

Correction

# Correction: Cakir, M.; Gunes, B. A Fitted Operator Finite Difference Approximation for Singularly Perturbed Volterra-Fredholm Integro-Differential Equations. *Mathematics* 2022, 10, 3560

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The authors wish to make the following corrections to this paper [1]:

1. In the fifth paragraph of the Introduction Section, the sentence “In [35], using composite trapezoidal rule, fitted mesh finite difference schemes have been established on Shishkin type mesh” should be “In [35], using composite trapezoidal rule, fitted mesh finite difference schemes have been established on Shishkin type mesh”.

2. In Section 2, (1) At the end of the Proof of Lemma 1, “Therefore, the proof of the lemma is complete” should be “Therefore, the proof of the lemma is completed”; (2) In the Proof of Lemma 1, the following reference is used to prove the relation (5): “Amiraliyev, G.M.; Durmaz, M.E.; Kudu, M. A numerical method for a second order singularly perturbed Fredholm integro-differential equation. *Miskolc Math. Notes* 2021, 22, 37–48.”

Therefore, this reference should be added to the “References” section, and it should be cited after the sentence “which hints at the proof of the relation (5)” at the end of the Proof of Lemma 1.

3. In Section 4, (1) In the expression of Lemma 2,

$$c_0 = \frac{1}{1 - |\lambda| \left( \max_{1 \leq i \leq N} \sum_{j=1}^i \tilde{h}_j |K_{1,ij}| + \max_{1 \leq i \leq N} \sum_{j=1}^N \tilde{h}_j |K_{2,ij}| \right)}$$

should be changed to

$$c_0 = \frac{\alpha^{-1}}{1 - \alpha^{-1} |\lambda| \left( \max_{1 \leq i \leq N} \sum_{j=1}^i \tilde{h}_j |K_{1,ij}| + \max_{1 \leq i \leq N} \sum_{j=1}^N \tilde{h}_j |K_{2,ij}| \right)}$$

(2) In the Proof of Lemma 2,

$$|z_i| \leq \alpha^{-1} \max_{1 \leq i \leq N} |R_i| + \alpha^{-1} \max_{1 \leq i \leq N} \sum_{j=1}^i \tilde{h}_j |K_{1,ij}| |z_j| + \alpha^{-1} |\lambda| \max_{1 \leq i \leq N} \sum_{j=1}^N \tilde{h}_j |K_{2,ij}| |z_j|$$

should be

$$|z_i| \leq \alpha^{-1} \max_{1 \leq i \leq N} |R_i| + \alpha^{-1} |\lambda| \max_{1 \leq i \leq N} \sum_{j=1}^i \tilde{h}_j |K_{1,ij}| |z_j| + \alpha^{-1} |\lambda| \max_{1 \leq i \leq N} \sum_{j=1}^N \tilde{h}_j |K_{2,ij}| |z_j|$$

(3) In the Proof of Lemma 2,

$$\|z\|_{\infty} \leq \alpha^{-1} \|R\|_{\infty} + \alpha^{-1} \max_{1 \leq i \leq N} \sum_{j=1}^i h_j |K_{1,ij}| \|z\|_{\infty} + \alpha^{-1} |\lambda|_1 \max_{1 \leq i \leq N} \sum_{j=1}^N h_j |K_{2,ij}| \|z\|_{\infty}$$

should be

$$\|z\|_{\infty} \leq \alpha^{-1} \|R\|_{\infty} + \alpha^{-1} |\lambda|_1 \max_{1 \leq i \leq N} \sum_{j=1}^i h_j |K_{1,ij}| \|z\|_{\infty} + \alpha^{-1} |\lambda|_1 \max_{1 \leq i \leq N} \sum_{j=1}^N h_j |K_{2,ij}| \|z\|_{\infty}$$

4. In Section 5, (1) In Example 1,

$$-\varepsilon^2 u'' + u + \int_0^x u(t) dt + \int_0^1 u(t) dt = -\varepsilon \left( e^{\frac{-x}{\varepsilon}} + e^{\frac{-1}{\varepsilon}} + 2 \right), \quad x \in (0, 1)$$

should be

$$-\varepsilon^2 u'' + u + \int_0^x u(t) dt + \int_0^1 u(t) dt = -\varepsilon \left( e^{\frac{-x}{\varepsilon}} + e^{\frac{-1}{\varepsilon}} - 2 \right), \quad x \in (0, 1)$$

(2) In Example 3,

$$u(0) = 0, \quad u(1) = e^{\frac{-1}{\varepsilon}} + \frac{1}{\varepsilon}$$

should be

$$u(0) = 0, \quad u(1) = e^{\frac{-1}{\varepsilon}} + 1$$

5. In the References Section, the following reference should be added after the Ref. 41 to the “References” section:

“Amiraliyev, G.M.; Durmaz, M.E.; Kudu, M. A numerical method for a second order singularly perturbed Fredholm integro-differential equation. *Miskolc Math. Notes* **2021**, *22*, 37–48.”

With this correction, the order of some references has been adjusted accordingly.

The authors apologize for any inconvenience caused and state that the scientific conclusions are unaffected. The original article has been updated.

## Reference

1. Cakir, M.; Gunes, B. A fitted operator finite difference approximation for singularly perturbed Volterra-Fredholm integro-differential equations. *Mathematics* **2022**, *10*, 3560. [[CrossRef](#)]