demand drops to $\beta s(0 \leq \beta \leq 1)$, where $\beta$ denotes the consumer's retention rate. In real life, if the restaurant distributes take-out by itself, consumers generally do not need to pay the delivery fee. Therefore, in the TOH model, we do not need to consider the impact of delivery fees on demand. We then use $p_{r}^{i}$ and $p_{e}^{i}$ to denote the dine-in price and take-out price under TOH model, respectively. The dine-in demand $q_{r}^{i}$ and take-out demand $q_{e}^{i}$ under the TOH model is formulated as follows:

$$
\begin{gather*}
q_{r}^{i}=(1-\theta)(a+\beta s)-b_{r} p_{r}^{i}  \tag{4}\\
q_{e}^{i}=\theta(a+\beta s)-b_{e} p_{e}^{i} \tag{5}
\end{gather*}
$$

## 4. Price and/or Commission Rate Decision

To analyze which model is better for the restaurant, we need to obtain the restaurant's profit under a different model. To this end, in this section, we will study how the restaurant and/or the take-out platform decide the price and commission rate, respectively, to maximize their own profits.

### 4.1. The NTO Model

In this scenario, the restaurant decides the dine-in price to maximize its own profits. The retailer's profit, denoted as $\Pi_{R}$, is determined by

$$
\begin{equation*}
\Pi_{R}=\left(p_{r}-c\right)\left(a-b_{r} p_{r}\right) \tag{6}
\end{equation*}
$$

In (6), $\left(p_{r}-c\right)$ denotes the restaurant's margin. $\left(a-b_{r} p_{r}\right)$ denotes the dine-in demand of restaurant.

Taking the first-order and second-order conditions of Equation (6) with respect to $p_{r}$ :

$$
\begin{gather*}
\frac{d \Pi_{R}}{d p_{r}}=a-2 b_{r} p_{r}+b_{r} c  \tag{7}\\
\frac{d^{2} \Pi_{R}}{d p_{r}^{2}}=-2 b_{r}<0 \tag{8}
\end{gather*}
$$

From (7) and (8), we then can obtain the following proposition:
Proposition 1. Under the NTO model, the restaurant's profit is concave in $p_{r}$, the optimal dine-in price $p_{r}^{*}$ is given by

$$
\begin{equation*}
p_{r}^{*}=\frac{a+b_{r} c}{2 b_{r}} \tag{9}
\end{equation*}
$$

Substituting $p_{r}^{*}$ into (6), we then obtain the maximal profit of the restaurant $\Pi_{R}^{*}$ under NTO model

$$
\begin{equation*}
\Pi_{R}^{*}=\left(\frac{a-b_{r} c}{2 b_{r}}\right)^{2} \tag{10}
\end{equation*}
$$

### 4.2. The TOF Model

In this scenario, the take-out platform, as Stackelberg leader, decides the commission rate $\lambda$ first, and then the restaurant as the follower determines dine-in price and take-out price. By backward induction, we first analyze the restaurant's best response, and then study the platform's commission rate decision.

Stage 1: Restaurant's best response.
In this subsection, the restaurant will decide the best response to a commission rate to maximize its own profits. The restaurant's profit, denoted as $\Pi_{R}^{j}$, is determined by

$$
\begin{equation*}
\Pi_{R}^{j}=\left(p_{r}^{j}-c\right)\left[(1-\theta)(a+s)-b_{r} p_{r}^{j}\right]+\left[(1-\lambda) p_{e}^{j}-c\right]\left[\theta(a+s)-b_{e} p_{e}^{j}-b_{e} c_{d}\right] \tag{11}
\end{equation*}
$$

In (11), the first and second terms denote the profit of dine-in and take-out, respectively. We then can get the restaurant's best response

$$
\begin{gather*}
p_{r}^{j}=\frac{(1-\theta)(a+s)+b_{r} c}{2 b_{r}}  \tag{12}\\
p_{e}^{j}=\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2(1-\lambda)} \tag{13}
\end{gather*}
$$

Stage 2: Take-out platform's commission rate decision.
The take-out platform's profit, denoted as $\Pi_{f}^{j}$, is determined by

$$
\begin{equation*}
\Pi_{f}^{j}=\lambda p_{e}^{j}\left[\theta(a+s)-b_{e} p_{e}^{j}-b_{e} c_{d}\right]+c_{d}\left[\theta(a+s)-b_{e} p_{e}^{j}-b_{e} c_{d}\right]-c_{t}\left[\theta(a+s)-b_{e} p_{e}^{j}-b_{e} c_{d}\right] \tag{14}
\end{equation*}
$$

In (14), the first term denotes the platform's revenue from the restaurant's commission, the second term denotes the platform's revenue from the consumer's delivery fee, and the last term denotes the total take-out service cost. $c_{t}$ denotes the unit take-out service cost, specifically including the platform's operating cost and delivery cost. Substituting (13) into (14) and simplifying, we can obtain

$$
\begin{equation*}
\Pi_{f}^{j}=\left[\lambda\left(\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2(1-\lambda)}\right)+c_{d}-c_{t}\right]\left[\frac{\theta(a+s)-b_{e} c_{d}}{2}-\frac{b_{e} c}{2(1-\lambda)}\right] \tag{15}
\end{equation*}
$$

From (15), we can obtain the following proposition. For clarity, all the proof is provided in the Appendix A.

