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Double Auction Offloading for Energy and Cost Efficient Wireless Networks

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Abstract: Network infrastructure sharing and mobile traffic offloading are promising technologies for Heterogeneous Networks (HetNets) to provide energy and cost effective services. In order to decrease the energy requirements and the capital and operational expenditures, Mobile Network Operators (MNOs) and third parties cooperate dynamically with changing roles leading to a novel market model, where innovative challenges are introduced. In this paper, a novel resource sharing and offloading algorithm is introduced based on a double auction mechanism where MNOs and third parties buy and sell capacity and roam their traffic among each other. For low traffic periods, Base Stations (BSs) and Small Cells (SCs) can even be switched off in order to gain even more in energy and cost. Due to the complexity of the scenario, we adopt the multi-objective optimization theory to capture the conflicting interests of the participating entities and we design an iterative double auction algorithm that ensures the efficient operation of the market. Additionally, the selection of the appropriate time periods to apply the proposed algorithm is of great importance. Thus, we propose a machine learning technique for traffic load prediction and for the selection of the most effective time periods to offload traffic and switch off the Base Stations. Analytical and experimental results are presented to assess the performance of the algorithm.

Keywords: auction; multi-objective optimization; offloading; heterogeneous networks; infrastructure sharing

MSC: 37M10

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1. Introduction

1.1. Motivation and Related Work

During the past decades, a tremendous growth occurred in mobile traffic forces Mobile Network Operators (MNOs) to extend their infrastructure by installing Base Stations (BSs) and, in an effort to support the emerging user demands, they lease resources from third parties, who deploy low-powered Small Cells (SCs) [1]. The network densification incurs huge capital and operational expenditures. The involvement of various entities (MNOs, third parties) of corporate nature with conflicting financial goals generates a new ecosystem, in which, offloading and network sharing seem appealing solutions.

Numerous works in the literature focused on algorithms for greening the wireless network through the BSs switching off [2–5]. However, it is difficult to maintain proper user experience and ubiquitous connectivity when a number of macro BSs are switched off; thus, offloading is investigated in [6–10]. Even though offloading boosts bandwidth, energy is still wasted. To address this issue, MNOs and third parties are motivated to share their resources and appropriate deactivation policies in HetNets have been thoroughly investigated in the literature [11–13]. The authors in [11] examine an algorithm where multiple levels of switching off exist and observe the tradeoff between energy consumption and throughput. Distance is considered as the critical parameter to deactivate the cells in a

HetNet in [12] and a more dynamic load aware algorithm is examined in [13]. However, these works do not consider cooperation between multiple operators, service providers and third parties. The switching off approach considering numerous operators is investigated in [14]. However, the cost and the incentives to motivate the operators to cooperate are not considered.

Optimization techniques and machine learning were also employed for the deactivation of cells in 5G networks and HetNets. Interesting approaches employing several sleep modes lead to great reduction in energy consumption [15–17], as well as a mathematical optimization switching off strategy that still satisfies Quality of Service (QoS) requirements [18]. With the use of evolutionary algorithms the effective operation and energy efficiency is achieved in [19,20]. Moreover, other approaches use machine learning to implement deactivation policies with good results [21–23]. These deactivation techniques, again, do not exploit the possible cooperation among the participating parties and rely on the exchange of a lot of information. However, MNOs and third parties are not willing to share their traffic information. Thus, auction schemes were proposed in the state-of-the-art.

The authors in [24] propose a reverse auction-based offloading scheme for a resource allocation problem. A double auction for mobile offloading that satisfies economic properties is investigated in [25]. The objective of the reverse auction in [26] maximizes the third party's income and a greedy algorithm is proposed to approach the optimal solution. The authors assume that through offloading, some BSs can be switched off; however, this solution can be applied to particular network and traffic scenarios and cannot be generalized in more complicated configurations. In [27], a reverse auction is applied and its goal is to minimize the total network cost; however, energy efficiency is neglected. In [28,29], the energy consumption parameter is considered in the objective function of reverse auctions. The offloading problem is formulated as a reverse auction that maximizes the revenue of the operator in [30]. Even though the experimental results of these works are promising, there are still open issues and challenges that can be explored, such as the employment of double auction.

1.2. Contributions

In this paper, we consider the realistic scenario of a market with asymmetric information, i.e., where the independent parties are not aware of the actual needs of their competitors. For example, operators with large numbers of subscribers often need more bandwidth during peak hours, whereas MNOs with few subscribers may have unused bandwidth and can either lease their spare resources or deactivate their BSs. In addition, the third party competitors are willing to participate in the market depending on the offers and their capabilities that may vary during time or depending on their network size. Therefore, we must employ an incentive-compatible mechanism that induces the buyers (MNOs) and the sellers (MNOs and third party) to truthfully reveal their needs. With this information, the idea is to maximize the efficiency of the market by properly matching the buyers' and the sellers' requests. Double auction is an appropriate tool for our market [31], since through the use of double auction a competitive and fair equilibrium can be found in an heterogeneous market and with low overhead (MNOs and third party do not need to reveal their market preferences). We design a capacity sharing, which considers the interests of the involved parties (MNOs and third party), the network configuration (BSs and SCs) and the time-varying traffic characteristics (high and low traffic conditions). We introduce an offloading mechanism, where MNOs can act either as buyers or sellers and the SCs act as sellers. We investigate the energy and cost saving potentials by allowing the involved parties to trade capacity for various traffic conditions. For low traffic, SC and BS deactivation may be allowed to attain even higher gains [28]. In the proposed scheme, multiple buyers bid for bandwidth and multiple sellers determine the prices for the desired demand through an iterative algorithm that gradually reaches the socially efficient solution, without any prior knowledge of the market.

The main technical contributions are as follows:

1. We present a general market model where MNOs compete to lease capacity from multiple BSs and SCs for offloading. Each MNO concurrently acts as buyer or seller and the third party as seller.
2. We apply an iterative double auction that is efficient (maximizes the social welfare), individually rational (MNOs and third party participate), and incentive-compatible (MNOs truthfully reveal their needs/ demands).
3. We validate the problem theoretically and we assess the effectiveness of the proposed algorithm to obtain useful insights for modern networks. The simulation results indicate the potential energy and financial gains in the network and give the necessary incentives to the MNOs to decide to enter in a resource allocation negotiation with the third party and the competing operators.

The remainder of this paper is organized as follows. The system and energy models are described in Section 2. In Section 3, we introduce the energy and cost efficient auction-based optimization approach. Section 4 includes the iterative double auction algorithm. The performance evaluation is provided in Sections 5 and 6 concludes this paper.

2. System Model and Operation

2.1. Network Configuration

We consider a scenario with a set of MNOs whose serving areas are overlapping in the same geographical area. Each operator is denoted by MNO_n, where $n \in \mathcal{N} = \{1, 2, \ldots, N\}$. In addition, we assume that each of the N operators provides coverage in $\mathcal{K}$ macro cells, hence there is a set of $\mathcal{K}_n$ BSs, denoted by $BS_{n,k}$, where $n \in \mathcal{N}$ and $k \in \mathcal{K} \subseteq \mathcal{N}$ characterize the operator and the corresponding cell. The set of all BSs is denoted with $\mathcal{K} = \bigcup_{n \in \mathcal{N}} \mathcal{K}_n$. Each BS serves one cell, hence the terms BS and cell are used interchangeably. We consider that each BS of one operator is similarly loaded and hence, the central cell could represent the network of the MNO. In this case, the BSs can be simply represented as BS_n.

Along with the traditional macro network, we consider one tier of SCs in the same geographical area. Each SC can be a Wi-Fi node or a femtocell that operates in a separate channel, and hence does not interfere with the macro cells [32]. Each SC is represented as SC_m, where $m \in \mathcal{M} = \{1, \ldots, M\}$. We assume that the M SCs are randomly distributed in the area, where hotspot traffic is observed. The SCs are deployed by third parties. The network configuration adopted in this work is illustrated in Figure 1.

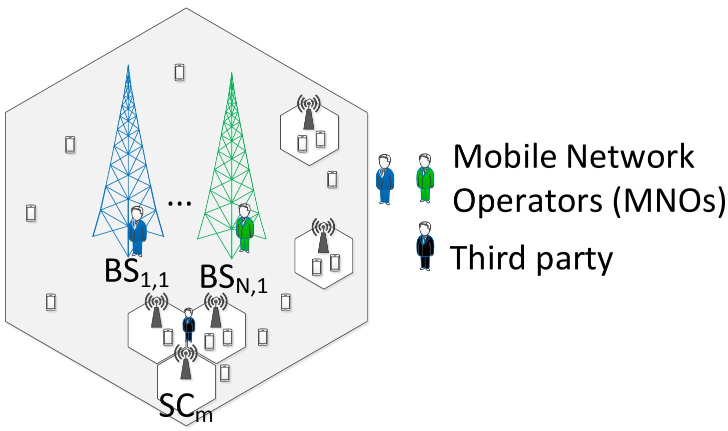


Figure 1. Network with a macro cell served by N MNOs and M SCs.

2.2. Traffic Load Model

We adopt a traffic pattern, where the peak hours are observed in the morning and during early afternoon, while during night hours the traffic is significantly lower. Our idea is to exploit the traffic fluctuations to provide energy saving solutions based on the different users' requirements throughout the day. For the sake of generality, we assume that the traffic volumes of different MNOs may be different, although they follow the same pattern.

