

Supplement A. Calculation Details

A.1 Solution to $\nabla Q_1(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) = \mathbf{0}_3$

By solving $\nabla Q_1(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)}) = \mathbf{0}_3$ defined in §2.2, we have

$$\left\{ \begin{array}{l} \theta_0^{(t+1)} = \frac{S_1^{(t)} + \sqrt{[S_1^{(t)}]^2 + 12nS_2^{(t)}}}{2S_2^{(t)}}, \\ \theta_1^{(t+1)} = \frac{S_3^{(t)} \pm \sqrt{[S_3^{(t)}]^2 + 4nS_4^{(t)}}}{2S_4^{(t)}}, \\ \theta_2^{(t+1)} = \frac{S_5^{(t)} \pm \sqrt{[S_5^{(t)}]^2 + 4S_6^{(t)}S_7^{(t)}}}{2S_6^{(t)}}, \end{array} \right. \quad (\text{S1})$$

where

$$\begin{aligned} S_1^{(t)} &= n \left(1 + \frac{\theta_1^{(t)}}{1 - \theta_1^{(t)}} + \frac{1 - \theta_2^{(t)}}{\theta_2^{(t)}} \right), \\ S_2^{(t)} &= \sum_{i=1}^n B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}) \left\{ 1 + \frac{(1 - x_{i1})[\theta_1^{(t)}]^2}{x_{i1}[1 - \theta_1^{(t)}]^2} + \frac{x_{i2}[1 - \theta_2^{(t)}]^2}{(1 - x_{i2})[\theta_2^{(t)}]^2} \right\}, \\ S_3^{(t)} &= n\theta_0^{(t)} - 2n, \\ S_4^{(t)} &= n\theta_0^{(t)} - n + [\theta_0^{(t)}]^2 \sum_{i=1}^n B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}) \frac{1 - x_{i1}}{x_{i1}}, \\ S_5^{(t)} &= -n\theta_0^{(t)} - 2[\theta_0^{(t)}]^2 \sum_{i=1}^n B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}) \frac{x_{i2}}{1 - x_{i2}}, \\ S_6^{(t)} &= n - n\theta_0^{(t)} - [\theta_0^{(t)}]^2 \sum_{i=1}^n B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}) \frac{x_{i2}}{1 - x_{i2}}, \\ S_7^{(t)} &= [\theta_0^{(t)}]^2 \sum_{i=1}^n B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}) \frac{x_{i2}}{1 - x_{i2}}. \end{aligned}$$

From (S1), we know that the value of $\boldsymbol{\theta}^{(t+1)}$ is complex.

A.2 Calculations for $\nabla G_1(\boldsymbol{\theta}_{-0}|\boldsymbol{\theta}^{(t)})$

We can obtain

$$\begin{aligned}\frac{\partial Q_1(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})}{\partial \theta_1} &= \frac{n\theta_0}{(1-\theta_1)^2} + \frac{n}{\theta_1(1-\theta_1)} - \frac{\theta_0^2\theta_1}{(1-\theta_1)^3} \sum_{i=1}^n \frac{1-x_{i1}}{x_{i1}} B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}), \\ \frac{\partial Q_1(\boldsymbol{\theta}|\boldsymbol{\theta}^{(t)})}{\partial \theta_2} &= -\frac{n\theta_0}{\theta_2^2} - \frac{n}{\theta_2(1-\theta_2)} + \frac{\theta_0^2(1-\theta_2)}{\theta_2^3} \sum_{i=1}^n \frac{x_{i2}}{1-x_{i2}} B_1(\mathbf{x}_i, \boldsymbol{\theta}^{(t)}).\end{aligned}$$

A.3 Calculations for $\nabla G_2(\boldsymbol{\vartheta}_{-0}|\boldsymbol{\vartheta}^{(t)})$

