

Article

An Algorithm for Business Management Based on Portfolio Optimization

Todor Stoilov *  and Krasimira Stoilova

Institute of Information and Communication Technologies-Bulgarian Academy of Sciences, 1113 Sofia, Bulgaria

* Correspondence: todor.stoilov@iict.bas.bg

Abstract: An algorithm is derived for active business management. The key component of the algorithm is the definition and solution of an appropriate portfolio problem. For the last one, the disbursements for the business management are regarded as potential portfolio resources. For the portfolio definition, the increases or decreases of the disbursements are assumed to be the assets, whose weights are found as solutions from the portfolio optimization problem. These solutions recommend the reallocation of the resources between different disbursements, which increases the income of the business entity. The definition and solution of this portfolio problem are made sequentially in time, and the obtained solutions are applied as recommendations for future business management steps. An algorithm for business management, based on the sliding mode of repetitive definitions and solutions of portfolio problems with historical data of last disbursements, gives recommendations for the reallocation of resources for the next future period of management. The algorithm is numerically tested with real data on animal husbandry from Bulgaria. The empirical results demonstrate an advantage in increasing the husbandry income in comparison with the lack of such an active business policy. The algorithm can be implemented as a software solution in an appropriate programming system, supporting fintech service for active business management.

Keywords: active business management; portfolio optimization; resource allocation

MSC: 91B28; 91B32; 68W01



Citation: Stoilov, T.; Stoilova, K. An Algorithm for Business Management Based on Portfolio Optimization. *Mathematics* **2022**, *10*, 4262. <https://doi.org/10.3390/math10224262>

Academic Editor: Sy-Ming Guu

Received: 20 October 2022

Accepted: 9 November 2022

Published: 14 November 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The main approach in this paper is to motivate that the business management decisions have to be based on quantitative assessments and solutions. Quantitative solutions can be found by applying an optimization approach by definition and solution of an appropriate optimization problem. This research targets the usage of quantitative models, which are applied to the business management of animal husbandry. It is chosen quantitative modeling and optimization to be performed in the framework of the portfolio theory. Thus, business management will be performed under two important criteria for maximization of return and minimization of risk. Our research targets an approach coming from the fintech domain for the usage and application of information technology solutions in business management activities. Thus, our application of the portfolio theory will result in deriving an algorithm for business management. The existence of a working algorithm gives the opportunity for parts of the business management decisions to be performed in an automatic way by the usage of software tools, which implement the algorithm derived in this paper.

This research is based on the overlapping points between the topics of business management, the fintech domain, and portfolio modeling. The overview below tries to point out cases that can illustrate the power of business solutions, which can benefit from the application of a quantitative business approach, applied in an algorithmic sequence for optimal management of a business and production entity.

To clear the content of business management, we use the definition of management science as “the broad interdisciplinary study of problem-solving and decision making in human organization, with strong links to management, economics, business, engineering, management consulting and other fields” [1]. Business management is an important domain of management science, and its roots come in the early years of the last century [2]. This kind of management applies methods that are related to analyzing and modeling business processes [3]. Business management, in general, contains different elements, which are described mainly in a qualitative way. For example, in [4], six general elements of business management are outlined: strategic alignment, governance, methods, information technology, people, and culture. These elements are qualitatively assessed and estimated by the components, which have to be performed for their satisfaction. In [5], one can find descriptions of six main functional areas for applications of business management: people management; operation management of production process; accounting for management of finances; strategy management as high-level business entity control; data analytics, targeting the optimization of the business performance; marketing, which includes advertising and social media presence. It exists wider forms of decomposition of the business functional management. For illustration, Ref. [6] defines purchase function; productivity; personnel function; research and development; financial management, and economics.

This paper does not claim an extended overview of the application domains of business management nor the content of its functions. This short reference tries to illustrate that the main contents of the business management activities are mainly described in a qualitative descriptive way.

The benefit of business management generally comes from quantitative decisions and recommendations. In our work, we generally pay attention to quantitative solutions, which can be derived and implemented in real business cases. Our approach to quantification of business decisions is directed towards the financial management of a business entity that has production activities of goods.

The contributions of this paper concern the quantitative approach, which is applied to considering decision making for business management. This is achieved by the definition and solution of a portfolio optimization problem. As portfolio assets, a set of disbursements of the production activities of a business entity are regarded. The portfolio problem addresses the increase or decrease in the disbursements by means of increasing the income without having additional resources for disbursements. The portfolio solutions recommend the reallocation of the available resource. This portfolio problem is incorporated in an algorithm, which targets the increase in income by reallocation of resources. The empirical results confirm the benefit of the derived algorithm. The rest of the paper is organized as follows: in Section 2, an analysis of the quantitative models for business management is given. In Section 3, key elements of the portfolio theory are discussed. In Section 4, our model for the portfolio problem, addressing the increase or decrease in income, is derived. In Section 5, the algorithm is presented, which incorporates the usage of the portfolio problem. In Section 6, the rules for the application of the algorithm are presented. Section 7 gives numerical results of the empirical experiments with real data. Section 8 discusses the obtained results and the potential for improving and complicating the algorithm. Finally, Section 9 concludes this paper.

2. Quantitative Models for Business Management and Applications of Portfolio Theory

The quantitative models in business management provide analysis of data and support decision making. The quantitative approach allows being found the best solutions, providing efficient decisions. The quantitative solutions are obtained mainly from the application of different parts of mathematics. As an example, in [7], the solutions to a set of problems for business management are illustrated with the application of a linear set of equations, matrix algebra, mathematics of finance, linear programming, network analysis and scheduling, transportation problems, and inventory planning. Quantitative methods

for management and decision making in the business domain are discussed in [8]. Our research addresses problems related to the financial management of enterprise resources. This makes it important to discuss the topic of fintech applications for financial services. Fintech refers mainly to the application of information technologies in financial services [9]. An overview of the fintech approaches one can find in [10–12]. Following [13], the term fintech appeared at first in business journals. It addresses problems and services in the financial sector. Fintech is defined as a service that applies information technology, supporting the financial domain [14,15]. An important part of fintech solutions is the development and implementation of algorithmic and software components for fintech services. Thus, information technologies are the backbone for the implementation of new financial services and decision solutions in this domain. For example, Fintech services are related to the domains of credit reports, digital banking, insurance, investing, lending, payments, real estate, and risk analysis [13]. Because our primary goal of this research is to provide info service to a production business management, our approach is to model the production process as a sequence of investment decisions. We assume that all disbursements made for a production process have to be assumed as an appropriate form of investment. The investment decisions are quantified under the approaches of the portfolio optimization theory [16]. Thus, our research deals with the development of models and algorithms, which basically use formal descriptions from the portfolio theory.

The portfolio theory has been launched with the famous works of [16]. We mention important stages of its development with the works of [17,18]. In [16], the formal optimization portfolio problem was introduced, which minimizes the risk of the investment, and supports an appropriate level of portfolio return. In [17], the portfolio theory introduced a new portfolio parameter as the market portfolio. Respectively, additional relations for the portfolio have been derived in the so-called Capital Asset Pricing Model. It has introduced new important relations, named Capital Market Line, Security Market Line, and characteristic line. A new step in the development of portfolio theory was the works of Black–Litterman, where additional new parameters were introduced, named “implied returns” [18–20]. These new parameters integrate the assessment both from the historical dynamics and the expert expectation for the future levels of the asset returns giving an assessment about the next future levels of the assets. The usage of the “implied returns” according to the Black–Litterman model gives better results in comparison with the classical portfolio model of Markowitz. The portfolio problem has been extended with additional constraints by means to reflect real business cases with transactional costs. Respectively the cardinality constraints, with nixed integer constraint, reduce the size of the assets included in the portfolio.

An extension of the term risk and its formal presentation was the definition of the parameter Value at Risk (VaR) [21]. Respectively, the portfolio model can accommodate risk with two formal descriptions with the value of variance and VaR of the portfolio return and/or loss and its derivative form Conditional VaR (CVaR). The portfolio problem is complicated in [22] by the application of CVaR in a goal function of the portfolio problem, defined as an appropriate Sharpe ratio. The portfolio problem takes additional cardinality constraints. The resulting portfolio problem is solved with neuro dynamic optimization due to its complexity.

The analysis above targets to convince the reader that the portfolio theory has its own way of development, which increases its potential for quantitative decisions and management recommendations. The portfolio theory is successfully applied for the assessment of the economic value of business management because it gives a universal metric for the comparison of different business applications [23].