Proposition 2. Under the TOF model, the take-out platform's profit is quasi-concave in $\lambda$, the optimal commission rate $\lambda^{*}$ is given by

$$
\begin{equation*}
\lambda^{*}=1-\sqrt[3]{A+\sqrt{A^{2}+\left(\frac{A\left[c+2\left(c_{t}-c_{d}\right)\right]}{3 c}\right)^{3}}}-\sqrt[3]{A-\sqrt{A^{2}+\left(\frac{A\left[c+2\left(c_{t}-c_{d}\right)\right]}{3 c}\right)^{3}}} \tag{16}
\end{equation*}
$$

where $A=\frac{b_{e}^{2} c^{2}}{\left[\theta(a+s)-b_{e} c_{d}\right]^{2}}$.
For clarity, all proofs, if not provided in the main text, are detailed in the Appendix A. Proposition 2 implies that the restaurant will only benefit from the increase of the commission rate if it is accompanied by additional revenues that more than compensate for the commission cost. The main reason is that the increased commission rate will prompt the restaurant to increase the take-out price, which will reduce the take-out demand.

We next study how system parameters $\left(\theta, s, c_{t}, c_{d}\right)$ impact the optimal commission rate. However, the expression of the commission rate is too complex. Thus, we conduct numerical studies to show the results, which are depicted in Figure 1. In the numerical studies of this paper, if not specified otherwise, the parameter values are: $\mathrm{a}=1000$, $\theta=0.35, s=600, b_{r}=4, b_{e}=6, \beta=0.6, c_{o}=10, c_{t}=5, c_{d}=3$ and $c=30$. In practice, the platform or restaurant generally provides take-out service through the online mode, so it is easy for consumers to compare prices with other take-out prices. Therefore, the demand price elasticity of take-out $b_{e}$ is generally greater than that of dine-in $b_{r}$. Based on it, we set $b_{e}=6>b_{r}=4$. According to a financial report by Meituan, the delivery fee charged by Meituan to its customers does not cover wages paid to delivery staff. Hence, we set $c_{d}=3<c_{t}=5$. In addition, based on the assumptions of this paper, we set $0 \leq \theta=0.35 \leq 1$ and $c_{o}=10>c_{t}=5$.

Observation 1. The commission rate $\lambda^{*}$ increases in the preference degree of take-out $\theta$ and incremental demand s, and decreases in the take-out service cost of platform $c_{t}$ and the delivery fee $c_{d}$.

The higher preference degree of take-out and incremental demand means that the restaurant can get higher revenues through take-out. Therefore, the take-out platform will also set a higher commission rate to share this part of the revenue. From (3) and (13), we can find that both commission rate and delivery fee have a negative impact on the take-out demand. Hence, the optimal commission rate will decrease as delivery fee increases. In practice, the take-out platforms provide take-out services for many restaurants and charge commissions at the same rate. However, different types of restaurants have different degrees of preference for take-out and incremental demand. From Proposition 2 and Observation 1, we know that the same commission rate for all restaurants is not a good strategy. The platform should set different commission rates according to the actual situation of different restaurants. For example, in the initial stage of joining the platform,
due to the lack of historical sales and consumer reviews, this restaurant's ranking on the platform search page is generally lower, which will lead to a lower incremental demand. In this case, the platform should set a lower commission rate. Generally, consumers are more acceptable of hamburger or pizza take-out than hotpot take-out. Therefore, the commission rate of a burger or pizza restaurant can be higher than that of a hotpot restaurant.


Figure 1. The impacts of system parameter on the commission rate $\lambda^{*}$. (a) The impact of $\theta$ on the commission rate. (b) The impact of $s$ on the commission rate. (c) The impact of $c_{t}$ on the commission rate. (d) The impact of $c_{d}$ on the commission rate.

Substituting (16) into (13), we can obtain the following proposition:
Proposition 3. Under the TOF model, the restaurant's optimal dine-in price $p_{r}^{j *}$ and take-out price is $p_{e}^{j *}$ are given by

$$
\begin{equation*}
p_{r}^{j *}=\frac{(1-\theta)(a+s)+b_{r} c}{2 b_{r}} \tag{17}
\end{equation*}
$$

$$
\begin{equation*}
p_{e}^{j *}=\frac{\theta(a+s)-b_{d} c_{d}}{2 b_{e}}+\frac{c}{2\left(\sqrt[3]{A+\sqrt{A^{2}+\left(\frac{A\left[c+2\left(c_{t}-c_{d}\right)\right]}{3 c}\right)^{3}}}+\sqrt[3]{A-\sqrt{A^{2}+\left(\frac{A\left[c+2\left(c_{t}-c_{d}\right)\right]}{3 c}\right)^{3}}}\right)} \tag{18}
\end{equation*}
$$

Substituting $p_{r}^{j *}, p_{e}^{j *}$ and $\lambda^{*}$ into (11) and (15), we then obtain the maximal profit of the restaurant and take-out platform under TOF model.

$$
\begin{gather*}
\Pi_{R}^{j *}=\left[\frac{(1-\theta)(a+s)-b_{r} c}{2 b_{r}}\right]^{2}+\frac{\left[\left(1-\lambda^{*}\right) \theta(a+s)-\left(1-\lambda^{*}\right) b_{e} c_{d}-b_{e} c\right]^{2}}{4 b_{e}\left(1-\lambda^{*}\right)}  \tag{19}\\
\Pi_{f}^{j *}=\left[\lambda^{*}\left(\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2\left(1-\lambda^{*}\right)}\right)+c_{d}-c_{t}\right]\left[\frac{\theta(a+s)-b_{e} c_{d}}{2}-\frac{b_{e} c}{2\left(1-\lambda^{*}\right)}\right] \tag{20}
\end{gather*}
$$

We then study the how system parameter $\left(\theta, s, c_{t}, c_{d}\right)$ impact the take-out platform's profit. We conduct numerical studies and plot the results in Figure 2.