The users are randomly distributed within the coverage cell area and during the peak hours there is hotspot traffic in some areas. The users are also covered by one or more SCs. In our study, it is assumed that the time is slotted, and our proposal focuses on one time period. For that time, the user location and traffic are considered fixed, but may change over time.

2.3. Energy Consumption Model

The energy consumption of a HetNet, with traffic L (measured in Mbps), is considered as the sum of the consumed energy of different cells, i.e., BSs and SCs, and is given below:

$$E(L) = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} E_{BS_{n,k}}(L_{BS_{n,k}}) + \sum_{m \in \mathcal{M}} E_{SC_m}(L_{SC_m}), \tag{1}$$

where $E_{BS_{n,k}}(L_{BS_{n,k}})$ and $E_{SC_m}(L_{SC_m})$ are the energy consumption of a BS and an SC, when serving traffic equal to $L_{BS_{n,k}}$ and L_{SC_m}, respectively, with $L = \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} L_{BS_{n,k}} + \sum_{m \in \mathcal{M}} L_{SC_m}$.

To calculate the average energy consumption of the BSs, we consider, first, the constant power consumption, P_{cnst}, which is independent of the traffic and is consumed for operations such as cooling, antenna feeding, etc. Second, when traffic is served, wireless transmission power, denoted by P_{tx}, is consumed and, last, the power during the idle state is denoted by P_{idle}. The total energy consumption for serving the offered load, L, of a BS is given by:

$$E_{BS_{n,k}}(L) = \begin{cases} P_{sleep}, & \text{sleep mode.} \\ P_{cnst} + \sum_{u \in \mathcal{U}} \lambda_u \cdot S_u \cdot \frac{1}{\mu_u} \cdot P_{tx} & \\ + \left(1 - \sum_{u \in \mathcal{U}} \lambda_u \cdot S_u \cdot \frac{1}{\mu_u}\right) \cdot P_{idle}, & \text{otherwise.} \end{cases} \tag{2}$$

where S_u, λ_u, μ_u are the average packet size, the packet arrival rate and service packet rate of a user u. P_{sleep} is the sleep mode power consumption that is typically smaller than $P_{cnst} + P_{tx}$. The energy consumption model for the SCs is equivalent to the one of the BS, so it is not presented here in detail.

3. Energy Efficient Auction-Based Mechanism

3.1. The Energy Market

We model an energy market for trading the network capacity among the MNOs-owned macro sites and the third party-owned SCs. The MNOs and the third party compete to lease capacity for offloading their traffic. For this reason, a centralized auctioneer is employed to realize the procedure. The MNOs and the third party participate in the decision process, each with a separate role. The MNOs submit both bids, i.e., offers to lease the SCs resources and opportunistically offload their traffic, and asks, i.e., offers to sell their capacity to other MNOs simultaneously, with the objective to maximize their profit. The MNOs participate in the auction repeatedly, but in any single round an MNO is allowed to either buy or sell. On the other hand, the SCs sell their capacity to the MNOs. The challenge is to design a market mechanism tailored to the offloading and deactivation problem that, at the same time, satisfies the desirable economic properties. We assume that the auctioneer is not aware of the actual needs of the MNOs and the third party and, thus, the market has asymmetric information. In addition, the values of bids and asks are private to their sources and a mechanism that induces the participants to truthfully reveal their needs is needed. Having this information, the bids and asks are matched by the auction mechanism to minimize energy consumption and reduce the cost of the participants. At the end of the process, the winning buyers and sellers, the specific trade prices and agreed quantities are determined. The auctioneer does not have the incentive to deviate from the overall goal imposed by the system designer (e.g., social welfare). Taking into account the aforementioned observations, a double auction strategy is the most suitable for our case.

3.2. The Auction Scheme

The proposed auction-based scheme involves four phases, whose process and respective decision makers (in parentheses) are presented below (Figure 2):

1. In the *bidding phase* (MNOs, third party), the MNOs place their bids and ask for the requested resources to the auctioneer. The third party places its asks, at the same time. Each bid and ask includes the information of the requested capacities and the corresponding prices.
2. In the *allocation phase* (auctioneer), the auctioneer collects the bids and asks. The determination of the winning bids and asks is conducted through the solution of the resource allocation problem that minimizes the energy consumption and maximizes the income of the involved parties. The auctioneer ends up with non-negative payments.
3. In the *pricing phase* (auctioneer), the auctioneer decides each winner's payment price, based on the resource allocation of the previous step.
4. In the *offloading phase* (MNOs, third party), the MNOs and the third party apply the decision of the resource allocation problem and offload or accept the traffic. If BSs or SCs end up with no traffic, they can be switched off.

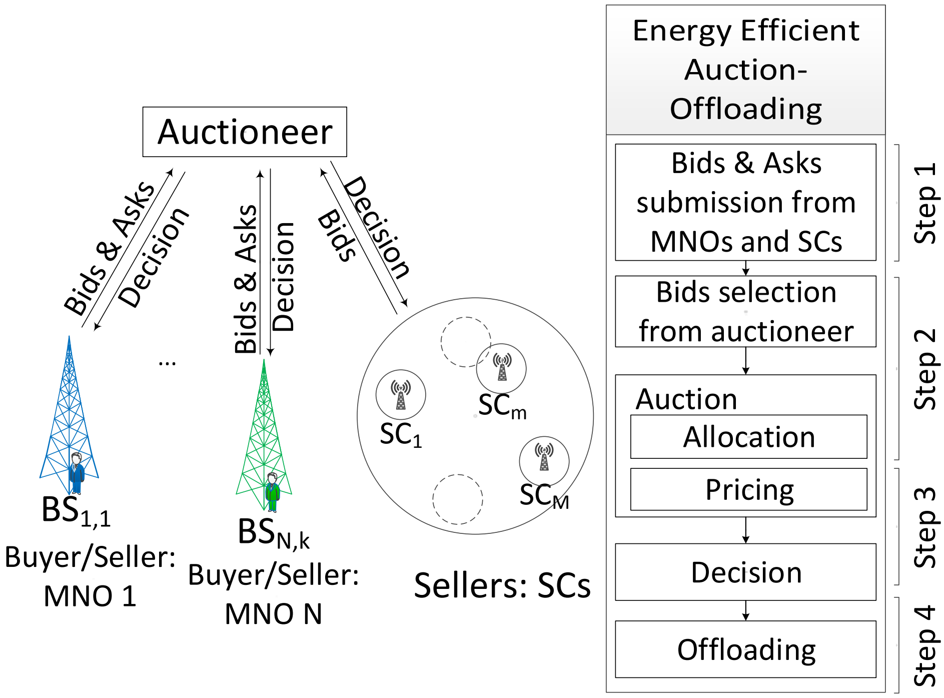


Figure 2. Auction illustration and proposed algorithm flowchart.

3.3. The Bidding Phase

3.3.1. Bids

Each operator $n \in \mathcal{N}$ is willing to offload $x_{n,m} \geq 0$ Mbps of data through the SC_m (where $m \in \mathcal{M} = \{1, \ldots, M\}$) or the BS_m (where $m \in \mathcal{N} = \{1, \ldots, N\}$ and $n \neq m$). We define the offloading request vector for BS_n to all SCs and BSs as $x_n = (x_{n,m}, \forall m \in \mathcal{M}, \forall m \in \mathcal{N}/n)$ and the total offloaded data of BS_n is $X_n = \sum\limits_{m \in \mathcal{M}, m \in \mathcal{N}/n} x_{n,m}$.

Each bid corresponds to the unit of the offloaded traffic. Thus, the MNOs determine how much energy can be saved by offloading this unit of the traffic. By offloading traffic, an operator saves a part of the dynamic energy, required to support that traffic. On one hand, the potential of energy saving decreases with the increase in offloaded data. If the MNO succeeds in completely offloading its whole traffic, it saves even the constant energy by completely shutting down the BS or by going to deep sleep mode. As a result, the MNOs need to consider their current traffic to decide their bidding strategy.

The MNOs are expected to submit offers for multi-unit resources based on the utility they receive when they offload their traffic. The offloading benefit in terms of the energy consumption can be calculated as:

$$J_n(x_{n.m}) = E_{BS_n}(L_{max}^{BS}) - E_{BS_n}(L_{max}^{BS} - x_{n.m}) - E_{tr}, \tag{3}$$

where $E_{BS_n}(L_{max}^{BS})$ is the energy consumption when the BS serves its whole traffic, $E_{BS_n}(L_{max}^{BS} - x_{n.m})$ corresponds to the energy consumption when the BS offloads $x_{n,m} \geq 0$ bytes of data and finally, E_{tr} is the energy consumption to offload the corresponding traffic (The k for the variable $E_{BS_{n,k}}(L)$ can be dropped, since as we already mentioned, the central cell could represent the network of the MNO.). The utility can be valuated by a price $b_{n,m}$, based on the electricity cost of the energy gain. This bidding price is submitted to the auctioneer.

3.3.2. Asks

The MNOs generate the asks to sell capacity to other MNOs. At the same time, the third party also generates asks to offer resources to operators. Similar to bids calculation, to generate the asks, the excess energy cost suffered by conceding the load from other MNOs is calculated.