We can obtain

$$\begin{aligned}\frac{\partial Q_2(\boldsymbol{\vartheta}|\boldsymbol{\vartheta}^{(t)})}{\partial \boldsymbol{\alpha}_1} &= \sum_{i=1}^n \left[1 + \theta_0 \exp(\mathbf{w}_i^\top \boldsymbol{\alpha}_1) - \frac{1-x_{i1}}{x_{i1}} \theta_0^2 \exp(2\mathbf{w}_i^\top \boldsymbol{\alpha}_1) B_2(\mathbf{x}_i, \mathbf{w}_i, \boldsymbol{\vartheta}^{(t)}) \right] \mathbf{w}_i, \\ \frac{\partial Q_2(\boldsymbol{\vartheta}|\boldsymbol{\vartheta}^{(t)})}{\partial \boldsymbol{\alpha}_2} &= -\sum_{i=1}^n \left[1 + \theta_0 \exp(-\mathbf{w}_i^\top \boldsymbol{\alpha}_2) - \frac{x_{i2}}{1-x_{i2}} \theta_0^2 \exp(-2\mathbf{w}_i^\top \boldsymbol{\alpha}_2) B_2(\mathbf{x}_i, \mathbf{w}_i, \boldsymbol{\vartheta}^{(t)}) \right] \mathbf{w}_i.\end{aligned}$$

A.4 Calculations for $\nabla \ell_3(\phi|Y_{\text{obs}_3})$

We can obtain

$$\begin{aligned}\frac{\partial \ell_3(\phi|Y_{\text{obs}_3})}{\partial \phi_0} &= n \left(\frac{\phi_1}{1-\phi_1} + \frac{1}{\phi_2} \right) \frac{\Gamma' \left(\frac{\phi_0\phi_1}{1-\phi_1} + \frac{\phi_0}{\phi_2} \right)}{\Gamma \left(\frac{\phi_0\phi_1}{1-\phi_1} + \frac{\phi_0}{\phi_2} \right)} - \frac{n\Gamma'(\phi_0)}{\Gamma(\phi_0)} - \frac{n\phi_1\Gamma' \left(\frac{\phi_0\phi_1}{1-\phi_1} \right)}{(1-\phi_1)\Gamma \left(\frac{\phi_0\phi_1}{1-\phi_1} \right)} \\ &\quad - \frac{n(1-\phi_2)\Gamma' \left(\frac{\phi_0}{\phi_2} - \phi_0 \right)}{\phi_2\Gamma \left(\frac{\phi_0}{\phi_2} - \phi_0 \right)} + \frac{\phi_1}{1-\phi_1} \sum_{i=1}^n \log \left(\frac{x_{i1}}{1-x_{i1}} \right) + \frac{1-\phi_2}{\phi_2} \sum_{i=1}^n \log \left(\frac{1-x_{i2}}{x_{i2}} \right) \\ &\quad - \left(\frac{\phi_1}{1-\phi_1} + \frac{1}{\phi_2} \right) \sum_{i=1}^n \log \left(1 + \frac{x_{i1}}{1-x_{i1}} + \frac{1-x_{i2}}{x_{i2}} \right), \\ \frac{\partial \ell_3(\phi|Y_{\text{obs}_3})}{\partial \phi_1} &= \frac{n\phi_0\Gamma' \left(\frac{\phi_0\phi_1}{1-\phi_1} + \frac{\phi_0}{\phi_2} \right)}{(1-\phi_1)^2\Gamma \left(\frac{\phi_0\phi_1}{1-\phi_1} + \frac{\phi_0}{\phi_2} \right)} - \frac{n\phi_0\Gamma' \left(\frac{\phi_0\phi_1}{1-\phi_1} \right)}{(1-\phi_1)^2\Gamma \left(\frac{\phi_0\phi_1}{1-\phi_1} \right)} \\ &\quad + \frac{\phi_0}{(1-\phi_1)^2} \sum_{i=1}^n \log \left(\frac{x_{i1}}{1-x_{i1}} \right) - \frac{\phi_0}{(1-\phi_1)^2} \sum_{i=1}^n \log \left(1 + \frac{x_{i1}}{1-x_{i1}} + \frac{1-x_{i2}}{x_{i2}} \right),\end{aligned}$$