Figure 2. The impacts of system parameter on the take-out platform's profit $\Pi_{f}^{j *}$. (a) The impact of $\theta$ on the platform's profit. (b) The impact of $s$ on the platform's profit. (c) The impact of $c_{t}$ on the platform's profit. (d) The impact of $c_{d}$ on the platform's profit.

Observation 2. The preference degree of take-out $\theta$, incremental demand $s$, delivery fee $c_{d}$ and take-out service cost of platform $c_{t}$ affects the take-out platform's profit $\Pi_{f}^{j *}$ as follows:
(1) The take-out platform's profit $\Pi_{f}^{j^{*}}$ increases in the preference degree of take-out $\theta$ and incremental demand $s$, and deceases in the take-out service cost of platform $c_{t}$.
(2) The take-out platform's profit $\Pi_{f}^{j *}$ is concave in the delivery fee $c_{d}$.

Observation 2 implies that the take-out platform can increase its profit by increasing the preference degree of take-out and incremental demand and decreasing the take-out service cost of the platform. In practice, the preference degree of take-out of consumers mainly depends on the delivery speed and food quality. The platform can improve the delivery
speed by optimizing the delivery task allocation and delivery path. The improvement of delivery speed can not only shorten the delivery time, but also reduce the difference in taste between take-out food and dine-in food, which will help to enhance the preference degree of take-out. The production process of take-out is not visible, and compared to dine-in, it also needs to be packaged and delivered, so some consumers are worried about the take-out food's quality. Therefore, the platform should strengthen the supervision of food quality throughout the entire process.

At present, due to the delivery cost, the delivery range provided by the take-out platform for restaurants is generally within 3 km . Expanding delivery range is the key to increase incremental demand. Simultaneously, the delivery cost is also the main expenditure of the platform. For example, in 2020, Meituan's delivery cost accounted for $74.8 \%$ of its total revenue. Therefore, reducing delivery cost is the key to increasing incremental demand and reducing the service cost of a platform. In addition to optimizing the delivery task allocation and delivery path, increasing the delivery scale and the delivery density are two main ways to reduce delivery cost. Therefore, the platform should open her delivery system to provide delivery service for more non-restaurant enterprises. In addition, different take-out platforms should strengthen cooperation and try to carry out joint delivery.

From Observation 2, we also find that either charging a higher or lower delivery fee is not a good choice for the platform. The main reason is that the lower delivery fee reduces the unit income, while the higher delivery fee greatly reduces the demand. Therefore, only moderate delivery fees benefit the platform.

### 4.3. The TOH Model

In this scenario, the restaurant decides the dine-in price and take-out price to maximize its own profits simultaneously. We use $c_{o}$ to denote the take-out service cost of the restaurant. Generally, the scale and specialization of the take-out platform is higher than those of the restaurant, so we assume $c_{o}>c_{t}$. The restaurant's profit, denoted as $\Pi_{R}^{i}$, is determined by

$$
\begin{equation*}
\Pi_{R}^{i}=\left(p_{r}^{i}-c\right)\left[(1-\theta)(a+\beta s)-b_{r} p_{r}^{i}\right]+\left[p_{e}^{i}+c_{d}-c-c_{o}\right]\left[\theta(a+\beta s)-b_{e} p_{e}^{i}-b_{e} c_{d}\right] \tag{21}
\end{equation*}
$$

In (21), the first term denotes the restaurant's revenue from the consumer's dine-in demand, the second term denotes the restaurant's revenue from the consumer's takeout demand.

From (21), we can then obtain the following proposition:
Proposition 4. Under the TOH model, the restaurant's profit is jointly concave in ( $p_{r}^{i}, p_{e}^{i}$ ), and the optimal dine-in price $p_{r}^{i *}$ and take-out price $p_{e}^{i *}$ are given by

$$
\begin{gather*}
p_{r}^{i *}=\frac{(1-\theta)(a+\beta s)+b_{r} c}{2 b_{r}}  \tag{22}\\
p_{e}^{i *}=\frac{\theta(a+\beta s)+b_{e}\left(c+c_{o}\right)-2 b_{e} c_{d}}{2 b_{e}} \tag{23}
\end{gather*}
$$

Substituting $p_{r}^{i *}$ and $p_{e}^{i *}$ into (21), we then obtain the maximal profit of the restaurant $\Pi_{R}^{i *}$ under the TOH model.

$$
\begin{equation*}
\Pi_{R}^{i *}=\left[\frac{(1-\theta)(a+\beta s)-b_{r} c}{2 b_{r}}\right]^{2}+\left[\frac{\theta(a+\beta s)-b_{e}\left(c+c_{o}\right)}{2 b_{e}}\right]^{2} \tag{24}
\end{equation*}
$$

## 5. The Take-Out Model Choice

In this section, we focus on the main research questions of this paper: how does the restaurant choose the take-out model? Therefore, we need to compare the restaurant's profit among the NTO model, TOF model and TOH model. To this end, we first compare
the profit between the NTO model and the TOH model. We then have the following proposition.

Proposition 5. When $s \geq \frac{D+\sqrt{D^{2}-4 B b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{0}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B \beta}$, the restaurant's profit under the TOH model is higher than that under the NTO model, where $B=(1-\theta)^{2} b_{e}^{2}+\theta^{2} b_{r}^{2}$ and $D=2 b_{e} b_{r}\left[b_{e} c(1-\theta)+b_{r}\left(c+c_{o}\right) \theta\right]$.

