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A Robust Optimization Method for Location Selection of Parcel Lockers under Uncertain Demands

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Abstract: Parcel lockers have continuously growing in popularity as an alternative mode for last-mile delivery services due to their capability of effectively alleviating the risk of a delivery failure, increasing the possibility of delivery consolidation, and reducing the number of drop-off sites. However, poorly located parcel lockers be less efficient. When determining the parcel locker location, inadequate consideration of uncertain demands can potentially increase the risk of unsatisfied demands. To remedy this issue, a robust optimization model is proposed in this paper with consideration of the demand uncertainties, including the large and small parcels to be received and sent. Not only can the collection locations be optimally determined, but so can the number of large and small parcel lockers for each location at the same time under various robust levels. Meanwhile, the sites whose demands are covered by one of the collection locations are also determined by the constraints of acceptable walking distance. A series of numerical experiments has been performed to evaluate the proposed model, with the main focus being on the solution robustness. Since the set of non-linear constraints are transformed into the linear counterparts, the robust solution can be obtained by the existing solvers within a reasonable time with moderate computing power. The experimental results also provide useful guidance for the practical application of the method, as slightly more conservative decision making can secure the solution robustness with only a marginal increase in costs.

Keywords: last-mile delivery; parcel lockers; location problem; robust optimization; uncertain demands

MSC: 65K02



Citation: Wang, Y.; Zhang, Y.; Bi, M.; Lai, J.; Chen, Y. A Robust Optimization Method for Location Selection of Parcel Lockers under Uncertain Demands. *Mathematics* **2022**, *10*, 4289. <https://doi.org/10.3390/math10224289>

Academic Editor: Ripon Kumar Chakraborty

Received: 7 October 2022

Accepted: 12 November 2022

Published: 16 November 2022

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1. Introduction

With the development of information technology and e-commerce, the market share of the B2C shopping mode continues to grow, which brings huge demand for express delivery. Statistics from China Post show that China's express delivery service companies handled 108.3 billion pieces of business in 2021, up 29.92% from 2020 [1]. However, the costliest stage in the supply chain is the last-mile delivery, which accounts for approximately 28% of the total cost [2]. In the traditional door-to-door delivery service, the failed delivery attempts is one of the most expensive resources for the last-mile delivery. In addition, customer security concerns over door-to-door delivery potentially degrade their satisfaction.

As compared with the traditional door-to-door delivery mode, the parcel locker mode is convenient, secure, and accessible 24/7 for customers collecting and sending parcels, while high degree of consolidation and a wide service time window can effectively delivery services extricate from the cost predicament of the last-mile delivery. At present, more than 20 countries around the world have deployed parcel lockers [3]. From 2011 to 2019, DHL added more than 10,000 parcel locker service points in Germany [4,5]. In recent years, the InPost parcel locker service has gained rapid popularity over Europe [4]. In China,

parcel lockers have been included as part of urban and rural public infrastructure to be constructed [6].

Most of the existing studies on parcel lockers focus mainly on the analysis of the current situation of the use of parcel lockers, and demonstrate the feasibility of the parcel locker from the following aspects. First, many studies have unveiled that the use of parcel lockers can significantly reduce the cost of delivery. If the failed first-time delivery attempts could be completely avoided, in London alone would save GBP 850,000,000 a year [7]. In addition, the shipping time and the corresponding exhaust emissions can benefit from the delivery mode of parcel lockers. Carotenuto et al. [8] used the established urban delivery model to compare the parcel locker delivery mode with the door-to-door mode and found that the parcel locker delivery mode can reduce total travel distance by more than 24%, total delivery time by more than 50%, and CO₂ emissions by more than 21%. According to a survey report by AGH University of Science and Technology in Poland, using parcel lockers can deliver 10 times more goods than door-to-door delivery, and save more than half the delivery miles [9]. Finally, the parcel locker delivery mode can also considerably improve customer satisfaction. Sun [10] found that using parcel lockers can save 1–5 min, as compared to the traditional service mode. Lemke et al. [11] conducted a survey on Polish customers and found that about 95% of users were satisfied with the parcel locker service provided. Choi et al. [12] optimized a last-mile fresh food delivery system for the underserved populations based on several alternative delivery networks for a number of modes, including parcel lockers. They found the parcel locker delivery mode to be less cost-effective in delivering fresh items as compared to other modes, such as third-party delivery or truck, etc.

The location and scale of parcel lockers are particularly critical to the promotion of the parcel locker delivery mode. A study on the parcel locker users in Poland by Iwan et al. [3] shows that reasonable location of parcel lockers is a decisive factor in improving customer utilization rate. Based on a survey conducted for the customers in the Netherlands, Weltevreden [13] found that five-minute driving distance should be taken as the key accessibility threshold in determining the location of parcel lockers. Lagorio and Pinto [14] found that the main factors influencing the choice of the positioning of parcel lockers were availability, accessibility, safety, environmental impact, costs, delivery times, usage and regulation. Peppel and Spinler [15] established a multinomial logit model to design parcel locker networks with the goal of minimizing CO₂-equivalent emissions and costs during delivery and pick-up. Yang et al. [16] constructed a bilevel programming model to optimize the location for a set of parcel lockers, aiming to improve delivery efficiency and customer satisfaction. With the goal to maximize the total profit, Deutsch and Golany [17] established an integer linear location model to determine the optimal location and number of parcel lockers without considering the capacity constraint. Wang et al. [18] proposed a robust optimization model for the mobile parcel lockers under uncertain demand, but only the demand uncertainty associated with one type of parcel lockers is assumed.

However, most studies assume that delivery demand is known and fixed when determining the optimal location for a set of parcel lockers. Obviously, a parcel locker network planned in this way will not be able to adapt effectively to changes in demand. As this paper is not intended to serve as an exhaustive review of the location methods with consideration of the demand uncertainty, we briefly summarize a number of research works which are closely related to our work. Atamtürk and Zhang [19] proposed a two-stage robust optimization approach to solve the network flow and design problems with uncertain demand. Tanonkou et al. [20] established a two-stage nonlinear discrete stochastic optimization model to determine the location of distribution centers. Ukkusuri et al. [21] developed an optimization method for the traffic robust network design problem under demand uncertainty. By reviewing the existing studies, it has been found that the current optimization methods to deal with uncertain demand can be classified into three categories, with the uncertainty characterized based on the empirical distribution [22,23], robust optimization [24–26] and uncertainty theory [27,28], respectively. In the work presented in

this paper, we employed robust optimization to tackle the uncertainty issue related to the delivery demand, as no assumption on the probability distribution of uncertain demand should be made.

