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Event-Triggered Asynchronous Filter of Nonlinear Switched Positive Systems with Output Quantization

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Abstract: This paper deals with a static/dynamic event-triggered asynchronous filter of nonlinear switched positive systems with output quantization. The nonlinear function is located in a sector. Both static and dynamic event-triggering conditions are established based on the 1-norm form. By virtue of the event-triggering mechanism, the error system is transformed into an interval uncertain system. An event-triggered asynchronous filter is designed by employing a matrix decomposition approach. The positivity and L_1 -gain stability of the error system are guaranteed by means of linear copositive Lyapunov functions and a linear programming approach. Finally, two examples are given to verify the effectiveness of the design.

Keywords: switched nonlinear positive systems; event-triggered filter; asynchronous switching; linear programming

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1. Introduction

As an important class of hybrid systems, switched positive systems composing of a series of positive subsystems and a switching rule to coordinate the operation of subsystems have attracted extensive attention [1,2]. Compared with the general (non-positive) switched systems [3–5], switched positive systems are more suitable to accurately model a kind of practical system consisting of nonnegative quantities, such as communication networks [6], chemical engineering [7], and water systems [8]. In [9,10], the stability and stabilization of switched positive systems were investigated based on linear copositive Lyapunov functions.

The study [11] dealt with the issue of L_1 -gain characterization for switched positive systems by virtue of copositive Lyapunov function and linear programming approach. The L_1 -gain analysis and control synthesis of switched positive systems was investigated in [12] using multiple linear copositive Lyapunov functions incorporated with the average dwell time approach. More results on a switched positive systems can be found in [13–15]. The above literature mainly investigated linear switched positive systems. In fact, nonlinear processes exist in most practical systems. A linear model cannot describe the nonlinear processes accurately.

Modeling such systems via nonlinear switched systems will have less error than linear switched systems. In [16], the distributed filter was proposed for nonlinear switched positive systems with stochastic nonlinearities and missing measurements based on switched Lyapunov function and linear programming. A sector nonlinearity was first introduced to ensure the positivity of nonlinear switched positive systems in [17,18], where the considered nonlinear functions are located in a sector.

A robust fault detection filter was designed for nonlinear switched systems with time-varying delay based on the average dwell time approach and the Lyapunov functional technique [19]. In [20], the issue of H_∞ filter of nonlinear switched systems with stable and unstable subsystems was solved by means of the mode-dependent average dwell time technique. Further results about nonlinear switched systems can refer to [21–25].

The filter design for switched systems in the literature mentioned above is mainly based on synchronous switching. It should be pointed out that it takes time to identify which subsystem is activated and which matched filter is activated. The transmission time delay of the switching signal or the impact of the external factors may result in asynchronous switching between the filters and the switched systems [26,27]. Therefore, an asynchronous filter is more practical than a synchronous one.

An asynchronous ℓ_1 positive filter of switched positive systems with modal-dwell-time was proposed in [28]. Using the average dwell time and linear matrix inequality, the study [29] was concerned with the H_∞ filtering problem of linear switched systems with asynchronous switching. An L_1 -gain filter of switched positive systems was investigated by introducing a clock-dependent Lyapunov function [30]. In addition, the issue of quantization was considered in [27], which can deal with the failure phenomenon of elements. The quantization can also guarantee the safety of information transmission [31–33].

The study [31] was concerned with feedback stabilization problems for linear time-invariant control systems with saturating quantized measurements. In [32], the authors investigated a design method of a time-varying quantizer to stabilize switched systems with quantized output and switching delays based on a dwell-time assumption and level sets of a common Lyapunov function. Using the sojourn probability-based switching law and parameterized Lyapunov functional, the literature [33] addressed the issue of quantized H_∞ filtering for switched linear parameter-varying systems with both sojourn probabilities and unreliable communication channels. How can we establish an asynchronous filter framework of nonlinear switched positive systems and solve the signal quantization based on a linear approach? These questions motivate the current investigation.

Up to now, many related results on event-triggering mechanism have been reported in [34–38]. Event-triggered communication mechanism provides a more effective and practical method for solving the control issues than time-triggered sample to reduce unnecessary signal transmission. The study [39] investigated the event-triggered L_1 -gain filter of switched positive systems subject to state saturation using linear programming and linear copositive Lyapunov function.

In [40], an event-triggered filter of positive systems was designed by adopting a matrix decomposition approach and linear copositive Lyapunov functions. An event-triggered filter of switched positive systems subject to state saturation was investigated by resorting to linear programming and average dwell time technology [41]. More recently, a dynamic event-triggered mechanism, which was developed from the static one, has been presented in [42,43]. In [44], the issue of recursive distributed filtering was investigated for nonlinear time-varying systems under a dynamic event-triggered mechanism.

With the help of the mathematical induction and Lyapunov theorem, the study [45] presented a dynamic event-triggered control scheme for linear time-invariant systems. The study [46] dealt with the stability of linear stochastic systems based on the dynamic event-triggered mechanism with an impulsive switched system approach. To the best of authors' knowledge, there have been no research achievements regarding the asynchronous filter design of nonlinear switched positive systems under a static/dynamic event-triggering mechanism. Therefore, applying static and dynamic event-triggering communication mechanisms to the asynchronous filter design of switched nonlinear positive systems is one motivation of this work.

In this paper, we focus on the event-triggered L_1 -gain asynchronous filter of nonlinear switched positive systems with output quantization. Static and dynamic event-triggering schemes based on 1-norm inequality are constructed for the considered systems, respectively. The filter gain matrices are designed by using the matrix decomposition technique to guarantee the positivity and L_1 -gain stability of the underlying systems. The outline of the paper is as follows: Section 2 provides the problem formulation; Section 3 presents the main results; Two examples are given in Section 4; and Section 5 concludes this paper.

Notation 1. Let $\mathbb{R}^n$ (or $\mathbb{R}_+^n$) and $\mathbb{R}^{n \times m}$ be sets of n -dimensional vectors (or, nonnegative) and $n \times m$ matrices, respectively. The symbols $\mathbb{N}$ and $\mathbb{N}_+$ denote the sets of nonnegative and positive integers, respectively. For a matrix $A = [a_{ij}]$, $A \succeq 0$ ($\succ 0$) indicates that $a_{ij} \geq 0$ ($a_{ij} > 0$), $\forall i, j = 1, \dots, n$, where a_{ij} is the element in the i th row and j th column of A . $A^\top$ stands for the transpose of matrix A .

For $v \in \mathbb{R}^n$, $v^{(i)}$ is the i th element of the vector. $v \succeq 0$ ($\succ 0$) means $v^{(i)} \succeq$ ($\succ 0$), $\forall i = 1, \dots, n$. The 1-norm of $x = (x_1, x_2, \dots, x_n)$ is defined by $\|x\|_1 = \sum_{i=1}^n |x_i|$, and the ℓ_1 norm of the vector is $\sum_{k=0}^\infty \|x(k)\|_1$. Define $1_n = (\underbrace{1, \dots, 1}_n)^\top \in \mathbb{R}^n$ and $1_n^{(i)} = (\underbrace{0, \dots, 0}_i, \underbrace{0, \dots, 0}_{n-i})^\top$. A matrix I denotes the identity matrix with appropriate dimensions, and $1_{n \times n} \in \mathbb{R}^{n \times n}$ is a matrix with all the elements being 1. The logic operator $a \vee b$ means that a is valid or b is valid.

2. Preliminaries

Consider the discrete-time nonlinear switched system:

$$\begin{aligned} x(k+1) &= A_{\sigma(k)}f(x(k)) + B_{\sigma(k)}g(\omega(k)), \\ y(k) &= C_{\sigma(k)}h(x(k)) + D_{\sigma(k)}l(\omega(k)), \\ z(k) &= E_{\sigma(k)}p(x(k)) + F_{\sigma(k)}q(\omega(k)), \end{aligned} \quad (1)$$

where $x(k) = (x_1(k), \dots, x_n(k))^\top \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^m$, $\omega(k) \in \mathbb{R}_+^m$, and $z(k) \in \mathbb{R}^s$ are the system state, measurable output, disturbance, and output to be estimated, respectively. The nonlinear functions satisfy that

$$\begin{aligned} f(x) &= (f_1(x_1), \dots, f_n(x_n))^\top, h(x) = (h_1(x_1), \dots, h_n(x_n))^\top, \\ p(x) &= (p_1(x_1), \dots, p_n(x_n))^\top, g(\omega) = (g_1(\omega_1), \dots, g_m(\omega_m))^\top, \\ l(\omega) &= (l_1(\omega_1), \dots, l_m(\omega_m))^\top, q(\omega) = (q_1(\omega_1), \dots, q_m(\omega_m))^\top. \end{aligned}$$

The function $\sigma(k)$ denotes the switching law taking values at a finite set $S = \{1, 2, \dots, N\}$, $N \in \mathbb{N}_+$, where N represents the number of subsystems. Assume that the i th subsystem is invoked when $\sigma(k) = i$.

Assumption 1. The system matrices satisfy that $A_i \succeq 0$, $B_i \succeq 0$, $C_i \succeq 0$, $D_i \succeq 0$, $E_i \succeq 0$, and $F_i \succeq 0$ for each $i \in S$.

Assumption 2. The nonlinear functions $f(x)$, $g(\omega)$, $h(x)$, $l(\omega)$, $p(x)$, and $q(\omega)$ are located in some sector fields with

$$\omega_1 x_i^2 \leq f_i(x_i)x_i \leq \omega_2 x_i^2, \omega_3 x_i^2 \leq h_i(x_i)x_i \leq \omega_4 x_i^2, \omega_5 x_i^2 \leq p_i(x_i)x_i \leq \omega_6 x_i^2, \quad (2)$$

$$\varepsilon_1 \omega_i^2 \leq g_i(\omega_i)\omega_i \leq \varepsilon_2 \omega_i^2, \varepsilon_3 \omega_i^2 \leq l_i(\omega_i)\omega_i \leq \varepsilon_4 \omega_i^2, \varepsilon_5 \omega_i^2 \leq q_i(\omega_i)\omega_i \leq \varepsilon_6 \omega_i^2, \quad (3)$$

where $i = 1, 2, \dots, n$, $\iota = 1, 2, \dots, m$, $0 < \omega_1 \leq \omega_2$, $0 < \omega_3 \leq \omega_4$, $0 < \omega_5 \leq \omega_6$, $0 < \varepsilon_1 \leq \varepsilon_2$, $0 < \varepsilon_3 \leq \varepsilon_4$, $0 < \varepsilon_5 \leq \varepsilon_6$, and $f_i(0) = 0$.

Some preliminaries about positive systems are introduced.

Definition 1 ([1,2]). A system is said to be positive if all its states and outputs are nonnegative for any nonnegative initial conditions and nonnegative inputs.

