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Pessimistic Multigranulation Roughness of a Fuzzy Set Based on Soft Binary Relations over Dual Universes and Its Application

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Abstract: The rough set model for dual universes and multi granulation over dual universes is an interesting generalization of the Pawlak rough set model. In this paper, we present a pessimistic multigranulation roughness of a fuzzy set based on soft binary relations over dual universes. Firstly, we approximate fuzzy set w.r.t aftersets and foresets of the finite number of soft binary relations. As a result, we obtained two sets of fuzzy soft sets known as the pessimistic lower approximation of a fuzzy set and the pessimistic upper approximation of a fuzzy set—the w.r.t aftersets and the w.r.t foresets. The pessimistic lower and pessimistic upper approximations of the newly proposed multigranulation rough set model are then investigated for several interesting properties. This article also addresses accuracy measures and measures of roughness. Finally, we give a decision-making algorithm as well as examples from the perspective of application.

Keywords: fuzzy set; roughness; soft set; soft binary relations; multigranulations

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1. Introduction

We come across various problems in our surroundings that involve some uncertainties. For example, the notion of beautiful guys is imprecise (uncertain), because we cannot uniquely classify all beautiful guys into two classes: beautiful guys and not beautiful guys. Thus the beauty is not exact but rather an uncertain (vague) concept. For this reason, uncertainty is important to philosophers, mathematicians, and recently also computer scientists have turned their interest to these vague (uncertainty) concepts. There are several theories for dealing with uncertainty, including probability theory, vague set theory, and interval mathematics. Each approach has its advantages and disadvantages.

Zadeh [1] developed the concept of a fuzzy set, which was the first successful approach to imprecision. Sets are defined by partial membership in this technique, as opposed to exact membership in the classical set. It can deal with problem uncertainties and solve the problems of decision-making. Each of these theories is well-known and frequently beneficial for characterizing imprecision, but each of them has its own number of difficulties, as indicated in [2]. In 1999, a Russian mathematician Molodtsov [2] presented a novel mathematical framework to deal with impression. This novel approach is know as soft set theory (SST). SST is a new technique that avoids the problems that present in existing theories. This theory has wider applications. Maji et al. [3,4] provided the first practical implementations of soft theory, as well as establishing many operations and a theoretical study on SST. Ali et al. [5] introduces various additional operations on SS and improves the concept of a SS complement. To solve a problem in soft sets theory, the parameters are usually ambiguous phrases or sentences involving vague terms. Maji et al. [6] defined a fuzzy soft set (FSS) as a combination of (FS) and (SS). FSS can deal with the problems of DM in real life. Roy and Maji [7] discussed an FSS theoretic approach towards a discussion DM, Yang et al. [8] presented the notion of interval valued FSS, and the interval valued FSS is being used to examine a DM problem, Bhardwaj et al. [9] recently discussed an advanced uncertainty measure based on FSS, as well as its application to DM problems. Yang et al. [10] presented the notion of FS matrices and their applications. Petchimuthu et al. [11] discussed the mean operators and generalized products of fuzzy soft matrices and their applications in MCGDM. They also discussed the adjustable approaches to multi-criteria group decision making based on inverse fuzzy soft matrices in [12].

Another mathematical method for dealing with problem containing imprecision is the rough set theory (RST), which was presented by Pawlak in 1982 [13]. RST is a frequently used method for dealing with imprecision. Similar to FST, it is not an alternative to traditional set theory, but rather an integrated part of it. RST has the advantage of requiring no preliminary or supplemental data knowledge, such as statistical probability. Many applications of RST have been discovered. Machine learning, information acquisition, decision making, knowledge production from databases, expert systems, inductive reasoning, and pattern recognition are just a few examples. The rough set technique is crucial in artificial intelligence and cognitive sciences [13–15]. Partition is the foundation of Pawlak's RST. Many application are restricted by such a partition, because it can only deal with complete data. To address these issues, tolerance relations, similarity relations, general binary relations, neighborhood systems, and others are used in place of partitions. Feng et al. [16] combined SS with FS and RS, the RSS and SRS are investigated in [17–19], the rough set approximation based on SBr and knowledge bases was discussed by Li et al. [20], Meng et al. [21] discussed SRFS and SFRS, Zhang et al. [22] presented novel FRS models and corresponding applications to MCDM. Many authors have blended the concepts of FS and RS in various ways, as demonstrated in [23–26].

In many practical situations, the usual RS model is built on a single equivalence relation, which has difficulties when dealing with multi-granulation information. To address these issue, Qian et al. [27–29] proposed a multi-granulation rough sets (MGRS) model to approximate a set in w.r.t finite number of equivalence relations rather than a single equivalence relation, Qaian et al. [30] also presented Pessimistic RS based decisions, a fusion strategy. Many scholars from all around the world have been drawn to MGRS and have contributed significantly to their development and applications. Xu et al. [31,32] discussed two types of MGRSs based on ordered and tolerance relations, FMGRS can be found in [33–35], new types of dominance based MGRS and their applications in conflict analysis problems were described by Ali et al. [36]. Xu et al. [37] created two new forms of MGRS., Lin, et al. [38] discussed neighborhood-based MGRS, MGCR was discussed by Liu et al. [39], Kumar et al [40] proposed a OMGRS based classification for medical diagnostics, and Huang et al. [41] combined the idea of MGRS and intuitionistic FS and defined intuitionistic FMGRSs.

In reality, many practical problems, such as disease symptoms and medications used in disease diagnostics, contain multiple universes of objects. The Pawlak RS model deals with the problems of a single universe. To address these issue, the RS model over dual universes was presented by Liu [42] and Yan et al. [43], establishing a relationship between the RS model over a single universe, and the RS model over dual universes was discussed. To measure the uncertainty of knowledge, Ma and Sun [44] proposed probabilistic RS over dual universes, the graded RS model based on dual universes and its features were addressed by Liu et al. [45], Shabir et al. [46] discussed approximation of a set based on SBr over dual universes and their application in the reduction of an information system, Zhang et al. [47] generalized FRS to dual universes with interval valued data, Wu et al. [48] discussed FR approximation over dual universes, and Sun et al. [49] presented MGRS over dual universes of objects. MGRS in two universes is a well-structured framework for dealing with a variety of decision-making problems. It has become a hot topic in the field of multiple decision problems, attracting a wide spectrum of theoretical and application studies. Zhang et al. [50] described the Pythagorean FMGRS and its applications in mergers and acquisitions, Sun et al. [51,52] described the MGFRS over dual universes and its application to DM and three way GDM. Multigranulation vague rough set and diversified binary relation based FMGRS over dual universes and application to multiple attribute GDM can be found in [53,54], Zhang et al. [55] proposed a steam turbine defect diagnostic model based on an interval valued hesitant FMGRS over dual universes, and Tan et al. [56] presented granulation selection and DM with MGRS over dual universes.

Qian et al. [27–30], presented the notion of MGRS based on multi equivalence relations over an universe, Sun et al. [49] generalized this notion and introduced optimistic and pessimistic MGRS over dual universes, replacing equivalence relations with general binary relations from an universe set U to V. On the other hand, Shabir et al. [46], generalized these concepts of RS and replaced relation by SBr from an universal U to V. The MGRS based on SBr was recently presented by Shabir et al. [57]. This paper mainly focuses on pessimistic MGRFS based on BSr over dual universes U and V and approximates an FS $\lambda \in F(V)$ by using the aftersets of SBr and approximates an FS $\gamma \in F(U)$ by using foresets of SBr, where λ, γ are fuzzy sets in U, V respectively. $F(U)$ and $F(V)$ represent a set of all fuzzy sets in U, V respectively. After that, we looked at some of the algebraic properties of our proposed model.

The rest of the paper is laid out as follows. Section 2 recalls the basic concepts of FS, Pawlak RS, MGRSs, SBr, and FSS. Section 3 presents the pessimistic MGR of a FS over dual universes by two SBrs and their basic algebraic properties and examples. Section 4 presents the pessimistic MGR of an FS over dual universes by multi SBrs and their basic algebraic properties. In Section 5 the accuracy measures of the pessimistic MGFSS are presented. In Section 6 we focus on algorithms and a practical example about DM problems. Finally, in Section 7 we conclude the paper.

2. Preliminaries

This section introduces the fuzzy set, rough set, multi-granulation rough set, soft set, soft binary relation, and fuzzy soft set concepts that will be used in subsequent sections.

Definition 1 ([1]). *A membership mapping $\lambda : U \to [0,1]$ is known as an fuzzy set, where $U \neq \emptyset$ is a set of objects. The value $\lambda(x)$ is known as the membership grade of the object $x \in U$. Let λ and γ be two FSs in U. Then $\lambda \leq \gamma$ if $\lambda(x) \leq \gamma(x)$, for all $x \in U$. Moreover $\lambda = \gamma$ if $\lambda \leq \gamma$ and $\gamma \geq \lambda$. An FS λ in U is know as null FS if $\lambda(x) = 0$ for all $x \in U$. An FS λ in U is known as a whole FS, if $\lambda(x) = 1$ for all $x \in U$. We usually denote the null FS by 0 and the whole FS by 1.*

Definition 2 ([1]). *Let λ and γ be two FSs in U. Then their intersection and union are defined as follows*

$$(\lambda \cap \gamma)(x) = \lambda(x) \wedge \gamma(x),$$
$$(\lambda \cup \gamma)(x) = \lambda(x) \vee \gamma(x),$$

for all $x \in U$, where $\wedge$ and $\vee$ means minimum and maximum, respectively.

Definition 3 ([1]). *For a number $0 < \alpha \leq 1$, the α cut of an FS λ in U is $\lambda_\alpha = \{x \in U : \lambda(x) \geq \alpha\}$ which is a subset of U.*

Definition 4 ([13]). *Let ρ be an equivalence relation on U. The Pawlak lower and upper approximations for any $M \subseteq U$ w.r.t ρ are defined by*

$$\underline{\rho}(M) = \{x \in U : [x]_\rho \subseteq M\}$$
$$\overline{\rho}(M) = \{x \in U : [x]_\rho \cap M \neq \emptyset\}.$$

where $[x]_\rho$ is the equivalence class of x w.r.t ρ. The set $BN_\rho(M) = \overline{\rho}(M) - \underline{\rho}(M)$, is the boundary region of M. If $BN_\rho(M) = \emptyset$ then we say that M is definable (exact), otherwise, M is rough w.r.t

ρ. To measure the exactness of a set M the accuracy measure is defined by $\alpha_\rho(M) = \frac{|\underline{\rho}(M)|}{|\overline{\rho}(M)|}$ and roughness measure by $\rho_\rho(M) = 1 - \alpha_\rho(M)$.

Qian et al. [27,30] extended the Pawlak RS model to an MGRS model, in which set approximations are established by multi-equivalence relations on the universe.