First, we assume that each MNO is willing to offer its resources and serve $y_{n,p} \geq 0$ bytes of traffic from other operators denoted with $p \in \mathcal{N} \setminus \{n\}$. The MNO needs to calculate how much additional energy will be consumed by receiving traffic from other MNOs. Note that the energy required to support the extra traffic depends on the current traffic load. The extra cost is represented as:

$$V_n(y_{n,p}) = E_{BS_n}(L_{max}^{BS} + y_{n,p}) - E_{BS_n}(L_{max}^{BS}) + E_{tr}, \tag{4}$$

where $E_{BS_n}(L_{max}^{BS} + y_{n,p})$ corresponds to the energy consumption when the BS receives the $y_{n,p} \geq 0$ bytes of traffic from another operator. The cost can be evaluated by a price $a_{n,p}$, based on the electricity cost of the consumed energy.

Second, from the third party's view, the asks correspond to the resources that can be offered in order for the third party to increase its income. The third party calculates how much additional energy will be consumed by receiving traffic from the MNOs. Thus, the ask for receiving $y_{m,n}$ units of traffic is calculated by:

$$V(y_{m,n}) = E_{SC}(y_{m,n}) + E_{tr}, \tag{5}$$

where $E_{SC}(y_{m,n})$ corresponds to the energy consumption when the m-th SC receives the $y_{m,n}$ units of traffic from the n-th operator. The corresponding ask is denoted with $\alpha_{m,n}$.

3.4. The Allocation Phase

Clearly, the objectives of the operators and the third party are conflicting. If they decide independently how much traffic to offload or admit, it is very unlikely that they can reach an agreement. Therefore, a market controller (an auctioneer) ensures that the market operates efficiently. The double auction mechanism that we employ satisfies the economic criteria of all the participants. Clearly, the buyers and the sellers try to maximize their own profits. The auctioneer aims to find a balancing point between the conflicting desires of the participating parties. Towards this direction, the maximization of the market efficiency is the key to properly match the buyers and the sellers. The offloading and deactivation decisions need to be coupled depending on the total network traffic, when at the same time the different costs of MNOs and third party SCs are taken into account. Consequently, these specific features of the market can be solved with the use of a social welfare maximization problem. Additionally, it is noted that the social welfare optimization is a general paradigm for achieving fairness in systems and networks where numerous participants are involved. Through social welfare optimization, the individual preferences are mapped and the decision represents the overall optimum.

In the beginning of the allocation phase, the auctioneer undertakes the task of determining the offloading request matrix:

$$x_n = (x_{n,m}, \forall m \in \mathcal{M}), \tag{6}$$

and the admitted traffic matrices:

$$y_n = \big(y_{n,p}, \forall p \in \mathcal{N} \setminus \{n\}\big), \tag{7}$$

$$y = (y_{m,n}, \forall m \in \mathcal{M}, \forall n \in \mathcal{N}), \tag{8}$$

that ensure the efficient market operation. This is achieved when the difference between the total benefit for the MNOs and their aggregated cost and of the third party is maximized.

The auctioneer defines the matrices for the bids and asks:

$$b_n = (x_{n,m}, \forall m \in \mathcal{M}), \tag{9}$$

$$a_n = \big(y_{n,p}, \forall p \in \mathcal{N} \setminus \{n\}\big), \tag{10}$$

$$a = (y_{m,n}, \forall m \in \mathcal{M}, \forall n \in \mathcal{N}). \tag{11}$$

To find the optimal bids, the MNOs solve the following maximization problem, where the goal is to maximize their individual offloading benefit:

$$\textbf{BIDS (Operator):} \quad \max_{\mathbf{b_n}} J(x_n) \tag{12}$$

$$\text{s.t.} \quad b_{n,m} \geq 0, \forall m \in \mathcal{M}. \tag{13}$$

For the asks, the MNOs tend to minimize the cost of admitting traffic and solve the following minimization problem:

$$\textbf{ASKS (Operator):} \quad \min_{\mathbf{a_n}} V(y_n) \tag{14}$$

$$\text{s.t.} \quad a_{n,p} \geq 0, \forall p \in \mathcal{N} \setminus \{n\}. \tag{15}$$

Equivalently, the third party solves the minimization problem below to find the optimal asks:

$$\textbf{ASKS (Third party):} \quad \min_{\mathbf{a}} V(y) \tag{16}$$

$$\text{s.t. } a_{m,n} \geq 0, \forall m \in \mathcal{M}, \forall n \in \mathcal{N}. \tag{17}$$

Having received the bids and asks, the auctioneer finds the optimal $\mathbf{x_n}$, $\mathbf{y_n}$ and $\mathbf{y}$ by solving the social welfare maximization problem:

$$\textbf{P1:} \max_{\mathbf{x_n}, \mathbf{y_n}, \mathbf{y}} \quad \sum_{n \in \mathcal{N}} \sum_{m \in \mathcal{M}} x_{n,m} \tag{18}$$

$$- \left(\sum_{n \in \mathcal{N}} \sum_{p \in \mathcal{N} \setminus \{n\}} y_{n,p} + \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} y_{m,n} \right)$$

$$\text{s.t.} \sum_{n \in \mathcal{N}} y_{m,n} \leq C_{SC}, \forall m \in \mathcal{M} \tag{19}$$

$$\sum_{p \in \mathcal{N} \setminus \{n\}} y_{n,p} + L_{max}^{BS} \leq C_{BS}, \forall n \in \mathcal{N}, \tag{20}$$

$$y_{n,p} + y_{m,n} \leq x_{n,m}, \forall n \in \mathcal{N}, \forall n \in \mathcal{N}, \tag{21}$$
$$\forall p \in \mathcal{N} \setminus \{n\}, \forall m \in \mathcal{M},$$

$$x_{n,m} + y_{n,p} \in \{x_{n,m}, y_{n,p}\}, \forall n \in \mathcal{N}, \tag{22}$$
$$\forall p \in \mathcal{N} \setminus \{n\}, \forall m \in \mathcal{M},$$

$$y_{n,p} \geq 0, y_{m,n} \geq 0, x_{n,m} \geq 0, \forall n \in \mathcal{N}, \tag{23}$$
$$\forall p \in \mathcal{N} \setminus \{n\}, \forall m \in \mathcal{M}.$$

The objective function (18) aims at maximizing the difference between the offloading benefit and the cost of the admitted traffic and is strictly concave. Constraints (19) and (20) ensure that the total number of allocated resources does not exceed their availability for the SCs and BSs, respectively. Constraint (21) indicates that the amount of offloaded data that the MNOs and the third party decide to admit should satisfy the respective amount requested by the operators. Constraint (22) ensures that an operator ends up only as buyer or seller. Finally, constraint (23) ensures that the offloaded and admitted data are non-negative. The constraint set is not convex, since the constraint (22) is not linear. By using the Big-M notation, the problem is transformed as follows:

$$\mathbf{P2:}\ \max_{\mathbf{x_n},\mathbf{y_n},\mathbf{y}} \quad \sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}} x_{n,m} \tag{24}$$
$$-\left(\sum_{n\in\mathcal{N}}\sum_{p\in\mathcal{N}\setminus\{n\}} y_{n,p} + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} y_{m,n}\right)$$
$$\text{s.t. } \sum_{n\in\mathcal{N}} y_{m,n} \leq C_{SC}, \forall m \in \mathcal{M} \tag{25}$$
$$\sum_{p\in\mathcal{N}\setminus\{n\}} y_{n,p} + L_{max}^{BS} \leq C_{BS}, \forall n \in \mathcal{N}, \tag{26}$$
$$y_{n,p} + y_{m,n} \leq x_{n,m}, \forall n \in \mathcal{N}, \forall n \in \mathcal{N}, \forall p \in \mathcal{N}\setminus\{n\}, \forall m \in \mathcal{M}, \tag{27}$$
$$x_{n,m} \leq C_{SC}\cdot c, \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \tag{28}$$
$$y_{n,p} \leq L_{max}^{BS}\cdot d, \forall n \in \mathcal{N}, \forall p \in \mathcal{N}\setminus\{n\}, \tag{29}$$
$$c + d \leq 1, \tag{30}$$
$$y_{n,p} \geq 0, y_{m,n} \geq 0, x_{n,m} \geq 0, \forall n \in \mathcal{N}, \forall p \in \mathcal{N}\setminus\{n\}, \forall m \in \mathcal{M}. \tag{31}$$

Given the maximization problem **P2**, it is infeasible for the auctioneer to derive the optimal solution, due to limited information; therefore, the double auction mechanism is used to elicit the hidden information. The bidders are induced to truthfully reveal their valuations (bids and asks) and the duty of the auctioneer is to find the optimal solution (social welfare maximization). Thus, problem **P2** is transformed as follows:

$$\mathbf{P3:}\ \max_{\mathbf{x_n},\mathbf{y_n},\mathbf{y}} \quad \sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{M}} b_{n,m}\cdot \log x_{n,m} \tag{32}$$
$$-\left(\sum_{n\in\mathcal{N}}\sum_{p\in\mathcal{N}\setminus\{n\}} \frac{\alpha_{n,p}}{2}\cdot y_{n,p}^2 + \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}} \frac{\alpha_{m,n}}{2}\cdot y_{m,n}^2\right)$$
$$\text{s.t. } \sum_{n\in\mathcal{N}} y_{m,n} \leq C_{SC}, \forall m \in \mathcal{M} \tag{33}$$
$$\sum_{p\in\mathcal{N}\setminus\{n\}} y_{n,p} + L_{max}^{BS} \leq C_{BS}, \forall n \in \mathcal{N}, \tag{34}$$
$$y_{n,p} + y_{m,n} \leq x_{n,m}, \forall n \in \mathcal{N}, \forall n \in \mathcal{N}, \forall p \in \mathcal{N}\setminus\{n\}, \forall m \in \mathcal{M}, \tag{35}$$
$$x_{n,m} \leq C_{SC}\cdot c, \forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \tag{36}$$
$$y_{n,p} \leq L_{max}^{BS}\cdot d, \forall n \in \mathcal{N}, \forall p \in \mathcal{N}\setminus\{n\}, \tag{37}$$
$$c + d \leq 1, \tag{38}$$
$$y_{n,p} \geq 0, y_{m,n} \geq 0, x_{n,m} \geq 0, \forall n \in \mathcal{N}, \forall p \in \mathcal{N}\setminus\{n\}, \forall m \in \mathcal{M}. \tag{39}$$