Proposition 5 implies that when the incremental demand brought by take-out is large, the TOH model is better than the NTO model. Otherwise, the NTO mode is a better choice for the restaurant. The main reason is that the take-out service is a double-edged sword for the restaurant. On the one hand, the take-out service increases the total base demand, which will help increase the restaurant's profit. On the other hand, the take-out service transforms a part of the original dine-in consumption into take-out consumption. The take-out service requires an additional service cost, such as a delivery fee. As a result, the service cost to the consumers increases, which leads to lower profits for the restaurant. If the take-out service adds more demand to the restaurant, the former plays a greater role than the latter so that the retailer's profit increases. Otherwise, the latter will dominate. The restaurant' profit function under the TOF model is too complicated. Thus, we also conduct numerical studies to compare the results. Our results are depicted in Figure 3.


(c)

Figure 3. The impacts of system parameter on the model choice. (a) The impact of $\theta$ on the model choice. (b) The impact of $\beta$ on the model choice. (c) The impact of $s$ on the model choice.

Observation 3. The preference degree of take-out $\theta$, incremental demand s and retention rate $\beta$ affect the restaurant's optimal model choice as follows:
(1) When the preference degree of take-out $\theta$ is low, the restaurant's optimal model choice is TOF. Conversely, the NTO model is the better choice.
(2) When the incremental demand s is high, the restaurant's optimal model choice is TOF. Conversely, the NTO model is the better choice.
(3) When the retention rate $\beta$ is high, the restaurant's optimal model choice is TOH. Conversely, the TOF model is the better choice.

The increased preference degree of take-out means that if the take-out service is provided, some dine-in consumption will transform into take-out consumption. Under this condition, the average unit cost will increase, which will lead to a decline in restaurant profits under the TOF and TOH model. Similar to the analysis of Proposition 5, when the incremental demand is low, the increased revenue cannot compensate for the increased cost, so the NTO mode is the best choice for restaurants.

From Figure 3a, we find that when the preference degree of take-out is lower, the restaurant's profit under the TOF model is higher than that under the TOH model. Compared with the TOF model, the advantage of the TOH model is that the restaurant does not need to pay commissions to the take-out platform, i.e., it does not need to share the take-out revenue with the take-out platform. The disadvantage of the TOH model is that the promotion ability is weaker than the take-out platform, i.e., incremental demand under the TOH mode is lower than that under the TOF model. When the preference degree of take-out is lower, more consumers who search for the restaurant through the take-out platform chooses dine-in, i.e., more traffic from the take-out platform is transferred to dine-in. The dine-in revenue does not need to pay commission, and the incremental demand under the TOF model is higher, so the restaurant should choose the TOF mode under this condition. When the preference degree of take-out is higher, the take-out revenue accounts for a larger proportion of the restaurant's total revenue. In addition, from Observation 1, we know that the commission rate also increases with the preference degree of take-out. It means that a large part of the restaurant's revenue will be transferred to the take-out platform as a commission fee. Therefore, under this condition, the restaurant's profit under the TOH model is higher than that under TOF model.

From Observation 1, we know that the commission rate increases as the incremental demand increases. It means that when the incremental demand is higher, the restaurant needs to pay a higher commission under the TOF model. Therefore, the take-out revenue under the TOH model will be higher than that under the TOF model. However, the higher incremental demand also brings higher dine-in consumption. Due to higher incremental demand under the TOF model, the dine-in revenue under the TOF model will be higher than that under the TOH model. When the incremental demand is higher, the latter plays a greater role than the former so that the restaurant should adopt the TOF model.

From Figure 3b, we find that when the consumer's retention rate is higher, the restaurant's profit under the TOH model is higher than that under the TOF model. Higher consumer retention rate means that when the restaurant leaves the platform to provide take-out service on its own, the loss of demand is not large. Simultaneously, the restaurant does not need to pay any commission. Therefore, under this condition, the TOH model is a better choice.

## 6. The Price and Model Choice Coordination

In this section, we mainly focus on these questions: From the perspective of a centralized supply chain, what is the optimal price and model choice decision? For the take-out platform, how do we establish a coordination mechanism to make the decisions under the decentralized supply chain consistent with those under the centralized supply chain?

In the centralized supply chain, the purpose of price and model choice decisions are to maximize the entire supply chain's profit. For clarity, we add superscript ()$^{k}$ to the
notation. Hence, for benchmark purposes, we first derive the supply chain's total profit $\Pi_{T}^{k}$ as follows:

$$
\begin{equation*}
\Pi_{T}^{k}=\left(p_{r}^{k}-c\right)\left[(1-\theta)(a+s)-b_{r} p_{r}^{k}\right]+\left(p_{e}^{k}+c_{d}-c-c_{t}\right)\left[\theta(a+s)-b_{e} p_{e}^{k}-b_{e} c_{d}\right] \tag{25}
\end{equation*}
$$

In (25), the first and second terms denote the profit of dine-in and take-out, respectively. From (25), we then can obtain the following proposition:

Proposition 6. Under the centralized supply chain and TOF model, the supply chain is jointly concave in $\left(p_{r}^{k}, p_{e}^{k}\right)$, the optimal dine-in price $p_{r}^{k *}$ and the take-out price $p_{e}^{k *}$ are given by

$$
\begin{gather*}
p_{r}^{k *}=\frac{(1-\theta)(a+s)+b_{r} c}{2 b_{r}}  \tag{26}\\
p_{e}^{k *}=\frac{\theta(a+s)+b_{e}\left(c+c_{t}\right)-2 b_{e} c_{d}}{2 b_{e}} \tag{27}
\end{gather*}
$$

Comparing (27) and (18), we find that the take-out price under a centralized supply chain is not consistent with the price under the decentralized. The take-out platform charges the commission, which causes the double-marginal effect. This is the main reason for the price difference.