Definition 5 ([27]). *Let $\hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_m$ be m equivalence relations on U and $M \subseteq U$. Then the lower and upper approximations of M are defined as*

$$\underline{M}_{\sum_{i=1}^{m}\hat{\rho}_i} = \{x \in U : [x]_{\hat{\rho}_i} \subseteq M \; for\ some\ i, 1 \leq i \leq m\},$$
$$\overline{M}^{\sum_{i=1}^{m}\hat{\rho}_i} = (\underline{M^c}_{\sum_{i=1}^{m}\hat{\rho}_i})^c.$$

Definition 6 ([30]). *Let $\hat{\rho}_1, \hat{\rho}_2, \ldots, \hat{\rho}_m$ be m equivalence relations on an universal set U and $M \subseteq U$. Then the pessimistic lower and upper approximations of M are defined as*

$$\underline{M}_{\sum_{i=1}^{m}\hat{\rho}_i} = \{x \in U : [x]_{\hat{\rho}_i} \subseteq M \; for\ all\ i, 1 \leq i \leq m\},$$
$$\overline{M}^{\sum_{i=1}^{m}\hat{\rho}_i} = (\underline{M^c}_{\sum_{i=1}^{m}\hat{\rho}_i})^c.$$

Definition 7 ([2]). *A soft set over U is a pair (ρ, A), where ρ is a mapping given by $\rho : A \to P(U)$, $U \neq \emptyset$ finite set and $A \subseteq E$ (set of parameters), where $P(U)$ is a power set of U.*

Definition 8 ([58]). *If (ρ, A) is a soft set over $U \times U$, then (ρ, A) is referred to as a soft binary relation on U.*

$SBr(U)$ will be used to represent the collection of all soft binary relations on U.

Li et al. [20] modified the notion of an SBr over a set U to include a SBr from U to V.

Definition 9 ([20]). *If (ρ, A) is a soft set over $U \times V$, then (ρ, A) is a soft binary relation (SBr) from U to V.*

We shall denote the collection of all soft binary relations from U to V by $SBr(U, V)$.

Definition 10 ([7]). *Let $F(U)$ be the set of all FSs on U. Then the pair (ρ, A) is known as FSS over U, where $A \subseteq E$ (set of parameters)*

Definition 11 ([7]). *Let (ρ_1, A) and (ρ_2, B) be two FSSs over a common universe, (ρ_1, A) is a fuzzy soft subset of (ρ_2, B) if $A \subseteq B$ and $\rho_1(e)$ is a fuzzy soft subset of $\rho_2(e)$ for each $e \in A$. The fuzzy soft sets (ρ_1, A) and (ρ_2, B) are equal if and only if (ρ_1, A) is a fuzzy soft subset of (ρ_2, B) and (ρ_2, B) is a fuzzy soft subset of (ρ_1, A).*

3. Pessimistic Roughness of a Fuzzy Set over Two Universes Based on Two Soft Binary Relations

In this section, we discuss the pessimistic roughness of an FS by two SBrs and approximate an FS of universe V in universe U and an FS of universe U in universe V by using aftersets and foresets of SBr from U to V, respectively. As a result, we have two FSSs that correspond to the FS in $V(U)$.

Definition 12. *Let (ρ_1, A) and (ρ_2, A), be two SBrs from U to V and λ be an FS in V. The pessimistic lower approximation (PLAP) $\underline{\rho_1 + \rho_2}^{\lambda}_{p}$ and pessimistic upper approximation (PUAP) ${}^{p}\overline{\rho_1 + \rho_2}^{\lambda}$, of FS λ w.r.t aftersets of (ρ_1, A) and (ρ_2, A) are defined as*

$$
{}_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e)(u) = \begin{cases} \bigwedge\{\lambda(v) : v \in \big(u\rho_1(e) \cap u\rho_2(e)\big)\}, & \text{if } u\rho_1(e) \cap u\rho_2(e) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}
$$
$$
{}^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e)(u) = \begin{cases} \bigvee\{\lambda(v) : v \in \big(u\rho_1(e) \cup u\rho_2(e)\big)\}, & \text{if } u\rho_1(e) \cup u\rho_2(e) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}
$$

where $u\rho_1(e) = \{v \in V : (u, v) \in \rho_1(e)\}, u\rho_2(e) = \{v \in V : (u, v) \in \rho_2(e)\}$ are aftersets of u for $u \in U$ and $e \in A$.
Obviously $\left({}_{p}\underline{\rho_1 + \rho_2}^{\lambda}, A\right)$ and $\left({}^{p}\overline{\rho_1 + \rho_2}^{\lambda}, A\right)$ are two FSSs over U.

Definition 13. *Let (ρ_1, A) and (ρ_2, A), be two SBrs from U to V and γ be an FS in U. The pessimistic lower approximation (PLAP) ${}^{\gamma}\underline{\rho_1 + \rho_2}_{p}$ and pessimistic upper approximation (PUAP) ${}^{\gamma}\overline{\rho_1 + \rho_2}^{p}$, of FS γ w.r.t foresets of (ρ_1, A) and (ρ_2, A) are defined as*

$$
{}^{\gamma}\underline{\rho_1 + \rho_2}_{p}(e)(v) = \begin{cases} \bigwedge\{\gamma(u) : u \in \big(\rho_1(e)(v) \cap \rho_2(e)(v)\big)\}, & \text{if } \rho_1(e)(v) \cap \rho_2(e)(v) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}
$$
$$
{}^{\gamma}\overline{\rho_1 + \rho_2}^{p}(e)(v) = \begin{cases} \bigvee\{\gamma(u) : u \in \big(\rho_1(e)(v) \cup \rho_2(e)(v)\big)\}, & \text{if } \rho_1(e)(v) \cup \rho_2(e)(v) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}
$$

where $\rho_1(e)v = \{u \in U : (u, v) \in \rho_1(e)\}, \rho_2(e)v = \{u \in U : (u, v) \in \rho_2(e)\}$ are foresets of v for $v \in V$ and $e \in A$.

Obviously $\left({}^{\gamma}\underline{\rho_1 + \rho_2}_{p}, A\right)$ and $({}^{\gamma}\overline{\rho_1 + \rho_2}^{p}, A)$ are two fuzzy soft sets over V.

Moreover ${}_{p}\underline{\rho_1 + \rho_2}^{\lambda} : A \to F(U), {}^{p}\overline{\rho_1 + \rho_2}^{\lambda} : A \to F(U)$ and ${}^{\gamma}\underline{\rho_1 + \rho_2}_{p} : A \to F(V), {}^{\gamma}\overline{\rho_1 + \rho_2}^{p} : A \to F(V)$ and we say that $(U, V, \{\rho_1, \rho_2\})$ is a generalized Soft Approximation Space (GSAS).

Next we add an example to elaborate the above defined concepts.

Example 1. *A franchise X wants to select the best allrounder for their team and there are 15 top allrounders who are available for the tournament. These allrounders are categorized into two groups—platinum and diamond. Ghe set $U = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8\}$ represents the players of the platinum group and $V = \{p'_1, p'_2, p'_3, p'_4, p'_5, p'_6, p'_7\}$ represents the players of the diamond group. Let $A = \{e_1, e_2\}$ be the set of parameters, where e_1 represents the batsmen and e_2 represents the bowler. Let the two different teams of coaches analyze and compare these players based on their performance in the different leagues these players play throughout the world, from these comparisons, we have,*

$\rho_1 : A \to P(U \times V)$ represents the comparison of the first team of coaches as defined by:

$$
\begin{aligned}
\rho_1(e_1) =& \{(p_1, p'_2), (p_1, p'_3), (p_2, p'_2), (p_2, p'_5), (p_3, p'_4), (p_3, p'_5), (p_4, p'_1), (p_4, p'_3), (p_5, p'_1), (p_5, p'_6), \\
& (p_7, p'_4), (p_7, p'_7)\}, \\
\rho_1(e_2) =& \{(p_1, p'_3), (p_1, p'_6), (p_2, p'_1), (p_2, p'_4), (p_3, p'_1), (p_4, p'_5), (p_4, p'_7), (p_5, p'_2), (p_5, p'_7), \\
& (p_7, p'_3), (p_7, p'_6), (p_8, p'_1), (p_8, p'_7)\},
\end{aligned}
$$

where $\rho_1(e_1)$ compares the batting performance of the players and $\rho_1(e_2)$ compares the bowling performance of the players.

$\rho_2 : A \to P(U \times V)$ represents the comparison of the second team of coaches as defined by:

$$\rho_2(e_1) = \{(p_1, p_2'), (p_2, p_3'), (p_2, p_5'), (p_3, p_4'), (p_4, p_3'), (p_4, p_5'), (p_4, p_6'), (p_5, p_4'), (p_6, p_7'), (p_7, p_3'), (p_7, p_7'), (p_8, p_2'), (p_8, p_5')\},$$
$$\rho_2(e_2) = \{(p_1, p_3'), (p_1, p_4'), (p_2, p_3'), (p_2, p_4'), (p_2, p_7'), (p_3, p_1'), (p_3, p_6'), (p_4, p_2'), (p_4, p_4'), (p_5, p_2'), (p_6, p_5'), (p_7, p_6'), (p_8, p_1'), (p_8, p_3')\},$$

where $\rho_1(e_1)$ compares the batting performance of the players and $\rho_1(e_2)$ compares the bowling performance of the players.

From these comparisons, we get two SBrs from U to V. Now the aftersets are:

$$\begin{array}{llll}
p_1\rho_1(e_1) = \{p_2', p_3'\}, & p_1\rho_1(e_2) = \{p_3', p_6'\}, & p_1\rho_2(e_1) = \{p_2'\}, & p_1\rho_2(e_2) = \{p_3', p_4'\} \\
p_2\rho_1(e_1) = \{p_2', p_5'\}, & p_2\rho_1(e_2) = \{p_1', p_4'\}, & p_2\rho_2(e_1) = \{p_3', p_5'\}, & p_2\rho_2(e_2) = \{p_3', p_4', p_7'\} \\
p_3\rho_1(e_1) = \{p_4', p_5'\}, & p_3\rho_1(e_2) = \{p_1'\}, & p_3\rho_2(e_1) = \{p_4'\}, & p_3\rho_2(e_2) = \{p_1', p_6'\} \\
p_4\rho_1(e_1) = \{p_1', p_3'\}, & p_4\rho_1(e_2) = \{p_5', p_7'\}, & p_4\rho_2(e_1) = \{p_3', p_5', p_6'\}, & p_4\rho_2(e_2) = \{p_2', p_4'\} \\
p_5\rho_1(e_1) = \{p_1', p_6'\}, & p_5\rho_1(e_2) = \{p_2', p_7'\}, & p_5\rho_2(e_1) = \{p_4'\}, & p_5\rho_2(e_2) = \{p_2'\} \\
p_6\rho_1(e_1) = \emptyset, & p_6\rho_1(e_2) = \emptyset, & p_6\rho_2(e_1) = \{p_7'\}, & p_6\rho_2(e_2) = \{p_5'\} \\
p_7\rho_1(e_1) = \{p_4', p_7'\}, & p_7\rho_1(e_2) = \{p_3', p_6'\}, & p_7\rho_2(e_1) = \{p_3', p_7'\}, & p_7\rho_2(e_2) = \{p_6'\} \\
p_8\rho_1(e_1) = \emptyset, & p_8\rho_1(e_2) = \{p_1', p_7'\}, & p_8\rho_2(e_1) = \{p_2', p_5'\}, & p_8\rho_2(e_2) = \{p_1'\},
\end{array}$$

