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Duty Cycle-Rotor Angular Speed Reverse Acting Relationship Steady State Analysis Based on a PMSG d–q Transform Modeling

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Abstract: Multilevel converters have been broadly used in wind energy conversion systems (WECS) to set the generator angular speed to a certain value, which allows maximizing wind power extraction; nevertheless, power that is drawn out from WECS strongly depends on the power coefficient and the ability to operate at the optimal tip speed ratio that corresponds to the maximum power coefficient. This work presents a novel and formal steady-state analysis to demonstrate the reverse relationship between the duty cycle of a multilevel boost converter (MBC) and the angular speed of a permanent magnet synchronous generator (PMSG). The study was based on the d–q transformation using the rotor reference frame. It was carried out by employing a reduced order dynamic system that included an equivalent electrical load resistance as a representation for the subsystems that were cascade-connected at the terminals of the PMSG. The steady-state characteristic was obtained by using the definition of equilibrium point. The set of nonlinear equations that represents the steady state of this WECS was solved by using the Newton method; besides, an analysis that considers the equivalent load as a bifurcation parameter demonstrates that the number of equilibria never changes.

Keywords: bifurcation analysis; d–q transform modeling; dynamic systems control; permanent magnet synchronous generator; multilevel boost converter

MSC: 37N35; 37M99

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1. Introduction

Wind energy conversion systems have multiplied considerably in recent decades. According to the latest GWEC report, their total installed power exceeds 650 GW [1]. Intelligent and innovative technologies based on various investigations related to the optimization of the extraction of wind power have been developed in recent years, such as optimization methodologies for avoiding low outputs, extreme-seeking control, yaw-based wake, angle control, and artificial neural networks [2–8]. Appropriately, there are several reviews on the state of the art of WECS regarding emerging technologies, innovative systems, trends, and challenges, demonstrating its importance [9–13].

In terms of wind energy extraction, the use of multilevel converters has been widely used [14–17]. In the more traditional models, a controller is in charge of sending a typically adjusted signal and then injecting it in the MBC's duty cycle. Regardless of the elements used in the control law, it is essential to guarantee optimization in wind energy extraction.

In general, for a given wind speed, there is an angular speed of the rotor of the associated turbine that allows optimization of the wind power extraction. The function of the control system is to supply the MBC with the appropriate signal so that the amount

of energy extracted from the WECS is such that the speed of the turbine rotor is optimal according to the wind speed at that time [18].

In this sense, it is essential to determine the relationship between the input and the system's output, this is, between the MBC duty cycle and the angular velocity of the wind turbine rotor, to allow for establishing a more appropriate control action for energy efficiency.

Within the WECS, there are two variants of coupling between the turbine and the generator. One is through a gearbox that multiplies the speed of the turbine rotor towards the generator, and the other is the direct-drive connection, where the angular speeds of the turbine and the generator have the same value [9]. It is common to use a PMSG in this mode of operation [19–23]; in fact, that is the configuration used in the analysis performed in this work. PMS generators present several advantages and have been widely used in WECS. Some interesting studies have been developed regardless of sensorless adaptive output feedback control, as shown in [24,25].

There are works in which a representation of variable load resistance is used at the output of the PMSG to analyze the phenomena in PMSG-based WECS. This resistance replaces a higher-order model for the uncontrolled rectifier, the MBC, the three-phase inverter, and the three-phase load. For example, in [26], a comprehensive MPPT was developed and validated for the full range of operating wind speed. In contrast, a unified dynamic model of the wind turbine generator was built to simplify the analysis. On the other hand, in [27], a neuro-adaptive generalized global sliding mode controller was used for maximizing wind power extraction using an equivalent model for the WECS.

For the particular case in which a purely resistive three-phase load is connected, it seems natural that the relationship between the input and the output of the mentioned system is inverse. However, no formal study has demonstrated this relationship, nor is there a complementary one which has demonstrated that a stable equilibrium exists and that it is unique. Using open-loop steady-state analysis, we studied those issues in this work.

This paper mainly presents a formal and innovative analysis that demonstrates the inverse relationship between the duty cycle of an MBC converter and the angular speed of the rotor of a PMSG in a direct-drive WECS for a purely resistive three-phase load. It also demonstrates the absence of the nonlinear bifurcation phenomenon. It shows the existence of an equilibrium for each value of the duty cycle and that it is stable.