Remark 1. There indeed exist some systems whose states and outputs are nonnegative for some non-positive initial conditions and inputs. The nonnegativity of these systems only holds for some of initial conditions and inputs rather than any nonnegative initial conditions and inputs. In

this paper, the definition of positive system means that the states and outputs are nonnegative for any nonnegative initial conditions and inputs. The definition follows the notions in [1,2]. Such a definition is to guarantee the essential nonnegativity of a system for any nonnegative initial conditions and inputs. Thus, the nonnegative initial conditions are required.

Lemma 1 ([1,2]). A system $x(k+1) = Ax(k)$ is positive if and only if $A \succeq 0$.

Lemma 2 ([1,2]). Given a matrix $A \succeq 0$, the following conditions are equivalent:

- (i) The matrix A is a Schur matrix.
- (ii) There exists some vector $v \succ 0$ such that $(A - I)v \prec 0$.

Definition 2 ([3]). For a switching signal $\sigma(k)$ and $0 \leq k_1 \leq k_2$, denote the number of the switching of $\sigma(k)$ by $N_\sigma(k_2, k_1)$. If there exist $N_0 \geq 0$ and $\tau_a > 0$ such that

$$N_\sigma(k_2, k_1) \leq N_0 + (k_2 - k_1) / \tau_a,$$

then τ_a is an average dwell time of the switching signal $\sigma(k)$.

Definition 3 ([41]). The system (1) is said to be ℓ_1 -gain stable if the following statements hold:

- (i) For $\omega(k) = 0$, the system (1) is asymptotically stable.
- (ii) Under zero-initial conditions, the following inequality holds for $\omega(k) \neq 0$,

$$\sum_{k=0}^{\infty} e^{-\hbar k} \|e(k)\|_1 \leq \gamma \sum_{k=0}^{\infty} \|\omega(k)\|_1,$$

where $\gamma > 0$ is the ℓ_1 -gain value and $\hbar > 0$.

3. Main Results

This section first explores the positivity of system (1). Then, a nonlinear asynchronous filter is designed under static event-triggering mechanism for system (1) with output quantization. Finally, a dynamic event-triggering filter for system (1) is proposed.

3.1. Positivity

Lemma 3. Under Assumption 2, system (1) is positive if and only if Assumption 1 holds.

Proof. *Necessity.* Let $x(0) = 0$. Then, $x(1) = B_i g(\omega(0))$ for some $i \in S$. By Assumption 2, $g(\omega(0)) \succeq 0$ for any $\omega(0) \succeq 0$. Since $x(1) \succeq 0$ for any $g(\omega(0)) \succeq 0$, then $B_i \succeq 0$.

Now, we prove that $A_i \succeq 0$ via reductio ad absurdum. Let $g(\omega(k)) = 0$. Suppose there exists an element $a_i^{(ij)} < 0$, then we find

$$x_i(k+1) = \sum_{j=1, j \neq i}^n (a_i^{(ij)} f_j(x_j(k)) + a_i^{(ij)} f_j(x_j(k))). \quad (4)$$

It is possible that $x_i(k+1) < 0$ if $a_i^{(ij)}$ takes a small value enough, which yields a contradiction with the positivity of system (1). Thus, $A_i \succeq 0$.

Sufficiency. Denote by F the set of indices that satisfies $x_i(k) = 0$ for $i \in F$. Then, for some $i \in S$

$$x_i(k+1) = \sum_{j \notin \Omega} a_i^{(ij)} f_j(x_j(k)) + \sum_{l=1}^m b_i^{(ul)} g_l(\omega_l(k)), i \in F, \quad (5)$$

where $a_i^{(ij)}$ is the i th row j th column element of A_i and $b_i^{(ul)}$ is the i th row l th column element of B_i . Note the condition (2), it follows that $f_j(x_j(k)) \geq 0$ for $k \in [0, +\infty)$. By Assumption 1, $a_i^{(ij)} \geq 0$ and $b_i^{(ul)} \geq 0$. From (3), we have $g(\omega(k)) \succeq 0$, $l(\omega(k)) \succeq 0$, and $q(\omega(k)) \succeq 0$ for $\omega(k) \in \mathbb{R}_+^m$. So, we have $x_i(k+1) \geq 0$ for $g(\omega(k)) \succeq 0$. By (2a), we have $h(x(k)) \succeq 0$ and $p(x(k)) \succeq 0$ for $x(k) \succeq 0$. Together these with $C_i \succeq 0$, $D_i \succeq 0$, $E_i \succeq 0$, and $F_i \succeq 0$ give $y(k) \succeq 0$ and $z(k) \succeq 0$. $\square$

The proof of Lemma 3 follows the proof of the positivity in [1,2]. The sector conditions in Assumption 2 are key to the positivity of system (1).

3.2. Static Event-Triggering Case

This subsection aims to design an event-triggered nonlinear asynchronous filter for a switched nonlinear positive system (1). A more general case is investigated, where the nonlinear function is unknown. This implies that the nonlinear function $f(x(k))$ cannot be used for the filter design. Thus, a nonlinear function $\hat{f}(x)$ is introduced to estimate the unknown nonlinear function $f(x)$. Then, the corresponding filter design is more difficult compared with the former filter design.

First, we introduce an event-triggering mechanism to detect and manage the transmission of output variables. Define $e_y(k) = \tilde{y}(k) - y(k)$, where $\tilde{y}(k) = y(k_\varphi)$, $y(k_\varphi)$ is the output of event generator at the event-triggering instant k_φ , $\varphi \in \mathbb{N}$. The measurement output will be released only when the following event-triggering condition is satisfied:

$$\|e_y(k)\|_1 > \beta \|y(k)\|_1, \quad (6)$$

where $k \in [k_\varphi, k_{\varphi+1})$ and $\beta \in [0, 1)$ is the event-triggering coefficient.

To further reduce the design cost and increase the practicability of the filter, we introduce a quantization technique to measure the output signal. Figure 1 is the event-triggered nonlinear quantization filter framework of switched nonlinear positive systems. The model of the quantized output signal is given as:

$$\bar{y}(k) = \bar{\mathcal{U}}(\tilde{y}(k)) = (\bar{\mathcal{U}}_1(\tilde{y}_1(k)), \bar{\mathcal{U}}_2(\tilde{y}_2(k)), \dots, \bar{\mathcal{U}}_m(\tilde{y}_m(k)))^\top,$$

where $\tilde{y}(k) \in \mathbb{R}^m$ denotes the quantized signal of the event generator's output signal $\tilde{y}(k)$ and $\bar{\mathcal{U}}(\tilde{y}(k))$ is the logarithmic quantizer. Moreover, the subquantizer $\bar{\mathcal{U}}_c(\tilde{y}_c(k))$ ($1 \leq c \leq m$) is characterized by the set of quantization levels:

Figure 1. The event-triggered nonlinear filter with output quantization.

$$u_c = \{\phi_c | \phi_c = \kappa_c \phi_{c0}\},$$

where $0 < \kappa_c < 1$, $\phi_{c0} > 0$, ϕ_c denotes the quantization level corresponding to a segment of c th component of the output signal $\tilde{y}(k)$. Then, the subquantizer $\bar{\mathcal{U}}_c(\tilde{y}_c(k))$ is defined as follows:

$$\bar{\mathcal{U}}_c(\tilde{y}_c(k)) = \begin{cases} \phi_c, & \frac{1}{1+\epsilon_c} \phi_c < \tilde{y}_c(k) \leq \frac{1}{1-\epsilon_c} \phi_c, \\ 0, & \tilde{y}_c(k) = 0, \\ \tilde{y}_c(k), & 0 < \tilde{y}_c(k) < \frac{1}{1+\epsilon_c} \phi_c, \tilde{y}_c(k) > \frac{1}{1-\epsilon_c} \phi_c, \end{cases} \quad (7)$$

where $\epsilon_c = \frac{1-\kappa_c}{1+\kappa_c}$. For any quantization error, the following sector-bound expression can be obtained:

$$\tilde{U}(\tilde{y}(k)) - \tilde{y}(k) = \Delta(k)\tilde{y}(k),$$

where $\Delta(k) = \text{diag}\{\Delta_1(k), \Delta_2(k), \dots, \Delta_m(k)\}$ and $|\Delta_c(k)| \leq \epsilon_c$. Then, the output quantization signal based on the event-triggering strategy received by the filter can be described as: $\tilde{y}(k) = (I + \Delta(k))\tilde{y}(k)$.

A nonlinear asynchronous filter with output quantization is constructed as follows:

$$\begin{aligned} x_f(k+1) &= A_{f\sigma_f(k)}\hat{f}(x_f(k)) + B_{f\sigma_f(k)}\tilde{y}(k), \\ z_f(k) &= E_{f\sigma_f(k)}p(x_f(k)) + F_{f\sigma_f(k)}\tilde{y}(k), \end{aligned} \quad (8)$$

where $\hat{f}(x_f(k)) = (\hat{f}_1(x_1), \dots, \hat{f}_n(x_n))^T$ satisfies $\vartheta_1 x_i^2 \leq \hat{f}_i(x_i)x_i \leq \vartheta_2 x_i^2$ with $\vartheta_2 > \vartheta_1 > 0$; $x_f(k)$ is the state of the filter; $z_f(k)$ is the estimation of $z(k)$; $\sigma_f(k)$ is switching law of the filter taking values in $S = \{1, 2, \dots, N\}$; The matrices $A_{f\sigma_f(k)}$, $B_{f\sigma_f(k)}$, $E_{f\sigma_f(k)}$, and $F_{f\sigma_f(k)}$ are to be determined.

Remark 2. Consider the interval $[k_r, k_{r+1})$, $r = 0, 1, \dots$, where the asynchronous phenomenon occurs in $[k_r, k_r + \Delta_r)$ and the synchronous switching arises in $[k_r + \Delta_r, k_{r+1})$. This indicates that the i th subsystem and the j th filter are active in $k \in [k_r, k_r + \Delta_r)$, and then the i th filter is active in $[k_r + \Delta_r, k_{r+1})$.

Remark 3. The filter (8) is a switched system, and the matrices of the filter depend on the system modes. In this paper, the filter is assumed to be switched asynchronously with the subsystems, which means that the switching instant of the filter lags behind the system (1) by Δ_r , where $\Delta_0 = 0$, $\Delta_r < k_{r+1} - k_r$, $r = 1, 2, \dots$, and k_r is the switching time instant. Therefore, $\sigma_f(k_r) = \sigma(k_r) + \Delta_r$.