Substituting $p_{r}^{k *}$ and $p_{e}^{k *}$ into (25), we then obtain the maximal profit of the supply chain $\Pi_{T}^{k *}$ under the TOF model.

$$
\begin{equation*}
\Pi_{T}^{k *}=\left[\frac{(1-\theta)(a+s)-b_{r} c}{2 b_{r}}\right]^{2}+\left[\frac{\theta(a+s)-b_{e}\left(c+c_{t}\right)}{2 b_{e}}\right]^{2} \tag{28}
\end{equation*}
$$

Comparing the results in (28) and (24), we find that the profit of the centralized supply chain under the TOF model is always higher than the restaurant's profit under the TOH model. It means that from the perspective of the centralized supply chain, the TOF model is better than the TOH model. The main reason is that there is no commission under the centralized supply chain and TOF model. Meanwhile, the incremental demand and takeout service cost under the TOF model is lower than that under the TOH model. Therefore, we just need to compare the TOF model and NTO model. Comparing (28) and (10), we can get the following proposition:

Proposition 7. Under the centralized supply chain, if $s \geq \frac{E+\sqrt{E^{2}-4 A b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{t}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B}$, the optimal model choice of supply chain is the TOF. Otherwise, the NTO model is better for the supply chain. Where $E=2 b_{e} b_{r}\left[b_{e} c(1-\theta)+b_{r}\left(c+c_{t}\right) \theta\right]$.

Comparing Propositions 5 and 7, we can find that the optimal model choice under the centralized model is also not consistent with that under the decentralized one. We then can easily check that
$\frac{E+\sqrt{E^{2}-4 A b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{t}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B} \leq \frac{D+\sqrt{D^{2}-4 B b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{o}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B \beta}$
Based on (29), Propositions 5 and 7, we can draw a comparison figure of model choice, which is shown in Figure 4.

To achieve coordination, we design a sales reward contract. Under this contract, the take-out platform sets up a sales target for the restaurant. If the restaurant's take-out sales (turnover, i.e., $p_{e} q_{e}$ ) are beyond the target, the platform will give the retailer a rebate, $G$. In order to ensure that the price decision under the decentralized model is consistent with that under the centralized model, the sales target should be set as $p_{e}^{k *} q_{e}^{k *}$.

$$
\begin{equation*}
p_{e}^{k *} q_{e}^{k *}=\frac{\theta^{2}(a+s)^{2}-b_{e}^{2}\left(c+c_{t}\right)^{2}-2 b_{e} c_{d}\left[\theta(a+s)-b_{e}\left(c+c_{t}\right)\right]}{4 b_{e}} \tag{30}
\end{equation*}
$$



Figure 4. The comparison of mode choice between Decentralized and Centralized. (where $h=$ $\frac{D+\sqrt{D^{2}-4 B b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{o}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B \beta}$ and $\left.l=\frac{E+\sqrt{E^{2}-4 A b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{t}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B}\right)$.

In practice, the platform and restaurant are willing to accept the coordination mechanism only when it can benefit all of them. Hence, an appropriate rebate $G$ should be adopted to ensure that everyone's expected profit can improve under the sales reward contract. We next analyze the value range of the rebate $G$.

Case 1: $l \leq s \leq h$.
In this case, if $\Pi_{R}^{j *} \geq \Pi_{R}^{*}$, the restaurant will adopt the TOF model under the decentralized scenario. Under this condition, the restaurant's optimal profit is equal to $\Pi_{R}^{j \text { * }}$, and the platform's optimal profit is equal to $\Pi_{f}^{j *}$. In order to ensure that the platform and restaurant accept this sales reward contract, the rebate $G$ needs to meet the following conditions

$$
\begin{align*}
& \Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)+G \geq \Pi_{R}^{j *}  \tag{31}\\
& \Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)-G \geq \Pi_{f}^{j *} \tag{32}
\end{align*}
$$

If $\Pi_{R}^{j *}<\Pi_{R}^{*}$, the restaurant will adopt the NTO model under the decentralized scenario. Under this condition, the restaurant's optimal profit is equal to $\Pi_{R}^{*}$, and the platform's optimal profit is equal to zero. To ensure that both of them can accept this contract, the rebate $G$ needs to meet the following conditions

$$
\begin{gather*}
\Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)+G \geq \Pi_{R}^{*}  \tag{33}\\
\Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)-G \geq 0 \tag{34}
\end{gather*}
$$

Case 2: $s>h$.
If $\Pi_{R}^{j *} \geq \Pi_{R}^{i *}$, the restaurant will adopt the TOF model under the decentralized scenario. Under this condition, the restaurant's optimal profit is equal to $\Pi_{R}^{j *}$, and the platform's optimal profit is equal to $\Pi_{f}^{j *}$. To ensure that both of them can accept this contract, the rebate $G$ needs to satisfy (31) and (32).

If $\Pi_{R}^{j *}<\Pi_{R}^{i *}$, the restaurant will adopt the TOH model under decentralized. Under this condition, the restaurant's optimal profit is equal to $\Pi_{R}^{i *}$, and the platform's optimal profit is equal to zero. To ensure that both of them can accept this contract, the rebate $G$ needs to meet the following conditions

$$
\begin{gather*}
\Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)+G \geq \Pi_{R}^{i *}  \tag{35}\\
\Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)-G \geq 0 \tag{36}
\end{gather*}
$$