where $p_i\rho_j(e_1)$ represents all those players of the diamond group whose batting performance is similar to p_i, and $p_i\rho_j(e_2)$ represents all those players of the diamond group whose bowling performance is similar to p_i. The foresets are:

$$\begin{array}{llll}
\rho_1(e_1)p_1' = \{p_4, p_5\}, & \rho_1(e_2)p_1' = \{p_2, p_3, p_8\}, & \rho_2(e_1)p_1' = \emptyset, & \rho_2(e_2)p_1' = \{p_3, p_8\} \\
\rho_1(e_1)p_2' = \{p_1, p_2\}, & \rho_1(e_2)p_2' = \{p_5\}, & \rho_2(e_1)p_2' = \{p_8\}, & \rho_2(e_2)p_2' = \{p_4, p_5\} \\
\rho_1(e_1)p_3' = \{p_1, p_4\}, & \rho_1(e_2)p_3' = \{p_7\}, & \rho_2(e_1)p_3' = \{p_2, p_4, p_7\}, & \rho_2(e_2)p_3' = \{p_1, p_2\} \\
\rho_1(e_1)p_4' = \{p_7\}, & \rho_1(e_2)p_4' = \{p_2\}, & \rho_2(e_1)p_4' = \{p_3, p_5\}, & \rho_2(e_2)p_4' = \{p_1, p_4\} \\
\rho_1(e_1)p_5' = \{p_2, p_3\}, & \rho_1(e_2)p_5' = \{p_4\}, & \rho_2(e_1)p_5' = \{p_2, p_4, p_8\}, & \rho_2(e_2)p_5' = \{p_6\} \\
\rho_1(e_1)p_6' = \{p_5\}, & \rho_1(e_2)p_6' = \{p_1, p_7\}, & \rho_2(e_1)p_6' = \{p_4\}, & \rho_2(e_2)p_6' = \{p_3, p_7\} \\
\rho_1(e_1)p_7' = \{p_7\}, & \rho_1(e_2)p_7' = \{p_4, p_5, p_8\}, & \rho_2(e_1)p_7' = \{p_6, p_7\}, & \rho_2(e_2)p_7' = \{p_2\},
\end{array}$$

where $\rho_j(e_1)p_i'$ represents all those players of the platinum group whose batting performance is similar to p_i', and $\rho_j(e_2)p_i'$ represents all those players of the platinum group whose bowling performance is similar to p_i'

Define $\lambda : V \to [0,1]$, which represents the preference of the players given by franchise X such that

$\lambda(p_1') = 0.9, \lambda(p_2') = 0.8, \lambda(p_3') = 0.4, \lambda(p_4') = 0, \lambda(p_5') = 0.3, \lambda(p_6') = 0.1, \lambda(p_7') = 1$ *and*

Define $\gamma : U \to [0,1]$, which represents the preference of the players given by franchise X such that

$\gamma(p_1) = 0.2, \gamma(p_2) = 1, \gamma(p_3) = 0.5, \gamma(p_4) = 0.9, \gamma(p_5) = 0.6, \gamma(p_6) = 0.7, \gamma(p_7) = 0.1, \gamma(p_8) = 0.3$. *Therefore, the pessimistic lower and upper approximations of λ (the w.r.t aftersets of ρ_1 and ρ_2) are:*

$$\begin{aligned}
{}_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e_1) &= \frac{0.8}{p_1} + \frac{0.3}{p_2} + \frac{0}{p_3} + \frac{0.4}{p_4} + \frac{0}{p_5} + \frac{0}{p_6} + \frac{1}{p_7} + \frac{0}{p_8} \\
{}^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e_1) &= \frac{0.8}{p_1} + \frac{0.8}{p_2} + \frac{0.3}{p_3} + \frac{0.9}{p_4} + \frac{0.9}{p_5} + \frac{1}{p_6} + \frac{1}{p_7} + \frac{0.8}{p_8} \\
{}_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e_2) &= \frac{0.4}{p_1} + \frac{0}{p_2} + \frac{0.9}{p_3} + \frac{0}{p_4} + \frac{0.8}{p_5} + \frac{0}{p_6} + \frac{0.1}{p_7} + \frac{0.9}{p_8} \\
{}^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e_2) &= \frac{0.4}{p_1} + \frac{1}{p_2} + \frac{0.9}{p_3} + \frac{1}{p_4} + \frac{1}{p_5} + \frac{0.3}{p_6} + \frac{0.4}{p_7} + \frac{1}{p_8}.
\end{aligned}$$

Hence, $_p\underline{\rho_1+\rho_2}^{\lambda}(e_i)(p_i)$ *gives the exact degree of the performance of the player to* λ *as a batsman and bowler and,* $^p\overline{\rho_1+\rho_2}^{\lambda}(e_i)(p_i)$ *gives the possible degree of the performance of the player to* λ *as a batsman and bowler w.r.t aftersets.*

The pessimistic lower and upper approximations of γ *(with respect to the foresets of* ρ_1 *and* ρ_2*) are:*

$$
\begin{aligned}
{}^{\gamma}\underline{\rho_1+\rho_2}_p(e_1) &= \frac{0}{p_1'}+\frac{0}{p_2'}+\frac{0.9}{p_3'}+\frac{0}{p_4'}+\frac{1}{p_5'}+\frac{0}{p_6'}+\frac{0.1}{p_7'}\\
{}^{\gamma}\overline{\rho_1+\rho_2}^p(e_1) &= \frac{0.9}{p_1'}+\frac{1}{p_2'}+\frac{1}{p_3'}+\frac{0.6}{p_4'}+\frac{1}{p_5'}+\frac{0.9}{p_6'}+\frac{0.7}{p_7'}\\
{}^{\gamma}\underline{\rho_1+\rho_2}_p(e_2) &= \frac{0.3}{p_1'}+\frac{0.6}{p_2'}+\frac{0}{p_3'}+\frac{0}{p_4'}+\frac{0}{p_5'}+\frac{0.1}{p_6'}+\frac{0}{p_7'}\\
{}^{\gamma}\overline{\rho_1+\rho_2}^p(e_2) &= \frac{1}{p_1'}+\frac{0.9}{p_2'}+\frac{1}{p_3'}+\frac{1}{p_4'}+\frac{0.9}{p_5'}+\frac{0.5}{p_6'}+\frac{1}{p_7'}.
\end{aligned}
$$

Hence, ${}^{\gamma}\underline{\rho_1+\rho_2}_p(e_i)(p_i')$ *gives the exact degree of the performance of the player to* γ *as a batsman and bowler and,* ${}^{\gamma}\overline{\rho_1+\rho_2}^p(e_2)(p_i')$ *gives the possible degree of the performance of the player to* γ *as a batsman and bowler w.r.t foresets.*

Next we study some properties of the above defined approximations.

Proposition 1. *Let* (ρ_1, A) *and* (ρ_2, A) *be two SBrs from universe* U *to* V*, that is* $\rho_1 : A \to P(U \times V)$ *and* $\rho_2 : A \to P(U \times V)$*. Then, the following holds.*

(1) $_p\underline{\rho_1+\rho_2}^{1}(e) = 1$ *for all* $e \in A$ *if* $u\rho_1(e) \cap u\rho_2(e) \neq \emptyset$

(2) $^p\overline{\rho_1+\rho_2}^{1}(e) = 1$ *for all* $e \in A$ *if* $u\rho_1(e) \neq \emptyset$ *or* $u\rho_2(e) \neq \emptyset$

(3) $_p\underline{\rho_1+\rho_2}^{0}(e) = 0 = {}^p\overline{\rho_1+\rho_2}^{0}(e)$

Proof.

(1) Consider $_p\underline{\rho_1+\rho_1}^{1}(e)(u) = \wedge\{1(v) : v \in u\rho_1(e) \cap u\rho_1(e)\} = \wedge\{1 : v \in u\rho_1(e) \cap u\rho_1(e)\} = 1$ because $u\rho_1(e) \cap u\rho_1(e) \neq \emptyset$.

(2) Consider $^p\overline{\rho_1+\rho_1}^{1}(e)(u) = \vee\{1(v) : v \in u\rho_1(e) \cup u\rho_1(e)\} = \vee\{1 : v \in u\rho_1(e) \cup u\rho_1(e)\} = 1$ because $u\rho_1(e) \neq \emptyset$ or $u\rho_1(e) \neq \emptyset$.

(3) Straightforward.

□

Proposition 2. *Let* (ρ_1, A) *and* (ρ_2, A) *be two SBrs from universe* U *to* V*, that is* $\rho_1 : A \to P(U \times V)$ *and* $\rho_2 : A \to P(U \times V)$*. Then, the following holds.*

(1) ${}^{1}\underline{\rho_1+\rho_2}_p(e) = 1$ *for all* $e \in A$ *if* $\rho_1(e)v \cap \rho_2(e)v \neq \emptyset$

(2) ${}^{1}\overline{\rho_1+\rho_2}^p(e) = 1$ *for all* $e \in A$ *if* $\rho_1(e)v \neq \emptyset$ *or* $\rho_2(e)v \neq \emptyset$

(3) ${}^{0}\underline{\rho_1+\rho_2}_p(e) = 0 = {}^{0}\overline{\rho_1+\rho_2}^p(e)$*.*

Proof. The proof is similar to the proof of Proposition 1. □

Proposition 3. *Let* (ρ_1, A) *and* (ρ_2, A) *be two SBrs from universe* U *to* V*, that is* $\rho_1 : A \to P(U \times V)$ *and* $\rho_2 : A \to P(U \times V)$ *and* $\lambda \in F(V)$*. Then* $_p\underline{\rho_1+\rho_2}^{\lambda} \leq {}^p\overline{\rho_1+\rho_2}^{\lambda}$*.*

Proof. Case 1: If $u\rho_1(e) \cap u\rho_2(e) = \emptyset$, then it is obvious.
Case 2: If $u\rho_1(e) \cap u\rho_2(e) \neq \emptyset$, then $_p\underline{\rho_1+\rho_2}^{\lambda}(e)(u) = \wedge\{\lambda(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\}$ $\leq \vee\{\lambda(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\} = {}^p\overline{\rho_1+\rho_2}^{\lambda}(e)(u)$. Hence $_p\underline{\rho_1+\rho_2}^{\lambda} \leq {}^p\overline{\rho_1+\rho_2}^{\lambda}$ □

Proposition 4. *Let* (ρ_1, A) *and* (ρ_2, A) *be two SBrs from universe* U *to* V*, that is* $\rho_1 : A \to P(U \times V)$ *and* $\rho_2 : A \to P(U \times V)$ *and* $\gamma \in F(U)$*. Then* ${}^{\gamma}\underline{\rho_1+\rho_2}_p \leq {}^{\gamma}\overline{\rho_1+\rho_2}^p$*.*

Proof. The proof is similar to the proof of Proposition 3. □

Proposition 5. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from universe U to V, that is $\rho_1 : A \to P(U \times V)$ and $\rho_2 : A \to P(U \times V)$ and $\lambda, \lambda_1, \lambda_2 \in F(V)$. Then the following properties hold the w.r.t aftersets.*

(1) *If $\lambda_1 \leq \lambda_2$ then ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \leq {}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}$,*
(2) *If $\lambda_1 \leq \lambda_2$ then ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \leq {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}$*
(3) ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2} = {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \cap {}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}$
(4) ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2} \geq {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \cup {}_{p}\underline{\rho_1+\rho_2}_{p}^{\lambda_2}$
(5) ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2} = {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \cup {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}$
(6) ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2} \leq {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \cap {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}$

Proof.