The robust optimization method proposed in this paper contributes to knowledge by extending the location optimization method for parcel lockers under uncertain demands. More specifically, we consider two sources of uncertainty for both the receiving and sending parcel demand. In addition, the optimal locations and number of lockers can be optimized simultaneously with various sizes of lockers. After formulating the robust counterpart for the bounded demands, a set of non-linear constraints are transformed into the corresponding linear constraints to accelerate the solving speed.

The remaining sections of this paper are structured as follows. The next section is devoted to formulating the location problem of parcel lockers under a number of assumptions. In Section 3, the models to determine the optimal location of parcel lockers are successively proposed under deterministic and stochastic demands, respectively. Following that, a series of experiments and the corresponding results are described to evaluate the robust optimization model proposed. This paper is concluded with a number of findings, which are summarized in Section 5.

2. Problem Formulation and Assumptions

Problem Description and Model Assumptions

In this study, we present the distribution network as $G(I, E)$, comprising a set of demand sites I and edges E , with the edge weight being the shortest distance between two connected sites. Although all demand sites could be equipped with a number of parcel lockers, the operating cost would be unacceptably high. Therefore, we need to select a set of demand sites to accommodate parcel lockers and determine the number of lockers for each collection location, at the minimal cost and within an acceptable walking distance for those customers whose communities are not equipped with parcel lockers. In other words, the customers can go to the nearest parcel locker site to collect their parcels within the acceptable walking distance. A diagram of this problem is shown in Figure 1. During the planning phase, it is unrealistic to accurately predict the exact demand for each site, but the demand, varying in a range, can be normally estimated with some knowledge of the future trend. Obviously, it rarely happens that all demand sites have the maximum demand, except for in boom seasons. Due to the fact that most parcel lockers are operated independently by a third party, we assume that the delivery task is accomplished by the courier companies. The determination of the number of parcel lockers also needs to consider the operation process of couriers in collecting and uploading parcels. Based on our survey on the courier operation, it is assumed that a courier empties the parcel lockers by collecting the parcels to be sent out and not be taken within a certain period (i.e., 48 h in this study) at first, and then stores the newly arrived parcels when reaching a parcel locker site. Further assumptions are made as follows:

- (1) The delivery demand, including the number of parcels to be received and sent by customers, of each site fluctuates within a known range.
- (2) Customers are always willing to go to the closest collection location of parcel lockers to collect and send parcels, and the distance to the closest parcel lockers must be within an acceptable walking distance.
- (3) Each collection location is equipped with two sizes of lockers for large and small parcels, respectively, and the small parcel can also be stored in a large locker if no small lockers are free.
- (4) For simplicity, we assume that the cost (including manufacturing, construction and maintenance cost, etc.) and the size of a large parcel locker are δ ($\delta > 1$, $\delta \in \mathbb{Z}$) times those of a small locker on average. The cost and the size of a small locker are known values (the parameters used for this study are provided in Section 4.1).
- (5) A parcel collection location can be equipped with θ large parcel lockers or, equivalently, $\delta\theta$ small ones.

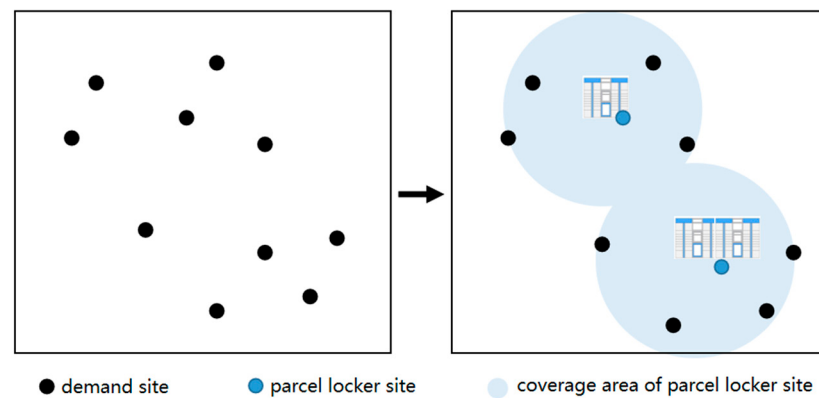


Figure 1. Location selection problem of parcel lockers.

Before presenting the optimal location models, the variables and parameters involved in the models are summarized in Table 1.

Table 1. Symbols and their descriptions.

Sets	Descriptions
I	The set of demand points
E	The edge set, and each edge associated with a weight being the shortest distance between two connected sites
Parameters	Descriptions
c^f	The purchase and maintenance cost of a small locker
c_i^g	The land rent cost per unit at the collection location i
δ	A positive integer greater than 1
θ	A positive integer
Q	The maximum number of lockers allowed at each site
W	A sufficiently large positive number
M	A sufficiently large positive number
l_{ij}	The distance from demand site j to collection site i
L	The maximum acceptable walking distance
d_j^b	The demand for large parcel services at site j
d_j^s	The demand for small parcel services at site j
r_j^b	The number of large parcels uncollected during service time at site j
r_j^s	The number of small parcels uncollected during service time at site j
$\bar{d}_j^b$	The average demand for large parcel services at site j
$\bar{d}_j^s$	The maximum variation in demand variation for large parcel services at site j
$\bar{d}_j^s$	The averaged demand for small parcel services at site j
$\bar{d}_j^s$	The maximum variation in demand for small parcel services at site j
$\bar{r}_j^b$	The average number of large parcels uncollected during service time at site j
$\hat{r}_j^b$	The maximum variation in large parcels uncollected during service time at site j
$\bar{r}_j^s$	The average number of small parcels uncollected during service time at site j
$\hat{r}_j^s$	The maximum variation in small parcels uncollected during service time at site j
Γ	A parameter to adjust the robustness degree
Decision variables	Descriptions
x_i^b	The number of large parcels to be set at site i
x_i^s	The number of small parcels to be set at site i
y_{ij}	A binary variable. If demand site j is assigned to demand site i , $y_{ij} = 1$, or 0 otherwise

3. Optimal Location Models

With the problem description and assumptions provided in the previous section, this section presents the optimization model to determine the collection locations and the

number of parcel lockers for each location under deterministic demand first, and then the one under uncertain demand.