Let $\tilde{x}(k) = (x^T(k) x_f^T(k) - x^T(k))^T$ and $e(k) = z_f(k) - z(k)$. Based on system (1) and filter (8), the following error system is obtained: For $k \in [k_r, k_r + \Delta_r)$,

$$\begin{aligned} \tilde{x}(k+1) &= \begin{pmatrix} A_i f(x(k)) + B_i g(\omega(k)) \\ A_{fj} \hat{f}(x_f(k)) + B_{fj}(I + \Delta(k))(C_i h(x(k)) + D_i l(\omega(k)) + e_y(k)) - A_i f(x(k)) - B_i g(\omega(k)) \end{pmatrix}, \\ e(k) &= E_{fj} p(x_f(k)) + F_{fj}(I + \Delta(k))(C_i h(x(k)) + D_i l(\omega(k)) + e_y(k)) - E_i p(x(k)) - F_i q(\omega(k)), \end{aligned} \quad (9)$$

and for $k \in [k_r + \Delta_r, k_{r+1})$,

$$\begin{aligned} \tilde{x}(k+1) &= \begin{pmatrix} A_i f(x(k)) + B_i g(\omega(k)) \\ A_{fi} \hat{f}(x_f(k)) + B_{fi}(I + \Delta(k))(C_i h(x(k)) + D_i l(\omega(k)) + e_y(k)) - A_i f(x(k)) - B_i g(\omega(k)) \end{pmatrix}, \\ e(k) &= E_{fi} p(x_f(k)) + F_{fi}(I + \Delta(k))(C_i h(x(k)) + D_i l(\omega(k)) + e_y(k)) - E_i p(x(k)) - F_i q(\omega(k)). \end{aligned} \quad (10)$$

Let $\Lambda = \text{diag}\{\epsilon_1, \epsilon_2, \dots, \epsilon_m\}$. Thus, we have $0 \preceq L \preceq I + \Delta(k) \preceq J$, where $L = I - \Lambda$ and $J = I + \Lambda$. Based on Assumption 2, we have that, for $k \in [k_r, k_r + \Delta_r)$,

$$\begin{aligned} \tilde{x}(k+1) &\succeq \tilde{A}_{1ij}\tilde{x}(k) + \tilde{B}_{1ij}\omega(k) + \tilde{D}_{1ij}e_y(k), \\ e(k) &\succeq \tilde{E}_{1ij}\tilde{x}(k) + \tilde{F}_{1ij}\omega(k) + F_{fj}Le_y(k), \end{aligned} \quad (11)$$

and

$$\begin{aligned} \tilde{x}(k+1) &\preceq \tilde{A}_{2ij}\tilde{x}(k) + \tilde{B}_{2ij}\omega(k) + \tilde{D}_{2ij}e_y(k), \\ e(k) &\preceq \tilde{E}_{2ij}\tilde{x}(k) + \tilde{F}_{2ij}\omega(k) + F_{fj}Je_y(k), \end{aligned} \quad (12)$$

where

$$\begin{aligned}\tilde{A}_{1ij} &= \begin{pmatrix} \omega_1 A_i & 0 \\ \vartheta_1 A_{fj} + \omega_3 B_{fj} L C_i - \omega_2 A_i & \vartheta_1 A_{fj} \end{pmatrix}, \\ \tilde{A}_{2ij} &= \begin{pmatrix} \omega_2 A_i & 0 \\ \vartheta_2 A_{fj} + \omega_4 B_{fj} J C_i - \omega_1 A_i & \vartheta_2 A_{fj} \end{pmatrix}, \\ \tilde{E}_{1ij} &= (\omega_5 E_{fj} + \omega_3 F_{fj} L C_i - \omega_6 E_i \quad \omega_5 E_{fj}), \\ \tilde{E}_{2ij} &= (\omega_6 E_{fj} + \omega_4 F_{fj} J C_i - \omega_5 E_i \quad \omega_6 E_{fj}), \\ \tilde{B}_{1ij} &= \begin{pmatrix} \varepsilon_1 B_i \\ \varepsilon_3 B_{fj} L D_i - \varepsilon_2 B_i \end{pmatrix}, \tilde{B}_{2ij} = \begin{pmatrix} \varepsilon_2 B_i \\ \varepsilon_4 B_{fj} J D_i - \varepsilon_1 B_i \end{pmatrix}, \\ \tilde{F}_{1ij} &= (\varepsilon_3 F_{fj} L D_i - \varepsilon_6 F_i), \tilde{F}_{2ij} = (\varepsilon_4 F_{fj} J D_i - \varepsilon_5 F_i), \\ \tilde{D}_{1j} &= \begin{pmatrix} 0 \\ B_{fj} L \end{pmatrix}, \tilde{D}_{2j} = \begin{pmatrix} 0 \\ B_{fj} J \end{pmatrix},\end{aligned}$$

and for $k \in [k_r + \Delta_r, k_{r+1})$,

$$\begin{aligned}\tilde{x}(k+1) &\succeq \tilde{A}_{1i} \tilde{x}(k) + \tilde{B}_{1i} \omega(k) + \tilde{D}_{1i} e_y(k), \\ e(k) &\succeq \tilde{E}_{1i} \tilde{x}(k) + \tilde{F}_{1i} \omega(k) + F_{fi} L e_y(k),\end{aligned}\quad (13)$$

and

$$\begin{aligned}\tilde{x}(k+1) &\preceq \tilde{A}_{2i} \tilde{x}(k) + \tilde{B}_{2i} \omega(k) + \tilde{D}_{2i} e_y(k), \\ e(k) &\preceq \tilde{E}_{2i} \tilde{x}(k) + \tilde{F}_{2i} \omega(k) + F_{fi} J e_y(k),\end{aligned}\quad (14)$$

where

$$\begin{aligned}\tilde{A}_{1i} &= \begin{pmatrix} \omega_1 A_i & 0 \\ \vartheta_1 A_{fi} + \omega_3 B_{fi} L C_i - \omega_2 A_i & \vartheta_1 A_{fi} \end{pmatrix}, \\ \tilde{A}_{2i} &= \begin{pmatrix} \omega_2 A_i & 0 \\ \vartheta_2 A_{fi} + \omega_4 B_{fi} J C_i - \omega_1 A_i & \vartheta_2 A_{fi} \end{pmatrix}, \\ \tilde{E}_{1i} &= (\omega_5 E_{fi} + \omega_3 F_{fi} L C_i - \omega_6 E_i \quad \omega_5 E_{fi}), \\ \tilde{E}_{2i} &= (\omega_6 E_{fi} + \omega_4 F_{fi} J C_i - \omega_5 E_i \quad \omega_6 E_{fi}), \\ \tilde{B}_{1i} &= \begin{pmatrix} \varepsilon_1 B_i \\ \varepsilon_3 B_{fi} L D_i - \varepsilon_2 B_i \end{pmatrix}, \tilde{B}_{2i} = \begin{pmatrix} \varepsilon_2 B_i \\ \varepsilon_4 B_{fi} J D_i - \varepsilon_1 B_i \end{pmatrix}, \\ \tilde{F}_{1i} &= (\varepsilon_3 F_{fi} L D_i - \varepsilon_6 F_i), \tilde{F}_{2i} = (\varepsilon_4 F_{fi} J D_i - \varepsilon_5 F_i), \\ \tilde{D}_{1i} &= \begin{pmatrix} 0 \\ B_{fi} L \end{pmatrix}, \tilde{D}_{2i} = \begin{pmatrix} 0 \\ B_{fi} J \end{pmatrix}.\end{aligned}$$

Theorem 1. If there exist constants $0 < \omega_1 \leq \omega_2, 0 < \omega_3 \leq \omega_4, 0 < \omega_5 \leq \omega_6, 0 < \varepsilon_1 \leq \varepsilon_2, 0 < \varepsilon_3 \leq \varepsilon_4, 0 < \varepsilon_5 \leq \varepsilon_6, 0 < \vartheta_1 \leq \vartheta_2, \gamma > 0, \lambda > 1, 0 \leq \beta < 1, 0 < \mu_1 < 1, \mu_2 > 1, \mathbb{R}^n$ vectors $\zeta_i \succ 0, \zeta_{(i,j)} \succ 0, \varphi_i \succ 0, \varphi_{(i,j)} \succ 0, \xi_i \succeq 0, \xi_{ii} \succeq 0, \xi_j \succeq 0, \xi_{ji} \succeq 0, \rho_{ij} \succeq 0$, and $\mathbb{R}^m$ vectors $\delta_i \succeq 0, \delta_{ii} \succeq 0, \delta_j \succeq 0, \delta_{ji} \succeq 0, \theta_{ij} \succeq 0$ such that

$$\vartheta_1 \sum_{i=1}^n 1_n^{(i)} \zeta_{ii}^\top + \omega_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top L M C_i - \omega_2 1_n^\top \varphi_i A_i \succeq 0, \quad (15)$$

$$\varepsilon_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top L M D_i - \varepsilon_2 1_n^\top \varphi_i B_i \succeq 0, \quad (16)$$

$$\omega_5 \sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top + \omega_3 \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top L M C_i - \omega_6 E_i \succeq 0, \quad (17)$$

$$\varepsilon_3 \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top L M D_i - \varepsilon_6 F_i \succeq 0, \quad (18)$$

$$\vartheta_1 \sum_{i=1}^n 1_n^{(i)} \zeta_{ji}^\top + \omega_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ji}^\top L M C_i - \omega_2 1_n^\top \varphi_j A_i \succeq 0, \quad (19)$$

$$\varepsilon_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ji}^\top L M D_i - \varepsilon_2 1_n^\top \varphi_j B_i \succeq 0, \quad (20)$$

$$\omega_5 \sum_{j=1}^s 1_s^{(j)} \rho_{jj}^\top + \omega_3 \sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top L M C_i - \omega_6 E_i \succeq 0, \quad (21)$$

$$\varepsilon_3 \sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top L M D_i - \varepsilon_6 F_i \succeq 0, \quad (22)$$

$$\begin{aligned} \omega_2 A_i^\top \zeta_i + \vartheta_2 \xi_i + \omega_4 C_i^\top H J \delta_i - \omega_1 A_i^\top \varphi_i - \mu_1 \zeta_i + (\omega_6 \sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top \\ + \omega_4 \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top J H C_i - \omega_5 E_i)^\top 1_s \leq 0, \end{aligned} \quad (23)$$

$$\omega_6 (\sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top)^\top 1_s + \vartheta_2 \xi_i - \mu_1 \varphi_i \leq 0, \quad (24)$$

$$\varepsilon_2 B_i^\top \zeta_i + \varepsilon_4 D_i^\top H J \delta_i - \varepsilon_1 B_i^\top \varphi_i + \varepsilon_4 D_i^\top H J (\sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top)^\top 1_s - \varepsilon_5 F_i^\top 1_s - \gamma 1_m \leq 0, \quad (25)$$