$$\begin{aligned}
\frac{\partial \ell_3(\phi|Y_{\text{obs}_3})}{\partial \phi_2} &= -\frac{n\phi_0\Gamma'\left(\frac{\phi_0\phi_1}{1-\phi_1} + \frac{\phi_0}{\phi_2}\right)}{\phi_2^2\Gamma\left(\frac{\phi_0\phi_1}{1-\phi_1} + \frac{\phi_0}{\phi_2}\right)} + \frac{n\phi_0\Gamma'\left(\frac{\phi_0}{\phi_2} - \phi_0\right)}{\phi_2^2\Gamma\left(\frac{\phi_0}{\phi_2} - \phi_0\right)} - \frac{\phi_0}{\phi_2^2} \sum_{i=1}^n \log\left(\frac{1-x_{i2}}{x_{i2}}\right) \\
&\quad + \frac{\phi_0}{\phi_2^2} \sum_{i=1}^n \log\left(1 + \frac{x_{i1}}{1-x_{i1}} + \frac{1-x_{i2}}{x_{i2}}\right).
\end{aligned}$$

A.5 Calculations for $\nabla Q_3(\phi|\phi^{(t)})$

We can obtain

$$\begin{aligned}
\frac{\partial Q_3(\phi|\phi^{(t)})}{\partial \phi_0} &= n\left(\frac{\phi_1}{1-\phi_1} + \frac{1}{\phi_2}\right) \frac{\Gamma'\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}} + \frac{\phi_0^{(t)}}{\phi_2^{(t)}}\right)}{\Gamma\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}} + \frac{\phi_0^{(t)}}{\phi_2^{(t)}}\right)} - \frac{n\Gamma'(\phi_0)}{\Gamma(\phi_0^{(t)})} - \frac{n\phi_1\Gamma'\left(\frac{\phi_0\phi_1}{1-\phi_1}\right)}{(1-\phi_1)\Gamma\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}}\right)} \\
&\quad - \frac{n(1-\phi_2)\Gamma'\left(\frac{\phi_0}{\phi_2} - \phi_0\right)}{\phi_2\Gamma\left(\frac{\phi_0^{(t)}}{\phi_2^{(t)}} - \phi_0^{(t)}\right)} + \frac{\phi_1}{1-\phi_1} \sum_{i=1}^n \log\left(\frac{x_{i1}}{1-x_{i1}}\right) + \frac{1-\phi_2}{\phi_2} \sum_{i=1}^n \log\left(\frac{1-x_{i2}}{x_{i2}}\right) \\
&\quad - \left(\frac{\phi_1}{1-\phi_1} + \frac{1}{\phi_2}\right) \sum_{i=1}^n \log\left(1 + \frac{x_{i1}}{1-x_{i1}} + \frac{1-x_{i2}}{x_{i2}}\right), \\
\frac{\partial Q_3(\phi|\phi^{(t)})}{\partial \phi_1} &= \frac{n\phi_0\Gamma'\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}} + \frac{\phi_0^{(t)}}{\phi_2^{(t)}}\right)}{(1-\phi_1)^2\Gamma\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}} + \frac{\phi_0^{(t)}}{\phi_2^{(t)}}\right)} - \frac{n\phi_0\Gamma'\left(\frac{\phi_0\phi_1}{1-\phi_1}\right)}{(1-\phi_1)^2\Gamma\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}}\right)} \\
&\quad + \frac{\phi_0}{(1-\phi_1)^2} \sum_{i=1}^n \log\left(\frac{x_{i1}}{1-x_{i1}}\right) - \frac{\phi_0}{(1-\phi_1)^2} \sum_{i=1}^n \log\left(1 + \frac{x_{i1}}{1-x_{i1}} + \frac{1-x_{i2}}{x_{i2}}\right), \\
\frac{\partial Q_3(\phi|\phi^{(t)})}{\partial \phi_2} &= -\frac{n\phi_0\Gamma'\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}} + \frac{\phi_0^{(t)}}{\phi_2^{(t)}}\right)}{\phi_2^2\Gamma\left(\frac{\phi_0^{(t)}\phi_1^{(t)}}{1-\phi_1^{(t)}} + \frac{\phi_0^{(t)}}{\phi_2^{(t)}}\right)} + \frac{n\phi_0\Gamma'\left(\frac{\phi_0}{\phi_2} - \phi_0\right)}{\phi_2^2\Gamma\left(\frac{\phi_0^{(t)}}{\phi_2^{(t)}} - \phi_0^{(t)}\right)} - \frac{\phi_0}{\phi_2^2} \sum_{i=1}^n \log\left(\frac{1-x_{i2}}{x_{i2}}\right) \\
&\quad + \frac{\phi_0}{\phi_2^2} \sum_{i=1}^n \log\left(1 + \frac{x_{i1}}{1-x_{i1}} + \frac{1-x_{i2}}{x_{i2}}\right).
\end{aligned}$$