3.1. Optimal Location Model under Deterministic Demands

The optimization goal is to minimize the total operating costs, including the cost of purchase, maintenance and land rent of the parcel locker network. Obviously, the cost of purchase and maintenance of a parcel locker is only related to the type and number of parcel lockers, which will be the same for different sites. However, this is not the case for the rental cost of the land used to accommodate a set of parcel lockers, since different rental rate may be applied at different areas. Based on the assumptions summarized in the previous section, the objective function can be expressed as (1).

$$\min \sum_{i \in I} c^f (x_i^s + \delta x_i^b) + c_i^g \left\lceil (x_i^s + \delta x_i^b) / (\delta \vartheta) \right\rceil \quad (1)$$

where the first part of the objective function represents the total costs of purchase and maintenance for all parcel lockers, and the second part the total land rental costs. x_i^b and x_i^s are decision variables, representing the number of large and small parcel lockers to be provided at site i . δ is a positive integer greater than 1, indicating that the size, purchase and maintenance cost of a large parcel locker is δ times that of a small one. c_i^g is the land rent of a parcel locker at site i . Since it is normal that several parcel lockers are stacked one above another, the land area occupied by a set of stacked parcel lockers (thereafter, called a unit of parcel lockers) is equal to the land needed to accommodate one single locker. It is assumed that a unit of parcel lockers consisting of ϑ (a positive integer) large parcel lockers or, equivalently, $\delta \vartheta$ small parcel lockers and, therefore, $(x_i^s + \delta x_i^b) / (\delta \vartheta)$, represents the number of parcel locker units to be deployed at site i . Note that we round up to the nearest integer when estimating the area to rent for a set of parcel lockers. However, for simplicity, we relax $\lceil * \rceil$ and then obtain (2).

$$\min \sum_{i \in I} c^f (x_i^s + \delta x_i^b) + c_i^g (x_i^s + \delta x_i^b) / (\delta \vartheta) \quad (2)$$

The constraints associated with the optimization model are shown in (3)–(11).

$$x_i^b \geq 0, \forall i \in I \quad (3)$$

$$x_i^s \geq 0, \forall i \in I \quad (4)$$

$$\left\lceil (x_i^s + \delta x_i^b) / (\delta \vartheta) \right\rceil \leq Q, \forall i \in I \quad (5)$$

$$y_{ij} \in \{0, 1\}, \forall i, j \in I \quad (6)$$

$$\sum_{i \in I} y_{ij} = 1, \forall j \in I \quad (7)$$

$$W y_{ij} \geq (x_i^s + x_i^b), \forall i, j \in I, i = j \quad (8a)$$

$$x_i^s + x_i^b \geq y_{ij}, \forall i, j \in I, i = j \quad (8b)$$

$$x_i^s + x_i^b \geq y_{ij}, \forall i, j \in I \quad (9)$$

$$y_{ij} \min_{i \in I, (x_i^s + x_i^b) \geq 1} \{l_{ij}\} > 0, \forall j \in I, i \neq j \quad (10)$$

$$l_{ij} y_{ij} \leq L, \forall i, j \in I \quad (11)$$

$$\sum_{j \in I} (d_j^b + r_j^b) y_{ij} \leq x_i^b, \forall i \in I \quad (12a)$$

$$\sum_{j \in I} (d_j^b + r_j^b) y_{ij} + \sum_{j \in I} (d_j^s + r_j^s) y_{ij} \leq x_i^b + x_i^s, \forall i \in I \quad (12b)$$

Constraints (3) and (4) are non-negative constraints on the decision variables x_i^b and x_i^s . When $x_i^s + x_i^b > 0$, demand point i is selected as a collection site. Due to the limited area available to accommodate parcel lockers at a site, the total number of large and small parcel lockers cannot be equipped without an up-bound, which is indicated by Constraint (5). In Constraint (6), $y_{ij} = 1$ indicates that those customers at demand site j need to go to site i to collect or send their parcels, or 0 otherwise. Furthermore, each demand site can only be associated to one collection location, which is expressed by Constraint (7). Constraint (8) states the fact that, if site i is selected as a collection location, the provision of parcel lockers should at least satisfy the demand from site i . Constraint (9) ensures that collection site i , associated with demand point j , has been equipped with parcel lockers. We assume that customers always tend to collect and send parcels at the nearest collection site, and this restriction is realized by Constraint (10). As shown in (11), we impose a constraint that the distance from demand site j to collection location i cannot exceed the maximum acceptable walking distance. Constraint (12a) ensures that the unoccupied large parcel lockers at collection site i can satisfy all demand on the large parcel delivery service from all demand sites associated to this collection site. As small parcels can also be accommodated by the unoccupied space typically reserved for large parcels, the requirements on storing small parcels can be satisfied jointly by the unoccupied small lockers and the large lockers. This is ensured by Constraint (12b).

Note that Constraint (10) contains $\min$ function, which can raise difficulty when solving this optimization problem. Therefore, we transform it into the corresponding linear constraint, as shown in (13).

$$My_{ij}^{j(1)} \geq x_i^{b,j(1)} + x_i^{s,j(1)}, \forall i, j \in I \quad (13a)$$

$$My_{ij}^{j(k)} \geq x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} [x_i^{b,j(m)} + x_i^{s,j(m)}], \forall i, j \in I, k = 2, \dots, |I| \quad (13b)$$

In (13), the sufficiently large positive number M should satisfy $\max_{i \in I} (x_i^b + x_i^s) \leq M$. When $y_{ij}^{j(1)} = 1$, demand site j is allocated to collection site i and “(1)” indicates that site j is the nearest site for the customers from site j to collect and send parcels. Correspondingly, “ $j(k)$ ” and “ $j(m)$ ” denote that site i represents the k th and m th nearest sites from site j , respectively.

To prove the equivalence of (13) to (10), Lemma 1 and Lemma 2 are given first.

Lemma 1. If $x_i^{b,j(m)} + x_i^{s,j(m)} \geq 1$ and $M \geq \max_{k \in K} (x_i^{b,j(k)} + x_i^{s,j(k)})$ hold for any $k > 1$, $m < k$, $K = \{1, \dots, |I|\}$, whether $x_i^{b,j(k)} + x_i^{s,j(k)} = 0$ or $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$, $x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) \leq 0$ is always true.

Proof. Since $x_i^{b,j(m)} + x_i^{s,j(m)} \geq 1$, $M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) \geq M$. Furthermore, based on the prerequisite, $M \geq \max_{k \in K} (x_i^{b,j(k)} + x_i^{s,j(k)})$, $K = \{1, \dots, |I|\}$, it can be easily concluded that $M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) \geq x_i^{b,j(k)} + x_i^{s,j(k)}$ is true. $\square$

Lemma 2. If $\sum_{i \in I} y_{ij} = 1$ and $y_{ij}^{j(k)} = 1$, $y_{ij}^{j(s)} = 0$, $s \neq k$, $\forall i, j \in I$, $\forall k, s \in K$ must hold. This is to say, demand site j can only be allocated to one collection site.

In the following proof, three cases are presented based on Lemma 1 and 2.

Case 1: When site I , which is the nearest site to site j , is selected as a collection location and all parcels of the customers at site j are stored in the parcel lockers at site i , i.e., $x_i^{b,j(1)} + x_i^{s,j(1)} \geq 1$, and $y_{ij}^{j(1)} = 1, \forall i, j \in I$.