$$\begin{aligned} \omega_2 A_i^\top \zeta_{(i,j)} + \vartheta_2 \xi_j + \omega_4 C_i^\top H J \delta_j - \omega_1 A_i^\top \varphi_{(i,j)} - \mu_2 \zeta_{(i,j)} + (\omega_6 \sum_{j=1}^s 1_s^{(j)} \rho_{jj}^\top \\ + \omega_4 \sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top J H C_i - \omega_5 E_i)^\top 1_s \leq 0, \end{aligned} \quad (26)$$

$$\omega_6 (\sum_{j=1}^s 1_s^{(j)} \rho_{jj}^\top)^\top 1_s + \vartheta_2 \xi_j - \mu_2 \varphi_{(i,j)} \leq 0, \quad (27)$$

$$\begin{aligned} \varepsilon_2 B_i^\top \zeta_{(i,j)} + \varepsilon_4 D_i^\top H J \delta_j - \varepsilon_1 B_i^\top \varphi_{(i,j)} + \varepsilon_4 D_i^\top H J (\sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top)^\top 1_s \\ - \varepsilon_5 F_i^\top 1_s - \gamma 1_m \leq 0, \end{aligned} \quad (28)$$

$$\begin{aligned} \zeta_i \preceq \lambda \zeta_{(i,j)}, \zeta_i \preceq \lambda \zeta_{(j,i)}, \zeta_{(i,j)} \preceq \lambda \zeta_i, \zeta_{(j,i)} \preceq \lambda \zeta_i, \\ \varphi_i \preceq \lambda \varphi_{(i,j)}, \varphi_i \preceq \lambda \varphi_{(j,i)}, \varphi_{(i,j)} \preceq \lambda \varphi_i, \varphi_{(j,i)} \preceq \lambda \varphi_i, \end{aligned} \quad (29)$$

$$\xi_{ii} \preceq \xi_i, \delta_{ii} \preceq \delta_i, \xi_{ji} \preceq \xi_j, \delta_{ji} \preceq \delta_j, \quad (30)$$

hold $\forall i, j \in S, i \neq j, i = 1, 2, \dots, n$ and $j = 1, 2, \dots, s$, then the error systems (9) and (10) are positive and stable with filter gain matrices

$$A_{fi} = \frac{\sum_{i=1}^n 1_n^{(i)} \xi_{ii}^\top}{1_n^\top \varphi_i}, B_{fi} = \frac{\sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top}{1_n^\top \varphi_i}, \quad (31)$$

$$E_{fi} = \sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top, F_{fi} = \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top, \quad (32)$$

and the switching law satisfying

$$\frac{\Gamma^-(k_0, k)}{\Gamma^+(k_0, k)} \geq \frac{\ln \mu_2 - \ln \mu_1}{\ln \mu_1^* - \ln \mu_1}, \mu_1^* \in (\mu_1, 1), \quad (33)$$

$$\tau_a \geq \tau_a^* = -\frac{2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{max}}{\ln \mu_1^*}, \quad (34)$$

where $M = I - \beta 1_{m \times m}$, $H = I + \beta 1_{m \times m}$, and Δ_{max} denotes the maximum of time lag Δ_r .

Proof. First, the positivity of the error systems (9) and (10) are considered. For $x(k_0) \succeq 0$, the output satisfies $y(k_0) \succeq 0$. Using event-triggering condition (6) gives

$$\|e_y(k_0)\|_1 \leq \beta 1_m^\top y(k_0), \quad (35)$$

which gives that

$$-\beta 1_{m \times m} y(k_0) \preceq e_y(k_0) \preceq \beta 1_{m \times m} y(k_0). \quad (36)$$

For $k \in [k_r, k_r + \Delta_r)$, it follows from (11) and (36) that

$$\begin{aligned} \tilde{x}(k_0 + 1) &\succeq \tilde{A}_{1ij} \tilde{x}(k_0) + \tilde{B}_{1ij} \omega(k_0), \\ e(k_0) &\succeq \tilde{E}_{1ij} \tilde{x}(k_0) + \tilde{F}_{1ij} \omega(k_0), \end{aligned} \quad (37)$$

where

$$\begin{aligned} \tilde{A}_{1ij} &= \begin{pmatrix} \omega_1 A_i & 0 \\ \vartheta_1 A_{fj} + \omega_3 B_{fj} L M C_i - \omega_2 A_i & \vartheta_1 A_{fj} \end{pmatrix}, \tilde{B}_{1ij} = \begin{pmatrix} \varepsilon_1 B_i \\ \varepsilon_3 B_{fj} L M D_i - \varepsilon_2 B_i \end{pmatrix}, \\ \tilde{E}_{1ij} &= (\omega_5 E_{fj} + \omega_3 F_{fj} L M C_i - \omega_6 E_i \quad \omega_5 E_{fj}), \tilde{F}_{1ij} = (\varepsilon_3 F_{fj} L M D_i - \varepsilon_6 F_i). \end{aligned}$$

For $k \in [k_r + \Delta_r, k_{r+1})$,

$$\begin{aligned}\tilde{x}(k_0 + 1) &\succeq \tilde{A}_{1i}\tilde{x}(k_0) + \tilde{B}_{1i}\omega(k_0), \\ e(k_0) &\succeq \tilde{E}_{1i}\tilde{x}(k_0) + \tilde{F}_{1i}\omega(k_0),\end{aligned}\quad (38)$$

where

$$\begin{aligned}\tilde{A}_{1i} &= \begin{pmatrix} \omega_1 A_i & 0 \\ \vartheta_1 A_{fi} + \omega_3 B_{fi} LMC_i - \omega_2 A_i & \vartheta_1 A_{fi} \end{pmatrix}, \tilde{B}_{1i} = \begin{pmatrix} \varepsilon_1 B_i \\ \varepsilon_3 B_{fi} LMD_i - \varepsilon_2 B_i \end{pmatrix}, \\ \tilde{E}_{1i} &= (\omega_5 E_{fi} + \omega_3 F_{fi} LMC_i - \omega_6 E_i \quad \omega_5 E_{fi}), \tilde{F}_{1i} = (\varepsilon_3 F_{fi} LMD_i - \varepsilon_6 F_i).\end{aligned}$$

Using (15), (16), (19) and (20) gives

$$\frac{\vartheta_1 \sum_{i=1}^n 1_n^{(i)} \xi_{ii}^\top}{1_n^\top \varphi_i} + \frac{\omega_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top}{1_n^\top \varphi_i} LMC_i - \omega_2 A_i \succeq 0, \quad (39)$$

$$\frac{\varepsilon_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top}{1_n^\top \varphi_i} LMD_i - \varepsilon_2 B_i \succeq 0, \quad (40)$$

$$\frac{\vartheta_1 \sum_{i=1}^n 1_n^{(i)} \xi_{ji}^\top}{1_n^\top \varphi_j} + \frac{\omega_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ji}^\top}{1_n^\top \varphi_j} LMC_i - \omega_2 A_i \succeq 0, \quad (41)$$

$$\frac{\varepsilon_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ji}^\top}{1_n^\top \varphi_j} LMD_i - \varepsilon_2 B_i \succeq 0. \quad (42)$$

Together with (17), (18), (21), (22), (31), and (32), we have

$$\vartheta_1 A_{fi} + \omega_3 B_{fi} LMC_i - \omega_2 A_i \succeq 0, \quad (43)$$

$$\varepsilon_3 B_{fi} LMD_i - \varepsilon_2 B_i \succeq 0, \quad (44)$$

$$\omega_5 E_{fi} + \omega_3 F_{fi} LMC_i - \omega_6 E_i \succeq 0, \quad (45)$$

$$\varepsilon_3 F_{fi} LMD_i - \varepsilon_6 F_i \succeq 0, \quad (46)$$

$$\vartheta_1 A_{fj} + \omega_3 B_{fj} LMC_i - \omega_2 A_i \succeq 0, \quad (47)$$

$$\varepsilon_3 B_{fj} LMD_i - \varepsilon_2 B_i \succeq 0, \quad (48)$$

$$\omega_5 E_{fj} + \omega_3 F_{fj} LMC_i - \omega_6 E_i \succeq 0, \quad (49)$$

$$\varepsilon_3 F_{fj} LMD_i - \varepsilon_6 F_i \succeq 0. \quad (50)$$

Due to $\xi_{ii} \succeq 0, \delta_{ii} \succeq 0, \rho_{ij} \succeq 0$, and $\theta_{ij} \succeq 0$, this yields $A_{fi} \succeq 0, B_{fi} \succeq 0, E_{fi} \succeq 0$, and $F_{fi} \succeq 0$. Thus, we have $\tilde{A}_{1i} \succeq 0, \tilde{B}_{1i} \succeq 0, \tilde{E}_{1i} \succeq 0$, and $\tilde{F}_{1i} \succeq 0$. Similarly, we can obtain $\tilde{A}_{1ij} \succeq 0, \tilde{B}_{1ij} \succeq 0, \tilde{E}_{1ij} \succeq 0$, and $\tilde{F}_{1ij} \succeq 0$. By (37), (38), and Lemma 1, we have $\tilde{x}(k_0 + 1) \succeq 0$ and $e(k_0) \succeq 0$. Using recursive derivation gives $\tilde{x}(k) \succeq 0$ and $e(k) \succeq 0$, that is to say, the error systems (9) and (10) are positive.