To sum up, we can get the following Proposition:
Proposition 8. The take-out platform can provide the retailer with a rebate: a reward of $G$ for the retailer's take-out turnover ( $p_{e} q_{e}$ ) above $\frac{\theta^{2}(a+s)^{2}-b_{e}^{2}\left(c+c_{t}\right)^{2}-2 b_{e} c_{d}\left[\theta(a+s)-b_{e}\left(c+c_{t}\right)\right]}{4 b_{e}}$, which can make the price decision and model choice under the decentralized supply chains consistent with that under the centralized supply chains. The value range of $G$ is as follows:
(1) When $l \leq s \leq h, G \in\left[\Pi_{R}^{j *}-\Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right), \Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)-\Pi_{f}^{j *}\right]$ if $\Pi_{R}^{j *} \geq \Pi_{R}^{*}$, and $G \in\left[\Pi_{R}^{*}-\Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right), \Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)\right]$ if $\Pi_{R}^{j *}<\Pi_{R}^{*}$.
(2) When $s>h, G \in\left[\Pi_{R}^{j *}-\Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right), \Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)-\Pi_{f}^{j *}\right]$ if $\Pi_{R}^{j *} \geq \Pi_{R}^{i *}$, and $G \in\left[\Pi_{R}^{i *}-\Pi_{R}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right), \Pi_{f}^{j}\left(p_{r}^{k *}, p_{e}^{k *}, \lambda^{*}\right)\right]$ if $\Pi_{R}^{j *}<\Pi_{R}^{i *}$.

Proposition 8 implies that both the restaurant and the take-out platform will be happy to enter this sales reward contract when the take-out platform chooses a proper rebate $G$.

## 7. Conclusions

In this paper, we study the take-out model choice and the coordination of an online take-out supply chain. To this end, we model a supply chain with one restaurant and one platform. We then derive the optimal decisions of the price and/or commission rate under each of the three possible take-out models: the TOF model, NTO model and the TOH model. By comparing the restaurant's profit under these models, we find that under the decentralized supply chain, the restaurant should adopt the NTO model if the incremental demand is low; otherwise, the TOF model and TOH model are better choices. Next, we conduct numerical studies to further compare the restaurant's profit under TOF and TOH, and find that when the incremental demand is high, the restaurant's optimal model choice is TOH only if the retention rate is high. Subsequently, we derive the optimal decisions of price and take-out model under centralization. We find that the TOF model is always better than the TOH model. From the perspective of a supply chain, when the incremental demand is high, the restaurant should choose the TOF model; otherwise, the NTO model is better. Lastly, by comparing centralization with decentralization, we propose a sales reward contract and prove that it can coordinate the take-out supply chain.

Furthermore, our study provides some managerial insights. First, the restaurant may not always benefit from continuously increasing the commission rate. Second, as the preference degree of take-out and incremental demand increases or the take-out service cost of the platform deceases, the platform can increase the commission rate. Lastly, charging a higher or lower delivery fee from consumers will have a negative impact on the profit of the platform.

There are a few interesting topics for further research. First, in this paper, we do not consider competition between restaurants or platforms. Therefore, how competition affects price decisions, commission rate decisions, take-out model choices and coordinating mechanism design are worthy of future investigation. Second, this paper considers a single-period setting. However, in practice, the take-out price and commission rate are often set dynamically, which can also be a future research direction. Third, in the model of this paper, we consider a deterministic demand function. However, in business practice, the demand is often uncertain. Hence, modeling this more realistic but complicated setting is a worthwhile research direction. Lastly, in this paper, we assume that the delivery service is provided by the platform. In reality, there are also some third-party logistics companies that provide delivery services. Research opportunities abound in this supply chain which consists of restaurant, platform and logistics service providers.

Author Contributions: Conceptualization, S.J. and P.Z.; methodology, H.H.; validation, P.Z. and H.H.; formal analysis, S.J.; investigation, H.H. and P.Z.; resources, S.J.; writing-original draft preparation, S.J.; writing—review and editing, P.Z.; visualization, H.H.; supervision, P.Z.; project administration, P.Z.; funding acquisition, H.H. and P.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the National Natural Science Foundation of China (Nos. 71801170, 72101117 and 72103178), Natural Science Foundation of Jiangsu Province of China (Grants BK20200485), the Humanity and Social Science Youth Foundation of Ministry of Education of China (No. 18YJC630246).

Institutional Review Board Statement: Not applicable.
Informed Consent Statement: Not applicable.
Data Availability Statement: Not applicable.
Conflicts of Interest: The authors declare that they have no conflict of interest.

## Appendix A

Proof of Proposition 2. The second-order conditions of $\Pi_{R}^{j}$ with respect to $p_{r}^{j}$ and $p_{e}^{j}$, respectively, are

$$
\begin{align*}
& \frac{\partial^{2} \Pi_{R}^{j}}{\partial p_{r}^{j^{2}}}=-2 b_{r}<0  \tag{A1}\\
& \frac{\partial^{2} \Pi_{R}^{j}}{\partial p_{r}^{j^{2}}}=-2 b_{e}<0 \tag{A2}
\end{align*}
$$

Based (A1) and (A2), we can get the restaurant's best response by taking the firstorder conditions.

The first- and second-order conditions of $\Pi_{R}^{j}$ with respect to $p_{r}^{j}$ and $p_{e}^{j}$, respectively, are

$$
\begin{align*}
\frac{d \Pi_{f}^{j}}{d \lambda} & =\left[\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2(1-\lambda)}+\frac{\lambda c}{2(1-\lambda)^{2}}\right]\left[\frac{\theta(a+s)-b_{e} c_{d}}{2}-\frac{b_{e} c}{2(1-\lambda)}\right]  \tag{A3}\\
& -\left[\lambda\left(\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2(1-\lambda)}\right)+c_{d}-c_{t}\right] \frac{b_{e} c}{2(1-\lambda)^{2}}
\end{align*}
$$

$$
\begin{align*}
\frac{d^{2} \Pi_{f}^{j}}{d^{2} \lambda} & =\left[\frac{c}{(1-\lambda)^{2}}+\frac{\lambda c}{(1-\lambda)^{3}}\right]\left[\frac{\theta(a+s)-b_{d} c_{d}}{2}-\frac{b_{e} c}{2(1-\lambda)}\right] \\
& -\left[\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2(1-\lambda)}+\frac{\lambda c}{2(1-\lambda)^{2}}\right] \frac{b_{e} c}{(1-\lambda)^{2}}-\left[\lambda\left(\frac{\theta(a+s)-b_{e} c_{d}}{2 b_{e}}+\frac{c}{2(1-\lambda)}\right)+c_{d}-c_{t}\right] \frac{b_{e} c}{(1-\lambda)^{3}} \tag{A4}
\end{align*}
$$