Nevertheless, that portfolio theory is applied in financial and investment domains, and its quantitative models are used in different cases and applications. In [24], portfolio formalism is used in solving the problem of structural choice, which gives a structural diagnosis for resource distribution. In [25], the Capital Asset Pricing Model is quantitatively applied for fintech decisions, increasing the number of factors taken into consideration in

the investment process. The portfolio mean-variance modeling is applied for the integration of healthcare services [26]. In [27], the portfolio theory is used to disperse the supply chain, finding efficient supplier combinations.

A trading algorithm based on portfolio theory is discussed in [28]. The portfolio theory is applied to choosing a combination of wheat, which enhances the profitability over the selection of a single sort of wheat [29]. Commercial real estate portfolios are analyzed with quantitative categories of the portfolio theory in [30]. A solution for decreasing the risk in the agriculture domain is performed by the application of the portfolio theory [31]. The application of quantitative modeling of business management is assessed and evaluated in [32,33]. The stock process of companies for the development of e-business services is predicted and managed, applying portfolio modeling and optimization to assess the future behavior of the e-business [34]. Purchasing decisions for procurement and supply chains by applying portfolio modeling are given in [35]. Improvements in pension plans by foreign assets are also evaluated on portfolio formalism [36].

This sequence of applications of the portfolio theory proves and gives evidence that portfolio formalism is a powerful tool for the quantification of decision making in managing different forms of business activities. Next, we introduce the key points of the portfolio theory, which we use in the next sections of the paper.

3. Formal Background of the Portfolio Theory

The basic input parameters for the portfolio problems are the records of the historical returns of the assets, which will participate in the portfolio

$$\mathbf{R}_1 = [R_1^{(1)}, R_1^{(2)}, \dots, R_1^{(n)}] \quad (1)$$

$$\mathbf{R}_N = [R_N^{(1)}, R_N^{(2)}, \dots, R_N^{(n)}], \mathbf{R} = \mathbf{R}_1, \dots, \mathbf{R}_N,$$

where N is the number of assets, and n is the length of the historical period. $R_i^{(j)}$ is the current return of asset i for the time (j) .

The initial set of data \mathbf{R} allows evaluating three important statistical characteristics of the assets returns:

- Mean value of the return:

$$E_i = \frac{1}{n} \sum_{k=1}^n R_i^{(k)}, \quad (2)$$

- Standard deviations:

$$\sigma_i^2 = \frac{1}{n} \sum_{k=1}^n (E_i - R_i^{(k)})^2, \quad i = 1, \dots, N. \quad (3)$$

- Covariance square matrix Σ , $N \times N$ between the asset returns $\mathbf{R}_i, \mathbf{R}_j, i, j, 1 = N$:

$$\Sigma_{|N \times N} = \frac{1}{n} \sum_{k=1}^n (R_i^{(k)} - E_i)(R_j^{(k)} - E_j). \quad (4)$$

The portfolio theory assumes that the standard deviations, together with the covariance, are quantitative measures for the risk of each asset as of the portfolio, which is a combination of the assets.

The risk interpretation comes from the statistics [37], given that the predominant set of the real returns $R_i(t)$ will belong to the set of values given by the inequality

$$Pr(E_i - \sigma_i \leq R_i(t) \leq E_i + \sigma_i) \approx 68.27\%.$$

This relation says that the probability that the values of returns $R_i(t)$ belong to the space $[E_i - \sigma_i, E_i + \sigma_i]$ is about 68%. Not all values of $R_i(t)$ will be there, but the predominant

amount of them will belong to this diapason. The graphical interpretation of this relation is given in Figure 1.

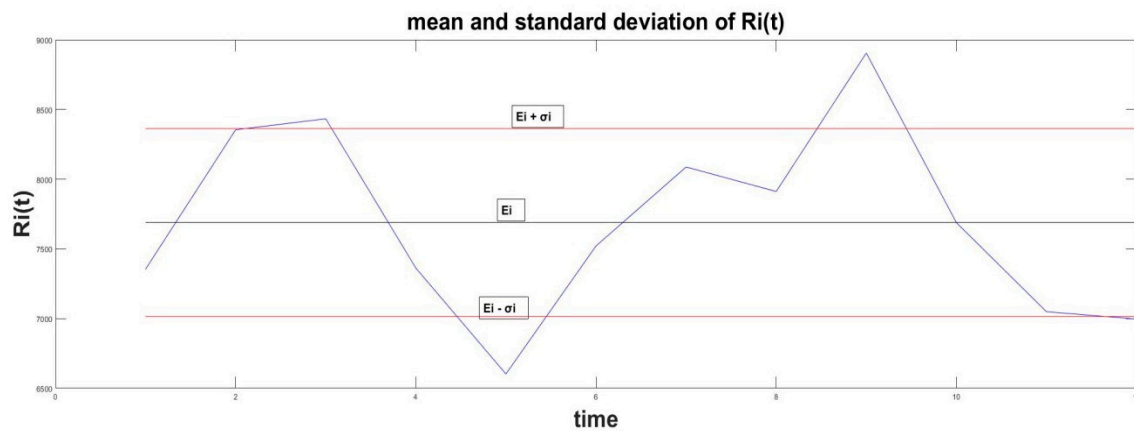


Figure 1. Illustration between the mean E_i , standard deviation σ_i , and stochastic value $R_i(t)$.

Hence, if the standard deviation σ_i has a small value, then the real value of the return $R_i(t)$ will be close to the mean estimation E_i and the decision maker can use this value E_i as a forecast for the future level of asset return.

These asset parameters are used for the evaluation of the portfolio return and risk. Because the portfolio is a combination of assets, its return and risk depend on the weights w_i , $i = 1, N$ of each asset, how many amounts of it participates in the portfolio

$$E_p = \sum_{i=1}^N w_i E_i \quad i = 1, N, \mathbf{w} = (w_1, \dots, w_N) \quad (5)$$

$$\sigma_p^2 = \mathbf{w}^T \mathbf{\Sigma} \mathbf{w},$$

where E_p is the portfolio return, σ_p^2 —portfolio risk, $\mathbf{\Sigma}$ —covariance matrix of the asset returns. For the weights w_i a normalization equation is applied

$$w_1 + w_2 + \dots + w_N = 1, \text{ or } \mathbf{w}^T \mathbf{1} = 1, w_i \geq 0, i = 1, \dots, N.$$

With such defined portfolio relation, the optimization problem, which evaluated the optimal weights of assets in the portfolio, can be evaluated by the portfolio problem

$$\min_{\mathbf{w}} \frac{\mathbf{w}^T \mathbf{\Sigma} \mathbf{w}}{\mathbf{E}^T \mathbf{w}} \quad (6)$$

$$\mathbf{w}^T \mathbf{1} = 1, w^T \geq 0.$$

The next step of the portfolio theory comes from the definition and inclusion of a new term, named market return E_M and market risk σ_M^2 [17]. This new parameter has been formally related to the risks and returns of the portfolio and assets. Thus, coming from the Capital Asset Pricing Model, the following formal relations are introduced in the portfolio theory:

$$\text{Capital Market Line (CML)} : E_p = r_f + \frac{E_M - r_f}{\sigma_M} \sigma_p,$$

where r_f is the return of the risky free asset.

The CML gives an assessment of the possible relation between the portfolio return E_p and risk σ_p for the particular market, which has market return E_M and market risk σ_M^2 . Particularly the assessment of these values (E_M, σ_M) comes from the values of the index of the corresponding market.

$$\text{Securit Market Line (SML)} : E_i = r_f + (E_M - r_f)\beta_i,$$

$$\text{where } \beta_i = \frac{\text{cov}(i,M)}{\sigma_M^2}.$$

The SML gives straight relation between the market return E_M and the return of the individual asset E_i . The coefficient β_i depends on the covariance between the return of asset i and this one of the market.

$$\text{Characteristic Line (HL)} : R_i = r_f + \beta_i(R_M - r_f),$$

It gives the relation between the current values of the asset return R_i and this one of the market R_M . This relation can be used to forecast the level of the asset return R_i according to the value of the current market return R_M .

In this research, we use these formal relations from the portfolio theory. Particularly the characteristic line plays an important role in the derived algorithm. This relation plays an important role in the analytical definition of the relations between the portfolio return and the disbursements in the business entity.

To have a more complex view of the analytical relations of portfolio theory, one can refer to the overview [38].

4. Portfolio Model for the Business Management

The particular case, which is under consideration, is the management of animal husbandry. It produces milk and meat from cows, but these products are related to a set of disbursements. The application of the portfolio theory targets the quantification of how much a type of disbursement is to be reduced or increased by means to increase the income of the husbandry. The initial data of incomes and disbursements come from the balance sheets of the husbandry for 3 previous years, 2019, 2020, and 2021.