The objective function (32) incorporates the bids and asks for the offloaded and admitted traffic to achieve the optimal solution and is motivated by the allocation rule in [33]. Since the objective function (32) is strictly concave and the constraints (33)–(39) are compact and convex, the allocation problem admits a unique optimal solution that can be characterized using the necessary and sufficient Karush–Kuhn–Tucker (KKT) conditions. Thus, we define the Lagrange function of the social welfare problem **P3** as:

$$
\begin{aligned}
\bar{L}(\boldsymbol{\lambda}, \boldsymbol{\mu}, \mathbf{x_n}, \mathbf{y_n}, \mathbf{y}) &= \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} b_{n,m} \cdot \log x_{n,m} \\
&- \left(\sum_{n\in\mathcal{N}} \sum_{p\in\mathcal{N}\setminus\{n\}} \frac{\alpha_{n,p}}{2} \cdot y_{n,p}^2 + \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \frac{\alpha_{m,n}}{2} \cdot y_{m,n}^2 \right) \\
&- \sum_{m\in\mathcal{M}} \lambda_m \left(\sum_{n\in\mathcal{N}} y_{m,n} - C_{SC} \right) \\
&- \sum_{n\in\mathcal{N}} \mu_n \left(\sum_{p\in\mathcal{N}\setminus\{n\}} y_{n,p} + L_{max}^{BS} - C_{BS} \right) \\
&- \sum_{n\in\mathcal{N}} \sum_{p\in\mathcal{N}\setminus\{n\}} \sum_{m\in\mathcal{M}} \lambda_{n,p,m} \left(y_{n,p} + y_{m,n} - x_{n,m} \right) \\
&- \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{M}} \lambda_{n,m} \left(x_{n,m} - L_{max}^{BS} \cdot c \right) \\
&- \sum_{p\in\mathcal{N}\setminus\{n\}} \sum_{n\in\mathcal{N}} \mu_{n,p} \left(y_{n,p} - C_{SC} \cdot d \right) - \lambda(c + d - 1),
\end{aligned} \tag{40}
$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the vectors of Lagrange multipliers corresponding to the constraints (33)–(39). The KKT conditions that yield the optimal variables $\boldsymbol{\lambda}^*, \boldsymbol{\mu}^*, \mathbf{x_n}^*, \mathbf{y_n}^*, \mathbf{y}^*$ for the problem are given by the following set of equations:

$$ b_{n,m} \cdot \frac{1}{x_{n,m}^*} + \lambda_{n,p,m}^* - \lambda_{n,m}^* = 0, \tag{41} $$

$$ a_{n,p} \cdot y_{n,p}^* - \mu_n^* - \lambda_{n,p,m}^* - \mu_{n,p}^* = 0, \tag{42} $$

$$ a_{m,n} \cdot y_{m,n}^* - \lambda_m^* - \lambda_{n,p,m}^* = 0, \tag{43} $$

$$ \lambda_m^* \left(\sum_{n\in\mathcal{N}} y_{m,n}^* - C_{SC} \right) = 0, \tag{44} $$

$$ \mu_n^* \left(\sum_{p\in\mathcal{N}\setminus\{n\}} y_{n,p}^* + L_{max}^{BS} - C_{BS} \right) = 0, \tag{45} $$

$$ \lambda_{n,p,m}^* \left(y_{n,p}^* + y_{m,n}^* - x_{n,m}^* \right) = 0, \tag{46} $$

$$ \lambda_{n,m}^* \left(x_{n,m}^* - L_{max}^{BS} \cdot c \right) = 0, \tag{47} $$

$$ \mu_{n,p}^* \left(y_{n,p}^* - C_{SC} \cdot d \right) = 0, \tag{48} $$

$$ \lambda^* (c + d - 1) = 0, \tag{49} $$

$$ \sum_{n\in\mathcal{N}} y_{n,m}^* \leq C_{SC}, \tag{50} $$

$$ \sum_{p\in\mathcal{N}\setminus\{n\}} y_{n,p}^* + L_{max}^{BS} \leq C_{BS}, \tag{51} $$

$$ y_{n,p}^* + y_{m,n}^* \leq x_{n,m}^*, \tag{52} $$

$$ x_{n,m}^* \leq L_{max}^{BS} \cdot c, \tag{53} $$

$$ y_{n,p}^* \leq C_{SC} \cdot d, \tag{54} $$

$$ c + d \leq 1, \tag{55} $$

$$ y_{n,p}^* \geq 0, y_{m,n}^* \geq 0, x_{n,m}^* \geq 0, \lambda_m^* \geq 0, \mu_n^* \geq 0, \tag{56} $$

$$ \lambda_{n,p,m}^* \geq 0, \lambda_{n,m}^* \geq 0, \mu_{n,p}^* \geq 0. \tag{57} $$

Equations (41)–(57) define the allocation rule of our mechanism that ensure the social welfare maximization.

4. Energy Minimization Iterative Offloading Mechanism (EMIO)

In this section, we present the Energy Minimization Iterative Offloading (EMIO) mechanism that solves the resource allocation problem. With Equations (41)–(57), the auctioneer computes the optimal prices for the bids and asks by using an iterative algorithm that gradually leads to an equilibrium.

The iterative mechanism is described in detail in Algorithm 1. The auctioneer announces the Lagrange multipliers of each round to the MNOs and the third party (Step 1). Then, the involved parties calculate their bids and asks that optimize their benefits according to their own optimization problems and they submit them to the auctioneer (Step 2). The auctioneer solves the maximization problem **P3** and yields the Lagrange multipliers (Step 3). At this step, the Lagrange multipliers are updated gradually through differential equations. Finally, the mechanism is repeated until convergence (Step 4). At this point, it is important to note that the algorithm is applied every hour of the day in order to decide in the beginning of the hour how the traffic can be offloaded and the number of BSs and SCs that can be deactivated. The period of the hour can be selected since the hour is the least time interval that the traffic fluctuates [13,34]. However, the algorithm can be applied in even smaller and bigger time periods according to the needs and the traffic data that are available. Thus, we can come to the conclusion that the algorithm is dynamic both regarding the traffic variations and the parameter of time.

Algorithm 1 Energy Minimization Iterative Offloading (EMIO) Mechanism

Require: $x_n^*, y_n^*, y^*, \lambda^*, \mu^*$

Start from time $t = 0$

Set starting values for $x_{n,m}^{(0)}, y_{n,p}^{(0)}, y_{m,n}^{(0)}, \lambda_m^{(0)}, \mu_n^{(0)}, \lambda_{n,p,m}^{(0)}, \lambda_{n,m}^{(0)}, \mu_{p,n}^{(0)}, \lambda^{(0)}, \mu^{(0)}, \epsilon_1, \epsilon_2, \epsilon_3$, $\forall n \in \mathcal{N}, \forall m \in \mathcal{M}, \forall p \in \mathcal{N}/n$

$termination\_condition = 0$

while $termination\_condition == 0$ **do** ▷ Step 1

  The auctioneer announces the Lagrange multipliers $\lambda^{(t)}, \mu^{(t)}$ at time t

  Set $t = t + 1$ ▷ Step 2

  Each MNO $n \in \mathcal{N}$ computes the optimal bids, $b_n^{(t)}$

  Each MNO $n \in \mathcal{N}$ computes the optimal asks, $\alpha_n^{(t)}$

  The third party computes the optimal asks, $\alpha^{(t)}$

  The bids and asks are submitted to the auctioneer ▷ Step 3

  The auctioneer computes the x_n^*, y_n^*, and y^* from Equations (41)–(57)

  The auctioneer computes the Lagrange multipliers

$$\lambda_m^{(t)} = \left(\lambda_m^{(t-1)} + \gamma^{(t)} \cdot \left(\sum_{n \in \mathcal{N}} y_{m,n}^{(t-1)} - C_{CS}\right)\right)^+$$

$$\mu_n^{(t)} = \left(\mu_n^{(t-1)} + \gamma^{(t)} \cdot \left(\sum_{p \in \mathcal{N}/n} y_{n,p}^{(t-1)} - L_{max}^{BS} - C_{BS}\right)\right)^+$$

$$\lambda_{n,p,m}^{(t)} = \left(\lambda_{n,p,m}^{(t-1)} + \gamma^{(t)} \cdot \left(y_{n,p}^{(t-1)} + y_{m,n}^{(t-1)} - x_{n,m}^{(t-1)}\right)\right)^+$$

$$\lambda_{n,m}^{(t)} = \left(\lambda_{n,m}^{(t-1)} + \gamma^{(t)} \cdot \left(x_{n,m}^{(t-1)} - L_{max}^{BS} \cdot c\right)\right)^+$$

$$\mu_{n,p}^{(t)} = \left(\mu_{n,p}^{(t-1)} + \gamma^{(t)} \cdot \left(y_{n,p}^{(t-1)} - C_{SC} \cdot d\right)\right)^+$$

$$\lambda^{(t)} = \left(\lambda^{(t-1)} + \gamma^{(t)}(c + d - 1)\right)^+$$ ▷ Step 4

  The auctioneer checks the termination condition

  if $\left(\left| \frac{b_{n,m}^{(t)} - b_{n,m}^{(t-1)}}{b_{n,m}^{(t-1)}} \right| < \epsilon_1 \&\& \left| \frac{\alpha_{n,p}^{(t)} - \alpha_{n,p}^{(t-1)}}{\alpha_{n,p}^{(t-1)}} \right| < \epsilon_2 \&\& \left| \frac{\alpha_{m,n}^{(t)} - \alpha_{m,n}^{(t-1)}}{\alpha_{m,n}^{(t-1)}} \right| < \epsilon_3 \right)$ **then**

    $termination\_condition = 1$

  end if

end while

Having proposed the EMIO algorithm, we now show that it is efficient, individually rational, incentive compatible and has small messaging overhead through Propositions 1 and 2.