PMSG was modeled using the d–q transform. The system of the uncontrolled rectifier, the 2N-MBC, the three-phase inverter, and the three-phase load were taken as variable equivalent electrical load resistances dependent on the duty cycle.

The paper is organized as follows: Section 2 provides a general overview of features of the WECS this analysis is based on. In Section 3, the mathematical model we solve is presented. Section 4 details the procedure for solving the mathematical model. Section 5 shows the results obtained, and finally, Section 6 summarizes the conclusions and future works.

2. General Features of the Wind Energy Conversion System

The WECS this study is based on is represented in Figure 1.

Figure 1. General overview of the WECS used in this research.

The block associated with the wind turbine emulator in Figure 1 was experimentally implemented by utilizing a Linux-based real-time platform, a variable speed drive, and an induction motor; see Figure 2. Once wind speed was specified as an input for this wind turbine emulator, the value for the mechanical torque was computed, and its corresponding signal was connected to the variable speed drive. This device was responsible for applying adequate voltages to the induction motor such that experimental mechanical torque at the motor shaft reached its desired value. Additionally, the value of rotor speed that maximizes wind power extraction (ω_{rsp}) was computed as required for the closed-loop implementation.

Figure 2. Wind turbine emulator.

The experimental wind turbine emulator that included the induction motor, was directly connected to the PMSG (gearless configuration). One of the advantages of this type of WECS is that installation of a gearbox is avoided, along with the problems related to its use [28–30], making it a less expensive configuration. Furthermore, since its operating speeds in the asynchronous mode are extensive, the PMSG is preferred over other electrical generators [26]. Table 1 shows the parameters associated with the experimental four-pole three-phase PMSG.

Table 1. Parameters associated with the PMSG.

Symbol	Value	Units	Description
R_s	0.69	Ω	Stator resistance
L_q	0.0068	H	q-axis stator inductance
L_d	0.0068	H	d-axis stator inductance
P	4	Poles	Number of poles
λ_m	0.4411	Vs	Flux linkages amplitude
R_{Lmin}	3.3647	Ω	Minimum equivalent electrical load resistance
R_{Lmax}	115	Ω	Maximum equivalent electrical load resistance
R_L	3.3647–115	Ω	Equivalent electrical load resistance
J	0.1488	kgm ²	Combined inertia

This WECS contained a three-phase full-rated full-wave diode rectifier for transforming the variable frequency voltage at the terminals of the generator into a varying magnitude DC voltage. Power was then transferred from the output of the diode rectifier to the custom-made full-rated 2N-MBC [31,32]. The experimental PWM frequency of the 2N-MBC was 50 kHz. The discrete-time controller in Figure 1 defined in real-time the duty cycle of the 2N-MBC. An IGBT-based full-rated voltage source inverter was utilized to transfer power from the output of the 2N-MBC to the wye-connected electrical load. The frequency at the output terminals of the source inverter was fixed at 60 Hz; in other words, the electric load was always fed by a set of 60 Hz three-phase voltages. The PMSG used in this work was manufactured by Wind-Blue Power. Its rated power and rated rotor speed are 300 W and 1800 RPM, respectively. Figure 3 shows the experimental emulator.

Figure 3. Experimental emulator used in this research.

More details regarding to the methodology and instrumentation used in the experimental setup, along with the controller performance to maximize wind power extraction, can be found in [33].

3. Mathematical Model

A formal, open-loop, steady-state analysis is described in this section, which was used to investigate the inverse relationship between the input and the system's output to be controlled and the occurrence of nonlinear bifurcation phenomena. The input was the duty cycle (D) applied to the 2N-MBC, and the output was the turbine-generator speed (ω_r). This study is significant, as the WECS extracted power is noticeably affected by the power coefficient (C_p), a function whose monotonicity may grow or diminish along the tip speed ratio axis (λ).

To simplify the high-order model representative of the uncontrolled rectifier, the 2N-MBC, and the three-phase inverter and load, as shown in Figure 1, a mathematical expression to calculate a variable equivalent electrical load resistance is proposed, as shown in Equation (1). It is worth mentioning that some works where similar topology employs a variant resistor to analyze phenomena in PMSG-based WECS have already been published [26,27].