(1) When $k = 1$

Obviously, if $M \geq \max_k (x_i^{b,j(k)} + x_i^{s,j(k)})$ ($k = 1, \dots, |I|$), $My_{ij}^{j(1)} \geq x_i^{b,j(1)} + x_i^{s,j(1)}$ is true, $\forall i, j \in I$.

(2) When $k > 1$

Lemma 2 says that $y_{ij}^{j(k)} = 0$. Suppose $x_i^{b,j(k)} + x_i^{s,j(k)} = 0$, since $x_i^{b,j(1)} + x_i^{s,j(1)} \geq 1$, it can be derived that $x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) \leq 0$. Suppose there exists k and $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$ holds, since $x_i^{b,j(1)} + x_i^{s,j(1)} \geq 1$, it can be obtained that $x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) \leq 0$ holds based on Lemma 1. Therefore, $My_{ij}^{j(k)} \geq x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)})$ is true.

Case 2: When site i , which is k th nearest site to site j , is chosen as an collection location and the customers at site j need to go to site i to receive the delivery services, i.e., $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$ ($\forall k > 1$), $x_i^{b,j(s)} + x_i^{s,j(s)} = 0$ ($1 \leq s \leq k-1$), and $y_{ij}^{j(k)} = 1$.

(1) When $k = 2$ (i.e., $s = 1$)

$y_{ij}^{j(2)} = 1$, according to Lemma 2, $y_{ij}^{j(1)} = 0$. Additionally, since $x_i^{b,j(1)} + x_i^{s,j(1)} = 0$, then $My_{ij}^{j(1)} \geq x_i^{b,j(1)} + x_i^{s,j(1)}$ holds. Since $y_{ij}^{j(2)} = 1$, $x_i^{b,j(2)} + x_i^{s,j(2)} \geq 1$ and $M \geq \max_k (x_i^{b,j(k)} + x_i^{s,j(k)})$, $k = 1, \dots, |I|$, then $My_{ij}^{j(2)} \geq x_i^{b,j(2)} + x_i^{s,j(2)} - M (x_i^{b,j(1)} + x_i^{s,j(1)})$ is true.

(2) When $k > 2$ (i.e., $s > 1$)

As $y_{ij}^{j(k)} = 1$, it can be derived directly that $y_{ij}^{j(s)} = 0$, which holds based on Lemma 2. Furthermore, since $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$ and $x_i^{b,j(s)} + x_i^{s,j(s)} = 0$, then $My_{ij}^{j(k)} \geq x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)})$ holds.

(3) When $|I| > h > k$

Similarly, we can obtain $y_{ij}^{j(h)} = 0$ based on Lemma 2, with $y_{ij}^{j(k)} = 1$. Since $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$ and $x_i^{b,j(s)} + x_i^{s,j(s)} = 0$, it can be derived that $x_i^{b,j(h)} + x_i^{s,j(h)} - M \sum_{m=1}^{h-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) \leq 0$ holds when $x_i^{b,j(h)} + x_i^{s,j(h)} = 0$ or $x_i^{b,j(h)} + x_i^{s,j(h)} > 0$, based on Lemma 1. Therefore, $My_{ij}^{j(h)} \geq x_i^{b,j(h)} + x_i^{s,j(h)} - M \sum_{m=1}^{h-1} (x_i^{b,j(m)} + x_i^{s,j(m)})$ holds.

Case 3: When $h \neq k$ and $h, k = 1, \dots, |I|$

Suppose $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$, $x_i^{b,j(s)} + x_i^{s,j(s)} = 0$ ($1 \leq s \leq k-1$) (i.e., site i , which is k th nearest site to site j , is selected as a site to provide parcel lockers) and $y_{ij}^{j(h)} = 1$ (i.e., site i is associated to site j , which is h th nearest site to site i), $y_{ij}^{j(k)} = 0$ holds, as $y_{ij}^{j(h)} = 1$ and $\sum_{i \in I} y_{ij} = 1$. Furthermore, it is apparent that $My_{ij}^{j(k)} \geq x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} (x_i^{b,j(m)} + x_i^{s,j(m)}) = x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$, since $x_i^{b,j(k)} + x_i^{s,j(k)} \geq 1$ and $x_i^{b,j(s)} + x_i^{s,j(s)} = 0$.

3.2. Demand Uncertainties Considered

This subsection provides a summary of the demand uncertainties considered in this study, and each of them is depicted as a random variable for the delivery demand from each site. Since it is not easy to accurately determine the probability distribution of these random variables during the planning phase, this study assumes that the delivery demands of large and small parcels to be received and sent by customers at each site vary within a known interval. More specifically, it is assumed that the number of large parcels d_j^b and small parcels d_j^s to be received by the customers at site j are in the range $[\bar{d}_j^b - \hat{d}_j^b, \bar{d}_j^b + \hat{d}_j^b]$ and $[\bar{d}_j^s - \hat{d}_j^s, \bar{d}_j^s + \hat{d}_j^s]$ respectively, while the large and small parcels to be sent by the customers are estimated to vary in the ranges of $[\bar{r}_j^b - \hat{r}_j^b, \bar{r}_j^b + \hat{r}_j^b]$ and $[\bar{r}_j^s - \hat{r}_j^s, \bar{r}_j^s + \hat{r}_j^s]$.

3.3. Optimization Location Model under Uncertain Demands

Based on the robust optimization approach proposed by Bertsimas and Sim (2004), this study extends the robust optimization model to the parcel locker location optimization problem containing multiple random variables representing the uncertainties in the number of small and large parcels demanded and uncollected. To this end, the constraints for the uncertain demands are formulated as (14a) and (14b), which correspond to (12a) and (12b) for the deterministic demands.

$$\sum_{j \in J_i} y_{ij} (\bar{d}_j^b + \bar{r}_j^b) + \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} y_{ij} (\hat{d}_j^b + \hat{r}_j^b) + (\Gamma_i - \lfloor \Gamma_i \rfloor) (\hat{d}_{t_i}^b + \hat{r}_{t_i}^b) y_{it_i} \right\} \leq x_i^b, \forall i \in I \quad (14a)$$

$$\begin{aligned} & \sum_{j \in J_i} y_{ij} [(\bar{d}_j^s + \bar{r}_j^s) + (\bar{d}_j^b + \bar{r}_j^b)] \\ & + \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} y_{ij} [(\hat{d}_j^s + \hat{r}_j^s) + (\hat{d}_j^b + \hat{r}_j^b)] \right. \\ & \left. + (\Gamma_i - \lfloor \Gamma_i \rfloor) [(\hat{d}_{t_i}^s + \hat{r}_{t_i}^s) + (\hat{d}_{t_i}^b + \hat{r}_{t_i}^b)] y_{it_i} \right\} \leq x_i^b + x_i^s, \quad \forall i \in I \end{aligned} \quad (14b)$$