Proposition 1 (Efficiency). *EMIO mechanism is efficient, individually rational and incentive compatible.*

Proof. The iterative algorithm converges to an equilibrium that satisfies the KKT conditions of the Lagrange multipliers, Equations (41)–(57). Since the solution combines the bidders' desires from the optimization problems (BIDS (Operator), ASKS (Operator) and ASKS (Third party)) within the optimization problem P3, the algorithm reaches the socially optimal solution. The utility of a winning participant is always non-negative, proving the individual rationality. Incentive compatibility means that reporting the true asks and bids is the dominant strategy for all players. We have two cases:

1. A buyer reveals his true bid and we assume that the individual utility is U_n,
2. The operator bids untruthfully and utility becomes U_n'.

When the player bids untruthfully, in order to win, the bidder must bid higher than the lower truthful bid; therefore, utility becomes lower than the utility that could be achieved if the bidder had revealed his bid truthfully. The untruthful bidder cannot maximize his utility, when he alters the bidding strategy. This proves that the buyer does not have an incentive to be untruthful. The same rationale applies for the asks. □

Proposition 2 (Polynomial overhead)**.** *The EMIO mechanism requires $O(N^3 \cdot M)$ communicating messages.*

Proof. There are only $N \cdot M$ bids (line 7) and $N \cdot M + N \cdot (N-1)$ asks (lines 8 and 9) that must be communicated from the operators and the third party to the auctioneer. Similarly, the auctioneer announces in total $2 \cdot N^3 \cdot M + N^3 \cdot M^2 + 2 \cdot N^2 \cdot M^2 + N \cdot M^2 - N^2 \cdot M + N \cdot M$ (lines 11 and 12) messages containing the Lagrange multipliers to the bidders. Thus, the messaging overhead is $O(N^3 \cdot M)$ at each t that the algorithm is applied. Taking into account that the N and M represent the number of MNOs and SCs, we conclude that the messaging overhead in not large and thus, does not pose a great obstacle in the applications of the algorithm. □

Proposition 3 (Computation Complexity)**.** *The EMIO mechanism reaches the optimal solution.*

Proof. The proposed algorithm converges to the optimal solution of the optimization problem under the examined conditions since it has a strictly concave objective function. More specifically, if we consider that the auctioneer announces the Lagrange multipliers every small time slots t, then the algorithm is approximated by its continuous time counterpart. By this, we mean that the Lagrange multipliers can be updated according to the differential equations. For example, the KKT conditions, namely Equations (44) and (45), are calculated as follows:

$$\frac{d\lambda_m}{dt} = \left(\sum_{n \in \mathcal{N}} y^*_{m,n} - C_{SC} \right)^+_{\lambda_m} \tag{58}$$

$$\frac{d\mu_n}{dt} = \left(\sum_{p \in \mathcal{N} \setminus \{n\}} y^*_{n,p} + L^{BS}_{max} - C_{BS} \right)^+_{\mu_n} \tag{59}$$

These equations are called the stationery conditions and when these are set to zero the unconstrained optimization problem can be solved. Given the constrained inequalities conditions (namely Equations (50)–(55), the iterative algorithm can reach to convergence in few steps. Additionally, the EMIO mechanism (referred in Algorithm 1) is computationally efficient and converges in few iterations. More specifically, the proposed algorithm searches the winners of the auction among the set of MNOs (set of $\mathcal{N}$) and a third-party [35]. Thus, the time of finding the winner among the participating parties is $O(N+1)$ and an iterative auction process is executed until convergence. Therefore, the proposed algorithm is performed in polynomial time with time complexity of $O((N+1)^2)$. In conclusion, Algorithm 1 is computationally efficient. □

The iterative algorithm is executed by a single auctioneer and requires small messaging overhead. The MNOs and the third party calculate their bids and asks simultaneously and submit the corresponding information to an auctioneer that is responsible for solving the problem. However, the algorithm could be applied in a decentralized network, where multiple synchronized auctioneers coexist. An improved mechanism in terms of scalability could be applied to large networks where each auctioneer will be responsible for a smaller area, but their communication will ensure a faster solution for the network.

5. Performance Evaluation

In this section, numerical results are presented to validate our analysis. We start by providing an example with a simple network configuration. Subsequently, we consider a bigger network to study a number of different parameters of the offloading market.

To assess the performance of our scheme, we compare the proposed auction-based offloading strategy (referred to as EMIO), to three benchmark solutions: (1) an auction-based switching off scheme, wherein the income of the third party is the unique objective to be maximized (referred to as ISO) [26], (2) an offloading strategy, where the deactivation is not considered and offloading leads to energy savings (referred to as SO) [10] and (3) one benchmark solution (Full Operational Topology (FOT), where each MNO serves its own traffic and none of the BSs and SCs are switched off).

5.1. Numerical Example

Here, we consider the simple network configuration with $N = 2$ MNOs and $M = 3$ SCs and we examine the behavior of the proposed strategy under various traffic conditions, when the network traffic is low (traffic around 10% of network capacity), medium (traffic around 50% of network capacity) and high (traffic equal to 100% of network capacity). Regarding the Cisco report about the mobile data traffic forecast [36], and the works in the literature [13,34], we concluded that the three different traffic scenarios (e.g., low, medium, high) are representative, since the traffic load periods are categorized into three distinctive cases:

1. High traffic: Between 13:00 and 21:00 the traffic load reached its peak, when the majority of network users works and returns home for the rest of the day.
2. Low traffic: The traffic is very low from 02:00 until 08:00, a time period when usually the network users go to sleep and thus, do not use their smartphones, laptops and PCs.
3. Medium traffic: The rest of the day, the traffic load fluctuates between heavier and lower traffic.

In Figure 3, first, we plot the social welfare achieved by the optimization problem **P3** in each iteration and, second, we show how the number of iterations is affected by the traffic. In the first subplot, we observe that the algorithm converges to the optimal after a small number of 60 iterations. The initial values for bids and asks are randomly selected. Notice that before convergence, the social welfare can be higher than the respective optimal value due to the problem constraints. The negative value of the social welfare is explained since the third party requests higher asks for leasing the capacity than the initial offered bids. The difference between the asks and the bids is even higher when the traffic load is higher, since as the admitted traffic of a BS increases, its operation cost for admitting one more traffic unit increases even higher, due to the congestion effect and because fewer of its resources are available for serving its own traffic. For low and medium traffic, the social welfare values are very close, since we investigate a network with only two operators, a scenario that does not allow switching off of more than 1 BS and thus, the social benefit is similar for both cases. In addition, as shown in both figures, the number of iterations is highly affected by the network traffic. Specifically, from the second subplot we observe that as traffic increases reaching 40%, the number of iterations increases, as well, since when the traffic is lower, there are more possible combinations for offloading. In contrary, the number of iterations decreases as the network traffic increases. When MNOs have higher traffic, they are not willing to admit extra traffic to serve from their neighbors and thus, the convergence is faster due to the smaller number of combinations of the optimization problem. It is important to point out that for low and high traffic, the number of iterations is small, and only when the load is medium, the required number of iterations is higher. However, for all cases the number of iterations is not prohibitive, since the offloading decision is reached very quickly in a few minutes.

We repeated the experiment for a relatively bigger network with $N = 4$ MNOs and $M = 15$ SCs under various traffic conditions (low traffic around 10% of network capacity, medium traffic around 50% and high traffic equal to 100%). Similarly to Figure 3, in Figure 4 we plot the social welfare achieved by the optimization problem **P3** in each iteration. We observe that the social welfare is higher as the traffic is lower, since the MNOs are given the opportunity to switch off higher number of BSs leading to higher energy and cost gains.

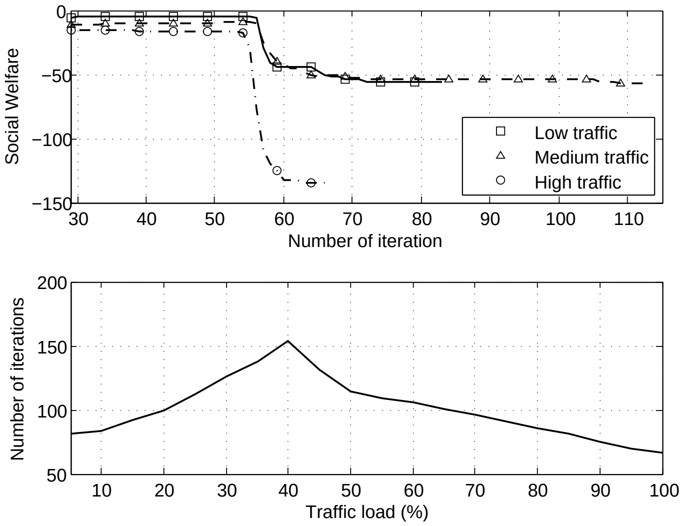


Figure 3. Evolution of social welfare and relation between traffic load and number of iterations for a small market of $N = 2$ operators and $M = 3$ SCs.