In Equation (1), it can be easily seen that if $D = 1$, then $R_L = R_{Lmin}$; on the other hand, if $D = 0$, then $R_L = R_{Lmax}$, as can be seen in Table 1, $R_{Lmax} \gg R_{Lmin}$, so for this case, the model has the intention of representing a load resistance that is open-circuited.

$$R_L = (R_{Lmin} - R_{Lmax})D + R_{Lmax} \quad (1)$$

Now, based on the assumptions related to the model and according to [34,35], first principles can be utilized to obtain the corresponding state-space representation, as is shown in Equations (2) and (3).

$$\dot{x} = f(x, p) \quad (2)$$

where

$$f(x, p) = \begin{bmatrix} \left[-(R_s + R_L)i_{qs}^r - \omega_r L_d i_{ds}^r - \omega_r \lambda_m \right] / L_q \\ \left[-(R_s + R_L)i_{ds}^r - \omega_r L_q i_{qs}^r \right] / L_d \\ P(T_e - T_{\omega t}) / (2J) \end{bmatrix} \quad (3)$$

$x = [i_{qs}^r \ i_{ds}^r \ \omega_r]^T$ represents the state vector and p is the bifurcation parameter. i_{qs}^r and i_{ds}^r are the stator currents in rotor's reference frame; ω_r is the turbine-generator speed; L_q , L_d ,

R_s , R_L , P , J , and λ_m are defined in Table 1. T_{wt} is the turbine torque, and it is defined by Equation (4). T_e represents the steady-state and transient electrical torque, which is defined by Equation (9).

$$T_{wt} = \frac{P_{wt}}{\omega_r} \quad (4)$$

P_{wt} is the wind turbine power that can be obtained by Equation (5), where $\rho = 1.2 \text{ kg/m}^3$ is the air density, $A_{wt} = \pi R_{wt}^2$ is the wind turbine swept area in m^2 , $R_{wt} = 0.6 \text{ m}$ is the wind turbine swept area radius, v_w represents the wind speed in m/s , and $C_p(\lambda, \beta)$ is the power coefficient that is obtained by Equation (6). λ is the tip speed ratio and can be obtained by Equation (7). β represents the pitch angle; in this case its assigned value was zero. It is worth mentioning that there are several mathematical models for C_p —for example, exponential, sinusoidal, and polynomial representations [36]. In this work a particular sinusoidal representation was chosen, because it was the one which showed the best performance.

$$P_{wt} = \frac{\rho}{2} A_{wt} C_p(\lambda, \beta) \quad (5)$$

$$C_p(\lambda, \beta) = [a_0 + a_1(b_0\beta + a_2)] \sin\left(\frac{\pi(\lambda + a_3)}{a_4 + a_5(b_1\beta + a_6)}\right) + a_7(\lambda + a_8)(b_2\beta + a_9) \quad (6)$$

$$\lambda = \frac{\omega_r R_{wt}}{v_w} \quad (7)$$

The values of the coefficients selected for Equation (6) are indicated in Table 2:

Table 2. Coefficients' values for Equation (6).

Coefficient	a_0	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	b_0	b_1	b_2
Value	0.5	−0.00167	−2	0.1	18.5	−0.3	−2	0.00184	−3	−2	1	1	1

By combining Equations (4)–(6), we obtained the final expression for T_{wt} used in this analysis, as shown in Equation (8)

$$T_{wt} = \frac{\rho \pi R_{wt}^2 v_w^3}{2 \omega_r} \left\{ [a_0 + a_1(\beta + a_2)] \sin\left(\frac{\pi\left(\frac{\omega_r R_{wt}}{v_w} + a_3\right)}{a_4 + a_5\left(\frac{\omega_r R_{wt}}{v_w} + a_6\right)}\right) + a_7\left(\frac{\omega_r R_{wt}}{v_w} + a_8\right)(\beta + a_9) \right\} \quad (8)$$

$$T_e = \left(\frac{3}{2}\right) \left(\frac{P}{2}\right) [\lambda_m i_{qs}^r + (L_d - L_q) i_{qs}^r i_{ds}^r] \quad (9)$$