(1) Since $\lambda_1 \leq \lambda_2$ so ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1}(e)(u) = \wedge\{\lambda_1(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\} \leq \wedge\{\lambda_2(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\} = {}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}(e)(u)$. Hence ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \leq {}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}$.

(2) Since $\lambda_1 \leq \lambda_2$, so ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1}(e)(u) = \vee\{\lambda_1(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\} \leq \vee\{\lambda_2(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\} = {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}(e)(u)$. Hence ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \leq {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}$.

(3) Consider ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2}(e)(u) = \wedge\{(\lambda_1 \wedge \lambda_2)(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\} = \wedge\{\lambda_1(v) \wedge \lambda_2(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\} = (\wedge\{\lambda_1(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\}) \bigwedge(\wedge\{\lambda_2(v) : v \in (u\rho_1(e) \cap u\rho_2(e))\})$
$= ({}_{p}\underline{\rho_1+\rho_2}^{\lambda_1}(e)(u)) \bigwedge({}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}(e)(u))$.
Hence ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2} = {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \cap {}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}$.

(4) Since $\lambda_1 \leq \lambda_1 \cup \lambda_2$ and $\lambda_2 \leq \lambda_1 \vee \lambda_2$, we have by part (1) ${}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \leq {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2}$ and
${}_{p}\underline{\rho_1+\rho_2}^{\lambda_2} \leq {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2} \Rightarrow {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1} \cup {}_{p}\underline{\rho_1+\rho_2}^{\lambda_2} \leq {}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2}$.

(5) Consider ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2}(e)(u) = \vee\{(\lambda_1 \cup \lambda_2)(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\} = \vee\{\lambda_1(v) \vee \lambda_2(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\} = \{\vee\{\lambda_1(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\}\} \cup \{\vee\{\lambda_2(v) : v \in (u\rho_1(e) \cup u\rho_2(e))\}\}$
$= \{ {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1}(e)(u)\} \cup \{{}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}(e)(u)\}$. Hence ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2} = {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \cup {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}$.

(6) Since $\lambda_1 \geq \lambda_1 \cap \lambda_2$ and $\lambda_2 \geq \lambda_1 \cap \lambda_2$, we have by part (2) ${}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \geq {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2}$ and
${}^{p}\overline{\rho_1+\rho_2}^{\lambda_2} \geq {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2} \Rightarrow {}^{p}\overline{\rho_1+\rho_2}^{\lambda_1} \cap {}^{p}\overline{\rho_1+\rho_2}^{\lambda_2} \geq \overline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2}$.

□

Proposition 6. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from universe U to V, that is $\rho_1 : A \to P(U \times V)$ and $\rho_2 : A \to P(U \times V)$ and $\gamma, \gamma_1, \gamma_2 \in F(U)$. Then, the following hold the w.r.t foresets.*

(1) *If $\gamma_1 \leq \gamma_2$ then ${}^{\gamma_1}\underline{\rho_1+\rho_2}_{p} \leq {}^{\gamma_1}\underline{\rho_1+\rho_2}_{p}$,*
(2) *If $\gamma_1 \leq \gamma_2$ then ${}^{\gamma_1}\overline{\rho_1+\rho_2}^{p} \leq {}^{\gamma_2}\overline{\rho_1+\rho_2}^{p}$*
(3) ${}^{\gamma_1\cap\gamma_2}\underline{\rho_1+\rho_2}_{p} = {}^{\gamma_1}\underline{\rho_1+\rho_2}_{p} \cap {}^{\gamma_2}\underline{\rho_1+\rho_2}_{p}$
(4) ${}^{\gamma_1\cup\gamma_2}\underline{\rho_1+\rho_2}_{p} \geq {}^{\gamma_1}\underline{\rho_1+\rho_2}_{p} \cup {}^{\gamma_2}\underline{\rho_1+\rho_2}_{p}$
(5) ${}^{\gamma_1\cup\gamma_2}\overline{\rho_1+\rho_2}^{p} = {}^{\gamma_1}\overline{\rho_1+\rho_2}^{p} \cup {}^{\gamma_2}\overline{\rho_1+\rho_2}^{p}$
(6) ${}^{\gamma_1\cap\gamma_2}\overline{\rho_1+\rho_2}^{p} \leq {}^{\gamma_1}\overline{\rho_1+\rho_2}^{p} \cap {}^{\gamma_2}\overline{\rho_1+\rho_2}^{p}$

Proof. The proof is similar to the proof of Proposition 5. □

The following example shows that the equality does not hold in parts 4 and 6 of Propositions 5 and 6, generally.

Example 2. *Suppose $U = \{u_1, u_2, u_3, u_4\}$ and $V = \{v_1, v_2, v_3, v_4\}$ are universes, (ρ_1, A) and (ρ_2, A) are two SBrs from U to V, whose aftersets are given below:*

$$\begin{array}{llll}
u_1\rho_1(e_1)=\{v_1,v_2,v_4\}, & u_1\rho_1(e_2)=\{v_2\}, & u_1\rho_2(e_1)=\{v_2,v_3,v_4\}, & u_1\rho_2(e_2)=\{v_1\} \\
u_2\rho_1(e_1)=\{v_2\}, & u_2\rho_1(e_2)=\{v_4\}, & u_2\rho_2(e_1)=\{v_2\}, & u_2\rho_2(e_2)=\{v_2,v_4\} \\
u_3\rho_1(e_1)=\{v_3,v_4\}, & u_3\rho_1(e_2)=\{v_1\}, & u_3\rho_2(e_1)=\{v_4\}, & u_3\rho_2(e_2)=\{v_2,v_4\} \\
u_4\rho_1(e_1)=\emptyset, & u_4\rho_1(e_2)=\{v_2\}, & u_4\rho_2(e_1)=\{v_2,v_3\}, & u_4\rho_2(e_2)=\{v_1,v_2\}
\end{array}$$

and the foresets are:

$$\begin{array}{llll}
\rho_1(e_1)v_1=\{u_1\}, & \rho_1(e_2)v_1=\{u_3\}, & \rho_2(e_1)v_1=\emptyset, & \rho_2(e_2)v_1=\{u_1,u_4\} \\
\rho_1(e_1)v_2=\{u_1,u_2\}, & \rho_1(e_2)v_2=\{u_1,u_4\}, & \rho_2(e_1)v_2=\{u_1,u_2,u_4\}, & \rho_2(e_2)v_2=\{u_2,u_3,u_4\} \\
\rho_1(e_1)v_3=\{u_3\}, & \rho_1(e_2)v_3=\emptyset, & \rho_2(e_1)v_3=\{u_1,u_4\}, & \rho_2(e_2)v_3=\emptyset \\
\rho_1(e_1)v_4=\{u_1,u_3\}, & \rho_1(e_2)v_4=\{u_2\}, & \rho_2(e_1)v_4=\{u_1,u_4\}, & \rho_2(e_2)v_4=\{u_2,u_3\}
\end{array}$$

Let $\lambda_1,\lambda_2,\lambda_1\cup\lambda_2,\lambda_1\cap\lambda_2\in F(V)$ be defined as follows:

$$\begin{aligned}
\lambda_1 &= \frac{0.2}{v_1}+\frac{0.7}{v_2}+\frac{0.3}{v_3}+\frac{0}{v_4} \\
\lambda_2 &= \frac{0.3}{v_1}+\frac{0.5}{v_2}+\frac{0}{v_3}+\frac{0.6}{v_4} \\
\lambda_1\cup\lambda_2 &= \frac{0.3}{v_1}+\frac{0.7}{v_2}+\frac{0.3}{v_3}+\frac{0.6}{v_4} \\
\lambda_1\cap\lambda_2 &= \frac{0.2}{v_1}+\frac{0.5}{v_2}+\frac{0}{v_3}+\frac{0}{v_4},
\end{aligned}$$

and $\gamma_1,\gamma_2,\gamma_1\cup\gamma_2,\gamma_1\cap\gamma_2\in F(U)$ are defined as follows:

$$\begin{aligned}
\gamma_1 &= \frac{0.1}{u_1}+\frac{0.2}{u_2}+\frac{0.3}{u_3}+\frac{0.5}{u_4} \\
\gamma_2 &= \frac{0.5}{u_1}+\frac{0}{u_2}+\frac{0.3}{u_3}+\frac{0}{u_4} \\
\gamma_1\cup\gamma_2 &= \frac{0.5}{u_1}+\frac{0.2}{u_2}+\frac{0.3}{u_3}+\frac{0.5}{u_4} \\
\gamma_1\cap\gamma_2 &= \frac{0.1}{u_1}+\frac{0}{u_2}+\frac{0.3}{u_3}+\frac{0}{u_4}.
\end{aligned}$$