Note that the uncollected parcels at collection site i are split for all sites associated with collection site i for simplicity. That is, $\sum_{j \in J} y_{ij} \tilde{r}_j^b = \tilde{x}_i^b$ and $\sum_{j \in J} y_{ij} \tilde{r}_j^s = \tilde{x}_i^s$. Constraint (14a) ensures that the number of large parcel lockers provided at site i should be adequate to accommodate the maximum potential demand on the large parcels (including the large parcels to be collected and uncollected) from all demand sites associated to site i (J_i denotes the set of demand sites from which customers have to go to site i to collect or send parcels) for given Γ_i . On the other hand, the supply of parcel lockers for all demands, including small and larger parcels, is ensured by (14b) in the same way as that in (14a). Among them, Γ_i , which can vary in the range of $[0, |J_i|]$, is an important parameter to adjust the robustness. More specifically, the larger Γ_i is, the smaller the chance the demand constraint will be violated. When $\Gamma_i = |J_i|$, the most robustness can be obtained for the problem. However, it is too conservative, as it hardly happens when the demands from all sites approach the upper bounds at the same time. On the other extreme, the optimization model will deteriorate to the deterministic one when $\Gamma_i = 0$. If Γ_i is not an integer, the integer part (i.e., $\lfloor \Gamma_i \rfloor = |S_i|$, S_i denotes that the demand at site i is assumed to be the maximum and $\{S_i\}$ is the set containing all sites where the demand reaches the maximum) represents the number of sites which have the maximum demand, while the demand variation at site t_i ($t_i \in J_i \setminus S_i$) is restricted to the fractional times its up-bound is unoccupied (i.e., t_i represents that the demand at site i is below the up-bound, and $\{t_i\}$ is the set containing all the sites at which the demand is below the maximum).

Let $\alpha_i(y_i^*, \Gamma_i^1) = \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i^1 \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} y_{ij}^* (\hat{d}_j^b + \hat{r}_j^b) + (\Gamma_i^1 - \lfloor \Gamma_i^1 \rfloor) (\hat{d}_{t_i}^b + \hat{r}_{t_i}^b) y_{it_i}^* \right\}$, given vector y_i^* and parameter Γ_i^1 ($\Gamma_i^1 \in [0, |J_i|]$), we can re-write it as, with an auxiliary variable z_{ij}^1 (which varies in the range of $[0, 1]$ when y_{ij}^* is one and 0 otherwise)

$$\beta_i(y_i^*, \Gamma_i^1) = \max \sum_{j \in J_i} (\hat{d}_j^b + \hat{r}_j^b) y_{ij}^* z_{ij}^1 \quad (15)$$

s.t.

$$\sum_{j \in J_i} z_{ij}^1 \leq \Gamma_i^1, \forall i \in I \quad (16)$$

$$0 \leq z_{ij}^1 \leq y_{ij}^*, \forall j \in J_i, i \in I \quad (17)$$

(15)–(17) indicate that the variation in the total demand (including the large parcels to be received and sent by customers) at the collection location i is maximized for the demand sites associated with y_i^* under parameter Γ_i^1 .

To convert the stochastic constraint into the corresponding deterministic form, we write the dual form for (15)–(17) as follows

$$\min \sum_{j \in J_i} p_{ij}^1 + \Gamma_i^1 q_i^1 \quad (18)$$

s.t.

$$p_{ij}^1 + q_i^1 \geq (\hat{d}_j^b + \hat{r}_j^b) y_{ij}^*, \forall i, j \in I \quad (19)$$

$$p_{ij}^1 \geq 0, \forall i, j \in I \quad (20)$$

$$q_i^1 \geq 0, \forall i \in I \quad (21)$$

Similarly, let $\beta_i(y_i^*, \Gamma_i^1) = \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = |\Gamma_i|, t_i \in J_i \setminus S_i\}} \{\sum_{j \in S_i} y_{ij}^* [(\hat{d}_j^s + \hat{r}_j^s) + (\hat{d}_j^b + \hat{r}_j^b)] + (\Gamma_i - |S_i|) [(\hat{d}_{t_i}^s + \hat{r}_{t_i}^s) + (\hat{d}_{t_i}^b + \hat{r}_{t_i}^b)] y_{ij}^*\}$ with a certain parameter Γ_i^2 ($\Gamma_i^2 \in [0, |J_i|]$) and the association indicator y_i^* for site i , with an auxiliary variable z_{ij}^2 (which varies in the range of $[0, 1]$ when y_{ij}^* is one and 0 otherwise). For convenience, we rearrange it as follows:

$$\beta_i(y_i^*, \Gamma_i) = \max \sum_{j \in J_i} [(\hat{d}_j^s + \hat{r}_j^s) + (\hat{d}_j^b + \hat{r}_j^b)] y_{ij}^* z_{ij}^2 \quad (22)$$

s.t.

$$\sum_{j \in J_i} z_{ij}^2 \leq \Gamma_i^2, \forall i \in I \quad (23)$$

$$0 \leq z_{ij}^2 \leq y_{ij}^*, \forall j \in J_i, i \in I \quad (24)$$

The dual form of (22)–(24) is

$$\min \sum_{j \in J_i} p_{ij}^2 + \Gamma_i^2 q_i^2 \quad (25)$$

s.t.

$$p_{ij}^2 + q_i^2 \geq [(\hat{d}_j^s + \hat{r}_j^s) + (\hat{d}_j^b + \hat{r}_j^b)] y_{ij}^*, \forall i, j \in I \quad (26)$$

$$p_{ij}^2 \geq 0, \forall i, j \in I \quad (27)$$

$$q_i^2 \geq 0, \forall i \in I \quad (28)$$

Therefore, the location problem of parcel lockers under uncertain demands can be expressed as an integer linear programming (ILP) model, presented as follows.

$$\min \sum_{i \in I} c^f (x_i^s + \delta x_i^b) + c_i^g (x_i^s + \delta x_i^b) / (\delta \vartheta) \quad (29)$$

s.t.