The disbursements are classified in $N = 12$ categories: $R_1 \equiv$ drugs; $R_2 \equiv$ electricity, $R_3 \equiv$ salaries, $R_4 \equiv$ insurances, $R_5 \equiv$ credits, $R_6 \equiv$ interest loans, $R_7 \equiv$ leasing contracts, $R_8 \equiv$ fuel, $R_9 \equiv$ repairs, $R_{10} \equiv$ fodders, $R_{11} \equiv$ others, and $R_{12} \equiv$ animal insemination products. The animal husbandry incomes are also estimated per month. Formally the input data of the husbandry production and business are illustrated in Table 1.

Table 1. Available input data of the husbandry production.

Disbursements	January 2019–December 2019	January 2020–December 2020	January 2021–December 2021
$R_1, n = 12 \text{ months}$	$R_{1,2019}^{(1)} \dots R_{1,2019}^{(12)}$	$R_{1,2020}^{(1)} \dots R_{1,2020}^{(12)}$	$R_{1,2021}^{(1)} \dots R_{1,2021}^{(12)}$
...
$R_N, N = 12$	$R_{12,2019}^{(1)} \dots R_{12,2019}^{(12)}$	$R_{12,2020}^{(1)} \dots R_{12,2020}^{(12)}$	$R_{12,2021}^{(1)} \dots R_{12,2021}^{(12)}$
Income R_M	$R_{M,2019}^{(1)} \dots R_{M,2019}^{(12)}$	$R_{M,2020}^{(1)} \dots R_{M,2020}^{(12)}$	$R_{M,2021}^{(1)} \dots R_{M,2021}^{(12)}$

Having these historical data, the management problem concerns the evaluation of the relative increases or decreases of the disbursements $R_i = |R_i^{(k)}, k = 1, n|$, $R_i, i = 1, N$, by means to increase the husbandry income R_M . The evaluation of the relative weights for increase or decrease is performed by a formal model based on portfolio relations.

From the initial data from Table 1, it is possible to evaluate the mean values of disbursements and the income as their standard deviations as a measure of risk or

$$E = (E_1, \dots, E_N), \sigma = (\sigma_1, \dots, \sigma_N), E_M, \sigma_M.$$

To simplify the notations, these values are calculated only for a period of one year, or $n = 12$ months. Following the portfolio formalism, the characteristic line HC formally is

defined as a linear relationship between the current market return R_M and the current asset return R_i

$$R_i = a_i^* + b_i^* R_M$$

where a_i^* and b_i^* are coefficients.

The assumptions made in this research take R_M to be the husbandry income and R_i is the corresponding i th disbursement. Because we want to find how disbursement R_i influences the husbandry return, we interchange the roles of R_i and R_M in a formal relation

$$R_M = a_i + b_i R_i \quad (7)$$

where a_i and b_i are unknown coefficients. For the evaluation of a_i , b_i one can use two possible solutions.

4.1. Evaluation by the Usage of the Covariance Matrix

Assuming that relation (7) holds for the current values of R_M and R_i this relation has to be valid also for the mean values of E_M and E_i of the income and disbursement i or

$$E_M = a_i + b_i E_i. \quad (8)$$

The covariance between M and i is equal to

$$\text{cov}_{Mi} = \sum_1^n \Pr(R_i)(R_M - E_M)(R_i - E_i). \quad (9)$$

Substituting (7) and (8) in (9) gives

$$\begin{aligned} \text{cov}_{Mi} &= \sum_1^n \Pr(R_i)(a_i + b_i R_i - b_i E_i)(R_i - E_i) \\ \text{cov}_{Mi} &= \sum_1^n \Pr(R_i)b_i(R_i - E_i)^2 = b_i \sum_1^n \Pr(R_i)(R_i - E_i)^2 = b_i \sigma_i^2 \\ \text{cov}_{Mi} &= b_i \sigma_i^2, \quad b_i = \frac{\text{cov}_{Mi}}{\sigma_i^2}. \end{aligned}$$

Respectively from (8)

$$a_i = E_M - \frac{\text{cov}_{Mi}}{\sigma_i^2} E_i.$$

The values b_i define the tendencies between the disbursement i in the income M . If $b_i > 0$, this means that the increase in the disbursement i will increase the income M . If $b_i < 0$, this means that the decrease in the disbursement i will decrease the income M .

4.2. Evaluation by Application of the Least Square Method

The analytical approximation between R_M and R_i is a linear one. The least square method evaluates the unknown coefficients a_i and b_i by a linear system of equations [39]

$$\begin{bmatrix} m & \sum_{k=1}^n R_i^{(k)} \\ \sum_{k=1}^n R_i^{(k)} & \sum_{k=1}^n R_i^{2(k)} \end{bmatrix} \begin{bmatrix} a_i \\ b_i \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^n R_M \\ \sum_{k=1}^n R_M R_i^{(k)} \end{bmatrix}. \quad (10)$$

Thus, using directly the initial data from Table 1, the least square method gives the values of the unknown coefficients a_i and b_i .

Relation (7) gives a formal description of the influence of each disbursement R_i towards the income R_M . Because we need to find the simultaneous influences of all R_i $i = 1, N$ to the income R_M we derive such analytical relation in the following way, taking into consideration that (7) is also valid for the mean values E_i and E_M :

$$\text{for } i = 1, E_1 = a_1 + b_1 E_1$$

$$\begin{aligned}
& \text{for } i = n, E_M = a_N + b_N E_N \\
& \text{or } \sum_{i=1}^N E_M = \sum_{i=1}^N a_i + \sum_{i=1}^N b_i E_i \\
& \text{or } E_M = \frac{1}{N} \sum_{i=1}^N a_i + \sum_{i=1}^N b_i E_i
\end{aligned} \tag{11}$$

4.3. Definition of a Portfolio Problem

Let us start from the derived analytical relation for particular disbursement i , given by the linear relation (8). We assume that due to the manager's decisions, the disbursement i will change its mean with a relative weight of w_i . Hence, the new value of the income will be equal to

$$E_{Mnew} = a_i + b_i E_i (1 + w_i) = E_M + \Delta E_M.$$

The increase in the income ΔE_M is related to the sign of the coefficient b_i :

- If $b_i > 0$, the increase in the income is $\Delta E_M = b_i E_i w_i$;
- If $b_i < 0$, the increase in the income is due to the decrease in disbursement i from relative 1 to $(1 - w_i)$, $w_i \geq 0$, or $\Delta E_M = b_i E_i (1 - w_i)$.

The increases due to all disbursements $i = 1, N$ can be used in a portfolio problem for the evaluation of the weights w_i for finding how much to increase or decrease the mean value of a corresponding disbursement.

Hence, the portfolio problem targets the maximization of the increase in the income ΔE_M given by all disbursements $i = 1, N$ or formally

$$\begin{aligned}
& \max_{w_i} \left\{ \sum_{i \in b_i > 0}^k b_i E_i w_i + \sum_{j \in b_j < 0}^l b_j E_j (1 - w_j) \right\}, k + l = N \text{ or} \\
& \max_{w_i} \left\{ \sum_{i \in b_i > 0}^k b_i E_i w_i - \sum_{j \in b_j < 0}^l b_j E_j w_j \right\} \text{ or} \\
& \max_{w_i} \left\{ \sum_{i=1}^N \text{abs}(b_i) E_i w_i \right\}.
\end{aligned}$$

Because w_i is a relative increase in disbursement i , such increase has to satisfy the relative relation

$$\sum_{i=1}^N w_i \leq 1, \quad w_i \geq 0,$$

because only a few changes of w_i could be eligible.

The portfolio problem takes an analytical form

$$\begin{aligned}
& \max_{w_i} \left\{ \sum_{i \in b_i > 0}^k b_i E_i w_i + \sum_{j \in b_j < 0}^l b_j E_j (1 - w_j) \right\}, k + l = N \\
& \sum_{i \in b_i > 0}^k w_i + \sum_{j \in b_j < 0}^l w_j \leq 1, \quad w_i \geq 0, \quad w_j \geq 0.
\end{aligned} \tag{12}$$

The portfolio problem (12) can accommodate additional constraints for resource allocation. Such a constraint can insist on the volume of resources, which decreases from disbursement j for which $b_j < 0$ is to be allocated for disbursement i where $b_i > 0$. Thus additional resources for business management are not used, but by a reorganization of the available ones, the income is increased according to the portfolio problem (12). Formally such additional constraint can take the form

$$\sum_{i \in b_i > 0}^k b_i E_i w_i - \sum_{j \in b_j < 0}^l (-b_j) E_j w_j \leq 0. \tag{13}$$

However, this form of the portfolio problem does not consider the risk of the income increase ΔR_M .