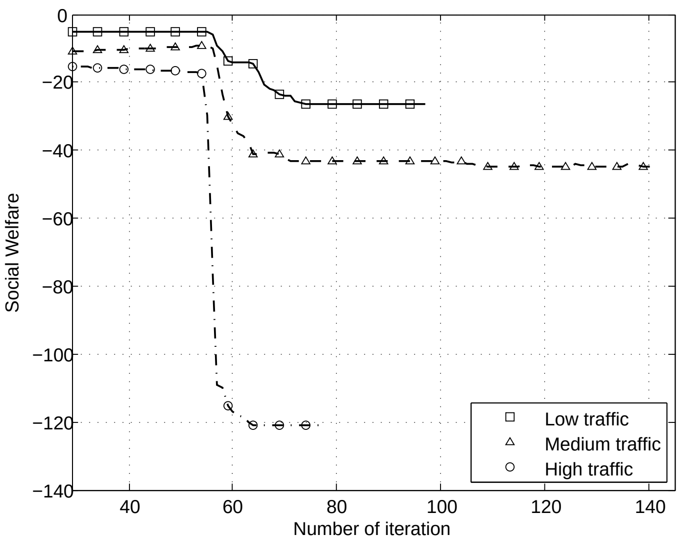


Figure 4. Evolution of social welfare for a bigger market of $N = 4$ operators and $M = 15$ SCs.

In Figure 5, we plot the bids and asks of the MNOs for each iteration of the EMIO, which converges to the optimal after few iterations. Even though the initial values are randomly selected, we observe that the optimal values that maximize the social welfare are quickly calculated. The bids increase as the traffic increases, since the MNOs are willing to offload more traffic. On the other hand, we notice that the asks are very small close to zero, meaning that the operators are not eager to accept traffic from their opponents. Since the MNOs have the same traffic, this result is expected. The bids increase as the traffic demand increases. When the traffic is low, the MNOs do not need to compete by bidding high since there is excess capacity in the network. However, as the traffic grows, the competition grows and MNOs are requested to pay more to offload their traffic.

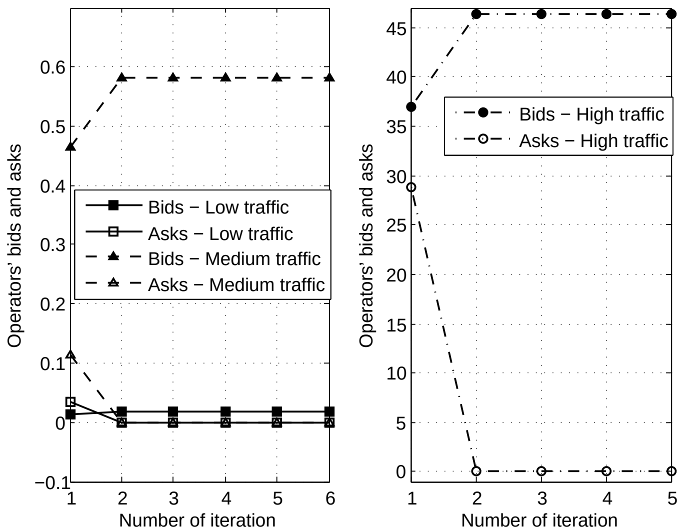


Figure 5. Evolution of bids and asks for a small market of $N = 2$ operators and $M = 3$ SCs.

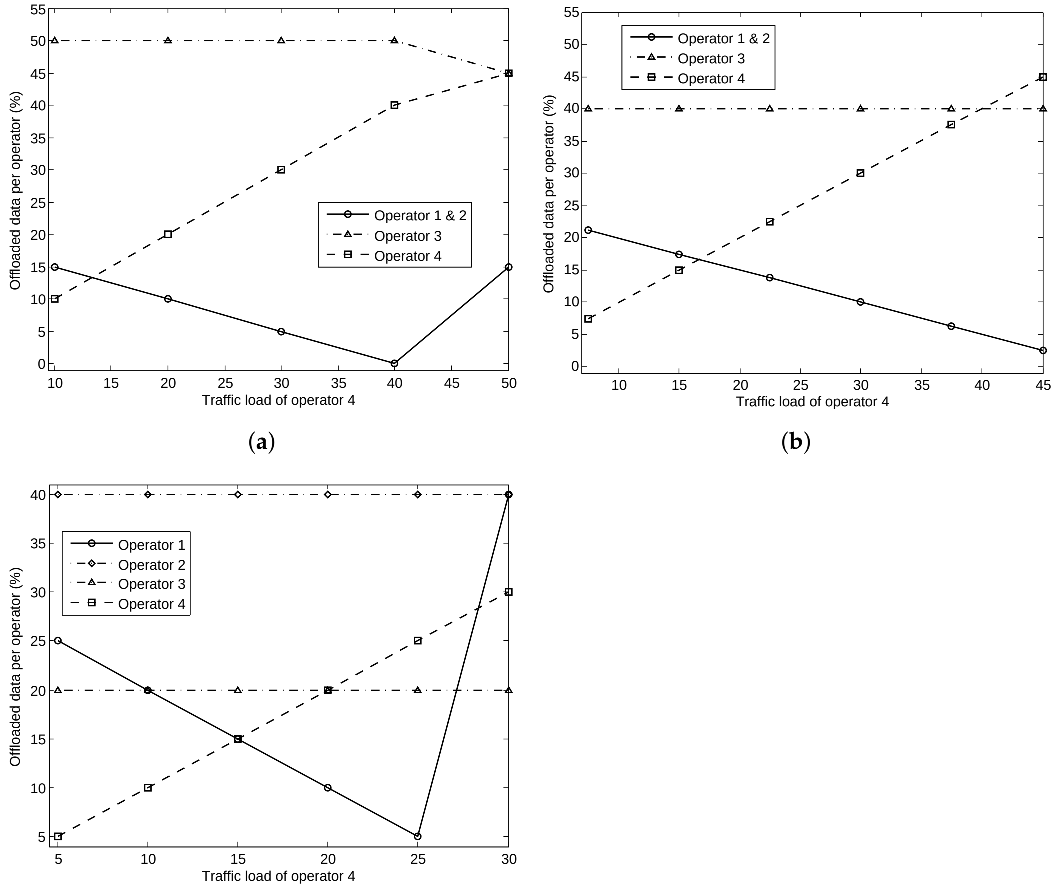
5.2. Simulation Results

We consider a network with $N = 4$ MNOs and $M = 15$ SCs uniformly distributed. In these experiments, we study a scenario where the MNOs have different traffic volumes and, thus, different requirements. According to recent studies [2], in most European countries, there are usually up to two telecommunication companies holding the major part of the market share (operators 1 and 2), one smaller MNO serving an intermediate portion of users (operator 3), and finally, up to one or two smaller MNOs with a very small slice of the market (operator 4). By using these insights, we consider three different scenarios.

1. Two operators are almost fully loaded (80% of the BSs capacity is used), another MNO has relatively medium traffic (50%) and the last operator has increasing traffic that begins from low load. This scenario could reflect the traffic of network during midday when the users demand is at their peak.
2. Two MNOs are highly loaded, but not at their maximum (60% of the BSs capacity), another MNO has relatively medium traffic (40%) and the last operator has increasing traffic that begins from low load. This scenario corresponds to the case that the network traffic is high but lower than peak hours during the afternoon.
3. We consider two MNOs with medium traffic (40% of the BSs capacity), another MNO has relatively lower traffic (20%) and the last operator has increasing traffic. This scenario reflects the off peak periods, especially during the night.

These scenarios correspond to realistic networks, during different hours of the day (higher network traffic for daytime and lower for night) and areas (business and urban areas have different traffic).