Then,

$$\begin{aligned}
{}_{p}\underline{\rho_1+\rho_2}^{\lambda_1}(e_1) &= \frac{0}{u_1}+\frac{0.7}{u_2}+\frac{0}{u_3}+\frac{0}{u_4} \\
{}^{p}\overline{\rho_1+\rho_2}^{\lambda_1}(e_1) &= \frac{0.7}{u_1}+\frac{0.7}{u_2}+\frac{0.3}{u_3}+\frac{0.7}{u_4} \\
{}_{p}\underline{\rho_1+\rho_2}^{\lambda_2}(e_1) &= \frac{0.5}{u_1}+\frac{0.5}{u_2}+\frac{0.6}{u_3}+\frac{0}{u_4} \\
{}^{p}\overline{\rho_1+\rho_2}^{\lambda_2}(e_1) &= \frac{0.6}{u_1}+\frac{0.5}{u_2}+\frac{0.6}{u_3}+\frac{0}{u_4} \\
{}_{p}\underline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2}(e_1) &= \frac{0.6}{u_1}+\frac{0.7}{u_2}+\frac{0.6}{u_3}+\frac{0}{u_4} \\
{}^{p}\overline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2}(e_1) &= \frac{0.5}{u_1}+\frac{0.5}{u_2}+\frac{0}{u_3}+\frac{0.5}{u_4}
\end{aligned}$$

and

$$
\begin{aligned}
{}^{\gamma_1}\underline{\rho_1+\rho_2}_p(e_1) &= \frac{0}{v_1}+\frac{0.1}{v_2}+\frac{0}{v_3}+\frac{0.1}{v_4}\\
{}^{\gamma_1}\overline{\rho_1+\rho_2}^p(e_1) &= \frac{0.1}{v_1}+\frac{0.5}{v_2}+\frac{0.5}{v_3}+\frac{0.5}{v_4}\\
{}^{\gamma_2}\underline{\rho_1+\rho_2}_p(e_1) &= \frac{0}{v_1}+\frac{0}{v_2}+\frac{0}{v_3}+\frac{0.5}{v_4}\\
{}^{\gamma_2}\overline{\rho_1+\rho_2}^p(e_1) &= \frac{0.5}{v_1}+\frac{0.5}{v_2}+\frac{0.5}{v_3}+\frac{0.5}{v_4}\\
{}^{\gamma_1\cup\gamma_2}\underline{\rho_1+\rho_2}_p(e_1) &= \frac{0}{v_1}+\frac{0.2}{v_2}+\frac{0}{v_3}+\frac{0.5}{v_4}\\
{}^{\gamma_1\cap\gamma_2}\overline{\rho_1+\rho_2}^p(e_1) &= \frac{0.1}{v_1}+\frac{0.1}{v_2}+\frac{0.3}{v_3}+\frac{0.3}{v_4}.
\end{aligned}
$$

Hence

$$
\begin{aligned}
&{}_p\underline{\rho_1+\rho_2}^{\lambda_1}(e_1)(u_1) \vee {}_p\underline{\rho_1+\rho_2}^{\lambda_2}(e_1)(u_1) = 0.5 \ngeq 0.6 = {}_p\underline{\rho_1+\rho_2}^{\lambda_1\cup\lambda_2}(e_1)(u_1)\\
&{}^p\overline{\rho_1+\rho_2}^{\lambda_1}(e_1)(u_1) \wedge {}^p\overline{\rho_1+\rho_2}^{\lambda_2}(e_1)(u_1) = 0.6 \nleq 0.5 = {}^p\overline{\rho_1+\rho_2}^{\lambda_1\cap\lambda_2}(e_1)(u_1),
\end{aligned}
$$

and

$$
\begin{aligned}
&{}^{\gamma_1}\underline{\rho_1+\rho_2}_p(e_1)(v_2) \vee {}^{\gamma_2}\underline{\rho_1+\rho_2}_p(e_1)(v_2) = 0.1 \ngeq 0.2 = {}^{\gamma_1\cup\gamma_2}\underline{\rho_1+\rho_2}_p(e_1)(v_2)\\
&{}^{\gamma_1}\overline{\rho_1+\rho_2}^p(e_1)(v_3) \wedge {}^{\gamma_2}\overline{\rho_1+\rho_2}^p(e_1)(v_3) = 0.5 \nleq 0.3 = {}^{\gamma_1\cap\gamma_2}\overline{\rho_1+\rho_2}^p(e_1)(v_3)
\end{aligned}
$$

In the next definition we define the level set or $\alpha - cut$ of lower approximation ${}_p\underline{\rho_1+\rho_2}^{\lambda}(e)$ and upper approximation ${}^p\overline{\rho_1+\rho_2}^{\lambda}(e)$. Approximations in Definitions 12 and 13 are pairs of FSS. If we associate $\alpha - cut$ of an fuzzy set, we can make a description of the lower approximation $\left({}_p\underline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha$ and upper approximation $\left({}^p\overline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha$.

Definition 14. *Let U and V be two non-empty universes, and $\lambda \in F(V)$. Let (ρ_1, A) and (ρ_2, A) be two SBrs from U to V. For any $0 < \alpha \leq 1$, the $\alpha - cut$ of lower approximation ${}_p\underline{\rho_1+\rho_2}^{\lambda}$ and upper approximation ${}^p\overline{\rho_1+\rho_2}^{\lambda}$ of λ w.r.t aftersets are defined, respectively, as follows:*

$$
\begin{aligned}
\left({}_p\underline{\rho_1+\rho_2}_p^{\lambda}(e)\right)_\alpha &= \{u \in U : {}_p\underline{\rho_1+\rho_2}^{\lambda}(e)(u) \geq \alpha\}\\
\left({}^p\overline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha &= \{u \in U : {}^p\overline{\rho_1+\rho_2}^{\lambda}(e)(u) \geq \alpha\}.
\end{aligned}
$$

These are soft sets over U.

Definition 15. *Let U and V be two non-empty universes, and $\gamma \in F(U)$. Let (ρ_1, A) and (ρ_2, A) be two SBrs from U to V. For any $0 < \alpha \leq 1$, the $\alpha - cut$ of lower approximation ${}^{\gamma}\underline{\rho_1+\rho_2}_p$ and upper approximation ${}^{\gamma}\overline{\rho_1+\rho_2}^p$ of λ w.r.t foresets are defined, respectively, as follows:*

$$
\begin{aligned}
\left({}^{\gamma}\underline{\rho_1+\rho_2}_p(e)\right)_\alpha &= \{v \in V : {}^{\gamma}\underline{\rho_1+\rho_2}_p(e)(v) \geq \alpha\}\\
({}^{\gamma}\overline{\rho_1+\rho_2}^p(e))_\alpha &= \{v \in V : {}^{\gamma}\overline{\rho_1+\rho_2}^p(e)(u) \geq \alpha\}.
\end{aligned}
$$

These are soft sets over V.

Proposition 7. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from universe U to V, $\lambda \in F(V)$ and $0 < \alpha \leq 1$. Then, the following properties hold the w.r.t aftersets:*

(1) ${}_p\underline{\rho_1+\rho_2}^{(\lambda_\alpha)}(e) = \left({}_p\underline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha$

(2) $^{p}\overline{\rho_1+\rho_2}^{(\lambda_\alpha)}(e) = \left({}^{p}\overline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha$

Proof.

(1) Let $\lambda \in F(V)$ and $0 < \alpha \le 1$. For the crisp set λ_α, we have

$$\begin{aligned}
{}_{p}\underline{\rho_1+\rho_2}^{(\lambda_\alpha)}(e) &= \{u \in U : (u\rho_1 \cap u\rho_2) \subseteq \lambda_\alpha\} \\
&= \{u \in U : \ \lambda(v) \ge \alpha \ for\ all\ v \in (u\rho_1(e) \cap u\rho_2(e))\} \\
&= \{u \in U : \ \wedge\{\lambda(v) : \ v \in (u\rho_1(e) \cap u\rho_2(e))\} \ge \alpha\} \\
&= \left({}_{p}\underline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha.
\end{aligned}$$

(2) Let $\lambda \in F(V)$ and $0 < \alpha \le 1$. For the crisp set λ_α, we have

$$\begin{aligned}
{}^{p}\overline{\rho_1+\rho_2}^{(\lambda_\alpha)}(e) &= \{u \in U : (u\rho_1 \cup u\rho_2) \cap \lambda_\alpha \neq \emptyset\} \\
&= \{u \in U : \ \lambda(v) \ge \alpha \ for\ some\ v \in (u\rho_1(e) \cup u\rho_2(e))\} \\
&= \{u \in U : \ \vee\{\lambda(v) : \ v \in (u\rho_1(e) \cup u\rho_2(e))\} \ge \alpha\} \\
&= \left({}^{p}\overline{\rho_1+\rho_2}^{\lambda}(e)\right)_\alpha.
\end{aligned}$$

□

Proposition 8. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from universe U to V, $\gamma \in F(U)$ and $0 < \alpha \le 1$. Then, the following properties hold the w.r.t foresets:*

(1) $^{(\gamma_\alpha)}\underline{\rho_1+\rho_2}_{p}(e) = \left({}^{\gamma}\underline{\rho_1+\rho_2}_{p}(e)\right)_\alpha$

(2) $^{(\gamma_\alpha)}\overline{\rho_1+\rho_2}^{p}(e) = \left({}^{(\gamma}\overline{\rho_1+\rho_2}^{p}(e)\right)_\alpha$

Proof. The proof is similar to the proof of Proposition 7. □

4. Pessimistic Roughness of a Fuzzy Set over Two Universes Based on Multi Soft Binary Relations

In this section, we generalize the concept of the pessimistic multigranulation roughness (PMGR) of an FS based on two SBr to pessimistic multigranulation roughness (PMGR) of an FS based on multi SBrs.

Definition 16. *Let U, V be two non-empty finite universes and θ be a family of SBrs from U to V. Then, we say (U, V, θ) a multigranulation generalized soft approximation space (MGGSAS) over two universes.*

It is easy to see that the multigranulation generalized soft approximation space (MGGSAS) (U, V, θ), is a generalization of soft approximation space over two universes (U, V, ρ).

Definition 17. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes and λ be a fuzzy set in V. The pessimistic lower approximation ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}$ and pessimistic upper approximation ${}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}$, of FS λ w.r.t aftersets of SBrs $(\rho_i, A) \in \theta$ are defined as*

$$\begin{aligned}
{}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e)(u) &= \begin{cases} \wedge\{\lambda(v) : v \in \cap_{i=1}^{m} u\rho_i(e)\}, & if\ \cap_{i=1}^{m} u\rho_i(e) \neq \emptyset \\ 0, & otherwise. \end{cases} \\
{}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e)(u) &= \begin{cases} \vee\{\lambda(v) : v \in \cup_{i=1}^{m} u\rho_i(e)\}, & if\ u\rho_i(e) \neq \emptyset,\ for\ some\ i \\ 0, & otherwise. \end{cases}
\end{aligned}$$

where $u\rho_i(e) = \{v \in V : (u, v) \in \rho_i(e)\}$, are the aftersets of u for $u \in U$ and $e \in A$. $({}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}, A)$ and $({}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}, A)$ are two FSSs over U.

Definition 18. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes and γ be a fuzzy set in U. The pessimistic lower approximation ${}^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p}$ and pessimistic upper approximation ${}^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p}$, of FS γ w.r.t foresets of SBrs $(\rho_i, A) \in \theta$ are defined as*

$$
{}^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p}(e)(v) = \begin{cases} \wedge\{\gamma(u) : u \in \cap_{i=1}^{m}\rho_i(e)(v)\}, & \text{if } \cap_{i=1}^{m}\rho_i(e)(v) \neq \emptyset \\ 0, & \text{otherwise.} \end{cases}
$$

$$
{}^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p}(e)(v) = \begin{cases} \vee\{\gamma(u) : u \in \cup_{i=1}^{m}\rho_i(e)(v)\}, & \text{if } \rho_i(e)(v) \neq \emptyset,\ \text{for some } i \\ 0, & \text{otherwise.} \end{cases}
$$

where $\rho_i(e)v = \{u \in U : (u, v) \in \rho_i(e)\}$ are the foresets of v for $v \in V$ and $e \in A$.