Substituting $\frac{d \Pi_{f}^{j}}{d \lambda}=0$ into (A4), we can get

$$
\begin{align*}
\frac{d^{2} \Pi_{f}}{d^{2} \lambda} & =\left[\frac{c}{(1-\lambda)}+\frac{\lambda c}{(1-\lambda)^{2}}\right]\left[\frac{\theta(a+s)-b_{d} c_{d}}{1}-\frac{b_{e} c}{(1-\lambda)}\right] \\
& -\left[\frac{\theta(a+s)-b_{d} c_{d}}{b_{e}}+\frac{c}{(1-\lambda)}+\frac{\lambda c}{(1-\lambda)^{2}}\right] \frac{\theta(a+s)-b_{d} c_{d}}{1}<0 \tag{A5}
\end{align*}
$$

From (A5), we can obtain that $\Pi_{f}^{j}$ is quasi-concave in $\lambda$ and the unique optimal $\lambda^{*}$ should satisfy the first-order condition. Hence, we can get the proposition.

## Proof of Proposition 5.

$$
\begin{equation*}
\Pi_{R}^{i *}-\Pi_{R}^{*}=\left[\frac{(1-\theta)(a+\beta s)-b_{r} c}{2 b_{r}}\right]^{2}+\left[\frac{\theta(a+\beta s)-b_{e}\left(c+c_{o}\right)}{2 b_{e}}\right]^{2}-\left(\frac{a-b_{r} c}{2 b_{r}}\right)^{2} \tag{A6}
\end{equation*}
$$

By rearranging, we can get that when $\Pi_{R}^{i *}>\Pi_{R}^{*}$, the following inequality must be satisfied

$$
\begin{gather*}
(1-\theta)^{2} b_{e}^{2}(a+\beta s)^{2}+b_{e}^{2} b_{r}^{2} c^{2}-2 b_{e}^{2} b_{r} c(1-\theta)(a+\beta s)+\theta^{2} b_{r}^{2}(a+\beta s)^{2}+b_{e}^{2} b_{r}^{2}\left(c+c_{o}\right)^{2}  \tag{A7}\\
-2 b_{e} b_{r}^{2}\left(c+c_{o}\right) \theta(a+\beta s)-\left(a-b_{r} c\right)^{2} b_{e}^{2}>0
\end{gather*}
$$

We then can get that when $s \geq \frac{D+\sqrt{D^{2}-4 B b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{o}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B \beta}, \Pi_{R}^{i *} \geq \Pi_{R}^{*}$.

## Proof of Proposition 7.

$$
\begin{equation*}
\Pi_{R}^{k *}-\Pi_{R}^{*}=\left[\frac{(1-\theta)(a+s)-b_{r} c}{2 b_{r}}\right]^{2}+\left[\frac{\theta(a+s)-b_{e}\left(c+c_{t}\right)}{2 b_{e}}\right]^{2}-\left(\frac{a-b_{r} c}{2 b_{r}}\right)^{2} \tag{A8}
\end{equation*}
$$

By rearranging, we can get that when $\Pi_{R}^{i *}>\Pi_{R}^{*}$, the following inequality must be satisfied

$$
\begin{gather*}
(1-\theta)^{2} b_{e}^{2}(a+s)^{2}+b_{e}^{2} b_{r}^{2} c^{2}-2 b_{e}^{2} b_{r} c(1-\theta)(a+s)+\theta^{2} b_{r}^{2}(a+s)^{2}+b_{e}^{2} b_{r}^{2}\left(c+c_{t}\right)^{2} \\
-2 b_{e} b_{r}^{2}\left(c+c_{t}\right) \theta(a+s)-\left(a-b_{r} c\right)^{2} b_{e}^{2}>0 \tag{A9}
\end{gather*}
$$

We then can get that when $s \geq \frac{E+\sqrt{E^{2}-4 A b_{e}^{2}\left[b_{r}^{2} c^{2}+b_{r}^{2}\left(c+c_{t}\right)^{2}-\left(a-b_{r} c\right)^{2}\right]}-2 B a}{2 B}, \Pi_{R}^{j *} \geq \Pi_{R}^{*}$.