Next, we will analyze the ℓ_1 -gain stability of the considered error systems. Construct a piecewise multiple copositive Lyapunov function candidate:

$$V_i(k) = \begin{cases} \tilde{x}^\top(k) v_i, & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r), \\ \tilde{x}^\top(k) v_{(i,j)}, & \forall k \in [k_r, k_r + \Delta_r), \end{cases} \quad (51)$$

where $v_i = (\zeta_i^\top \varphi_i^\top)^\top$ and $v_{(i,j)} = (\zeta_{(i,j)}^\top \varphi_{(i,j)}^\top)^\top$. From (12) and (36), for $k \in [k_r, k_r + \Delta_r)$, it follows that

$$\begin{aligned}\tilde{x}(k+1) &\preceq \tilde{A}_{1ij}\tilde{x}(k) + \tilde{B}_{1ij}\omega(k), \\ e(k) &\preceq \tilde{E}_{1ij}\tilde{x}(k) + \tilde{F}_{1ij}\omega(k),\end{aligned}\quad (52)$$

where

$$\begin{aligned}\bar{\bar{A}}_{1ij} &= \begin{pmatrix} \omega_2 A_i & 0 \\ \vartheta_2 A_{fj} + \omega_4 B_{fj} JHC_i - \omega_1 A_i & \vartheta_2 A_{fj} \end{pmatrix}, \bar{\bar{B}}_{1ij} = \begin{pmatrix} \varepsilon_2 B_i \\ \varepsilon_4 B_{fj} JHD_i - \varepsilon_1 B_i \end{pmatrix}, \\ \bar{\bar{E}}_{1ij} &= (\omega_6 E_{fj} + \omega_4 F_{fj} JHC_i - \omega_5 E_i \quad \omega_6 E_{fj}), \bar{\bar{F}}_{1ij} = (\varepsilon_4 F_{fj} JHD_i - \varepsilon_5 F_i).\end{aligned}$$

For $k \in [k_r + \Delta_r, k_{r+1})$, we have

$$\begin{aligned}\tilde{x}(k+1) &\preceq \bar{\bar{A}}_{1i} \tilde{x}(k) + \bar{\bar{B}}_{1i} \omega(k), \\ e(k) &\preceq \bar{\bar{E}}_{1i} \tilde{x}(k) + \bar{\bar{F}}_{1i} \omega(k),\end{aligned}\quad (53)$$

where

$$\begin{aligned}\bar{\bar{A}}_{1i} &= \begin{pmatrix} \omega_2 A_i & 0 \\ \vartheta_2 A_{fi} + \omega_4 B_{fi} JHC_i - \omega_1 A_i & \vartheta_2 A_{fi} \end{pmatrix}, \bar{\bar{B}}_{1i} = \begin{pmatrix} \varepsilon_2 B_i \\ \varepsilon_4 B_{fi} JHD_i - \varepsilon_1 B_i \end{pmatrix}, \\ \bar{\bar{E}}_{1i} &= (\omega_6 E_{fi} + \omega_4 F_{fi} JHC_i - \omega_5 E_i \quad \omega_6 E_{fi}), \bar{\bar{F}}_{1i} = (\varepsilon_4 F_{fi} JHD_i - \varepsilon_5 F_i).\end{aligned}$$

From the upper bound systems (52) and (53), the forward difference of (51) along the trajectories satisfies

$$\Delta V_i(k) \leq \begin{cases} \tilde{x}^\top(k) Y_1 + \omega^\top(k) \Theta_1, & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r), \\ \tilde{x}^\top(k) Y_2 + \omega^\top(k) \Theta_2, & \forall k \in [k_r, k_r + \Delta_r), \end{cases}\quad (54)$$

where

$$\begin{aligned}Y_1 &= \bar{\bar{A}}_{1i}^\top v_i - v_i = \begin{pmatrix} \omega_2 A_i^\top \zeta_i + (\vartheta_2 A_{fi}^\top + \omega_4 C_i^\top HJB_{fi}^\top - \omega_1 A_i^\top) \varphi_i - \zeta_i \\ \vartheta_2 A_{fi}^\top \varphi_i - \varphi_i \end{pmatrix}, \\ Y_2 &= \bar{\bar{A}}_{1ij}^\top v_{(i,j)} - v_{(i,j)} = \begin{pmatrix} \omega_2 A_i^\top \zeta_{(i,j)} + (\vartheta_2 A_{fj}^\top + \omega_4 C_i^\top HJB_{fj}^\top - \omega_1 A_i^\top) \varphi_{(i,j)} - \zeta_{(i,j)} \\ \vartheta_2 A_{fj}^\top \varphi_{(i,j)} - \varphi_{(i,j)} \end{pmatrix}, \\ \Theta_1 &= \bar{\bar{B}}_{1i}^\top v_i = (\varepsilon_2 B_i^\top \zeta_i + (\varepsilon_4 D_i^\top HJB_{fi}^\top - \varepsilon_1 B_i^\top) \varphi_i), \\ \Theta_2 &= \bar{\bar{B}}_{1ij}^\top v_{(i,j)} = (\varepsilon_2 B_i^\top \zeta_{(i,j)} + (\varepsilon_4 D_i^\top HJB_{fj}^\top - \varepsilon_1 B_i^\top) \varphi_{(i,j)}).\end{aligned}$$

Using (30) and (31), it derives that

$$A_{fi} \preceq \frac{\sum_{i=1}^n 1_n^{(i)} \xi_i^\top}{1_n^\top \varphi_i} = \frac{1_n \xi_i^\top}{1_n^\top \varphi_i}, \quad B_{fi} \preceq \frac{\sum_{i=1}^n 1_n^{(i)} \delta_i^\top}{1_n^\top \varphi_i} = \frac{1_n \delta_i^\top}{1_n^\top \varphi_i}.$$

Together with the fact $\varphi_i \succ 0$ gives

$$A_{fi}^\top \varphi_i \preceq \frac{\xi_i 1_n^\top}{1_n^\top \varphi_i} \varphi_i = \xi_i, \quad B_{fi}^\top \varphi_i \preceq \frac{\delta_i 1_n^\top}{1_n^\top \varphi_i} \varphi_i = \delta_i. \quad (55)$$

Similarly,

$$A_{fj}^\top \varphi_{(i,j)} \preceq \frac{\xi_j 1_n^\top}{1_n^\top \varphi_{(i,j)}} \varphi_{(i,j)} = \xi_j, \quad B_{fj}^\top \varphi_{(i,j)} \preceq \frac{\delta_j 1_n^\top}{1_n^\top \varphi_{(i,j)}} \varphi_{(i,j)} = \delta_j. \quad (56)$$

Define $\Xi(k) = \gamma \|\omega(k)\|_1 - \|e(k)\|_1$. From (23)–(28), (54), (55), and (56), we can obtain

$$V_i(k) \leq \begin{cases} \mu_1 V_i(k-1) + \Xi(k-1), & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r), \\ \mu_2 V_{(i,j)}(k-1) + \Xi(k-1), & \forall k \in [k_r, k_r + \Delta_r). \end{cases}\quad (57)$$

Thus,

$$V_i(k) \leq \begin{cases} \mu_1^{k-k_{r-1}-\Delta_{r-1}} V_i(k_{r-1} + \Delta_{r-1}) + \sum_{\zeta=k_{r-1}+\Delta_{r-1}}^{k-1} \mu_1^{k-1-\zeta} \Xi(\zeta), & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r), \\ \mu_2^{k-k_r} V_{(i,j)}(k_r) + \sum_{\zeta=k_r}^{k-1} \mu_2^{k-1-\zeta} \Xi(\zeta), & \forall k \in [k_r, k_r + \Delta_r). \end{cases} \quad (58)$$

Noting the condition (29), then

$$V_i(k) \leq \begin{cases} \lambda \mu_1^{k-k_{r-1}-\Delta_{r-1}} V_i(k_{r-1} + \Delta_{r-1}) + \sum_{\zeta=k_{r-1}+\Delta_{r-1}}^{k-1} \mu_1^{k-1-\zeta} \Xi(\zeta), & \forall k \in [k_{r-1}, k_{r-1} + \Delta_{r-1}), \\ \lambda \mu_2^{k-k_r} V_{(i,j)}(k_r) + \sum_{\zeta=k_r}^{k-1} \mu_2^{k-1-\zeta} \Xi(\zeta), & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r). \end{cases} \quad (59)$$

For $T \in [k_{N_\sigma(T, k_0)} + \Delta_{N_\sigma(T, k_0)}, k_{N_\sigma(T, k_0)+1})$, repeating (58) and (59) follows that:

$$\begin{aligned} V_{\sigma(k_{N+1})}(T) &\leq \mu_1^{T-k_N-\Delta_N} V_{\sigma(k_N)}(k_N + \Delta_N) + \sum_{\zeta=k_N+\Delta_N}^{T-1} \mu_1^{T-1-\zeta} \Xi(\zeta) \\ &\leq \lambda \mu_1^{T-k_N-\Delta_N} \mu_2^{\Delta_N} V_{\sigma(k_N+\Delta_N)}(k_N) + \sum_{\zeta=k_N+\Delta_N}^{T-1} \mu_1^{T-1-\zeta} \Xi(\zeta) \\ &\quad + \lambda \mu_1^{T-k_N-\Delta_N} \sum_{\zeta=k_N}^{k_N+\Delta_N-1} \mu_2^{k_N+\Delta_N-1-\zeta} \\ &\leq \lambda^2 \mu_1^{T-k_{N-1}-\Delta_{N-1}-\Delta_N-1} \mu_2^{\Delta_N} V_{\sigma(k_{N-1})}(k_{N-1} + \Delta_{N-1}) + \sum_{\zeta=k_N+\Delta_N}^{T-1} \mu_1^{T-1-\zeta} \Xi(\zeta) \\ &\quad + \lambda \mu_1^{T-k_N-\Delta_N} \sum_{\zeta=k_N}^{k_N+\Delta_N-1} \mu_2^{k_N+\Delta_N-1-\zeta} \\ &\quad + \lambda^2 \mu_2^{\Delta_N} \sum_{\zeta=k_{N-1}+\Delta_{N-1}}^{k_N-1} \mu_1^{T-1-\Delta_N-\zeta} \Xi(\zeta) \\ &= \lambda^2 e^{(T-k_{N-1}-\Delta_{N-1}-\Delta_N-1) \ln \mu_1} e^{\Delta_N \ln \mu_2} V_{\sigma(k_{N-1})}(k_{N-1} + \Delta_{N-1}) + \sum_{\zeta=k_N+\Delta_N}^{T-1} e^{(T-1-\zeta) \ln \mu_1} \Xi(\zeta) \\ &\quad + \lambda e^{(T-k_N-\Delta_N) \ln \mu_1} \sum_{\zeta=k_N}^{k_N+\Delta_N-1} e^{(k_N+\Delta_N-1-\zeta) \ln \mu_2} \\ &\quad + \lambda^2 e^{\Delta_N \ln \mu_2} \sum_{\zeta=k_{N-1}+\Delta_{N-1}}^{k_N-1} e^{(T-1-\Delta_N-\zeta) \ln \mu_1} \Xi(\zeta) \\ &\leq \dots \\ &\leq e^{2N \ln \lambda} e^{(T-k_0-N\Delta_{\max}) \ln \mu_1} e^{N\Delta_{\max} \ln \mu_2} V_{\sigma(k_0)}(k_0) \\ &\quad + \sum_{\zeta=k_0}^{T-1} e^{2N_\sigma(T, \zeta) \ln \lambda} e^{N_\sigma(T-1, \zeta) \Delta_{\max} \ln \mu_2} e^{(T-1-N_\sigma(T-1, \zeta) \Delta_{\max}-\zeta) \ln \mu_1} \Xi(\zeta), \end{aligned} \quad (60)$$

where $N = N_\sigma(T, k_0)$ and $\Delta_{\max} = \max\{\Delta_1, \Delta_2, \dots, \Delta_N\}$. Under zero initial conditions, we have

$$0 \leq \sum_{\zeta=k_0}^{T-1} e^{2N_\sigma(T, \zeta) \ln \lambda} e^{N_\sigma(T-1, \zeta) \Delta_{\max} \ln \mu_2} e^{(T-1-N_\sigma(T-1, \zeta) \Delta_{\max}-\zeta) \ln \mu_1} \Xi(\zeta), \quad (61)$$

that is,

$$\begin{aligned} &\sum_{\zeta=k_0}^{T-1} e^{2N_\sigma(T, \zeta) \ln \lambda} e^{N_\sigma(T-1, \zeta) \Delta_{\max} \ln \mu_2} e^{(T-1-N_\sigma(T-1, \zeta) \Delta_{\max}-\zeta) \ln \mu_1} \|e(\zeta)\|_1 \\ &\leq \gamma \sum_{\zeta=k_0}^{T-1} e^{2N_\sigma(T, \zeta) \ln \lambda} e^{N_\sigma(T-1, \zeta) \Delta_{\max} \ln \mu_2} e^{(T-1-N_\sigma(T-1, \zeta) \Delta_{\max}-\zeta) \ln \mu_1} \|\omega(\zeta)\|_1. \end{aligned} \quad (62)$$