$$x_i^b \geq 0, \forall i \in I \quad (30)$$

$$x_i^s \geq 0, \forall i \in I \quad (31)$$

$$y_{ij} \in \{0, 1\}, \forall i, j \in I \quad (32)$$

$$\sum_{i \in I} y_{ij} = 1, \forall j \in I \quad (33)$$

$$W y_{ij} \geq (x_i^s + x_i^b), \forall i, j \in I, i = j \quad (34a)$$

$$x_i^s + x_i^b \geq y_{ij}, \forall i, j \in I, i = j \quad (34b)$$

$$x_i^s + x_i^b \geq y_{ij}, \forall i, j \in I \quad (35)$$

$$\lceil (x_i^s + \delta x_i^b) / (\delta \vartheta) \rceil \leq Q, \forall i \in I \quad (36)$$

$$My_{ij}^{j(1)} \geq x_i^{b,j(1)} + x_i^{s,j(1)}, \forall i, j \in I \quad (37a)$$

$$My_{ij}^{j(k)} \geq x_i^{b,j(k)} + x_i^{s,j(k)} - M \sum_{m=1}^{k-1} [x_i^{b,j(m)} + x_i^{s,j(m)}], \forall i, j \in I, k = 2, \dots, |I| \quad (37b)$$

$$l_{ij}y_{ij} \leq r, \forall i, j \in I \quad (38)$$

$$\sum_{j \in I} y_{ij} (\bar{d}_j^b + \bar{r}_j^b) + \sum_{j \in I} p_{ij}^1 + \Gamma_i q_i^1 \leq x_i^b, \forall i \in I \quad (39)$$

$$p_{ij}^1 + q_i^1 \geq (\hat{d}_j^b + \hat{r}_j^b) y_{ij}, \forall i, j \in I \quad (40)$$

$$p_{ij}^1 \geq 0, \forall i, j \in I \quad (41)$$

$$q_i^1 \geq 0, \forall i \in I \quad (42)$$

The above models can be solved by the existing commercial solvers (e.g., CPLEX or Gurobi, etc.). As we coded this model in the MATLAB environment and, therefore, the solver provided for ILP in MATLAB is used in this study.

4. Numerical Experiments

Before presenting the experiments designed to evaluate the proposed location method for parcel lockers, a set of parameters involved in the simulated experiments are introduced first.

4.1. Parameter Settings

In this study, the primary parameters, shown in Table 1, for a unit of 100 parcel lockers provided by a parcel locker management company are adopted as the basis for the numerical experiments. Based on the information shown in Table 2, it can be directly derived that δ is 2, θ is 60, the land rent cost c_i^g of a parcel locker is about 16.44 yuan per day, the purchase and maintenance cost c^f of a small locker, and communication cost and electricity charging cost is 0.22 yuan/day. Since the land rent rate at each demand point may not necessarily be the same, we assume that the land rent rates in the experiments varies randomly with the mean value of 16.44 yuan/parcel locker/day.

Table 2. Primary parameters of a unit of parcel lockers.

Parameter	Value
Number of large parcel lockers	20
Number of small parcel lockers	80
The ratio of large and small lockers in size	2
Selling price	28,000 yuan
Service life	10 years
Average communication cost	650 yuan/year
Average power consumption	0.5 degree/day
Commercial electricity cost	0.78 yuan/kWh
Charging efficiency	80%
Average land rent	16.44 yuan/parcel locker/day

In the experiments, the developed robust optimization model has been evaluated based on the location problems consisting of 50 demand sites, with 150 m as the maximum acceptable walking distance. The demand generation process for each experiment is as follows. The large parcel demand for each site is randomly generated in the range [30, 50], with the demand variation restricted to the range [2, 12]. On the other hand, it is assumed that the number of large parcels uncollected at each site varies in the range [4, 7], with minimum and maximum variation bounded to 2 and 5, respectively. The same generation process is adopted for the small parcel demand for each site, with the mean demand and variation restricted to the range [50, 150] and [10, 40], respectively. Furthermore, the ranges

[15, 25] and [10, 20] are used to generate the mean and variation values in the small parcels which go uncollected.

In each experiment, the time for searching a location solution is restricted to 10 min and the parameters remain unaltered apart from the one whose influence on the solution is examined. All experiments were performed on a x64-PC with Intel Core i7-8550U 1.80 GHz CPU and 8.0 GB RAM.

4.2. Experimental Results

It should be noted that, due to the need to analyze the total number of demand points from which the demand fluctuation value is obtained, this paper uses Γ (that is, the sum of all pick-up points Γ_i and Γ_i varies in the range $[0, |J_i|]$) as an indicator in the experimental analysis.

For ease of explaining the experimental results, we first present an example for the experiment conducted with $\Gamma = 20$ (i.e., the summation of Γ_i for all sites) for a set of delivery requirements, randomly generated based on the parameters summarized in the previous sub-section. The results obtained are shown in Table 3.

Table 3. Results obtained for the parcel locker location problem with $\Gamma = 20$.

Collection Site	Demand Sites Covered	Number of Large Parcel Lockers	Unsatisfied Large Parcel Demand	Number of Small Parcel Lockers	Unsatisfied Small Parcel Demand
2	2, 6, 18	179	0	537	0
8	8	36	0	75	1
15	13, 15, 34	185	0	543	0
19	19	47	0	124	6
21	3, 21, 28	157	0	433	0
24	10, 11, 16, 24, 35, 40	317	0	889	0
25	1, 25, 41	181	0	529	0
26	26, 37	128	0	292	0
27	20, 27, 32	177	0	513	0
29	12, 29	110	0	311	0
30	23, 30	108	0	304	0
31	4, 9, 17, 22, 31, 33, 38, 47	454	0	1312	0
36	14, 36	111	0	321	0
39	39, 48	95	0	247	0
42	42, 45	103	0	284	0
43	43	56	1	162	6
44	44	47	0	121	5
49	5, 7, 49	170	0	484	0
50	46, 50	99	0	257	0

Not only the solution of the collection locations, the demand sites associated with each collection site, and the number of large and small parcel lockers, but also unsatisfied demands are listed in Table 3. For example, site 8 has been selected as a collection location which covers only the demand from its own site, and the number of large and small parcel lockers determined as 36 and 75, respectively. Since the actual small parcel demand at site 8 is 76, the unsatisfied demand is 1. Although Γ is set as 20 which indicates only 40% of uncertain demands are considered during the location optimization, most demands are satisfied by the solution generated from our approach in general.

To gain an insight into the converging process, the major operation of the solving process for the above example is briefly summarized here. The problem solving starts by relaxing the ILP problem into a linear programming (LP) problem, which is solved by the dual simplex algorithm. In this example, the optimal objective value obtained for the relaxed problem is 2279.38, which is an initial lower bound and can never be reached for the ILP problem. Subsequently, the cut generation is performed to tighten the LP relaxation of the ILP problem so that the solutions get closer to integers. After obtaining a

feasible solution or one which exceeds expectations, the branch-and-bound procedure is invoked with the iterative construction of a set of subproblems which attempt to converge to a solution of the ILP. The number of nodes explored, running time, number of integer solutions found, objective value and relative gap are briefly summarized in Table 4. It is apparent that the objective value decreases as more nodes are explored. Although a quick convergence can be observed at the initial stage after applying the branch-and-bound algorithm, no further reduction in the objective value can be found at the later stage.