## References

1. ZhiYan Consulting Company. Analysis of China's Take-Out Industry Market Transaction Volume, Industry Market Penetration, Online Takeout User Scale and Industry Development Trend from 2019 to 2020. Available online: http:/ /www.chyxx.com/ industry/202005/865307.html. 2020 (accessed on 1 January 2020).
2. Zhang, S.; Pauwels, K.; Peng, C. The Impact of Adding Online-to-Offline Service Platform Channels on Firms' Offline and Total Sales and Profits. J. Interact. Mark. 2019, 47, 115-128. [CrossRef]
3. Heo, C.Y. Exploring group-buying platforms for restaurant revenue management. Int. J. Hosp. Manag. 2016, 52, 154-159. [CrossRef]
4. Zheng, X.; Guo, X. E-retailing of restaurant services: Pricing strategies in a competing online environment. J. Oper. Res. Soc. 2016, 67, 1408-1418. [CrossRef]
5. $\mathrm{Xu}, \mathrm{J} . ; \mathrm{Hu}, \mathrm{L} . ; \mathrm{Guo}, \mathrm{X}$.; Yan, X. Online cooperation mechanism: Game analysis between a restaurant and a third-party website. J. Revenue Pricing Manag. 2020, 19, 61-73. [CrossRef]
6. Zhang, H.; Luo, K.; Ni, G. The effects of price subsidy and fairness concern on pricing and benefits of take-away supply chain. J. Comb. Optim. 2020. [CrossRef]
7. Liu, Y.; Guo, B.; Chen, C.; Du, H.; Yu, Z.W.; Zhang, D.Q.; Ma, H.D. FooDNet: Toward an Optimized Food Delivery Network based on Spatial Crowdsourcing. IEEE. Trans. Mobile. Comput. 2019, 18, 1288-1300. [CrossRef]
8. Wang, L.; Yu, Z.; Han, Q.; Guo, B.; Xiong, H.Y. Multi-objective Optimization based Allocation of Heterogeneous Spatial Crowdsourcing Tasks. IEEE. Trans. Mobile. Comput. 2018, 17, 1637-1650. [CrossRef]
9. Yan, L.; Xu, F.; Liu, J.; Teo, K.L.; Lai, M.Y. Stability strategies of demand-driven supply networks with transportation delay. Appl. Math. Model. 2019, 76, 109-121. [CrossRef]
10. Bai, L.B.; Shi, C.M.; Guo, Y.T.; Du, Q.; Huang, Y.D. Quality risk evaluation of the food supply chain using a fuzzy comprehensive evaluation model and failure mode, effects, and criticality analysis. J. Food Qual. 2018, 2018, 2637075. [CrossRef]
11. He, Y.; Huang, H.F.; Li, D.; Shi, C.M.; Wu, S.J. Quality and operations management in food supply chains: A literature review. J. Food Qual. 2018, 2018, 7279491. [CrossRef]
12. Young, J.A.; Clack, P.W.; Mcintyre, F.S. The web as an e-commerce medium: An exploratory study of consumer perceptions in a restaurant setting. J. Mark. Channels 2006, 14, 5-22. [CrossRef]
13. Tahir, M.; Guru, P. What factors determine-satisfaction and consumer spending in e-commerce retailing. J. Retail. Consum. Serv. 2017, 39, 135-144.
14. Wang, L.; Yi, B. Research on O2O take-away restaurant recommendation system: Taking ele.me APP as an example. Cluster. Comput. 2019, 22, 6069-6077. [CrossRef]
15. Hatami, M.; Pashazadeh, S. Enhancing prediction in collaborative filtering-based recommender systems. Int. J. Comput. Sci. Eng. 2014, 2, 48-51.
16. Yu, X.Q.; Ren, X.X. The Impact of Food Quality Information Services on Food Supply Chain Pricing Decisions and Coordination Mechanisms Based on the O2O E-Commerce Mode. J. Food Qual. 2018, 2018, 8956820. [CrossRef]
17. Wu, S.S.; Li, Q. Emergency Quantity Discount Contract with Suppliers Risk Aversion under Stochastic Price. Mathematics 2021, 9, 1791. [CrossRef]
18. Becker-Peth, M.; Katok, E.; Thonemann, U.W. Designing buyback contracts for irrational but predictable newsvendors. Manag. Sci. 2013, 59, 1800-1816. [CrossRef]
19. Guo, P.; Jia, Y.L.; Gan, J.W.; Li, X.F. Optimal Pricing and Ordering Strategies with a Flexible Return Strategy under Uncertainty. Mathematics 2021, 9, 2097. [CrossRef]
20. Nosoohi, I.; Nookabadi, A.S. Approaches to designing the revenue sharing contract under asymmetric cost information. IMA. J. Manag. Math. 2018, 29, 69-97. [CrossRef]
21. Taboubi, S.; Zaccour, G.; Leopoldwildburger, U. Pricing decisions in marketing channels in the presence of optional contingent products. Cent. Eur. J. Oper. Res. 2020, 28, 167-192.
22. Zhang, P.; He, Y.; Shi, C.M. Transshipment and coordination in a two-echelon supply chain. RAIRO-Oper. Res. 2017, 51, 729-747. [CrossRef]
23. Chen, J.; Zhang, H.; Sun, Y. Implementing coordination contracts in a manufacturer Stackelberg dual-channel supply chain. Omega-Int. J. Manag. Sci. 2012, 40, 571-583. [CrossRef]
24. He, Y.; Zhang, P.; Yao, Y.L. Unidirectional transshipment policies in a dual-channel supply chain. Econ. Model. 2014, 40, 259-268. [CrossRef]
25. Zhu, B.L.; Wen, B.; Ji, S.F. Coordinating a dual-channel supply chain with conditional value-at-risk under uncertainties of yield and demand. Comput. Ind. Eng. 2020, 139, 106181. [CrossRef]
26. Zha, Y.; Zhang, J.H.; Yue, X.H.; Hua, Z.S. Service supply chain coordination with platform effort-induced demand. Ann. Oper. Res. 2015, 235, 785-806. [CrossRef]
27. Kurata, H.; Yao, D.Q.; Liu, J.J. Pricing policies under direct vs. indirect channel competition and national vs. store brand competition. Eur. J. Oper. Res. 2007, 180, 262-281. [CrossRef]
28. Hua, G.; Wang, S.; Cheng, T.C.E. Price and lead time decisions in dual-channel supply chains. Eur. J. Oper. Res. 2010, 205, 113-126. [CrossRef]