$({}^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p}, A)$, and $({}^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p}, A)$ are two fuzzy soft sets over V.

Moreover ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda} : A \to F(U)$, ${}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda} : A \to F(U)$ and ${}^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p} : A \to F(V)$, ${}^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p} : A \to F(V)$.

Proposition 9. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes. The following properties hold the w.r.t aftersets.*

(1) ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{1}(e) = 1$ *for all $e \in A$ if $\cap u\rho_i(e) \neq \emptyset$.*

(2) ${}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{1}(e) = 1$ *for all $e \in A$ if $u\rho_i(e) \neq \emptyset$ for some $i \leq m$*

(3) ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{0}(e) = 0 = {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{0}(e)$.

Proof. The proof is similar to the proof of Proposition 1. □

Proposition 10. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes. The following properties hold the w.r.t forersets.*

(1) ${}^{1}\underline{\sum_{i=1}^{m}\rho_i}_{p}(e) = 1$ *for all $e \in A$ if $\cap\rho_i(e)v \neq \emptyset$.*

(2) ${}^{1}\overline{\sum_{i=1}^{m}\rho_i}^{p}(e) = 1$ *for all $e \in A$, if $\rho_i(e)v \neq \emptyset$ for some $i \leq m$*

(3) ${}^{0}\underline{\sum_{i=1}^{m}\rho_i}_{p}(e) = 0 = {}^{0}\overline{\sum_{i=1}^{m}\rho_i}^{p}(e)$.

Proof. The proof of this is similar to the proof of Proposition 1. □

Proposition 11. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes and $\lambda, \lambda_1, \lambda_2 \in F(V)$. The following properties hold the w.r.t aftersets.*

(1) *If $\lambda_1 \leq \lambda_2$, then* ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \leq {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_2}$,

(2) *If $\lambda_1 \leq \lambda_2$, then* ${}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \leq {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_2}$

(3) ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_1\cap\lambda_2} = {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \cap {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_2}$

(4) ${}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_1\cup\lambda_2} \geq {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \cup {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_2}$

(5) ${}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_1\cup\lambda_2} = {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \cup {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_2}$

(6) ${}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_1\cap\lambda_2} \leq {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \cap {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_2}$

Proof. The proof is similar to the proof of Proposition 5. □

Proposition 12. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes and $\gamma, \gamma_1, \gamma_2 \in F(U)$. The following properties hold the w.r.t foresets.*

(1) *If $\gamma_1 \leq \gamma_2$, then* ${}^{\gamma_1}\underline{\sum_{i=1}^{m}\rho_i}_{p} \leq {}^{\gamma_2}\underline{\sum_{i=1}^{m}\rho_i}_{p}$,

(2) *If $\gamma_1 \leq \gamma_2$, then* ${}^{\gamma_1}\overline{\sum_{i=1}^{m}\rho_i}^{p} \leq {}^{\gamma_2}\overline{\sum_{i=1}^{m}\rho_i}^{p}$

(3) ${}^{\gamma_1\cap\gamma_2}\underline{\sum_{i=1}^{m}\rho_i}_{p} = {}^{\gamma_1}\underline{\sum_{i=1}^{m}\rho_i}_{p} \cap {}^{\gamma_2}\underline{\sum_{i=1}^{m}\rho_i}_{p}$

(4) ${}^{\gamma_1\cup\gamma_2}\underline{\sum_{i=1}^{m}\rho_i}_{p} \geq {}^{\gamma_1}\underline{\sum_{i=1}^{m}\rho_i}_{p} \cup {}^{\gamma_2}\underline{\sum_{i=1}^{m}\rho_i}_{p}$

(5) $^{\gamma_1\cup\gamma_2}\overline{\sum_{i=1}^{m}\rho_i}^{p} = {}^{\gamma_1}\overline{\sum_{i=1}^{m}\rho_i}^{p} \cup {}^{\gamma_2}\overline{\sum_{i=1}^{m}\rho_i}^{p}$

(6) $^{\gamma_1\cap\gamma_2}\overline{\sum_{i=1}^{m}\rho_i}^{p} \leq {}^{\gamma_1}\overline{\sum_{i=1}^{m}\rho_i}^{p} \cap {}^{\gamma_2}\overline{\sum_{i=1}^{m}\rho_i}^{p}$

Proof. The proof is similar to the proof of Proposition 5. □

Proposition 13. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes and $\lambda_1, \lambda_2, \lambda_3, \cdots \lambda_n \in F(V)$, be such that $\lambda_1 \subseteq \lambda_2 \subseteq \lambda_3 \subseteq \cdots \subseteq \lambda_n$. Then, the following properties hold the w.r.t aftersets.*

(1) $_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \subseteq {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_2} \subseteq {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_3} \subseteq \cdots \subseteq {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda_n}$

(2) $^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_1} \subseteq {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_2} \subseteq {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_3} \subseteq \cdots \subseteq {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda_n}$

Proof. Straightforward. □

Proposition 14. *Let (U, V, θ) be a multigranulation generalized soft approximation space over two universes and $\gamma_1, \gamma_2, \gamma_3, \cdots \gamma_n \in F(U)$, be such that $\gamma_1 \subseteq \gamma_2 \subseteq \gamma_3 \subseteq \cdots \subseteq \gamma_n$. Then, the following properties hold the w.r.t foresets.*

(1) $^{\gamma_1}\underline{\sum_{i=1}^{m}\rho_i}_{p} \subseteq {}^{\gamma_2}\underline{\sum_{i=1}^{m}\rho_i}_{p} \subseteq {}^{\gamma_3}\underline{\sum_{i=1}^{m}\rho_i}_{p} \subseteq \cdots \subseteq {}^{\gamma_n}\underline{\sum_{i=1}^{m}\rho_i}_{p}$

(2) $^{\gamma_1}\overline{\sum_{i=1}^{m}\rho_i}^{p} \subseteq {}^{\gamma_2}\overline{\sum_{i=1}^{m}\rho_i}^{p} \subseteq {}^{\gamma_3}\overline{\sum_{i=1}^{m}\rho_i}^{p} \subseteq \cdots \subseteq {}^{\gamma_n}\overline{\sum_{i=1}^{m}\rho_i}^{p}$

Proof. Straightforward. □

Definition 19. *Let (U, V, θ) be a multi-granulation generalized soft approximation space over two universes, $\lambda \in F(V)$. For any $0 < \alpha \leq 1$, the α cut set of lower approximation $_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}$ and upper approximation $^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}$ of λ are defined, respectively, as follows:*

$$(_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e))_{\alpha} = \{u \in U : {}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e)(u) \geq \alpha\}$$

$$(^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e))_{\alpha} = \{u \in U : {}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e)(u) \geq \alpha\}.$$

These are the soft sets over U.

Definition 20. *Let (U, V, θ) be a multi-granulation generalized soft approximation space over two universes, $\gamma \in F(U)$. For any $0 < \alpha \leq 1$, the α cut set of lower approximation $^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p}$ and upper approximation $^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p}$ of γ about the α are defined, respectively, as follows:*

$$(^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p}(e))_{\alpha} = \{v \in V : {}^{\gamma}\underline{\sum_{i=1}^{m}\rho_i}_{p}(e)(v) \geq \alpha\}$$

$$(^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p}(e))_{\alpha} = \{v \in V : {}^{\gamma}\overline{\sum_{i=1}^{m}\rho_i}^{p}(e)(u) \geq \alpha\}.$$

These are soft sets over V.

Proposition 15. *Let (U, V, θ) be a multi-granulation generalized soft approximation space over two universes, $\lambda \in F(V)$. For $0 < \alpha \leq 1$,. The following properties hold the w.r.t aftersets:*

(1) $_{p}\underline{\sum_{i=1}^{m}\rho_i}^{(\lambda_{\alpha})}(e) = ({}_{p}\underline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e))_{\alpha}$

(2) $^{p}\overline{\sum_{i=1}^{m}\rho_i}^{(\lambda_{\alpha})}(e) = ({}^{p}\overline{\sum_{i=1}^{m}\rho_i}^{\lambda}(e))_{\alpha}$

Proof. The proof is similar to the proof of Proposition 7. □

Proposition 16. *Let (U, V, θ) be a multi-granulation generalized soft approximation space over two universes, $\gamma \in F(U)$. For $0 < \alpha \leq 1$,. The following properties hold the w.r.t foresets:*

(1) $^{(\gamma_\alpha)}\underline{\sum_{i=1}^{m} \rho_i}_p(e) = (^{\gamma}\underline{\sum_{i=1}^{m} \rho_i}_p(e))_\alpha$

(2) $^{(\gamma_\alpha)}\overline{\sum_{i=1}^{m} \rho_i}^p(e) = (^{(\gamma}\overline{\sum_{i=1}^{m} \rho_i}^p(e))_\alpha$

Proof. The proof is similar to the proof of Proposition 7. □

5. Measures of Pessimistic Multigranulation Roughness of a Fuzzy Set

In this section, we discuss the accuracy measure and rough measure of pessimistic multigranulation roughness of fuzzy sets with respect to aftersets and foresets and their basic properties.

Definition 21. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a nonempty universe U to V and $0 < \beta \leq \alpha \leq 1$. Then the accuracy measure (or degree of accuracy) of membership $\lambda \in F(V)$, with respect to α, β and the w.r.t aftersets of (ρ_1, A), (ρ_2, A) is defined as*

$$PA(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)} = \frac{|(_p\underline{\rho_1 + \rho_2}^{\lambda}(e_i))_\alpha|}{|(^p\overline{\rho_1 + \rho_2}^{\lambda}(e_i))_\beta|}, for\ all\ e_i \in A,$$

where $|.|$ means the cardinality of the set, where PA means the pessimistic accuracy measure. It is obvious that $0 \leq PA(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)} \leq 1$. When $PA(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)} = 1$, the FS $\lambda \in F(V)$ is definable with respect to the aftersets. The pessimistic rough measure is defined as

$$PR(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)} = 1 - PA(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)}$$

Definition 22. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a non-empty universe U to V and $0 < \beta \leq \alpha \leq 1$, the accuracy measure (or degree of accuracy) of membership $\gamma \in F(U)$, w.r.t α, β with respect to foresets of (ρ_1, A), (ρ_2, A) is defined as*

$$PA(^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)} = \frac{|(^{\gamma}\underline{\rho_1 + \rho_2}_p(e_i))_\alpha|}{|(^{\gamma}\overline{\rho_1 + \rho_2}^p(e_i))_\beta|}, for\ all\ e_i \in A,$$

where $|.|$ means the cardinality of the set, where PA means the pessimistic accuracy measure. It is obvious that $0 \leq PA(^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)} \leq 1$. When $PA(^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)} = 1$, the FS $\gamma \in F(U)$ is definable as the w.r.t foresets. The pessimistic rough measure is defined as

$$PR(^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)} = 1 - PA(^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}$$