The mathematical model given by Equations (2), (3), and (9) is indicated in actual quantities (not per-unit), and it is distinctly expressed in the $qd0$ rotor reference frame utilizing standard notation [34]. Voltages at the generator terminals are implicitly accounted for in Equations (2) and (3), since they have been represented as functions of flowing currents through R_L . In particular,

$$\begin{cases} v_{qs}^r &= -r_L i_{qs}^r \\ v_{ds}^r &= -r_L i_{ds}^r \end{cases} \quad (10)$$

4. Methodology

In order to solve the nonlinear system shown in Equation (3), the Newton method was used. Thus, three functions are defined as shown in Equations (11)–(13), where T_{wt} is defined by Equation (8). The system was to be solved in terms of $x = [i_{qs}^r \ i_{ds}^r \ \omega_r]^T$. Initial conditions were established as $x_0 = [-10 \ -1.77 \ -18]^T$.

$$f_1(x, p) = \frac{-(R_s + R_L)i_{qs}^r - \omega_r L_d i_{ds}^r - \omega_r \lambda_m}{L_q} \quad (11)$$

$$f_2(x, p) = \frac{-(R_s + R_L)i_{ds}^r - \omega_r L_q i_{qs}^r}{L_d} \quad (12)$$

$$f_3(x, p) = \frac{P \left(\frac{3P}{4} [\lambda_m i_{qs}^r + (L_d - L_q) i_{qs}^r i_{ds}^r] - T_{\omega t} \right)}{2J} \quad (13)$$

The associated Jacobian matrix is given by expression (14)

$$J_M = \begin{bmatrix} \frac{\delta}{\delta i_{qs}^r} f_1(x, p) & \frac{\delta}{\delta i_{ds}^r} f_1(x, p) & \frac{\delta}{\delta \omega_r} f_1(x, p) \\ \frac{\delta}{\delta i_{qs}^r} f_2(x, p) & \frac{\delta}{\delta i_{ds}^r} f_2(x, p) & \frac{\delta}{\delta \omega_r} f_2(x, p) \\ \frac{\delta}{\delta i_{qs}^r} f_3(x, p) & \frac{\delta}{\delta i_{ds}^r} f_3(x, p) & \frac{\delta}{\delta \omega_r} f_3(x, p) \end{bmatrix} \quad (14)$$

The equilibrium point is obtained by repeatedly applying Equation (15), where x_n is the new state vector to be obtained in each iteration, x_{n-1} is the previous state vector, J_M^{-1} represents the inverse Jacobian matrix which must be evaluated in x_{n-1} , and $f(x_{n-1}, p)$ is $\dot{x}$ evaluated in x_{n-1} .

$$x_n = x_{n-1} - \left(J_M^{-1} \Big|_{x_{n-1}} \right) f(x_{n-1}, p) \quad (15)$$

The system can be solved for different values of wind speed (v_w); nevertheless, in this paper, graphics and associated tables were obtained for ($v_w = 7$ m/s). On the other hand, an external loop was established for variable equivalent electrical load resistance, and the vector was built from R_{Lmin} to R_{Lmax} —these values correspond to a specific MBC duty cycle (D). In the program, 214 iterations were established; then, for each specific value of D, an inner loop containing the Newton method to obtain the equilibrium point was set, so an epsilon error condition was established to end the calculation for the every equilibrium point corresponding to each duty cycle. In the end, 214 equilibrium points were obtained, one for each specific 2B-MBC duty cycle.

Eigenvalues for final iteration Jacobian matrices corresponding to each value of D were also obtained, in order to set a bifurcation and for stability analysis. Finally, instructions for plotting and generating of tables associated with the graphics were established.

Figure 4 shows a flow diagram indicating the procedure for solving the nonlinear system.

Figure 4. Flow diagram for solving the nonlinear system.

5. Results and Discussion

Table 3 contains values that represent the relationship between the electrical equivalent load resistance and the turbine-PMSG rotational speed corresponding to 23 chosen iterations. As can be appreciated in Figure 5, there is a nonlinear, inverse relationship between the variables involved. Limits for the x-axis are $[R_{Lmin}, R_{Lmax}]$.

Figure 5. Turbine-PMSG rotational speed as electrical equivalent load resistance function.

Table 3. Values associated with Figure 5.