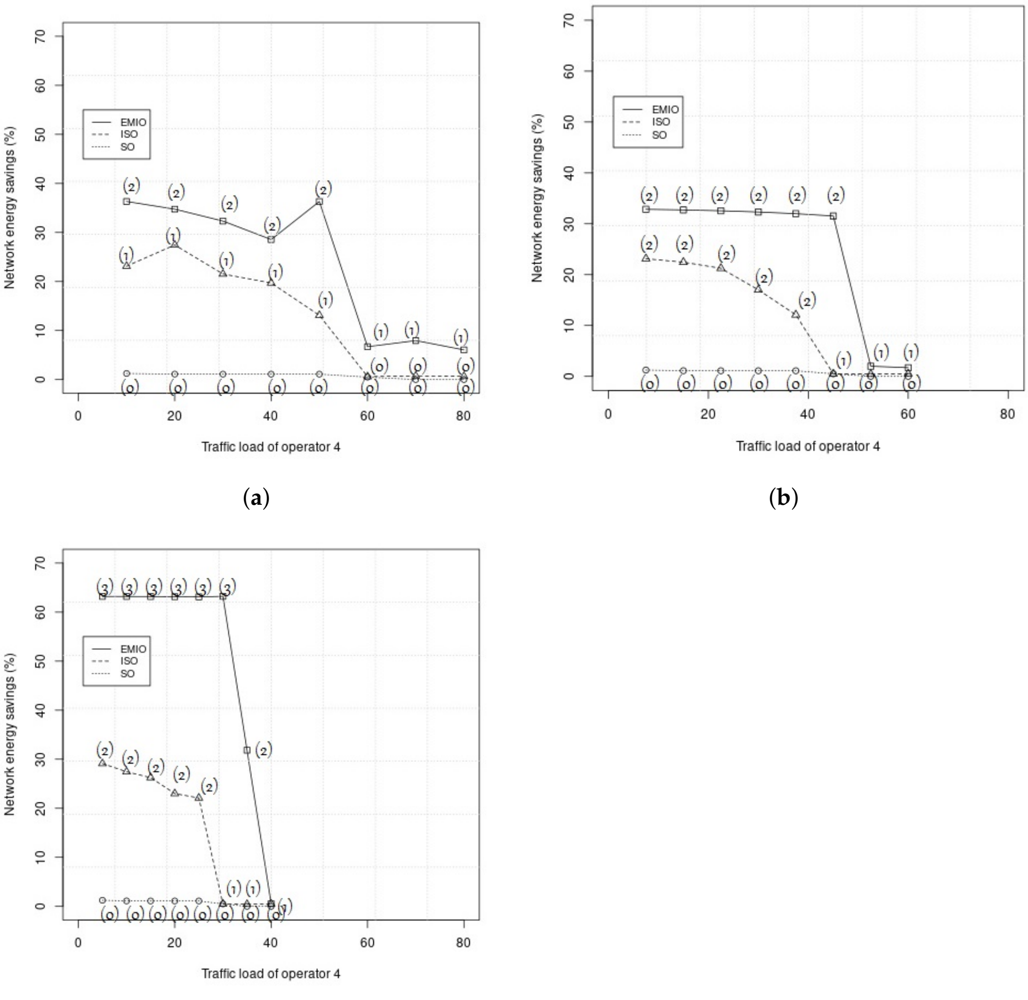
In Figure 6, the offloaded data per operator are plotted versus the increasing traffic of MNO 4. By observing the three figures, we can conclude the following. First, the operators 4 and 3 manage to offload their whole traffic and eventually switch off their BS. We notice that when the total network traffic is high or medium (scenarios 1 and 2), the MNO with the intermediate traffic appears to be the winner of the auction, since he/she manages to offload the highest portion of the traffic. In scenario 3, where the traffic of the network is low, even one of the operators with the highest traffic (MNO 2) offloads a big portion of its traffic. Finally, operators with the highest traffic (operators 1 and 2) offload a decreasing portion of traffic. Thus, as the traffic of the fourth operator increases without exceeding the maximum traffic of the operators 1 and 2, their offloaded traffic decreases. From the above remarks, we conclude that the MNOs with lower and intermediate traffic have the advantage to offload more traffic and are given the opportunity to switch off their BSs. In addition, the auction achieves to balance the traffic of the MNOs that keep their BSs active. The traffic balance leads to lower energy consumption.



(**a**) (**b**) (**c**)

Figure 6. Impact of network traffic on the offloaded data of each operator. (**a**) Scenario 1—High traffic; (**b**) Scenario 2—Medium traffic; (**c**) Scenario 3—Low traffic.

To gain some further insight into the potential of energy saving by capacity sharing, in Figure 7, we plot the network energy savings for the three scenarios as the network traffic increases compared to FOT. The number on each point in parenthesis in the plots corresponds to the number of the switched off BSs. To begin with, we compare the performance of EMIO algorithm in the three subplots. From the figure, we observe that the energy savings are higher as the traffic is low (Scenario 1). For medium and high traffic (Scenario 2 and 3, respectively), the energy savings are almost equal, since the number of the switched off BSs are the same and the load balancing has similar impact on the energy consumption. The fluctuations of the energy savings are due to the different offloading potentials. This is the reason that, for some cases, the energy saving in Scenario 1 is higher than in Scenario 2, since more offloading is achieved leading to higher energy efficiency. Moreover, it is observed from the figure that as the network traffic increases, the energy savings decrease. Specifically, when the traffic of operator 4 is over 30%, 45% and 50% for low, medium and high traffic, respectively, the gains start to decrease, since the total network traffic increases and the BSs cannot be deactivated (only one MNO can switch off its BS) and remain active to serve the excess traffic. With the application of the proposed algorithm, we still have some gains through the traffic offloading, however, the gains are not high, since switching off can not be performed. The scalability of the algorithm is not although affected, since the scenarios where all the MNOs have very high traffic at the same time are rare. Additionally, when observing each of the subplots individually, we note that the EMIO outperforms the related work (referred as ISO and SO) and gives higher overall energy savings. The reason behind this is the number of switched off BSs that is larger in the proposed strategy. The ISO approach has good performance in terms of energy efficiency since deactivation is also taken into consideration; however, the SO scheme has low performance in terms of energy efficiency since only offloading is considered.



(**a**) (**b**) (**c**)

Figure 7. Network energy savings. (**a**) Scenario 1—High traffic; (**b**) Scenario 2—Medium traffic; (**c**) Scenario 3—Low traffic.

Along with the network energy efficiency, it is interesting to study the individual energy efficiency of MNOs. The individual gains for the three scenarios are quantified in Table 1, where interesting conclusions are extracted. Additionally, the table shows the comparison between the energy savings between the proposed EMIO algorithm, i.e., the works from the literature (ISO and SO) with respect to the FOT scheme. Particularly, the proposed algorithm is beneficial for the group of operators that switch off their BSs (MNO 3 and MNO 4 for high and medium traffic scenario and MNO 2, MNO 3 and MNO 4 for low traffic scenario). Specifically, MNOs that switch off their BSs theoretically achieve infinite energy gains, as they have their traffic served at zero energy cost, whereas the active operators serve their own traffic and only offload a smaller portion of their traffic to SCs. For those operators, the EMIO algorithm achieves energy consumption reduction. A second observation derives when EMIO is compared to ISO and SO. Firstly, the ISO algorithm proposes offloading of the network traffic with main goal the income of the third party instead of the energy efficiency and deactivation of unused BSs may occur is possible. From the table, it is observed that ISO achieves lower energy gains for all the operators and in every scenario (Operator 1 has slightly better performance and only in the case of low traffic). For every case study, the performance of EMIO is significantly superior, since larger number of BSs are switched off compared to ISO. Secondly, in the SO strategy, offloading is initiated in order to a achieve a balance between the traffic of all MNOs. Thus, the energy savings are due to offloading and not because of the deactivation. Based on this comparison, it is noted that MNO1 and MNO2 achieve higher energy gains since they can offload more of their traffic to the lower used BSs and SCs, however, operators 3 and 4 (also operator 2 in the third scenario) gain very little in energy and thus, they have no incentive to participate in a cooperative algorithm that is based only in offloading.

Table 1. Operator energy savings (%) with respect to the FOT scheme.

Network Traffic	Algorithm	MNO1	MNO2	MNO3	MNO4
High—Scenario 1	**EMIO**	11	11	∞	∞
	ISO	9	9	8	∞
	SO	20	17	9	8
Medium—Scenario 2	**EMIO**	2.7	2.7	∞	∞
	ISO	2	1.9	∞	∞
	SO	17	16	6	4
Low—Scenario 3	**EMIO**	0.5	∞	∞	∞
	ISO	3	3	∞	∞
	SO	12	10	5	5

6. Conclusions

In this paper, motivated by the coexistence of multiple operators and third party SCs in the same area, we proposed a novel double auction-based offloading algorithm that achieves energy savings and cost reduction by encouraging MNOs to offload their traffic, and potentially switch off BSs and SCs. An iterative algorithm that satisfies the desired economic requirements of each involved party was proposed. The novel scheme has been evaluated for various traffic conditions. The results have shown that our proposal significantly improves the network energy efficiency. The energy savings reach up to 63% reduction. Regarding the offloaded data and the individual energy gains, the proposed scheme provides fairness and high energy benefits, motivating the operators and the third party to participate in a double auction offloading.

The market study that we investigated differs considerably from other schemes. Previous works in the literature quantify the benefits of offloading or deactivation. In other works, markets with complete information are studied and competitive strategies are chosen. Nonetheless when the participants are not willing to reveal their desires, the market becomes more challenging. Such markets are resolved with the use of double auction, where an auctioneer tries to match the buyers and sellers' desires. The double auction mechanism is still difficult and different approaches have been examined in the literature. In our work, we adopted a market mechanism based on the social welfare optimization. The limited market information and the conflicting objectives of the participants is solved through an iterative double-auction mechanism that satisfies all the desirable economic properties. Thus, the proposed algorithm leads to maximized energy and cost efficiency in the emerging and future wireless networks. Our promising solution can be applied in a dynamic and scalable fashion in multi-operator HetNets.

A possible extension of the work could include the examination of other algorithms (i.e., backwards induction) for solving the problem and the comparison among different schemes. Additionally, the study of the market considering the frequency of bidding phases and the different behaviors of the participating parties is in our plans.