Multiplying both sides of the inequality (62) with $e^{(\ln \mu_1 - \ln \mu_2) N_\sigma(T, k_0) \Delta_{\max} - 2N_\sigma(T, k_0) \ln \lambda}$ gives

$$\begin{aligned} &\sum_{\zeta=k_0}^{T-1} e^{(T-1-\zeta) \ln \mu_1 + (\ln \mu_1 - \ln \mu_2) N_\sigma(\zeta, 0) \Delta_{\max} - 2N_\sigma(\zeta, 0) \ln \lambda} \|e(\zeta)\|_1 \\ &\leq \gamma \sum_{\zeta=k_0}^{T-1} e^{(T-1-\zeta) \ln \mu_1 + (\ln \mu_1 - \ln \mu_2) N_\sigma(\zeta, 0) \Delta_{\max} - 2N_\sigma(\zeta, 0) \ln \lambda} \|\omega(\zeta)\|_1. \end{aligned} \quad (63)$$

From Definition 2 and (34), it is clear that

$$N_\sigma(\zeta, k_0) \leq N_0 + \frac{(\zeta-k_0) \ln \mu_1^*}{2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{\max}}. \quad (64)$$

Then, (63) can be transformed into

$$\begin{aligned} &\sum_{\zeta=k_0}^{T-1} e^{(T-1-\zeta) \ln \mu_1 - [2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{\max}] (N_0 + \frac{(\zeta-k_0) \ln \mu_1^*}{2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{\max}})} \|e(\zeta)\|_1 \\ &\leq \gamma \sum_{\zeta=k_0}^{T-1} e^{(T-1-\zeta) \ln \mu_1 - [2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{\max}] (N_0 + \frac{(\zeta-k_0) \ln \mu_1^*}{2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{\max}})} \|\omega(\zeta)\|_1. \end{aligned} \quad (65)$$

The above inequality can be further written as

$$\begin{aligned} & \sum_{\zeta=k_0}^{T-1} e^{(T-1)\ln\mu_1 - k_0\ln\mu_1^*} e^{(\ln\mu_1^* - \ln\mu_1)\zeta} \|e(\zeta)\|_1 \\ & \leq \gamma \sum_{\zeta=k_0}^{T-1} e^{(T-1)\ln\mu_1 - k_0\ln\mu_1^*} e^{(\ln\mu_1^* - \ln\mu_1)\zeta} \|\omega(\zeta)\|_1. \end{aligned} \quad (66)$$

From (66), we can obtain

$$\sum_{\zeta=k_0}^{T-1} e^{-(\ln\mu_1^* - \ln\mu_1)\zeta} \|e(\zeta)\|_1 \leq \gamma \sum_{\zeta=k_0}^{T-1} \|\omega(\zeta)\|_1. \quad (67)$$

Summing from 0 to ∞ for both sides of (67), it yields that

$$\sum_{k=0}^{\infty} e^{-\hbar k} \|e(k)\|_1 \leq \gamma \sum_{k=0}^{\infty} \|\omega(k)\|_1, \quad (68)$$

where $\hbar = \ln\mu_1^* - \ln\mu_1$. By Definition 3, the error systems (9) and (10) satisfy the ℓ_1 -gain performance index (68). $\square$

Remark 4. Generally, the signal can be quantized during the actual transmission process due to various reasons, such as energy consumption issues, intermittent sensor fault, limited digital communication resource, and so on. It is necessary to incorporate the quantization technique with event-triggering mechanism to generate the quantized output. Theorem 1 first introduces the quantization technique to the filter design of positive systems, where a sector restriction is adopted to analyze and mitigate the quantization effect [47–49]. Currently, few efforts have been devoted to positive systems, though the quantization approach is effective and practical for dealing with many practical problems. It is interesting to develop the quantization approach in Theorem 1 for other issues of positive systems.

3.3. Dynamic Event-Triggering Case

In this section, we propose a dynamic event-triggering mechanism as an alternative of the static event-triggering mechanism for system (1). Define the sampling error of the event generator as $e_y(k) = \tilde{y}(k) - y(k)$, where $\tilde{y}(k) = y(k_\varphi)$, $y(k_\varphi)$ is the output signal of the event generator at the event-triggering instant k_φ , $\varphi \in \mathbb{N}$. Following this, the output will be released by the following dynamic event-triggering condition:

$$\|e_y(k)\|_1 > \beta \|y(k)\|_1 + \frac{1}{\psi} \eta(k) \vee \eta(k) > \|y(k)\|_1, \quad (69)$$

where β and ψ are given positive constants, and $\eta(k)$ is an internal dynamic variable satisfying

$$\eta(k+1) = \varrho \eta(k) + \beta \|y(k)\|_1 - \|e_y(k)\|_1, \quad (70)$$

with $\eta(k_0) = \eta_0$ as the initial value and $\varrho \in (0, 1)$ as a given constant.

Theorem 2. If there exist constants $0 < \omega_1 \leq \omega_2, 0 < \omega_3 \leq \omega_4, 0 < \omega_5 \leq \omega_6, 0 < \varepsilon_1 \leq \varepsilon_2, 0 < \varepsilon_3 \leq \varepsilon_4, 0 < \varepsilon_5 \leq \varepsilon_6, 0 < \vartheta_1 \leq \vartheta_2, \psi > 0, \gamma > 0, \lambda > 1, 0 \leq \beta < 1, 0 < \mu_1 < 1, \mu_2 > 1$, $\mathbb{R}^n$ vectors $\zeta_i \succ 0, \zeta_{(i,j)} \succ 0, \varphi_i \succ 0, \varphi_{(i,j)} \succ 0, \xi_i \succeq 0, \xi_{ii} \succeq 0, \xi_j \succeq 0, \xi_{ji} \succeq 0, \rho_{ij} \succeq 0$, and $\mathbb{R}^m$ vectors $\delta_i \succeq 0, \delta_{ii} \succeq 0, \delta_j \succeq 0, \delta_{ji} \succeq 0, \theta_{ij} \succeq 0$ such that

$$\vartheta_1 \sum_{i=1}^n 1_n^{(i)} \zeta_{ii}^\top + \omega_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top L \Psi C_i - \omega_2 1_n^\top \varphi_i A_i \succeq 0, \quad (71)$$

$$\varepsilon_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ii}^\top L \Psi D_i - \varepsilon_2 1_n^\top \varphi_i B_i \succeq 0, \quad (72)$$

$$\omega_5 \sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top + \omega_3 \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top L \Psi C_i - \omega_6 E_i \succeq 0, \quad (73)$$

$$\varepsilon_3 \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top L \Psi D_i - \varepsilon_6 F_i \succeq 0, \quad (74)$$

$$\vartheta_1 \sum_{i=1}^n 1_n^{(i)} \zeta_{ji}^\top + \omega_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ji}^\top L \Psi C_i - \omega_2 1_n^\top \varphi_j A_i \succeq 0, \quad (75)$$

$$\varepsilon_3 \sum_{i=1}^n 1_n^{(i)} \delta_{ji}^\top L \Psi D_i - \varepsilon_2 1_n^\top \varphi_j B_i \geq 0, \quad (76)$$

$$\omega_5 \sum_{j=1}^s 1_s^{(j)} \rho_{jj}^\top + \omega_3 \sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top L \Psi C_i - \omega_6 E_i \geq 0, \quad (77)$$

$$\varepsilon_3 \sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top L \Psi D_i - \varepsilon_6 F_i \geq 0, \quad (78)$$

$$\omega_2 A_i^\top \zeta_i + \theta_2 \zeta_i + \omega_4 C_i^\top \Phi J \delta_i - \omega_1 A_i^\top \varphi_i - \mu_1 \zeta_i + (\omega_6 \sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top + \omega_4 \sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top J \Phi C_i - \omega_5 E_i)^\top 1_s + \beta \omega_4 C_i^\top 1_m \leq 0, \quad (79)$$

$$\omega_6 (\sum_{j=1}^s 1_s^{(j)} \rho_{ij}^\top)^\top 1_s + \theta_2 \zeta_i - \mu_1 \varphi_i \leq 0, \quad (80)$$

$$\varepsilon_2 B_i^\top \zeta_i + \beta \varepsilon_4 D_i^\top 1_m + \varepsilon_4 D_i^\top \Phi J \delta_i - \varepsilon_1 B_i^\top \varphi_i + \varepsilon_4 D_i^\top \Phi J (\sum_{j=1}^s 1_s^{(j)} \theta_{ij}^\top)^\top 1_s - \varepsilon_5 F_i^\top 1_s - \gamma 1_m \leq 0, \quad (81)$$

$$\omega_2 A_i^\top \zeta_{(i,j)} + \theta_2 \zeta_j + \omega_4 C_i^\top \Phi J \delta_j - \omega_1 A_i^\top \varphi_{(i,j)} - \mu_2 \zeta_{(i,j)} + (\omega_6 \sum_{j=1}^s 1_s^{(j)} \rho_{jj}^\top + \omega_4 \sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top J \Phi C_i - \omega_5 E_i)^\top 1_s + \beta \omega_4 C_i^\top 1_m \leq 0, \quad (82)$$

$$\omega_6 (\sum_{j=1}^s 1_s^{(j)} \rho_{jj}^\top)^\top 1_s + \theta_2 \zeta_j - \mu_2 \varphi_{(i,j)} \leq 0, \quad (83)$$

$$\varepsilon_2 B_i^\top \zeta_{(i,j)} + \beta \varepsilon_4 D_i^\top 1_m + \varepsilon_4 D_i^\top \Phi J \delta_j - \varepsilon_1 B_i^\top \varphi_{(i,j)} + \varepsilon_4 D_i^\top \Phi J (\sum_{j=1}^s 1_s^{(j)} \theta_{jj}^\top)^\top 1_s - \varepsilon_5 F_i^\top 1_s - \gamma 1_m \leq 0, \quad (84)$$