Iteration	R_L (Ω)	ω_r (rad/s)
1	3.3647	13.24289364
11	8.7356	27.4005822
21	14.106	40.41648182
31	19.477	52.17736821
41	24.848	62.66736099
51	30.219	71.96855457
61	35.59	80.20869496
71	40.961	87.52521686
81	46.332	94.04717091
91	51.703	99.8882987
101	57.074	105.1457687
111	62.444	109.9013685
121	67.815	114.2235352
131	73.186	118.1694878
141	78.557	121.7871656
151	83.928	125.1168832
161	89.299	128.192701
171	94.67	131.0435415
181	100.04	133.6940936
191	105.41	136.1655434
201	110.78	138.4761643
211	116.15	140.6417963
214	117.76	141.2653233

Table 4 contains values associated with the relationship between the 2N-MBC duty cycle and the Jacobian matrix determinant evaluated at the state state vector equilibrium point corresponding to 23 chosen iterations. As can be seen in Figure 6, the determinant increases as the number of duty cycles increases, so there is a nonlinear direct relationship between D and the Jacobian matrix determinant.

Table 4. Values associated with Figure 6.

Iteration	Duty Cycle	Jacobian Determinant ($\times 10^6$)
1	1	−2.453
11	0.95327	−5.5013
21	0.90654	−8.9032
31	0.85981	−12.822
41	0.81308	−17.337
51	0.76636	−22.48
61	0.71963	−28.26

Table 4. Cont.

Iteration	Duty Cycle	Jacobian Determinant ($\times 10^6$)
71	0.6729	−34.669
81	0.62617	−41.696
91	0.57944	−49.325
101	0.53271	−57.54
111	0.48598	−66.328
121	0.43925	−75.675
131	0.39252	−85.568
141	0.34579	−95.996
151	0.29907	−106.95
161	0.25234	−118.42
171	0.20561	−130.4
181	0.15888	−142.87
191	0.11215	−155.85
201	0.065421	−169.31
211	0.018692	−183.25
214	0.0046729	−187.53

Figure 6. Jacobian determinant as a function of electrical equivalent load resistance.

Table 5 shows the corresponding values between the electrical equivalent load resistance and the module of the state vector's derivative. This table is relevant since it shows the absolute errors associated with different values of R_L for the same number of iterations. As can be seen in Figure 7, there is an oscillating, increasing trend between R_L and $\dot{x}$. The order of the absolute error is 10^{-12} .

Table 5. Values associated with Figure 7.

Iteration	$R_L(\Omega)$	Derivative Module ($\times 10^{-14}$)
1	3.3647	6.1968
11	8.7356	10.578
21	14.106	4.4067
31	19.477	7.5534
41	24.848	41.163
51	30.219	39.107
61	35.59	52.314
71	40.961	41.255
81	46.332	3.3455
91	51.703	28.366
101	57.074	78.76
111	62.444	5.907
121	67.815	77.856
131	73.186	47.463
141	78.557	15.151
151	83.928	24.73
161	89.299	146.16
171	94.67	82.665
181	100.04	80.601
191	105.41	32.787
201	110.78	94.37
211	116.15	23.906
214	117.76	9.2541

Figure 7. Derivative magnitude as a function of electrical equivalent load resistance.

Table 6 indicates the corresponding values between the electrical equivalent load resistance and the tip speed ratio. The minimum and maximum values for λ are within a typical tip speed ratio range. Figure 8 shows a directly nonlinear relationship between R_L and λ .

Table 6. Values associated with Figure 8.

Iteration	$R_L(\Omega)$	Tip Speed Ratio
1	3.3647	1.1351
11	8.7356	2.3486
21	14.106	3.4643
31	19.477	4.4723
41	24.848	5.3715
51	30.219	6.1687
61	35.59	6.875
71	40.961	7.5022
81	46.332	8.0612
91	51.703	8.5619
101	57.074	9.0125
111	62.444	9.4201
121	67.815	9.7906
131	73.186	10.129
141	78.557	10.439
151	83.928	10.724
161	89.299	10.988
171	94.67	11.232
181	100.04	11.459
191	105.41	11.671
201	110.78	11.869
211	116.15	12.055
214	117.76	12.108