4.4. Evaluation of the Risk of the Income Increase ΔE_M

The portfolio theory states that the real increase ΔR_M will belong predominantly in the space

$$\Delta R_M \in [\Delta E_M - \sigma_{\Delta M}; \Delta E_M + \sigma_{\Delta M}].$$

For the portfolio problem, it is needed to find an analytical description of the standard deviation of the income increase $\sigma_{\Delta M}$, which is a measure of the risk for the income increase ΔR_M . By definition, the volatility of the increase ΔR_M is evaluated as

$$\sigma_{\Delta M}^2 = \sum_1^n \Pr(\Delta R_M) (\Delta R_M - \Delta E_M)^2. \quad (14)$$

However,

$$\Delta E_M = \sum_{i \in b_i > 0}^k b_i E_i w_i - \sum_{j \in b_j < 0}^l b_j E_j w_j. \quad (15)$$

Respectively,

$$\Delta R_M = \sum_{i \in b_i > 0}^k b_i R_i w_i - \sum_{j \in b_j < 0}^l b_j R_j w_j. \quad (16)$$

After substitution (15) and (16) in (14), the last takes the form

$$\sigma_{\Delta M}^2 = \sum_1^n \Pr(\Delta R_M) \left(\sum_{i \in b_i > 0}^k b_i (R_i - E_i) w_i + \sum_{j \in b_j < 0}^l b_j (R_j - E_j) w_j \right)^2.$$

To simplify the notations, a matrix form of relations is used

$$\sigma_{\Delta M}^2 = \mathbf{w}^T \mathbf{b}^T \Sigma \mathbf{b} \mathbf{w}, \quad (17)$$

where $\mathbf{b} = (b_1, \dots, b_N)$.

$$\mathbf{b}^T \Sigma \mathbf{b} = \begin{vmatrix} b_1^2 \sigma_1^2 & b_1 b_2 cov_{12} & \dots & b_1 b_N cov_{1N} \\ b_1 b_2 cov_{12} & b_2^2 \sigma_2^2 & \dots & b_2 b_N cov_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ b_N b_1 cov_{N1} & b_N b_2 cov_{N2} & \dots & b_N^2 \sigma_N^2 \end{vmatrix}.$$

The covariance matrix Σ is evaluated from the initial data from Table 1 according to relations (4). Using relation (17), the portfolio problem, which minimizes the risk of the increase ΔR_M and maximizes the mean increase ΔE_M takes the analytical form

$$\min_{\mathbf{w}} \left\{ \frac{\mathbf{w}^T \mathbf{b}^T \Sigma \mathbf{b} \mathbf{w}}{\mathbf{w}^T \mathbf{b}^T \mathbf{E}} \right\} \quad (18)$$

$$\sum_{i=1}^N w_i \leq 1, \quad w_i \geq 0.$$

This portfolio problem incorporates both the risk and return in a single analytic relation, known as the Sharpe ratio [17]. Towards this portfolio, problem relation (13) can be added as a constraint, which means that there are no additional investment resources for the portfolio. Portfolio problem (18) is incorporated in an algorithm, which evaluates the relative weights w_i , $i = 1, N$ and recommends which and how many to increase or decrease the appropriate disbursement by means to maximize the mean income of the production.

5. Algorithm for Business Management of a Husbandry

5.1. Initial Data

The historical records of monthly disbursements and income are in the form given in Table 1. These data are available for a period of three years: 2019, 2020, and 2021. For illustration purposes, the historical records are illustrated for 2021, given in Table 2. It is seen that the scales of these values differ in a wide range. That is why we conducted

normalization of these values considering the influence of each disbursement towards the husbandry income.

Table 2. Illustration of the real disbursement and income data.

2021	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_M
January	2706	5940	1530	0	4750	1174	2585	7354	627	101,570	170	586	194,220
February	1523	5290	1596	577	19,523	3281	2596	8354	1158	116,815	388	0	193,159
March	2930	6089	1558	502	18,447	1480	2607	8433	1009	148,100	1040	0	216,364
April	2745	5549	1463	563	20,598	1932	2618	7362	0	146,156	860	10	277,303
May	623	4858	1498	467	0	1237	2629	6602	0	100,629	260	0	199,754
June	2933	5931	1447	0	0	2399	2277	7521	2657	112,743	0	0	176,796
July	3887	8440	1395	3157	58,569	8673	2652	8087	998	115,077	223	10	158,392
August	6605	7455	1153	971	14,824	3283	2297	7911	1171	100,484	0	79	185,403
September	3301	7302	1153	0	0	112	367	8905	3521	91,070	0	0	196,243
October	5783	10,617	1275	226	19,471	3462	0	7688	940	82,855	186	0	207,144
November	4624	11,643	1280	0	0	112	1049	7049	4456	118,442	0	10	199,756
December	5110	13,688	1312	5419	63,315	6391	713	6996	502	128,525	0	0	455,050

5.2. Normalization of the Values of Disbursements

The normalization of the disbursements R_i , $i = 1, N$ is made for each month $k = 1, n$ or for each row of the matrix in Table 2.

$$r_i^{(k)} = \frac{R_i^{(k)}}{\sum_{j=1}^N R_j^{(k)}}, \quad i = 1, N = 12, k = 1, n$$

where $r_i^{(k)}$ normalizes disbursement i for the month k .

5.3. Normalization of Income

The normalization of the income R_M is performed individually for each month according to the income for the overall year

$$r_M^{(k)} = \frac{R_M^{(k)}}{\sum_{j=1}^n R_M^{(k)}}, \quad k = 1, n, n = 12.$$

5.4. Rearrangement of the Income towards the Increasing Values of the Disbursements

The modification of the portfolio problem insists on the sequence of the values of the income R_M be arranged according to the increasing values of the disbursements R_i , $i = 1, N$ because the relation (8) defines R_i as an argument for the linear function of R_M or $R_M = R_M(R_i)$.

5.5. Evaluation of Mean Values

The normalized data are used for the evaluation of the disbursements and income for the entire year

$$E_i = \sum_{k=1}^n R_i^{(k)}, \quad i = 1, N; \quad E_M = \sum_{k=1}^n R_M^{(k)}.$$

Their numerical values are given in Table 3.

Table 3. Mean values of disbursements and incomes for 2021.

E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}	E_{11}	E_{12}
0.023	0.0496	0.0091	0.005	0.0967	0.0162	0.012	0.0505	0.0099	0.7259	0.0016	0.0005

5.6. Evaluations of the Coefficients of the Linear Approximations a_i and b_i

These coefficients are calculated according to the least square method, formally given by the linear set of Equation (10). The resulting evaluations are given in Table 4.

Table 4. Values of the coefficients a_i and b_i .

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}
0.0879	0.0756	0.1521	0.0693	0.0694	0.0783	0.1031	0.1536	0.0917	0.1679	0.0836	0.0848
b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	b_{11}	b_{12}
−0.1983	0.1567	−7.5686	2.8104	0.1443	0.3093	−1.6516	−1.3908	−0.8489	−0.1165	−0.1593	−2.9610

Graphical illustrations of the linear approximation (8) are given in Figures 2–4.

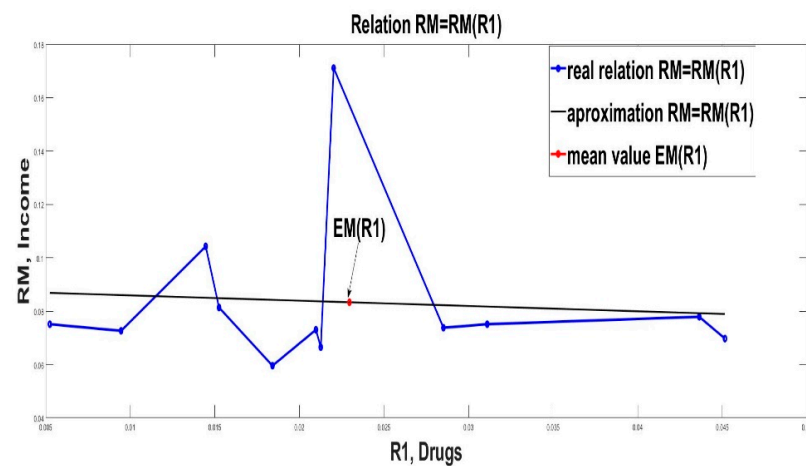


Figure 2. Linear approximation of income R_M from R_1 (drugs).