$$\zeta_i \leq \lambda \zeta_{(i,j)}, \zeta_i \leq \lambda \zeta_{(j,i)}, \zeta_{(i,j)} \leq \lambda \zeta_i, \zeta_{(j,i)} \leq \lambda \zeta_i, \quad (85)$$

$$\varphi_i \leq \lambda \varphi_{(i,j)}, \varphi_i \leq \lambda \varphi_{(j,i)}, \varphi_{(i,j)} \leq \lambda \varphi_i, \varphi_{(j,i)} \leq \lambda \varphi_i, \quad (86)$$

$$\zeta_{ii} \leq \zeta_i, \delta_{ii} \leq \delta_i, \zeta_{ji} \leq \zeta_j, \delta_{ji} \leq \delta_j, \quad (86)$$

hold $\forall i, j \in S, i \neq j, i = 1, 2, \dots, n$ and $j = 1, 2, \dots, s$, then the error systems (9) and (10) are positive and stable with filter gain matrices (31) and (32) and the switching law satisfying

$$\frac{\Gamma^-(k_0, k)}{\Gamma^+(k_0, k)} \geq \frac{\ln \mu_2 - \ln \mu_1}{\ln \mu_1^* - \ln \mu_1}, \mu_1^* \in (\mu_1, 1), \quad (87)$$

$$\tau_a \geq \tau_a^* = -\frac{2 \ln \lambda + (\ln \mu_2 - \ln \mu_1) \Delta_{\max}}{\ln \mu_1^*}, \quad (88)$$

where $\Psi = I - (\beta + \frac{1}{\psi}) 1_{m \times m}$, $\Phi = I + (\beta + \frac{1}{\psi}) 1_{m \times m}$, and $\Delta_{\max}$ denotes the maximum of time lag Δ_r .

Proof. First, the positivity of the error systems (9) and (10) are considered. For $x(k_0) \geq 0$, the output satisfies $y(k_0) \geq 0$. We can obtain from the dynamic event-triggering condition (69) and (70) that

$$\|e_y(k_0)\|_1 \leq \beta \|y(k_0)\|_1 + \frac{1}{\psi} \eta(k_0) \leq (\beta + \frac{1}{\psi}) 1_m^\top y(k_0), \quad (89)$$

which leads to

$$-(\beta + \frac{1}{\psi}) 1_{m \times m} y(k_0) \leq e_y(k_0) \leq (\beta + \frac{1}{\psi}) 1_{m \times m} y(k_0). \quad (90)$$

From (11), (13), and (90), it is clear that, for $k \in [k_r, k_r + \Delta_r)$,

$$\begin{aligned} \tilde{x}(k_0 + 1) &\geq \tilde{A}_{2ij} \tilde{x}(k_0) + \tilde{B}_{2ij} \omega(k_0), \\ e(k_0) &\geq \tilde{E}_{2ij} \tilde{x}(k_0) + \tilde{F}_{2ij} \omega(k_0), \end{aligned} \quad (91)$$

where

$$\begin{aligned}\tilde{A}_{2ij} &= \begin{pmatrix} \omega_1 A_i & 0 \\ \vartheta_1 A_{fj} + \omega_3 B_{fj} L \Psi C_i - \omega_2 A_i & \vartheta_1 A_{fj} \end{pmatrix}, \tilde{B}_{2ij} = \begin{pmatrix} \varepsilon_1 B_i \\ \varepsilon_3 B_{fj} L \Psi D_i - \varepsilon_2 B_i \end{pmatrix}, \\ \tilde{E}_{2ij} &= (\omega_5 E_{fj} + \omega_3 F_{fj} L \Psi C_i - \omega_6 E_i \quad \omega_5 E_{fj}), \tilde{F}_{2ij} = (\varepsilon_3 F_{fj} L \Psi D_i - \varepsilon_6 F_i).\end{aligned}$$

and for $k \in [k_r + \Delta_r, k_{r+1})$,

$$\begin{aligned}\tilde{x}(k_0 + 1) &\preceq \tilde{A}_{2i} \tilde{x}(k_0) + \tilde{B}_{2i} \omega(k_0), \\ e(k_0) &\preceq \tilde{E}_{2i} \tilde{x}(k_0) + \tilde{F}_{2i} \omega(k_0),\end{aligned}\quad (92)$$

where

$$\begin{aligned}\tilde{A}_{2i} &= \begin{pmatrix} \omega_1 A_i & 0 \\ \vartheta_1 A_{fi} + \omega_3 B_{fi} L \Psi C_i - \omega_2 A_i & \vartheta_1 A_{fi} \end{pmatrix}, \tilde{B}_{2i} = \begin{pmatrix} \varepsilon_1 B_i \\ \varepsilon_3 B_{fi} L \Psi D_i - \varepsilon_2 B_i \end{pmatrix}, \\ \tilde{E}_{2i} &= (\omega_5 E_{fi} + \omega_3 F_{fi} L \Psi C_i - \omega_6 E_i \quad \omega_5 E_{fi}), \tilde{F}_{2i} = (\varepsilon_3 F_{fi} L \Psi D_i - \varepsilon_6 F_i).\end{aligned}$$

Using a similar method in Theorem 1 gives that the error systems (9) and (10) are positive. From (12) and (90), for $k \in [k_r, k_r + \Delta_r)$, we have

$$\begin{aligned}\tilde{x}(k + 1) &\preceq \tilde{A}_{2ij} \tilde{x}(k) + \tilde{B}_{2ij} \omega(k), \\ e(k) &\preceq \tilde{E}_{2ij} \tilde{x}(k) + \tilde{F}_{2ij} \omega(k),\end{aligned}\quad (93)$$

where

$$\begin{aligned}\tilde{A}_{2ij} &= \begin{pmatrix} \omega_2 A_i & 0 \\ \vartheta_2 A_{fj} + \omega_4 B_{fj} J \Phi C_i - \omega_1 A_i & \vartheta_2 A_{fj} \end{pmatrix}, \tilde{B}_{2ij} = \begin{pmatrix} \varepsilon_2 B_i \\ \varepsilon_4 B_{fj} J \Phi D_i - \varepsilon_1 B_i \end{pmatrix}, \\ \tilde{E}_{2ij} &= (\omega_6 E_{fj} + \omega_4 F_{fj} J \Phi C_i - \omega_5 E_i \quad \omega_6 E_{fj}), \tilde{F}_{2ij} = (\varepsilon_4 F_{fj} J \Phi D_i - \varepsilon_5 F_i).\end{aligned}$$

For $k \in [k_r + \Delta_r, k_{r+1})$, we have

$$\begin{aligned}\tilde{x}(k + 1) &\preceq \tilde{A}_{2i} \tilde{x}(k) + \tilde{B}_{2i} \omega(k), \\ e(k) &\preceq \tilde{E}_{2i} \tilde{x}(k) + \tilde{F}_{2i} \omega(k),\end{aligned}\quad (94)$$

where

$$\begin{aligned}\tilde{A}_{2i} &= \begin{pmatrix} \omega_2 A_i & 0 \\ \vartheta_2 A_{fi} + \omega_4 B_{fi} J \Phi C_i - \omega_1 A_i & \vartheta_2 A_{fi} \end{pmatrix}, \tilde{B}_{2i} = \begin{pmatrix} \varepsilon_2 B_i \\ \varepsilon_4 B_{fi} J \Phi D_i - \varepsilon_1 B_i \end{pmatrix}, \\ \tilde{E}_{2i} &= (\omega_6 E_{fi} + \omega_4 F_{fi} J \Phi C_i - \omega_5 E_i \quad \omega_6 E_{fi}), \tilde{F}_{2i} = (\varepsilon_4 F_{fi} J \Phi D_i - \varepsilon_5 F_i).\end{aligned}$$

Choose a linear copositive Lyapunov function:

$$V_i(k) = \begin{cases} \tilde{x}^\top(k) v_i + \eta(k), & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r), \\ \tilde{x}^\top(k) v_{(i,j)} + \eta(k), & \forall k \in [k_r, k_r + \Delta_r). \end{cases}$$

Combining (93) and (94), we find

$$\Delta V_i(k) \leq \begin{cases} \tilde{x}^\top(k) \Omega_1 + \omega^\top(k) \Gamma_1 + (\varrho - 1) \eta(k), & \forall k \in [k_{r-1} + \Delta_{r-1}, k_r), \\ \tilde{x}^\top(k) \Omega_2 + \omega^\top(k) \Gamma_2 + (\varrho - 1) \eta(k), & \forall k \in [k_r, k_r + \Delta_r), \end{cases}$$

where

$$\begin{aligned}\Omega_1 &= \bar{A}_{2i}^\top v_i - v_i = \begin{pmatrix} \omega_2 A_i^\top \zeta_i + (\vartheta_2 A_{fi}^\top + \omega_4 C_i^\top \Phi J B_{fi}^\top - \omega_1 A_i^\top) \varphi_i + \beta \omega_4 C_i^\top 1_m - \zeta_i \\ \vartheta_2 A_{fi}^\top \varphi_i - \varphi_i \end{pmatrix}, \\ \Omega_2 &= \bar{A}_{2ij}^\top v_{(i,j)} - v_{(i,j)} = \begin{pmatrix} \omega_2 A_i^\top \zeta_{(i,j)} + (\vartheta_2 A_{fj}^\top + \omega_4 C_i^\top \Phi J B_{fj}^\top - \omega_1 A_i^\top) \varphi_{(i,j)} + \beta \omega_4 C_i^\top 1_m - \zeta_{(i,j)} \\ \vartheta_2 A_{fj}^\top \varphi_{(i,j)} - \varphi_{(i,j)} \end{pmatrix}, \\ \Gamma_1 &= \bar{B}_{2i}^\top v_i = \varepsilon_2 B_i^\top \zeta_i + \beta \varepsilon_4 D_i^\top 1_m + (\varepsilon_4 D_i^\top \Phi J B_{fi}^\top - \varepsilon_1 B_i^\top) \varphi_i, \\ \Gamma_2 &= \bar{B}_{2ij}^\top v_{(i,j)} = \varepsilon_2 B_i^\top \zeta_{(i,j)} + \beta \varepsilon_4 D_i^\top 1_m + (\varepsilon_4 D_i^\top \Phi J B_{fj}^\top - \varepsilon_1 B_i^\top) \varphi_{(i,j)}.\end{aligned}$$

Using a similar method to Theorem 1, it is not difficult to obtain (68) under average dwell time (88), which means that the error systems (9) and (10) are ℓ_1 -gain stable with performance γ . $\square$

Remark 5. Compared with the existing results on static event-triggering strategies of positive systems [39–41], the dynamic strategy proposed in Theorem 2 is more flexible and releases fewer data. For the dynamic event-triggered issues of general systems [42–46], it is clear that dynamic event-triggering conditions cannot be directly used for positive systems. Therefore, we proposed a dynamic event-triggering condition (69), and the lower bound of the error systems can be obtained from an interval system. Based on this point, the dynamic event-triggering condition can be applied to other issues of positive systems, such as output feedback control, observer design, etc.