Example 3 (Continued from Example 1). *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a non-empty universal set U to V as given in Example 1. Then, for $\lambda \in F(V)$ defined in Example 1, and $\alpha = 0.4$ and $\beta = 0.2$ cut sets the w.r.t aftersets are as follows, respectively.*

$$(_p\underline{\rho_1 + \rho_2}^{\lambda}(e_1))_{0.4} = \{u_1, u_4\}$$
$$(_p\underline{\rho_1 + \rho_2}^{\lambda}(e_2))_{0.4} = \{u_1, u_3, u_5, u_8\}$$

$$(^p\overline{\rho_1 + \rho_2}^{\lambda}(e_1))_{0.2} = \{u_1, u_2, u_3, u_4, u_5, u_6u_7, u_8\}$$
$$(^p\overline{\rho_1 + \rho_2}^{\lambda}(e_2))_{0.2} = \{u_1, u_2, u_3, u_4, u_5, u_6u_7, u_8\}.$$

The pessimistic accuracy measures for λ with respect to $\alpha = 0.4$ and $\beta = 0.2$ and the w.r.t aftersets of SBrs $(\rho_1, A), (\rho_2, A)$ are calculated as

$$PA(\rho_1 + \rho_2{}^{\lambda}(e_1))_{(\alpha,\beta)} = \frac{|({}_p\underline{\rho_1 + \rho_2}^{\lambda}(e_1))_{0.4}|}{|({}^p\overline{\rho_1 + \rho_2}^{\lambda}(e_1))_{0.2}|} = \frac{2}{8} = 0.25,$$
$$PA(\rho_1 + \rho_2{}^{\lambda}(e_2))_{(\alpha,\beta)} = \frac{|({}_p\underline{\rho_1 + \rho_2}^{\lambda}(e_2))_{0.4}|}{|({}^p\overline{\rho_1 + \rho_2}^{\lambda}(e_2))_{0.2}|} = \frac{4}{8} = 0.5.$$

$PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}$ shows the degree to which the FS $\lambda \in F(V)$ is accurate constrained to the parameters $\alpha = 0.4$ and $\beta = 0.2$ for $i = 1, 2$ w.r.t aftersets. Similarly for $\gamma \in F(U)$ defined in Example 1, the $\alpha = 0.4$ and $\beta = 0.2$ cut sets with respect to foresets are as follows, respectively.

$$({}^{\gamma}\underline{\rho_1 + \rho_2}_p(e_1))_{0.4} = \{v_3\}$$
$$({}^{\gamma}\underline{\rho_1 + \rho_2}_p(e_2))_{0.4} = \{v_1\},$$

and

$$({}^{\gamma}\overline{\rho_1 + \rho_2}^p(e_1))_{0.2} = \{v_1, v_2, v_3, v_4, v_5, v_6 v_7\}$$
$$({}^{\gamma}\overline{\rho_1 + \rho_2}^p(e_2))_{0.2} = \{v_1, v_2, v_3, v_4, v_5, v_6 v_7\}.$$

The pessimistic accuracy measures for $\gamma \in F(U)$ with respect to $\alpha = 0.4$ and $\beta = 0.2$ and the w.r.t foresets of SBrs $(\rho_1, A), (\rho_2, A)$ are calculated as

$$PA({}^{\gamma}\rho_1 + \rho_2(e_1))_{(\alpha,\beta)} = \frac{|({}^{\gamma}\underline{\rho_1 + \rho_2}_p(e_1))_{0.4}|}{|({}^{\gamma}\overline{\rho_1 + \rho_2}^p(e_1))_{0.2}|} = \frac{1}{8} = 0.125,$$
$$PA({}^{\gamma}\rho_1 + \rho_2(e_2))_{(\alpha,\beta)} = \frac{|({}^{\gamma}\underline{\rho_1 + \rho_2}_p(e_2))_{0.4}|}{|({}^{\gamma}\overline{\rho_1 + \rho_2}^p(e_2))_{0.2}|} = \frac{1}{8} = 0.125.$$

$PA({}^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}$ shows the degree to which the FS $\gamma \in F(U)$ is accurately constrained to the parameters $\alpha = 0.4$ and $\beta = 0.2$ for $i = 1, 2$ w.r.t foresets.

Proposition 17. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a non-empty universe U to V, $\lambda \in F(V)$ and $0 < \beta \leq \alpha \leq 1$. Then*

(1) $PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}$ increases with the increase in β, if α stands fixed.
(2) $PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}$ decreases with the increase in α, if β stands fixed.

Proof.

(1) Let α stand fixed and $0 < \beta_1 \leq \beta_2 \leq 1$. Then we have $|({}^p\overline{\rho_1 + \rho_2}^{\lambda}(e_i))_{\beta_2}| \leq |({}^p\overline{\rho_1 + \rho_2}^{\lambda}\ (e_i))_{\beta_1}|$. This implies that $\frac{|({}_p\underline{\rho_1+\rho_2}^{\lambda}(e_i))_{\alpha}|}{|({}^p\overline{\rho_1+\rho_2}^{\lambda}(e_i))_{\beta_1}|} \leq \frac{|({}_p\underline{\rho_1+\rho_2}^{\lambda}(e_i))_{\alpha}|}{|({}^p\overline{\rho_1+\rho_2}^{\lambda}(e_i))_{\beta_2}|}$, that is $PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta_1)} \leq PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta_2)}$.
This shows that $PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}$ increases with the increase in β for all $e_i \in A$.

(2) Let β stands fixed and $0 < \alpha_1 \leq \alpha_2 \leq 1$. Then we have $|({}_p\underline{\rho_1 + \rho_2}^{\lambda}(e_i))_{\alpha_2}| \leq |({}^p\overline{\rho_1 + \rho_2}^{\lambda}\ (e_i))_{\alpha_1}|$. This implies that $\frac{|({}_p\underline{\rho_1+\rho_2}^{\lambda}(e_i))_{\alpha_2}|}{|({}^p\overline{\rho_1+\rho_2}^{\lambda}(e_i))_{\beta}|} \leq \frac{|({}_p\underline{\rho_1+\rho_2}_p^{\lambda}(e_i))_{\alpha_1}|}{|({}^p\overline{\rho_1+\rho_2}^{\lambda}(e_i))_{\beta}|}$, that is $PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha_2,\beta)} \leq PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha_1,\beta)}$.
This shows that $PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}$ increases with the increase in α for all $e_i \in A$.

□

Proposition 18. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a non-empty universe U to V, $\gamma \in F(U)$ and $0 < \beta \leq \alpha \leq 1$. Then*

(1) $PA({}^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}$ increases with the increase in β, if α stands fixed.
(2) $PA({}^{\gamma}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}$ decreases with the increase in α, if β stands fixed.

Proof. The proof is similar to the proof of Proposition 17. □

Proposition 19. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a non- empty universe U to V, $0 < \beta \leq \alpha \leq 1$ and $\lambda, \mu \in F(V)$, with $\lambda \leq \mu$. Then the following properties hold the w.r.t aftersets.*

(1) $(PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}) \leq PA(\rho_1 + \rho_2{}^{\mu}(e_i))_{(\alpha,\beta)}$, whenever $({}^{p}\overline{\rho_1 + \rho_{2_o}}^{\lambda})_\beta = ({}^{p}\overline{\rho_1 + \rho_2}^{\mu})_\beta$.
(2) $(PA(\rho_1 + \rho_2{}^{\lambda}(e_i))_{(\alpha,\beta)}) \geq PA(\rho_1 + \rho_2{}^{\mu}(e_i))_{(\alpha,\beta)}$, whenever $({}_{p}\underline{\rho_1 + \rho_2}^{\lambda})_\beta = ({}_{p}\underline{\rho_1 + \rho_2}^{\mu})_\beta$.

Proof.

(1) Let $0 < \beta \leq \alpha \leq 1$ and $\lambda, \mu \in F(V)$ be such that $\lambda \leq \mu$. Then $({}_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e_i))_\alpha \leq ({}_{p}\underline{\rho_1 + \rho_2}^{\mu}(e_i))_\alpha$, that is $|({}_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e_i))_\alpha| \leq |({}_{p}\underline{\rho_1 + \rho_2}^{\mu}(e_i))_\alpha|$. This implies that $\frac{|({}_{p}\underline{\rho_1+\rho_2}^{\lambda}(e_i))_\alpha|}{|({}^{p}\overline{\rho_1+\rho_2}^{\lambda}(e_i))_\beta|} \leq \frac{|({}_{p}\underline{\rho_1+\rho_2}^{\mu}(e_i))_\alpha|}{|({}^{p}\overline{\rho_1+\rho_2}^{\mu}(e_i))_\beta|}$. Hence $PA(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)} \leq PA(\rho_1 + \rho_2^{\mu}(e_i))_{(\alpha,\beta)}$ for all $e_i \in A$.

(2) Let $0 < \beta \leq \alpha \leq 1$ and $\lambda, \mu \in F(V)$ be such that $\lambda \leq \mu$. Then $({}^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e_i))_\beta \leq ({}^{p}\overline{\rho_1 + \rho_2}^{\mu}(e_i))_\beta$, that is $|({}^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e_i))_\beta| \leq |({}^{p}\overline{\rho_1 + \rho_2}^{\mu}(e_i))_\beta|$. This implies that $\frac{|({}_{p}\underline{\rho_1+\rho_2}^{\lambda}(e_i))_\alpha|}{|({}^{p}\overline{\rho_1+\rho_2}^{\lambda}(e_i))_\beta|} \geq \frac{|({}_{p}\underline{\rho_1+\rho_2}^{\mu}(e_i))_\alpha|}{|({}^{p}\overline{\rho_1+\rho_2}^{\mu}(e_i))_\beta|}$. Hence $PA(\rho_1 + \rho_2^{\lambda}(e_i))_{(\alpha,\beta)} \geq PA(\rho_1 + \rho_2^{\mu}(e_i))_{(\alpha,\beta)}$ for all $e_i \in A$.

□

Proposition 20. *Let (ρ_1, A) and (ρ_2, A) be two SBrs from a non- empty universe U to V, $0 < \beta \leq \alpha \leq 1$ and $\gamma, \delta \in F(U)$, with $\gamma \leq \delta$. Then the following properties hold the w.r.t foresets.*

(1) $(PA({}^{\lambda}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}) \leq PA({}^{\mu}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}$, whenever $({}^{\gamma}\overline{\rho_1 + \rho_2}^{p})_\beta = ({}^{\mu}\overline{\rho_1 + \rho_2}^{p})_\beta$.
(2) $PA({}^{\lambda}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)} \geq PA({}^{\mu}\rho_1 + \rho_2(e_i))_{(\alpha,\beta)}$, whenever $({}^{\gamma}\underline{\rho_1 + \rho_2}_{p})_\beta = ({}^{\mu}\underline{\rho_1 + \rho_2}_{p})_\beta$.