A.6 Calculations for $\nabla \ell_4(\boldsymbol{\varphi}|Y_{\text{obs}_4})$

We can obtain

$$\begin{aligned}\ell_4(\boldsymbol{\varphi}|Y_{\text{obs}_4}) &= c_3 + \sum_{i=1}^n \left\{ \log \Gamma [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)] - \log \Gamma(\phi_0) \right. \\ &\quad - \log \Gamma [\phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1)] - \log \Gamma [\phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)] \\ &\quad + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) \log \left(\frac{x_{i1}}{1 - x_{i1}} \right) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2) \log \left(\frac{1 - x_{i2}}{x_{i2}} \right) \\ &\quad \left. - \phi_0 [1 + \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)] \log \left(1 + \frac{x_{i1}}{1 - x_{i1}} + \frac{1 - x_{i2}}{x_{i2}} \right) \right\}.\end{aligned}$$

Then we have

$$\begin{aligned}\frac{\partial \ell_4(\boldsymbol{\varphi}|Y_{\text{obs}_4})}{\partial \phi_0} &= \sum_{i=1}^n \left\{ \frac{\Gamma' [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]}{\Gamma [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]} \cdot [1 + \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)] \right. \\ &\quad - \frac{\Gamma'(\phi_0)}{\Gamma(\phi_0)} - \frac{\Gamma' [\phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1)]}{\Gamma [\phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1)]} \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) - \frac{\Gamma' [\phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]}{\Gamma [\phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]} \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2) \\ &\quad + \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) \log \left(\frac{x_{i1}}{1 - x_{i1}} \right) + \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2) \log \left(\frac{1 - x_{i2}}{x_{i2}} \right) \\ &\quad \left. - [1 + \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)] \cdot \log \left(1 + \frac{x_{i1}}{1 - x_{i1}} + \frac{1 - x_{i2}}{x_{i2}} \right) \right\}, \\ \frac{\partial \ell_4(\boldsymbol{\varphi}|Y_{\text{obs}_4})}{\partial \boldsymbol{\beta}_1} &= \phi_0 \sum_{i=1}^n \left\{ \frac{\Gamma' [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]}{\Gamma [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]} - \frac{\Gamma' [\phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1)]}{\Gamma [\phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1)]} \right. \\ &\quad \left. + \log \left(\frac{x_{i1}}{1 - x_{i1}} \right) - \log \left(1 + \frac{x_{i1}}{1 - x_{i1}} + \frac{1 - x_{i2}}{x_{i2}} \right) \right\} \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) \mathbf{v}_i, \\ \frac{\partial \ell_4(\boldsymbol{\varphi}|Y_{\text{obs}_4})}{\partial \boldsymbol{\beta}_2} &= -\phi_0 \sum_{i=1}^n \left\{ \frac{\Gamma' [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]}{\Gamma [\phi_0 + \phi_0 \exp(\mathbf{v}_i^\top \boldsymbol{\beta}_1) + \phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]} - \frac{\Gamma' [\phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]}{\Gamma [\phi_0 \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2)]} \right. \\ &\quad \left. + \log \left(\frac{1 - x_{i2}}{x_{i2}} \right) - \log \left(1 + \frac{x_{i1}}{1 - x_{i1}} + \frac{1 - x_{i2}}{x_{i2}} \right) \right\} \exp(-\mathbf{v}_i^\top \boldsymbol{\beta}_2) \mathbf{v}_i.\end{aligned}$$

Supplement B. Descriptions of Real Data

The data included 41 patients and 40 controls. The sample statistics of $\mathbf{x} = (X_1, X_2)^\top$ are shown in the following subsections. We give the statistics of common covariates (age & gender) in Table S1 and some descriptions of symbols in the following tables: (i) Var. is short for variables; (ii) Std stands for standard deviation of samples; (iii) Sample correlations are calculated based on *Spearman correlation coefficient* (Spearman's r), and the p -values of significance tests are denoted by p .