Table 4. Converging results obtained for the parcel locker location problem with $\Gamma = 20$.

Number of Nodes Explored	Time (s)	Number of Integer Solutions Found	Objective Value	Relative Gap (%)
5209	105.59	2	2794.16	0.31
7130	124.17	3	2789.98	0.16
8768	140.09	4	2789.46	0.14
18,768	263.49	4	2789.46	0.14
28,768	398.27	4	2789.46	0.14
38,768	518.05	4	2789.46	0.14

4.3. Robustness Analysis

The role of parameter Γ is to regulate the robustness degree for the optimal solution, since the amount of uncertain demand is controlled implicitly by Γ , thus resulting in different violating degree. To evaluate the adaptability and robustness of the proposed method under various uncertain demand levels, a series of numerical experiments have been conducted by varying parameter Γ from 0 to 45 (corresponding to 0 to $0.9|J_i|$) with an interval of 5, and the unsatisfied parcel requirements for the solution under each Γ value have been obtained. Note that the influence of each Γ value on solution has been examined by performing 20 independent runs under different demands randomly generated and the averaged results are presented in Table 5.

Table 5. Results on the examination of the influence of Γ on the solution.

Γ	Optimal Cost (RMB)	Relative Cost Ratio	Number of Large Parcel Lockers	Relatively Large Parcel Locker Ratio	Unsatisfied Large Parcel Demand	Number of Small Parcel Lockers	Relatively Small Parcel Locker Ratio	Unsatisfied Small Parcel Demand
0	2538.97	/	2286	/	209.5	6052	/	697.0
5	3153.78	24.21%	2725	19.20%	11.0	7548	24.72%	74.5
10	3187.71	25.55%	2733	19.55%	7.5	7579	25.23%	80.5
15	3198.21	26.36%	2785	20.08%	2.0	7648	26.37%	40.0
20	3209.70	26.42%	2760	20.73%	0.5	7738	27.86%	10.0
25	3214.03	26.59%	2773	21.30%	0.0	7848	29.68%	0.0
30	3215.35	26.64%	2779	21.57%	0.0	7870	30.04%	0.0
35	3232.97	27.33%	2791	22.05%	0.0	7936	31.10%	0.0
40	3261.27	28.45%	2809	22.88%	0.0	7969	31.68%	0.0
45	3267.07	28.68%	2815	23.18%	0.0	8017	32.50%	0.0

Relative cost ratio listed in Table 5 denotes the gap between the cost of the optimal solution obtained for a certain Γ value (i.e., $\Gamma > 0$) and that for the deterministic demand (i.e., $\Gamma = 0$). Similarly, a relatively large parcel locker ratio and relatively small parcel locker ratio represent the difference between the number of large and small parcel locker under a certain Γ value (i.e., $\Gamma > 0$) and those under deterministic demand (i.e., $\Gamma = 0$).

As shown in Table 4, the unsatisfied numbers of large and small parcel demands under the deterministic situation (i.e., $\Gamma = 0$) are 209.5 and 697.0, respectively. When Γ increases to 5, the cost increases by 24.21% (614.81 RMB), but only 11 large parcel demands and 74.5 small parcel demands are not satisfied on average, as compared to those under the deterministic demand. When Γ continuously increases from 5 to 45, the total cost rises slightly, but the satisfied demand diminishes rapidly. Furthermore, it can be seen that all demands can be satisfied when $\Gamma \geq 25$. This implies that it would be too conservative if the worst case (i.e., demands at all sites reach to the maximum at the same time) is taken

into account when determining the parcel locker location, though the increase in the total cost is relatively moderate. Although it seems that the value of Γ can be set as half of the number of sites (i.e., $\Gamma = 25$) used in this experiment, a slightly over-conservative value is recommended in practice to secure that all demands can be satisfied with high probability.

Therefore, based on the experimental results, the following conclusions can be made. First of all, the robust optimization method proposed in this study can deliver a flexible solution to the location problem of parcel lockers in terms of robustness to uncertain demands. Furthermore, high robustness can be achieved for the location solution without considerably sacrificing the total cost. However, it would be recommended that higher value should be selected for parameter Γ for the cost-insensitive cases. Due to the transform of the nonlinear constraints into the corresponding linear counterparts, the existing commercial solver can be employed to generate a high-quality solution for the parcel locker location problem within a reasonable time with moderate computing power.

5. Conclusions

The parcel locker delivery mode is an effective solution to the problem of last-mile delivery, which has been promoted and deployed in many countries around the world. A reasonable location is critical to promoting the further development of parcel lockers and better to meet the customer demand. However, no reports on the robust optimization model for the parcel locker location problem with consideration of multiple uncertain demands have been found in the literature. Therefore, the study presented in this paper attempts to extend the robust optimal method to the parcel locker location problem by formulating an optimization model to determine both the collection locations and the number of parcel lockers for each collection, with consideration of demand uncertainties. The optimization model has been evaluated through a series of experiments, with the main focus being on the analysis of the solution robustness. The following findings can be obtained from this study.

(1) The optimization model developed is capable of dealing with a set of uncertainties in the numbers of small and large parcels to be received and sent, respectively.

(2) The collection locations and the number of parcel lockers can be determined simultaneously by the robust optimization model proposed, with all demand sites to be covered by the nearest collection site within acceptable walking distance.

(3) The solution robustness and unsatisfied demand risk can be reconciled by adjusting parameter Γ . Although the experimental findings imply that a moderately high level of Γ is able to provide high robustness for the solution, it is recommended that Γ could take a slightly large value to avoid any risk of unmet demands, since the cost resulted from the improved robustness is not significantly exacerbated.

(4) The robust solution can be obtained by the existing solver within a reasonable timescale with moderate computing power thanks to the linearization of several nonlinear constraints, which considerably simplify the programming model.

However, this study also has several limitations, which should be addressed in future studies. First of all, a sharp increase in the delivery demand during special periods (e.g., COVID-19 pandemic) is not considered as a source of demand uncertainty in this study, as it is not easy to accurately estimate the potential bounds of demand. On the other hand, the predication bias on the long-term trend of the parcel locker demand is not explicitly taken into account when constructing the robust optimization model. In practice, several delivery modes, such as attended self-pick-up points and parcel lockers, may be jointly employed in the last-mile delivery service, a practice which also deserves a further investigation.