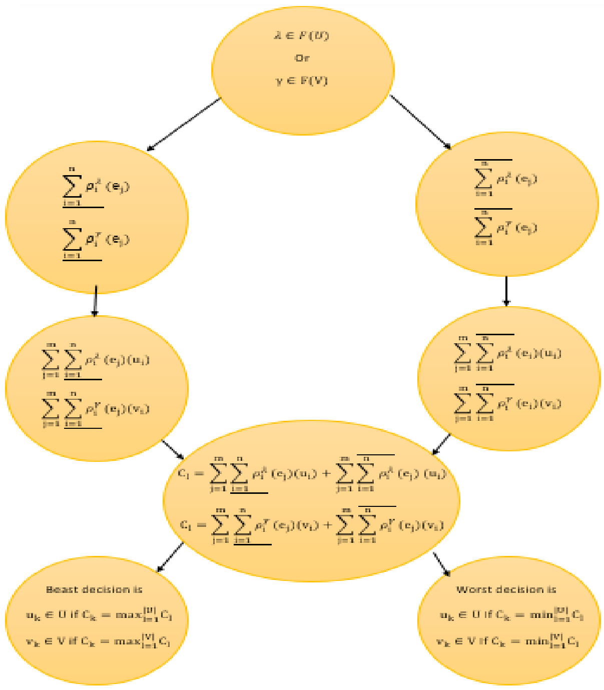
Proof. The proof is similar to the proof of Proposition 19. □

6. Decision Making

In this section, we defined an algorithm for the above-proposed model. We know that FSS have a wide application in decision-making problems. In most cases the approaches to decision-making based on FSS are dependent on choice value "C_k". It is simply reasonable to select the object with the maximum choice value as the optimal alternative. So, we redefine the choice value C_j for the decision alternative u_j of the universe U with respect to the aftersets (foresets) of soft binary relations, to deal with decision-making problems based on RFSS. We know the lower and upper approximations are the two most close-sets to the approximated subsets of a universe. Therefore, we obtain two most corresponding values ${}_{p}\underline{\sum_{k=1}^{n}\rho_k}^{\lambda}(e_i)(u_j)$ and ${}^{p}\overline{\sum_{k=1}^{n}\rho_k}^{\lambda}(e_i)(u_j)$ w.r.t aftersets, the decision alternative $u_j \in U$ by the FS lower and upper approximations of an FS $\lambda \in F(V)$.

Here, we present two algorithms for the proposed model, which consist of the following steps.

Flowchart for Algorithms 1 and 2.



Algorithm 1: An algorithm for the approach to a decision-making problem of thew.r.t aftersets is presented in the following.

Step 1: Compute the lower pessimistic multigranulation fuzzy soft set approximation $_{P}\underline{\sum_{i=1}^{n} \rho_i}^{\lambda}$ and upper pessimistic multigranulation fuzzy soft set approximation $^{P}\overline{\sum_{i=1}^{n} \rho_i}^{\lambda}$, of fuzzy set λ with respect to the aftersets.

Step 2: Compute the sum of a lower pessimistic multigranulation fuzzy soft set approximation $\sum_{j=1}^{n}(_{P}\underline{\sum_{i=1}^{n} \rho_i}^{\lambda}(e_j)(u_l))$ and the sum of an upper pessimistic multigranulation fuzzy soft set approximation $\sum_{j=1}^{n}(^{P}\overline{\sum_{i=1}^{n} \rho_i}^{\lambda}(e_j)(u_l))$, corresponding to j with respect to aftersets.

Step 3: Compute the choice value $C_l = \sum_{j=1}^{n}(_{P}\underline{\sum_{i=1}^{n} \rho_i}^{\lambda}(e_j)(u_l)) + \sum_{j=1}^{n}(^{P}\overline{\sum_{i=1}^{n} \rho_i}^{\lambda}(e_j)(u_l))$, $u_l \in U$ with respect to the aftersets.

Step 4: The best decision is $u_k \in U$ if $C_k = max_{l=1}^{|U|} C_l$.

Step 5: The worst decision is $u_k \in U$ if $C_k = min_{l=1}^{|U|} C_l$.

Step 6: If k has more than one value, then any one of the u_k may be chosen.

Algorithm 2: An algorithm for the approach to a decision-making problem with respect to the foresets is presented in the following.

Step 1: Compute the lower pessimistic multigranulation fuzzy soft set approximation ${}^{\gamma}\underline{\sum_{i=1}^{n}\rho_i}_{p}$ and upper pessimistic multigranulation fuzzy soft set approximation ${}^{\gamma}\overline{\sum_{i=1}^{n}\rho_i}^{p}$, of fuzzy set γ with respect to foresets.
Step 2: Compute the sum of lower pessimistic multigranulation fuzzy soft set approximation $\sum_{j=1}^{n}({}^{\gamma}\underline{\sum_{i=1}^{n}\rho_i}_{p}(e_j)(v_l))$ and the sum of upper pessimistic multigranulation fuzzy soft set approximation $\sum_{j=1}^{n}({}^{\gamma}\overline{\sum_{i=1}^{n}\rho_i}^{p}(e_j)(v_l))$, corresponding to j with respect to foresets.
Step 3: Compute the choice value $C_l = \sum_{j=1}^{n}({}^{\gamma}\underline{\sum_{i=1}^{n}\rho_i}_{p}(e_j)(v_l)) + \sum_{j=1}^{n}({}^{\gamma}\overline{\sum_{i=1}^{n}\rho_i}^{P}(e_j)(v_l))$, $v_l \in V$ with respect to the foresets.
Step 4: The best decision is $v_k \in V$ if $C_k = max_{l=1}^{|V|} C_l$.
Step 5: The worst decision is $v_k \in V$ if $C_k = min_{l=1}^{|V|} C_l$.
Step 6: If k has more than one value, then any one of v_k may be chosen.

An Application of the Decision-Making Approach

Example 4 (Continued from Example 1)**.** *Consider the soft binary relations of Example 1 again, where a franchise X wants to pick a best foreign player (allrounder) for their team from the platinum and diamond categories.*

Define $\lambda : V \to [0,1]$, *which represents the preference of the player given by franchise X such that* $\lambda(v_1) = 0.9, \lambda(v_2) = 0.8, \lambda(v_3) = 0.4, \lambda(v_4) = 0, \lambda(v_5) = 0.3, \lambda(v_6) = 0.1, \lambda(v_7) = 1,$
and
Define $\gamma : U \to [0,1]$, *which represents the preference of the player given by franchise X such that* $\gamma(u_1) = 0.2, \gamma(u_2) = 1, \gamma(u_3) = 0.5, \gamma(u_4) = 0.9, \gamma(u_5) = 0.6, \gamma(u_6) = 0.7, \gamma(u_7) = 0.1, \gamma(u_8) = 0.3.$

Consider Tables 1 and 2 after applying the above algorithms.

Table 1. The pessimistic result of the decision algorithm with respect to the aftersets.

	$_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e_1)$	$_{p}\underline{\rho_1 + \rho_2}^{\lambda}(e_2)$	$^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e_1)$	$^{p}\overline{\rho_1 + \rho_2}^{\lambda}(e_2)$	**Choice Value C_k**
p_1	0.8	0.4	0.8	0.4	2.4
p_2	0.3	0	0.8	1	2.1
p_3	0	0.9	0.3	0.9	2.1
p_4	0.4	0	0.9	1	2.3
p_5	0	0.8	0.9	1	2.7
p_6	0	0	1	0.3	1.3
p_7	1	0.1	1	0.4	2.5
p_8	0	0.9	0.8	1	2.7

Table 2. The pessimistic result of the decision algorithm with respect to the foresets.

	${}^{\gamma}\underline{\rho_1+\rho_2}_p(e_1)$	${}^{\gamma}\underline{\rho_1+\rho_2}_p(e_2)$	${}^{\gamma}\overline{\rho_1+\rho_2}^p(e_1)$	${}^{\gamma}\overline{\rho_1+\rho_2}^p(e_2)$	**Choice Value** C'_k
p'_1	0	0.3	0.9	1	2.1
p'_2	0	0.6	1	0.9	2.5
p'_3	0.9	0	1	1	2.9
p'_4	0	0	0.6	1	1.6
p'_5	1	0	1	0.9	2.9
p'_6	0	0.1	0.9	0.5	1.5
p'_7	0.1	0	0.7	1	1.8

Here the choice value $C_l = \sum_{j=1}^{n}({}_{p}\underline{\sum_{i=1}^{n}\rho_i}^{\lambda}(e_j)(u_l)) + \sum_{j=1}^{n}({}^{p}\overline{\sum_{i=1}^{n}\rho_i}^{\lambda}(e_j)(u_l))$, $u_l \in U$ *with respect to aftersets and* $C'_l = \sum_{j=1}^{n}({}^{\gamma}\underline{\sum_{i=1}^{n}\rho_i}_p(e_j)(v_l)) + \sum_{j=1}^{n}({}^{\gamma}\overline{\sum_{i=1}^{n}\rho_i}^{p}(e_j)(v_l))$, $v_l \in V$ *with respect to foresets.*

From Table 1 it is clear that the maximum choice-value $C_k = 2.7 = C_5 = C_8$ *is scored by the players* p_5 *and* p_8*, and the decision is in the favor of selecting the players* p_5 *or* p_8*. Moreover, player* p_6 *is ignored. Hence franchise X will choose any one of the players* p_5 *and* p_8 *from the platinum category with respect to the aftersets. Similarly, from Table 2, the maximum choice-value* $C'_k = 2 = C'_3 = C'_5$ *scored by the players* p'_3, p'_5*, and the decision is in the favor of selecting any one of the players* p'_3, p'_5*. Moreover, player* p'_6 *is ignored. Hence franchise X will choose any one of the players* p'_3 *or* p'_5 *from the diamond category with respect to the foresets.*

7. Conclusions

This article studies the pessimistic multigranulation roughness of a fuzzy set based on SBrs over two universes. Initially, we defined the pessimistic roughness of a fuzzy set with respect to the aftersets and foresets of two soft binary relations and approximate a fuzzy set $\lambda \in F(V)$ in universe U, and a fuzzy set $\gamma \in F(U)$ in universe V, by using the aftersets and foresets of binary relations from which we got two fuzzy soft sets over U and over V, with respect to the aftersets and foresets. We also investigate some fundamental properties of pessimistic multigranulation roughness of a fuzzy set. Then we generalized these definitions to the pessimistic multigranulation roughness of a fuzzy set based on a finite number of soft binary relations. In addition, we define the accuracy measures and roughness measures for this proposed pessimistic multigranulation roughness. Moreover, we presented two algorithms in decision-making with respect to the afterset and foresets. We also give an example to apply the above algorithm. The main advantage of this approach over other existing approaches is that we can approximate a fuzzy set of a universe in some other universe and we are able to take decision on the basis of each parameter. Future studies will focus on the practical applications of the proposed method in solving a wider range of selection problems, such as disease symptoms and medications used in disease diagnostics.

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