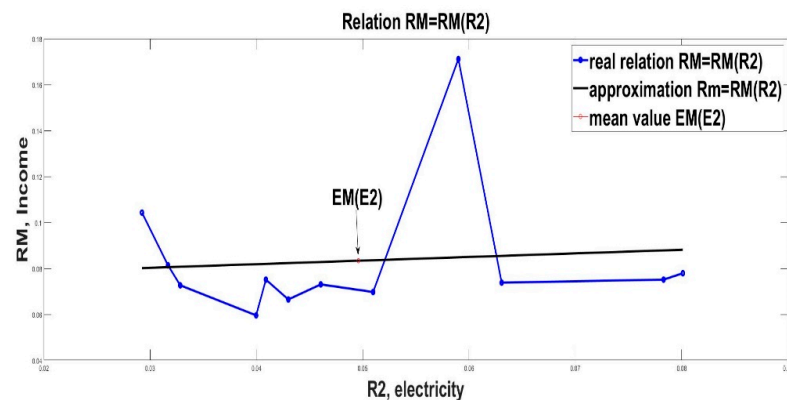


Figure 3. Linear approximation of income R_M from R_1 (electricity).

5.7. Definition and Solution of Portfolio Problem (18)

Relation (13) for zero new investment resources is added as an additional constraint to the portfolio problem (18). The solutions to this portfolio problem are illustrated in Table 5.

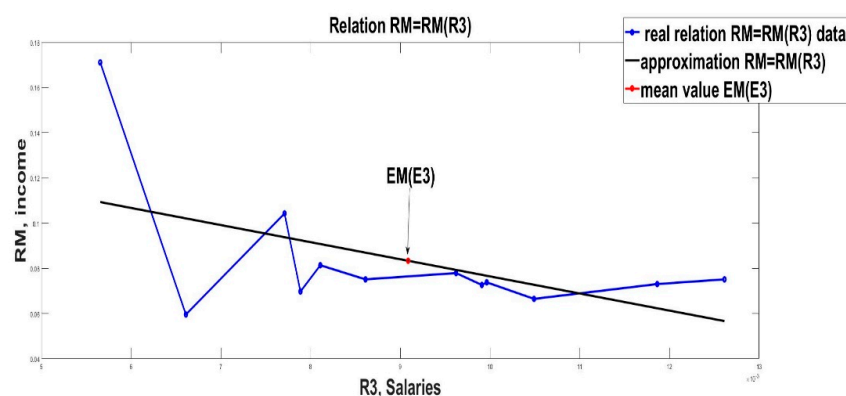


Figure 4. Linear approximation of income R_M from R_3 (salaries).

Table 5. Solutions of the portfolio problem (18) and constraint (13).

b_1	b_2	b_3	$+b_4$	$+b_5$	b_6	b_7	b_8	b_9	$-b_{10}$	b_{11}	b_{12}
w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}
0	0	0	0.8557	0.0023	0	0	0	0	0.142	0	0

These results are interpreted according to the signs of the parameters b_i , which is given in Table 5. For w_4 and w_5 , the coefficients are $b_4 > 0$ and $b_5 > 0$, and the portfolio solutions recommend the disbursements of categories 4 and 5 to be increased by 85% and 0.2%, but the disbursement of category 10 must be decreased by 14% from its current value.

Without the portfolio solutions, the mean value of the income is $E_M^{without} = 0.1196$. If the solutions of the portfolio problem \mathbf{w}^{opt} are applied to increase or decrease the disbursements the income of the husbandry will be $E_M^{opt} = 0.1425$. Hence, the optimal management of the disbursements will increase the income with

$$\frac{E_M^{opt} - E_M^{without}}{E_M^{without}} = 19\%.$$

These simulation results prove that the portfolio management of the disbursements can increase the mean husbandry income. The drawback of the algorithm comes from the sequence of usage of the initial portfolio data. For the definition of the parameters of the portfolio problem, the decision maker has to use available data for disbursements and income, but for the real case of business management, the real levels of the disbursements and income are not known currently, and they can be calculated hardly at the end of the management period. That is why in this research, we modify the algorithm above with the inclusion of a sliding procedure for active business management.

6. Active Business Management Applying the Derived Algorithm

Active business management applies in a repeated way derived algorithm with different initial data. A window with a set of initial data is moving sequentially ahead for each step of management by definition and solving the appropriate portfolio problem. This rolling procedure is graphically illustrated in Figure 5.

For this research, the historical period for the data in the window is chosen for 3 months. These historical data are used for the definition and solution of a corresponding portfolio problem. The optimal solutions \mathbf{w}^{opt} are applied for the next future period by correcting the disbursements and or providing recommendations for the future limitations of the mean disbursements per category. Thus, the business management will have reference data about the recommendations for the mean amount of disbursement per category by means to achieve the maximal level of income.

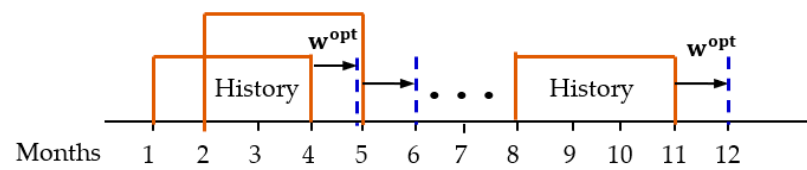


Figure 5. Rolling procedure for definition and solutions of portfolio problems.

This active management procedure is assessed by calculating three types of income values: E_M^{real} —For the management period k , it is used the available historical data. The recommendations from the previous period $\mathbf{w}^{opt}(k-1) = \mathbf{w}_{old}^{opt}$, which were evaluated by the portfolio problem, are applied by changing the current mean disbursements for the period k . The resulting income E_M^{real} is evaluated. This is the real income found by the application of the portfolio problem.

$E_M^{without}$ —This income is evaluated according to the current historical data. The resulting income is calculated without the application of portfolio optimization.

E_M^{opt} —This is the optimal possible value of the income. It is evaluated according to the actual optimal weights $\mathbf{w}^{opt}(k)$, which are solved by the portfolio problem. This evaluation can be performed only at the end of period k , when the real mean values of disbursements should be available. This value gives the maximum income, which can be obtained if the weights $\mathbf{w}^{opt}(k)$ were applied for business management for period k .

The evaluated $\mathbf{w}^{opt}(k)$ are given as old values for the next $k+1$ period and $\mathbf{w}^{opt}(k) = \mathbf{w}_{old}^{opt}(k+1)$. The graphical interpretation of this active management is presented in Figure 6.

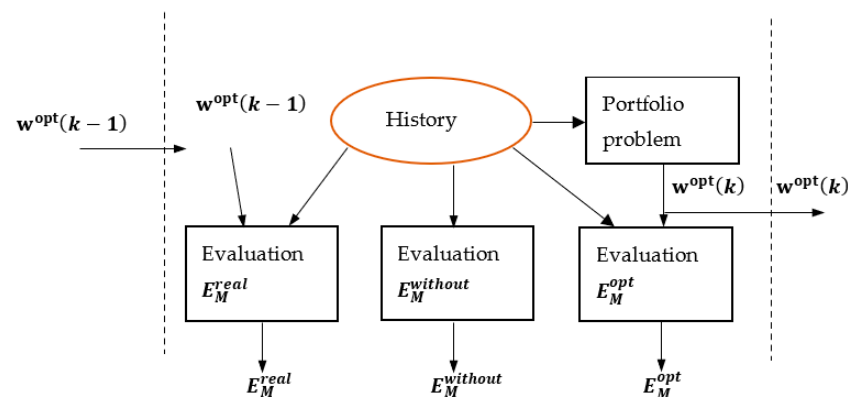


Figure 6. Rolling active business management by changing the recommendations for mean levels of disbursements.