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References

1. Bao, X.; Lin, Y.; Lee, U.; Rimac, I.; Choudhury, R.R. Dataspotting: Exploiting naturally clustered mobile devices to offload cellular traffic. In Proceedings of the 2013 Proceedings IEEE INFOCOM, Turin, Italy, 14–19 April 2013; pp. 420–424. [CrossRef]
2. Marsan, M.A.; Meo, M. Network sharing and its energy benefits: A study of european mobile network operators. In Proceedings of the 2013 IEEE Global Communications Conference (GLOBECOM), Atlanta, GA, USA, 9–13 December 2013; pp. 2561–2567. [CrossRef]
3. Bousia, A.; Kartsakli, E.; Antonopoulos, A.; Alonso, L.; Verikoukis, C. Game-theoretic infrastructure sharing in multioperator cellular networks. *IEEE Trans. Veh. Technol.* **2016**, *65*, 3326–3341. [CrossRef]
4. Rengah, D.; Hassan, H.A.H.; Meo, M.; Nuayami, L. Energy management and base station on/off switching in green mobile networks for offering ancillary services. *IEEE Trans. Green Commun. Netw.* **2018**, *2*, 868–880. [CrossRef]
5. Euttamarajah, S.; Ng, Y.H.; Tan, C.K. Energy-efficient joint base station switching and power allocation for smart grid based hybrid-powered CoMP-enabled HetNet. *Future Internet* **2021**, *13*, 213. [CrossRef]
6. Gao, L.; Iosifidis, G.; Huang, J.; Tassiulas, L. Economics of mobile data offloading. In Proceedings of the 2013 Proceedings IEEE INFOCOM, Turin, Italy, 14–19 April 2013; pp. 3303–3308. [CrossRef]
7. Dimatteo, S.; Hui, P.; Han, B.; Li, V.O.K. Cellular traffic offloading through wifi networks. In Proceedings of the 2011 IEEE Eighth International Conference on Mobile Ad-Hoc and Sensor Systems, Valencia, Spain, 17–22 October 2011; pp. 192–201. [CrossRef]
8. Fan, W.; Han, J.; Yao, L.; Wu, F.; Liu, Y. Latency-energy optimization for joint WiFi and cellular offloading in mobile edge computing networks. *Comput. Netw.* **2020**, *181*, 107570. [CrossRef]
9. Shen, H. Congestion-aware WiFi offload algorithm for 5G heterogeneous wireless networks. *Comput. Commun.* **2020**, *164*, 69–76. [CrossRef]
10. Ayub, A.; Jangsher, S.; Butt, M.M.; Maud, A.R.; Bhatti, F.A. A comparative analysis of Wi-Fi offloading and cooperation in small-cell network. *Electronics* **2021**, *10*, 1493. [CrossRef]
11. Liu, C.; Natarajan, B.; Xia, H. Small cell base station sleep strategies for energy efficiency. *IEEE Trans. Veh. Technol.* **2016**, *65*, 1652–1661. [CrossRef]
12. Cai, S.; Che, Y.; Duan, L.; Wang, J.; Zhou, S.; Zhang, R. Green 5G Heterogeneous Networks Through Dynamic Small-Cell Operation. *IEEE J. Sel. Areas Commun.* **2016**, *34*, 1103–1115. [CrossRef]
13. Wu, J.; Jin, S.; Jiang, L.; Wang, G. Dynamic switching off algorithms for pico base stations in heterogeneous cellular networks. *EURASIP J. Wirel. Commun. Netw.* **2015**, *2015*, 117. [CrossRef]
14. Tan, X.; Xiong, K.; Lu, Y.; Zhang, Y.; Fan, P.; Letaief, K.B. Roaming-cost-based base station switching-off in MISO networks: From a joint energy saving and profit guarantee perspective. In Proceedings of the 2022 IEEE International Conference on Communications (ICC), Seoul, Korea, 16–20 May 2022; pp. 147–152. [CrossRef]
15. El-Amine, A.; Iturralde, M.; Haj Hassan, H.A.; Nuaymi, L. A Distributed Q-Learning Approach for Adaptive Sleep Modes in 5G Networks. In Proceedings of the 2019 IEEE Wireless Communications and Networking Conference (WCNC), Marrakesh, Morocco, 15–18 April 2019; pp. 1–6. [CrossRef]
16. Feng, M.; Mao, S.; Jiang, T. Base station on-off switching in 5g wireless networks: Approaches and challenges. *IEEE Wirel. Commun.* **2014**, *24*, 46–54. [CrossRef]
17. Luna, F.; Zapata-Cano, P.H.; Valenzuela-Valdes, J.F.; Padilla, P. A robust approach to the cell switch-off problem in 5G ultradense networks. In Proceedings of the 2019 International Conference on High Performance Computing & Simulation (HPCS), Dublin, Ireland, 15–19 July 2019; pp. 129–135. [CrossRef]
18. Feng, M.; Mao, S.; Jiang, T. BOOST: Base Station on-off Switching Strategy for Green Massive MIMO HetNets. *IEEE Trans. Wirel. Commun.* **2017**, *16*, 7319–7332. [CrossRef]
19. He, Z.; Yen, G.G.; Yi, Z. Robust multiobjective optimization via evolutionary algorithms. *IEEE Trans. Evol. Comput.* **2019**, *23*, 316–330. [CrossRef]
20. Salem, A.A.; El-Rabaie, S.; Shokair, M. Energy efficient ultra-dense networks (UDNs) based on joint optimisation evolutionary algorithm. *IET Commun.* **2019** 13, 99–107. [CrossRef]

21. Guerra, I.; Yin, B.; Zhang, S.; Cheng, Y. Optimization of Base Station ON-Off Switching with a Machine Learning Approach. In Proceedings of the IEEE International Conference on Communications, Montreal, QC, Canada, 14–23 June 2021; pp. 1-6. [CrossRef]
22. Li, R.; Zhao, Z.; Chen, X.; Zhang, H. Energy saving through a learning framework in greener cellular radio access networks. In Proceedings of the IEEE Global Communications Conference (GLOBECOM), Anaheim, CA, USA, 3–7 December 2012; pp. 1556–1561. [CrossRef]
23. Gan, X.; Wang, L.; Feng, X.; Liu, J.; Yu, H.; Zhang, Z.; Liu, H. Energy efficient switch policy for small cells. *China Commun.* **2015**, *12*, 78–88. [CrossRef]
24. Zhuo, X.; Gao, W.; Cao, G.; Hua, S. An incentive framework for cellular traffic offloading. *IEEE Trans. Mob. Comput.* **2014**, *13*, 541555. [CrossRef]
25. Iosifidis, G.; Gao, L.; Huang, J.; Tassiulas, L. A double-auction mechanism for mobile data-offloading markets. *IEEE/ACM Trans. Netw.* **2015**, *23*, 1634–1647. [CrossRef]
26. Paris, S.; Martignon, F.; Filippini, I.; Capone, A. A bandwidth trading marketplace for mobile data offloading. In Proceedings of the 2013 IEEE INFOCOM, Turin, Italy, 14–19 April 2013. [CrossRef]
27. Dong, W.; Rallapalli, S.; Jana, R.; Qiu, L.; Ramakrishnan, K.K.; Razoumov, L.; Zhang, Y.; Cho, T.W. ideal: Incentivized dynamic cellular offloading via auctions. *IEEE/ACM Trans. Netw.* **2014**, *22*, 1271–1284. [CrossRef]
28. Bousia, A.; Kartsakli, E.; Antonopoulos, A.; Alonso, L.; Verikoukis, C. Multiobjective auction-based switching-off scheme in heterogeneous networks: To bid or not to bid? *IEEE Trans. Veh. Technol.* **2016**, *65*, 9168–9180. [CrossRef]
29. Bousia, A.; Kartsakli, E.; Antonopoulos, A.; Alonso, L.; Verikoukis, C. Auction-based offloading for base station switching off in heterogeneous networks. In Proceedings of the 2016 European Conference on Networks and Communications (EuCNC), Athens, Greece, 27–30 June 2016; pp. 335–339. [CrossRef]
30. Zhou, H.; Chen, X.; He, S.; Chen, J.; Wu, J. DRAIM: A novel delay-constraint and reverse auction-Based incentive mechanism for WiFi offloading. *IEEE J. Sel. Areas Commun.* **2020**, *38*, 711–722. [CrossRef]
31. Krishna, V. *Auction Theory*, 1st ed.; Academic Press: Cambridge, MA, USA, 2002. Available online: https://EconPapers.repec.org/RePEc:eee:monogr:9780124262973 (accessed on 19 September 2022)
32. Cheung, W.C.; Quek, T.Q.S.; Kountouris, M. Throughput optimization, spectrum allocation, and access control in two-tier femtocell networks. *IEEE J. Sel. Areas Commun.* **2012**, *30*, 561–574. [CrossRef]
33. Kelly, F.; Maulloo, A.; Tan, D. Rate control for communication networks: Shadow prices, proportional fairness and stability. *J. Oper. Res. Soc.* **1998**, *49*, 237–252. [CrossRef]
34. Paul, U.; Liu, J.; Troia, S.; Falowo, O.; Maier, G. Traffic-profile and machine learning based regional data center design and operation for 5G network. *J. Commun. Netw.* **2019**, *21*, 569–583. [CrossRef]
35. Jiao, Y.; Wang, P.; Niyato, D.; Xiong, Z. Social Welfare Maximization Auction in Edge Computing Resource Allocation for Mobile Blockchainh. In Proceedings of the IEEE International Conference on Communications, Kansas City, MO, USA, 20–24 May 2018. [CrossRef]
36. Cisco White Paper. Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 2017–2022. Cisco. 2019. Available online: http://media.mediapost.com/uploads/CiscoForecast.pdf (accessed on 10 October 2022)