Author Contributions: Conceptualization, Y.W. and J.L.; methodology, Y.W., M.B. and Y.Z.; formal analysis, M.B.; investigation, M.B. and Y.Z.; resources, Y.C. and J.L.; data curation, M.B. and J.L.; writing—original draft preparation, Y.W., M.B. and Y.Z.; writing—review and editing, Y.Z., M.B. and Y.W.; visualization, Y.Z. and M.B.; supervision, Y.W. and Y.C.; project administration, Y.C.; funding acquisition, Y.W. and J.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Beijing Municipal Natural Science Foundation 8212002 and Beijing Social Science Foundation 21SRC017.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: All important parameters for the experiments reported here have been included in this paper. The proposed optimization model and its corresponding codes that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

References

- 2021 Postal Industry Development Statistical Bulletin State. Available online: <https://www.spb.gov.cn/gjyzj/c100276/202206/d58b4f4fb1414177a6988b6054e40529.shtml> (accessed on 6 September 2022).
- Pronello, C.; Camusso, C.; Valentina, R. Last mile freight distribution and transport operators' needs: Which targets and challenges? *Transp. Res. Procedia* **2017**, *25*, 888–899. [CrossRef]
- Iwan, S.; Kijewska, K.; Lemke, J. Analysis of parcel lockers' efficiency as the last mile delivery solution—the results of the research in Poland. *Transp. Res. Procedia* **2016**, *12*, 644–655. [CrossRef]
- Zenezini, G.; Lagorio, A.; Pinto, R.; De Marco, A.; Golini, R. The collection-and-delivery points implementation process from the courier, express and parcel operator's perspective. *IFAC-PapersOnLine* **2018**, *51*, 594–599. [CrossRef]
- DHL. Annual Report 2019. 2019. Available online: <https://www.dpdhl.com/en/about-us.html> (accessed on 23 June 2020).
- Jang, Y.X. Smart Parcel Locker Industry Welcomes Good News: Each Group Subsidy Does not Exceed 10,000 yuan. In *China Business Herald*; 2020; Available online: <https://m.gmw.cn/baijia/2020-03/11/1301038986.html> (accessed on 11 March 2020).
- Francke, J.; Visser, J. Internet shopping and its impacts on mobility. In Proceedings of the 25th World Road Congress (PIARC), Seoul, Korea, 2–6 November 2015.
- Carotenuto, P.; Gastaldi, M.; Giordani, S.; Rossi, R.; Rabachin, A.; Salvatore, A. Comparison of various urban distribution systems supporting e-commerce. Point-to-point vs collection-point-based deliveries. *Transp. Res. Procedia* **2018**, *30*, 188–196. [CrossRef]
- Log4.pl. Paczkomaty InPost-ekspertyza AGH. 2013. Available online: http://log4.pl/aktualnosci-2609-Paczkomaty_InPost_ekspertyza_AGH.html (accessed on 19 April 2019).
- Sun, H. Study on the “Last Kilometer” Delivery Model of Intelligent Parcel Lockers. Master's Thesis, An Hui University of Science and Technology, Huainan, China, 2018. (In Chinese).
- Lemke, J.; Iwan, S.; Korczak, J. Usability of the parcel lockers from the customer perspective—the research in Polish Cities. *Transp. Res. Procedia* **2016**, *16*, 272–287. [CrossRef]
- Choi, Y.; Schonfeld, P.M.; Lee, Y.J.; Shin, H.S. Innovative methods for delivering fresh food to underserved populations. *J. Transp. Eng. Part A Syst.* **2021**, *147*, 04020140. [CrossRef]
- Weltevreden, J.W. B2C e-commerce logistics: The rise of collection-and-delivery points in The Netherlands. *Int. J. Retail. Distrib. Manag.* **2008**, *36*, 638–660. [CrossRef]
- Lagorio, A.; Roberto, P. The Parcel Locker Location Issues: An Overview of Factors Affecting Their Location. In Proceedings of the 8th International Conference on Information Systems, Logistics and Supply Chain: Interconnected Supply Chains in an Era of Innovation, ILS, Austin, TX, USA, 22–24 April 2020.
- Peppel, M.; Spinler, S. The impact of optimal parcel locker locations on costs and the environment. *Int. J. Phys. Distrib. Logist. Manag.* **2022**, ahead-of-print. [CrossRef]
- Yang, G.; Huang, Y.; Fu, Y.; Huang, B.; Sheng, S.; Mao, L.; Yin, Q. Parcel locker location based on a bilevel programming model. *Math. Probl. Eng.* **2020**. [CrossRef]
- Deutsch, Y.; Golany, B. A parcel locker network as a solution to the logistics last mile problem. *Int. J. Prod. Res.* **2018**, *56*, 251–261. [CrossRef]
- Wang, Y.; Bi, M.; Lai, J.; Chen, Y. Locating movable parcel lockers under stochastic demands. *Symmetry* **2020**, *12*, 2033. [CrossRef]
- Atamtürk, A.; Zhang, M. Two-stage robust network flow and design under demand uncertainty. *Oper. Res.* **2007**, *55*, 662–673. [CrossRef]
- Tanoukou, G.A.; Benyoucef, L.; Xie, X. A scenario analysis of a location problem with uncertain demand. *Int. J. Comput. Appl. Technol.* **2008**, *32*, 290–297. [CrossRef]
- Ukkusuri, S.V.; Mathew, T.V.; Waller, S.T. Robust transportation network design under demand uncertainty. *Comput. -Aided Civ. Infrastruct. Eng.* **2007**, *22*, 6–18. [CrossRef]
- Bieniek, M. A note on the facility location problem with stochastic demands. *Omega* **2015**, *55*, 53–60. [CrossRef]
- Albareda-Sambola, M.; Fernández, E.; Saldanha-da-Gama, F. Heuristic solutions to the facility location problem with general Bernoulli demands. *INFORMS J. Comput.* **2017**, *29*, 737–753. [CrossRef]
- Bertsimas, D.; Sim, M. The price of robustness. *Oper. Res.* **2004**, *52*, 35–53. [CrossRef]
- Gabrel, V.; Lacroix, M.; Murat, C.; Remli, N. Robust location transportation problems under uncertain demands. *Discret. Appl. Math.* **2014**, *164*, 100–111. [CrossRef]

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26. Sundarakani, B.; Pereira, V.; Ishizaka, A. Robust facility location decisions for resilient sustainable supply chain performance in the face of disruptions. *Int. J. Logist. Manag.* **2021**, *32*, 357–385. [[CrossRef](#)]
 27. Huang, M.; Ren, L.; Lee, L.H.; Wang, X.; Kuang, H.; Shi, H. Model and algorithm for 4PLRP with uncertain delivery time. *Information Sciences* **2016**, *330*, 211–225. [[CrossRef](#)]
 28. Sun, Y. A fuzzy multi-objective routing model for managing hazardous materials door-to-door transportation in the road-rail multimodal network with uncertain demand and improved service level. *IEEE Access* **2020**, *8*, 172808–172828. [[CrossRef](#)]