Table 7 is very important, since it indicates the values between the 2N-MBC duty cycle and the turbine-PMSG angular speed, demonstrating the inverse relationship between these two variables, which was one of the main goals of this research. Figure 9 shows the inverse nonlinear relationship between D and ω_r . The stator current in the original coordinates for phase “a” was calculated by Equation (16); then, three particular orbits were selected, as shown in Figure 9, to represent the current behavior. The red sine wave represents i_{as} when $D = 0.25701$, the green function is i_{as} when $D = 0.63084$, and the purple sine function shows i_{as} when $D = 0.88785$. It can be appreciated that the higher the value of D , the the lower frequency and higher amplitude of i_{as} , as a consequence of a higher voltage in the 2N-MBC output terminals. Table 8 contains the corresponding values for the phase “a” stator currents in original coordinates for the three orbits previously described, and the 33 values of the time vector in the interval $[0, 0.4996]$ seconds.

$$i_{as} = i_{qr}^s \cos(\omega_r t) + i_{ds} \sin(\omega_r t) \quad (16)$$

Figure 8. Tip speed ratio as a function of electrical equivalent load resistance.

Table 7. Relationship data for turbine-PMSG angular speed and duty cycle.

Iteration	Duty Cycle	ω_r (rad/s)
1	1	13.24289364
11	0.95327	27.4005822
21	0.90654	40.41648182
31	0.85981	52.17736821
41	0.81308	62.66736099
51	0.76636	71.96855457
61	0.71963	80.20869496
71	0.6729	87.52521686
81	0.62617	94.04717091
91	0.57944	99.8882987
101	0.53271	105.1457687
111	0.48598	109.9013685
121	0.43925	114.2235352
131	0.39252	118.1694878
141	0.34579	121.7871656
151	0.29907	125.1168832
161	0.25234	128.192701
171	0.20561	131.0435415
181	0.15888	133.6940936
191	0.11215	136.1655434
201	0.065421	138.4761643
211	0.018692	140.6417963
214	0.0046729	141.2653233

Table 8. Relationship data for phase A stator current in original coordinates and time for three specific values of duty cycle.

Iteration	Time (s)	$I_{as1}(A)$	$I_{as2}(A)$	$I_{as3}(A)$
1	0	−1.1782	−0.88638	−0.63061
28	0.015613	−0.90981	−0.11125	0.25499
55	0.031225	−0.20545	0.86148	0.41988
82	0.046838	0.59735	0.30409	−0.60199
109	0.06245	1.1139	−0.79341	0.077613
136	0.078063	1.0967	−0.4817	0.53785
163	0.093675	0.55396	0.68558	−0.5221
190	0.10929	−0.25423	0.63517	−0.10638
217	0.1249	−0.9406	−0.54339	0.61001
244	0.14051	−1.1762	−0.75681	−0.39775
271	0.15613	−0.84825	0.37398	−0.2813
298	0.17174	−0.11378	0.84052	0.63023
325	0.18735	0.67521	−0.18582	−0.23952
352	0.20296	1.1406	−0.88212	−0.43228
379	0.21858	1.0595	−0.01164	0.59677
406	0.23419	0.47065	0.87951	−0.060901
433	0.2498	−0.34372	0.20852	−0.54644
460	0.26541	−0.99339	−0.83284	0.51249
487	0.28103	−1.167	−0.39496	0.12291
514	0.29664	−0.78145	0.74442	−0.61406
541	0.31225	−0.021407	0.5616	0.38456
568	0.32786	0.7489	−0.61871	0.29625
595	0.34348	1.1603	−0.7001	−0.62939
622	0.35909	1.0158	0.46199	0.22389
649	0.3747	0.38444	0.80351	0.44437
676	0.39031	−0.4311	−0.28212	−0.59112
703	0.40593	−1.0401	−0.86667	0.044146
730	0.42154	−1.1506	0.088114	0.55464
757	0.43715	−0.70984	0.88639	−0.50251
784	0.45276	0.071099	0.11031	−0.13935
811	0.46838	0.81797	−0.8617	0.61767
838	0.48399	1.1729	−0.3032	−0.3711
865	0.4996	0.96576	0.79383	−0.31099

Figure 9. Turbine-generator speed steady-state as a function of the duty cycle and three representative steady-state orbits in original coordinates.