A flow chart of the operational rules of the algorithm is graphically interpreted in Figure 7. The initial data contain the monthly disbursements $R_i^{(k)}$ and income $R_M^{(k)}$ where $k=1$, n is the period of available data, $i=1, N$ are the type of of disbursement. These sets of data are normalized to $r_i^{(k)}$ and $r_M^{(k)}$ by means of operating with fewer values of the data. For the active management, the historical period is chosen to be $t=1$ year, and the investment period T is fixed to $T=3$ years for Case1: 2019–2021, with results in Table 6. For Case 2, it takes $t=3$ months history and $T=12$ months (one year) with results in Table 7. According to the length and sequence of the widow, the corresponding initial data are taken from the set of normalized values $r_i^{(k)}$ and $r_M^{(k)}$, $k=1, t$. The corresponding means of disbursements and income $E_i(t)$, $E_M(t)$, and covariance $\Sigma(t)$ are calculated. Using these data, the corresponding characteristics lines are estimated by evaluation of the coefficients a_i and b_i , $i=1, N$. They are used to evaluate the parameters for the portfolio problem (18) and constraints (13), where $\mathbf{w}^T \mathbf{b}^T \Sigma \mathbf{b} \mathbf{w}$ concerns the risk of the income increase, and $\mathbf{w}^T \mathbf{b}^T \mathbf{E}$ is the mean value of the income increase. The solutions to the portfolio problem

$\mathbf{w}^{opt}(t)$ give recommendations on which type of disbursement to be increased (for $b_i > 0$) and which to be decreased (for $b_i < 0$). Then the comparison is made between the mean income for the defined period $E_M^{without}$, the real achieved income, with the application of the evaluated weights from the previous period $E_M^{real}(\mathbf{w}^{opt}(t-1))$ and the maximal potential case $E_M^{opt}(\mathbf{w}^{opt}(t))$ assuming precise forecast for the means values for the time window t . These three values are used as benchmarks for the evaluation of the benefit of the derived algorithm for the reallocation of resources between different types of disbursements.

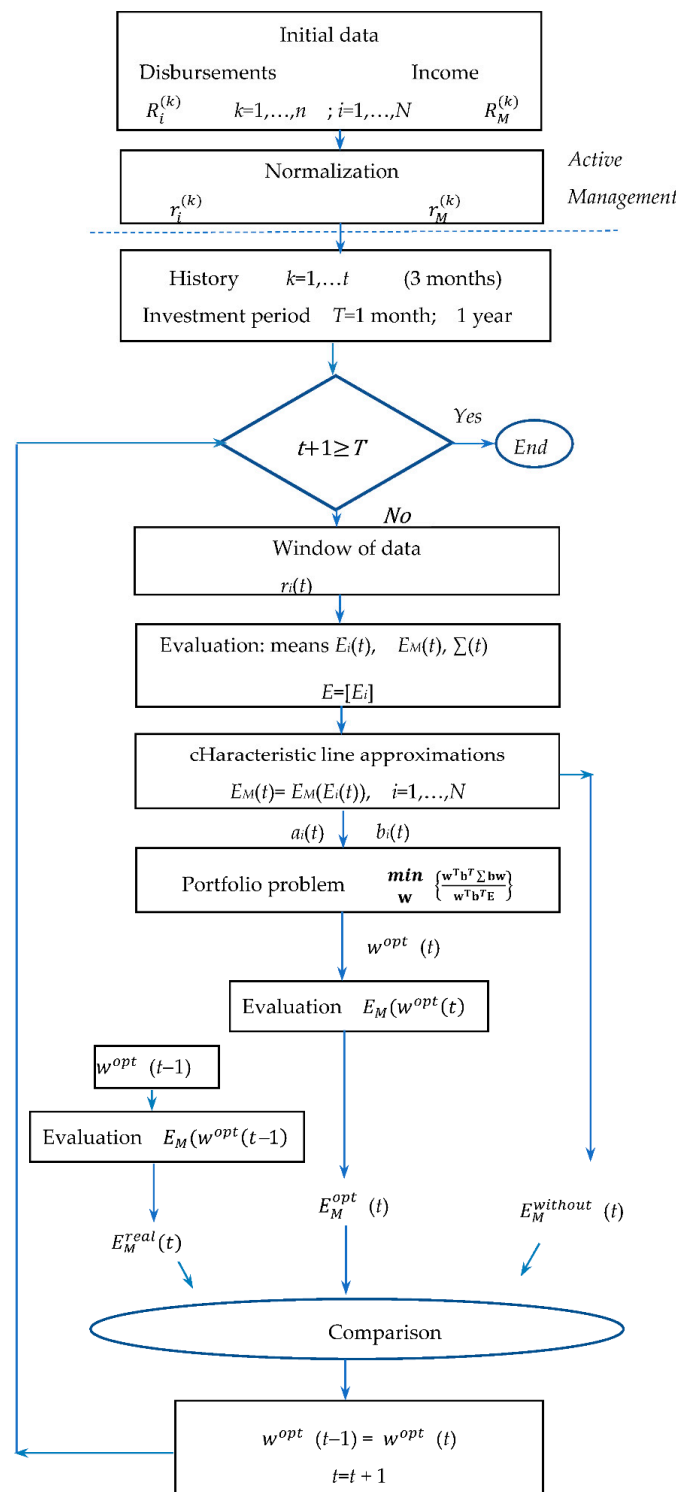


Figure 7. Flow chart diagram of active business management.

Table 6. Numerical results for the case of one-year historical period.

Year	$E_M^{without}$	E_M^{real} with $\mathbf{w}^{opt}(k-1)$	E_M^{opt} with $\mathbf{w}^{opt}(k)$
2019	0.1222	0.1222	0.1542
2020	0.0722	0.0836	0.0868
2021	0.1195	0.1245	0.1420

Table 7. Comparisons between the income's values with and without the application of active business management.

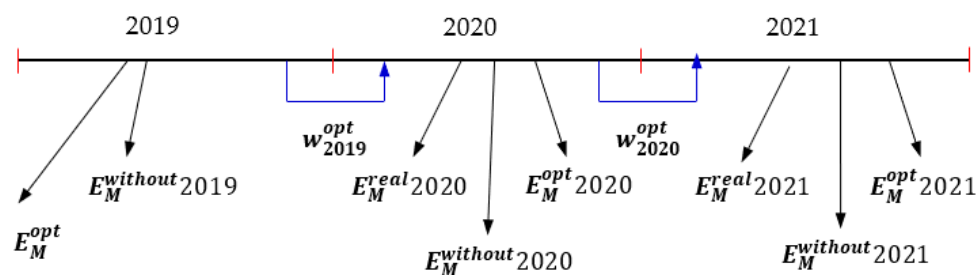
k	1	2	3	4	5	6	7	8	9	Mean
$E_M^{without}$	0.0854	0.1684	0.162	0.197	0.0583	0.0567	0.0616	0.0733	0.0775	0.1045
E_M^{real}	0.0854	0.1359	0.1451	0.1912	0.0446	0.0457	0.0494	0.0689	0.0753	0.0935
E_M^{opt}	0.0966	0.2031	0.1856	0.2515	0.0604	0.0575	0.0627	0.0746	0.0784	0.1189

7. Empirical Simulations and Results

The amount of the historical period influences the solutions obtained by the portfolio problem. Respectively, the evaluated weights $\mathbf{w}^{opt}(k)$ recommend the allocation of resources per disbursement for the mean values for the defined historical period. Here is the illustration of the results from the active management, applying different durations of the historical period.

7.1. Case 1: 2019–2021

The historical period is with duration of one year. The evaluated weights are applied for the next year as a management period. The graphical illustration of this active management is illustrated in Figure 8.

**Figure 8.** Evaluation of the different categories of income for a historical period of one year.

The numerical results are summarized in Table 6.

These results demonstrate that the application of active business management gives income $E_M^{real} \mathbf{w}^{opt}(k-1) \geq E_M^{without}$. The value of $E_M^{without}$ is calculated without the application of active management. The value of $E_M^{opt}(\mathbf{w}^{opt}(k))$ is the maximal potential income if actual weights $\mathbf{w}^{opt}(k)$ have been applied, which will increase the real income or $E_M^{real} \mathbf{w}^{opt}(k-1) \geq E_M^{opt}(\mathbf{w}^{opt}(k))$.

For illustration purposes, the evaluated weights per period k are given as follows. Only the nonzero values $w_i^{opt} > 0$ are denoted.

$$\mathbf{w}(k=2019): b_5 < 0; w_5^{opt} = 0.7031; b_{10} > 0; w_{10}^{opt} = 0.2966$$

These results recommend for the next year 2020 the mean disbursements for E_5 (credits) be decreased by 70% and increase the mean E_{10} (crops) by 29.66%.

$$\mathbf{w}(k=2020) : b_4 > 0 ; w_4^{opt} = 0.4396 ; b_{10} < 0 ; w_{10}^{opt} = 0.2423.$$

These results recommend for 2021 the mean disbursements for E_4 (insurance) increase by 43.96% and the mean disbursements for E_{10} (crops) decrease by 24.23%.

$$\mathbf{w}(k=2021) : b_4 > 0 ; w_4^{opt} = 0.8579 ; b_{10} < 0 ; w_{10}^{opt} = 0.1421.$$

These recommendations are for 2022 again to increase the mean E_4 (insurance) and to decrease the mean disbursements for E_{10} (crops) with the evaluated coefficients. Unfortunately, the real and optimal value of the income is not possible to evaluate in advance because the actual mean disbursements E_i , $i = 1, N$ for 2022 are not available.