4. Illustrative Examples

Two examples are provided to verify the effectiveness of the proposed design.

Example 1. Consider the system (1) with two subsystems:

$$\begin{aligned}A_1 &= \begin{pmatrix} 0.1503 & 0.1452 & 0.0231 \\ 0.2046 & 0.1729 & 0.1967 \\ 0.1050 & 0.2130 & 0.1714 \end{pmatrix}, B_1 = \begin{pmatrix} 0.0519 & 0.2034 \\ 0.1107 & 0.0257 \\ 0.0173 & 0.1322 \end{pmatrix}, C_1 = \begin{pmatrix} 0.3 & 0.3 & 0.2 \\ 0.4 & 0.1 & 0.1 \end{pmatrix}, \\ D_1 &= \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.4 \end{pmatrix}, E_1 = (0.1 \quad 0.2 \quad 0.2), F_1 = (0.3 \quad 0.6), \\ A_2 &= \begin{pmatrix} 0.2214 & 0.0496 & 0.1183 \\ 0.1426 & 0.1651 & 0.1742 \\ 0.1357 & 0.2073 & 0.0137 \end{pmatrix}, B_2 = \begin{pmatrix} 0.2641 & 0.2026 \\ 0.1273 & 0.1797 \\ 0.2065 & 0.1842 \end{pmatrix}, C_2 = \begin{pmatrix} 0.2 & 0.4 & 0.2 \\ 0.2 & 0.3 & 0.1 \end{pmatrix}, \\ D_2 &= \begin{pmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \end{pmatrix}, E_2 = (0.1 \quad 0.2 \quad 0.2), F_2 = (0.4 \quad 0.5),\end{aligned}$$

where $f_i(x_i(k)) = 2e^{-0.2k}x_i(k)$, $\hat{f}_i(x_i(k)) = e^{-k}x_{fi}(k)$, $h_i(x_i(k)) = x_i(k) + \frac{x_i(k)}{x_i^2(k)+1}$, $p_i(x_i(k)) = x_i(k) + \frac{x_i(k)}{x_i^2(k)+5}$, the disturbance signal is $\omega_i(k) = \left(\frac{0.4}{(k+1)^{2/3}} \quad 0.4e^{-0.1k}\right)^\top$, and the nonlinear disturbance is given as $g_i(\omega_i(k)) = 0.3e^{-0.03k}\omega_i(k)$, $l_i(\omega_i(k)) = \omega_i(k)$, $q_i(\omega_i(k)) = 0.75\omega_i(k)$. Then, $\omega_1 = 0.20$, $\omega_2 = 0.30$, $\omega_3 = 0.30$, $\omega_4 = 0.50$, $\omega_5 = 0.20$, $\omega_6 = 0.30$, $\varepsilon_1 = 0.10$, $\varepsilon_2 = 0.20$, $\varepsilon_3 = 1$, $\varepsilon_4 = 1$, $\varepsilon_5 = 0.30$, $\varepsilon_6 = 0.50$. Choose $\mu_1 = 0.69$, $\mu_1^* = 0.80$, $\mu_2 = 1.30$, $\beta = 0.15$, and $\lambda = 1.20$. By Theorem 1, the filter gain matrices are:

$$\begin{aligned}A_{f1} &= \begin{pmatrix} 0.3132 & 0.1982 & 0.2287 \\ 0.2897 & 0.2154 & 0.2727 \\ 0.2698 & 0.2372 & 0.2524 \end{pmatrix}, B_{f1} = \begin{pmatrix} 0.5950 & 0.0317 \\ 0.5807 & 0.0258 \\ 0.5845 & 0.0259 \end{pmatrix}, E_{f1} = \begin{pmatrix} 0.0013 \\ 0.0014 \\ 0.0190 \end{pmatrix}^\top, F_{f1} = \begin{pmatrix} 1.2703 \\ 0.9366 \end{pmatrix}^\top, \\ A_{f2} &= \begin{pmatrix} 0.1622 & 0.0921 & 0.2278 \\ 0.1468 & 0.0894 & 0.2468 \\ 0.1493 & 0.0946 & 0.2309 \end{pmatrix}, B_{f2} = \begin{pmatrix} 1.3793 & 1.5699 \\ 1.5275 & 1.4030 \\ 1.5288 & 1.3936 \end{pmatrix}, E_{f2} = \begin{pmatrix} 0.0055 \\ 0.0046 \\ 0.0126 \end{pmatrix}^\top, F_{f2} = \begin{pmatrix} 1.4756 \\ 0.6147 \end{pmatrix}^\top,\end{aligned}$$

and the ℓ_1 -gain value is $\gamma = 0.9644$ and average dwell time switching satisfies $\tau_a \geq 4.4728$. Figures 2 and 3 denote the event-triggered output signal $\tilde{y}(k)$ and the quantified output signal $\bar{y}(k)$ for nonlinear switched positive systems. The simulations of the output signal $\tilde{y}(k)$ and $\bar{y}(k)$

under different initial conditions are shown in Figure 4. Figure 5 provides the simulations of the output $z(k)$ and the estimated output $z_f(k)$ under the asynchronous switching signal. Figure 6 shows the event-triggering release interval. The simulations of $z(k)$ and $z_f(k)$ under different initial conditions are given in Figure 7.

Figure 2. Event-triggered output signal $\tilde{y}_1(k)$ and quantized output signal $\bar{y}_1(k)$.

Figure 3. Event-triggered output signal $\tilde{y}_2(k)$ and quantized output signal $\bar{y}_2(k)$.

Figure 4. The simulations of $\tilde{y}(k)$ and $\bar{y}(k)$ under different initial conditions.

Figure 5. The simulations of $z(k)$ and $z_f(k)$ with an asynchronous switching signal.

Figure 6. The event-triggering release instants and release intervals.

Figure 7. The simulations of $z(k)$ and $z_f(k)$ under different initial conditions.

Example 2. Consider the system (1) with two subsystems:

$$\begin{aligned}
 A_1 &= \begin{pmatrix} 0.0894 & 0.1462 & 0.2763 \\ 0.1945 & 0.1573 & 0.2645 \\ 0.1497 & 0.0154 & 0.1637 \end{pmatrix}, B_1 = \begin{pmatrix} 0.1361 & 0.1104 \\ 0.1476 & 0.0632 \\ 0.0248 & 0.1353 \end{pmatrix}, C_1 = \begin{pmatrix} 0.2 & 0.2 & 0.3 \\ 0.3 & 0.2 & 0.1 \end{pmatrix}, \\
 D_1 &= \begin{pmatrix} 0.3 & 0.2 \\ 0.2 & 0.4 \end{pmatrix}, E_1 = (0.1 \quad 0.1 \quad 0.2), F_1 = (0.4 \quad 0.5), \\
 A_2 &= \begin{pmatrix} 0.1523 & 0.0496 & 0.1346 \\ 0.1817 & 0.1977 & 0.0412 \\ 0.1264 & 0.0741 & 0.1255 \end{pmatrix}, B_2 = \begin{pmatrix} 0.1450 & 0.2144 \\ 0.2145 & 0.1562 \\ 0.1855 & 0.1786 \end{pmatrix}, C_2 = \begin{pmatrix} 0.3 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.2 \end{pmatrix}, \\
 D_2 &= \begin{pmatrix} 0.3 & 0.3 \\ 0.2 & 0.1 \end{pmatrix}, E_2 = (0.2 \quad 0.1 \quad 0.2), F_2 = (0.3 \quad 0.3).
 \end{aligned}$$

Choose the same parameters as in Example 1. Furthermore, under dynamic event-triggering condition (41) and (42), we give $\eta_0 = 0.60$, $\varrho = 0.50$. By Theorem 2, the filter gain matrices can be obtained:

$$A_{f1} = \begin{pmatrix} 0.2388 & 0.2588 & 0.3569 \\ 0.2685 & 0.2962 & 0.3488 \\ 0.2454 & 0.2393 & 0.3122 \end{pmatrix}, B_{f1} = \begin{pmatrix} 0.2952 & 0.0460 \\ 0.2951 & 0.0451 \\ 0.3005 & 0.0550 \end{pmatrix}, E_{f1} = \begin{pmatrix} 0.0039 \\ 0.0034 \\ 0.1127 \end{pmatrix}^\top, F_{f1} = \begin{pmatrix} 1.3585 \\ 0.8493 \end{pmatrix}^\top,$$

$$A_{f2} = \begin{pmatrix} 0.1447 & 0.1751 & 0.2787 \\ 0.1513 & 0.1986 & 0.2497 \\ 0.1386 & 0.1620 & 0.2379 \end{pmatrix}, B_{f2} = \begin{pmatrix} 0.6693 & 0.7391 \\ 0.9234 & 0.6583 \\ 0.8987 & 0.6241 \end{pmatrix}, E_{f2} = \begin{pmatrix} 0.0660 \\ 0.0385 \\ 0.1896 \end{pmatrix}^\top, F_{f2} = \begin{pmatrix} 0.8772 \\ 0.5213 \end{pmatrix}^\top,$$

and the ℓ_1 -gain value is $\gamma = 1.0796$ and average dwell time switching satisfies $\tau_a \geq 4.4728$. The dynamic event-triggered output signal $\tilde{y}(k)$ and the quantified output signal $\bar{y}(k)$ are given in Figures 8 and 9. Figure 10 shows the simulation results of the output $z(k)$ and the estimated output $z_f(k)$ under the asynchronous switching signal. Figure 11 shows the dynamic event-triggering release interval. The simulation results of $z(k)$ and $z_f(k)$ under different initial conditions are shown in Figure 12.

Figure 8. The dynamic event-triggered output signal $\tilde{y}_1(k)$ and quantified output signal $\bar{y}_1(k)$.

Figure 9. The dynamic event-triggered output signal $\tilde{y}_2(k)$ and quantified output signal $\bar{y}_2(k)$.

Figure 10. The simulations of $z(k)$ and $z_f(k)$ with asynchronous switching signal.

Figure 11. The dynamic event-triggering release instants and release intervals.

Figure 12. The simulations of $z(k)$ and $z_f(k)$ under different initial conditions.

5. Conclusions

In this paper, we investigated an event-triggered asynchronous filter of nonlinear switched positive systems with output quantization. Based on static and dynamic event-triggering mechanisms, an asynchronous filter was proposed using the matrix decomposition technique. The positivity and L_1 -gain stability of the underlying systems were guaranteed by using a linear copositive Lyapunov function and linear programming approach. Then, the issue of output quantization is solved under a quantizer.

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