Table 9 presents the equilibrium points for different duty cycle values. For a particular D value, there are three elements for the equilibrium point; they are represented by three asterisks in Figure 10. Two are complex conjugate and one is located in the real axis. As can be seen, the bifurcation analysis shows that there is neither a saddle node, nor a pitchfork bifurcation, nor a transcritical bifurcation, nor a Hopf bifurcation occur, when the equivalent electrical load resistance is considered as the bifurcation parameter. This means that there is no eigenvalue for the Jacobian matrix evaluated in the equilibrium point for each duty cycle value that crosses the imaginary axis. As a result, the critical boundary in the parametric space is non-existent and the number of equilibria is constant. Besides, each equilibrium represented in Figure 10 is locally asymptotically stable, as all of the corresponding eigenvalues are always located on the open left half complex plane. It is important to note that the real axis in Figure 10 has a logarithmic scale.

Figure 10. Eigenvalues as the duty cycle is varied.

Table 9. Eigenvalues associated with different duty cycle values.

Iteration	Duty Cycle	Real Part 1	Imaginary Part 1	Real Part 2	Imaginary Part 2	Real Part 3	Imaginary Part 3
1	1	−593.35	−13.048	−593.35	13.048	−6.9643	0
11	0.95327	−1384.9	−27.422	−1384.9	27.422	−2.8673	0
21	0.90654	−2175.2	−40.438	−2175.2	40.438	−1.8811	0
31	0.85981	−2965.2	−52.195	−2965.2	52.195	−1.4578	0
41	0.81308	−3755.2	62.681	−3755.2	−62.681	−1.2291	0
51	0.76636	−4545.1	71.98	−4545.1	−71.98	−1.0879	0
61	0.71963	−5335	80.218	−5335	−80.218	−0.99268	0
71	0.6729	−6124.8	87.533	−6124.8	−87.533	−0.92399	0
81	0.62617	−6914.7	94.054	−6914.7	−94.054	−0.8719	0
91	0.57944	−7704.6	99.894	−7704.6	−99.894	−0.83079	0
101	0.53271	−8494.4	105.15	−8494.4	−105.15	−0.79733	0
111	0.48598	−9284.3	109.91	−9284.3	−109.91	−0.76938	0
121	0.43925	−10074	114.23	−10074	−114.23	−0.74556	0
131	0.39252	−10864	118.17	−10864	−118.17	−0.72491	0
141	0.34579	−11654	121.79	−11654	−121.79	−0.70676	0
151	0.29907	−12444	125.12	−12444	−125.12	−0.69062	0
161	0.25234	−13234	128.2	−13234	−128.2	−0.67613	0
171	0.20561	−14023	131.05	−14023	−131.05	−0.66302	0
181	0.15888	−14813	133.7	−14813	−133.7	−0.65106	0
191	0.11215	−15603	136.17	−15603	−136.17	−0.6401	0
201	0.065421	−16393	138.48	−16393	−138.48	−0.62999	0
211	0.018692	−17183	140.64	−17183	−140.64	−0.62063	0
214	0.0046729	−17420	141.27	−17420	−141.27	−0.61795	0

6. Conclusions

This research demonstrated a nonlinear inverse relationship between a 2N-MBC duty cycle and the turbine-PMSG rotational speed of a WECS. It was also concluded that for higher duty cycles, there will be smaller stator current frequencies and larger amplitudes. This is a logical fact because of the increasing voltage at the 2N-MBC output terminals. The steady-state analysis also showed that every eigenvalue of the linearized system was located in the left half of the complex plane; therefore, every equilibrium is locally asymptotically stable. It is important to recall that the coefficient matrix of the linearized system is obtained by evaluating the Jacobian matrix at the nominal steady state point (equilibrium) for each particular duty cycle. In our case, eigenvalues never cross the imaginary axis of the complex plane; therefore, the number of equilibria never changes, and the bifurcation surface does not appear in the parametric space. The proposed model that represents the uncontrolled rectifier, the 2N-MBC, the three-phase inverter, and the three-phase load allowed the system to be simplified in order to obtain valuable results for pure resistive electric loads. In practice, nonlinear dynamic electric loads are usually connected to WECS, so different and further analysis will be carried out as future work.

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Abbreviations

The following abbreviations are used in this manuscript:

MBC	Multilevel Boost Converter
GW	Gigawatt
GWEC	Global Wind Energy Council
WECS	Wind Energy Conversion Systems

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