7.2. Case 2: 2021

For this case, the historical period is shortened to 3 months. The evaluated weights are applied one month ahead of the management period. In that case, the sequence of evaluation and sliding the management period is graphically presented in Figure 5. The initial historical period is for months 1–3 in 2021. The evaluated portfolio weights $w^{opt}(k=1)$ are applied for month 4. The corresponding incomes $E_M^{without}$, E_M^{real} , and E_M^{opt} are evaluated with and without the application of the weights are evaluated and compared. Then the active management makes a move for the historical period from months 2 till 4. The newly evaluated weights $w^{opt}(k=2)$ are applied for month 5, and the corresponding values of the income are evaluated. For the 2021 year, such steps and sliding evaluations are $k = 12 \text{ months} - 3 \text{ months history} = 9 \text{ steps}$. The results for the 2021 year are given in Table 7.

The increase in the mean real income E_M^{real} towards the mean increase without the application of active management $E_M^{without}$ is

$$\Delta E_M^{real} = \frac{0.1045 - 0.0935}{0.0935} \equiv 11.76\%.$$

7.3. Case 3: 2019–2021

Here, the historical period is again 3 months. The application of the weights w^{opt} is for the next month, but the duration of the active management is for the entire period from 2019 to 2021. The behavior of the values of the three categories of incomes $E_M^{without}$, E_M^{real} , and E_M^{opt} are given in a graphical way in Figure 9.

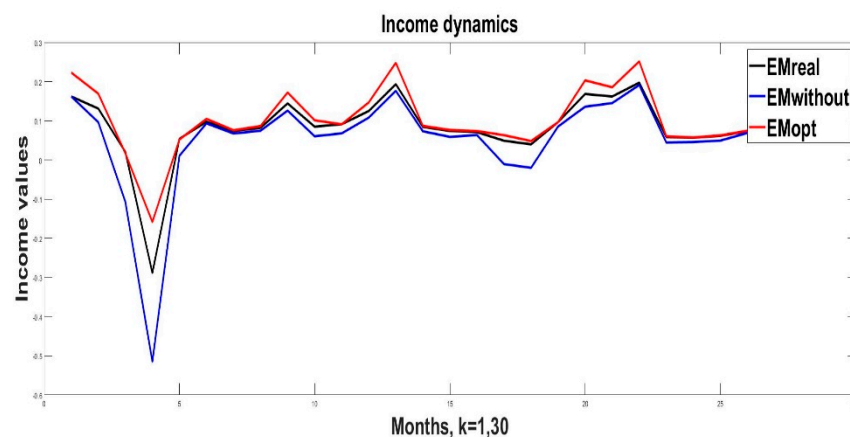


Figure 9. The behavior of the three categories of income $E_M^{without}$, E_M^{real} , and E_M^{opt} .

To simplify the description and comparison of the resulting evaluations, here only the resulting mean values of the three categories of income are presented for the entire 3-year period.

$$\text{mean } E_M^{\text{real}} (2019\text{--}2021) = 0.1053,$$

$$\text{mean } E_M^{\text{without}} (2019\text{--}2021) = 0.0935,$$

$$\text{mean } E_M^{\text{opt}} (2019\text{--}2021) = 0.1189.$$

It is seen again that the real mean income with application of the active management is bigger than the mean income without the application of the derived algorithm for recommending the levels of disbursements $E_M^{\text{real}} \geq E_M^{\text{without}}$, but the value of E_M^{real} a potential to increase till the level of E_M^{opt} . Even for this longer period of active management, the relations $E_M^{\text{without}} \leq E_M^{\text{real}} \leq E_M^{\text{opt}}$ is kept. The increase in income, due to the application of the derived algorithm for active management, is

$$\Delta E_M^{\text{real}} = \frac{0.1053 - 0.0935}{0.0935} \equiv 12.62\%.$$

8. Discussion

The derived policy for active business management is based on the algorithm, which performs sequential definition and solution of portfolio problems. The obtained solutions $\mathbf{w}^{\text{opt}}(k)$ are applied for the next future period $(k + 1)$. Because in the future time the mean values of the disbursements will be different, the real mean income E_M^{real} will differ, and it will be less than the potential optimal one E_M^{opt} . The solutions $\mathbf{w}^{\text{opt}}(k)$ recommend a decrease or increase in corresponding disbursements according to the current levels of mean disbursements. The production technology of the business entity can insist limits on the resources that are needed; such constraints can be added to complicate the portfolio problem. This can be implemented simply by adding low and upper bounds to the values of disbursement in the portfolio problem or more complex mutual equalities or inequalities, but the algorithm does not estimate the future levels of such disbursements. Nevertheless, of the unknown values of the means E_i , the recommendations $\mathbf{w}^{\text{opt}}(k)$ give an increase in the income when the means E_i are estimated at the end of the active future period $k + 1$. A potential exists by defining the portfolio problem with parameters, which are evaluated not only from the available data from the history of the disbursements but by the application of forecast approaches. Such approaches can be based on ARMA modeling (Auto-Regressive and Moving Average) [40], GARCH modeling (General Auto Regressive Conditional Heteroskedasticity) process [41], application of the Black–Litterman model for integration of historical data with expert subjective assessments about the future behavior of disbursements. Hence, a sequence of forecasting the levels of future means of disbursement can benefit the real obtained income E_M^{real} , which should be closer to the potential optimal value E_M^{opt} . It is expected that such additional improvements and modifications of the derived algorithm will make less the difference between the real income E_M^{real} and the potential maximal one E_M^{opt} for the used management period.

9. Conclusions

This research derives an algorithm that provides active management of the production policy of a business entity such as animal husbandry. The management is concerned with the appropriate redistribution of available resources between different types of disbursements. Thus, the income from production can increase without investing additional resources for production. The key component of this algorithm is the definition and solution of appropriate portfolio problems. Their solutions recommend the redistribution of resources between the different types of disbursements.

The numerical simulations illustrate that the application of the derived algorithm in the sequential application of the portfolio problem gives an increase in the husbandry

income in comparison with the lack of such active business management. The numerical simulation has been performed with real data on animal husbandry from the region of Stara Zagora of Bulgaria. This active business management allows being automated the needed calculations by appropriate software suit. It has been discussed that the algorithm can be complicated with forecast modeling of the levels of disbursements, which will increase additionally the real husbandry income.

Author Contributions: Conceptualization, T.S.; methodology, T.S.; software, K.S.; validation, K.S.; formal analysis, T.S. and K.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Ministry of education and science of Bulgaria under the National Science Program INTELLIGENT ANIMAL HUSBANDRY, grant agreement N D01-62/18.03.2021.