Table S1. Statistics for the common covariates (age & gender) between controls and patients.

| Var. | Controls | | | Patients | | |
|---------|----------|-------|--------|----------|-------|--------|
| | Mean | Range | Std | Mean | Range | Std |
| Age | 33.6341 | 19~49 | 9.2351 | 33.6341 | 19~42 | 9.2351 |
| Gender* | 0.2750 | 0, 1 | 0.4522 | 0.2439 | 0, 1 | 0.4348 |

* Male=0; Female=1.

B.1 Lateral and Suborbital Sulcus

The sample statistics for the thickness difference of horizontal ramus of the anterior segment of the lateral sulcus (X_1), suborbital sulcus (X_2) in right hemisphere and covariates (age & gender) in controls and patients are shown in Table S2, while the Spearman's r for X_1 & X_2 with covariates between controls and patients are given in Table S3.

Table S2. Statistics for the thickness difference of X_1 & X_2 in §5.1 and covariates (age & gender) between controls and patients.

| Var. | Controls | | | Patients | | |
|-----------------------------------|----------|------------------|---|----------|------------------|--------|
| | Mean | Range | Std | Mean | Range | Std |
| X_1 | 0.4982 | (0.0721, 0.9910) | 0.2176 | 0.4091 | (0.0090, 0.9640) | 0.1966 |
| X_2 | 0.5022 | (0.0977, 0.9953) | 0.1958 | 0.4199 | (0.0047, 0.8279) | 0.1967 |
| Spearman's r: 0.07 ($p > 0.05$) | | | Spearman's r: −0.49 ($p \ll 0.01$) | | | |

Table S3. Spearman's r for the thickness difference of X_1 & X_2 in §5.1 with covariates (age & gender) between controls and patients. * Male = 0; Female = 1.

| Var. | Controls | | Patients | |
|-------|-------------------------------|----------------------|----------------------|----------------------|
| | Age | Gender* | Age | Gender* |
| X_1 | -0.33 ($p = 0.04$) | -0.09 ($p = 0.59$) | -0.22 ($p = 0.17$) | 0.02 ($p = 0.89$) |
| X_2 | -0.53 ($p \ll 0.01$) | 0.17 ($p = 0.31$) | -0.02 ($p = 0.89$) | -0.26 ($p = 0.11$) |

B.2 Cingulate Gyrus and Lateral Occipito-Temporal Sulcus

The sample statistics for the thickness difference of left posterior-dorsal part of the cingulate gyrus (X_1) and right lateral occipito-temporal sulcus (X_2) in controls and patients are shown in Table S4, while the Spearman's r for X_1 & X_2 with covariates (age & gender) between controls and patients are given in Table S5.

Table S4. Statistics for the thickness difference of X_1 & X_2 in §5.2 and covariates (age & gender) between controls and patients.

| Var. | Controls | | | Patients | | |
|-------|--|------------------|--------|---|------------------|--------|
| | Mean | Range | Std | Mean | Range | Std |
| X_1 | 0.4862 | (0.1748, 0.9903) | 0.1904 | 0.3891 | (0.0097, 0.7767) | 0.1740 |
| X_2 | 0.5384 | (0.2721, 0.8529) | 0.1498 | 0.4647 | (0.0074, 0.9926) | 0.2122 |
| | Spearman's r: -0.43 ($p \ll 0.01$) | | | Spearman's r: 0.19 ($p > 0.05$) | | |

Table S5. Spearman's r for the thickness difference of X_1 & X_2 in §5.2 with covariates (age & gender) between controls and patients. * Male = 0; Female = 1.

| Var. | Controls | | Patients | |
|-------|-------------------------------|---------------------|-----------------------------|----------------------|
| | Age | Gender* | Age | Gender* |
| X_1 | -0.50 ($p \ll 0.01$) | 0.19 ($p = 0.24$) | -0.36 ($p = 0.02$) | 0.05 ($p = 0.78$) |
| X_2 | 0.18 ($p = 0.26$) | 0.03 ($p = 0.87$) | -0.20 ($p = 0.21$) | -0.01 ($p = 0.96$) |