Data Availability Statement: Bulgarian Science Program INTELLIGENT ANIMAL HUSBANDRY, grant agreement N D01-62/18.03.2021.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Management Sciences. 2022. Available online: <https://www.bing.com/search?q=management+sciences&cvid=99dcb5fc5e864436a573840efab06843&aqs=edge..69i57j0l8.4909j0j1&pqlt=41&FORM=ANNTA1&PC=U531> (accessed on 17 October 2022).
2. Taylor, F.W. *The Principles of Scientific Management*; Harper & Brothers: New York, NY, USA; London, UK, 1911.
3. Vom Brocke, J.; Rosemann, M. Business Process Management. In *Wiley Encyclopedia of Management*; Wiley: Hoboken, NJ, USA, 2015.
4. Rosemann, M.; vom Brocke, J. The Six Core Elements of Business Process Management. In *Handbook on Business Process Management*; Springer: Cham, Switzerland, 2010.
5. Fundamentals of Business Management. Available online: <https://blog.edx.org/6-fundamentals-of-business-management#:~:text=Fundamentals%20of%20Business%20Management%3A%20Explore%206%20Functional%20Areas,5%20Data%20Analytics%20...%206%20Marketing%20> (accessed on 17 October 2022).
6. Business Functions. Available online: <https://www.businessmanagementideas.com/business/functions-business/business-functions-7-main-types-of-business-function/3744> (accessed on 17 October 2022).
7. Ngugi, D. Quantitative Methods for Business Management. 2018, p. 420. Available online: https://www.researchgate.net/publication/329031752_Quantitative_Methods_For_Business_Management (accessed on 17 October 2022).
8. Fred, S.; Nassuna, A.; Byarugaba, P.; Arinaitwe, A. Quantitative Methods for Managerial Decision Making. 2020. Available online: https://www.researchgate.net/publication/340754384_quantitative_methods_for_managerial_decision_making (accessed on 17 October 2022).
9. Alshater, M.M.; Saba, I.; Supriani, I.; Rabbani, M.R. Fintech in islamic finance literature: A review. *Heliyon* **2022**, *8*, e10385. [CrossRef] [PubMed]
10. Soriano, R.R.; Budi, I.; Purwandari, B. Challenges and Trends of Financial Technology (Fintech): A Systematic Literature Review. *Information* **2020**, *11*, 590. [CrossRef]
11. Sarhan, H. *Fintech: An Overview*; ResearchGate: Berlin, Germany, 2020; pp. 1–34. [CrossRef]
12. Khotinskay, G.I. Fin Tech: Fundamental Theory and Empirical Features. In Proceedings of the Social and Behavioural Sciences GCPMED 2018 International Scientific Conference, 2019; Conference: GCPMED 2018—International Scientific Conference “Global Challenges and Prospects of the Modern Economic Development, Samara, Russia, 6–8 December 2018; pp. 222–229. Available online: https://www.researchgate.net/publication/331905682_Fin_Tech_Fundamental_Theory_And_Empirical_Features (accessed on 17 October 2022).
13. Varga, D. Fintech, the new era of financial services. *Vez. Bp. Manag. Rev.* **2017**, *48*, 22–32. Available online: http://unipub.lib.uni-corvinus.hu/3170/1/VT_2017n11p22.pdf (accessed on 17 October 2022). [CrossRef]
14. Kim, Y.; Choi, J.; Park, Y.; Yeon, J. The Adoption of Mobile Payment Services for “Fintech”. *Int. J. Appl. Eng. Res.* **2016**, *11*, 1058–1061.
15. Yong, C.; Tham, J.; Azam, S.M.F.; Khatibi, A. Empirical study on acceptance of mobile payment and its competitive advantage in Malaysia. *Eur. J. Soc. Sci. Stud.* **2021**, *7*. [CrossRef]
16. Markowitz, H. Portfolio selection. *J. Financ.* **1952**, *7*, 77–91.
17. Sharpe, W. *Portfolio Theory and Capital Market*; McGraw Hill: London, UK, 2000; 316p.
18. Black, F.; Litterman, R. Asset Allocation: Combining investor views with market equilibrium. *J. Fixed Income* **1991**, *1*, 7–18. [CrossRef]
19. Satchell, S.; Scowcroft, A. A demystification of the Black–Litterman model: Managing quantitative and traditional portfolio construction. *J. Asset Manag.* **2000**, *1*, 138–150. [CrossRef]
20. Meucci, A. The Black–Litterman approach: Original model and extensions. In *The Encyclopedia of Quantitative Finance*; Wiley: New York, NY, USA, 2010.

21. Chen, J.M. On Exactitude in Financial Regulation: Value-at-Risk, Expected Shortfall, and Expectiles. *Risks* **2018**, *6*, 61. [CrossRef]
22. Leung, M.; Wang, J.; Che, H. Cardinality-constrained portfolio selection via two-timescale duplex neurodynamic optimization. *J. Neural Netw.* **2022**, *153*, 399–410. [CrossRef]
23. Dobrowolski, Z.; Drozdowski, G.; Panait, M.; Babczuk, A. Can the Economic Value Added Be Used as the Universal Financial Metric? *Sustainability* **2022**, *14*, 2967. [CrossRef]
24. Sukharev, O.S. Portfolio Theory in Solving the Problem Structural Choice. *J. Risk Financ. Manag.* **2020**, *13*, 195. [CrossRef]
25. Kato, Y. AI/Fintech and Asset Management Businesses. Policy Research Institute, Ministry of Finance, Japan. *Public Policy Rev.* **2020**, *16*, 1–28.
26. Fagefors, C.; Lantz, B. Application of Portfolio Theory to Healthcare Capacity Management. *Int. J. Environ. Res. Public Health* **2021**, *18*, 659. [CrossRef]
27. Lao, B.; Liu, N. Application of Portfolio Theory in Controlling Supply Chain Crisis. In *Integration and Innovation Orient to E-Society 1*; Wang, W., Ed.; IFIP International Federation for Information Processing, 251; Springer: Boston, MA, USA, 2007; pp. 9–16. Available online: https://link.springer.com/content/pdf/10.1007/978-0-387-75466-6_2.pdf (accessed on 17 October 2022).
28. Cui, Y.; Cheng, C. Modern Portfolio Theory and Application in Australia. *J. Econ. Bus. Manag.* **2022**, *10*, 128–132. Available online: <http://joebm.com/vol10/686-EM4270.pdf> (accessed on 17 October 2022). [CrossRef]
29. Barkley, A.; Peterson, H.H. Wheat Variety Selection: An Application of Portfolio Theory to Improve Returns. In Proceedings of the NCCC-134 Conference on Applied Commodity Price Analysis, Forecasting, and Market Risk Management, St. Louis, MO, USA, 21–22 April 2008.
30. Hines, E. Application of Portfolio Theory to Commercial Real Estate. Johns Hopkins University, 2009. Available online: https://jscholarship.library.jhu.edu/bitstream/handle/1774.2/35837/Hines_Application%20of%20Portfolio%20Theory%20to%20Commercial%20Real%20Estate_2009_Gouline.pdf (accessed on 17 October 2022).
31. Paut, R.; Sabatier, R.; Tchamitchian, M. Reducing risk through crop diversification: An application of portfolio theory to diversified horticultural systems. *Agric. Syst.* **2019**, *168*, 123–130. [CrossRef]
32. Pulido-Rojano, A.; Pizarro-Rada, A.; Padilla-Polanco, M.; Sánchez-Jiménez, M.; De-la-Rosa, L.L. An optimization approach for inventory costs in probabilistic inventory models: A case study. *Ing. Rev. Chil. Ing.* **2020**, *28*, 383–395. [CrossRef]
33. Mondal, R.; Shaikh, A.A.; Bhunia, A.K.; Hezam, I.M.; Chakraborty, R.K. Impact of Trapezoidal Demand and Deteriorating Preventing Technology in an Inventory Model in Interval Uncertainty under Backlogging Situation. *Mathematics* **2022**, *10*, 78. [CrossRef]
34. Chaiboonsri, C.; Wannapan, S. Multi-Process Analysis and Portfolio Optimization Based on Quantum Mechanics (QM) Under Risk Management in ASEAN Exchanges: A Case Study of Answering to the E-Commerce and E-Business Direction. In *Handbook of Research on Innovation and Development of E-Commerce and E-Business in ASEAN*; Almunawar, M.N., Ali, M.A., Lim, S.A., Eds.; IGI Global: Hershey, PA, USA, 2021; pp. 46–61. [CrossRef]
35. Liu, X.; Luo, G.; Xu, X. Optimal Purchasing Decisions with Supplier Default in Portfolio Procurement. *Mathematics* **2022**, *10*, 3155. [CrossRef]
36. Tang, M.-L.; Wu, T.-P.; Hung, M.-C. Optimal Pension Fund Management with Foreign Investment in a Stochastic Environment. *Mathematics* **2022**, *10*, 2468. [CrossRef]
37. Standard Normal Distribution Showing Standard Deviations. Available online: <https://www.statisticshowto.com/wp-content/uploads/2013/09/standard-normal-distribution.jpg> (accessed on 17 October 2022).
38. Kolm, P.N.; Tutuncu, R.; Fabozzi, F.J. 60 Years of portfolio optimization: Practical challenges and current trends. *Eur. J. Oper. Res.* **2014**, *234*, 356–371. [CrossRef]
39. Kantar, Y.M. Generalized least squares and weighted least squares estimation methods for distributional parameters. *REVSTAT Stat. J.* **2015**, *13*, 263–282.
40. Liu, C.; Shi, H.; Wu, L.; Guo, M. The Short-Term and Long-Term Trade-off between Risk and Return: Chaos vs Rationality. *J. Bus. Econ. Manag.* **2020**, *21*, 23–43. [CrossRef]
41. Engle, R.F. Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *J. Econom.* **1982**, *50*, 987–1007. [CrossRef]